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BY
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Toronto. Ont.

July 1870.

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Astron. W

Josephus Morrison M.D. Toronto. 1870.

A

COMPREHENSIVE

SYSTEM OF ASTRONOMY,

BOTH THEORETIC AND PRACTICAL,

WITH EXTENSIVE

TABLES OF THE SUN, MOON, AND PLANETS;

TO WHICH ARE PREFIXED,

PRECEPTS AND EXAMPLES FOR COMPUTING ALL THE REQUISITES FOR FILLING UP THE COLUMNS IN THE NAUTICAL ALMANAC, WHITE'S EPHEMERIS, &c. &c.

BY

THOMAS WHITING,

ASSISTED BY THE ADVICE AND OBSERVATIONS OF THE ASTRONOMERS ROYAL, AND IN THE LABOUR BY SEVERAL SCIENTIFIC GENTLEMEN.

Into the wide expanse of Heaven I
Have presumed to enter, although an
Earthly guest, and thence to gather
Fruit scattered by fair Urania's hand.
In spiral course I then proceed
Toward Terra, there to impart
Celestial food to all who chuse to taste.

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Errata.

Page 22, For "the Sunday which falls *upon or* next after the first full Moon which happens next after the twenty-first day of March; that is, the Sunday which falls *upon or* next after the first full Moon after the vernal equinox."

Read, "the Sunday which falls next after the full Moon which happens *upon or* next after the twenty-first day of March; that is, the Sunday which falls next after the first full Moon after the vernal equinox."

Page 36, Table for finding the number of direction:—

Under 1 and opposite A, for 29 read 26.
Under 1 and opposite C, for 8 read 28.
Under 1 and opposite D, for 19 read 29.
Under 6 and opposite E, for 20 read 30.
Under 7 and opposite G, for 25 read 18.

Plate 3, for *r* read T. Character ⊙ wanting. For *b* read *d*.

Page 103, Prob. 5, line 4 from top, for "3 equal pyramids," read "3 equal triangular pyramids."

Page 107, Prop. 3, line 20 from top, for "OC :: GZ" read "OC + GZ."

Page 150, 151, Theor. 11, cor. 3, 4, wherever *c* occurs read C.

Page 152, Theor. 1, line 17 from bottom, for "$ct\sqrt{\frac{2R}{s}}$," read "$ct\sqrt{\frac{2R}{S}}$," and line 15 from bottom, for "$\sqrt{2Rs}$" read "$\sqrt{2RS}$." Line 4 from bottom, for "2CR," read "2*c*R."

Page 181, col. 1, line 6, for "M⊙N" read "*m*⊙*n*."

Page 194, col. 1, line 16, for "S*m*" read "*sm*."

PREFACE.

ASTRONOMY is a science of such an exalted nature, that the wisest and greatest of men both among the ancients and moderns, have been charmed with its beauties. To contemplate the grand spectacle of the celestial concavity, has ever been considered as the noblest privilege of man. It is here that we discover the wonders of the Supreme Being, and see his wisdom in the works of creation. Nor is there any knowledge, attained by the light of nature, that gives us juster ideas of this Great Being, or furnishes us with stronger arguments by which to demonstrate his existence and attributes. No doubt can exist but that the Creator of the universe intended that we should become acquainted with its structure, or he would never have given us the light of reason, and minds capable of acquiring scientific knowledge sufficient to enable us to enter deeply into this sublime study. No created being except man, has been gifted with an upright position and mental endowments sufficient to enable him to look upwards with delight on those beauties which present themselves in the celestial regions. It would be difficult to enumerate all the pleasures and advantages arising from the study of Astronomy. Almost all the improvements which have been made in geography, commerce, and navigation, have their origin *in*, and are derived *from* this exalted science. Many introductory books have been published, some of which are very good as far as they go, but they do not teach us to make a single accurate calculation of any consequence. Others abound in theory; but cannot be understood by any but very good Mathematicians. This performance will be found to contain an adequate portion of theory, with all the necessary precepts and examples for finding the places of the Sun, Moon, Planets, &c. as put down in the Nautical Almanac, White's Ephemeris, &c. This work also contains all the new solar and lunar tables, with those of the planets, and are the same as now used by the Astronomer Royal and his assistants for computing the Nautical Ephemeris, *the whole of which are not contained in any other book.*

The Tables of the Moon are those of M. Burckhardt, fitted to the Meridian of Greenwich, and are deserving the greatest praise. Mr. Vince adopted those of M. Burg before M. Burckhardt published his, and what is most extraordinary, the Editor of the new edition of Mr. Vince's work, has thought fit to republish all the old tables, although every one acquainted with the subject must see the great superiority of the new lunar tables, in point of arrangement, as well as the great saving of labour. The Nautical Almanac was first published in 1767, under the direction and superintendence of the late very learned Astronomer Royal, Dr. Maskelyne, and has been continued annually to the present time. This is the most useful astronomical work ever published, and is the only safe guide by which the longitude can be found at all times and in all places on the Earth.

Astronomy has undergone many improvements within a very few years, and in nothing more than in the tabular part; all the old tables having run out, and become useless. The very full and complete alphabetical list of definitions, &c. will be found useful to many, as every learner should make himself well acquainted with the terms made use of. Those students who have not read Euclid, and have not learned Plane and Spherical Trigonometry, will do well to make themselves acquainted with those subjects; at least as much of the Elements as are contained in this work. There will also be found as much of the Conic Sections as is useful in Astronomy; which should also be understood by the learner, if he wish to be a proficient in the theory. The laws of motion, &c. should be well attended to; after this the student will proceed with much pleasure to the practical part. Those who wish to enter deeply into the Physical part of Astronomy, will find it largely treated on by Mr. Vince; and more ingeniously by Mr. Woodhouse, in his work on the subjects.

Notwithstanding we live in a country, where the arts and sciences are brought to great perfection, there are some weak enough to advocate the cause of Judicial Astrology; and to suppose the Moon to have an influence on the human intellect, and on the weather! The immense distance of the stars affords a convincing proof that they are too remote from us, for their influence to have any effect upon our globe; much less on the actions and passions of men. But how vain soever their pretensions may be, who would persuade us they can read future events in the stars; or foretel the state of the weather for twelve months to come; yet all those who are acquainted with science are convinced, that there are no principles in nature, nor any regular series of observations, on which their opinions can be supported.

It is unnecessary to trouble the reader with an explanation and refutation of the systems of Ptolemy and Tycho Brahe, as they are without defenders, exploded, and almost forgotten. As all the examples in this work are original, the labour of making the calculations has been very great; and to ensure the accuracy of the results they have been performed by three persons: viz. myself, Mr. Benjamin Workman, and my very ingenious friend, Mr. J. A. Dotchen, who has rendered me very essential service without any pecuniary compensation whatever.

THOMAS WHITING.

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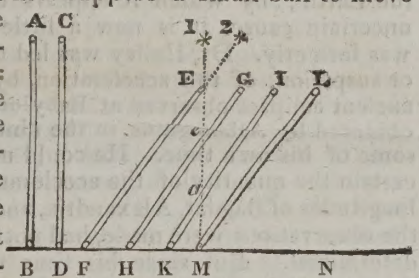
ASTRONOMICAL DEFINITIONS, &c.

ABENEL CAUBY, a fixed star of the second or third magnitude, in the south scale of Libra.

ABERRATION, an apparent motion of the celestial bodies, occasioned by the progressive motion of light, and the annual motion of the Earth in its orbit.

As this subject has often been misunderstood, and we wish to make all things as clear, and plain, as possible, we will first suppose a small ball to move down the tube AB, with an uniform velocity: and at the same time, suppose the tube to be carried with the same velocity along the line BN, at right angles to the direction of the ball; it is evident, that the ball will strike the side of the tube, long before it arrives at the bottom at B. Or, we will suppose a drop of rain to enter the center of the tube at A; and to move uniformly down towards B; it is manifest, that the drop will come in contact with the side of the tube before it arrives in the place CD. But if we incline the tube in an angle of 45°: namely, in the direction EF; and then suppose the ball, or drop of water, to enter as before, and the tube to move from EF to GH, then the drop will be in the axis of the tube at *e*: and when the tube is carried to IK, the drop will be at *a*: and when the tube is at LM, the ball or drop will have passed down the axis of the tube to M, without touching the side. Now, instead of making the velocity of the tube and that of the ball, or drop, equal to each other; or the lines FM and EM equal; we will suppose EM to FM as the velocity of light to that of the Earth in its orbit: Then, if instead of a ball, or drop of water, we suppose a particle of light, coming from a star, to enter the tube at E, and when the tube is carried by the Earth's motion to H, the particle will be at *e*, in the axis of the tube; and when the tube is at K, the particle will be at *a*; and, lastly, when the tube is at M, the particle will be at M, and will there enter the eye of the observer, who will see the * in the place 2, instead of 1, its true place. It is to be observed, that there is a constant succession of these particles, following each other, which form a ray of light; and a number of these rays, passing into the eye, is called *a pencil of rays*. The angle FEM, or its equal, * E *, is the quantity of aberration. This apparent motion will be a circle when the star is in the pole of the Ecliptic; that is to say, if the Earth's motion were uniform; but this is too small a quantity to be taken notice of. When the star is in the Ecliptic, the apparent motion is a straight line, or arch of a major circle. In all other situations of the star the apparent motion is elliptical.



ABERRATION *of the planets*, is equal to their geocentric motion, or the space they appear to move through as seen from the Earth, during the time light is in passing from the planet to the Earth. Thus, in the sun, the aberration in longitude is constantly about 20 seconds, that being the space the sun appears to move through in the same time light is in passing from the sun to the Earth. In like manner, as light passes from the sun to the Earth in 8 minutes and 7 seconds; say as the distance of the sun from the Earth, is to the distance of the planet from the Earth, at that time: so is 8m, 7s, to the

time light is in passing from the planet to the Earth: then compute the geocentric motion of the planet in this time, and you have the aberration of the planet, whether it be in longitude, latitude, right ascension, or declination.

It is evident that the aberration will be greatest in longitude, and very small in latitude, because the planes of the orbits of the planets, differ but little from the plane of the Ecliptic; or plane of the Earth's orbit: so that the aberration in the latitude of any planet is usually neglected. The quantity of aberration, in right ascension, and declination, depends on the situation of the planet in the zodiac. The aberration in longitude will be least when the planet is stationary; and greatest in the exterior planets when they are in opposition to the sun; but in the interior planets, the aberration is greatest when they are in their superior conjunction.

ABSIS, ABSES, ABSIDES. See Apsis, &c.

ABSOLUTE *Equation*, is the sum of the optic and excentric equations. The apparent inequality of the planet's motion, arising from its not being equally distant from the Earth at all times, is called its optic equation; and this would subsist if the motion of the planet were uniform. The excentric inequality is caused by the planet's motion being variable. To illustrate this, suppose the Earth placed in the centre of a circle, and the Sun to move uniformly in its circumference. It is evident that as the Sun moves uniformly there can be no optic nor excentric equation. But if we suppose the Earth to be removed out of the centre; and the Sun's motion to remain uniform; his motion would not appear to be so to a spectator on the Earth; and in this case there will be an optic equation, but no excentric one. But if we suppose the Sun to move in an ellipsis, and the Earth in its focus; it will be evident that the Sun's motion will not be uniform in such an orbit, and consequently he will have two equations; namely the optic equation, and the excentric one. See *Equation*, and *Optic* inequality.

ACCELERATED *Motion*, is that which receives fresh accessions of velocity; and the acceleration may be either equable or variable: if the accessions of velocity be always equal in equal times, the motion is said to be uniformly accelerated: But if the accessions, in equal times, either increase, or decrease, then the motion is variably accelerated. Retardation is directly opposed to acceleration, and denotes a diminution of velocity.

ACCELERATION. The diurnal acceleration of the fixed stars, is the time in which the stars, in one diurnal revolution, anticipate the mean diurnal revolution of the Sun; which is 3 minutes, 55 seconds, and 9 tenths of mean time, or nearly 3m. 56s. that is, a star rises, or sets, or passes the meridian, about 3m. 56s. sooner each day. This acceleration of the stars, which is only apparent in them, arises from the real motion of the Earth in its orbit; or the apparent motion of the Sun from west to east. The diurnal acceleration serves to regulate the lengths of pendulums in clocks. If I observe a fixed star to set or pass behind a chimney, building, hill, steeple, or other fixed object, when the clock marks, for instance, 10 minutes past 9; and the next day, the eye being in the same place as before, the passage be at 9h. 6m. 4s. I then conclude that the clock is well regulated, and measures mean time truly. The sun at a mean passes over 59′ 8″ and 2 tenths every day; the mean quantity of this anticipation, or acceleration, is found by saying, As 360° 59′ $8\frac{1}{2}$″ : 59′ $8\frac{1}{2}$″ :: 24 hours: 3m. $55\frac{1}{2}$s. the fourth term of which is the mean daily acceleration.

Acceleration *of a Planet.* A planet is said to be accelerated in its motion, when its real diurnal motion exceeds its mean diurnal motion. And on the contrary, the planet is said to be retarded in its motion, when the mean exceeds the real diurnal motion. This inequality arises from the change in the distance of the planet from the Sun, which is continually varying; the planet always moves quicker when it is nearer to the Sun, and slower when farther off.

Acceleration *of the Moon*, is a term used to express the increase of the Moon's mean motion from the Sun, compared with the diurnal motion of the Earth; by which it appears that, from some uncertain cause, it is now a little quicker than it was formerly. Dr. Halley was led to the discovery, or suspicion, of this acceleration, by comparing the ancient eclipses observed at Babylon, &c. and those observed by Albategnius, in the ninth century, with some of his own time. He could not, however, ascertain the quantity of the acceleration, because the longitudes of Bagdat, Alexandria, and Aleppo, where the observations were made, had not been accurately determined. But since his time the longitude of Alexandria has been ascertained by Chazelles; and Babylon, according to Ptolemy's account, lies 50′ east of Alexandria. From these data, Mr. Dunthorne compared the recorded times of several ancient and modern eclipses, with the calculations of them by his own tables, and thereby verified the suspicion of Dr. Halley; for he found that the same tables gave the Moon's place more backward than her true place in ancient eclipses; and more forward than her true place in later eclipses; and hence he concluded that her motion was slower in ancient times, and quicker in later times than the tables give it. Not content with barely ascertaining the fact, he proceeded to determine, as well as the observations would allow, the quantity of the acceleration; and by means of an eclipse, of which the most authentic account remains, observed at Babylon in the year 721 be-

fore Christ, he found that the beginning of this eclipse was about an hour and three quarters sooner than the beginning by the tables; and that therefore the moon's true place preceded her place by computation, by about 50′ of a degree at that time.

Then admitting the acceleration to be uniform, and the aggregate of it as the square of the time, it will be at the rate of from 9 to 10 seconds by later computations, in 100 years. M. de la Place announced to the Academy of Sciences the cause of this interesting discovery in physical astronomy. The Academy had at three different times in 1762, 70, and 72, a premium for the elucidation of this subject, but Euler himself could find nothing in the theory of attraction, which could explain it; and the learned were ready to conclude, that the difference between the ancient and modern observations was to be regarded as doubtful, and that the Arabs might have been mistaken: but M. Delambre, having calculated in 1783 the observations made by M. le Paute d'Agelet at the Military Academy, has found a new proof of this curious phenomenon. We were waiting for a satisfactory account from theory, when Laplace, who so ingeniously explained in 1786 the secular equations of Jupiter and Saturn, succeeded so far, after many attempts, as to reduce that of the Moon to the laws of universal weight. It is the diminution which exists, and has existed for many centuries in the equation of the solar orbit, that is the cause of this inequality in the Moon: but as the diminution, produced by the attraction of planets, will become, in the revolution of ages, a real augmentation, so what appears now to us an acceleration in the Moon's motion, will likewise become a retardation, and it is nothing more than a periodical inequality. M. Laplace finds it 11 seconds for the first century, and as this increases nearly as the square of the time, so for the year 720 before our æra, the nearest date for the most ancient observations that have reached us, it produces a difference that perfectly agrees with the observation. Thus then the Moon's mean motion appears to accelerate continually; and the accelerations appear to be in the duplicate ratio of the times. The acceleration for the end of the 17th century was about 9″; so that the whole motion of the Moon during the 17th century must be increased 9″, to obtain its motion during the 18th; and as much must be taken from it, or added to the computed longitude, to obtain its motion during the 16th; and the double of this must be taken from the motion during the 16th to obtain its motion during the 15th, &c.—To explain the theory, it must be considered, that the lunar period which we observe, is not that which would have obtained, had the moon been influenced by the Earth alone. We should not have known that her natural period was increased, had the disturbing force of the Sun remained the same; whereas this varies, and that in the inverse triplicate ratio of the Earth's distance from the Sun, and is therefore greater in our winter, when the Earth is nearer to the Sun. This is the source of the annual equation, by which the lunar period in January is made to exceed that in July almost 24 minutes. The angular velocity of the Moon in general is diminished $\frac{1}{179}$, and this varies inversely as the cube of the Earth's distance from the Sun. If this ratio be expanded into a series arranged according to the sines and cosines of the Earth's mean motion, making the mean distance of the Earth 1, then the series will contain a term equal to $\frac{1}{179}$ of this velocity multiplied by $\frac{3}{2}$ of the square of the Earth's excentricity. Had this excentricity remained constant, this product would also be constant, and would still be confounded with the general diminution, making a constant part of it: but it is known that the excentricity of the Earth's orbit diminishes, and its diminution is the result of the universality of the Newtonian law of the planetary deflections. Though this diminution is very small, its effect on the lunar motion becomes sensible in the course of many ages. The excentricity diminishing, the diminution of the Moon's angular motion must also diminish, that is, the angular motion must increase,

During the 18th century, the square of the Earth's excentricity has diminished .0000015325, the mean distance from the Sun being 1. In that time this has increased the Moon's angular motion .00000001285. As this augmentation is gradual, we must multiply the angular motion during the century by half of this quantity, in order to obtain its accumulated effect. This will be found 9″ very nearly; which exceeds that deduced from a most careful comparison of the motion of the last two centuries, only by a fraction of a second! While the diminution of the square of the excentricity of the Earth's orbit can be supposed proportional to the time, this effect will be as the squares of the times. The effect on the Moon's motion is periodical, as the change on the solar excentricity is, and its period includes millions of years. Its effect on the Moon's longitude will amount to several degrees before the secular acceleration changes to a retardation. By the solution of this problem, which had for a long time engaged the attention of mathematicians, M. Laplace has given the last degree of perfection to theoretical astronomy; as there is now no anomaly, in the motion of the heavenly bodies, that is not satisfactorily accounted for, on the simple Newtonian laws of gravitation.

ACHERNER, or ACHARNER, a star of the first magnitude in the southern extremity of the constellation Eridanus, marked *a* by Bayer.

ACHRONICAL, or ACHRONYCAL. See ACRONYCHAL.

ACONTIAS, a name used by some authors for

a sort of comet, or meteor, whose head appears round or oblong, and its tail very long and slender, resembling a javelin.

ACRONYCHAL, or ACRONYCAL, is when a star or planet is opposite to the Sun. It comes from a Greek word signifying the point or extremity of night, because the star rose at sunset, or the beginning of night, and set at sun rise, or the end of night; and so as they supposed it shone all night. The acronycal is one of the three Greek poetic risings and settings of the stars; and stands distinguished from Cosmical and Heliacal. It is to be observed that a star, at London for example, may have a great south declination, and rise when the Sun sets; and the consequence will be, that it will set long before morning; or perhaps before midnight.

ADVENT, in the Calendar, the time immediately preceding Christmas. It was anciently employed in pious preparation for the coming on of the feast of the nativity. Advent includes four sundays, or weeks; commencing either with the sunday which falls on St. Andrew's day, viz. the 30th of November, or the nearest sunday to that day, either before or after.

ÆRA, is the same as epoch, or epocha, and means a fixed point of time, from which to begin a computation of years ensuing.

ÆSCULAPIUS, the ancient name of Ophiucus.

ÆSTIVAL, or ESTIVAL, of or belonging to summer. Thus we say, the Æstival Solstice, in opposition to Brumal.

ÆSTIVAL *Point* is that whereby the Sun's ascent above the Equinoctial is determined.

ÆSTIVAL *Signs* are those extended from the solstitial point: *i. e.* the Sun's greatest declination northward, to the intersection of the Ecliptic and Equinoctial southward, including Cancer, Leo, and Virgo.

AGE, is used for a century. The time since the creation of the world is divided into three ages: The first, from Adam to Moses, which is called the age of nature; the second from Moses to Christ, called the age of the law; and the third, or age of grace, from Christ to the present time.

AGE *of the Moon*, the number of days elapsed since the last new Moon.

ALCOR, a small star adjoining to the large bright one in the middle of the tail of the greater bear. The word is Arabic.—It is a proverb among the Arabians, applied to one who affects to see small things, but overlooks much greater, "Thou canst see Alcor, and yet not see the full Moon."

ALDEBARAN, the Arabian name of a fixed star, of the first magnitude, in the sign Taurus, or the Bull, and because it is just in the eye of this constellation, it is called the Bull's eye.

ALDHAFERA, the Arabian name of a fixed star of the third magnitude, in the Lion's Mane.

ALGENIB, a fixed star of the second magnitude, on the right side of Perseus.

ALGOL, or *Medusa's head*, a fixed star of the third magnitude, in the constellation Perseus.

ALGORAB, a fixed star of the third magnitude, in the right wing of Corvus.

ALHIRTO, a fixed star of the third magnitude, in the constellation Capricorn. This is otherwise called *rostrum gallinæ*. Near this star, in the year 1600, it is said a new star appeared, which lasted 21 years, and then disappeared again.

ALLIOTH, a star in the tail of the greater Bear. The word is Arabic and denotes a horse; and they gave this name to each of the three stars, in the tail, as they are placed like three horses, drawing the waggon, commonly called Charles's Wain, represented by the four stars on the body of the same constellation.

ALMACANTAR, or ALMICANTAR, from the Arabic, are circles parallel to the horizon, and conceived to pass through every degree of the meridian: serving to shew the height of the celestial bodies; and are the same as parallels of altitude.

ALMAGEST, the name of a celebrated book composed by Ptolemy; containing a collection of a great number of the observations and problems of the ancients, relating principally to astronomy. This is the first work of the kind which has come down to us, it contains a catalogue of fixed stars, with their places, besides numerous records and observations of eclipses, the motions of the planets, &c.

Riccioli, an Italian Jesuit, also published, in 1651, a body of astronomy which he called The New Almagest.

ALMANAC, a calendar or table, in which are printed the days, feasts, fasts, &c. of the year; the age and phases of the Moon, and other celestial phenomena. It answers to the *fasti* of the ancient Romans.

ALPHABET, Greek:—

Figure.	Name.	Power.	Figure.	Name.	Power.
Α α	Alpha.	a	Ν ν	Nu.	n
Β β ϐ	Beta.	b	Ξ ξ	Xi.	x
Γ γ ſ	Gamma.	g	Ο ο	Omicron.	o *little*
Δ δ	Delta.	d	Π ϖ π	Pi.	p
Ε ε	Epsilon.	e *short*	Ρ ρ ϱ	Rho.	r
Ζ ζ ϛ	Zeta.	z	Σ σ ς	Sigma.	s
Η η	Eta.	e *long*	Τ τ 𝜏	Tau.	t
Θ ϑ θ	Theta.	th	Υ υ	Upsilon.	u
Ι ι	Iota.	i	Φ ϕ	Phi.	ph
Κ κ	Kappa.	k, c	Χ χ	Chi.	ch
Λ λ	Lambda.	l	Ψ ψ	Psi.	ps
Μ μ	Mu.	m	Ω ω	Omega.	o *great*

ALPHETA, a fixed star of the second magnitude, in the Northern Crown, otherwise called *Lucida Coronæ*.

ALRAMECH, or ARAMECH, the Arabic name of a star of the first magnitude, otherwise called Arcturus.

ALTITUDE is the arch of a vertical circle, contained between the horizon and any celestial object. This altitude may be either *true* or *apparent*. The apparent altitude is that which appears by sensible observations, made at any place on the Earth's surface. And the true altitude is that which results by correcting the apparent on account of refraction, parallax, &c.

The quantity of refraction is different at different altitudes, and these vary with the different states of the atmosphere. The quantity of the parallax is different in different luminaries; and this is occasioned by their different distances from the Earth. The fixed stars have no sensible parallax: the Sun's is about $8\frac{3}{4}$ seconds; but in the Moon it is about 52 minutes: but this varies on account of her distance from the Earth being variable.

ALTITUDE *of the Nonagessimal*, is the altitude of the highest point of the Ecliptic above the horizon, at any time; and is equal to the angle formed by the intersection of the horizon and Ecliptic at the same time.

ALTITUDE *of the Cone of the Shadow of the Earth or Moon*, the height of the shadow of the body, made by the Sun, and measured from the centre of the body. To find it say, As the tangent of the Sun's apparent semidiameter : radius : : 1 : the height of the shadow in semidiameters of the body.

Equal ALTITUDE *Instrument*, is an instrument used to observe a celestial object, when it has the same altitude on both sides of the meridian, or before and after it passes the meridian. This instrument is very useful for adjusting clocks, &c. and for comparing equal and apparent time. It is to be observed, that the greatest altitude of the Sun and Moon is a little before or after they come to the meridian.

AMPHISCII, or AMPHISCIANS, are the people who inhabit the torrid zone; which are so called, because they have their shadow at noon turned sometimes one way and sometimes another, namely at one time of the year their meridian shadow is towards the South; and at another towards the North.

AMPLITUDE, is an arch of the horizon, intercepted between the true east or west point and the centre of any celestial body at its rising or setting: so that the amplitude is of two kinds; ortive or eastern, and occiduous or western. Each of these two is either northern or southern, according as the point of rising or setting is in the northern or southern half of the horizon.

ANABIBAZON, a name sometimes given to the Dragon's Head, or northern Node of the Moon.

ANALEMMA, a planisphere, or projection of the sphere, orthographically made on the plane of the meridian, by perpendiculars from every point of that plane; the eye supposed to be at an infinite distance, and in the east or west point of the horizon. In this projection, the solstitial colure, and all its parallels, are projected into concentric circles, equal to real circles in the sphere; and all circles whose planes pass through the eye, as the horizon and its parallels, are projected into right lines equal to their diameters; but all oblique circles are projected into ellipses, having the diameter of the circle for the transverse axes. This instrument, having the furniture drawn on a plate of wood or brass, with an horizon fitted to it, is used for resolving many astronomical problems; as the time of the Sun's rising and setting, the length and hour of the day, &c. It is also used in dialling, for laying down the signs of the zodiac, with the lengths of days, and other matters of furniture, upon dials.

ANALOGY, the same as proportion, or equality, or similitude of ratios.

ANASTROUS *signs*, a name given to the 12 portions of the Ecliptic, which the signs possessed anciently, but have now deserted by the precession of the equinoxes.

ANDROMEDA, a constellation of the northern hemisphere, representing the figure of a woman almost in a state of nudity, and her arms extended and chained, being one of the original 48 asterisms, or figures under which the ancients comprehended the stars, as derived to us from the Greeks, who probably had them from the Egyptians or Indians, and who, it is suspected, altered their names, and accompanied them with fabulous stories of their own. According to them, Cepheas, the father of Andromeda, was obliged to give her up to be devoured by a monster, to preserve his kingdom from the plague; but that she was delivered by Perseus, who slew the monster, and espoused her. And the family were all translated by Minerva to the skies, the mother being the constellation Cassiopeia. The Arabians, whose religion did not permit them to draw the figure of the human body on any occasion whatever, have changed this constellation into the figure of a sea-calf. This constellation contains 66 stars, viz. three of the 2nd mag. two of the 3d mag. 12 of the 4th, &c. The most brilliant star in Andromeda is of the 2nd mag. situated in the side of her head. Mirach or Mizar in the girdle, Alhamak or Alhames on the south foot.

ANGLE, the opening or mutual inclination of two lines, or two planes, meeting in a point, called the vertex or angular point, and may be of any quantity less than 180°: it is frequently made use of by astronomers in these particulars, viz. Angle of incidence in a solar eclipse is formed by a line

drawn from the centre of a *penumbra* at the beginning of the eclipse to the centre of the Earth's disc; and this is called the first angle of incidence. The second is formed by a line drawn from the centre of the *penumbra* to the centre of the disc at the beginning of the central eclipse; that is, when the centre of the *penumbra* first touches the Earth's disc. And the third angle of incidence happens when all the *penumbra* falls within the disc, and it is formed by a line drawn from the centre of the disc to the perimeter of the disc in that point where the perimeter of the *penumbra* last touches it in its total obscurity within the disc.

ANGLES are divided into *plane*, *spherical*, and *solid*.

Plane ANGLES are those which are formed by two right lines meeting in a point.

Spherical ANGLE, is an angle formed on the surface of a sphere by the intersection of two major circles; or, it is the inclination of the planes of those circles to each other.

Solid ANGLE, is the mutual inclination of more than two planes, or plane angles, meeting in a point and not contained in the same plane: like the angles or corners of solid bodies.

ANGLE of commutation, Angle of elongation, see the words.

Parallactic ANGLE, or the *parallax*, is the angle formed at the centre of any heavenly body by two lines, one to the surface of the Earth and the other to its centre.

ANGLE of the position of the Sun, of the Sun's apparent semidiameter, &c. see the respective words.

ANGLE at the Sun, is the angle under which the distance of a planet from the Ecliptic is seen from the Sun.

ANGLE of the east. See Nonagessimal.

ANGLE of the obliquity, of the Ecliptic, or the angle of the inclination of the axis of the Earth to the axis of the Ecliptic, is now 23° 28′ nearly.

ANGLE of longitude, is the angle which the circle of a star's longitude makes with the meridian, at the pole of the Ecliptic.

ANNUAL *Equation*, of the mean motion of the Sun and Moon, and of the Sun's Apogee and Nodes.

ANNUS *Magnus*, the Great Year, is the time in which the Equinoctial points move from the first point of the Constellation Aries to the same point again. This, by moving at the rate of 50.2″ in a year is 25816 Tropical years.

ANOMALISTICAL year, is the space of time the Earth takes in moving from the aphelion to the same point again. This time is about 365d. 6h. 15m. 20s.

ANOMALY, is an irregularity in the motion of a planet, by which it deviates from the aphelion or apogee; or it is the angular distance of the planet from the aphelion or apogee; that is, the angle formed by the line of the apses, and another line drawn through the planet.

ANSÆ, *Anses*, The seemingly prominent parts of Saturn's ring, which appear like handles; when the eye of the spectator is but a little out of the plane of the ring: or when the side farthest from us is hid behind the body of the planet, and that nearest to us, is projected on his disc.

ANTARCTIC *Pole*, the southern extremity of the Earth's axis: this is the Terrestrial pole, and when produced to the fixed stars its extremity is the celestial one.

ANTARCTIC *circle*, a minor circle parallel to the Equator, and at the distance of 23° 28′, nearly, from the south pole.

ANTARES, a fixed star of the first magnitude in the heart of Scorpio.

ANTECEDENTIA, a term used by Astronomers to denote the motion of a planet when it appears to move westward; and when its motion is eastward it is said to move *in consequentia*.

ANTŒCI, is the place which lies under the same meridian as another place, one of which is as far south of the Equator as the other is north. The inhabitants of these two places are called antecians to each other. When it is summer at one of these places, it is winter at the other; when spring with one, it is autumn with the other, &c: but they have their noon at both places, or any other hour of the day at the same time.

ANTILOGARITHM, is what any logarithm of a sine, tangent, &c. wants of 10.

ANTINOUS, a part of the constellation Aquila.

ANTIPODES, the inhabitants of two places on the Earth which are diametrically opposite to each other, or that walk feet to feet.

ANTISCIANS, or ANTISCII, are those persons who inhabit different hemispheres as to north and south; and whose shadows at noon fall in contrary directions. But with deference to those who give the above definition we think they should have confined it to those who inhabit those parts of the two hemispheres lying without the tropics; because those in the torrid zone will sometimes have their shadows at noon directed the same way, although they may live on opposite sides of the Equator. This term is more general than the former, though often confounded with it.

APHELION, or APHELIUM, that point in the orbit of a planet which is at the greatest distance from the Sun. The aphelia of the planets are not fixed; for the disturbing forces exerted by all of them, keep those points in continual motion which is greater or less in each of them. This motion is

from west to east, or according to the order of the signs, and for this reason the Anomalistic year is longer than the siderial.

APHELLAN, the name of a bright star in Gemini.

APOGEE, *Apogæum*, that point in the orbit of the Sun, Moon, &c. which is farthest from the Earth, and the point opposite to it is called the Perigee, where the distance from the Earth is the least.

The Apogee and Perigee, would be two fixed points, if the Moon, or planet, were acted on only by the single body about which it revolves in an elliptic orbit. But by the joint actions of all the planets, those points become changed, and the orbit itself has a motion about its lower focus. The Moon and all other secondary planets, have their regular orbits about their primaries much disturbed, and that in an irregular manner according to their different positions with regard to their primaries and the Sun.

The quantity of the motion of the apogee may be found very nearly by observing its place at two very remote periods of time; converting the difference into minutes, and dividing them by the number of years elapsed between the two observations, the quotient gives the mean annual motion of the apogee. By those means the Sun's apogee has been found to move 1′ 2″ nearly in a year. And the mean annual motion of the Moon's apogee is about 40° 39′ 52.″

But the moon's apogee is very irregular. When she is in syzygy with the Sun, it moves forwards; but in the quadratures backwards; and these progressions and regressions are not equable, but it goes forward slower when the Moon is in the quadratures, or perhaps in a retrograde direction; and when the Moon is in the syzygy, it goes forward the fastest of all.

APPARENT, that which is visible, or evident to the eye, or the understanding.

Apparent *conjunction* of the planets, is when a right line, supposed to be drawn through their centres, passes through the eye of the spectator, and not through the Earth's centre. In general, the apparent *altitude, diameter, distance, horizon, magnitude, motion, place, &c.* of a heavenly body, are such as they appear to be, as seen from the surface of the Earth, before any corrections are applied.

APPARITION, is when a star, or any other celestial object becomes visible, which before was hid.

APPULSE, the near approach of two luminaries, so as to be seen for instance in the same telescope at the same time.

APSES, or APSIDES, the two points in the orbit of a planet which are at the greatest and least distance from the Earth, or Sun. The point at the greatest distance is called the higher apsis, and that at the least distance, the lower apsis. Also the higher apsis is called the aphelion, or apogee; and the lower apsis, the perihelion, or perigee. The line joining those two points is the transverse diameter of the elliptical orbit, and is called the line of the apses, or apsides. Some suppose the lines of the greatest and least distances not to be in the same right line; but it appears better to conceive the whole orbit to move about the lower focus, and then these lines will form but one right line; and the apses will move forwards and backwards as this line moves. Apsis is the singular of Apses.

APUS, *Avis Indica*, a constellation of the southern hemisphere, placed near the pole of the world, supposed to represent the Bird of Paradise. The Apus is supposed to be one of those birds called *apodes*, as having no feet. This constellation contains 11 stars, the largest of which is of the 5th magnitude.

AQUARIUS, one of the 12 signs of the zodiac, and is marked by the character ♒, representing part of a stream of water, issuing from the vessel of Aquarius, or the Water-pourer. The poets feign that Aquarius was Ganymede, whom Jupiter carried up to heaven to serve as a cup-bearer, instead of Hebe and Vulcan. Others say, that this sign was thus called, because that when it appears in the horizon, the weather commonly proves rainy. This constellation contains 108 stars.

AQUILA, the Eagle, or *Vulture* as it is sometimes called, is a constellation of the northern hemisphere, usually joined with Antinous. It is one of the 48 old constellations, according to the division of which Hipparchus made his Catalogue of the fixed stars, and which are described by Ptolomy. Aquila is thought to have been Merops, a king of the island of Cos, now called Lango, one of the cyclades in the Archipelago. This king, according to Ovid, was changed into an eagle, and placed among the constellations.

Antinous et Aquila are depicted together on the globe, and generally considered as one constellation.

This constellation contains about 74 stars, viz. one of the first magnitude, ten of the third, three of the fourth, &c. The chief star in this constellation is *Athair* or *Attair*, of the first magnitude, situated in the neck of Aquila.

ARA, *the Altar*, one of the 48 old constellations, mentioned by the ancient astronomers, and is situated in the southern hemisphere: containing 9 stars, viz. three of the third magnitude, four of the fourth, &c.

ARC, *or Arch*, part of any curved line. In astronomy there are various kinds of arcs. Thus, the latitude, elevation of the pole, and the declination

are measured by an arc of the meridian; and the longitude of a place by an arc of the Equator.

Diurnal Arc, is that part of a minor circle which is above the horizon in an oblique sphere; and that part which is below the horizon, is called the nocturnal arc.

Arc, of *progression*, or *direction*, is an arc of the Ecliptic, which a planet seems to pass over, when its motion is direct.

Arc, of *retrogradation*, is an arc of the Ecliptic passed over while a planet is moving in a direction contrary to the order of the signs.

Arc *between the centres*, is an arc passing from the centre of the Earth's shadow, perpendicular to the Moon's orbit, meeting her centre at the middle of an eclipse.

Arc of *vision*, is that which measures the Sun's depression below the horizon, when a star, before lost in his rays, begins to appear again. The quantity of this arc varies according to circumstances.

ARCTIC *circle*, is a minor circle of the sphere, parallel to the Equator, passing through the north pole of the Ecliptic at the distance of 23° 28′ nearly from the north pole of the world. The distance of the polar circles from the poles of the world, the obliquity of the Ecliptic, and the Sun's greatest declination, are always equal to the same quantity; but this is constantly varying on account of the nutation of the Earth's axes, and its secular diminution.

Arctic *pole*, the north pole of the world.

ARCTOPHYLAX, the same as Boötes.

ARCTURUS, a fixed star of the first magnitude, between the thighs of the constellation Boötes. This star is twice mentioned in the Book of Job, viz. c. 9. v. 9. and c. 38. v. 32, by the name Aish, if the translation be right; and by many of the ancients under its Greek name Arcturus. The Greeks gave the fabulous history of this star, or constellation, to this purport; Calisto, who was afterwards, in the form of the Greater Bear placed in the constellation, they tell us, brought forth a son to Jupiter, whom they called Arcas. That Lyacon, when Jupiter afterwards came to visit him, cut the boy in pieces, and served him up at table. Jupiter, in revenge, as well as by way of punishment, called down lightning to consume the palace, and turned the monarch into a wolf. The limbs of the boy were gathered up, to which the god gave life again; and he was taken and educated by some of the people. His mother, who was all this time a bear in the woods, fell in his way: he chased her, ignorant of the fact, and, to avoid him she threw herself into the temple of Jupiter: he followed her thither to destroy her; and this being death by the laws of the country, Jupiter placed them both in the skies, to prevent the punishment, making her the constellation of the greater bear, and converting the youth into this single star behind her. Dr. Hornsby, the Savilian Professor of Astronomy, concludes that Arcturus is the nearest star to our system visible in the northern hemisphere, because the variation of its place, in consequence of its own proper motion, is more remarkable than that of any other of the stars; and by comparing a variety of observations respecting both the quantity and direction of the motion of this star, he infers, that the obliquity of the Ecliptic decreases at the rate of 58″ in 100 years: a quantity which nearly corresponds with the computations framed by Euler, Lalande, and Laplace, on the principles of attraction. Philos. Trans. Vol. 63.

ARCTUS, a name given by the Greeks to two Constellations of the northern hemisphere; by the Latins called *Ursa Major* and *Ursa Minor*, and by us the *Greater* and *Less Bear*.

ARGO NAVIS, or the Ship, is a constellation in the southern hemisphere, being one of the 48 ancient constellations, containing 64 stars, viz. one of the 1st mag. six of the 2nd, nine of the 3d, nine of the 4th, &c. The most brilliant star in it is Canopus, of the first mag. situated near the keel.

The Greeks tell us that this was the noted ship in which the Argonauts performed that celebrated expedition, which has been so famous in their history.

ARGUMENT, is an arch given, by which another arch is found.

Argument of *inclination*, or Argument of *latitude*, of any planet, is an arch of a planet's orbit, intercepted between the ascending node, and the place of the planet from the Sun, numbered according to the succession of the signs.

Menstrual Argument of *latitude*, is the distance of the Moon's true place, from the true place of the Sun.—By this the quantity of the real obscuration in eclipses is found, or how many digits are darkened in any place.

Annual Argument of *the Moon's apogee*, or simply *annual* Argument, is the distance of the Sun's place from the place of the Moon's apogee; that is the arch of the Ecliptic comprised between those two places.

ARIES, the Ram, one of the constellations of the northern hemisphere, being the first of the old 12 signs of the zodiac, and marked ♈ in imitation of a ram's head. This constellation contains 66 stars, viz. one of the second mag. one of the third, and 2 of the 4th, &c. The most brilliant star is Arietis, situated in the northern horn, near the Tropic of Cancer.

The fabulous account of this asterism, as given by the Greeks, is to this effect. That Nephele gave Phryxus, her son, a ram, which bore a golden fleece, as a guard against the greatest dangers. Juno the step-mother both of him and Helle, laid designs

against their lives; but Phryxus, remembering the admonition of his mother, took his sister with him, and getting upon the back of the ram, they were carried to the sea. The ram plunged in, and the youth was carried over; but Helle, dropping off, was drowned, and so gave name to the Hellespont. When he arrived in Colchis, Æeta the king received him kindly; and, sacrificing the ram to Jupiter, dedicated the fleece to the god; which was afterwards carried off by Jason. The animal itself, they say, Jupiter snatched up into the skies, and made of it the constellation Aries. They have other fables also to account for its origin. But it is most probable that the inventors of this sign, placed it there as the father of those animals which are brought forth about the time the sun approaches to that part of the heavens, and so marking the beginning of spring.

ARMILLARY *sphere*, a name given to the artificial sphere, composed of a number of circles of metal, wood, or paper, which represent the several circles of the system of the world, put together in their natural order. It serves to assist the imagination to conceive the disposition of the heavens, and the motion of the heavenly bodies.

ARISTA, the same as *spika Virginis*.

ASCENDING, a term used to denote any star, or degree, or any point of the heavens, rising above the horizon.

Ascending *latitude*, is the latitude of a planet when moving towards the north.

Ascending *node*, is that point of a planet's orbit which crosses the Ecliptic, when the planet is passing from the south into north latitude.

Ascending *Signs*, are such as are ascending from the lowest point of the Zodiac to the highest.

ASCENSION *day*, otherwise called *Holy Thursday*, is a festival of the Church, held 10 days before Whitsunday, in memory of Christ's ascension.

Ascension, *Right*, of a heavenly body, is that point of the Equinoctial which comes to the meridian at the same moment with the celestial object.

Right Ascension of *the Mid-heaven*, or of the *Medium Cœli*, that point of the Equinoctial which is on the meridian; and that point of the Ecliptic which is on the meridian at any time is called the Medium Cœli.

Oblique Ascension, of a heavenly body, is an arch of the Equator intercepted between the first point of Aries, and that point of the Equinoctial which rises with the star, &c. in an oblique sphere.

ASCENSIONAL *difference*, is the difference between the right, and oblique ascension of the same point on the celestial sphere.

ASCII, are those inhabitants of the Earth, who at certain times of the year, have no shadow at noon. Such are the inhabitants of the torrid zone.

ASELLI, two fixed stars of the fourth magnitude, in the constellation Cancer.

ASHWEDNESDAY, the first day of lent, so called from the custom of sprinkling ashes on the heads of the penitents.

ASPECT, is the situation of the stars or planets in respect to each other. The ancients reckoned five, which are as under.

Names.	*Character.*	*Dif. of Longitude.*
Conjunction.	☌	0° = 0s.
Sextile.	⚹	60 = 2
Quartile.	□	90 = 3
Trine.	△	120 = 4
Opposition.	☍	180 = 6

These intervals are reckoned according to the longitudes of the planets; so that the aspects are the same whether the planet be in the Ecliptic or not.

ASSUMPTION, a feast of the Romish Church, in honour of the Virgin Mary's ascent, body and soul into Heaven.

ASTERION, one of the Canes Venatici.

ASTERISM, the same with constellation, or a collection of many stars, which are usually represented on globes by some particular image, or figure, to distinguish the stars contained in this constellation from those of others.

ASTRAL, something relating to the stars.

ASTROLABE, an ancient circular instrument for taking the altitudes, &c. of the stars.

ASTRONOMICAL, something relating to astronomy.

Astronomical Hours, begin at the meridian, and are reckoned from noon to noon.

Astronomical Calendar, may be any instrument, or tables made to solve Astronomical problems.

Astronomical Place, of a star or planet, is its longitude or place in the Ecliptic, reckoned from the beginning of Aries, *in consequentia*, or according to the natural order of the signs.

ASTRONOMY, the doctrine of the Heavens, and their phenomena. Astronomy is properly a mixed mathematical science, by which we become acquainted with the celestial bodies, their motions, periods, eclipses, magnitudes, distances, and other phenomena relating thereto.

ASTRUM, or ASTRON, a constellation, or collection of stars; in which sense it is distinguished from Aster, which denotes a single star.

ATMOSPHERE, that invisible elastic fluid which surrounds our globe to un unknown height, and partakes of all its motions.

ATTRACTION, a general term used to denote the cause, power or principle, by which all bodies mutually tend towards each other, and cohere, till separated by some other power. The laws, phenomena, &c. of attraction, form the chief subject of

the Newtonian philosophy, being the chief agent of nature, in almost all her wonderful operations. Not only large bodies, such as the Sun and planets, attract each other, but likewise comparatively smaller ones; and even the particles of matter that compose them. It has been proved by experiment, that a large mountain will draw a weight suspended by a string towards itself, so that the thread will form an angle with a line standing perpendicular to the Earth's surface. By the attraction of the Sun, Moon, and planets, the waters of the Earth are drawn up so as to form tides, and as they are easily transported or removed from one place to another, the particles which compose these fluids have no tendency to produce a nutation in the Earth's axis: But all solid particles which cannot be removed by the exertions of the Moon, &c. must have such a tendency. Now all mountains, hills, buildings, and even trees, being acted upon, are so many levers, which act the more powerfully because they are farther from the Earth's center than the surface of the Seas. Because there is a great deal more land on the north side of the Equator than on the south; and as the solid immovable particles all act as levers, and as there are a great many more acting on the north side than on the south, it follows, that the Moon has more power to draw the north pole towards herself than she has to remove the south pole. But as the Moon will be on all the different celestial meridians in her passage round the Earth, at the times those large continents pass by her; the poles of the world will describe a vermicular curve about the poles of the Ecliptic.

AUGES, two points in a planet's orbit, otherwise called *Apsides*, see Apsis. One of the Auges is particularly denominated the *Apogee*, the other the *Perigee*.

AURIGA, the Waggoner; a constellation in the northern hemisphere; whose stars, in the Britannic catalogue are 66. This is one of the 48 asterisms, mentioned by all ancient astronomers; and represented by an old man in a kind of sitting posture, with a goat and her kids in his left hand, and a bridle in his right. Besides the Hoedi this constellation includes another of the stars which the ancients distinguished by peculiar names, that is, Capella, the goat Capra, and Amalthœa Capra, which is the bright one near the shoulder, and supposed to be the mother of the Hoedi and the nurse of Jupiter.

The Hoedi, or the two stars in the arm of Auriga, were regarded by the ancients as affording presages of the weather: and they were so much dreaded on account of the storms, and tempests, that succeeded their rising, that they are said to shut up the navigation of the sea at this season. When the day of their peculiar influence was past, they celebrated a festival with sports and games, under the denomination of "*Natalis Navigationis.*" Germanicus calls them unfriendly stars to mariners; and Virgil joins them with Arcturus, mentioning their rising and setting as circumstances of the most important presage. To the same purpose all the ancient critics represent a part of the constellation Auriga, if not the whole of it, as deserving particular attention, and as much an object of terror as the blazing Arcturus.

AURORA, the morning twilight.

AURORA BOREALIS, *northern light;* a kind of meteor appearing in the northern part of the heavens, mostly in the winter, and in frosty weather. It is generally of a reddish colour, inclining to yellow, and sends out frequent corruscations of pale light, which seem to shoot up towards the zenith. This meteor has not been observed near the Equator, but it frequently appears in high south latitudes, much the same as here, only of a lighter colour.

Those appearances are much more beautiful in high latitudes than with us; and in some parts this appearance is attended with a hissing, and cracking noise, sufficient to strike terror into those who have never heard and seen it before. Philosophers are not agreed about the cause of those appearances.

AUSTRAL, the same with southern. Thus, austral signs are the last 6 signs of the zodiac: and are so named because they are on the south side of the Equinoctial.

AUTUMN, the third season of the year. This begins when the Sun enters the sign Libra, about the 22d of September, and ends about the same time in December: But by a reference to the ephemeris it will appear that it does not always happen on the same day of the month, on account of the leap years. An instance of this may be found between 1796 and 1804.

AUTUMNAL, something relating to Autumn.

AUTUMNAL *Equinox*, the time when the Sun enters the descending point of the Ecliptic, where it crosses the Equinoctial. The point where this intersection is made is called the autumnal point.

AUTUMNAL *signs*, are those signs through which the Sun appears to move during Autumn.

AUX, see AUGES. Some use Aux to denote the arch of the Ecliptic, intercepted between the first point of Aries, and the point wherein the Sun, or a planet, is at its greatest distance from the earth.

AXIS, *of the Earth*, a right line conceived to be drawn from one pole to the other through the center of the Earth; and when continued as far as some of the fixed Stars it points out the celestial poles, and this line is called the celestial axis. This line is always perpendicular to the plane of the Equator; though it is not quite parallel to itself in its motion

round the Sun. It is somewhat surprising that some of our best Mathematicians, who are able to write on Nutation and Precession, should affirm that this line is always parallel to itself!

AZIMUTH, of any celestial body, is an arch of the horizon intercepted between the meridian of the place, and a vertical, or azimuth circle, passing through the center of the object; this horizontal arch is equal to the angle at the zenith, contained between the meridian and vertical circle.

AZIMUTH, or *vertical circles*, are major circles of the sphere intersecting each other in the zenith and nadir; and cutting the horizon at right angles.

BASILICA, or BASILICUS, the same as Regulus.

BEAR, a name given to two constellations, called the *greater* and *less bear*, or ursa *major* and ursa *minor*. The pole star is in the tail of the less bear, and is within two degrees of the north pole; but as the pole is now moving in a vermicular spiral, round the pole of the Ecliptic, this distance is variable.

BELLATRIX, a ruddy glittering star of the second magnitude, in the left shoulder of Orion.

BELTS, *Fasciæ*, two or more zones or girdles surrounding the planet Jupiter, resembling parallel lines passing over his body. Those belts sometimes increase and at other times diminish in breadth, and they do not remain exactly on the same parts of his disc. Saturn is also said to be surrounded by belts, which do not adhere to his body like those of Jupiter, but are supposed to be large dark rings, surrounding the planet at a distance.

BERENICE'S Hair; see *coma Berenices*.

BETELGEUSE, a star of the first magnitude in the right shoulder of Orion.

BIQUINTILE, an aspect of the planets, when they are 144 degrees or $\frac{2}{5}$ of a circle asunder.

BISSEXTILE, or *leap-year*, a year consisting of 366 days, and happening every fourth year, in consequence of the addition of 1 day to February, when that month contains 29 days. This is done to compensate for the 6 hours which the Sun takes up nearly in his course, more than the 365 days commonly allowed for it in common years.

The day thus added was by Julius Cæsar appointed to be the day before the 24th of February, which among the Romans was the 6th of the calends, and which on this occasion was reckoned twice; whence it was called bissextile. When the two right hand figures of the given year can be divided by 4 without a remainder it is leap year; if 1 remain it is the first after leap year; if 2 the second, and if 3 the third. But when the two right hand places are nought, cut them off and divide the left hand figures by 4, and when nothing remains it is a leap year; but if 1, 2, or 3 remain it is a common year. This Julian year is a little too long, on which account the centuries which are not divisible by 4 are made common years, and hence there are only 97 intercalary days in 400 years. The length of a solar year is 365 days, 5 hours, 48 minutes and 48 seconds. $365 \times 400 + 97 = 146097$ days in 400 years by the Gregorian reckoning. But $365d.\ 5h.\ 48m.\ 48s. \times 400 = 146096$ days, 21 hours, 20 minutes = 400 solar years: the difference is 2 hours and 40 minutes; hence in 3600 years there will be a whole day, and one of the centuries which is a leap year by the rule must be made a common year, and this will bring the two reckonings exactly together.

BOÖTES, a constellation of the northern hemisphere, being one of the 48 ancient ones; containing 54 stars; of which one, in the skirt of his coat, is of the first magnitude, and named Arcturus. We have some account of the respective lustre of the stars in Boötes, by Dr. Herschel, in the Phil. Trans. for 1797. Boötes is represented as a man in a walking posture, with a club in his right hand, and his left raised upwards and holding the cord of the two dogs, which seem barking at the Greater Bear. The Greeks give no certain account of the origin of this constellation. The poets say, that Boötes was Icarus, the father of Erigonus, whom Jupiter placed in the skies. Others suppose him to be Arcas, the son of Calisto. Those, who, at a very early period, supposed the stars, which were afterwards formed into the Greater Bear, to represent a waggon drawn by oxen, made this Boötes the driver of them; from which circumstance he was called the waggoner; others continued the office when the waggon was destroyed, and made it his office to drive the two bears round about the pole; and some, when the greater waggon was changed into the greater bear, wished still to preserve the form of that machine in the stars which constitute Boötes. This constellation is called by various other names, as Arcas, Arctophylax, Arcturus minor, Bubulcus, Bubulus, Clamator, Icarus, Lycaon, Plorans, Plaustri-Custos, Philometus, Thegnis, and Vociferator; by others, Ceginus, Lanceator, Septentrio; by Hesychius, Orion; by others, Canis Latrans; by the Arabs, Armech, or Arcamech. Schiller, instead of Boötes, makes the figure of St. Sylvester; Schickhard, that of Nimrod; and Weigelius, that of the Three Swedish Crowns.

BOREAL *signs*, are the first six signs of the Zodiac, and are so called because they lie on the north side of the Equinoctial.

BRANDENBURGIUM *Sceptrum*, a small constellation on the west of Lepus and Orion, containing five stars, one of which is of the third magnitude, two of the fourth, &c.

CALENDAR, or KALENDAR, a distribution

of time as accommodated to the uses of life; or an almanac, or table, containing the order of days, weeks, months, feasts, &c. occurring in the course of the year; it is so called from the word calendæ, which among the Romans denoted the first days of every month.

CANCER, *the Crab*, one of the 12 signs of the Zodiac, and one of the 48 old constellations; ordinarily represented on the globe in form of a crab, and in astronomical books denoted by a figure much resembling that of the number sixty-nine; thus ♋. The reason generally assigned for its name, as well as figure, is a supposed resemblance which the Sun's motion in this sign bears to that of the crab-fish. As the latter walks backwards, or rather sideways, so the former in this part of his course begins to go sideways, at the same time that it recedes gently from us; though the disposition of the stars in this sign is by others supposed to have given the first hint to the representation of a crab. The Greeks pretend that when Hercules was contending with the Lernæan hydra, a crab which crawled upon the marsh seized his foot. The hero, however, crushed the reptile to pieces under his heel; but Juno, in gratitude for the offered service, though inconsiderable, advanced the creature to the skies. This sign contains 83 stars, viz. seven of the fourth magnitude, the rest smaller. The chief star in Cancer is Acubene, situated in the middle of the great southern claw.

Tropic of CANCER, a minor circle of the sphere, parallel to the Equinoctial, at the distance of about 23° 28′ therefrom.

CANDLEMAS, or the *purification*, a feast of the Church, held on the second of February, in memory of the purification of the Virgin. It is supposed to have taken its name from the number of lighted candles used by the Romish church in the procession of that day, or from the candles being consecrated on that day for the whole year.

CANES *venatici*, the *hounds*, one of the new constellations of the northern hemisphere, formed by Hevelius. These two dogs are also distinguished by the names of Asterion and Chara.

CANICULAR, a name given by some of the earlier Astronomers to the constellation which we call *Canis minor*. It is also used for one of the stars of the constellation *Canis major* or the *greater dog*; called also simply the *Dog-star*, and by the Greeks Sirius, it is situated in the nose of the Dog, and is the largest in appearance of all the fixed stars in the firmament. From the heliacal rising of this star; that is, its immersion from the Sun's rays, which now happens about the 11th of August, the ancients reckoned their *Dies caniculares* or Dog-days.

CANICULAR *days*, or *Dog-days*, denote a certain number of days, before and after the heliacal rising of the dog-star, in the morning. The ancients imagined that this rising of Sirius occasioned the sultry weather usually felt in July, with all the diseases attendant on that sickly season. The Romans were so firm in the belief of the many ills attending this circumstance that they sacrificed a brown dog to Canicula every year to appease his anger. All this arose from the ill founded notion that the dog-star so rising was the occasion of the extreme heat and the consequent diseases which prevailed in that season.

On account of the precession of the Equinoxes this star rises heliacally later now than it did to the ancients, and it will continue to do so till at length the south pole will approach so near to the dog-star that it will not rise at all to those who live more than about 27° north of the Equator; and this will happen to the inhabitants of London much sooner than to those who live in 27° or 28° of north latitude. Some writers on the Globes propose a problem to find the time this star will rise heliacally at Christmas, without having considered that the precession of the Equinoxes will have carried the south pole of the world so near to Sirius, that it will cease to rise at all to the inhabitants of London; and even if this had not been the case, they must put a new axis to their artificial globe before they can solve the problem.

CANOPUS, a star of the first magnitude in the rudder of Argo, one of the southern constellations.

CAPELLA, a bright star of the first magnitude, in the left shoulder of the constellation Auriga.

CAPRA, or *the she goat*, a name given to the star Capella; and sometimes to the constellation Capricorn. Some represent Capra as a constellation in the northern hemisphere, consisting of 3 stars, comprised between the 45th and 55th degrees of latitude.

Tropic of CAPRICORN, a minor circle of the sphere parallel to the Equinoctial, passing through the beginning of Capricorn, or the winter solstice, the point of the Sun's greatest declination.

CAPRICORNUS, the goat, the tenth sign of the zodiac, from which also the tenth part of the Ecliptic takes the same denomination. This was one of the 48 constellations received from the Egyptians by the Greeks. As to the figure of this asterism, the Greeks pretend that Pan, in order to avoid the terrible giant Typhon, threw himself into the Nile, and was changed into this figure; and that, in commemoration of this exploit, Jupiter took it up to the skies. But it seems more probable, as Macrobius observes, that the Egyptians marked the portion of the heavens appropriated to this sign, where the sun begins to ascend towards the north, with the figure of a goat, which is an animal that delights in climbing the sides of mountains. The ancients accounted

Capricornus the tenth sign, and when the sun arrived thereat, it made the winter solstice with regard to our hemisphere; but the stars having advanced about a whole sign towards the east, this is now rather the 11th sign; and it is at the sun's entry into Sagittarius, that the solstice happens: though the ancient manner of speaking is still retained. This constellation contains fifty-one stars, viz. three of the third magnitude, three of the fourth, &c. The chief star is situated near the root of his left horn. It is represented on ancient monuments, medals, &c. as having the fore part of a goat, and the hind part of a fish; which is the form of an Ægipan; and sometimes, simply, under the form of a goat.

CAMELEON, one of the constellations of the southern hemisphere, near the south pole, and invisible in our latitude. There are ten stars in this asterism, none of which are above the 5th magnitude.

CAMELOPARDALUS, a new constellation of the northern hemisphere, formed by Hevelius, consisting of thirty-two stars first observed by him. It is situated between Cepheus, Cassiopeia, Perseus, the two Bears, and Draco; and it contains fifty-eight stars.

CANIS MAJOR, the greater dog, a constellation of the southern hemisphere, below the feet of Orion, and one of the ancient forty-eight constellations. The Greeks, as usual, have many fables of their own about the exaltation of the dog into the skies; but the origin of this asterism, as well as its other name Sirius, lies more probably among the Egyptians, who carefully watched the rising of this star, and by it judged of the swelling of the Nile, calling the star the sentinel and watch of the year; and hence, according to their manner of hieroglyphic writing, represented it under the figure of a dog. They also called the Nile Siris; and hence their Osiris. This constellation contains thirty-one stars, viz. one of the first magnitude, two of the second, four of the third, four of the fourth, &c. The most brilliant star in Canis Major is Sirius, and it is thought to be nearer to the Earth than any of the fixed stars.

CANIS MINOR, a constellation of the northern hemisphere, just below Gemini: it is one of the forty-eight original constellations. The Greeks fabled that this is one of Orion's hounds; but the Egyptians were most probably the inventors of this constellation, and they may have given it this figure to express a little dog, or watchful creature, going before, as conductor of the larger, or rising before it: and hence the Latins have called it Anti Canis, the star before the dog.

This constellation contains fourteen stars, viz. one of the first magnitude, one of the second, one of the fourth, &c. Procyon, of the first magnitude, is the chief star in Canis Minor.

CAPUT *draconis*, or dragon's head, a name given by some to a star of the first magnitude in the head of Draco.

CARDINAL *points*, the east, west, north and south points of the horizon. The cardinal *points of the heavens* or *of a nativity*, are the rising and setting of the Sun, the zenith and nadir.

CARDINAL *Signs*, are those at the four quarters, or the equinoxes and solstices.

CASSIOPEIA, one of the 48 old constellations, placed near Cepheus, not far from the north pole. The Greeks probably received this figure, as they did the rest, from the Egyptians, and in their fables added it to the family in the neighbouring part of the heavens, making her the wife of Cepheus, and the mother of Andromeda. They pretend that she was placed in this situation, to behold the destruction of her favourite daughter Andromeda, who is chained just by her on the shore, to be devoured; and this as a punishment for her pride and vanity in presuming to stand the comparison of beauty with the Nereids. In the year 1572 there appeared a new star, which at first surpassed Jupiter himself in magnitude and brightness; but it diminished by degrees, till it quite disappeared at the end of eighteen months. This star alarmed all the astronomers of that age, many of whom wrote dissertations upon it; among the rest Tycho Brahé, Kepler, Maurolycus, Lycetus, Gramineus, and others. Beza, the Landgrave of Hesse, Rose, and some more, write to prove it a comet, and the same that appeared to the Magi at the birth of Christ, and that it came to declare his second coming: these were answered by Tycho. The stars in the constellation of Cassiopeia are in Flamsteed's Catalogue 55, five of which are of the third magnitude, five of the fourth, &c.

CASTOR, a moiety of the constellation Gemini; called also Apollo. Also a star in this constellation.

CATABIBAZON, the Moon's descending node; called also the dragon's tail.

CAUDA *draconis*, or dragon's tail, the moon's southern, or descending node.

CAUDA *leonis*, a fixed star of the first magnitude in the lion's tail; called also by the Arabs, Dineb Eleced.

CENTAURUS, one of the forty-eight old constellations, being a southern one, and is in form half man and half horse. It is fabled by the Greeks that it was Chiron the Centaur, who was the tutor of Achilles and Esculapius. This constellation contains thirty-five stars, viz. two of the first magnitude, one of the second, six of the third, nine of the fourth, &c. The chief star is situated on the side of his right foot.

CENTRAL *Eclipse*, is when the centres of the luminaries exactly coincide, and fall directly in a line with the eye. This can happen only upon one

point of the Earth at the same instant, in a solar eclipse. There is only one place where the Sun can be centrally eclipsed on the meridian: but there are many where it appears so at different instants of time. Or rather the center of the shadow describes a curve upon the surface of the Earth, so that a person standing on any point of this curve, will see the sun centrally eclipsed, when the Sun and Moon are on the same vertical circle.

CENTRE, or CENTER, in a general sense, signifies a point equally remote from the extremes of a line, plane, or solid; or a middle point dividing them so that some certain effects are equal on all sides of it.

CENTER *of attraction* or *gravitation*, is the point to which bodies tend by gravity; or that point to which a revolving planet or comet is impelled or attracted, by the force or impetus of gravity.

CENTRIFUGAL *force*, is that by which a body revolving about a centre, or about any other body, endeavours to recede from it.

CENTRIPETAL *force*, is that by which a moving body is prepetually urged towards a centre, and made to revolve in a curve instead of moving in a right line.

CEPHEUS, a constellation of the northern hemisphere, being one of the forty-eight old asterisms. The Greeks fabled that Cepheus was king of Ethiopia, and the father of Andromeda, the princess who was delivered up to be destroyed by a sea monster, from which she was rescued by Perseus. Cepheus contains thirty-five stars, viz. three of the third mag. seven of the fourth, &c. The largest star in this constellation is called Alderamin, of the third magnitude, situated near his left shoulder.

CERBERUS, this is a small northern constellation, held in the left hand of Hercules, and is often considered as a part of that constellation; it contains only eight stars, which are numbered with those of Hercules. The fabulous history of Cerberus according to Hesiod, is, that he was a dog with fifty heads, belonging to Pluto, the chief of the infernal regions, at whose gates he was stationed, as a faithful keeper. Other mythologists give him only three heads; and in this way he is generally represented. The last exploit of Hercules, was to drag Cerberus from the infernal gates, and take him before Eurystheus king of Argos, a city of Greece.

CERES, a newly discovered primary planet, moving in an orbit between those of Mars and Jupiter. It was discovered by M. Piazzi, Astronomer Royal at Palermo in Sicily, on the first of January, 1801.

CETUS, the whale, a southern constellation, and one of the forty-eight old asterisms. The Greeks pretend that this was the sea monster sent by Neptune to devour Andromeda, but was killed by Perseus. In the neck of the whale is a remarkable star, Collo Ceti, which appears and disappears periodically, or rather becomes brighter and fainter by turns, arising as some suppose from its rotation on its axis, by which it turns its bright side, and dark one to us alternately. Others have supposed that those periodical stars are flat like mill-stones, or rather oblate spheroids, which they suppose move in such a manner as to present a flat side, or one of their poles to us, and then the edge or equator. The former supposition is not very probable, and the latter quite absurd, and against the laws of rotatory motion. Cetus contains ninety-seven stars, viz. two of the second magnitude, eight of the third, and nine of the fourth, &c. The most brilliant star in this constellation is Menkar, situated near the mouth, of the second magnitude.

CHARLES'S WAIN, a name by which some call Ursa Major or the *greater bear;* though some say the *less bear.* Indeed both of the bears have been called waggons or wains.

CIRCINUS, the compasses. This small constellation is situated S. E. of Centaurus; containing only four stars, one of which is of the fourth magnitude.

CIRCLE, a plane figure, bounded by a curve line which returns into itself, called its circumference, and which is every where equally distant from a point within, called its centre. The circumference or periphery itself is sometimes improperly called a circle; but the name belongs only to the part contained within the circumference.

CIRCLES *of the sphere*, are such as cut the mundane sphere, and have their circumferences on its surface.

The circles of the sphere are either *major* or *minor.*

All the writers on Geography and Astronomy have called those circles great and lesser. But as there are but two kinds of circles we are taught in our grammars to put them in the comparative degree: now great is in the positive, and lesser in the comparative. Again, the words are not opposed to each other, for if we use great, this is opposed to little; and little cannot be used to express a minor circle, for one of these may be infinitely near to a major circle; and that on the celestial sphere is great beyond our conception.

The Author hopes to be pardoned for making inroads on any thing which has been long established by custom; but he has spoken to several cassical gentlemen on the subject, and they generally agree that it should be altered. The critics often remark with some severity on the impropriety of the word lesser: an instance of which may be found in the Review of a little book called "A grammar for the use of the lesser schools". "Here (the reviewers say) is an Author writes a book to teach grammar, and makes a fault in the title page"! So much by way of excuse, and apology; but now we are about

it, there seems to be an impropriety in the custom of calling the planets *superior* and *inferior*; for how can the Earth, Venus, &c. be inferior to the four small, newly discovered planets? If we say becuse they are nearer the Sun, this is no reason. It seems therefore better to use the words *exterior* and *interior*, as some have done before.

A major CIRCLE of the sphere is that which divides it into two equal parts, or hemispheres.

A minor CIRCLE of the sphere, divides the sphere into two unequal parts.

CIRCLES *of declination*, are major circles crossing the Equinoctial and intersecting each other in the poles of the world.

CIRCLES *of altitude* or *almucantars*, are minor circles parallel to the horizon, having their common pole in the zenith.

Diurnal CIRCLES are parallel to the Equinoctial, supposed to be described by the several stars, and other points in the heavens, in their apparant diurnal motion about the Earth.

CIRCLES of excursion, are minor circles, parallel to the Ecliptic, and at such a distance from it, that the excursions of the planets towards the poles of the Ecliptic, may be included within them; being usually fixed at about 10 degrees.

CIRCLE of *illumination*, is that imaginary circle on the surface of the Earth, which separates the illuminated side of the Earth from the dark side.

CIRCLES *of latitude*, or *secondaries of the Ecliptic*, are major circles perpendicular to the plane of the Ecliptic, intersecting each other in its poles, and passing through every star and planet. These are so called because they serve to measure the latitude of the stars.

CIRCLES *of longitude*, are minor circles parallel to the Ecliptic, and diminish more and more as they approach the pole of that circle. They are so called because the longitudes of the stars, &c. are reckoned upon them.

CIRCLES *of perpetual apparition*, are minor circles parallel to the Equator, described by the most northern point of the horizon, as the sphere revolves by its diurnal motion. All the stars within this circle are continually above the horizon.

CIRCLE *of perpetual occultation*, this is another minor circle, at a like distance from the Equator, but on the other side of it, this circle is described by the most southern point of the horizon, and contains all those stars which never rise. At the Equator all the stars rise and set: but at the poles they continue to move round parallel to the horizon without setting, except the Sun, Moon, and other objects which are subject to a change of declination.

Polar CIRCLES, see Arctic and Antarctic circles.

CIRCLES *of position*, are circles passing through the common intersections of the horizon and meridian, and through any degree of the Ecliptic, or the centre of any star, or other point in the heavens; and are used for finding the situation or position of any star.

CIRCULAR *velocity*, signifies that velocity of a planet, or other revolving body, which is measured by the arch of a circle.

CIRCUMGYRATION, is the revolving motion of any body about a centre; as of the planets about the Sun, &c.

CIRCUMPOLAR *stars*, are those stars which continually move round the pole without setting.

CIVIL Day, Month, Year, &c. see the substantives.

CLIMATE, or *clime*, a part of the surface of the Earth bounded by two minor circles parallel to the Equator; and of such a breadth, as that the longest day in the parallel nearer the pole, exceeds the longest day in that next the Equator, by some certain space, as half an hour, an hour, or a month. The beginning of a climate, is that part nearest the Equator and the end where the day is longest; that is in the parallel nearer the pole. Those climates are so many girdles or bands, terminated by lines parallel to the Equator. The climates all differ in breadth, and the same climate in different parts is very different with regard to heat and cold. As the climates commence at the Equator, at the beginning of the first climate, that is, at the Equator, the day is just 12 hours long; but at the end of it, or at the beginning of the second climate, the longest day is 12 hours and a half long; and at the end of the second, or beginning of the third, the longest day is 13 hours long; and so of the rest, as far as the polar circles, where the climates differ by a whole month. The shortest rule for finding the breadth of the climates between the Equator and polar circles is, add the cotangent of the Sun's greatest declination to the sine of his ascensional difference, and the sum of those two logarithms is the tangent of the latitude of the boundary nearest to the pole.

10.3623894 = cotangent of 23° 28′
8.8155985 = sine of 3° 45′ = 15 min.
9.1779879 = the tangent of 8° 34′ for the latitude of the parallel where the first climate ends and the second begins.

For the boundary of the second.

10.3623894 = cotangent of 23° 28′
9.1156977 = sine of 7° 30′ = 30 min. of time.
9.4780871 = the tangent of 16° 44′ for the latitude where the second climate ends and the third begins. The ascensional difference of the Sun is the time he rises before 6, or sets after 6; and this time

turned into degrees is the ascensional difference. By subtracting the 8° 34′ from 16° 44′ we get the breadth of the second climate.

COLUMBA NOACHI, Noah's dove, a small constellation of the southern hemisphere, on the W. of Argo Navis and Canis Major; containing ten stars, one of which is of the second magnitude, one of the third, two of the fourth, &c.

COLURES, two major circles intersecting each other in the poles of the celestial sphere, and passing through the equinoctial and solstitial points.

COMA BERENICES, *Berenice's hair*, a modern constellation of the northern hemisphere; composed of unformed stars between the lion's tail and Boötes. It is said that this asterism was formed by Conon, an astronomer, to console the queen of Ptolemy Euergetes, for the loss of a lock of her hair, which was stolen out of the temple of Venus, where she had dedicated it on account of a victory obtained by her husband. This constellation contains forty-three stars, ten of which are of the fourth magnitude.

COMET, a heavenly body, belonging to the solar system, moving in a very excentric elliptical orbit. See the account given of it in the description of the solar system.

COMMUTATION, *angle of*, is the distance between the Sun's true place seen from the Earth, and the place of a planet reduced to the Ecliptic: this is found by taking the difference between the Sun's longitude and the heliocentric longitude of the planet.

COMPLEMENT, is what is wanting to complete some certain quantity or thing.

COMPLEMENT, *of an arch* or *angle*, is what it wants of 90 degrees; thus when an angle is 50°, its complement is 40°.

Arithmetical COMPLEMENT, of a logarithm, is what the logarithm wants of 10; and the easiest way to find it is to begin at the left hand and subtract each figure from 9, and the last, or right hand one from 10.

Thus, the arith. comp. of the log. 9.5329714

Is by subtracting from nines, &c. -- 0.4670286.— It is commonly used in trigonometrical calculations, when the first term is not radius; in that case add the logarithms of the second and third terms to the arith. comp.

COMPLEMENT, *of altitude*, is the zenith distance of a heavenly body, and is found by taking the correct altitude from 90°.

CONJUNCTION, is the meeting of the stars or planets in the same place, or point in the heavens. There are several kinds of conjunctions, viz. True, when a line drawn from the Earth's centre passes through the centres of the two bodies. Apparent, when a line drawn from the eye of an observer on any point of the Earth's surface passes through the centres of the objects. Ecliptic, when the two objects have the same longitude. Equatorial, when the objects are on the same meridian. And Azimuth, when they are on the same vertical circle.

CONSTELLATIONS, certain imaginary figures of men, birds, beasts, &c. in the heavens, within which are arranged certain stars; these are sometimes called *asterisms*.

The use of these is very apparent to Astronomers; but as those unacquainted with the subject think them ridiculous, it may be proper to remark, that their use is much the same as countries, cities, towns and streets on the surface of the Earth.

COPERNICAN *system*, is that system of the world, in which the Sun is supposed to be at rest in the centre, and the Earth, comets and other planets moving round him, in their respective orbits.

COR CAROLI, Charles's heart, an extra constellated star of the second magnitude in the northern hemisphere, so called by Sir Charles Scarborough, in honour of King Charles I. It is situated between the Coma Berenices and Ursa Major.

COR HYDRÆ, the Hydra's heart, a star of the second magnitude, in the heart of the constellation Hydra.

COR LEONIS, the Lion's heart, or Regulus, a star of the first magnitude in the constellation Leo.

COR SCORPII. See Antares.

CORONA *Borealis*, or *Septentrionalis*, the *northern crown* or *garland*, a constellation of the northern hemisphere, being one of the forty-eight old ones. This constellation contains twenty-one stars, one of the second magnitude, called Alphecca Gemma. There are also six of the fourth magnitude, and some minor ones. The historical account of it is, that Bacchus the son of Jupiter, gave an elegant crown to Ariadne, the daughter of Minos, king of Crete; she married Theseus, king of Athens; who basely deserting her, the crown was turned into a constellation.

CORONA *Australis*, or *Meridionalis*, the *southern crown*, a constellation of the southern hemisphere, containing twelve small stars.

CORVUS, the Crow or Raven, a southern constellation, fabled by the Greeks, was raised to the skies by Apollo, to whom it tattled that the beautiful maid Coronis, the daughter of Phlegeos, and mother of Esculapius by Apollo, played the deity false with Ischys, under a tree upon which the animal happened to be perched. This constellation contains nine stars, viz. three of the third magnitude, two of the fourth, &c. The most brilliant star in this asterism is Algorab.

COSMICAL *rising* or *setting*, is said of a star when it rises or sets at the same time with the Sun.

CRATER, the Cup, a constellation of the southern hemisphere, containing thirty-one stars, viz. seven of

the fourth magnitude; the most noted of them is called Alkes, situated at the bottom of the cup.

The name according to some, is from the Bowl of Bacchus, in which he treated his company.

CREPUSCULUM, twilight; the time from the first dawning or appearance of the morning, to the rising of the Sun; and again from the setting of the Sun till the last remains of day.

CRESCENT, the new moon, which, as it begins to recede from the Sun, shews a small rim of light, terminating in horns or points, which increase till it becomes full.

CROSIER, four stars, in the form of a cross; by the help of which, those who sail in the southern hemisphere find the south pole.

CROWN, a name given to two Constellations, the southern and the northern.

CRUX, CRUSERO, or CROSIER, the *Cross*, a small constellation of the southern hemisphere containing five stars, one of the first magnitude, two of the second, one of the third, and one of the fourth. Four of them are in the form of a cross, the northernmost and southernmost of which are always in a line with the south pole, and therefore serve for a direction to discover it.

CULMINATION, the passage of a star or planet over the meridian.

CURTATE distance, is the distance of a planet's place from the Sun, reduced to the Ecliptic; or the interval between the Sun and that point where a perpendicular, let fall from a planet, meets the Ecliptic.

CURTATION, the interval between a planet's distance from the Sun, and the curtate distance. From the foregoing article it is easy to find the curtate distance; whence the manner of constructing tables of curtation is obvious: the quantity of inclination, reduction, and curtation of a planet, depending on the argument of latitude.

CUSP, the points or horns of the Moon, or other luminary.

CYCLE, a certain period or series of numbers proceeding orderly from first to last, then returning again to the first, and so circulating continually. Cycles have chiefly arisen from the incommensurability of the periods of revolutions of the Earth and celestial bodies to each other. The apparent diurnal revolution of the Sun about the Earth has been arbitrarily divided into 24 hours, which is the basis of all our mensuration of time, whether days, years, &c. But neither the annual motion of the Sun, nor that of the other heavenly bodies, can be measured exactly, and without any remainder by hours or their multiples. That of the Sun, for Example, is 365 days, 5 hours, 48 minutes, 48 seconds, and perhaps a small fraction; that of the Moon, 29 days, 12 hours, 44 minutes nearly.

Hence, in order to find equivalent expressions for these fractions in whole numbers, and yet in numbers which only express days and years, cycles have been invented; which, comprehending several revolutions of the same body, replace it, after a certain number of years, in the same points of the heavens from whence it first departed; or, in the same place of the civil calendar. There are various cycles; as, that of indiction, the cycle of the Moon, that of the Sun, &c.

CYCLE *of Indiction*, is a series of 15 years, returning constantly the same as the other cycles; and it commenced from the third year before Christ; whence it happens that if 3 be added to any given year of Christ, and the sum be divided by 15, the remainder is the year of the indiction.

CYCLE *of the Moon*, or the *lunar cycle*, is a period of 19 years; in which time the new and full moons return to the same day of the Julian year. This cycle is also called the *Metonic period* or *cycle*, from its inventor Meton, the Athenian; and also the *Golden number*, from its excellent use in the calendar: though, properly speaking, the golden number is the particular number which shews the year of the lunar cycle, which any given year is in. This cycle of the Moon only holds true for $310\frac{7}{10}$ years: for, though the mean new moons do return on the same day at the end of 19 years; yet not on the same hour of the day, but near an hour and a half sooner. Which supposing the first cycle to have begun on the first hour of the day, it will fail at the end of about 16 cycles; or if the 16th prove true the 17th will fail. Yet those employed in reforming the Calendar made their calculations on a supposition that the lunations return precisely at the end of 19 years for ever. Some suppose that the times of eclipses may be determined by means of this cycle; but as the true times of new and full Moon differ from the mean; the eclipse may not happen on the same day as the mean time gives it; and consequently we must calculate for the time as well as for the magnitude of the eclipse. *To find the year of the lunar cycle* is the same as to find the Golden Number, which see.

CYCLE *of the Sun* or *Solar cycle*, is a period of 28 years, beginning with 1, and ending with 28; which elapsed, the dominical or Sunday letters, and those which express the other feasts, &c. return into their former place, and proceed in the same order as before. The days of the month return again to the same days of the week; the Sun's place to the same signs and degrees of the Ecliptic on the same months and days, so as not to differ one degree in 100 years; and the leap-years begin the same course with respect to the days of the week on which the days of the month fall.

This is called the cycle of the Sun, not from any

regard to the Sun's course; but from Sunday, which the ancients called *Dies Solis;* the Sun's days; as the dominical or Sunday letter is sought for from this revolution. The reformation of the calendar has occasioned an alteration in this cycle; but it will answer from the commencement of any century to the beginning of the next. The epoch, or beginning of this cycle is the 9th year before Christ. And consequently,

To find the cycle of the Sun for any given year: Add 9 to the given year, and divide the sum by 28; the remainder will be the number of the cycle, and the quotient the number of revolutions since Christ; or rather since 9 years before Christ.

Solar cycle, and Sunday letters, from 1800 *to* 1900.

1 ED	5 GF	9 BA	13 DC	17 FE	21 AG	25 CB
2 C	6 E	10 G	14 B	18 D	22 F	26 A
3 B	7 D	11 F	15 A	19 C	23 E	27 G
4 A	8 C	12 F	16 G	20 B	24 D	28 F

Great Paschal CYCLE, is the same as the Victorian or Dionysian period, which see.

CYGNUS, *the Swan*, a constellation of the northern hemisphere, being one of the forty-eight old ones, and fabled by the Greeks to be the swan, under the form of which Jupiter deceived Leda.

This asterism contains eighty-one stars, viz. one of the second magnitude, six of the third, twelve of the fourth, &c. The most brilliant star in Cygnus is called Deneb, of the second magnitude.

DAY, a division of time arising from the appearance and disappearance of the Sun. The day is either natural or artificial.

Artificial DAY, is the time the Sun remains above the horizon; though some include the twilight.

Natural DAY, is the time in which the Sun appears to make one revolution round the Earth; or the time in which the Earth makes one revolution and a fraction of another round its axis.

Astronomical DAY, commences at noon, and ends on the following noon, and begins 12 hours later than the civil day; and is counted up to twenty-four hours, instead of twelve twice over.

Civil DAY, is the time allotted for day in civil purposes, and begins differently in different Nations.

DAY *siderial*, is the time in which any star appears to revolve from any meridian, to the some meridian again, which is 23*h*. 56*m*. 4.1″ of mean solar time; being the time in which the Earth makes one revolution round its axis. It should be remarked that the siderial days are not all strictly equal, because the quantity of abberation of the fixed stars is not a constant quantity.

DECEMBER, the last month of the year; in which the Sun enters the tropic of Capricorn, making the winter solstice. In the time of Romulus, December was the tenth month; whence the name, viz. from Decem, Ten; for the Romans began their year in March, from which December is the tenth month. The month December was under the protection of Vesta. Romulus assigned it thirty days; Numa reduced it to twenty-nine; which Julius Cæsar increased to thirty-one. At the latter part of this month they had the juveniles Ludi, and the country-people kept the feast of the goddess Vacuna in the fields, having then gathered in their fruits, and sown their corn: whence it appears is derived our popular festival called Harvest-home.

DECLINATION, is the distance of any celestial object from the Equinoctial, measured in degrees, minutes, &c.

Circles of DECLINATION, are secondaries to the Equinoctial, passing through the poles of the world; these are the same as meridians in geography.

Parallax of DECLINATION, is an arch of the circle of declination by which the parallax in altitude diminishes or increases the declination of an object.

Parallels of DECLINATION, are minor circles parallel to the Equinoctial.

Refraction of the DECLINATION, an arch of the circle of declination, by which the declination of a star, &c. is increased or diminished by means of the refraction.

DECHOTOMIZED, the Moon or other planet is said to be so, when only half illuminated.

DEFLECTION, the turning any thing aside from its former course by some adventitious, or external cause.

DEFLECTION *of the planets*, is brought about in three different ways.

1. When a comparatively small body, B, revolves about a very large one, A, the small one is drawn from the tangent towards the centre of A, and this is the first kind of deflection, produced by what is called a centripetal force.

2. When two or more small bodies, B, C, &c. revolve about a large one, A, in orbits inclined to each other, there will be a deflection produced in all the revolving bodies; and this kind of side deflection will cause a displacement in the plane of the orbit of each moving body, A, B, &c. This deflection is caused by the mutual actions of those revolving bodies upon each other. This is the kind of deflection which causes the Obliquity of the Ecliptic to increase or decrease, and it is from this that some Astronomers endeavour to account for the diminution of the Obliquity of the Ecliptic; and this they say will continue to decrease for many ages, after which it will again increase. But whoever considers the nature of these deflections will find that they produce a kind of vibratory motion in the planes of the orbits of the planets, and are consequently insufficient to produce a constant secular diminution of the angle formed by the planes of the Ecliptic and Equinoctial.

3. When a comparatively small body, such as the

Earth, moves about a large spheroidal one, such as the Sun, not in the plane of his Equator, nor yet at right angles to it; the redundant matter about the Sun's Equator (which I call the Solannulus) will produce a deflection of the Earth in such a way that the node or point where the Ecliptic crosses the Sun's Equator will continually fall back a little, and this falling back will cause the poles of the Ecliptic to have a slow motion round the solar poles, or about those two points in the heavens directly over the poles of the Sun. It may be said that this motion is too slow to be detected even by our best instruments. Be it so; but if it should be no more than half a second in 100 years, it will be sufficient. For if the poles of the Ecliptic move round the solar poles in a circle of about 15° in diameter, and the poles of the world move round this circle of 15° in diameter, in another circle which we at present suppose to be about 47° in diameter, it must follow that two points in their circumferences, moving with unequal angular velocities, must approach *to* and recede *from* each other alternately, and this would be the case, if the pole of the Ecliptic were to be 300 millions of years in performing one revolution.

Hence, it is evident that it is the third kind of deflection which causes the secular inequality of the Obliquity of the Ecliptic, and the second kind is nothing more than a perturbation continually augmenting or diminishing the secular variation produced by the third kind. As the action of the Moon on the Terrannulus produces the greater part of the nutation of the Earth's axis; so the reaction of the Terrannulus on the Moon must cause her to be deflected towards the Equator, and this deflection must cause a small part of the recession of the lunar nodes. This increase and decrease of the Obliquity of the Ecliptic will not be uniform; as the poles of the world and those of the Ecliptic will sometimes move parallel *to*; directly *from*; or *towards* each other, according to their respective situations. The solar poles may be considered as fixed, there being no cause to disturb them except a small nutation, produced by the actions of the several planets on the Solannulus. Hence by knowing the angle formed by the plane of the Ecliptic and that of the Sun's Equator, we know the diameter of the circle described by the pole of the Ecliptic about the solar pole. But as the pole of the Ecliptic is in motion we have no data to find the point about which the pole of the world moves, except the quantity of the diminution of the obliquity of the Ecliptic, during about 2160 years, and this perhaps is not correctly known. See Nutation, Orbit, Poles, &c.

DELPHINUS, the Dolphin, a constellation of the northern hemisphere, containing eighteen stars, five of which are of the third magnitude.

DENEB, a bright star in the lion's tail.

DEPRESSION *of the pole*, So many degrees, &c. as you move on a meridian towards the Equator, so many you depress the pole, because it becomes so much nearer to the horizon.

DEPRESSION *of a celestial body*, is its angular distance below the horizon.

DEPRESSION *of the visible horizon*, or *dip of the horizon*, denotes its sinking or dipping below the true horizontal plane, by the observer's eye being raised above the surface of the sea, in consequence of which, the observed altitude of an object is by so much too great.

DESCENDING *latitude*, is the latitude of a planet in its return from the nodes of the Equinoctial.

DESCENSION, is either *right*, or oblique.

Right DESCENSION, is a point, or arch, of the Equator, which descends with a star, or sign, below the horizon, in a right sphere.

Oblique DESCENSION, is a point, or arch, of the equator, which descends at the same time with a star, or sign, below the horizon, in an oblique sphere.

DESCENSIONS both right and oblique, are counted from the first point of Aries, or the vernal intersection according to the order of the signs, namely from west to east. And as they are unequal, when it happens that they answer to equal arcs of the Ecliptic, as per example, to the twelve signs of the zodiac, it follows, that sometimes a greater part of the Equator rises or descends with a sign, in which case the sign is said to ascend or descend rightly: and sometimes, a less part of the Equator, rises or sets with a sign, in which case it is said to ascend or descend obliquely.

DESCENSIONAL *difference*, is the difference between the right and oblique descension of the same point.

DIACENTROS, a word used by *Kepler*, to signify the shorter diameter of the elliptical orbit of any planet.

DIAMETER, *of a heavenly body*, is either apparent, *i. e.* such as they appear to an eye, or real, *i. e.* such as they are in themselves. The apparent diameters are best measured with a micrometer, and are estimated by the measure of the angle they subtend at the eye. These are different in different circumstances, and parts of the orbits, or according to the different distances of the luminary; being in the inverse ratio of the distance.

The Sun's vertical diameter is found by taking the heights of the upper and lower edges of his disc, when he is on, or near the meridian; correcting the altitude of each edge on account of refraction and parallax; then the difference between the true altitudes of the two is the true apparent diameter sought. Or the apparent diameter may be determined

by observing with a good clock, the time which the Sun's disc takes in passing over the meridian: and here when the Sun is in, or near the Equinoctial, the following proportion may be used; viz. As the time between the Sun's leaving the meridian and returning to it again: 360°:: the time of the Sun's passing over the meridian: number of minutes and seconds of a degree contained in his apparent diameter: this supposes the motion of the Sun to be uniform during one diurnal revolution, and no change in the nutation. But when the Sun is in a parallel at some distance from the Equinoctial, his diameter measures a greater number of minutes and seconds in that parallel than it would do in a major circle; and takes up proportionably more time in passing over the meridian; in which case use this proportion: As radius: cosine of the Sun's declination:: the time of the Sun's passing over the meridian reduced to minutes and seconds of a degree: an arc of a major circle which measures the Sun's apparent horizontal diameter. This is on the supposition that the declination and nutation remain constant; and also that the two circles of declination drawn from the pole as tangents to the Sun are parallel to each other. The Sun's apparent diameter may also be taken by the projection of his image in a dark room. There are several ways of finding the apparent diameters of the planets; but the best is that with the micrometer. The Sun's diameter varies from 31′33.8″ to 32′ 38.6″. The apparent diameter of the Moon varies from about 29′ 28″ to 33′ 36″.

DIGIT, is the measure by which the part of the apparent diameter of a luminary in eclipses is estimated, being the 12th part either of the Sun or Moon. Thus, an eclipse is said to be of 9 digits, when 9 parts out of the 12 of the diameter are in the eclipsed part; when the whole of the luminary is exactly covered, the digits eclipsed are just 12. When all the parts are any distance within the shadow, as often happens in lunar eclipses, then more than 12 digits are said to be eclipsed: Thus, if the chord line of the Earth's shadow through which the moon passes be sufficiently long to inscribe a circle equal to twice the Moon's diameter; with its center in that chord, then the Moon would be said to be 24 digits eclipsed. The chord line cannot be the measure of the digits as some suppose: but when the Moon passes through the centre of the shadow, then the chord becomes the diameter, and measures the digits truly.

DIHELIOS, is that ordinate of the ellipsis, that passes through the focus in which the Sun is supposed to be placed.

DIOASTRODOXON, the name of Mr. Lloyd's transparent Orrery, a compound Greek word, which signifies, shewing the motions of the heavenly bodies.

DIONYSIAN, or *Victorian period.* See Period.

DIOPTRICS, the doctrine of refracted vision; or that part of optics which explains the effects of light as refracted by passing through different mediums, as air, water, glass, and especially lenses.

DIP *of the horizon.* See Depression.

DIRECT. A planet is said to have its motion direct when it appears to a spectator on the Earth to move according to the order of the signs.

DIRECTION, the motion and other phenomena of a celestial object when direct.

Number of DIRECTION, is the number of days that Septuagesima Sunday falls after the 17th of January.

DISC, or *Disk*, the body or face of the Sun or Moon; such as it appears to us, when viewed with the unassisted eye: but when viewed through a good telescope, they appear of their true spherical form.

DISTANCE *of the Sun from the Moon's node*, or *apogee*, is an arch of the Ecliptic intercepted between the Sun's true place and the Moon's node, or apogee.

Apparent DISTANCE, that distance which we judge an object is placed at when seen afar off. This is usually very different from the true distance; because we are apt to think that all very remote objects, whose parts cannot well be distinguished, and which have no other object in view near them, are at the same distance from us, though perhaps the one is millions of miles nearer than the other, as is the case with regard to the Sun and Moon.

Apparent DISTANCE of two heavenly bodies is the angle they subtend; or the angle subtended by two lines supposed to be drawn from the eye of the spectator, to the apparent places of the two objects.

DIURNAL, something relating to the day.

DIURNAL *arch*, is the arch which a celestial object seems to describe from its rising to its setting.

DIURNAL *circle*, is the apparent circle, described by a heavenly object, in consequence of the Earth's rotation.

DIURNAL *motion of a planet*, is so many degrees and minutes, &c. as any planet moves in a day.

DIURNAL *motion of the Earth*, is its daily motion round its axis; which constitutes the length of a siderial day: this is the only motion in the universe which is uniform.

DODECATEMORY, the twelve equal parts into which the Ecliptic is supposed to be divided.

DOG, a name common to two constellations called the greater and the less Dog; or canis major and minor.

DOMINICAL *letter*, otherwise called the Sunday letter, is one of the first seven letters of the alphabet used in almanacs, &c. to mark or denote the sundays throughout the year. The reason for using seven letters, is because that is the number of days in a

week; and the method of using them is this: the first letter, A, is set opposite the first day of the year, the second letter, B, opposite the second day of the year, and so on through the seven letters; after which they are repeated over and over again to the end of the year, the letter A denoting the 8th day, the letter B the 9th, &c. Then, whichever of the letters so placed, falls opposite the first Sunday in the year, the same letter will fall opposite every future Sunday throughout the year, because the number of letters is equal to the number of days in a week; that is, in common years; but in leap years, an interruption of the order takes place in them. For, on account of the intercalary day, either the letters must be put out of their places for the whole year afterwards, so as that the letter, for example, which answers to the 1st of March, shall likewise answer to the second, &c. or else the intercalary day must be denoted by the same letter as the preceding one.

This latter expedient was judged to be the best, and accordingly all the Sundays in the year after the intercalary day have another dominical letter.

The dominical letters were introduced into the calendar by the primitive Christians, instead of the nundinal letters in the Roman calendar; and in this manner were those seven letters set opposite the days of the year, to denote the days in the week, in most of our common almanacs, till the year 1771, when the initial letters of the days of the week were generally introduced instead of them, excepting the Sunday letter itself, which is still retained.

From the foregoing account it follows that,

1st. As the common year consists of 365 days, or 52 weeks, and one day over; the letters go one day backwards every common year: so that in such a year, if the beginning or first day fall on a Sunday, the next year it will fall on a Saturday, the next on a Friday, and so on. Hence, if G be the dominical letter for the present year, F will be that for the next year; and so on in a retrograde order.

2d. As the bissextile or leap-year consists of 366 days, or 52 weeks, and 2 days over, the beginning of the year next after goes back two days. Whence, if in the beginning of the bissextile year the dominical letter were G, that of the following year will be E.

3d. Since in leap-years the intercalary day falls on the 24th of February, in which case the 24th and 25th days are considered as one day, and denoted by the same letter; after the 24th day of February the dominical letter goes back one place: thus, if in the beginning of the year the Dominical letter be G, it will afterwards change to the letter F for the remaining Sundays of the year. With us, however, this day is now added at the end of February, and from thence it is that the change takes place.

4th. As every fourth year is bissextile, or leap-year, and as the number of letters is 7; the same order of dominical letters only returns in 4 times 7, or 28 years; which, without the interruption of bissextile, would return in 7 years.

5th. Hence the invention of the solar cycle of 28 years; on the expiration of which the dominical letters are restored successively to the same days of the month, or the same order of the letters returns.

To find the Dominical *letter of any given year.*

Find the Cycle of the Sun for that year as directed under cycle; and the dominical letter is found corresponding to it. When there are two letters, the proposed year is bissextile; the former of them serving till the end of February, and the latter for the rest of the year.

The dominical letter for any year of the present century may be found thus:

To the number of years elapsed since 1800, add 8 and divide the sum by 4; to the quotient add the first named number and divide the sum by 7, then subtract the remainder from 7, and the last number will be that answering to the dominical letter; that is, if the remainder be 1, it answers to A, 2 to B, &c. and 0 to G.

Example. What is the dominical letter for 1833?

Here $33 + 8 = 41$, this divided by 4 is 10, and $10 + 33 = 43$, this divided by 7 leaves a remainder of 1, which taken from 7, gives 6; answering to F, which is the dominical letter for that year.

When the year is bissextile, that is, when the date of it is divisible by 4, or the 2 right hand figures can be divided by 4 without a remainder, then the letter obtained by this rule is that used from February to the end of the year, and the Sunday letter from the beginning of the year, to the end of February, will be that, next in order in the alphabet, unless when the former is G, in which case the latter will be A.

By the reformation of the calendar under Pope Gregory the 13th, the order of the dominical letters was again disturbed in the Gregorian year; for the year 1582, which had G for its letter at the beginning, by retrenching 10 days after the 4th of October, came to have C: by which means the dominical letter of the ancient Julian calendar is 4 places before that of the Gregorian, the letter A in the former answering to D in the latter.

DORADO, XIPHIAS, or the *sword-fish:* this small asterism is situated at the south pole of the Ecliptic, and contains six stars; one of which is of the third magnitude, two of the fourth, &c.

DOUBLE *Stars.* See Star.

DRACO, the *Dragon,* a constellation of the northern hemisphere, whose origin is variously fabled by the Greeks; some of them representing it as the

Dragon which guarded the Hesperian fruit, or golden apples, but being killed by Hercules, Juno, as a reward for its faithful services, took it up to the skies, and so formed this constellation: while others say, that in the war of the giants, this dragon was brought into the combat, and opposed to Minerva, when the goddess taking the dragon in her hand, threw him, twisted as he was, up to the skies, and fixed him to the axis of the heavens, before he had time to unwind his contortions. Draco contains eighty stars, viz. four of the second magnitude, seven of the third, ten of the fourth, &c.

DRAGON'S *head* and *tail*, are the nodes of the planets, but more particularly of the Moon, being the points of the Ecliptic which are intersected by her orbit, in an angle of about 5° 18′. One of these points is where the Moon crosses from south latitude into north, this is represented by the knot ☊ for the head, and is the same as the ascending node, the other is the point where the Moon is supposed to cross from north, into south latitude; this is represented by ☋ for the tail, and is the same as the descending node. These points are at 180° from each other, and are supposed to continue in motion, so that when we find the place of the node at any time when she is not actually in it; we do not find the point where the Moon last crossed the Ecliptic, but the place where this moving point has arrived at; and this is the place of the node at that instant.

DUPLICATE *Ratio*, is the square of a ratio, or the ratio of the squares of two quantities.

EAGLE, the same as Aquila.

EARTH, the planet or globe on which we live, consisting of land and water. It is now agreed by most of our philosophers, that the Earth is in the form of an oblate spheroid, or more properly elipsoid: Though some are of opinion that it is not a solid of rotation; that is, that all the meridians are not similar and equal; this difference arises from the different results brought out by different persons, who were sent to distant parts to measure a degree of the meridian. There is also some doubt and difference of opinion relating to the uniform density of the Earth. We are not able to penetrate far into the Earth, and consequently shall never know what materials the central parts are composed of; nor even those at the depth of ten miles. But it is reasonable to suppose that generally the heaviest parts are nearest the centre: and that the different metals, &c. comparatively near the surface, have been produced by some chemical process which, in a variety of ways, is carrying on a little below the surface, which does not take place at greater depths. With respect to the figure of the Earth, the universal laws of gravitation give it the figure of an elipsoid; and it must of necessity be of this form; for if it were a sphere its rotation about its axis would throw all the waters on its surface up towards the equator. There is also a necessity that the parts of which it is composed, should have been either in a chaotic state; or that the land and water should not have been separated before it began to revolve; for had it been a perfect sphere and the dry part as it now is; it could not have taken its present form; but all the regions about the Equator would have been covered with water. That is, it must have been chaotic, or something like paste, before it began to move round its axis, or that motion could not have removed the solid parts, if they had been such as they now are.

EAST, one of the four cardinal points of the horizon.

EASTER, a feast of the church, held in memory of our Saviour's resurrection. This feast has been annually celebrated ever since the time of the Apostles, and is one of the most considerable festivals in the Christian calendar; being that which regulates and determines the times of all the other moveable feasts.

The rule for the celebration of Easter, fixed by the council of Nice, in the year 325, is, that it be held on the Sunday which falls upon, or next after, the full Moon which happens next after the twenty-first of March; that is the Sunday which falls upon or next after the first full Moon after the vernal equinox. The reason of which decree was, that christians might avoid celebrating their Easter at the same time with the Jewish passover, which was held on the day of the full Moon.

To find EASTER *according to the New or Gregorian Style, till the year* 1899 *inclusive.*

Look for the Golden Number of the year in the first column of the table, against which stands the day of the Paschal full moon; then look in the third column for the Sunday letter next after the day of the full moon, and the day of the month standing against that Sunday letter is Easter-day. When the full moon happens on a Sunday, then the next Sunday after is Easter-day.

For Example. For the year 1813, the Golden Number is 9; against which stands April 15th, and the next Sunday letter, which is C, below that, stands opposite April 18, which is therefore the Easter-day for the year 1813.

Though the Gregorian calendar be much preferable to the Julian, it is not without its defects. It

Gol. No.	Paschal full Moon.		Sun. Let.
14	March	21	C
3	——	22	D
		23	E
11	——	24	F
		25	G
19	——	26	A
8	——	27	B
		28	C
16	——	29	D
5	——	30	E
		31	F
13	April	1	G
2	——	2	A
		3	B
10	——	4	C
		5	D
18	——	6	E
7	——	7	F
		8	G
15	——	9	A
4	——	10	B
		11	C
12	——	12	D
1	——	13	E
		14	F
9	——	15	G
		16	A
17	——	17	B
6	——	18	C

cannot, for instance, keep the equinox always fixed on the 21st of March, but it will vary between the 19th, and the 23d. To which we may add, that the full moon happening the 20th of March, might sometimes be Paschal; yet it is not allowed as such in the Gregorian computation; as on the contrary, the full moon on the 22d of March may be allowed for Paschal, which it is not. See Number of Direction.

EASTER *Term*, see Term.

ECCENTRIC *Place of a planet*, the same with Orbit place.

ECLIPSE, a privation of a part, or the whole of the light, of one of the luminaries, by the interposition of some opaque body, which comes between the eye and the Sun, as that of the Moon, in a Solar Ecilpse, and that of the Earth, in a Lunar one. Or when the opaque body falls between the Sun and the one eclipsed, as is the case, in the eclipses of Jupiter's satellites.

Duration of an ECLIPSE, is the time of continuance.

Immersion, or *incidence, of an* ECLIPSE, is the moment when the Sun, Moon, or planet begins to be obscured.

Emersion, or *Expurgation, of an* ECLIPSE, is the time when the eclipsed luminary begins to reappear.

Quantity of an ECLIPSE, is the part of the diameter of the luminary which falls into the shadow; or is covered by the opaque body. This is done by supposing the apparent diameter of the Sun, or Moon, to be divided into 12 equal parts, called digits; hence the Eclipse is said to be of so many digits. See Digit.

Annular ECLIPSE, is when the whole is eclipsed, except a ring, which appears round the Sun's edge.

Central ECLIPSE, is one in which the centres of the Sun and Moon are in the same right line with the eye of the spectator.

Partial Eclipse, when only a part of the luminary is eclipsed.

Total ECLIPSE, is that in which the whole luminary is eclipsed.

ECLIPSE *of the Moon*, is a privation of the light of the Moon occasioned by the interposition of the body of the Earth, directly, (or very nearly so,) between the Sun and Moon, and so intercepting the Sun's rays that they cannot arrive at the Moon to illuminate her. Or, the obscuration of the Moon may be considered as a section of the Earth's conical shadow, by the Moon passing through some part of it.

ECLIPSE *of the Sun*, is an occultation of the Sun's body, occasioned by the interposition of the Moon between the Earth and Sun.

ECLIPSES *of the satellites of Jupiter, &c.* The satellites of Jupiter undergo three sorts of eclipses; the first is when they fall into his shadow, the same as our Moon does into that of the Earth; and it is by this, that the longitude of places on the Earth are easily found, because this appears to all the inhabitants of the earth, at the same instant of time. The second is a sort of transit, because it passes over the disc of its primary, and the third an occultation, because it passes behind the body of Jupiter.

ECLIPTIC, a major circle of the sphere conceived to pass through the middle of the Zodiac. This is sometimes called *via solis*, or the Sun's way or path, because they say he always appears in this circle among the fixed stars. Others tell us that the Earth's orbit and the Ecliptic are the same thing. The former of these is nearly true, but the latter is not. If the Earth and Sun were the only bodies in the system, and they revolved about their common centre of gravity, then the Sun would truly appear to a spectator on the Earth to describe the Ecliptic among the fixed stars, and the orbit of the Earth would lie in the plane of the Ecliptic; but as things are, neither of these suppositions is strictly correct. For as the Earth and Moon move round a common centre of gravity; and the Sun and planets all revolve about the common centre of gravity of the whole system, and as they do not move in the same plane, it follows, that the Earth passes out of the plane of the Ecliptic, first on one side, and then on the other, every time the Moon passes round the Earth. The Sun is also generally out of this plane on account of his motion round the common centre of gravity. If we suppose a line drawn from the centre of gravity of the whole system, through the centre of gravity of the Earth and Moon, till it meet the sphere of the fixed stars; and this line being carried once round by the centre of gravity between the Earth and Moon, its extremity will describe the Ecliptic among the fixed stars. See more original remarks on the secular diminution of the obliquity of the Ecliptic under the words Nutation, Obliquity, and Precession.

ECLIPTIC *bounds*, or *limits*, are the greatest distances from the nodes at which the Sun or Moon can be eclipsed; namely, near 18° for the Sun, and 12° for the Moon.

Poles of the ECLIPTIC, are the two opposite points of the sphere which are 90° distant from every point of the Ecliptic. The distance between the poles of the Ecliptic and those of the Equinoctial is equal to the variable distance of the Tropics, or Sun's greatest declination, or the measure of the Obliquity of the Ecliptic, which according to Dr. Maskelyne, was in 1800, 23° 27′ 56.6″.

Reduction to the ECLIPTIC, see Reduction.

EIDOURANION, tho name of Mr. Walker's grand transparent orrery; signifying a view of the heavens, from Εἴδω, to see, and Οὐρανος, Heaven.

ELEMENTS, are those principles deduced from astronomical observations and calculations, and such fundamental numbers, as are employed in the construction of tables exhibiting the planetary motions.

ELEVATION, the altitude, or height of any object, as of the Equinoctial, of the Pole, of a Star, &c. and is measured in degrees, &c.

ELLIPSE, or ELLIPSIS, is one of the conic sections, commonly though improperly called an oval. The Ellipse is a plane figure made by cutting a cone by a plane obliquely through the opposite sides of it, or so as that the plane makes a less angle with the base than the side of the cone makes with it.

ELLIPSOID, is an elliptical spheroid, being the solid generated by the revolution of an ellipse, or a semi-ellipse, about either axis. When it is made about the minor, or less axis, it is an oblate ellipsoid, or as it is commonly called, an oblate spheroid. This is the figure of the Earth; and of all such of the heavenly bodies, as revolve about an axis.

ELIPTICITY, of the terrestrial spheroid, is the ratio, or the difference between the two semiaxes. It is usually expressed in terms of the greater, or radius of the Equator, that is, by what fractional part this exceeds the polar semidiameter, or semiaxes. The quantity of this ellipticity has been variously assigned by different mathematicians. Sir I. Newton, supposing the Earth of uniform density, gave $\frac{1}{230}$, Boscovich, from a mean of several admeasurements, stated it at $\frac{1}{248}$; Lalande, at $\frac{1}{337}$; Laplace, $\frac{1}{321}$; Sajour $\frac{1}{387}$; Carouge $\frac{1}{300}$; Kroft, $\frac{1}{297}$; and Playfair, from a theorem of Clairaut, applied to the heterogeneous spheroid, states it at $\frac{1}{304}$. This problem is of such a nature as not to admit of an exact solution; so that if we take a mean of those which are not deduced from the supposition of an uniform density, we shall be pretty near the truth. viz $\frac{1}{313}$.

ELONGATION, the distance of a planet from the Sun, as seen from the Earth; or the angle formed by two lines drawn from the Earth, the one to the Sun, and the other to the planet.

EMBER *Days*, are certain days observed by the Church at four different seasons of the year: viz, the Wednesday, Friday, and Saturday next after Quadragesima Sunday, or the first Sunday in Lent; after Whitsunday; after Holyrood, or Holycross, the 14th day of September; and after St. Lucy, the 13th day of December. The name it seems, is derived from embers, or ashes, which it is supposed the primitive Christians strewed on their heads at these solemn fasts.

EMBER *Weeks*, are those weeks in which the ember days fall. These ember-weeks are now chiefly noticed on account of the ordination of priests and deacons; because the canon appoints the Sundays next after the ember-weeks for the solemn accasion of ordination; though the bishops may ordain on any Sunday, or holiday.

EMERSION, the reappearance of the Sun, Moon or other planet, after having been eclipsed, or hid by the interposition of the Moon, Earth, or other body.

The immersions and emersions of Jupiter's first satellite, are particularly useful for finding the longitudes of places; the immersions being observed from Jupiter's conjunction with the Sun, till his opposition; and the emersions from the opposition till the conjunction. For about 15 days before, till 15 days after the conjunction Jupiter and his moons are too near the Sun to be observed.

EMERSION is also used when a star, after being hid by the Sun, begins to reappear, and emerges out of his rays.

EPACT, the excess of the solar month above the lunar synodical month; or the solar year above the lunar year of twelve synodical months; or of several solar years above as many periods, each consisting of twelve synodical months.

The Epacts are therefore either annual or menstrual.

Menstrual EPACTS, are the excesses, of the solar calendar month above the lunar month. Suppose, for example, it were new moon on the first day of Jannary: thence as the month of January contains 31 days,

And the lunar month - - 29*d*. 12*h*. 44*m*. 3*s*.
The menstrual Epact is - - 1 11 15 57

Annual EPACTS, are the excesses of the solar year above the lunar. Hence,

As the Julian solar year is - - 365*d*. 6*h*. 0*m*. 0*s*.
And the Julian lunar year - - 354 8 48 38
The annual Epact will be - - 10 21 11 22

that is, almost eleven days. Consequently the epact of two years is twenty-two days; of three years, thirty-three days; or rather three, since thirty days make an embolismic, or intercalary month; therefore adding still eleven, the epact of four years is fourteen days; and so of the rest, as in the following table, where it may be observed, that they do not become thirty or 0 again, till the nineteenth year; so that at the twentieth year the epact is eleven again; and hence the cycle of epacts expires with the golden number, or lunar cycle of nineteen years, and begins with the same again.

Table of Julian Epacts.

Golden Numb.	Epacts.	Golden Numb.	Epacts.	Golden Numb.	Epacts.
1	11	8	28	15	15
2	22	9	9	16	26
3	3	10	20	17	8
4	14	11	1	18	19
5	25	12	12	19	30 or 0
6	6	13	23		
7	17	14	4		

Again, as the new Moons are the same, or fall on the same day, every nineteen years, so the difference between the solar and lunar years is the same every nineteen years. And because the said difference is always to be added to the lunar year, to adjust or make it equal to the solar year; therefore the said difference respectively belonging to each year of the Moon's cycle, is called the epact of the said year, that is, the number to be added to the said year, to make it equal to the solar year; and upon this mutual relation between the cycle of the Moon and the cycle of the epacts, is founded this

Rule for finding the Julian epact, belonging to any year of the Moon's cycle.—Multiply the golden number, or the given year of the Moon's cycle, by eleven, and the product will be the epact, if it be less than 30; but if it be more than 30, then reject as many thirties as the product contains, and the remainder will be the epact.

To find the Gregorian Epact.—The difference between the Gregorian and Julian years being equal to the difference between the solar and lunar year, or eleven days, therefore the Gregorian epact for any year is the same with the Julian epact for the preceding year; and hence the Gregorian epact will be found, by subtracting one from the golden number, multiplying the remainder by eleven, and rejecting the 30s; which rule will serve till the year 1900; but after that year, the Gregorian epact will be found by this rule. Divide the centuries of the given year by 4; multiply the remainder by 17; and to this product add 43 times the quotient, as also the number 86, and divide the whole sum by 25, reserving the quotient: next multiply the golden number by 11, and from the product take the reserved quotient; so shall the remainder, after rejecting the 30s contained in it, be the epact sought. The following table contains the golden numbers, with their corresponding epacts, till the year 1900.

Golden Numb.	*Epacts.*	*Golden Numb.*	*Epacts.*	*Golden Numb.*	*Epacts.*
1	0	8	17	15	4
2	11	9	28	16	15
3	22	10	9	17	26
4	3	11	20	18	7
5	14	12	1	19	18
6	25	13	12		
7	6	14	23		

EPHEMERIS, EPHEMERIDES, tables calculated by astronomers, shewing the present state of the heavens every day at noon; that is, the places in which all the planets are found at that time, differing but little from an astronomical almanac. There are now published such ephemerides by the Academies of several nations; but that which is in most esteem for its accuracy and use in finding the longitude, is the Nautical Almanac, or Astronomical Ephemeris, published in England by the Board of Longitude, under the direction of the Astronomer Royal, which commenced with the year 1767, by Dr. Maskelyne: besides many other smaller ones by Partridge, Parker, White, &c. the last mentioned seems to be the best, and most useful companion to the globes.

EPIPHANY, a christian festival, otherwise called the Manifestation of Christ to the Gentiles, observed on the 6th of January, in honour of the appearance of Christ to the three magi or wise men, who came to adore him and bring him presents.

EPOCH, or EPOCHA, a term or fixed point of time, whence the succeeding years are numbered or reckoned.

Different nations make use of different epochs. The Christians chiefly use the epoch of the nativity or incarnation of Jesus Christ; the Mahometans, that of the Hegira; the Jews, that of the Creation of the World, or that of the Deluge; the ancient Greeks that of the Olympiads; the Romans, that of the Building of their City; the ancient Persians and Assyrians, that of Nabonassar; &c. The doctrine and use of epochs is of very great extent in Chronology. To reduce the years of one epoch to those of another, *i. e.* to find what year of one corresponds to a given year of another; a period of years has been invented, which commencing before all the known epochs, is the common receptacle of them all, called the Julian period. To this period all the epochs are reduced; that is, the year of this period when each epoch commences is so determined; that adding the given year of one epoch to the year of the period corresponding with its rise, and from the sum taking the year of the same period corresponding to the other epoch, the remainder is the year of the required epoch.

EPOCH *of Christ*, is the common epoch throughout Europe, commencing at the supposed time of his nativity, December 25; or rather, according to the usual account from his circumcision, or first of January. The author of this epoch was an abbot of Rome, one Dionysius Exiguus, a Scythian, about the year 507 or 527. Dionysius began his account from the conception or incarnation, usually called the Annunciation, or Lady-day; which method obtained in the dominions of Great Britain till the year 1752, before which time the Dionysian was the same as the English epoch: but in that year the Gregorian calender having been admitted by act of parliament, they now reckon from the first of January, as in the other parts of Europe, except in the court of Rome, where the epoch of the incarnation still obtains for the date of their bulls.

EQUATED *Anomaly.* See Anomaly.

EQUATION, a quantity or arc, to be added to, or subtracted from another quantity or arc, in order to correct it.

EQUATION, *of Time*, denotes the difference between mean and apparent time, or the reduction of the apparent unequal time, or motion of the Sun or a planet, to equal and mean time, or motion. Apparent time is that which takes its beginning from the passage of the Sun's centre over the meridian of any place. The Sun's motion in right ascension is affected, 1st, by his unequal motion in the Ecliptic; 2d, by the obliquity of the Ecliptic; and 3d, by the Nutation of the Earth's axis. The first of these inequalities is occasioned by the orbit of the Earth being an ellipsis; the second by the Ecliptic crossing the circles of declination in variable angles, and also by those circles approaching nearer to each other as they approach the poles; the third is because the nutation brings the Sun nearer to the pole, or throws him farther off; and hence it is evident that the nearer he is to the poles, the nearer those circles of declination are together; and consequently the Sun passes over any two of them in less time than he would do if nearer the Equinoctial: And when the nutation diminishes the declination, the Sun runs down the right ascension more slowly. The rule given by the late Royal Astronomer, Dr. Maskelyne, in the Philosophical Transactions, vol. 54, p. 336, is, "Take the difference between the Sun's true right ascension and his mean longitude, corrected by the equation of the equinoxes in right ascension, and turn this into time by allowing 1*m.* of time to 15′ of right ascension.

EQUATOR, a major circle of the Earth, equally distant from its two poles, and dividing it into two equal parts, or hemispheres, the northern and southern. The circle in the heavens conceived directly over the Equator, is the Equinoctial.

EQUINOCTIAL, a major circle in the heavens under which the terrestrial equator moves in its diurnal motion. The Equator is fixed with respect to the Earth, but the Equinoctial is constantly moving in the heavens, on account of the nutation and precession.

EQUINOCTIAL *points*, are the two opposite points where the Ecliptic and Equinoctial cross each other; the one point being in the beginning of Aries, and called the vernal point, or vernal equinox; and the other in the beginning of Libra, and called the Autumnal point, or Autumnal equinox. It is found by observation, that the equinoctial points are continually moving towards the west. This motion is not uniform, but at a mean it is about 50.2″ every year.

EQUINOXES, the times when the Sun enters the equinoctial points; that is, about the 21st of March, and twenty-second of September. As the Sun's motion is unequal, being sometimes faster and sometimes slower, and as the Sun moves slowest when he is farthest from the Earth, and this happens in the summer; hence he remains about eight days longer on the north side of the Equinoctial than on the south side: but as the line of the apsides is moving eastward, the time will come when this difference will vanish, and after that, it will increase on the other side.

EQUINUS *Barbatus*, a kind of comet. See Hippeus.

EQUULEUS, EQUICULUS, and EQUUS *Minor*, *Equisectio*, the head and fore part of a little horse, one of the 48 old constellations, in the northern hemisphere, situated on the W. of Pegasus, and said to be his brother; it contains ten stars; four of which are of the fourth magnitude, and the rest smaller.

EQUULEUS *Pictorius*, the painter's Easel, is a constellation of the southern hemisphere containing eight small stars, situated by the keel of Argo Navis; and has its name from the machine, or stand, that the painters stretch their canvas on.

ERIDANUS, *the river*, a constellation of the southern hemisphere, and one of the 48 old asterisms. This is the ancient name of the River Po in Italy, and was made a constellation of, for receiving Phaëton, the son of Sol and Clymene, who asked the guidance of his father's chariot for one day, as a proof of his divine descent; but being unskilful he set the world on fire; for which he was struck by Jupiter with a thunderbolt into the river Po. This constellation winds from Orion, by Cetus, Phœnix, &c. it contains 84 stars, viz. one of the first magnitude, named Achernar, one of the second, eight of the third, and twenty-one of the fourth.

ERRATIC, an epithet applied to the planets, because they move from one part of the heavens to another.

ESTIVAL *occident*, *orient*, or *solstice.*

EVECTION, that equation of the Moon's motion, which is proportional to the sine of double the distance of the Moon from the Sun, diminished by the Moon's Anomaly. It is the greatest of all the Moon's Equations except that of the centre.

EXCENTRIC *place of a planet*, in its orbit, is that in which it appears as seen from the Sun.

EXCENTRIC *place in the ecliptic*, is the point of the Ecliptic to which the planet is referred as viewed from the Sun; and which coincides with the heliocentric longitude.

EXCENTRICITY, is the distance between the Sun and the centre of the elliptical orbit of a planet; this is called the simple, or single excentricity. The distance between the foci of the elliptic orbit is called the double excentricity.

EXTRA *constellary stars*, such as are not properly included in any constellation.

FALCATED, one of the phases of the planets, commonly called horned. Astronomers say, the Moon, or any planet is falcated, when the enlightened part appears in form of a crescent, like a sickle, or reaping hook, which by the Latins is called falx.

FASCIÆ, certain stripes or rows of bright parts, observed on the bodies of some of the planets, like swathes, bands, or belts; especially on the planet Jupiter. The belts of Jupiter, are more lucid than the rest of his disc, and are terminated by parallel lines. They are sometimes broader and sometimes narrower; and do not always possess the same part of the disc. M. Huygens also observed a very large one in Mars, in the year 1656; but it was darker than the rest of the disc, and occupied the middle part.

FEBRUARY, the second month in the year, containing twenty-eight days for three years, and every fourth year twenty-nine days. These years being called Bissextile, or leap-years; and return every fourth year, except at the commencement of a new century, which are not Bissextile unless the number be divisible by 400, or the two left hand figures by 4; so 1800 cannot be divided by 400, nor 18 the number of centuries by 4, nor will 1900 be so, but 2000 will be so, because 20 may be divided by 4 without a remainder.

In the first ages of Rome, February was the last month of the year, till the Decemviri made an order that February should be the second month, and come after January.

FIXED *stars*, are such as constantly retain the same distance and position with respect to each other, or very nearly so; by which they are contradistinguished from erratic or wandering stars, which are continually varying their situation and distance. The fixed stars only are properly called stars; the other celestial bodies falling under the denomination of planets or comets.

FOMAHAUT, or FOMALHAUT, a star of the first magnitude in the water of the constellation Aquarius, or in the mouth of the southern fish.

FORNAX CHEMICA, the Chemical Furnace, a constellation of the southern hemisphere, on the south of Cetus, containing fourteen stars, two of which are of the third magnitude, and the rest smaller.

FRIDAY, the 6th day of the week, so called from Friga, a goddess worshipped by the Saxons on this day. It is a fast day in the Church of England, in memory of Christ's Crucifixion, except when Christmas-day falls on a Friday, which is always a festival.

Good FRIDAY, the Friday next before Easter, representing the day of our Saviour's crucifixion.

FRIGID *zone*, the space about either pole of the Earth, contained within the arctic and antarctic circles: the breadth of this zone varies with the change in the obliquity of the ecliptic.

GALAXY, or *milky-way*, or *via lactea*, that long, whitish, luminous track, which appears to encompass the heavens like a girdle; and which is easily seen in a clear night. It is of a considerable, though unequal breadth; being also in some parts double, but in other single.

Many ridiculous opinions prevailed concerning this appearance before the invention of telescopes; but since that time Astronomers have found by directing a good telescope to any part of the milky-way, an innumerable multitude of small stars, where before was only observed a confused whiteness, arising from the assemblage and union of their joint light.

GEMINI, the Twins; a constellation, or sign of the Zodiac, the third in order, representing Castor and Pollux; and it is marked thus, ♊. This constellation was anciently depicted by a couple of young kids, by the Egyptian and eastern nations, as denoting that part of spring, when those animals appear; but the Greeks altered them to two Children, which some of them represented by Castor and Pollux, some by Hercules and Apollo, and others by Triptolemus and Jason; but the Arabians afterwards changed them into two peacocks, their religion not allowing them to paint or draw any human figure. Sir Isaac Newton thought, that the figures had some reference to the Argonautic expedition. The ancients attributed to every sign of the Zodiac one of the principal deities for its tutelary power. Phœbus had the care of Gemini, and thence arose all the jargon of astrologers about the agreement of the Sun with this constellation. This asterism contains 85 stars, viz, one of the 1st mag. two of the 2nd, four of the 3d, six of the 4th, &c. The chief star in Gemini, is Castor, of the first magnitude.

GEOCENTRIC, is said of a planet, or its orbit, to denote its having the Earth for its centre. The Moon alone is properly geocentric. And yet the motions of all the planets may be considered in respect to the Earth, or as they appear from the Earth, and thence called their geocentric places.

GEOCENTRIC *place*, of a planet, is the place where it appears to us, from the Earth; or it is a point in the ecliptic, to which a planet, seen from the Earth, is referred.

GEOCENTRIC *Latitude*, of a planet, is its latitude as seen from the Earth; or the inclination of a line, connecting the planet and the Earth, to the plane of the Ecliptic. Or it is the angle which the said line makes with a line drawn to meet a perpendicular let fall from the planet to the plane of the ecliptic.

GEOCENTRIC *Longitude*, of a planet, is the distance measured on the Ecliptic, in the order of the

signs, between the geocentric place and the first point of Aries.

GEORGIUM SIDUS, see Herschel, and Uranus.

GIBBOUS, is used for the shape of one state of the enlightened part of the Moon, being that in which she appears more than half full or enlightened, which is the time between the first quarter and the full Moon, and from the full Moon to the last quarter; appearing then gibbous, that is, bunched out, or convex on both sides of the enlightened part; as contradistinguished from the state when she is less than half full, when she is said to be horned, or a crescent.

GOLDEN *Number*, is the particular year of the Metonic or lunar cycle. See lunar cycle.

To find the Golden Number.— Add 1 to the given year, and divide the sum by 19, and what remains is the Golden number. Thus the Golden Number for the year 1818 is 14; as by the operation in the margin.

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1818
   1
19)1819(95
 171
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 109
  95
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Golden No. 14

GREATER BEAR, one of the constellations in the northern hemisphere. See Ursa Major.

GREAT CIRCLES, see Major circles.

GREGORIAN CALENDAR, so called from Pope Gregory the 13th, is the new or reformed calendar, shewing the new and full Moons, with the time of Easter, and the other moveable feasts depending on it, by means of epacts disposed through the several months of the Gregorian year.

GREGORIAN *Epoch*, is the epoch or time, from which the Gregorian calendar, or computation took place. This began in 1582; so that the year 1818 is the 236th of this epoch.

GREGORIAN *year*, the new account, or new style, introduced on the reformation of the calendar, by Pope Gregory the 13th in the year 1582, and from whom it took its name. This was introduced to reform the old, or Julian year, established by Julius Cæsar, which consisted of 365 days 6 hours, that is three years of 365 days each, and the fourth year of 366 days. But as the mean tropical year consists only of 365d. 5h. 48m. 48s. the former lost 11 min. 12 sec. every year, which in the time of Pope Gregory had amounted to 10 days, and who, by adding those 10 days, brought the account of time to its proper day again, and at the same time enacted, that every century after, except when divisible by 4, a day more should be added, thereby making the years of the complete centuries, viz. the 17th, 18th, 19th, 21st, &c. which are not exactly divisible by 4, to be common years of 365d. each, instead of leap years of 366 days.

This computation was not introduced into the account of time in England, till the year 1752, when the Julian account had lost 11 days, and therefore the 3d of September was in that year, by act of parliament accounted the 14th, thereby restoring the 11 days which had thus been omitted.

GRUS, the Crane, one of the new constellations of the southern hemisphere, containing 13 stars, viz. one of the 2nd mag. two of the 3d, two of the 4th, &c. Grus lies S. of Aquarius, and Pisces Australis.

GUARDS, a name that has been sometimes applied to the two stars nearest the north pole; being in the hind part of the chariot, at the tail of Ursa Minor, or less bear, one of them being also called the pole star.

HARDING, the same as Juno.

HELIACAL, something relating to the Sun.

HELIACAL *rising*, of a star or planet, is when it rises with, or at the same time as the Sun. And heliacal setting, the same as the setting with the Sun. Or, a star rises heliacally, when after it has been in conjunction with the Sun, and so invisible, it gets at such a distance from him, as to be seen in the morning before the Sun's rising. And it is said to set heliacally, when it approaches so near the Sun as to be hid by his beams. So that in strictness, the heliacal rising and setting are only an apparition and occultation.

HELICE, *Major* and *Minor:* the same as the Bears.

HELIOCENTRIC *Place of a Planet*, is the place in which a planet would appear to be when viewed from the Sun; or the point of the Ecliptic, in which a planet viewed from the Sun would appear to be. And therefore the heliocentric place coincides with the longitude of a planet viewed from the Sun.

HELIOCENTRIC *Latitude of a Planet*, is the inclination of the line drawn between the centre of the Sun and the centre of a planet, to the plane of the Ecliptic. The greatest heliocentric latitude is equal to the inclination of the planet's orbit to the plane of the Elicptic.

HEMISPHERE, one half of the mundane sphere. The Equinoctial divides the sphere into two equal parts, called the northern and southern hemispheres. The Horizon divides it into upper and lower; and the brazen meridian on an artificial globe divides the sphere into two equal parts, called the eastern and western hemispheres.

HEMISPHERE is also used for half the celestial sphere on a plane; being more frequently called a planisphere: or for a map or projection of half the terrestrial globe, through which are drawn the Equator and (though very improperly) the Ecliptic, with the principal minor circles. There can be no reason for drawing the Ecliptic on maps: and the only ex-

cuse which can be made for placing it on terrestrial globes is, because it enables us to work a few problems in the absence of a celestial globe; which problems properly belong to the latter.

HENIOCHAS, or HENIOCUS, the same as Auriga.

HERCULES, a constellation of the northern hemisphere, and one of the 48 old ones mentioned by ancient writers. He is represented in a kneeling posture, with the skin of the Nemæan lion over his shoulders, holding a club in his right hand, and in his left Cerberus. This asterism contains 113 stars, viz. seven of the third magnitude, seventeen of the fourth, &c. The largest star in Hercules is called Ras Algethi.

HERSCHEL. See Uranus.

HESPER, or HESPERUS, the evening star; an appellation given to Venus when she follows, or sets after the Sun. The word is from the Greek, and is supposed to have been originally the proper name of a man, brother to Atlas, and father or grandfather of the Hesperides. Diodorus relates, that Hesperus, distinguished by his piety, justice, and generosity, having ascended to the top of Mount Atlas, the better to observe and contemplate the stars, was carried away by an impetuous wind, and never returned more; and that hence he was fabled to have been changed into this star.

HETEROSCII, are such inhabitants of the Earth as have their shadows at noon projected always the same way with regard to themselves, or always contrary ways with regard to each other. Thus an inhabitant of London always has his noon day shadow projected north of him, and an inhabitant of Botany Bay will always have his shadow at noon projected south of him: and these two persons are Heteroscii to each other, having their shadows projected contrary ways at all times of the year.

HIRCUS, the same as *Capella*. This name is used by some writers for a comet, encompassed with a mane, seemingly rough and hairy.

HOLY *Thursday*, otherwise called Ascension-Day, being the 39th day after Easter-Sunday, and kept in commemoration of Christ's ascension into heaven.

HOLY *Rood*, or *Holy Cross*, a festival kept on the 14th of September, in memory of the exaltation of the holy cross.

HOLY *Week*, is the same as Passion-Week.

HORARY *Circles*, hour-lines, marking the hours, or circles drawn at the distance of an hour asunder.

HORARY *Motion*, is the space moved in an hour.

HORIZON, a major circle dividing the heavens into two equal parts, called the upper and lower hemispheres.

HORIZON, *Rational*, *True*, or *Astronomical*, called also simply and absolutely the horizon, is a major circle, having its plane passing through the Earth's centre, and its poles are the zenith and nadir.

HORIZON, *Apparent*, *Sensible*, or *Visible*, is a minor circle of the sphere, parallel to the rational horizon, dividing the visible part of the sphere from the invisible, the plane of which touches the spherical surface of the Earth. These two horizons, though their planes are at the distance of a semidiameter of the Earth, will appear to coincide when continued to the sphere of the fixed stars; because the Earth's radius is but a point when viewed from that great distance: that is, the angle of parallax vanishes.

HOROLOGIUM, the Clock, a new constellation of the southern hemisphere, containing 12 stars, all under the 4th magnitude. It is situated between Phœnix and the S. pole of the Ecliptic.

HYADES, are 7 stars in the face of Taurus, famous among the poets for bringing rain. The principal of them is in the Bull's left eye, by the Arabs called Aldebaran. The poets feign them to have been the daughters of Atlas and Pleione; and we have the names of six of them, viz. Eudora, Ambrosia, Prodice, Coronis, Phillito, and Poliso; others add a seventh, viz. Thione. Their brother Hyas being torn in pieces by a lioness, they wept his death with such vehemence, that the gods, in compassion to them, translated them into the skies, and placed them in the Bull's forehead; where, as they say, they continue to weep; this constellation being supposed to presage rain. Others represent the Hyades as Bacchus's nurses, and the same with Dodonides, who fearing the resentment of Juno, and flying from the cruelty of king Lycurgus, were translated by Jupiter into the skies. It is probable that these pretended Hyades, which word is a Greek term signifying rainy, are merely poetical personages, whose names were given to certain stars discovered by Atlas.

HYDRA, the water serpent, a southern constellation, extending from Cancer by Crater, Corvus, &c. to the head of Centaurus; it contains 60 stars, the chief of which is named Alphard, or Cor Hydra, of the 2nd mag. three of the 3d, twelve of the 4th, &c.

HYDRUS, a southern constellation including 10 stars, two of which are of the 2nd mag. one of the 3d, one of the 4th, &c.

HYPOTHESIS, a kind of system laid down from our own imagination, by which to account for some phenomenon or appearance of nature. Hypothesis is more particularly applied to the several systems of the heavens; or the divers manners in which different astronomers have supposed the heavenly bodies to be ranged, or moved.

ST. JAMES's *Day*, a festival in the calendar, ob-

served on the 25th of July, in honour of St. James the apostle.

JANUARY, the first month of the year, containing 31 days; so called by the Romans from Janus, one of their divinities, to whom they gave two faces; because, on the one side, the first day of this month looked towards the new year, and on the other towards the old one. January and February were introduced into the year by Numa Pompilius; the year of Romulus beginning with March.

IDES, in the Roman Calendar, a name given to a series of eight days in each month; which, in the full months, March, May, July, and October, commenced on the 15th day; and in the other months, on the 13th day; from thence reckoned backward, so as in those four months to terminate on the 8th day, and the rest on the 6th. These came between the calends and the nones. And this way of counting is still used in the Roman Chancery, and also in the calendar of the Breviary.

ILLUMINATIVE *Lunar Month*, the space of time that the Moon is visible between one conjunction and another.

IMMERSION, is when a star or planet comes so near the Sun, that it cannot be seen; being as it were enveloped, and hid in the rays of that luminary.

IMMERSION also denotes the beginning of an eclipse, or of an occultation, when the body, or any part of it, just begins to disappear, either behind the edge of another body, or in its shadow.

INCLINATION *of the Axis of the Earth*, is the angle it makes with the plane of the Ecliptic; or the angle contained between the planes of the Equinoctial and Ecliptic.

INCLINATION *of the orbit of a planet*, is the angle formed by the plane of the Ecliptic and the plane of the orbit of the planet.

INCLINATION *of a planet*, is the arch or angle comprehended between the Ecliptic and the place of the planet in its orbit.

INDICTION, or *Roman* Indiction, a kind of epoch, or manner of counting time, among the Romans; containing a cycle or revolution of 15 years. The Popes have dated their acts by the year of the Indiction, which was fixed to the first of January, A. D. 813, ever since Charlemagne made them sovereign; before that time, they dated them by the years of the emperors.

To find the Roman Indiction: Add 3 to the given year of Christ; divide the sum by 15, and the remainder after the division, will be the year of the Indiction: If there be no remainder, the Indiction is 15. The Indiction for 1818 is found to be 6, as in the margin.

1818	
3	
15)1821(121	
15	
32	
30	
21	
15	
Indiction, 6	

INDUS *et Pavo*, the Indian and Peacock, are southern constellations, situated on the antarctic circle, and are so near together or interwoven, that it appears most convenient to describe them together. These constellations contain 26 stars, viz. one of the 2nd mag. three of the 3d, four of the 4th, &c. the most brilliant one is in the eye of Pavo.

INFORMED *stars*, or informes *stellæ*, are such stars as have not been reduced into any constellation; otherwise called sporades. A great many of this kind were left by the ancient astronomers; but Hevelius and some others of the moderns have provided for the greater part of them, by forming new constellations.

INGRESS, the Sun's entrance into one of the signs; or the entrance of a planet upon the Sun's disc at the commencement of a transit.

INNOCENTS' *Day*, the 28th day of December, the day on which Herod caused all the children to be slain, in hopes Christ would be among them.

INTERCALARY *Day*, denotes the odd day inserted in the leap year, see Bissextile.

INTERSTELLAR, intervening between the stars; a word used by some, to express those parts of the universe, that are beyond the limit of our solar system. In the interstellar regions, it is supposed there are an infinite number of systems of planets, moving round the fixed stars, as the centres of their respective motions.

JULIAN *Calender*, is that invented by Julius Cæsar.

JULIAN *Epoch*, is that of the institution of the Julian reformation of the calendar, which began the 46th year before Christ.

JULIAN *period*, is a cycle of 7980 consecutive years, invented by Julius Scaliger, from whom it was named, though some say his name was Joseph Scaliger, and that it was called the Julian period, because he made use of Julian years. This period is formed by multiplying continually together the three following cycles, viz. that of the Sun of 28 years, that of the Moon of 19 years, and that of the Indiction of 15 years; so that this epoch, though but artificial or feigned, is of good use; in that every year within the period is distinguishable by a certain peculiar character; for the year of the Sun, Moon, and Indiction, will not be the same again till the whole 7980 years have revolved. Scaliger fixed the beginning of this period 764 years before the creation, or rather the period naturally reduces to that year, taking the numbers of the three given cycles as he then found them; and accounting 3950 years from the creation to the birth of Christ, this makes the first year of the Christian æra answer to the 4714th year of the Julian period; therefore, to find the year of this period answering to any proposed

year of Christ: *Rule,*—To the constant number 4713, add the given year of Christ, and the sum will be the year of the Julian period: thus, to 4713 add 1818, the sum 6531 is the year of this period for the year of Christ 1818. Hence the first revolution of this period will not be completed till the year of Christ 3267, after which a new revolution of the Julian period will commence.

JULIAN *Year*, is the old account of the year, established by Julius Cæsar, and consisted of 365¼ days. This year continued in use in all Europe, till it was superseded in most parts by the new or Gregorian account, in the year 1582. In England, however, it continued to be used till the year 1752, when it was abolished by Act of Parliament, and eleven days were added to the account, to bring it up to the new style. In Russia, the old or Julian year and style are still in use.

JULY, the 7th month of the year, consisting of 31 days. It was so named by Mark Anthony, from Julius Cæsar, who was born in this month.

JUNO, the name of a small planet, discovered by M. Harding, in the year 1804.

JUPITER, one of the exterior planets.

LACERTA, the Lizard, a constellation of the northern hemisphere, being one of the new ones formed by Hevelius; it contains 16 stars, three of which are of the 4th magnitude. It is situated between Cygnus and Andromeda.

LADY-*Day*, the 25th of March, being the Annunciation.

LAMMAS-*Day*, the 1st day of August; so called, according to some, because lambs are then out of season, as being too large. Others derive it from a Saxon word, signifying loaf-mass, because on that day our forefathers made an offering of bread prepared with new wheat. It is celebrated by the Romish Church in memory of St. Peter's imprisonment.

LAMPÆDIAS, a kind of bearded comet, resembling a burning lamp, being of several shapes; sometimes its flame runs tapering upwards like a sword, and at others it is double or treble pointed.

LATITUDE *of a place on the Earth's surface*, is its distance from the Equator, measured in degrees and minutes, on the meridian.

LATITUDE *of a star or planet*, is its distance from the Ecliptic, being an arch of a circle of latitude, reckoned from the Ecliptic towards its poles, either north or south. The Sun is said to have no latitude, but this is not strictly correct, because he has a motion about the common centre of gravity of the whole system; and again because the great orbit is described by the common centre of gravity of the Earth and Moon; hence both the Sun and Earth have latitude, except when they pass through the plane of the Ecliptic, alternately from one side to the other. All the Stars have their latitude, except any one chance to lie just in the plane of the Ecliptic, and the planets and comets are continually changing their latitudes, sometimes north, and sometimes south, passing through the plane of the Ecliptic from one side to the other; the points in which they cross or pass through being called the nodes of the planets, and in or near these points it is that they may appear to pass over the face of the Sun, or behind his body, viz. when they come both to this point, or near it, at the same time.

Circles of LATITUDE, are major circles passing through the poles of the Ecliptic, and consequently at right angles to it, as the Meridians are perpendicular to the Equator.

LATITUDE, *of the Moon, North ascending*, is when she proceeds from the ascending node towards her northern limit, or greatest elongation.

LATITUDE, *North descending*, is when the Moon returns from her northern limit towards the descending node.

LATITUDE, *South descending*, is when she proceeds from the descending node towards her southern limit.

LATITUDE, *South ascending*, is when she returns from her southern limit towards her ascending node. And the same is to be understood of the planets.

Heliocentric LATITUDE of a planet, is its latitude, or distance from the Ecliptic, such as it would appear from the Sun.

Geocentric LATITUDE of a planet, is its latitude as seen from the Earth. This, though the planet be in the same point of its orbit, is not always the same, but alters according to the position of the Earth, in respect to the planet. The latitude of a fixed star is altered only by the aberration of light, its own proper motion, and the displacement of the Ecliptic. The two last are very small, and are called secular variations, because the change in a year is too small to be observed.

Difference of LATITUDE on the Earth, is an arc of the meridian contained between the two places, if on the same meridian; and is the nearest distance between the two parallels when the places are in different longitudes. Parallax and Refraction of LATITUDE, see the substantives.

LEAP-YEAR, the same as Bissextile. This is so called because of leaping over a day in that year.

LEO, the Lion, a considerable constellation of the northern hemisphere, being one of the 48 old constellations, and the 5th sign of the Zodiac. It is marked thus ♌, as a rude sketch of the animal. The Greeks fabled that this was the Nemæan Lion, which had dropped from the Moon, but being slain by

Hercules, was raised to the skies by Jupiter, in commemoration of the dreadful conflict, and in honour of that hero. But the hieroglyphical meaning of this sign, so depicted by the Egyptians long before the invention of the fables of Hercules, was probably no more than to signify, by the fury of the lion, the violent heats occasioned by the Sun when he entered that part of the Ecliptic. This sign contains 95 stars, viz. two of the 1st mag. two of the 2nd, six of the 3d, thirteen of the 4th, &c. The chief star in this asterism is Regulus, or Cor Leonis. Denebalceed in the Lion's Tail, is likewise a brilliant star of the first magnitude; it is also called Denebola.

LEO *Minor*, the Less Lion, a constellation of the northern hemisphere, and one of the new ones. This constellation contains 53 stars, viz. one of the 3d mag. six of the 4th, &c. It is bounded on the N. by Ursa Major, E. by Coma Berenices, S. by Leo, and W. by Lynx.

Cor LEONIS, *Lion's heart*, a fixed star of the first magnitude, in the sign Leo; called also Regulus, Basilicus, &c.

LEPUS, *the Hare*, a constellation of the southern hemisphere, and one of the 48 old ones. The Greeks fabled, that this animal was placed in the heavens, near Orion, as being one of the animals which he hunted. But it is probable, their masters, the Egyptians, had some other meaning in this hieroglyphic. This constellation contains 19 stars, viz. three of the third magnitude, seven of the fourth, &c.

LIBRA, *the Balance*, is one of the 48 old constellations, and the seventh sign of the zodiac, being opposite to Aries, and marked like a part of a pair of scales: thus, ♎. The figure of the balance was probably given to this part of the Ecliptic, because when the Sun arrives at this part, which is at the time of the autumnal Equinox, the days and nights are equal, as if weighed in a balance. This sign contains 51 stars, viz. two of the second magnitude, one of the third, eight of the fourth, &c. The star that has the greatest lustre in Libra, is Zubeneschemali of the second magnitude, situated on the Ecliptic, in the southern scale.

LIBRATION, *of the Moon*, is an apparent irregularity in her motion, by which she seems to liberate, or waver about her own axis, at one time towards the east, and at another towards the west. Hence it is that some parts, near the Moon's western edge, at one time recede from the centre of the disc, while those on the opposite side approach nearer to it: and on the contrary, at another time the western parts are seen to be nearer the centre, and the eastern parts farther from it: by which means it happens that some of the parts, which were before visible, disappear behind the Moon, and afterwards return and appear again, on the nearer or visible side. This apparent motion is owing to her equable motion round her own axis, once in a month, in conjunction with her unequal motion in the perimeter of her orbit round the Earth. For if the Moon moved equably in a circle, having its centre in the centre of the Earth, while it turned on its axis in the precise time of its period round the Earth, then the plane of the same lunar meridian would always pass through the centre of the Earth, and the same face of the Moon would be constantly and exactly turned towards us: and this would obtain if the Earth and Moon described concentric circles about their common centre of gravity. But since these two bodies approach towards, and recede from this common centre of gravity, their angular velocity is not equable, and consequently the plane of no one meridian constantly passes through the Earth's centre.

The Libration of the Moon is of several kinds.

LIBRATION *in Longitude*, or a seeming vibrating motion according to the order of the signs of the Zodiac. This is occasioned by the Moon's unequal motion in her orbit.

LIBRATION *in Latitude*. In order to explain this, we will suppose two Moons similar and equal to each other, with their polar diameters, or axes parallel, or pointing to the same place in the celestial sphere; one placed at a certain number of degrees north of the Ecliptic; and the other as far south of it; on the same circle of celestial latitude, which circle we will suppose passes through the zenith of a spectator. Now as these moons have their axes parallel to each other, such spectator will see a portion of the under limb of that moon which is most elevated, and which cannot be seen on the lower margin of the other: and a part of the upper limb of the farther moon will be visible which cannot be seen by the spectator on the nearer. Now that part of the disc, which is visible on one, and does not appear on the other, is strictly the libration in latitude, and the quantity of this libration will vary according to circumstances.

LIBRATION *in declination*. Here we suppose the two moons placed on the same meridian, one many degrees above the other, and a spectator, suppose at Greenwich, observes these moons pass his meridian, at the same instant. Now it is plain that these moons, having their axes, &c. as before described, will exhibit portions of their under and upper margins in the same manner as described above, and those parts which appear on one, and do not on the other, form what may be called the Libration in declination.

LIBRATION *occasioned by the Sun*. In this fourth kind of libration it happens, that though another part of the Moon be not turned to the Earth, yet another

is illuminated by the Sun. The Moon's axis is nearly perpendicular to the Ecliptic, and for the better explanation of the thing, we will suppose it to be strictly so. Now it is evident that as the Moon passes through the plane of the Ecliptic to her most southern bounds, and back again to her most northern limit, her two poles will fall into the light and dark hemispheres alternately. For when she has reached her southern limit her north pole will be enlightened and her south pole will be in her dark hemisphere; and on the contrary when she has passed to the north side of the plane of the Ecliptic her north pole will have dipped into the dark hemisphere, and the other into the light. We have supposed the Moon's axis to be parallel to itself in all situations, and this will be the case very nearly. For as the Moon revolves on her axis but once in a month; her form must be nearly spherical, and consequently the nutation of her axis will be insensible.

LIBRATION *of the Earth.* Copernicus and other astronomers before the nutation was discovered, considered the axis of the Earth to remain perfectly parallel in all situations; and this parallelism they called libration.

LIMB, the outermost border or edge of the Sun or Moon; as the upper limb or edge; the lower limb, or edge; the preceding limb, or side; the the following limb.

LIMIT *of a planet*, has been sometimes used for its greatest heliocentric latitude.

LINE *of the Apses* or *Apsides*, the line joining the two apses, or the longer axis of the orbit of a planet.

LINE *of the Nodes*, that which joins the nodes of the orbit of a planet, being the common section of the plane of the orbit with the plane of the Ecliptic.

LIZARD, see Lacerta.

LONGITUDE, *of a Star, &c.* is an arch of the Ecliptic, counted from the beginning of Aries, to the place where it is cut by a circle perpendicular to it, and passing through the place of the star.

LUCIDA CORONÆ, a fixed star of the second magnitude, in the Northern Crown.

LUCIDA HYDRÆ, see Cor Hydræ.

LUCIDA LYRÆ, a bright star of the first magnitude, in the constellation Lyra.

LUCIFER, a name given to the planet Venus, when she appears in the morning before the Sun rises

LUMINARIES, a term used for the Sun and Moon, by way of eminence, for their extraordinary lustre.

LUNA, the Moon; which see.

LUNAR, something belonging to the Moon.

LUNAR *cycle*, see cycle.

LUNAR *Month*, is either periodical, synodical, or illuminative: which see; also month.

LUNAR *Year*, consists of 354 days, or 12 synodical months, of 29½ days each. See Year. In the early ages, the lunar year was used by all nations; the variety of course being more frequent and conspicuous in this planet, and consequently better known to men, than those of any other. The Romans regulated their year, in part, by the Moon, even to the time of Julius Cæsar. The Jews also had their lunar month and year.

LUNATION, the period or time between one new Moon and another; it is also called the synodical month.

LUPUS, the Wolf, a southern constellation joined to the Centaur, and bounded on the north by Scorpio; it contains 24 stars, viz. 4 of the 3d mag. 5 of the 4th, &c.

LYNX, a constellation of the northern hemisphere composed by Hevelius out of the unformed stars. Lynx contains 44 stars, none of them exceeding that of the 4th magnitude. It is situated between Ursa Major and Auriga.

LYRA, the Harp: this constellation is of the northern hemisphere, and is sometimes drawn as a harp, sometimes like a vulture, and originally like a tortoise. It is said that this is the Lyre, or harp, which Orpheus received of Apollo, when he went to the infernal regions in search of his wife Eurydice; and that it was at his death made a constellation. It is bounded on the N. by Draco, on the E. by Cygnus, and on the W. by Hercules. Lyra contains 21 stars, viz. one of the 1st mag. two of the 3d, one of the 4th, &c. The most brilliant star in Lyra is Vega.

MACHINA PNEUMATICA, the air pump, is a southern constellation, situated on the N. of Argo Navis, containing 24 stars, the largest not exceeding that of the 5th magnitude.

MACULÆ, dark spots appearing on the surfaces of the Sun and Moon, and sometimes on the planets. The solar spots are of an irregular figure, and very changeable. It is by the rotation of these spots that the rotation of the Sun on his axis is ascertained. But as those spots vary in magnitude, and sometimes two form themselves into one, may they not have local motion, and thereby in some measure deprive us of the advantage of ascertaining the exact time of his revolution?

MAGELLANIC CLOUDS, whitish appearances, like clouds, seen in the heavens near the south pole, and having the same apparent motions as the stars. They are three in number, two of them near each other. The largest is the farthest from the south pole, but the other two are only about 11 degrees. Mr. Boyle conjectures that if those clouds were seen through a good telescope, they would be found to contain multitudes of small stars like the milky-way. But it seems that as Doctor Herschel's large one has not been able to determine the point,

we must be satisfied with conjecture. Doctor Herschel thinks rather that Nebulæ are often owing to a self luminous fluid.

MAJOR CIRCLES, the same as great circles.

MARCH, MARTIUS, the third month of the year, according to the common way of counting, and consists of 31 days. Among the Romans, March was the first month; and in some ecclesiastical computations, that order is still preserved. In England, before the alteration of the stile, March was the first month in order, the year always commencing with the 25th day of the month. It has been said that it was Romulus who first divided the year into months; to the first of which he gave the name of his supposed father Mars. But according to Ovid it is otherwise.

MARS, one of the primary planets, and the first of the exterior ones. In the Philosophical Transactions for 1781, Doctor Herschel gave a series of observations on the rotation of this planet about its axis, from which he concluded that one mean sidereal rotation was between 24*h*. 39*m*. 5*s*. and 24*h*. 39*m*. 22*s*.; and in the Trans. for 1784, the same Gentleman has given a paper, on the remarkable appearances at the polar regions of the planet Mars, the inclination of its axis, the position of its poles, and its spheroidal figure; with some hints relating to its real diameter and atmosphere. He also observed several remarkable bright spots near both poles, which had a small motion.

MAUNDAY THURSDAY, is the Thursday in passion week; which was called Maunday or Mandate Thursday, from the command which Christ gave to his Apostles to commemorate him in the Lord's Supper, which he instituted on this day; or from the new commandment which he gave them to love each other, after he had washed their feet as an example of love and humility.

MAY, MAIUS, the fifth month in the year, reckoning from our first, but the third according to the ancient Romans. It was called Maius by Romulus, in respect to the senators and nobles of his city, who were named Majores; as the following month was called Junius, in honour of the youth of Rome, who served him in the war.

MEAN *conjunction* or *opposition*, is when the mean place of the Sun is in conjunction or opposition with the mean place of the Moon in the Ecliptic.

MEAN *Anomaly* of a planet, is an angle which is always proportional to the time of the planet's motion from the aphelion or perihelion, or proportional to the area described by the radius vector; that is, as the whole periodic time in one revolution of the planet, is to the time past the aphelion or perehelion, so is 360° to the mean anomaly.

MEAN *Distance*, of a planet from the Sun, is an arithmetical mean between the planet's greatest and least distances; and this is equal to the semitransverse axis of the elliptic orbit in which it moves, or to the right line drawn from the focus to the extremity of the conjugate axis of the same.

MEAN *Motion*, is that by which a planet is supposed to move equably in its orbit; and is always proportionable to the time.

MEAN *Time*, or equal time, is that which is measured by an equable motion, as a well going clock; as distinguished from apparent time, arising from the unequal motion of the Earth in its orbit.

MERCURY, one of the interior planets, and the nearest to the Sun. This and the other planets are described in the account hereafter given of the solar system.

MERIDIAN, is a major circle passing through the north and south poles of the Earth, and crossing the Equator at right angles.

Celestial MERIDIAN, is the same as a circle of declination.

First MERIDIAN, is that from which the rest are counted, reckoning both east and west, and is the beginning of longitude.

MERIDIAN altitude of a celestial object, is its altitude, when on the meridian of the place where it is observed.

METONIC Cycle, the same as golden number.

MICHAELMAS, the feast of St. Michael the archangel; held on the 29th of September.

MICROSCOPIUM, the Microscope, a southern constellation, containing a few small stars, none of which are greater than the 5th magnitude. It is situated on the E. of Centaurus.

MID HEAVEN, *Medium Cœli*, is that point of the Ecliptic which culminates, or is on the meridian at any time.

MIDSUMMER-DAY, is held on the 24th of June, and is the same day as the nativity of St. John the Baptist.

MILKY-WAY, the same as Galaxy, which see.

MINUTE, is the 60th part of a degree, or of an hour.

The minutes of a degree are marked with the acute accent; the seconds by two; the thirds by three, &c. The minutes, seconds, thirds, &c. in time, are sometimes marked the same way; but to avoid confusion, the better way is to use the initials of the words, as under,

Of an arch.	*Marked.*	Of an hour.	*Marked.*
Degrees - - - -	°	Minutes - - - -	*m*
Minutes - - - -	′	Seconds - - - -	*s*
Seconds - - - -	″	Thirds - - - - -	*t*
Thirds - - - - -	‴		

MONDAY, the second day in the week.

MONOCERAS, the Unicorn, a constellation of the southern hemisphere, formed by Hevelius. This

asterism contains 31 stars, viz. ten of the 4th magnitude, and the rest smaller; therefore Monoceras may be best known by its following Orion, and also by its being between Canis Major and Canis Minor.

MONS MÆNALUS, a constellation of the northern hemisphere, lying on the north of Virgo and Libra, it contains 11 stars, of which one is of the 4th mag. and the rest smaller; it is depicted on the globe like a mountain, and has its name from Mænalus, a mountain of Arcadia in the Peloponnesus, in Greece, sacred to the god Pan; and named after Mænalus, son of Lycaon, a king of Arcadia.

MONS MENSÆ, the table mountain, a constellation of the southern hemisphere, situated between the south pole of the world and that of the Ecliptic; containing 30 small stars.

MONTH, the twelfth part of the year, and takes its name from the Moon, by whose motions it was formerly regulated; being properly the time in which the Moon runs through the Zodiac. The lunar month is either illuminative, periodical, or synodical.

Illuminative MONTH, is the interval between the first appearance of one new moon and that of the next following. The times between the new moons vary with circumstances.

Lunar Periodical MONTH, is the time in which the Moon moves through the Zodiac.

Lunar Synodical Month, called also a lunation, is the time between two conjunctions of the Moon with the Sun.

Solar MONTH, is the time in which the Sun appears to move through one entire sign of the Ecliptic.

MOON, one of the secondary planets, and the constant attendant of the Earth, both in its journey round the Sun in its annual orbit; and about the common centre of gravity in their menstrual orbits. This point about which they move is as much nearer to the Earth's centre than to the Moon's as the mass of the greater body exceeds that of the less.

Some philosophers of great repute say, that the curious circumstance of the Moon's turning round her axis precisely in the same time that she moves round the Earth is owing to her form: for supposing her at first to have been a fluid like the sea, in that case the Earth's attraction would have drawn up the fluid in such a way, that the Moon would have put on the form of a prolate ellipsoid, the longer diameter of which continued would pass through the Earth's centre; and hence it happens that we always see the same face of the Moon; for she cannot rest in any other position, but always conforms herself to this situation. This is very correct reasoning if the Moon were a fluid; or indeed a very soft substance capable of being drawn up by the action of the Earth. But supposing this to have been the case, how does it come to pass that the Moon's motion about her axis is uniform, and that her libration answers to her unequal motion in her orbit? Should it not follow that this miniscus which is raised up next the Earth, should always continue to point to the primary, and thereby destroy the libration of the Moon in longitude? If we suppose the Moon to be as our telescopes lead us to believe, covered with hills, valleys, pits, seas, volcanoes and other diversities, it will be difficult to conceive how she could have put on this spheroidal form. It is also said that the action of the Earth is sufficient to raise tides on her to the height of 90 feet; this might have been the case had the Moon been covered with water to a considerable depth; or if her seas were very large; but we do not perceive this to be the case. We know that the Moon produces little or no effect on comparatively small collections of water on our globe, and this will always be the case when all the parts of the collection of water are at nearly the same distance from the attracting body. It is also a query whether the Earth could have the power of giving her a rotary motion even if she were covered with water to the depth of a mile; for except this aqueous miniscus could be fixed to the moon, and act as a lever, the waters would be transported from one part to another, without communicating any motion to the more solid body of the Moon.

The figure of the Moon's orbit about the Earth is nearly that of an ellipse, but the true paths of both the Earth and Moon are made up of a number of epicycloids described upon the great orbit, and are all concave to the Sun; but they do not make loops, as some have supposed.

MUSCA, the Fly, or Bee, a constellation of the northern hemisphere, on the N. of Aries, containing six stars; one of which is of the 3d mag. two of the 4th, &c.

MUSCA *Australis*, the southern Fly, or Bee, situated S. of Crux, near the Antarctic Circle; it contains 4 stars of the 4th magnitude.

NADIR, that point of the heavens directly under our feet, or opposite to the Zenith, which is directly over our heads. The Zenith and Nadir are the two poles of the horizon, each being 90 degress distant from it.

The Sun's NADIR, is the axis of the cone projected by the shadow of the Earth: so called, because that axis being produced to the heavens, gives a point in or very near the Ecliptic directly opposite to the Sun.

NODES, the two opposite points where the orbit of a planet passes through the plane of the Ecliptic.

NONAGESIMAL, or NONAGESIMAL *degree*, is the highest point, or 90th degree of the Ecliptic, reckoned from its intersection with the horizon at any time; and its altitude is equal to the angle that

the Ecliptic makes with the horizon at their intersection, or equal to the distance of the Zenith from the pole of the Ecliptic.

NORMA *vel Quadra Euclidis*, Euclid's square and rule, a constellation of the southern hemisphere, on the south of the tail of Scorpio, containing 12 stars, all less than the fourth magnitude.

NORTH *Star*, called also the pole-star, is the last in the tail of Ursa Minor, or the less bear.

NORTHERN *Signs*, are those six that are on the north side of the Equinoctial, beginning with Aries.

NOVEMBER, the 11th month in the Julian year, but the ninth in the year of Romulus.

NUCLEUS, the body or head of a comet.

NUMBER *of Direction*, some one of the 35 numbers between the Easter limits, or between the earliest and latest day on which it can fall, *i. e.* between March 22 and April 25, which are 35 days; being so called, because it serves as a direction for finding Easter for any year; being the number that expresses how many days after March 21, Easter day falls. Thus, Easter-day falling as in the first line following, the number of direction will be as on the second line.

March.	*April.*
Easter-day, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31,	1, 2, &c.
No. of dir. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,	&c.

and so on till the number of direction on the lower line be 35, which will answer to April 25, being the latest that Easter can happen. Therefore add 21 to the number of direction, and the sum will be so many days in March for the Easter-day: if the sum exceed 31, the excess will be the day of April.

To find the NUMBER *of Direction.*—Enter the following table (which is adapted to the new style), with the dominical letter on the left hand, and the golden number at the top, then where the columns meet is the Number of Direction for that year.

G. N.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Dom. Let.																			
A	29	19	5	26	12	33	19	12	26	19	5	26	12	5	26	12	33	19	12
B	27	13	6	27	13	34	20	13	27	20	6	27	13	6	20	13	34	20	6
C	8	14	7	21	14	35	21	7	28	21	7	28	14	7	21	14	28	21	7
D	19	15	8	22	15	29	22	8	29	15	8	29	15	1	22	15	29	22	8
E	30	16	2	23	16	20	23	9	30	16	9	23	16	2	23	9	30	23	9
F	24	17	3	24	10	31	24	10	31	17	10	24	17	3	24	10	31	17	10
G	25	18	4	25	11	32	25	11	32	18	4	25	18	4	25	11	32	18	11

Thus, for the year 1818, the dominical letter being D, and the golden number 14; on the line of D, and under 14, is 1 for the number of direction. To this add 21, the sum is 22, and March the 22d for Easter-day that year.

Remark. This rule supposes that the Vernal Equinox always falls on the 21st of March, and also that the mean full moon, and the true one happen on the same day. But according to the astronomical way of beginning the day at noon, the Vernal Equinox may fall on the 19th, 20th, 21st, or 22d, of March: and by the civil reckoning the Sun may pass the Equinoctial on the 20th, 21st, or 22d of March; hence it appears that a full moon may be Paschal and not be reckoned so; or it may not be Paschal and yet be found to be so by the rule. Again, if none but a true full moon be allowed to be Paschal, the rule may fail, because the true full moon and the mean one, may not happen on the same day.

The fathers of the Church were not so well acquainted with the irregularities of the Moon's motions, as the astronomers of the present day; and perhaps did not see the possibility of their rule failing. We have seen some crude remarks in the newspapers, to account for the rule having failed this year (1818), the writers of which lay the blame on the Gregorian manner of reckoning, but it is plain the fault lies in the rule itself; and it is equally evident, that no mode of reckoning which does not introduce a great deal of confusion, can fix the Equinox to the 21st of March: and it is quite impossible to make the true full moon and the mean one, to fall always on the same day. If it be of any real importance to have the rule altered, there is no way to be depended on when the number of direction is either 1, 2, 34, or 35, but to leave the almanac-makers to calculate the time of the true full moon, and also that of the Sun's entrance into the point Aries.

NUTATION, a kind of libratory motion of the Earth's axis; by which its inclination to the plane of the Ecliptic is continually varying, by a certain number of seconds, backwards and forwards. This is produced by the joint effect of the Sun and Moon upon the redundant parts of the Earth, so that the pole is made to describe a small vermicular curve about, or rather upon, the spiral which is forming round the pole of the Ecliptic by the precession of the Equinoxes.

If the Earth's orbit were a circle, and if the Earth's form were that of a perfect sphere, the attraction of the Sun would have no power to disturb the Earth's axis. But the Earth is surrounded by a large quantity of redundant matter, from 13 to 17 miles thick all round the Equator, and this ring of matter becomes thinner, the nearer we approach the poles, where its thickness vanishes. Now if an attracting body be placed any where over the Equator, or over either of the poles, it will produce no effect in changing the position of the axis. But if the Sun or Moon be placed in any other situation, the axis will be drawn out of its former position, and the nearer the Sun is to the tropics, or the Moon to her greatest northern or southern limit, the greater effect they produce. It should be remarked, that there is a certain declination, which an attracting body may have, so that its effect may be the greatest, and after that

it diminishes till it comes to the poles, where it vanishes. The reason of this effect is, because when the Sun, or Moon, is out of the Equinoctial, one side of this ring of matter is nearer to the heavenly body than the other: that is, that part of the ring which is on the same meridian with the Moon is nearer to her than that part which is on the opposite meridian, hence the nearer part is acted upon as a lever, and by that means turns the whole body of the Earth, more or less, out of its former position.

That part of the Earth's orbit, which is comprised between the autumnal and vernal Equinoxes, is less than that which lies on the northern side of the Equinoctial; consequently the Sun is longer in the northern signs than in the southern, and this difference is about 8 days. Now, since the Sun is a longer time in those signs, it has been said that the nutation caused by the Sun is greater on that side than it is on the other: but when we consider, that though he is longer in those signs, yet as he is then at a greater distance from the Earth, it remains to be proved, that the nutation is not as great on the south side of the Equinoctial as on the north.

OBLATE, flattened, or shortened, as an oblate ellipsoid, or spheroid, having the polar diameter shorter than the equatorial; being generated by the rotation of a curve about its minor axis.

OBLATENESS, *of the Earth*, the flatness at, and near the poles, caused by the revolution round its axis. The ratio of the axis to the equatorial diameter has been determined in a variety of ways; such as by the mensuration of a degree of latitude, or longitude, and sometimes by the length of a pendulum vibrating seconds in any given latitude, compared with that of another pendulum which vibrates once in a second, in another given latitude. See Earth, and Degree.

OBLIQUE *Ascension*, is that point of the Equinoctial which rises with the Sun's centre, or with any other celestial object, in an oblique sphere.

Oblique *Circle*, in the stereographic projection, is any circle that is oblique to the plane of projection.

Oblique *Descension*, is that point of the Equinoctial which sets with the centre of the Sun, Moon, Star or other heavenly object in an oblique sphere.

Oblique *Sphere*, is that in which the axis is neither in the horizon nor in the zenith of the place.

OBLIQUITY *of the Ecliptic*, is the angle contained between the Ecliptic and Equinoctial.

OBSERVATION, is the observing with an instrument some celestial phenomenon, as the altitude of the Sun, Moon, or Stars: or their distances asunder, &c.

OBSERVATORY, a building usually erected on some eminence, and furnished with instruments for making correct astronomical observations.

OCCIDENT, or OCCIDENTAL, west, or westward; a planet is said to be occident when it sets after the Sun.

Occident, *Equinoctial*, that point of the horizon where the Sun sets, when he crosses the Equinoctial.

Occident *Æstival*, that point of the horizon where the Sun sets when he enters the sign Cancer.

Occident *Hybernal*, that point of the horizon where the Sun sets when in the first point of Capricorn.

OCCULTATION, the obscuration of any celestial object, by the obscuration of the Moon or any other planet. The occultation of a fixed Star by the Moon, when observed in a place whose latitude and longitude are correctly ascertained, may be applied to the correction of the lunar tables; but if observed in a place whose latitude only is well known, it may be applied in finding the longitude of that place.

OCTANS HADLEIANUS, *Hadley's Octant*. This constellation is situated near the south pole of the world, and contains 43 stars, viz. one of the 3d magnitude, and the rest all under the 4th.

OCTANT, the 8th part of a circle.

Octant, or Octile, is the position, or aspect of two planets, when they are 45°, or the 8th part of a circle asunder.

OCTOBER, the 8th month of the year, in Romulus's calendar; but the 10th in that of Numa, Julius Cæsar, &c. after the addition of January and February.

OFFICINA SCULPTORIA, the sculptor's stand, is situated on the south of Cetus; and contains 12 small stars, spread over a considerable space of the firmament, not any of them exceeding the 5th magnitude.

OLBERS, the same as Pallas.

OLYMPIAD, a period of 4 years, by which the Greeks reckoned their time: so called from the Olympic games, which were celebrated every 4th year, during five days, near the summer solstice, on the banks of the river Alpheus, near Olympia, a town of Elis. The first year of those Olympiads commenced with the nearest new Moon to the summer solstice. The first Olympiad began the 3938th year of the Julian period, the 3208th of the creation, 776 years before Christ, or 24 years before the foundation of Rome. The computation by these ended with the 304th Olympiad, and the 440th of the Christian era.

OPHIUCUS, the same as Serpentarius.

OPPOSITION, is that aspect, or situation of two celestial objects, when their difference of longitude is 180°.

OPTIC *Inequality*, is an apparent inequality in the motion of the heavenly bodies; so called be-

cause it is not really in the moving bodies, but arising from the situation of the eye of the observer.

OPTIC *Place*, of a star, &c. is that point of its orbit, which is determined by our sight, when the star is seen there. This is either true or apparent; true when the eye is supposed to be at the centre of the motion, or apparent, when the eye is at the circumference of the Earth, and the body not in the zenith.

ORB, a spherical shell, hollow sphere, or space contained between two concentric spherical surfaces. The ancient astronomers conceived the heavens to be divided into several vast transparent orbs or spheres, inclosing each other, and including the bodies of the planets.

The ORBIS *Magnus*, or *great* Orb, is that in which the Sun is supposed to move in one siderial year; or that in which the Earth makes its annual round: or more strictly, that great ellipse which is described in one siderial year, by the common centre of gravity of the Earth and Moon.

ORBIT, is the path of a planet or comet; being the curve line described by its centre, in its proper motion in the heavens. The orbits of all the planets are ellipses, some of which are far more eccentric than others.

The orbits of the planets are not all in the same plane with the ecliptic, but are variously inclined to it, and to each other: but still the plane of the ecliptic, in which plane the Earth is supposed to move, and does so very nearly, intersects the plane of the orbit of every other planet, in a right line which is supposed to pass through the Sun, called the line of the nodes, and the points where those orbits pass through the plane of the ecliptic are called the nodes. The plane of the ecliptic was formerly supposed to be fixed, but later observations, as well as theory, prove that it is subject to a small displacement; but if we do not allow this, we must allow latitude to the Sun, when the earth is drawn out of this plane by the action of the other planets, or thrown out of it, by her motion round the common centre of gravity of the Earth and Moon. Besides this instability of the Ecliptic, it will be shewn under the word pole, that it suffers another regular motion or displacement, not yet pointed out by astronomers.

ORIENT, *the east*, or eastern point of the horizon.

ORIENT *Equinoctial*, that point of the horizon where the Sun rises when he is in the first point of Aries and Libra.

ORIENT *Æstival*, is the point where the Sun rises in the middle of summer when the days are longest.

ORIENT *Hybernal*, is the point where the Sun rises in the middle of winter, when the days are shortest.

ORIENTAL, situated towards the east.

ORION, a constellation of the southern hemisphere, with respect to the ecliptic, but part on the northern, and part on the southern side of the Equinoctial, which runs across his body. The stars in this constellation are 78 in Flamstead's catalogue, but some telescopes have discovered thousands more, of which there are 2 of the first magnitude, 4 of the second, with many of the third and fourth. One of those two stars of the first magnitude is on the middle of the left foot, named Regel; the other is on the right shoulder, named Betelguese: of the 4 of the second magnitude, one is on the left shoulder, named Bellatrix, and the other three are in the belt, nearly in a right line, at equal distances between the middle one, and each of the other two, forming what is commonly called the yard-wand.

This constellation is one of the 48 old asterisms, and one of the most remarkable in the firmament. It is drawn in the likeness of a man, with a sword by his side, and seems attacking the bull with a club, in his right hand, his left holding a shield.

No constellation was so terrible to mariners as Orion. He is mentioned in this way by all the Greek and Latin poets, and even by their historians; his rising and setting being attended by storms and tempests: and as the northern constellations are made the followers of the Pleiades, so are the southern ones made the attendants of Orion.

The name of this constellation is also met with in scripture. In Job it is asked, "Canst thou bind the sweet influence of the Pleiades, or loose the bands of Orion?" And Amos says, "Seek him that maketh the seven stars and Orion, and turneth the shadow of death into morning."

ORION'S *River*, the same as the constellation Eridanus.

ORTHOGRAPHIC, or ORTHOGRAPHICAL *Projection of the Sphere*, is the projection of its surface or of the sphere on a plane, passing through the middle of it, by an eye vertically at an infinite distance.

ORTIVE, or *Eastern Amplitude*, is an arch of the horizon contained between the point where a star rises, and the east point of the horizon.

OVAL, an Egg-like figure, improperly called an Ellipse.

PALILICUM, the same as Aldebaran.

PALLAS, one of the four newly discovered planets, too small to be observed by the unassisted eye, or even with a telescope of an inferior kind. It was discovered by Dr. Olbers, at Bremen, on March the 28th, 1802.

PALM SUNDAY, the last Sunday in Lent, or the Sunday next before Easter-day. So called from the primitive days, on account of a pious ceremony then in use, for bearing palms, in memory of the triumphant entry of Christ into Jerusalem, eight days before the feast of the passover.

PARACENTRIC *Motion*, is the difference between the radius vector of a planet, at any one time, and that of any other time. Or it is the space by which a planet approaches nearer to, or recedes farther from the Sun, or centre of attraction.

PARALLACTIC *Angle*, called also simply Parallax, is the angle contained between two lines drawn from a celestial body, one to the Earth's centre, and the other to the eye of the observer on the Earth's surface.

Horizontal PARALLAX, is the greatest parallax a heavenly body can have, and if a tangent be drawn to the Earth, from the place of observation, and produced till it meets the object, and another line be drawn from the Earth's centre to the same point of the object, the angle contained between those two lines is the horizontal parallax.

PARALLAX *in Altitude*, if a line be drawn from the place of observation to a celestial object, and another line be drawn from the Earth's centre to the same point of the object, the angle contained between those two lines is the parallax in altitude.

PARALLAX *in Right Ascension*, is an arch of the Equinoctial contained between two meridian lines, one of which passes through the true place of the object, and the other through that point where the object is brought down to by the effect of parallax.

PARALLAX *of Declination*, is an arch of a celestial meridian which is equal to the difference between the polar distance of the true place of the star, and of that of the place to which it is brought down by parallax.

PARALLAX *of Longitude*, is an arch of the Ecliptic contained between two circles of latitude, one of which passes through the true place of the star, and the other through the point to which it is brought down by parallax.

PARALLAX *of Latitude*, is an arch of a circle of latitude, which is equal to the difference between the distance of the true place of a star from the pole of the Ecliptic, and of that of the place to which it is brought down by the effect of parallax.

Menstrual PARALLAX *of the Sun*. If two right lines be drawn from the centre of the Sun, one to the Earth's centre, and the other to that of the Moon when in the quadratures, the angle contained between those lines is the parallax required.

PARALLAX *of the Annual Orbit of the Earth*, is the angle contained between two right lines, drawn from a star or planet, one of which passes to the Sun's centre, and the other to that of the Earth.

PARALLELS *of Altitude*, or Almacantars, are circles parallel to the horizon, conceived to pass through every minute of the meridian, between the zenith and horizon; having their common pole in the zenith.

PARALLELS, or *Parallel Circles*, called also parallels of latitude, and circles of latitude, are minor circles of the sphere, the former are parallel to the terrestrial Equator, and the latter are parallel to the Ecliptic.

PARALLELS *of Declination*, are minor celestial circles parallel to the Equinoctial.

PARALLELISM *of the Earth's Axis*, is that situation of the Earth's axis, in its progress through the annual orbit, by which it always keeps nearly parallel to itself; so that if a line be drawn parallel to its axis, while in any one part of its orbit, the axis in all other parts of its orbit will be very nearly parallel to the same line. To this parallelism joined to the inclination of the axis to the plane of the Ecliptic we owe all the change and variety of seasons, with the inequality of days and nights.

PAVO *et Indus*, see Indus.

PEGASUS, the Horse, a constellation of the northern hemisphere, drawn in the form of a flying horse; being one of the 48 ancient constellations. It is fabled by the Greeks to have been the offspring of Neptune and the Gorgon Medusa; and to have been that on which Bellerophon rode when he overcame the Chimera; and that flying from Mount Helicon to heaven, he there became a constellation, having thrown his rider in the flight; and that the stroke of his hoof on the mount opened the sacred fountain Hippocrene. There are 89 stars in this constellation, viz. 3 of the second magnitude, 3 of the third, 9 of the fourth, &c. The chief star in this constellation is named Markab.

PENUMBRA, a faint or partial shade in an eclipse, observed between the perfect shadow and the full light. In a solar eclipse, all those persons are within the penumbra who see the Sun partially eclipsed, and those who have the eclipse total, or the Sun quite obscured, are within the dark conical shadow.

PERIGÆUM, or PERIGEE, is that point of the orbit of the Sun or Moon, which is the nearest to the Earth.

PERIHELION, PERIHELIUM, that point in the orbit of a planet or comet which is nearest to the Sun.

PERIOD, the time in which a planet or comet makes one complete revolution in its orbit.

PERIOD, in Chronology, an epoch, or interval of time, by which the years are reckoned; or a series of years by which time is measured in different nations.

PERIODIC, or PERIODICAL, appertaining to period, or going by periods. Thus, the periodical motion of the Moon, is that of her menstrual course round the Earth, called her periodical month.

PERIŒCI, or PERIŒCIANS, are such as live on the same parallel of latitude, but differ in their longitude 180°. Hence they have the same seasons at the same time, but their time of the day differs at

all times by twelve hours, it being noon with the one when it is midnight with the other.

PERISCII, or PERISCIANS, those inhabitants of the Earth whose shadows do in the same day move quite round them, so that the Sun on a clear day may be seen upon every point of the compass, without disappearing. Those who live within the polar circles answer this description, except they live quite at the poles, and then, at the N. pole the Sun as well as their shadows always are on the south point of the horizon.

PERSEUS, a constellation of the northern hemisphere, being one of the 48 ancient asterisms. The Greeks fabled, that this is Perseus whom they make the son of Jupiter by Danae. The father of that lady had been told, that he should be killed by his grandchild, and having only Danae to take care of, he locked her up; but Jupiter found his way to her in a shower of gold, and Perseus verified the oracle. He cut off the head of the Gorgon, and fixed it to his shield: and after many other very great exploits he rescued Andromeda, the daughter of Cassiopeia, whom the sea nymphs, in revenge for that lady's boasting of superior beauty, had fastened to a rock to be devoured by a monster. Jupiter his father in honour of the exploit, they say, afterwards took up the hero, and the whole family with him, into the skies. The number of stars in this constellation is 59, viz. two of the second magnitude, four of the third, twelve of the fourth, &c. The largest star in this constellation is Algenib on his right side.

PERTICA, a kind of comet, the same with Veru.

PHASES, the various appearances or quantities of illumination of the Moon, planets, &c. by the Sun. These phases are seen in the Moon without the assistance of glasses; by which she sometimes increases, sometimes wanes, is now bent into horns, and again appears a half circle; at other times she is gibbous, and again a full circular face. By the help of a telescope the like variety of phases is observed in Venus, &c. Copernicus a little before the use of telescopes, foretold, that after ages would find that Venus undergoes all the changes of the Moon; which prophecy was first fulfilled by Galileo, who, directing his glass to Venus, observed her appearances to resemble those of the Moon.

PHASES *of an Eclipse.* To determine these for any time: Find the Moon's place in her visible way, for that moment, and with that point as a centre, with the interval of the Moon's semidiameter, describe a circle: in like manner find the Sun's place in the ecliptic, from which, with the semidiameter of the Sun, describe another circle: The intersection of the two circles, shews the phases of the eclipse, the quantity of obscuration, and the position of the cusps or horns.

PHENOMENON, or PHŒNOMENON, an appearance in physics, an extraordinary appearance in the Heavens, or on the Earth: either discovered by observation of the celestial bodies, or by physical experiments, the cause of which is not obvious. Such as meteors, comets, uncommon appearance of stars and planets, earthquakes, &c. Such also are the effects of the magnet, phosphorus, &c.

PHŒNIX, a new constellation of the southern hemisphere; being one of the asterisms unknown to the ancients, and is not visible in our northern parts of the globe. It contains 13 stars, viz. one of the second magnitude, one of the third, three of the fourth, &c. The brightest star in it is situated near its right eye.

PHOSPHORUS, the morning star, or the planet Venus, when she rises before the Sun. The Latins call it Lucifer, the French the Shepherd's Star, and the Greeks Phosphorus.

PIAZZI, a small newly discovered primary planet; it was observed for the first time by the Astronomer Piazzi of Palermo on Jan. 1, 1800. It is also called Ceres: which see.

PISCES, the 12th sign or constellation of the Zodiac; in the form of two fishes tied together by the tails. The Greeks, who were never at a loss for a fable to account for the origin of every constellation, tell us that when Venus and Cupid were one time on the banks of the Euphrates, there appeared before them that terrible giant Typhon, who was so long a terror to all the Gods. These deities, they say, immediately threw themselves into the water, and were changed into two fishes, by which they escaped the danger. But the Egyptians used the signs of the Zodiac as part of their hieroglyphic language, and by the 12 they conveyed an idea of the proper employment during the 12 months of the year. The Ram and the Bull had, at that time, taken to the increase of their flock, the young of those animals being then growing up; the maid Virgo, a reaper in the field, an emblem of harvest; Sagitarius declared autumn the time for hunting; and Pisces, or fishes tied together, in token of their being taken, reminded men that the approach of spring was the season for fishing.

The ancients, as they gave one of the twelve months of the year to the patronage of each of the twelve superior deities, so they also dedicated to, or put under the tutelage of each, one of the twelve signs of the Zodiac. In this division, the fishes naturally fell to the share of Neptune; and hence arises that rule of the astrologers, which throws every thing that regards the fate of fleets, and merchandize, under the more immediate patronage and protection of this constellation. The stars in this constellation are 113, viz. one of the third magnitude, five of the fourth, &c.

PISCES *Australis*, the Southern Fish, is a constellation of the southern hemisphere, being one of the 48 old asterisms mentioned by the ancients. The

Greeks have here again the fable of Venus and her son throwing themselves into the water to escape from the terrible Typhon. This fable is probably borrowed from the hieroglyphics of the Egyptians. With them a fish was the representative of the sea, its element; and Typhon was probably a land-flood, perhaps represented by the sign Aquarius, or the Water-Pourer, whose stream or river is represented as swallowed up by this fish, as the land-floods and rivers are by the sea. And Venus was some queen, perhaps Semiramis, otherwise called Hamamah, who took to the river or sea with her son, in a vessel, to avoid the floods, &c. The remarkable star Fomahaut, of the first magnitude, is just in the mouth of this fish. The stars in this constellation are 24, viz. one of the first magnitude, two of the third, five of the fourth, &c.

PISCES *Volans*, the Flying Fish, a small constellation of the southern hemisphere, unknown to the ancients. It is on the antarctic circle, and contains only eight stars, all under the fourth magnitude.

Optic PLACE of a star, is a point in the surface of the mundane sphere in which a spectator sees the centre of the star, &c.

This is divided into true and apparent.

True or *real* OPTIC *Place*, is that point of the surface of the sphere, where a spectator at the centre of the Earth would see the star, &c.

Apparent, or *Visible* OPTIC *Place*, is that point of the surface of the sphere, where a spectator at the surface of the Earth sees the star, &c. The difference between these two is produced by the effect of refraction and parallax.

PLACE *of the Sun*, or *Moon*, or *star*, or *planet*, simply denotes the sign and degree of the Zodiac which the luminary is in; and is usually expressed by latitude and longitude, or right ascension and declination.

PLANET, or Wandering Star, is a celestial body revolving about the Sun, or some other planet, as a centre, or focus, in nearly a circular orbit; or in an ellipsis of small excentricity.

The planets are usually distinguished into primary and secondary.

Primary PLANETS, are those that revolve about the Sun as a centre, or focus: such as Venus, the Earth, &c.

Secondary PLANETS, are such as revolve round a primary planet as a centre, as the primary ones do about the Sun; being more commonly called satellites; such is our Moon, the satellites of Jupiter, Saturn, and Uranus.

The primary planets are again distinguished into interior and exterior.

The *exterior* planets are those which are farther from the Sun than the Earth is; as Mars, Juno, &c.

The *interior* Planets are those which are nearer the Sun than the Earth is; as Mercury and Venus.

PLATONIC YEAR, or *Great Year*, is a period of time determined by the revolution of the equinoctial points, or the time in which the stars and constellations return to their former places, in respect to the equinoxes.

PLEIADES, an assemblage of 7 stars in the neck of the constellation Taurus, though there are now only six of them visible to the unassisted eye. The largest of these is of the third magnitude, called Lucida Pleiadum. The Greeks fabled, that the name Pleiades was given to these stars from seven daughters of Atlas and Pleione, one of the daughters of Oceanus, who having been the nurses of Bacchus, were for their services taken up to heaven and placed there as stars, where they still shine. The meaning of which fable may be, that Atlas first observed these stars, and called them by the names of the daughters of his wife Pleione.

PLENILUNIUM, the full Moon.

PLOUGH-MONDAY, the first after Epiphany or January 6.

POLAR CIRCLES, or two minor circles of the sphere, or globe, one about each pole, and at the same distance from it as is equal to the Sun's greatest declination, or the obliquity of the Ecliptic; that is, at present, almost 23° 28′.

POLES, the extremities of the Earth's axis, and these continued to the heavens point out the celestial poles. These are on every side at the distance of a quadrant, or 90°, from every point of the Equinoctial, and are called, by way of eminence, the poles of the world. That which is visible to us in Europe, or raised above our horizon, is called the Arctic, or North Pole; and the opposite one, the Antarctic, or South Pole.

The pole of the world has a motion round the pole of the Ecliptic, occasioned by the precession of the equinoxes; and this is occasioned by the action of the Sun, Moon, and planets, on the redundant parts about the terrestrial Equator, which circumscribes it like a broad hoop or ring, which I shall in future call by the name of Terrannulus. As I have something to introduce in this place which has not been noticed by Astronomers, it may be better to lay down some simple mechanical propositions and consequences which flow from the rotatory and orbicular motions of the celestial bodies.

1. If a large body, A (such as the Sun), be perfectly spherical, and a comparatively small spherical body, B, be projected in an orbit round A as a centre, there will be neither nutation nor precession in either of them. This is quite evident.

2. If A be quite spherical and B spheroidal, and it be projected with its polar axis inclined to its orbit

in any angle less than 90°, then B will suffer a nutation and a recession. For as one part of the ring about the Equator is nearer to A than any other part; and as all the particles of matter are acted upon in the inverse ratio of the squares of their distances from A, it follows that the equator of B will be translated out of its former position; and by this angular motion its poles will point to a different part of the heavens. And as B's equator has an angular motion towards A, it will be drawn up so as to have A directly over it, before B has proceeded 180° in its orbit, and just as much as this falls short of 180°, so much will be the precession.

3. If A and B are both oblate spheroids, and B proceed with its axis inclined as before, and the plane of B's orbit be inclined in an acute angle with the plane of A's equator, then A and B will both suffer a nutation, and B a recession as before. Moreover, the plane of B's orbit will suffer a displacement.

As action and reaction are equal and contrary, it follows, that the particles of matter which compose the ring of A will be acted upon by the whole body of B, in a similar manner to the action of A upon B, hence a nutation of A's polar axis must follow; and the precession of B remains the same as before.—Now it remains to prove the displacement of the plane of the orbit. Since B is attracted towards the centre of A by the inscribed sphere only; and by the redundant matter to a point out of the centre of this ring, the point to which the whole body of B is solicited, does not lie in the centre of A, but somewhere between A's equator and its centre: hence it follows that B will cross the equator of A before it has gone 180° from the point where it last crossed it; and this will continue to happen, every time B crosses the equator of A: and consequently the poles of its orbit will revolve about a point in the heavens, in a similar manner to that of the poles of its equator. But the poles of the equator are carried round by a conical angular motion of its axis, while those of the ecliptic are moved by a constant side motion of the whole body of B in a kind of spiral orbit.

All this being premised, it remains to be shewn, that as the Sun moves about his axis, he must be in the form of an oblate spheroid; and it is known that the plane of the Sun's equator forms an angle with the plane of the ecliptic; hence the plane of the ecliptic will suffer a secular translation, and this motion will cause the poles of the ecliptic to describe a circle round the solar poles, the diameter of which will be equal to twice the inclination of the aforesaid planes. On account of the great distance of the Sun, and the smallness of his ellipticity, this motion is extremely slow.

It is with the greatest deference that the author has given this to the learned for their further consideration; and he presumes that they will find that the pole of the world, and that of the ecliptic, are moving round a point in two concentric circles; or about two points not very distant asunder. Hence the cause of the diminution of the obliquity of the ecliptic is apparent; and will at some future period increase again. For though the pole of the world has a very slow motion; and the pole of the ecliptic a very much slower: yet they must approach *to*, and recede *from* each other alternately. There are other causes pointed out by astronomers for the displacement of the ecliptic, viz.—the action of the several planets; some of which draw the earth a little to the north, and others to the south of the mean ecliptic. But as those planets are constantly changing from north latitude to south, and *vice versa*, it does not seem, that this is a sufficient cause for producing a secular diminution of the obliquity of the ecliptic. It is more easy to conceive this translation of the plane of the ecliptic, which is caused by the action of the planets, by considering that all the bodies in the system move about a common centre of gravity; and if we conceive the planes of their orbits to be fixed, all those bodies will continually shift from one side to the other, of the respective planes of their own orbits.

The altitude or elevation of the POLE, is an arch of the meridian contained between the pole and the horizon, and is equal to the latitude of the place.

POLE-STAR, is a star of the second magnitude near the north pole, in the end of the tail of Ursa Minor.

The distance of this star from the pole in 1810 was 1° 42′ 19″, but this distance is continually varying on account of the precession of the equinoxes. The proximity of this star to the pole makes it very useful in navigation, to those who sail in the northern hemisphere.

POLLUX, the hind twin, or the posterior part of the constellation Gemini. It is also the name of a fixed star of the second magnitude, in the constellation Gemini.

PRAXITELES, *vel Cœla Sculptoria*, the Engraver's or Sculptor's Tools, a constellation of the southern hemisphere, on the west of Columba, containing a few small stars.

PRIMARY *Planets*, are those which revolve round the Sun as a centre. Such are the Planets Mercury, Venus, Terra or Earth, Mars, Vesta, Juno, Ceres, Pallas, Jupiter, Saturn and Uranus. They are thus called in contradistinction from the secondary planets or satellites which revolve about their respective primaries.

PRIME *vertical*, is that vertical circle, or azimuth, which is perpendicular to the meridian, and passes through the east and west points of the horizon.

PRIME *of the Moon*, is the new Moon at her first appearance, for about three days after her change. It means also the Golden Number; which see.

PRIMUM *Mobile*, in the Ptolemaic system, is supposed to be a vast sphere, whose centre is that of the world, and in comparison of which the earth is but a point. This they decribe as including all other spheres within it, and giving motion to them, turning itself and all the rest quite round in 24 hours.

PROCYON, a fixed star, of the second magnitude, in Canis Minor, or the Less Dog.

PROSTHAPHERESIS, the difference between the true and mean motion, or between the true and mean place of a planet, or between the true and equated anomaly; called also the equation of the orbit, or equation of the centre, or simply the equation. It is so called, because it is sometimes to be added to, and sometimes to be taken from the mean motion to give the corrected one.

PSEUDO-*Stella*, any kind of meteor or phenomenon, appearing in the heavens, and resembling a star.

PYTHAGOREAN, or PYTHAGORIC, *System*, the same as the Copernican, or Newtonian system.

PYXIS NAUTICA, the seaman's compass. It is situated in the rigging of Argo Navis, containing a few small stars, two of which are of the 5th magnitude, and the rest smaller.

QUADRAGESIMA, a denomination given to the time of Lent, from its consisting of about 40 days; commencing on Ash-Wednesday.

QUADRAGESIMA *Sunday*, is the first Sunday in Lent, or the first Sunday after Ash-Wednesday.

QUADRATURE, that aspect or position of the Moon when she is 90° distant from the Sun. Or, the quadratures or quarters are the two middle points of the Moon's orbit between the points of conjunction and opposition, viz. the points of the first and third quarters; at which times the Moon appears half full, being dichotomized or bisected.

QUARTER. The Moon's period, or lunation, is divided into four quarters, each containing between seven and eight days. The first quarter is from the new Moon to the quadrature; the second is from thence to the full Moon, and so on.

QUARTILE, an aspect of the planets when they are at the distance of 3 signs or 90° from each other.

QUINCUNX, is that position, or aspect, of the planets, when they are 5 signs or 150° asunder.

QUINQUAGESIMA-*Sunday*, is the same as Shrove-Sunday, and is so called as being about the 50th day before Easter, being the 7th Sunday before it. The term was used anciently for Whitsunday, and for the 50 days between Easter and Whitsunday; but to distinguish this from that before Easter, it was called the paschal quinquagesima.

QUINTILE, an aspect of the planets when they are distant the fifth part of a circle, or 72°.

RATIONAL *horizon*, or *true horizon*, is that whose plane is supposed to pass through the centre of the Earth; and which therefore divides the globe into two equal parts or hemispheres. It is called the rational horizon, because only conceived by the understanding; in opposition to the sensible or apparent horizon, or that which is visible to the eye.

RECESSION *of the Equinoxes*, the going back of the equinoctial points, commonly called precession; but this may be on account of the signs themselves preceding the points or intersections of the Ecliptic and Equinoctial.

REDUCTION *to the Ecliptic*, is the difference between the argument of latitude, and an arc of the Ecliptic, intercepted between the place of a planet and the node. To find this reduction, or difference, there are given, the distance of the planet from the node reckoned in its orbit; and the angle contained between the orbit and ecliptic: to find the arch of the Ecliptic, contained between the place of the node, and the point where a perpendicular let fall from the planet meets the Ecliptic.

REFRACTION, *of the heavenly bodies*, is an inflexion of the rays of light coming from those luminaries, in passing through our atmosphere; by which means they appear higher than they really are. The atmosphere is unequally dense at different heights: that part nearest the earth is the heaviest, and as we ascend it continually decreases, and is lightest of all at the top; which inequality in the same medium, makes it equivalent to several unequal mediums, by which the course of the ray of light is continually bent into a continued curve line. And Sir Isaac Newton has shewn, that a ray of light, in passing from the highest and rarest part of the atmosphere, down to the lowest, and most dense, undergoes the same quantity of refraction that it would do in passing immediately, at the same obliquity, out of a vacuum into air of equal density with that in the lowest part of the atmosphere.

REFRACTION *of altitude*, is an arc of a vertical circle, by which the altitude of a star is increased by refraction.

REFRACTION *of right ascension*, is an arc of the Equinoctial, by which the right ascension of a star is increased or diminished by refraction.

REFRACTION *of declination*, is an arc of a circle of declination, by which the declination of a star is increased or diminished by refraction.

REFRACTION *of latitude*, is an arc of a circle of latitude, by which the latitude of a star is increased or diminished by the refraction.

REFRACTION *of longitude*, is an arc of the Eclip-

tic, by which the longitude of a star is increased or diminished by means of the refraction.

REGEL, or RIGEL, a fixed star of the first magnitude, in the left foot of Orion.

REGULUS, a star of the first magnitude, in the constellation Leo; called also, from its situation, Cor Leonis, or the Lion's Heart; by the Arabs, Alhabor; and by the Chaldeans, Kalbeleced, or Karbeleceid; from an opinion of its influencing the affairs of the Heavens.

RETICULA RHOMBOIDALIS, the Rhomboidal Net, contains 10 stars; part of it lies on the antarctic circle, and the two largest stars in it are of the 4th magnitude.

RETROCESSION *of the equinox*, See precession.

RETROGRADATION, or RETROGRESSION, an apparent motion of the planets, by which they seem to go backwards in the Ecliptic, and to move contrary to the order of the signs.

RETROGRADATION *of the Moon's Nodes*, is a motion of the line of the nodes of her orbit, by which it continually shifts its situation from east to west contrary to the order of the signs, which is completed in about 19 years: that is, if the ascending node be in Aries at any time, it will continue to move backwards almost 19 degrees in a year, so that in something less than 19 years the same node will be in Aries again. Sir Isaac Newton has demonstrated in the *Principia*, that this retrogradation is caused by the action of the Sun, which continually draws the Moon from her orbit, and deflects the orbit from a plane, and causes its intersection with the Ecliptic continually to vary. Besides this deflection caused by the Sun, I think it will be found that the Terrannulus acts on the Moon, in a similar manner to the Sun's action on the Earth, described under the word pole. For if the Moon disturb the Earth's axis, so as to produce a nutation, there must be a reaction on the part of the Earth, by which the Moon must continually be deflected towards the Earth's equator, and this deflection must accelerate the motion of the Moon's nodes.

RETROGRADATION *of the Sun*, a motion by which in some situations, in the torrid zone, he retrogrades or moves backwards. This respects the azimuth only; and it can happen no where but in the torrid zone, when his declination is greater than the latitude of the place.

RETROGRESSION, or RETROCESSION. The same as Retrogradation.

REVOLUTION, a turning round a fixed axis; or a motion round a point at a distance from the moving body. The planets have both of these revolutions, the first of which is called the diurnal revolution, and the second the annual.

RIGEL, the same as Regel.

RING *of Saturn*, is a thin, broad, opaque, circular arch, encompassing the body of that planet, without touching it; and appears double, when seen through a good telescope. Late observations have thrown much light on this curious phenomenon, especially respecting its dimensions and rotation, and division into two or more parts. Lalande and Laplace say, that Cassini saw the breadth of the ring divided into two separate parts that are equal, or nearly so. Mr. Short assured M. Lalande, that he had seen many divisions on the ring, with his twelve feet telescope. And Mr. Hadley, with an excellent 5½ feet reflector, saw the ring divided into two parts. Some excellent theories have been given in the French Memoirs, by Laplace and others, contending for the division of the ring into many parts. But, the observations of Dr. Herschel, in many parts of the Philosophical Transactions, seem to confirm the division into two concentric parts only.

The dimensions of these two rings, and the space between them, he states to be as under;—

Inner diam. of the less ring -	146345	Miles.
Outer diam. of ditto. - - -	184393	
Inner diam. of the greater ring	190248	
Outer diam. of ditto. - - -	204883	
Breadth of the inner ring - -	20000	
Breadth of the outer ring - -	7200	
Breadth of the vacant space -	2839	

Ring revolves in its own plane, in 10*h*. 32*m*. 15.4*s*.

Dr. Herschel says, "Some theories and observations, of other persons, lead us to consider the question, whether the construction of this ring is of a nature so as permanently to remain in its present state? or whether it be liable to continual and frequent changes, in such a manner as in the course of a few years, to be seen subdivided into narrow slips, and then again as united into one or two circular planes only. Now, without entering into a discussion, the mind seems to revolt, even at first sight, against an idea of the chaotic state in which so large a mass as the ring of Saturn must needs be, if phenomena like these can be admitted. Nor ought we to indulge a suspicion of this being a reality, unless repeated and well confirmed observations had proved, beyond a doubt, that this ring was actually in so fluctuating a condition." From his own observations he concludes, "It does not appear to me, that there is a sufficient ground for admitting the ring of Saturn to be of a very changeable nature; and I guess that its phenomena will hereafter be so fully explained, as to reconcile all observations. In the mean while, we must withhold a final judgment of its construction, till we can have more observations. Its division, however, into two very unequal parts, can admit of no doubt."

RISING, the appearance of the heavenly bodies

above the horizon, which the moment before were hid beneath it. By reason of the refraction of the atmosphere, the celestial objects always appear to rise before their time.

ROBUR CAROLI, Charles's Oak, commonly called Royal Oak. See Royal Oak.

ROYAL OAK, *Robur Carolinum*, one of the new constellations, in the southern hemisphere. This constellation was named in commemoration of the oak tree, that Charles II. was concealed in, and saved by, when pursued, after the battle of Worcester, in 1651. This asterism contains 12 stars, viz. one of the first magnitude, (or the second) two of the second, three of the third, three of the fourth, &c.

SAGITTA, *the Arrow* or *Dart*, a constellation of the northern hemisphere, near the eagle, and one of the 48 old asterisms. The Greeks say that this constellation owes its origin to one of the arrows of Hercules, with which he killed the eagle or vulture that gnawed the liver of Prometheus. This constellation contains 18 stars, viz. four of the fourth magnitude, and the rest smaller.

SAGITTARIUS, *the Archer*, one of the signs of the Zodiac, being the 9th in order. This constellation is drawn in the figure of a centaur, or an animal half man and half horse, in the act of shooting an arrow. The Greeks feign this to be Erotus, the son of Eupheme, the nurse of the Muses. Among more ancient nations the figure was probably meant for a hunter to denote the hunting season, when the Sun enters this sign. This constellation contains 69 stars, viz. five of the third magnitude, nine of the fourth, &c.

SAROS, a period of 223 lunar months. The etymology of the word is said to be Chaldean, signifying restitution, or return of eclipses; that is, conjunctions of the Sun and Moon in nearly the same place of the ecliptic.

SATELLITES, secondary planets, or such as move round other planets, as the Moon does round the Earth. They are so called because always attending their respective primaries from rising to setting, and making their journey round the Sun together. The words Moon and Satellite are sometimes used indifferently; thus we say, either Jupiter's Moons, or Jupiter's Satellites; but generally we use the term Moon for the Earth's attendant, and apply that of Satellite, to the apparently small attendants of Jupiter, Saturn, and Uranus.

The Satellites move round their primary planets, as a centre, by the same laws as those primary ones do round their centre, the Sun; viz. in such a manner that, in the satellites of the same planet, the squares of the periodic times are proportional to the cubes of their distances from the primary planet.

There are 4 little moons, or satellites, now known to perform their revolutions round Jupiter, as that planet does about the Sun.

The periods of Jupiter's satellites, are discovered from their conjunctions with that planet, after the same manner as those of the primary planets are from their oppositions to the Sun. And their distances from the body of Jupiter are measured by a micrometer, and estimated in semidiameters of that planet, and thence reduced into miles.

SATELLITES *of Saturn*, are seven in number, moving round him. Dr. Herschel observes, that the fifth satellite turns once round its axis exactly in the time in which it revolves about its primary; in which respect it resembles our Moon.

SATELLITES *of Uranus*, are 6 apparently little moons that revolve about him, like those of Jupiter and Saturn. These are all the satellites we at present know of; what other discoveries may be made by further improvements in telescopes, time only can bring to light.

SATURDAY, the 7th or last day of the week, so called, from the idol Seater, worshipped on this day by the ancient Saxons, and thought to be the same as the Saturn of the Latins. In Astronomy, every day of the week is denoted by some one of the planets, and this day is marked with the ♄ Saturn. Saturday is the ancient Sabbath of the Jews.

SATURN, one of the primary planets, being the 10th in order from the Sun, and was supposed to be outermost of all, before Dr. Herschel discovered Uranus. Those satellites whose orbits do not lie in the plane of the rings must be much disturbed in their motions by the action of those rings, as well as by the disturbing forces of each other. The rings themselves must also suffer a nutation or constant vibration in different directions.

SCHOLIUM, a note, remark, or annotation occasionally made on some passage, proposition, &c.

SCORPIO, *the Scorpion*, the 8th sign of the Zodiac.

The Greeks, who wished to be thought the founders of astronomy, and who have, with that intent, applied some sort of story of their own to each of the old constellations, give a very singular account of the origin of this sign. They say that this is the creature which killed Orion; and according to them, the famous hunter of that name boasted to Diana and Latona, that he would destroy every animal that was upon the earth. The Earth, they say, enraged at this, sent forth the poisonous reptile the Scorpion, which insignificant creature stung him and he died. Jupiter then raised the Scorpion to the heavens, giving him this place among the constellations; and that afterwards Diana requested of him to do the same honour to Orion, to which he at last consented, but

placed him in such a situation, that when the Scorpion rises he sets.

But the Egyptians, or some other early nation who framed the Zodiac, probably placed this poisonous reptile in that part of the heavens to denote when the Sun arrived at it, fevers and sickness, the maladies of autumn, would begin to rage. This they represented by an animal whose sting was of such a nature as to occasion some of them; and it was thus they formed all the constellations.

The ancients allotted one of the 12 principal among their deities to be the guardian of each of the 12 signs of the Zodiac. The Scorpion, as their history of it made it a fierce and fatal animal that had killed the great Orion, fell naturally to the protection of the God of War; Mars is therefore its tutelary deity; and to this single circumstance is owing all that jargon of the astrologers, who tell us that there is a great analogy between the planet Mars and the constellation Scorpio. To this also is owing the doctrine of the alchymists, that iron, which they call Mars, is also under the dominion of the same constellation, and that the transmutation of that metal into gold can only be performed when the Sun is in this sign. Scorpio contains 44 stars, viz. 1 of the first magnitude, 1 of the second, 11 of the third, 8 of the fourth, &c. The most considerable star in this sign is Antares, (or cor Scorpio) of the first magnitude.

SCRUPLES, *Eclipsed*, denote that part of the Moon's diameter which enters the shadow, expressed in the same measure as the apparent diameter is.

SCRUPLES *of half duration*, an arc of the Moon's orbit, described by her from the beginning of an eclipse to its middle.

SCRUPLES *of immersion* or *incidence*, an arc of the Moon's orbit which her centre describes from the beginning of the eclipse, to the time when the centre falls into the shadow.

SCRUPLES *of emersion*, an arc of the Moon's orbit, which her centre describes in the time from the first emersion of the Moon's limb, to the end of the eclipse.

SCUTUM SOBIESKI, Sobieski's Shield, a constellation of the northern hemisphere, situated on the N. of Sagittarius. It contains 7 stars, two of which are of the 4th magnitude, and the rest smaller. This constellation was named by Hevelius, in honour of John Sobieski, king of Poland, who so heroically raised the siege of Vienna, when invested by the Turks, in 1683. The Emperor Leopald, being at this period driven from his capital, owed his preservation solely to this great monarch.

SEASONS, certain portions or quarters of the year, distinguished by the signs which the Sun enters at those periods. The year is divided into 4 seasons; spring, summer, autumn, and winter, which commence when the Sun enters the first point of the signs Aries, Cancer, Libra, and Capricorn.

SECONDARY *circles of the ecliptic*, are circles of longitude of the stars; or circles passing through the poles of the ecliptic, and are at right angles to it.

By means of these secondaries, all points in the heavens are referred to the ecliptic; that is any star, &c. is understood to be in that point of the ecliptic, which is cut by the secondary circle which passes through such star, &c.

When two stars are thus referred to the same point of the ecliptic, they are said to be in conjunction; if in opposite points, they are in opposition; and so of all the other aspects.

In general, all circles that intersect one of the six major circles of the sphere at right angles, may be called secondary circles.

SECULAR *Equations*, or Century Equations, are corrections required to compensate such inequalities in the celestial motions as occur in the course of a century, or 100 years. Thus, there are secular inequalities in the Moon's motion which require for their correction as many distinct secular equations.

SELENOGRAPAY, the description and representation of the Moon, with all the parts and appearances of her disc, such as we see on the globe of the Moon, or on a map of this luminary.

SEMIQUINTILE, is an aspect of the planets when distant from each other $\frac{1}{2}$ of $\frac{1}{5}$ of a circle, or 36°.

SEMISEXTILE, an aspect of two planets when they are 30° or the half of a sextile, or 2 signs asunder.

SEPTUAGESIMA, the 9th Sunday before Easter, so called, it is supposed, because it is nearly 70 days, though it is really no more than 63 days, before it.

SERPENS, a constellation in the northern hemisphere, being one of the 48 old ones, and is called more particularly Serpens Ophiuchi, being grasped in the hands of the constellation Ophiucus. The Greeks, in their fables, have ascribed it sometimes to one of Triptolemus's dragons, killed by Carnabus; and sometimes to the serpent of the river Segaris, destroyed by Hercules. This is supposed by some to be the same as the author of the book of Job calls the crooked serpent; but this expression probably meant the Constellation Draco, near the north pole. This constellation contains 64 stars, none of which are remarkable.

SERPENTARIUS, a constellation of the northern hemisphere, and one of the 48 old ones. It is also called Ophiucus, and anciently Æsculapius. It is in the figure of a man grasping a serpent; viz. Serpens. The Greeks had many fables about this, and other constellations, because they were ignorant of the true meaning of them. Some of them say it represents Carnabus, who killed one of the dragons

of Triptolemus. Others say, it was Hercules, killing the serpent of the river Segaris. And others say, it represents the celebrated physician Æsculapius, to denote his skill in medicine in curing the bite of the serpent. There are 74 stars in this constellation, one of which is of the second magnitude, in the head of the asterism, called Ras Alhague.

SESQUIQUADRATE, an aspect of the planets when they are 4½ signs, or 135° asunder.

SESQUIQUINTILE, is an aspect of the planets when they are distant ½ and a half of the circle, or 108°.

SETTING, the sinking of a star or planet below the horizon. Astronomers and poets count 3 different kinds of setting of the stars, viz. Achronical, Cosmical, and Heliacal.

SEVEN STARS, a common denomination given to the cluster of Stars in the neck of Taurus, properly called Pleiades. They are called seven from the number which appear to the unassisted eye; though some persons can see only six; but by the help of a telescope, a great number may be seen.

SEXAGESIMA, the 8th Sunday before Easter; it is so called because near 60 days before it.

SEXTANS, the sextant, a new constellation, made up of some unformed stars by Hevelius. This constellation is situated on the equator, at the foot of Leo, and contains 41 stars, all of them except one being under the 4th magnitude.

SEXTILE, the aspect or position of two planets, when they are the 6th part of a circle, or 60° asunder.

SIDERIAL, something relating to the stars.

Siderial *Day*, is the time in which any star appears to revolve from any meridian to the same meridian again: or the time in which the earth makes one complete revolution on its axis, which is 23*h*. 56*m*. 4*s*. 6*t*. of mean solar time; there being about 366 siderial days in a year.

SIGN, the 12th part of the Ecliptic, or Zodiac; or a portion containing 30° of the same. The ancients divided the Zodiac into 12 equal parts called signs; commencing at the point where the Ecliptic and Equinoctial intersect, and so counting forward from west to east, according to the apparent course of the Sun; these signs they named from the 12 constellations which possessed those signs in the time of Hipparchus. But the constellations have since so changed their places, by the precession of the equinox, that Aries is now found to be in the sign Taurus, and Taurus in that of Gemini, &c.

The signs are distinguished, with regard to the season of the year when the Sun is in them, into Vernal, Æstival, Autumnal, and Brumal.

Vernal or *spring* Signs, are Aries, Taurus, Gemini.

Æstival or *summer* Signs, are Cancer, Leo, and Virgo.

Autumnal Signs, are Libra, Scorpio, and Sagittarius.

Brumal or *winter* Signs, are Capricorn, Aquarius and Pisces.

The Vernal and Summer signs are also called northern signs, because they are on the north side of the equinoctial; and the Autumnal and Winter signs are called southern ones, because they are on the south side of the equinoctial.

The signs are also distinguished into ascending and descending, according as they are ascending towards the north, or descending towards the south. Thus the

Ascending Signs, are the winter and spring signs, or those 6 from the winter solstice to the summer solstice, viz. Capricornus, Aquarius, Pisces, Aries, Taurus, Gemini.

Descending Signs, are the summer and autumn signs, or the signs Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius.

SIRIUS, *the dog star*; a very bright star of the first magnitude, in the nose of the constellation *Canis Major*, or the greater dog. This is the brightest of all the stars in our firmament, and therefore probably, says Dr. Maskelyne, the nearest to us of them all, in a paper recommending the discovery of its parallax, Phil. Trans. Vol. 51, p. 889. Some however suppose Arcturus to be the nearest.

The Arabs called it Aschers, Elschure, Scera; the Greeks, Sirius; and Latins, Canicula, or Canis candens.

This is one of the earliest named stars in the heavens. Hesiod and Homer mention only four or five constellations, or stars, and this is one of them. Sirius and Orion, the Hyades, Pleiades, and Arcturus, are almost the whole of the old poetical astronomy. The three last the Greeks formed of their own observation, as appears by the names; the two others were Egyptian. Sirius was so called from the Nile, one of the names of that river being Sirius; and the Egyptians, seeing that river begin to swell at the time of a particular rising of this star, paid divine honours to the star, and called it by the name derived from that of the river, expressing the Star of the Nile.

SOLAR, something relating to the Sun. Thus we say, solar fire, as opposed to culinary fire.

Solar *Civil Month, Cycle*, &c. see the substantives.

Solar *Comet*, see Discus.

Solar *Eclipse*, is a privation of the light of the Sun, by the interposition of the opaque body of the Moon.

Solar *System*, the order and disposition of the several heavenly bodies, which move round the Sun as the centre of their motion; viz. the planets and comets.

SOLANNULUS, the redundant matter raised up about the equator of the Sun, by his rotatory motion about his axis.

SOLSTICE, the time when the Sun is in one of the solstitial points, that is, when he is at the greatest distance from the Equator, which is now not quite 23° 28′ on either side of it. It is so called because the Sun then seems to stand still, or not to change the place, as to declination either way. There are two solstices in each year, when the Sun is at the greatest distance on the north and south sides of the Equator; viz. the æstival or summer solstice, and the hyemal or winter solstice.

The summer solstice is when the Sun is in the tropic of Cancer; which is about the 21st of June, when he makes the longest day. The winter solstice is when he enters the first point of Capricorn, which is about the 21st of December, when he makes the shortest day. This is only applicable to the northern hemisphere; for in the southern, the Sun's entrance into Capricorn makes their summer solstice, and that into Cancer the winter one. So that it is better to say, the northern and southern solstice.

SOLSTITIAL *Points*, are the most western extremities of Cancer and Capricorn.

SOLSTITIAL *Colure*, is that which passes through the solstitial points.

SPHERE, a solid body contained under one single uniform surface, every point of which is equally distant from a certain point in the middle called its centre.

SPHERE *in Astronomy*, that great concave orb or expanse which invests our globe, and in which the heavenly bodies appear to be fixed at an equal distance from the eye. This is also called the sphere of the world; and it is the subject of spherical astronomy.

This sphere is conceived to be so extremely large, that if the whole solar system were viewed from its surface it would vanish into a mere point!

SPHERICAL *Astronomy*, that part of Astronomy which considers the universe such as it appears to the eye. Under Spherical Astronomy are included all the phenomena and appearances of the heavens and heavenly bodies, such as we perceive them, without any inquiry into the reason, the theory, or truth of them. In spherical astronomy, the world is conceived to be a concave spherical surface, in the centre of which is the Earth, or rather the eye, about which the visible frame appears to revolve, with stars and planets fixed in its circumference. And on this supposition all the other phenomena are determined. Theorical Astronomy teaches us from the laws of optics, &c. to correct this scheme.

SPHEROID, a solid body supposed to be generated by the rotation of an ellipsis or oval figure about one of its axes.

If it revolve about its longer diameter it is an oblong spheroid, and sometimes prolate, resembling a lemon: but if the oval revolve about its shorter axis, the solid will be an oblate spheroid, which resembles an orange, and which is the figure of the Earth, and all other celestial bodies which revolve about an axis. When the revolving oval is a perfect ellipse, the solid generated by the revolution is properly called an ellipsoid, as distinguished from the spheroid, which is generated by the rotation of any oval whatever. But, generally speaking, in the common acceptation of the word, the term spheroid is used for an ellipsoid.

SPORADES, a name by which the ancients distinguished such stars as were not included in any constellation. These the moderns call unformed stars. Many of these sporades of the ancients have since been formed into constellations.

SPOTS, are dark places observed on the discs or faces of the Sun, Moon, and planets. Many have been the opinions of philosophers and astronomers concerning the spots on the Sun: but as Dr. Herschel has the advantage of a very good telescope, he has a better opportunity of forming a more correct judgment on the subject. The Sun, he supposes, has an atmosphere; and this atmosphere consists of various elastic fluids, some of which exhibit a shining brilliancy, while others are merely transparent. Whenever the lucid fluid is removed, the body of the Sun may be seen through those that are transparent, as a dark spot. Like as an observer, placed on the Moon, sees the solid body of our Earth, only in those places where the transparent fluids of our atmosphere permit him. In others, the opaque vapours reflect the light of the Sun, without permitting his view to penetrate to the surface of our globe. By changes in the atmosphere of Jupiter, Dr. Herschel accounts for the phenomena of his belts, and on the same principle he illustrates the various appearances of a spot, which he observed on the Sun in 1779. This spot extended more than 50 thousand miles; and he says that "the idea of its being occasioned by a volcanic explosion, violently driving away a fiery fluid, which on its return would gradually fill up the vacancy, and thus restore the Sun in that place to its former splendour, ought to be rejected on many accounts." Dr. Herschel apprehends there are considerable inequalities in the surface of the Sun; and there may be elevations not less than 5 or 600 miles high. "A very high country, or chain of mountains, may oftener become visible, by the removal of the obstructing fluid, than the lower regions, on account of its not being so deeply covered with it; and some of the solar mountains may be high enough occasionally to project above the shining elastic fluid, when by some agitation, or

other cause, it is not of the usual height." And this opinion is much strengthened by the return of some remarkable spots, which served Cassini to ascertain the period of the Sun's rotation. According to Dr. Herschel's hypothesis, the dark spots are the opaque body of the Sun, and the luminous part is an atmosphere, which, being broken, gives us a glimpse of part of the Sun himself.

Lucid SPOTS, *in the heavens*, are several little whitish spots, that appear magnified, and more luminous when seen through telescopes; and yet without any stars in them. One of these is in the girdle of Andromeda: it has some whitish rays near its middle, is liable to several changes, and is sometimes invisible. Another is near the ecliptic, between the head and bow of Sagittarius; it is small and very luminous. A third is in the back of Centaur. A fourth of a smaller size, is before Antinous's right foot, having a star in it, which makes it appear more bright. A fifth is in Hercules, which may be seen on a clear night in the Moon's absence without a telescope. Perhaps these lucid spots may, when telescopes are more improved, be found to be clusters of small stars.

Planetary SPOTS, are those found on the discs of the planets. Jupiter, Mars, and Venus, when seen through a telescope, show some very remarkable ones: and it is by the motion of these spots, that the rotation of the planets about their axes is determined, in the same manner as that of the Sun is deduced from the apparent motion of his maculæ.

SPRING, one of the seasons of the year; commencing in the northern hemisphere, on the day the Sun enters the first point of Aries, which is about the 21st day of March, and ending when the Sun enters Cancer.

STAR, STELLA, a general name for all those lucid points we see in the firmament, either with, or without the assistance of glasses.

Erratic or *wandering* STARS, are those which are continually changing their places and distances, with regard to each other. These are what are properly called planets. Though comets or blazing stars are also of the same class.

Fixed STARS, are those which are observed to keep the same distance, or very nearly so, with regard to each other. The chief circumstances observable in the fixed stars, are their distance, magnitude, number, nature and motion.

The fixed stars are so extremely remote from us, that we have no distances in the solar system, to compare them with. The magnitudes of the stars appear to be very different from each other; which apparent difference may arise, partly from a diversity in their real magnitudes, but chiefly from their different distances.

The stars, on account of their apparent sizes, have been formed into classes called magnitudes. The stars of the first magnitude, are those that appear largest, and if they are nearly of the same real size, they are the nearest to us. Next to these are those of the second magnitude, and so on to the 6th, which are the smallest we can see without glasses. All beyond these that can be perceived by the help of instruments are called telescopic stars. Tyro is not to understand that all the stars put into any one class are of the same apparent magnitude; for there is a great difference in this respect, for there are scarcely any two which agree in size and colour. There are some of which it is doubtful to which class they belong, and therefore astronomers place them between the two. Procyon, for instance, Ptolemy makes of the first mag. and Tycho of the 2nd, but Flamstead lays it down between the first and second. So that instead of 6 or 7 magnitudes, we may say there are almost as many as there are stars, on account of the great variations observable in the magnitudes, colour, and brightness of them.

The number of stars appears to be immensely great, most likely infinite; yet have astronomers long since ascertained the number of such as are visible to the unassisted eye, which are much fewer than we at first sight could imagine. The immense distance of the stars leaves us greatly in the dark about the nature of them. What we can gather for certain is: 1st, That the fixed stars are greater than our Earth; because if that were not the case, they could not be visible at such immense distances. 2nd, The fixed stars are farther distant from the Earth than the farthest of the planets. For we often find the fixed stars hid behind the body of the planets: and moreover they have no parallax which the planets have. 3d, The fixed stars shine with their own light; for they are much farther from the Sun than Saturn is, and appear much smaller than he does; but yet, they are found to shine much brighter than that planet, it is evident they cannot borrow their light from the same source as Saturn does; but since we know of no other luminous body but the Sun, from which they might derive their light, it follows that they shine with their own native light.

The fixed stars have several kinds of apparent motion; the first is the diurnal motion, arising from the Earth's rotation about its axis; by which they seem to be carried along with the sphere or firmament, in which they seem to be fixed round the Earth, from east to west, in the space of a siderial day.

The second, is that by which they appear to go backwards from west to east, round the poles of the Ecliptic, with a very slow motion, owing to the recession of the equinoctial points. In consequence

of this apparent motion their longitude continues to increase till it arrives at 360°, when it commences the same circuit over again. It has been supposed that the latitudes of the stars never vary; but it is now well known that this is not the case. For as the Earth is deflected constantly towards the plane of the Sun's equator, by the Solannulus; and again by the disturbing forces of all the planets and comets, the plane of the Ecliptic suffers a constant displacement, consequently the latitudes of the stars are variable. Besides those apparent motions modern observations have shown that some of the stars have a proper motion of their own; and others are suspected to have such a motion. Now the Sun being one of the numberless central bodies, similar to the stars, Astronomers suspect that he with all his attendants partake of a proper motion; indeed it is suspected that a great number of those systems constantly move round one common centre of gravity; and that every distant cluster of small stars, is an assemblage of those systems obeying the same laws.

STATION or STATIONARY, the position or appearance of a planet, in the same point of the Zodiac, for several days. This happens from the observer being situated on the earth, which is far out of the centre of their orbits, by which they seem to move irregularly; being seen for a time to move direct, or according to the order of the signs, then to stand still, and after that to move in a contrary order. This is explained with a diagram in its proper place.

STATIONARY, the state of a planet, when to an observer on the earth, it appears for sometime to stand still, or remain immoveable in the same place in the heavens. For as the planets, to such an observer, have sometimes a progressive motion, and sometimes a retrograde one, there must be some point between the two where they must appear stationary. Now a planet will be seen stationary, when the line that joins the centers of the Earth and planet is continually directed to the same point in the heavens, which is when it keeps parallel to itself. For all right lines drawn from any point of the Earth's orbit, parallel to each other, do all point to the same star; the distance of these lines being insensible, in comparison of the great distance of the fixed stars.

STYLE, a particular manner of counting time.

Old STYLE, is the Julian manner of counting time, in which the mean year consists of 365*d*. 6*h*.

New STYLE, is the Gregorian manner of computation, instituted by Pope Gregory the 13th, in the year 1582, and is used by most catholic countries, and many other states in Europe. The Gregorian, or new style, agrees nearly with the true solar year, which contains only 365*d*. 5*h*. 48*m*. 48*s*. In the old style there were 100 leap years in 400 years: and in the new style there are 97 leap years; hence there is a difference of 3 days in that time. The mean length of a Gregorian year is 365*d*. 5*h*. 49*m*. 12*s*. consequently the Solar time will only differ one day from the Gregorian in 3600 years.

SUCCESSION *of Signs*, is the order in which they are reckoned, or follow each other, and according to which the Sun enters them, called also *consequentia*. When a planet goes according to the order of the signs, or *in consequentia*, it is said to be direct; but retrograde when they move the contrary way, or *in antecedentia*.

SUMMER, the name of one of the seasons of the year, being one of the quarters when the year is divided into four parts, or one half when the year is divided only into two, summer and winter.

SUMMER *Solstice*, the time or point when the Sun attains his greatest declination, or is nearest the elevated pole, and is nearest the zenith of the place.

SUN, SOL, ☉, the great luminary, about which all the planets in our system revolve, in different periods of time, and at different distances. The comets also revolve about the Sun. The reader will find an account of the Sun in the description of the Solar System, and under the word Spots.

Dr. Herschel has made many interesting experiments on the nature of the Sun's rays, and has thus firmly established a fact which had been long disputed, namely, the separate identity of light and heat: that they are both subject to the laws of reflection and refraction; that they are each of different refrangibility, are liable to be stopped in certain proportions when transmitted through diaphanous bodies; and that they are liable to be scattered on rough surfaces. These curious facts were discovered by the Doctor in his optical experiments on coloured glass, in which he was led to examine the difference between the coloured rays of the Sun with regard to their heating power. He thereby discovered that the most refracted rays of light, the violet rays, possess the lowest heating power; the least refracted, the red, the greatest power, and the intermediate rays of the prismatic spectrum shewed an intermediate power. Thus, in the red rays the thermometer, by the means of different experiments, rose 6¾ degrees; in the green rays 3¼ degrees; and the violet 2 degrees.

Pursuing these experiments, the same philosopher found the range of dispersion of the rays of heat by the prism, to differ most essentially from that of light; for on applying thermometers of great sensibility and successively in a line, beginning at the violet rays, and proceeding along the prismatic spectrum, he found not only the heat increased by advancing towards the red, or least refracted rays, but that the heat was greatest at a small distance beyond the extreme limits of the spectrum, that is, where no rays of light at all fell; and still continu-

ing to advance the thermometer in the same line, the heat then gradually diminished, till it became too small to be noticed. This most curious and important discovery shews, therefore, both an entire separation of heat from light in the solar ray, and refrangibility of one from the other, which together go near to establish the separate identity of caloric and light, and cause precisely the same arguments used to demonstrate the materiality of light to apply to the materiality of heat. These experiments of Dr. Herschel have been fully confirmed by Sir H. Englefield, whose apparatus were somewhat different and more accurate.

The Sun, like the planets, is not perfectly globular, but spheroidal, being higher about his equator than at the poles: the reason of which is, because he has a rotatory motion about his axis; and therefore the solar matter will endeavour to recede from the axis, and that with the greater force as the circles in which the particles move are greater: but the equator is the greatest circle; and the parallels towards the poles continually decrease; therefore, the solar matter, though at first in a spherical form, will endeavour to recede from the centre of the equator further than from the centre of the parallels. Consequently, since the gravity by which it is retained in its place, is supposed to be uniform throughout the whole Sun, it will really recede from the centre more at the equator, than at any of the parallels; and hence the Sun's diameter will be greater through the equator, than through the poles: that is, his figure will be that of an oblate spheroid.

SUNDAY, the first day of the week; thus called by our idolatrous ancestors, because set apart for the worship of the Sun. It is sometimes called the Lord's day, because kept as a feast in memory of Christ's resurrection on this day; and also Sabbath-day, because substituted under the new law instead of the sabbath in the old law. It was Constantine the great who first made a law for the observation of Sunday, and who, according to Eusebius, appointed that it should be regularly celebrated throughout the Roman empire.

SUNDAY-LETTER, see Dominical Letter.

SYNODICAL *Month*, is the period, or interval of time, in which the Moon passes from one conjunction with the Sun to another. This period is also called a lunation, since in this time the Moon puts on all her phases, or appearances, as to increase and decrease.

SYSTEM, in astronomy denotes an hypothesis, or a supposition of a certain order and arrangement of the several parts of the universe; by which astronomers explain all the phenomena or appearances of the celestial bodies, their motions, changes, &c. This is more particularly called the System of the World, and sometimes the Solar System. The systems of Ptolemy and Tycho Brahé were mere hypotheses, and consequently not deserving the name; but the Copernican is built on the firmest principles, and agrees with astronomical observations.

SYZYGY, a term used both for the conjunction and opposition of a planet with the Sun.

TARANDUS, *the Rein Deer*, a constellation of the northern hemisphere, extending from the north pole of the world nearly to the arctic circle, on the W. of Camelopardalis: it is lately introduced, and not exhibited on the old globes: it contains a few small stars, one of them in the left branch of its horn is of the fourth magnitude.

TAURUS, *the Bull*, one of the 12 signs of the Zodiac, and the second in order. The Greeks fabled, that this was the famous bull which carried Europa safely across the seas to Crete; and that Jupiter, to reward him for this signal service, placed the creature, whose form he had assumed on that occasion, among the stars, and that this is the constellation formed of it. But it is most likely that the Egyptians, or Babylonians, or whoever invented the signs of the Zodiac, placed this figure in that part of it which the Sun entered about the time of the bringing forth of calves; like as they placed the Ram in the first part of spring, as the lambs appear before them, and two kids (the original figure of the sign Gemini), afterwards, to denote the time of goats bringing forth their young.

In Taurus there are some remarkable stars: as Aldebaran in the south or left eye, the cluster called the Pleiades in the neck, and the cluster called Hyades in the face.

This constellation contains 141 stars, of which one is of the first magnitude, six of the third, nine of the fourth, &c.

TAURUS PONIATOWSKI, a northern constellation, named in honour of Count Poniatowski, a brave Polish officer, who saved the life of Charles the 12th, king of Sweden, at the battle of Poltowa, near the Dnieper, and also in the isle of Rugen, in the Baltic, near Stralsund. This constellation lies between Serpentarius and Aquila, it contains seven stars; 4 of them being of the fourth magnitude, the rest smaller. On the globe there is a star of the third magnitude in the tail of Serpens, which appears to belong to this asterism, though it does not.

TELESCOPIUM, the Telescope, a southern constellation, on the S. of Sagittarius, containing nine small stars, the largest of which is of the fourth magnitude.

TERMINATOR, a name sometimes given to the circle of illumination, from its property of terminating the boundaries of light and darkness.

TERRANNULUS, that broad hoop or ring, surrounding the Earth, which is from 13 to 17 miles thick at the equator, which thickness continues to

decrease till we come to the poles, where it is nothing. This redundant matter is the cause of the nutation of the Earth's axis, and the chief cause of the recession of the equinoctial points. A small part of the recession being produced by the deflection of the Earth which causes the poles of the Ecliptic to move round the celestial poles of the Sun. See Poles.

THURSDAY, the fifth day of the week. The name is from Thor, one of the Saxon idols.

TIME, a succession of phenomena in the universe; or a mode of duration, marked by certain periods and measures; chiefly by the motion and revolution of the Sun; and sometimes by that of other celestial bodies.

Astronomical TIME, is that which is taken from the motion of the heavenly bodies, without any other regard.

Civil TIME, is the former time accommodated to civil uses, and formed into years, months, days, &c.

Apparent TIME, is that deduced from the motion of the Sun, which is unequal.

Equal, Mean, or *True* TIME, is that which is shewn by a well-going clock, which never varies.

Equation of TIME is the difference between true and apparent time.

TORRID *Zone*, is that round the middle of the Earth, extending to about 23° 28′ on both sides the Equator.

TOUCAN, or American Goose, a modern constellation of the southern hemisphere, on the antarctic circle, containing 9 stars, viz. one of the third magnitude, two of the fourth, &c.

TRAJECTORY *of a Comet*, is its path or orbit, or the line it describes in its motion. This path Hevelius will have to be very nearly a right line; but Dr. Halley justly concludes it to be a very excentric ellipsis; though its place may often be well computed on the supposition of its being a paraboloid. Sir Isaac Newton, in prop. 41, of his third book, shews how to determine the trajectory of a comet from three observations; and in his last prop. how to correct a trajectory graphically described.

TRANSIT, denotes the passage of any planet, just before or over any other celestial body; or the passage of a star or planet over the meridian, or before an astronomical instrument.

The Transit of Mercury and Venus over the Sun's disc are very interesting and curious phenomena, not only on account of their rare and singular appearance, but also because of their use in finding the quantity of the Sun's parallax, and thence the real distance of this luminary from the Earth. The following are the times when there will be transits of Mercury.

1815	Nov. 11.	1835	Nov. 7.
1822	Nov. 4.	1845	May 8.
1832	May 5.	1848	Nov. 9.
1861	Nov. 11.	1881	Nov. 7.
1868	Nov. 4.	1891	May 9.
1878	May 5.	1894	Nov. 10.

It appears from this table, that the transits of Mercury always happen either in May or November; but most frequently in the latter; depending on the position of the elliptic projection of Mercury's orbit on the plane of the Ecliptic. This ellipse is now so placed, that it presents to us its perihelion, during the winter, and its aphelion during the summer; and as his orbit is very excentric, Mercury is much nearer the Sun in Nov. than he is in May. Now it is evident that a greater portion of that part of the orbit, which is nearest the Sun, will be projected upon his disc, than of that which is nearest to the earth: and as Mercury passes through that part nearest the Sun, in the winter half year; it follows that he has a greater chance to pass over the Sun's disc in November, than in May, when he is farther from the Sun.

From the observations of the transit of Nov. 8, 1802, the node appeared to be in 1*s*. 15° 58′ 56″.

The Transits of Venus are less frequent than those of Mercury because she is more distant from the Sun. The following are all that will happen before the year 2110.

1874	Dec. 8	2004	June 7
1882	Dec. 6	2109	Dec. 10

The chief use of these transits is, to determine the Sun's distance from the Earth, or the quantity of his horizontal parallax, which astronomers have in vain attempted to find by other methods; for the minuteness of the requisite angles easily eludes the nicest instruments. But by observing the ingress of Venus upon the Sun's disc, and her egress from the same, the interval between the moments of the internal contacts, observed to a second of time, may be obtained by the assistance of a moderately good telescope, and a pendulum clock which may be depended on to go uniformly for the third or fourth part of a day. Now from two such observations rightly made in proper places, the distance of the Sun, within a 500th part, may be determined. The only observations that have been made, were those of 1639, 1761, and 1769; from the last of these the Sun's parallax appears to be about $8\frac{5}{7}$ seconds of an arch. From this angle the distance of the Sun is found in semidiameters of the earth. Notwithstanding we agree pretty well about the distances of the heavenly bodies in semidiameters, it is to be lamented that mathematicians cannot so well agree, about the dimensions of our own globe, and as the Sun's distance depends on the length of the Earth's radius, we find a difference in this distance exists in most books on astronomy. It should also be determined on, whether we are to use the equatorial radius, the polar one, or a mean semidiameter.

TRINITY *Sunday*, is the next after Whitsunday;

so called, because on that day was anciently held a festival in honour of the Holy Trinity.

TRIANGULUM, the triangle, a northern constellation, lying on the N. of Aries. It consists of two triangles, a greater and less, by some considered as two constellations; containing 16 stars, three of which are of the 4th magnitude, and the rest smaller.

TRIANGULUM *Australis*, the southern triangle, a constellation of the southern hemisphere, lying across the Antarctic circle, containing 5 stars, viz. one of the 2d magnitude, two of third, &c.

TRIONES, a kind of constellation, or assemblage of 7 stars in Ursa Major, popularly called Charles' Wain. From the *septem Triones* the north pole takes the denomination Septentrio.

TROPICAL, something relating to the tropics.

TROPICAL *Year*, the space of time in which the Sun passes round from a tropic, till he returns to it again.

TROPICS, two minor circles of the sphere drawn parallel to the equinoctial, through the solstitial points, or at such a distance from the equinoctial as is equal to the Sun's greatest declination, or to the obliquity of the ecliptic, and also to the distance of the polar circles from the poles.

TRUE *place of a planet or star*, is a point in the heavens in which the star would appear, if not affected by refraction, parallax, or aberration.

TUESDAY, the 3d day of the week, so called from Tuesco, one of the Saxon idols, similar to Mars; for which reason ♂ is used for this day.

TWELFTH-DAY, the festival of the Epiphany, or the Manifestation of Christ to the Gentiles, so called, as being the 12th day, exclusive, from the nativity or Christmas-day; this falls on the 6th day of January.

TWILIGHT, is that faint light which is observed before sun-rising, and after sun-setting. The twilight is occasioned by the earth's atmosphere refracting or bending the rays of the Sun, and reflecting them among its particles.

VECTOR, or *Radius Vector*, is a line conceived to be drawn from any planet moving round a centre, or the focus of an ellipse, to that centre, or focus. It is so called, because it is that line by which the planet seems to be carried round its centre; and with which it describes areas proportional to the times.

VENUS, one of the interior planets, but the brightest, and to appearance the largest, of all the planets. Venus is easily distinguished from the other planets, by her brightness and whiteness, in which she exceeds even Jupiter himself. According to Doctor Herschel's observations, Venus is a little larger than the earth.

VERNAL, something belonging to the spring.

VERTEX, the same as Zenith.

VERTICAL *point*, the same with vertex, or zenith.

VERTICAL *circle*, is a major circle of the sphere, passing through the zenith and nadir of a place. The vertical circles are also named azimuths. The meridian of any place is a vertical circle, passing through the north and south points of the horizon. All those circles intersect each other in the zenith and nadir.

Prime VERTICAL, is that vertical circle, or azimuth, which passes through the poles of the meridian, or which is perpendicular to the meridian, and passes through the equinoctial points.

VERU, a comet, according to some resembling a spit, being nearly the same as the Lonchites, only its head is rounder, and its train longer and sharper pointed.

VESPER, called also Hesperus, and the evening star, is the planet Venus, when she is eastward of the Sun, sets after him, and shines as an evening star.

VESPERTINE, is when a planet is descending to the west after sun-set, or shines as an evening star.

VESTA, one of the small primary planets, revolving between Mars and Jupiter. It was discovered by M. Olbers, the 29th of March, 1807.

VIA LACTEA, the Milky Way, or Galaxy.

VIA SOLIS, or Sun's Way, is used by astronomers for the ecliptic, or path in which the Sun seems to move.

VINDEMIATRIX, or VINDEMIATOR, a fixed star of the 3d magnitude in the northern wing of Virgo.

VIRGO, one of the signs of the Zodiac; being one of the 48 old constellations, and is mentioned by the astronomers of all ages and nations, whose works have reached us. Antiently the figure was that of a girl, almost in a state of nudity, with an ear of corn in her hand, evidently to denote the time of harvest among the people who invented this sign. But the Greeks much altered the figure, with clothes, wings, &c. and variously explained the origin of it by their own fables: thus they tell us that the Virgin now exalted into the skies, was, while on earth, that Justitia, the daughter of Astræus and Anchora, who lived in the golden age, and taught mankind their duty; but who, when their crimes increased, was obliged to leave the earth, and take her place in the heavens. Again, Hesiod gives the celestial maid another origin, and says she was the daughter of Jupiter and Themis. There are also others who depart from both these accounts, and make her to have been Erigone, the daughter of Icarius; while others make her Parthene, the daughter of Apollo, who placed her there; others from the ear of corn, make it a representation of Ceres; and others from the obscurity of her head, of Fortune.

The ancients, as they gave each of the 12 months of the year to the care of some one of the 12 prin-

cipal deities, so they also threw into the protection of each of these one of the 12 signs of the Zodiac. Hence Virgo, from the ear of corn in her hand, naturally fell to the lot of Ceres, and we accordingly find it called Signum Cereris. Virgo contains 110 stars, viz. one of the first magnitude, six of the third, ten of the fourth, &c. The most brilliant star in Virgo is Spica Azimech, near the ecliptic: this star is also called Spica Virginis.

UNFORMED *Stars*, are such as were not contained in the constellations. But on the modern celestial globes, the new constellations include most of those unformed stars.

UNICORN, the same as Monoceros.

UNIVERSE, a collective name, signifying the assemblage of heaven and earth, with all things in them. The ancients, and after them the Cartesians, imagined the universe to be infinite; and the reason they give is, that it implies a contradiction to suppose it finite or bounded; since it is impossible not to conceive space beyond any limits that can be assigned; which space, according to the Cartesians, is body, and consequently part of the universe.

URANUS, a new primary planet, discovered by Doctor Herschel, in River-street, Bath, in the night of March 13, 1781. It is sometimes also called the Georgian planet, and the New planet, from its having been newly or lately discovered, also Herschel's planet, from the name of its discoverer, and the planet Herschel, or simply Herschel. This planet is the most remote of all those that are yet known, though not the largest, being in point of magnitude less than Saturn and Jupiter. Its light, says Doctor Herschel, is of a bluish white colour, and its brilliancy between that of Venus and the Moon. With a telescope that magnifies about 300 times, it appears to have a very well defined disc; but with instruments of a small power, it can hardly be distinguished from a fixed star of the 6th or 7th magnitude. In a very fine clear night, when the Moon is absent, a good eye will perceive it without a telescope.

URSA MAJOR, or the Greater Bear, one of the 48 old constellations, and probably one of the most ancient; being familiarly known and alluded to by the oldest writers, and is mentioned by Homer as observed by Navigators. It is supposed that this constellation is that mentioned in the Book of Job, under the name of Chesil, which our translation has rendered Orion, where it is said, "Canst thou loose the bands of Chesil (Orion)?" It is farther said that the ancients represented each of the two Bears under the form of a waggon drawn by a team of horses, and the Greeks originally called them waggons and two Bears; they are to this day popularly called the wains, or waggons, and the greater of them Charles's wain. Hence is remarked the propriety of the expression, "Loose the bands, &c," the binding and loosing being terms very applicable to a harness, &c.

Perhaps the Egyptians, or whoever invented the constellations, placed those stars, which are near the pole, in the figure of a Bear, as being an animal inhabiting the polar regions, and making neither long journies, nor swift motions. But the Greeks in their usual way, have adapted some of their fables to it. They say this bear was Callisto, daughter of Lycaon, king of Arcadia; that being defiled by Jupiter, he afterwards placed her in the heavens, as well as her son Arcturus. The Greeks called this constellation Arctos and Helice, from its turning round the pole. The Latins from the name of the nymph, as variously written, Calisto, Megisto, and Flemisto, and from the Arabians, sometimes Feretrum Majus, the Great Bier. And the Ursa Minor, they called Feretrum Minos, the little Bier. The Italians have followed the same custom, and call them Cataletto. They spoke also of the Phœnicians being guided by the less Bear, but the Geeks by the greater. There are two remarkable stars in this constellation, viz. those in the middle of his body, considered as the two hindermost of the wain, and called the pointers, because they always point nearly in a direction towards the north pole star, and so are useful in finding out this star.

This constellation contains 87 stars, viz. one of the first magnitude, four of the second, three of the third, ten of the fourth, &c. The chief star in Ursa Major is Dubhe, situated in its back, being the uppermost of the pointers and of the first magnitude, or between the first and second.

URSA MINOR, the less Bear, called also Arctos Minor, Phœnice, and Cynosura, one of the 48 old constellations, and near the north pole: the large star in the tip of its tail being very near to it, and thence called the pole-star. The Phœnicians guided their navigations by this constellation, for which reason it was called Phœnice, or the Phœnician constellation. It was also called Cynosura by the Greeks, because, according to some, that was one of the dogs of the huntress Callisto, or the Greater Bear; but according to others Cynosura was one of the Idæan nymphs that nursed the infant Jupiter; and some say that Callisto was another of them, and that, for their care, they were taken up together to the skies.

This constellation contains 24 stars, viz. one of the second magnitude, two of the third, four of the fourth, &c. Alruccabah the pole star, is the most noted.

VULPECULA *et* ANSER, the Fox and Goose, one of the new constellations of the northern hemisphere, made out of the unformed stars by Hevelius, and contains 35 stars, viz. four of the 4th magnitude, and the rest of inferior magnitudes.

WEDNESDAY, the 4th day of the week, formerly consecrated by the inhabitants of the northern nations to Woden or Oden; who being reputed the author of magic, and inventor of all the arts, was thought to answer to the Mercury of the Greeks and Romans, in honour of whom the same day was by them called *dies Mercurii*; and hence it is denoted by Astronomers ☿.

WEEK, a division of time containing 7 days. The origin of this division is much controverted. It has often been thought to have taken its rise from the 4 quarters or intervals of the Moon, between her changes of phases, which, being about 7 days distant gave occasion to the division: but others more probably from the 7 planets. Be this as it may, the division is certainly very ancient. The Syrians, Egyptians, and most of the oriental nations, appear to have used it from the earliest ages: though it did not get footing in the west till introduced by the Christians.

The Romans reckoned their days by ninths; and the Greeks by Decads, or tenths; in which they were imitated by the French in their new Calendar.

The Jews divided their time into weeks of 7 days each, as prescribed by the law of Moses; and some authors will have the use of weeks, among the other eastern nations, to have proceeded from the Jews; while others suppose the use of weeks, among the eastern nations, to be a remnant of the tradition of the creation, or else from the number of the planets. The Jews denominated the days of the week, first, second, third, &c. The like method is still kept up by the Christian Arabs, Persians, Ethiopeans, &c.

The ancient heathens denominated the days of the week from the 7 planets, which names are mostly retained among the Christians of the West: thus, the first day was called *Dies Solis*, Sun-Day; the second *Dies Lunæ*, Moon-Day; &c. a practice the more natural, on Dion's principle, that the Egyptians took the division of the week itself from the 7 planets. In fact, the true reason of these denominations seems to be founded in astrology. For the astrologers distributed the government and direction of all the hours in the week among the 7 planets, ♄ ♃ ♂ ☉ ♀ ☿ ☽, so as that the government of the first hour of the first day fell to Saturn, that of the second to Jupiter, &c. they gave each day the name of the planet which, according to their doctrine, presided over the first hour of it, and that according to the order above stated. So that the order of the planets in the week bear little relation to that in which they follow in the heavens; the former being grounded on an imaginary power each planet has, in its turn, on the first hour of each day.

Dion Cassius gives another reason for the denomination, drawn from the celestial harmony. For it being observed that the harmony of the diatessaron, which consists in the ratio of 4 to 3, is of great force and effect in music: it was thought proper to proceed directly from Saturn to the Sun; because, according to the old system, there are three planets between Saturn and the Sun, and four from the Sun to the Moon.

Our Saxon ancestors, before their conversion to Christianity, named the 7 days of the week from the Sun and Moon and some of their deified heroes, to whom they were peculiarly consecrated, and representing the ancient gods or planets; which names we have received and still retain: Thus, Sunday was devoted to the Sun; Monday to the Moon; Tuesday to Tuesco; Wednesday to Woden; Thursday to Thor, the Thunderer; Friday to Friga or Friya of Fræa, the wife of Thor; and Saturday to Seater. And nearly according to this order, the modern astronomers express the days of the week by the seven planets as annexed.

☉ Sunday.
☽ Monday.
♂ Tuesday.
☿ Wednesday.
♃ Thursday.
♀ Friday.
♄ Saturday.

In the same order and number also do these obtain in the Hindu days of the week. See Kindersley's specimens of Hindu literature, lately published.

WEST, one of the 4 cardinal points of the horizon.

WHALE, See Cetus.

WHITSUNDAY, the 50th day or 7th Sunday from Easter.

The season properly called Pentecost, is popularly called Whitsuntide; because, it is said, in the primitive church, the newly baptized persons came to church between Easter and Pentecost in white garments.

WINTER, one of the 4 seasons or quarters of the year. Winter properly commences on the day when the Sun's distance from the zenith of the place is the greatest, or when his declination is the greatest on the contrary side of the equator; and it ends when he next enters the equinoctial.

XIPHIAS, is the Dorado or Sword-Fish, one of the new constellations of the southern hemisphere. This small constellation is near the south pole of the ecliptic, and contains 6 stars; one of them is of the 3d mag. two of the fourth, &c.

YEAR, in the full extent of the word, is a system or cycle of several months, usually 12. Some define year, in the general, a period or space of time, measured by the revolution of some celestial body in its orbit. Thus, the time in which the fixed stars make a revolution, is called the great year; and the times in which the Moon and each of the planets complete one revolution, and return to that circle of latitude from which they began their circuit, are respectively called their siderial years. And the time in which any planet takes, in passing from that circle of latitude, which runs through its point Aries,

to its return to that circle, which passes through Aries again, is equal to such planet's tropical year. As year denoted originally a revolution, and was not limited to that of the Sun; accordingly we find by the oldest accounts, that people have, at different times, expressed other revolutions by it, particularly that of the Moon; and consequently that the years of some accounts, are to be reckoned only months, and sometimes periods of 2, 3, or 4 months. This will assist us greatly in understanding the accounts that certain nations give of their own antiquity, and perhaps also of the age men. We read expressly, in some of the old Greek writers, that the Egyptian year, at one period was only a month; and we are further told that at other periods it was 3 months, or 4 months; and it is probable that the children of Israel followed the Egyptian account of their years. The Egyptians boasted, near 2000 years ago, of having accounts of events 48000 years distant. The Egyptians appear to have had, in the time of the Greeks the same ambition which the Chinese have at present, and wished to pass themselves on that people, as those do upon us, for the most ancient inhabitants of the earth. They had recourse also to the same means, and both the present and the early impostors have pretended to ancient observations of the heavenly bodies, and recounted eclipses in particular, to vouch for the truth of their accounts. Since the time in which the solar year, or period of the earth's revolution round the Sun, has been received, we may account with certainty; but for these remote ages, in which we do not precisely know what is meant by the term year, it is impossible to form a correct judgment of the duration of time in the accounts. The Babylonians pretend to an antiquity of the like romantic kind; they boast of 47000 years in which they kept observations; but we may judge of those as of the others, and of the observations as of the years.

YEAR, or *Solar* YEAR, properly, and by way of eminence so called, is the space of time in which the Sun moves from the point Aries, round to the point Aries again; but during that interval this point has moved backwards a little; hence the Sun has not quite completed a circle in that period. This is called a tropical year, and contains 365 days, 5 hours, 48 minutes, 48 seconds. The length of a mean Gregorian year is 365*d*. 5*h*. 49*m*. 12*s*. which is a little more than the true length of the year; but it will not make a day difference in less than 36 centuries.

Siderial or *Astral* YEAR, is the space of time the Sun takes in passing from any circle of celestial latitude, till his return to the same circle again.

Lunar YEAR, is the space of 12 lunar months. Hence, from the two kinds, of synodical lunar months, there arise two kinds of lunar years; viz. astronomical, and civil.

Lunar Astronomical YEAR, consists of 12 lunar synodical months, and contains 354*d*. 8*h*. 48*m*. 38*s*. and is therefore 10*d*. 21*h*. 0*m*. 10*s*. shorter than the solar year. This difference is the foundation of the epact.

Lunar Civil YEAR, is either common or embolismic.

Common Lunar YEAR, consists of 12 lunar civil months; and therefore contains 354 days.

Embolismic or *Intercalary* YEAR consists of 13 lunar civil months, and contains 384 days.

Civil YEAR, is that form which different nations have contrived or adopted for computing their time by.

Common Civil YEAR, is that which consists of 365 days.

Bissextile or *Leap* YEAR, consists of 366 days; having one day extraordinary; named the intercalary, or bissextile day; was first introduced by Julius Cæsar; who to make the civil years keep pace with the tropical ones, contrived that the six hours should make one day in 4 years, and be added between the 22d and 23d of February, which was their 6th of the Calends of March; and as they then counted this day twice over, or had bissexto calendas, hence the year itself came to be called bissextus, and bissextile. This intercalary day is now made the 29th day of February.

Julian YEAR. This is in effect a solar year, commonly containing 365 days; though every fourth contains 366 days.

The mean quantity of the Julian year is 365 days, 6 hours; exceeding the true solar year by something more than 11 minutes; an excess which amounts to a whole day in about 130 years. Hence the times of the equinoxes go backwards, and fall earlier by one day in 130 years nearly.

Gregorian YEAR. This is the Julian year corrected, thus instead of every secular or 100th year being a bissextile, as it would be in the former mode, in the new way three of them are common years, and only the fourth is bissextile. The error of 11 minutes and 12 seconds in the Julian year, by continual repetition, had accumulated to an error of 13 days from the time when Cæsar made his correction; by which means the equinoxes were greatly disturbed. In the year 1582, the equinoxes were fallen back ten days, and the full Moons 4 days, more than they were in the time of the Nicene council, which was in the year 325; viz. the former from the 20th of March to the 10th, and the latter from the 5th to the 1st of April. To remedy this increasing irregularity, Pope Gregory the 13th, in

the year 1582, called together the best astronomers of his time, and concerted this correction, omitting the ten days above mentioned. He exchanged the lunar cycle for that of the epacts, and made the 5th of October of that year to be the 15th; by that means restoring the vernal equinox to the 21st of March. It was also provided, by the omission of 3 intercalary days, in 400 years, to make the civil year keep pace nearly with the tropical year, for the time to come. In the year 1700, the error of 10 days was increased to 11; upon which, the protestant states of Germany, to prevent farther confusion, adopted the Gregorian correction. And the same was accepted also in England in the year 1752, when 11 days were thrown out after the 2d of September that year, by accounting the 3d to be the 14th day of the month: calling this the new style, and the former the old style. The mean length of a Gregorian year is 365*d*. 5*h*. 49*m*. 12*s*. and that of a solar year 365*d*. 5*h*. 48*m*. 48*s*. the difference is 24*s*. This amounts to one day in 3600 years: hence if one day more be dropped once in 36 centuries, it will just restore the equinox to the 21st of March.

ZENITH, the vertical point, or point in the heavens directly over our heads.

Zenith *Distance*, is the distance of a celestial body from our zenith; and is the complement of the altitude, or what it wants of ninety degrees.

ZODIAC, an imaginary ring or broad circle, in the heavens, in form of a belt or girdle, within which the planets make their excursion round the Sun. The 4 small planets Vesta, Juno, Ceres and Pallas are said to wander sometimes out of the Zodiac. The Ecliptic passes through the middle of this zone.

In consequence of the motion of the poles of the Ecliptic about the solar poles; it will follow that the Ecliptic itself will be carried round in a similar manner to the Equinoctial, and in the course of perhaps many thousands of ages it will leave the signs through which it now passes; and by continually moving on, it will in a very long time, fall into the middle of them again.

The Zodiac is divided into 12 equal parts, of 30° each, called the signs of the Zodiac, being so named from the constellations which anciently passed through them. But, the stars having an apparent motion from west to east, arising from the recession of the equinoctial points, or the precession of the signs themselves; these constellations do not now correspond to their proper signs. And therefore, when a star is said to be in such a sign of the Zodiac, it is not to be understood of that constellation, but only of that dodecatemory or 12th part of it.

ZONE, a division of the Earth's surface, by means of parallel circles, chiefly with respect to the degree of heat in the different parts of that surface.

The ancient astronomers used the term zone, to explain the different appearances of the Sun and other heavenly bodies, with the length of the days and nights; and the geographers, as they used the climates to mark the situation of places, using the term climate when they were able to be more exact, and the term zone when less so.

The zones were commonly accounted five in number; one a broad girdle round the middle of the Earth, having the equator in the middle of it, and bounded, on the north and south sides, by parallel circles passing through the first points of Cancer and Capricorn. This they called the torrid zone, which they supposed not habitable, on account of its extreme heat. They sometimes divided this into two equal torrid zones, by the equator, one to the north, and the other south; and then the whole number of zones was accounted six.

Next, from the tropics of Cancer and Capricorn to the two polar circles, were two other spaces called temperate zones, as being moderately warm; and these they supposed to be the only habitable parts of the earth.

Lastly, the two spaces beyond the temperate zones, about either pole, bounded within the polar circles, and having the poles in the middle of them, are the two frigid or frozen zones, and which they supposed not habitable, on account of the extreme cold there.

Hence the breadth of the torrid zone is equal to twice the greatest declination of the Sun, or twice the obliquity of the Ecliptic, equal to 46° 56′ nearly. Each frigid zone is also of the same breadth. Now if we take the breadth of one of these zones from 90° the remainder, 43° 4′, will be the breadth of the two temperate zones.

The difference of zones is attended with a variety of curious particulars.

1. In the torrid zone, (except on its verge) the Sun passes through the zenith of every place in it twice a year; making as it were two summers in the year: the inhabitants of this zone are called amphiscians, because they have their noon day shadows projected different ways in different times of the year, northward at one season, and southward at the other.

2. In the temperate zones, the Sun rises and sets every natural day of 24 hours. Yet every where, but at the equator, the artificial days are of unequal lengths, and the inequality is the greater, as the place is farther from the equator. The inhabitants of the temperate zones are called heteroscians, because their meridian shadow is cast the same way all the year round.

3. Within the frigid zones, the inhabitants have their artificial days and nights extended to a great length; the Sun sometimes skirting round a little

above the horizon for many days together: and at another season never rising above the horizon at all, but making continual night for a considerable time. The inhabitants of these zones are called periscians, because sometimes they have their shadows going quite round them in the space of 24 hours.

END OF THE ASTRONOMICAL DEFINITIONS.

ASTRONOMY.

PART I.

CHAPTER I.

INTRODUCTION.

MAN, in this his fleeting mortal state, is a circumscribed finite creature, he cannot embrace infinity; yet from what passes within his own mind, his perceptions demonstrate to him, that his Soul is capable of advancing in knowledge from step to step as long as it exists; and hence he is led to the pleasing hope, that although, for the present, his Soul be imprisoned in a material mortal body, yet it will continue to exist throughout eternity; and with the aid of *omnipotence*, still advance for ever in the progress of knowledge and improvement. It is to be hoped, that so soon as the Soul shakes off this mortal clay mansion, it bids defiance to Gravitation, it shall no longer be confined to this Earth, by the matter of the body gravitating towards the Earth's centre, and thereby confining it to the earth's surface. It can no more be affected thereby. Matter influences it no longer. Distance of place is commensurate to it, and local motion within its sphere. It flies with the rapidity of thought from system to system, and employs and delights itself in contemplating the beauty, order, harmony, and excellence of each; and through eternity finds still more and more reason, to admire, love, and adore the *Supreme Being*, whose unbounded omnipotence, goodness, and mercy, it is for ever employed to acknowledge.

For finite man to give an adequate idea to the human mind, of the *infinity of space*, is impossible. It is boundless in every direction, its centre is every where, but its limits no where. In itself, it is an *infinite void* or *vacuum*. Were we to fly with one million times the rapidity of the particles of light, whose swiftness is some millions of times greater than that of a cannon-ball, and continue our flight in the same direction for millions of millions of years, still we could never reach the end. The same unbounded space lie before us: Here then our finite conceptions are lost! *Deity* alone can comprehend infinity!

In this infinite space it hath pleased the Omnipotent to create an infinite number of Suns with their systems of worlds or planets moving round them, and nourished by the vivifying light and heat of their respective Suns. But, at present, as we know nothing of any of these innumerable systems, but of that of our own; yet we conclude from analogy that they are all in a great degree similiar to ours; and regulated by the same immutable laws. We shall therefore proceed to give an account of the *Solar System*, that is, the system of our Sun, which is also called the *Mundane System*, from the Latin, *mundus*, the world; because the Earth on which we live belongs to it. And then infer that all the other numberless systems are similarly circumstanced and regulated in their respective motions, &c.

OF THE SOLAR SYSTEM.

The nature of our plan does not authorise us to take up the student's time, by entertaining him with

a description of all the different visionary Hypotheses and Systems imagined by ancient Astronomers, to account for the various phenomena of the heavenly bodies. These he may meet with, if led by curiosity, in the writings of our superficial or historical Astronomers. Our design being nothing more or less than to instruct him in pure and demonstrative Astronomy, we shall therefore proceed immediately to the description of this true system; sometimes called the Pythagorean or Copernican system; these two eminent men Pythagoras and Copernicus, having taught it, the former above 2000 years ago, and the latter revived it 300 years ago, after it had been disused or denied by the ignorant for more than 1700 years.

The Sun with the planets and comets which move round him as their centre, constitute what we call the Solar System. Those planets which are near the Sun not only finish their circuits sooner, but likewise move faster in their respective orbits, than those which are more remote from him. All the motions of the planets are performed from west to east round the Sun, in orbits that are nearly circular, but not exactly so; for they are truly Ellipses.* Their names, order, magnitudes, distances, and times of their periodical revolutions, are as follow:

The Sun, ⊙, an immense and an amazing globe of fire,† is 1500000 times larger than our Earth, and placed near the common centre or lower focus of the orbits of all the planets: He turns round his axis from west to east, in 25 days, 14 hours, nearly; as is evident by the motion of spots seen on his surface. His diameter is computed to be 883240 English miles; and by the various attractions of the circumvolving planets he is agitated by a small motion round the common centre of gravity of the whole system. All the planets as seen from him, move the same way, to wit, from west to east; but as seen from the Earth or any other planet, the others appear also sometimes to move from west to east, at other times stationary or at rest, and again appear to move in a contrary direction, namely, from east to west: but their real motion in the heavens, as seen from the Sun, is constantly from west to east, the other apparent motion from east to west, or rest, is only *relative*, and is caused by the observer being situated on the Earth and carried with it round the Sun. But here it seems necessary to remark, that *Comets* are another species of planets, which move round the Sun in all directions; they therefore do not follow the general order of the others, of moving from west to east; for some have been found to move in a quite opposite direction, viz. from east to west, others from north to south, others obliquely, &c. But they all follow the universal law of gravitating towards the Sun.

Having mentioned the Sun's turning round on his axis, and as there will be frequent occasion to speak of the like motion of the Earth and other planets, it may be proper here to inform the young tyro in astronomy, that the Sun and planets have no material axis to turn upon or support them, as in the little imperfect machines contrived to represent them; for the axis of a planet is a right line conceived to be drawn through its centre, about which it revolves as if on a real axis. The extremities of this line, terminating in opposite points of the planet's surface, are called its *Poles*; that pointing towards the north part of the heavens, the North Pole, and the opposite one, the South Pole. A bowl or globe whirled from one's hand into the open air, turns round such a line within itself, whilst it moves forward; and such are the lines we mean, when we speak of the axes of the heavenly bodies.

Let us suppose the plane of the Earth's orbit to be a circular leaf of paper perfectly even, without the

* To such of our readers, who are unacquainted with the Conic Sections, it may be necessary to define an Ellipsis for his information: it is described thus,—If a thread be tied loosely round two pins in a table, and moderately stretched by the point of a black-lead pencil carried round by an even and light pressure of the hand; an Oval or Ellipsis will be described. The two points where the two pins are fixed, are called the *foci* or *focuses* of the Ellipsis. Now, the orbits of all the planets are not perfect circles, but Ellipses: and the Sun is placed in or very near one of the foci of each orbit. The distance of either focus from the middle of the Ellipsis, or half the distance between the two focuses, is called the *Excentricity* of the orbit. In the orbits of some of the planets the excentricity is but small, but in others it is considerable, as shall be shewn farther on.

Astronomers are not far from the truth, when they assume the Sun's centre to be in the lower focus of all the planetary orbits; yet the Sun himself has a motion round the common centre of gravity of the whole system. The Sun gravitates towards each planet with the same momentum or force that the planet gravitates towards him; and by this means his centre is removed sometimes more and sometimes less from the lower focus of the planetary orbits. But we shall have occasion to speak more largely on this subject, which involves the most delicate and abstruse theory pertaining to astronomy.

† We have followed Mr. Ferguson and most other Astronomers in calling the Sun a globe of *fire*; but many of our modern philosophers deny that the Sun is a body of *fire*, and assert that the sensation of heat excited in the animal world, is caused only by the Action of the particles of light, issuing from the Sun, on the particles of the atmosphere: and also that vegetation is produced from the same cause. Probably these Gentlemen are right in their conjecture; but we cannot enter into such indeterminate speculations. All we know of the matter is, that the particles of light from the Sun fly with wonderful rapidity; for they are found to travel from the Sun to the Earth in the space of 8 minutes, which is a distance of more than 95000000 of miles: no wonder then that their action on the particles of the atmosphere should excite the sensation of heat; and produce many more effects, with which we are unacquainted.

least eminence or cavity in it; then this plane will pass through the Sun's centre; and let us conceive this plane to be extended to the starry heavens, where it will mark out that circle in the heavens which the Earth, viewed from the Sun, would seem to describe,* in its revolution round him; and which the Sun appears to us to describe round the Earth. This great circle in the heavens is called the *Ecliptic;* and the circumference of this and every other circle, we suppose to be divided into 360 equal parts called degrees, each degree into 60 equal parts called minutes, each minute into 60 equal parts called seconds, &c. The Ecliptic also is supposed to be divided into 12 equal parts of 30° each, called *Signs;* the Sun in his apparent motion round the Earth, travelling through about one sign of the ecliptic every month.

The planes of the orbits of all the other planets likewise pass through the Sun's centre; but these planes extended, form great circles in the heavens different from one another, and from the Ecliptic; one half of each being on the north side of the Ecliptic, and the other half on the south side: consequently the orbit of each planet cuts the Ecliptic in two opposite points, which are called the planet's *Nodes.* The nodes of the several planets are in various parts of the Ecliptic, and therefore, if the planetary tracts remained visible in the heavens, they would in some measure resemble the different ruts of waggon-wheels crossing one another in different parts, but never going far asunder. That node or intersection of the planet's orbit produced to the heavens and the ecliptic, from which the planet ascends northward above the ecliptic, is called the *ascending node* of the planet: and the other which is directly opposite to it, the *descending node.*

On the 1st of January 1813, the ascending node of the Georgium Sidus was in 12° 25′ of Gemini nearly, Saturn's in 22° 3′ of Cancer nearly, Jupiter's in 8° 31′ of Cancer nearly, Mar's in 18° 8′ of Taurus nearly, Venus's in 14° 59′ of Gemini nearly, and Mercury's in 16° 6′ of Taurus: But in our future astronomical calculations we shall shew how to determine *truly* the place of either node of the planets at all times.

It may be necessary here to caution the Student from the confounding the orbit of the Earth and the Ecliptic, as one and the same circle; for they are very different; true they lye both in the same plane passing through the Sun and Earth's centres; but the Ecliptic is conceived to be infinitely higher up in the heavens than the orbit of the Earth: and the whole distance between the Earth and Sun is supposed to vanish in this case, and to be no more than a mere point in the centre of the Ecliptic. The same thing is to be understood when we conceive the planes of the orbits of the other planets to be extended to the heavens, and there to mark out their respective Ecliptics or *celestial orbits.* The real orbit of a planet is that circular or rather elliptical tract or path described by its centre in its revolution round the Sun: And all the planets are kept in their respective orbits by the attractive power of the Sun, and the projectile force impressed upon them at first; between which power and force there is so exact an adjustment, that they continue to move in the same tracts, without calling in the aid of any other principle or agent to assist them to return to their former paths.

Mercury, the nearest planet to the Sun, (that we know of,) is distant from him 36800000 of miles; his diameter is 3224 miles, and he goes round the Sun, (as in the circle marked ☿,) in 87 days, 23 hours, nearly of our time, which is the length of his year; he revolves round on his axis from west to east in 24*h.* 5*m.* according to the latest and best observations as yet made. In his rapid motion round the Sun, he travels at no less a rate than 110000 miles an hour. His light and heat from the Sun are almost seven times as great as ours; and the Sun appears almost seven times as large to him as to us. The great heat on this planet is no argument against its being inhabited; because the Almighty could as easily suit the bodies and constitutions of its inhabitants to the heat of their dwelling, as he has done ours to the temperature of our Earth. And it is very probable that the people there have the same opinion of the cold we endure, as we have of the inhabitants of the Georgium Sidus. If, however, the new philosophy be right, namely, that there is no real heat in the Sun's rays, but that heat is excited by the action of the particles of light on the particles of the atmosphere; then the atmosphere of Mercury, and those of all the other planets, may by the Almighty have been so adjusted in quality and density, that an equality of heat may prevail in all the planets; which seems very probable.

Mercury appears to us with all the various phases of the Moon, when viewed at different times through a good telescope; save only that he never appears quite full; because his enlightened side is never turned directly towards us, but when he appears so

* This would be the case exactly, if the Earth were not disturbed in her orbit by the attractions of the other planets; and also by the motion of the Earth and Moon about the common centre of gravity of the two bodies. Setting aside these perturbations, which produce an exceedingly small effect, this common centre of gravity of the Earth and Moon would describe the great orbit; and the Sun if viewed from this point during a complete revolution would appear to describe the Ecliptic among the fixed stars. But Astronomers consider the Ecliptic and its plane as fixed and permanent, and consequently the Sun and Earth are sometimes a little removed out of it, from one side to the other.

near the Sun as to be lost to our sight through the intensity of the Sun's beams. And as his enlightened side is always turned towards the Sun, it is evident that he shines not by any light of his own, but by the reflected light of the Sun. That he moves about the Sun in an orbit within the orbit of the Earth is also evident; because he is never seen opposite to the Sun, in the heavens, nor more than 28° or 56 times the breadth of the Sun from the Sun's centre. His orbit is inclined to the plane of the Ecliptic in an angle of 7° nearly; and the place of his ascending node as has been already mentioned, is in 16° 6' of Taurus, the opposite one in 16° 6' of Scorpio. The Earth is in these points on the 6th of November and 4th of May; and when Mercury comes to either of these points about these times, at his inferior conjunction, he will appear to pass over the Sun's face or disc, like a dark round spot: Such a phenomenon is called a *Transit*. But in all other parts of his orbit, his conjunctions are invisible; because he either goes above or below the Sun. We shall hereafter shew how to calculate the times when these Transits of Mercury are to happen, with their quantity and duration: and furnish a Table of all those that are to happen during the present century.

Venus, the next planet in order from the Sun, is distant from him, 68800000 miles, and by moving at the rate of 80000 miles an hour in her orbit, (as in the circle marked ♀) she goes round the Sun in 224 days and 17 hours nearly of our time; which constitutes her year: And from the latest and best observations as yet made, she revolves round on her axis from west to east, in the space of 23*h*. 21*m*. Her diameter is 8648 miles: By her diurnal revolution round her axis her equatorial inhabitants are carried round from west to east at the rate of 1200 miles per hour. Her orbit includes that of Mercury; but the orbit of our Earth includes both hers and that of Mercury. For in the same manner, that we proved the orbit of Mercury to be included within the Earth's orbit, can we prove that the orbit of Venus is also included within the Earth's orbit; for both these planets are similarly circumstanced in their apparent motions to us: but with this difference, however, that Venus is seen by us, sometimes 48° east or west of the Sun, whereas Mercury is never seen above 28° from him. The inclination of the axis of this planet to the plane of her orbit is not as yet satisfactorily determined; but if it be as is generally supposed equal to that of our Earth, then her zones are in magnitude similar to ours, and her several seasons also similar to ours, but proportionally shorter. Consequently what we shall advance hereafter on the theory of the Earth and the various phenomena relating thereto, must likewise be applicable to Venus: There is this difference, however, that the face or disc of the Sun appears twice as large to Venus as to us, and consequently the quantity of light and heat from the Sun, at Venus, is double to what it is with us. Mr. Ferguson has entertained his reader, in giving an ingenious theory of the planet Venus, founded on the hypothesis, that her axis is inclined to the axis of her orbit in an angle of 75°, and that she takes upwards of 23 of our days to revolve round on her axis; but later and more accurate observations have furnished us with very different *data*; we are now assured that she performs her diurnal revolution in as many hours as he supposed her to take days; and although the obliquity of her equator to the plane of her orbit has not as yet been satisfactorily determined, yet we know this much already, that its quantity differs but little from the obliquity of the equator of our Earth with the Ecliptic. Mr. Ferguson's whole theory of Venus, therefore, falls to the ground, and is no more than visionary unfounded speculation.

When Venus appears west of the sun she rises before him in the morning, and is then called the morning star; when she appears east of the sun, she shines in the evening after he sets, and is called the evening star; being each in its turn, for the space of 290 of our days nearly. It may at first sight perhaps appear surprising to the reader, that Venus should appear longer on the east or west side of the Sun, than her whole time of revolving round him: but the difficulty vanishes, when we consider that we are carried with the Earth all the while round the Sun the same way, though not so quickly as Venus; and therefore her relative motion to the Earth, must in every period be no more than the excess of her velocity above that of the Earth's. During the time of Venus completing her orbit round the Sun, which is 224*d*. 17*h*. nearly, the Earth advances in the ecliptic 220 degrees nearly, which deducted from 360°, the whole circumference, leaves only 140°, the quantity that Venus is in advance of the Earth, in the time of her revolving round the Sun: then say as 140°, is to 360°, so is 224*d*. 17*h*. to 580 days nearly; the time she takes to complete a whole circumference round the Sun, more than the Earth takes: the half of which, viz. 290 days, is therefore the time of her appearing to us east and west of the Sun.

Venus as well as Mercury, when viewed through a good telescope, exhibits all the various phases of the Moon; but in a much stronger degree than Mercury: as shall be shewn more particularly hereafter. Venus also transits the Sun's disc, like Mercury; but her transits are less frequent than those of Mercury. She seldom transits the Sun more than twice in a century. Two of her transits happened in the course of the last century, viz. on the 5th of June 1761, and on the 3d of June 1769; two more will happen in the

course of the present century; the first on the 18th of December 1874, and the second on the 6th of December 1882. But we shall in the course of our practical Astronomy shew how these transits are calculated, and furnish a table of all those that are to happen for several centuries to come.

The Earth is the next planet above Venus in the system, and the third from the Sun; it is 95200000 miles distant from the Sun, and goes round him from west to east, (as in the circle marked ⊕,) in the space of 365*d*. 5*h*. 48*m*. 48*s*. nearly, from an equinox or solstice to the same again: but from a fixed star to the same again, as seen from the Sun, in 365*d*. 6*h*. 9*m*. nearly; the former being the length of the *tropical* year, the latter that of the *syderial*. It travels at the rate of 67000 miles per hour round the Sun: which motion though more than 130 times swifter than that of a cannon-ball, is little more than half the velocity of Mercury in his orbit. The diameter of the Earth is 7953 miles at a mean, though its equatorial diameter is 17 miles more, and its polar diameter or axis 17 miles less: the equatorial diameter being in proportion to the axis as 230 to 229. The Earth therefore is not a perfect sphere or globe but a *Spheroidal* figure, resembling an orange or turnip, which we call an *oblate spheroid*: this spheriodal figure is produced by the Earth's revolving round on its axis connected with the laws of gravitation; as shall be more fully explained by and by. But it may be proper to inform the young tyro in astronomy, that although the Earth is not a perfect sphere or globe* yet it differs so very little from a perfect sphere, that it is, in most astronomical calculations, taken as one; the difference being for the most part insensible.

The Earth turns round on its axis in something less than 24 hours, from west to east, and thereby causes an apparent diurnal motion of all the heavenly bodies from east to west, and at the same time produces the vicissitudes of day and night. By this quick motion of the Earth on its axis, the inhabitants of the equatorial parts are carried round, from west to east, 1042 miles every hour, whilst those on the parallel of London are carried only 580; besides the 67000 per hour, by the annual motion above mentioned, which is common to all places of the Earth.

The axis of the Earth makes an angle, of 23° 28′ nearly, with the axis of the ecliptic, and keeps always the same oblique direction to it nearly. This oblique position of the Earth's axis to the axis of the ecliptic, makes the celestial equator and ecliptic cut each other in an angle of 23° 28′ (commonly called the *obliquity of the ecliptic*,) which produces our change of seasons, Spring, Summer, Autumn, and Winter, as we shall more fully explain, in the next chapter. That the Earth is round like a globe, appears, 1st, By its shadow in eclipses of the moon, which shadow is always bounded by a circular line; and no other solid body, except a globe, can have a circular shadow in all situations. 2d, By our seeing the masts only of a ship at a distance at sea, while her hull is hid from our sight by the convexity of the water. The same thing happens at sea at a distance from the land; the shore will be hid from the sight by the convexity of the earth, while the tops of the mountains can be seen. 3d, By its having been sailed round by many Navigators. 4th, And lastly. From the universal laws of Gravitation. All the particles of matter of which it is composed, mutually attract each other, and by this principle of attraction they accumulate together, and naturally form themselves into a *round mass*, to make the attraction equal on all sides, and to keep all the parts in equalibrio. The hills and mountains on its surface take off no more from its roundness, in comparison, than grains of dust do from the roundness of a 2 feet globe. Something more than two thirds of the Earth's surface is covered with water, the other small third being land. The exact proportion of the land and water, cannot easily be determined.

The Moon is not a *primary planet*, but a Secondary or Satellite to the Earth; she is an attendant of the Earth; going round it from change to change, in 29*d*. 12*h*. 44*m*. nearly; and round the Sun in company with the Earth every year. The Moon's diameter is 2180 miles; her distance from the Earth is 240000 miles, at a mean; but she is at one time, one fourteenth part of that distance nearer to the Earth than at another time. She goes round her orbit in 27*d*. 7*h*. 43*m*. nearly; moving about 2290 miles every hour; and turns round her axis exactly in the same time that she takes to go round the Earth; which is the reason of her always keeping the same side towards us: and that her day and night, taken together, is as long as our lunar month.

* To enable the Student to conceive this matter in a familiar manner. Let us suppose a Terrestrial Globe of 2 feet diameter, (which is one of the largest size made,) to be reduced down, by the Globe-maker, to the true oblate spheroidal figure of our Earth; then, in that case, it would have its polar diameter or axis lessened only by one tenth part of an Inch, that is, $\frac{1}{20}$ part of an Inch would only be taken off at each of its poles; which is a quantity so small, that it would be, almost, imperceptible in so large a globe. In all problems, therefore, relative to Navigation, we assume the figure of the Earth to be that of a perfect sphere or globe, from which it differs not sensibly: And, in most problems of astronomy, we do the same; but in those accurate and nice problems relative to the moon's horizontal parallax, the exact magnitude of the several radii of the Earth must be specially used.

The Moon is an opaque spherical body like our Earth, and shines only by reflecting the light of the Sun: therefore whilst that half of her, which, is towards the Sun, is enlightened, the other half must be dark and invisible. Hence, she disappears, when she comes between us and the Sun; because her dark side is then towards us. When she is gone a little way forward, we see a little of her enlightened side, which still encreases to our view as she advances forward, until she comes to be opposite to the Sun; and then her whole enlightened side is towards the Earth, and she appears with a round illumined orb, which we call the *full moon*; her dark side being then turned away from the Earth. From the full she seems to decrease gradually, as she goes through the other half of her course; shewing us less and less of her enlightened side every day, till her next change or conjunction with the Sun, and then she disappears as before. That is, in the *crescent*, or first half of her course, from the change to the full; she appears first *falcated*, (horned,) then *dichotomized*, (semicircular or half full,) again *gibbous*, (round-backed,) and lastly *orbicular*, (circular or quite full,) at which time she is in opposition to the Sun: in the *decrease* or *wane*, in the last half of her course; she appears, first *gibbous*, then *dichotomized*, again *falcated*, and lastly dark, at the change, being then invisible to us, and in conjunction with the Sun.

This continual change of the Moon's phases, demonstrates, that she shines not by any light of her own; for if she did, being globular, we should always see her with a round full orb like the Sun. It also proves that she shines upon the Earth by the reflected light of the Sun only. In the diagram of the solar system, her orbit round the Earth, is represented by the little circle marked ☾ upon the earth's orbit; but it is drawn 60 times too large in proportion to the Earth's; and yet is almost too small to be seen by the naked eye. *The true proportion* of the magnitudes, distances, and times of the periodical revolutions of the celestial bodies to one another, can only be represented by *numbers*; but never can be exhibited by Orreries, or geometrical diagrams, is obvious. These, however, help the imagination, though they do not shew the exact proportion.

The Moon has scarcely any difference of seasons, her axis being almost perpendicular to the plane of her orbit: and what is very singular, one half of her has no real darkness at all; the Earth always affording it a strong reflected light in the absence of the sun; while the other half has a fortnight's darkness and a fortnight's light by turns. Our Earth is a moon to the Moon, waxing and waning regularly, but appearing 13 times as big, and affording 13 times as much light as she does to us. When the Moon changes to us, the Earth appears full to her; and when she is in her first quarter to us, the Earth is in its third quarter to her, and *vice versa*. But from one half of the Moon the Earth is never seen at all.* From the middle of the other half, the Earth is always seen overhead in the zenith, turning round almost thirty times as quickly as the Moon does. From the Moon's limb or circle which limits our view of her surface the Earth appears in the horizon. To the Moon's inhabitants the Earth seems to be the largest body in the universe; for it appears 13 times as big to them as the Moon does to us, and nearly 13 times larger than the Sun. Mr. Ferguson endeavours to prove that the Moon has no atmosphere of any *visible density*: but his arguments on the subject are inconclusive, and are refuted by the observations and reasonings of our best modern Astronomers, who have satisfactorily demonstrated that she really has an atmosphere; but its density and height above her surface are not as yet accurately determined. The same Gentleman, also labours to prove that there is no sea in the Moon; but his reasonings here, are as inconclusive as those against the existence of her atmosphere. But whether there be or be not seas in the Moon, at present we cannot decide: probably future observations will determine the matter with certainty.

As the Earth turns round on its axis, the several continents, seas and islands, appear to the Moon's inhabitants like so many spots of different brightness moving over its surface; but probably much fainter at sometimes than others; as our clouds obscure them or leave them: by these spots the Lunarians can determine the time of the Earth's diurnal revolution on its axis; just as we do that of the Sun and other planets. The Moon's axis is so nearly perpendicular to the plane of her orbit, that the Sun never removes sensibly from her equator; the obliquity of her orbit to the ecliptic, which is almost nothing, as seen from the Sun cannot cause the Sun's declination from her equator to have any sensible effect in producing any change of seasons in her.

* This is not exactly the case, for we sometimes see a portion of the Moon's eastern hemisphere, which was not seen in a former part of her revolution, and lose sight of a similar part of her western hemisphere. This is called the Moon's Libration, and is principally caused by her equable motion round her axis, and unequal motion in her orbit. If the Moon moved uniformly in a circle round the Earth concentric with the Ecliptic she would have no libration; but she sometimes moves in her orbit much faster than her mean motion, and sometimes slower, and her motion round her axis being always uniform, she therefore necessarily turns a proportional part of her back hemisphere to us, as in the difference of her true place in her orbit and her mean place. Besides this principal Libration, she has another small one in Latitude, which we shall notice hereafter.

We shall now remark that we have followed Mr. Ferguson in such parts of his book as will bear the test of mathematical demonstration, and modern accurate astronomical observations: but scientific accuracy compels us to dissent from him in all cases in which his speculations will not admit of such a criterion. He tells us that, "the inhabitants of the "moon, on the side next the earth, may as easily find "the longitude of their places, as we find the latitude "of ours; for the earth keeping constantly, or very "nearly so, over one meridian of the moon, the east "or west distances of places from that meridian are "as easily found as we can find our distance from "the equator, from the altitude of our celestial pole." In this he is partly right, and partly mistaken. Let us call that celestial meridian which passes through the Earth's centre to the Lunarians, (as he does) the *first meridian*; and let us suppose that the Earth's centre always keeps over the same point in the Moon's equator (as it nearly does;) then the complement of the Earth's altitude above the horizon of an observer situated on the Moon's equator will certainly be the longitude from the first meridian; but to an observer situated on the first meridian the complement of the Earth's altitude above his horizon will be his latitude from the equator and not his longitude altitude of the Earth's centre; therefore, taken any where between the equator and first meridian towards either pole can neither determine the latitude nor the longitude; but if the latitude of the place be known, the longitude can be found by the resolution of a spherical triangle, except the place be situated on or near the first meridian; in which case the altitude of the Earth's centre will not contribute towards finding the longitude. But of this enough—*humanum est errare.*

The planet Mars is the next in order, being the first above the Earth's orbit, and distant from the Sun 145000000 miles, and by travelling at the rate of 54000 miles an hour, (as in the circle marked ♂,) he goes round the Sun in 686*d.* 23*h.* nearly, of our time; which is the length of his year. He revolves round on his axis from west to east in 24*h.* 40*m.* nearly; and his diameter is 5150 miles. His quantity of light and heat from the Sun is but one half of ours, and the Sun appears but half as big to him as to us. He is of a fiery red colour, and appears sometimes *gibbous*, but never *falcated*; which proves that his orbit includes that of the Earth, and that he shines not by his own light, but by the reflected light of the Sun. To Mars, our Earth and Moon appear like two small Moons, a greater and a less, changing places with one another, and appearing sometimes *falcated*, sometimes half or three quarters illuminated, but never full; nor at most above a quarter of a degree from each other, although they are truly 240000 miles asunder. Our Earth appears almost as large to Mars as Venus does to us; and at Mars it is never seen above 48° from the Sun. The Earth also exhibits all the different phases to Mars, that Venus does to us. To Mars the planet Venus appears as near the Sun, as Mercury appears near him to us; and Mercury can rarely be seen by such eyes as ours unassisted by proper instruments, as he will be always at or so near the Sun as constantly to be involved in his rays. Jupiter, Saturn, and the Georgian planet, are as visible to Mars as to us; or rather a little more visible, Mars being a little nearer those planets than we are. The axis of Mars is perpendicular to the plane of the ecliptic, and his orbit is two degrees inclined to it. In Mars therefore, there is very little change of seasons, so that his inhabitants have their days and nights nearly equal at all times of the year.

Ceres, Pallas, Juno, and Vesta, four small planets, have been discovered within these few years: Their orbits are situated between those of Mars and Jupiter: But a series of actual astronomical observations, for some years yet to come, must be taken, before their exact magnitudes, distances, and times of their periodical revolutions, can be accurately ascertained: This much, however, we are certain of: that their magnitudes or quantities of matter are very small indeed, when compared to those of the other planets: For their perturbating forces upon the motions of the other planets, must, long before their discovery, have been felt in nice astronomical observations, had they been sensible, but no such effects have ever been discovered. Astronomical Tables and calculations made from tables that took no notice of the disturbing forces of these planets, could never have agreed so well with actual observations, had the aggregate quantity of their disturbing forces, upon the motions of the other planets, amounted to any thing sensible. But these planets, frequently passing near each other, must have their motions greatly disturbed, by their gravitating towards one another, and likewise gravitating towards the larger planets; which must render the calculations of their correct places exceedingly intricate and troublesome. Ceres is computed to be 1681*d.* 12*h.* in revolving round the Sun: Pallas 1703*d.* 16*h*; Juno, 1589*d.* 5*h*; and Vesta 1155*d.* 20*h*: But these times want farther observations to correct and confirm them.

Jupiter, the largest of all the planets, is still higher in the system, being about 495000000 miles from the Sun, and by travelling at the rate of 30000 miles an hour, goes round the Sun, (as in the circle marked ♃) in 11*y.* 314*d.* 12*h.* nearly of our time, which is the length of his year. Jupiter is 1500 times larger than our Earth, but the Sun is 1000 times larger than he. The diameter of Jupiter is 94150 miles,

and he turns round his axis from west to east in 9*h*. 56*m*. The prodigious velocity of this planet round his axis is such, that his equatorial parts revolve more rapidly round from west to east, than they are carried round the Sun, the hourly motion, at his equator, from west to east round his axis, being no less than 30940 miles, which exceeds his hourly motion in his orbit, by 940 miles. The gravity of the equatorial parts of this planet are so much diminished by the centrifugal force, caused by his immense velocity round his axis, that the figure of Jupiter differs more from that of a perfect sphere or globe than that of any other planet, Saturn excepted, whose figure is nearly similar to that of Jupiter, and from the same cause. Jupiter is therefore an *oblate spheroid;* his equatorial or greatest diameter, is to his least diameter or axis, as 13 to 12. The axis of this planet is nearly perpendicular to the ecliptic, only making angle of 1° 19′ with the axis of the ecliptic; and the Sun always appears to his inhabitants to be nearly over his equator; so that there is very little change of seasons in Jupiter.

Jupiter is surrounded by faint substances called *belts*, in which so many changes appear, that they are generally supposed to be clouds floating in his atmosphere; for they sometimes appear more faint and broken than at other times, and frequently disappear altogether; then again re-appear, and by turns increase and decrease in magnitude and density. The Sun appears but $\frac{1}{25}$ part so big to Jupiter as to us; so that we receive 28 times more light and heat from the Sun than the inhabitants of Jupiter do. But to make up for this apparent deficiency, Jupiter has a quick succession of day and night: besides he is attended by four Satellites or Moons, some larger and some less than our Earth; so that there is scarcely any part of this huge planet that is not enlightened during the whole night by one or more of these Moons.

The orbits of these Moons are represented by four small circles, marked 1, 2, 3, 4, on Jupiter's orbit, but they are drawn 60 times too large in proportion to it. The magnitudes, distances from Jupiter, and times of the periodical revolutions of Jupiter's Moons, will be seen in our table thereof; which appearing at one view to the reader, must afford him much more satisfactory information than we give him in a verbal exhibition of them. The three nearest Satellites to Jupiter pass through his shadow in every revolution, and are eclipsed: but the orbit of the fourth Satellite is so much inclined to Jupiter's orbit, that it passes by it in its opposition to him without falling into his shadow, during every two years out of six. By these eclipses of Jupiter's Moons, Astronomers have discovered the velocity of the particles of light, and have found that they take eight minutes in passing from the Sun to the Earth: These eclipses also afford an accurate method of determining the longitude of places on our Earth; of which we shall take an occasion to speak in a subsequent part of the work.

Saturn, the most distant from the Sun of all the planets known to ancient Astronomers, is no less than 907700000 of miles from the Sun; and travelling round the Sun at the rate of 20800 miles an hour, (as in the circle marked ♄,) performs his annual circuit round him in 29*y*. 167*d*. 5*h*. nearly of our time, which is the length of his year: Saturn's diameter is 78000 miles, he is therefore 750 times as large as our Earth, and half as large as Jupiter.—Saturn is attended by seven Moons (or probably more) which constantly move round him as their primary, in the same manner as those of Jupiter do. To this planet, the Sun appears only $\frac{1}{90}$ part of what he does to us, and consequently the light and heat he receives from the Sun are 90 times weaker than at the Earth: But his seven Moons in some measure compensate for the deficiency: Besides this planet is encompassed by a *prodigious ring;* much after the same manner that an artificial globe is by the wooden horizon. This ring is extremely *thin*, though it appears double when viewed through a good telescope, and is represented in the figure in such an oblique view as it generally appears to us. It is inclined 30° to the ecliptic; its breadth is equal to Saturn's semidiameter nearly, and its inner edge is distant from Saturn's surface, about the same quantity; so that it touches Saturn in no part, but it is every where elevated above the surface of that planet almost 39000 miles, and its breadth is also 39000 miles nearly: but we shall hereafter give the dimensions of this stupendous appendage of Saturn, according to the latest observations.

There is reason to believe, that the ring turns round on its axis, and that the axis of the ring is perpendicular to the plane of the ring; for when it is edgeways to us, that is, when the plane of the ring passes through the Earth, its thickness seems to increase and decrease sensibly in a short space of time: which variety of thickness must be in the ring itself; and the several thick and thin parts are in succession then turned to us by the revolution of the ring round its axis. Doctor Herschel thinks that the ring revolves round its axis in the same time that Saturn himself takes to revolve on his axis, namely, in 10*h*. 16*m*. The Sun shines almost 15 of our years on one side of the ring and an equal time on the other side; so that each side of the ring is illuminated and dark for the space of 15 years in turn. And if Saturn's axis be perpendicular to the plane of his orbit, as seems most probable, and inclined to the axis of the ring in an angle of 30°; then there is no change of

seasons in this planet; and the shadow of the ring will obscure sometimes a part of one of his hemispheres and sometimes a part of the other, and for a time involve his polar regions in absolute darkness by turns; but if his axis coincides with the axis of the ring: then he has a regular change of seasons, and his torrid zone will extend 30 degrees on each side of his equator; his temperate zones will be each 30 degrees broad; and his frigid zones extend 30 degrees from his poles, and the shadow of the ring be always cast on his winter hemisphere: but it is vain to indulge in speculations of this nature upon mere hypothesis. Doctor Maskelyne as well as Dr. Herschel's observations seem, however, to favour the latter hypothesis, and fix the plane of the ring directly over Saturn's equator. This matter may, probably, hereafter be settled with certainty.

Jupiter is the only planet that can be seen from Saturn, of all those included within his orbit; for, Mars, and much more so, the Earth, Venus, and Mercury, will appear to him so near the Sun as to be constantly in the Sun's beams. But the Georgium Sidus' orbit being beyond his, therefore that planet must have been seen and known by his inhabitants, for ages before we discovered it. Saturn's orbit is inclined to the ecliptic in an angle of 2° 30′ nearly. We have noticed already that Saturn has 7 Satellites or Moons moving round him. Two of these Moons are not long discovered. Dr. Herschel, who has made so many new and wonderful discoveries in the heavens, and who discovered these last two Moons of Saturn, thinks, that probably this planet may have more than seven.

The Georgium Sidus, (or Herschel, as it is called by some Astronomers,) is the most distant planet from the Sun, that we know of, at present; this planet was unknown to astronomers till the year 1781, when it was discovered by Dr. Herschel. And although this planet was not discovered till that period, yet Doctors Halley, Bradley, and several other eminent astronomical observers had asserted, that Saturn was agitated by some disturbing force beyond his orbit, the reason of which they could not then account for, on the principles of gravitation; but the discovery of this planet has obviated the doubt. The distance of the Georgium Sidus from the Sun, is more than double that of Saturn; so that it is no less than 1827000000 miles from the Sun; which is more than 19 times the distance of the Earth from him. The Georgium Sidus performs its revolution round the Sun in 83*y*. 150*d*. 18*h*. syderial time; its diameter is 35120 miles, or probably more. Six Satellites have been discovered by Dr. Herschel moving round this planet.*

Having given a brief account of the seven principal regular planets with their Satellites, and the four newly discovered minor ones, belonging to the solar system: we come now to the description of another species of planets, called Comets, which also belong to it; and move round the Sun in all directions. The orbits of the Comets, as well as those of all the other planets, are Ellipses; but with this difference, however; namely, that the orbits of the planets differ but little from perfect circles, the *excentricity* of each bearing only a small proportion to the diameter; but the orbits of the comets differ widely indeed from circles, they are very *oblong ellipses*, being exceedingly *long and narrow*, and the excentricity of each is so great, that the two foci of the ellipses are so far removed from each other, that each focus is situated almost at the extremity of the longest diameter of the ellipsis. The Sun's centre is placed in the lower focus of each orbit, as it is in the case of the other planets. That point, in a planet's orbit, nearest the Sun, is called the *Perihelion*, and the one opposite to it, being most distant from him, the *Aphelion*. The comets in their perihelions come very near the Sun, and in their aphelions are at an immense distance from him. Some comets in their perihelions have passed so near the Sun as not to be more than the length of his diameter from his centre. In their coming down in their orbits towards the Sun, their velocity constantly encreases till they reach their *perihelion*, when it is the greatest; (and almost beyond calculation in magnitude) but having passed their perihelions, in their flight from the Sun, their velocity gradually decreases till they reach their aphelions, when it is the least; and in the opposite points of the orbit, the velocity of the comet in its descent to and ascent from the Sun is the same. In their ascent from the Sun they fly off into the remote regions of space, probably far beyond the orbit of the Georgium Sidus. But the motion of the comets as well as that of all the other planets round the Sun, is regulated by one *universal law*, common to all bodies moving round a fixed point: Namely, *the cen-*

* Doctor Herschel has discovered that the motion of the Satellites of the Georgium Sidus is retrograde; that is, that they move round this planet from east to west; and as the information is from such high authority, we can scarce doubt the fact; yet it seems strange, that these Satellites should not travel in their journies in the same direction that those of the other planets do, and their own primary does, but take quite an opposite direction. True we can assign no reason why a planet should rather move from west to east, than from east to west; yet since the fact is, that all the other planets, whether they be primaries or secondaries, do certainly move in their respective orbits from west to east, there is, therefore something uncommon in the disposition of these *little remote worlds* that causes them to take an opposite direction to their primary in journeying round it. The immense distance of these small bodies from our Earth, renders the solution of the problem, for determing the direction of their motion, almost impossible.

tre of the moving body is always in one and the same plane, which plane passes through the fixed point; and that right line which connects the centre of the moving body and the fixed point, called the Radius Vector, always describes equal Areas or spaces in equal times. And hence the velocity of the moving body will always be inversely as the length of the Radius Vector. To make this more plain, let us suppose the Radius Vector, of a planet moving round the Sun, to be at one time double of what it is at another; then it will describe an equal Area, in this case, in the same time with half the velocity it required in the other; and if the Radius Vector of a comet be 1000 times longer in its aphelion than in its perihelion, then its velocity in its aphelion will be 1000 times slower than in its perihelion.

Our plan will not allow us to enter into hypothetical speculations on the nature and use of these erratic bodies to our system, nor can we positively say that they are or they are not inhabited. If, according to the old philosophy, the Sun be an *immense body of fire*, what a prodigious heat must the comets endure in their perihelions, and a proportional quantity of cold in their aphelions? Sir Isaac Newton computed the heat of one comet in its perihelion to be 2000 times greater than that of *red hot iron*, and that it would require some centuries in cooling: but if the new philosophy be right, and there be no real heat in the Sun's rays, then all this vast inequality of heat and cold which the comets were supposed to endure, vanishes; and therefore their inhabitants (if they have any) can suffer no inconvenience on that account. The tails of comets are always projected towards that part of the heavens which is opposite to the Sun. The comet that appeared towards the end of 1811 and disappeared in the beginning of 1812 must have been seen by most of our readers; therefore any further description of these bodies seems unnecessary in this place.

Having now given as comprehensive a view of the solar system as was consistent with our plan; we now propose that the student shall take his eye off this single system, and direct his sight to the starry heavens; that boundless expanse, gloriously bespangled with stars, infinite in number, and incomprehensibly distant from us, and from one another. These we call *Fixed Stars;* because they never change their positions or place in the heavens in respect to one another; and likewise to distinguish them from those little bodies belonging to our system called planets or wandering stars; which, although several of them appear to us to be much larger than any of the fixed stars, yet Jupiter, which is the largest of the planets, is, probably, as far inferior in magnitude to any fixed star as he is to the Sun. The distance of the nearest fixed star is so great, that no telescope has ever yet been made capable of magnifying it. Dr. Herschel's great telescope, which has the highest magnifying power of any we know of, is found incapable of magnifying any fixed star, in the least. True indeed it shews millions of fixed stars that are invisible to the naked eye, yet does not magnify one of them. The whole solar system seen from one of the nearest of the fixed stars, would appear but a mere point; and were not the Sun a luminous body in the centre of the system, the whole system would be invisible. That this is an absolute fact we demonstrate mathematically: for the Earth in its annual circuit round the Sun, must at one time of the year be 190400000 miles nearer to those fixed stars that are situated over its orbit at one season of the year than at another; yet astronomers were never able to find any sensible change in the distance. The nearest of the fixed stars to us must be many thousands of times more distant from us than we are from the sun. What then must be the distance of those which we call telescopic stars, which are hid from the unassisted eye. Sirius, which we suppose, on account of its magnitude, to be the nearest fixed star to our system; yet is so immensely distant from us, that a cannon-ball, continuing to fly with the same swiftness it left the cannon's mouth, would require, at least, one million of years, in passing from the Earth to that star. Yet this distance, notwithstanding its immensity, is but a mere point compared with that of others, which is still so much greater, that were the cannon-ball to continue its flight for ever, it could never reach them. The particles of light are the swiftest bodies we know of; for they fly from the sun to the Earth in the space of eight minutes, and a cannon-ball would require almost 29 years in performing the same journey; yet there must be stars, in infinite space, such, that their light has never yet reached the Earth, and which never can in any finite time, were it millions of millions of years. One drop of water bears a determinate and fixed proportion to the magnitude of the whole Earth, and the proportion can be exactly assigned in numbers: but the proportion between the solar system and one thousand more of equal magnitude added to it; to the aggregate of all the infinite number of systems comprehended in boundless space, can never be assigned by numbers, no more than one minute of time can measure eternity: Infinity is incommensurate to finiteness: and the finite and limited understanding of man is unable to embrace unlimited infinity. For let us figure to ourselves, thousands of thousands of systems of worlds moving round their central suns as in our system; yet will such an immense assemblage of systems and worlds, be no more than a mere point compared to the *infinite whole!* So that were our Sun with all his attendant worlds, and ten thousand other systems

similar to it, to be annihilated, they could never be missed by an eye that could take in the boundless whole! *God* alone, the *omniscient Deity*, is acquainted with the whole, and all its infinite parts!

Every fixed star we see in the firmament of heaven, together with the infinite number of others hid from our sight, we have the justest reasons to conclude, is a Sun like ours, dispensing his nourishing light and heat to a system of worlds moving round him; all regulated in their several periodical revolutions by perfect and immutable laws, founded in infinite wisdom and beneficence.

Astronomers have divided the fixed stars into certain classes called *Magnitudes*. Those that appear the brightest and largest to us are called Stars of the *First Magnitude;* those next in brightness and size, Stars of the *Second Magnitude*, and so on to the *Sixth Magnitude*, which contains stars of the least brightness or size visible to the naked eye; and lastly Stars of the *Seventh Magnitude* or *Telescopic Stars;* to this seventh and last class, belong all those stars that are invisible to the naked eye, and can only be seen with the assistance of a telescope: the number in this class is infinite. But it is necessary to inform the student, that this classification of the fixed stars into *magnitudes* does not suppose that all the stars of the *first* or any other magnitude are exactly of the same size and brightness; because very few can be found so exactly correspondent; but they are those that were judged to come the nearest to an equality in brightness and size; besides different astronomers have put some stars in the first class, which other astronomers put in the second, &c. The fixed stars are also, by astronomers, divided into certain parcels or contiguous collections called *Constellations*; to which ancient astronomers gave the names of men, women, birds, beasts, fishes, and other things; as their wild fancies and superstitious notions led them. But we shall treat more at large on the fixed stars, their magnitudes, constellations, apparent motions, &c. in our third chapter, to which we refer.

And now having in as comprehensive a manner as the brevity of our plan and size of our book would admit, laid open to the student's sight an unclouded view of the universe in general, and of the solar system in particular; it naturally follows that the young tyro's mind begins to expand and to be roused into action; it has been touched by a spark from the fountain of knowledge; and now it burns with ardent desire, to approach still nearer and nearer to that inexhaustible reservoir, and to drink largely and unremittingly of its delectable waters: he has commenced the study of that science, which of all others appears best suited to answer his wishes and to satisfy his desires; and it is to be hoped, that as he advances, his regard for astronomy will increase with his progress in the science. By this sublime science we are perfectly convinced that our Earth is no more than a *single atom* in infinite creation; nay, the whole solar system and a thousand others added to it, are but as nothing to the infinite whole. Astronomy, therefore, enlarges the human mind to proper and elevated ideas of the *Deity*, far beyond those it can attain from any other study: it compares and shews things in their just and proper light. And seeing this noble science reduces our Earth to what it really is, a mere insignificant atom in creation; what then are continents, islands, empires, and kingdoms on its surface? These sink still farther in the scale, and become, as it were, less than nothing in the *great whole!* Surely, then, the study of astronomy must tend to check the ambitious; to humble, mortify, and reprove the proud, haughty, and supercilious; to shew the wordly minded and avaricious man the insignificance of his pursuits; to rouse the unthoughtful, and to alarm the sinner: but to encourage and comfort the virtuous.

An astronomer compares the fall and redemption of man with each other, and views them in their proper light. If man had not fallen, probably, he should never have died, but have continued in a state of happiness in the terrestrial paradise for ever; and therefore must have remained in a state of confinement in this little world during eternity; the gravity of his material body always keeping him confined to it: but thanks to the beneficent *Deity*, who hath ordered it otherwise. True, that by reason of man's fall *death* must pass upon all; but this only delivers us from our imprisonment: eternal life and immortality then lye before us, through our redemption. Our condition is then infinitely more desirable than probably it was before the fall. We shall then be at full liberty to traverse the boundless regions of space; view and comprehend the exact number, magnitudes, orders, distances, and periodical revolutions of the several bodies belonging to each system, as we travel on; and be continually increasing our stock of knowledge in eternity. There will not then be found any thing so minute or so large as to escape our notice, nor of which we shall not be able to form a just and perfect conception. And, for our comfort, we shall have the assistance of the beneficent *Deity* himself, in our delightful research. Astronomy is therefore the most sublime, delightful, extensive, useful, and interesting study, that can be pursued and cultivated by man!

CHAP. II.

Of the Phenomena of the Interior Planets, as seen from the Earth.

Introduction.—BEFORE we begin to illustrate the phenomena of the planets as seen from the Earth; it

is necessary that the student should previously be made acquainted with certain circles and their dependencies, which astronomers have conceived to be marked out in the heavens. The first of these circles which demands his attention is called the *Horizon*, (from a Greek word signifying, to limit or terminate) because it limits or bounds our view of the heavens, and divides the visible from the invisible part of the heavens. Astronomers make mention of two horizons, as belonging to the same point on the Earth's surface, namely the *rational* and the *sensible;* but these two horizons coincide in the starry heavens and become one and the same circle.

ILLUSTRATION.—PLATE 2. FIG. 1.

Let DK, represent the Earth, the point C its centre; and let AZBN, be a great circle in the heavens concentric with the Earth, and passing through Z, the *Zenith* or highest point of the visible hemisphere, and N, the *Nadir* or lowest point of the invisible hemisphere; also, let D be a point on the Earth's surface on which an observer is situated; conceive a plane AB, to touch the Earth in the point D, and this will be the plane of the *sensible horizon*, and will coincide with a small portion of the Earth's surface at the point D; let another plane HO, passing through the Earth's centre be parallel to the plane of the sensible horizon, and this will be the plane of the *true* or *rational horizon:* this plane of the rational horizon being produced to the heavens, will there mark out a great circle that will divide the heavens into two equal parts; the plane of the sensible horizon being also produced to the heavens will mark out a circle in the heavens parallel to the former, and only distant from it by the Earth's semidiameter; but this distance vanishes, and becomes a mere insensible quantity, or rather nothing, in the sphere of the fixed stars. Thus, AH, or BO, which are equal to DC, the semidiameter of the Earth, becomes a quantity inconceivably small among the fixed stars, that is, an absolute nothing. Every fixed star we see is many times larger than our Earth, yet they appear no more to us than mere lucid points, at the same time the sensible and rational horizons must strike the same fixed star; the distance between the sensible and rational horizons is therefore nothing at all in the heavens; and hence the sensible or rational horizon is one and the same great circle; which we define briefly thus. *The Horizon is that great circle in the heavens which limits or bounds our view, and divides the heavens into two equal parts, called the visible and the invisible hemispheres; it is every where 90° from the Zenith and Nadir, which are its poles.*

The four cardinal points of the horizon are the north, east, south, and west points of it: and these divide the horizon into four equal parts of 90° each; so that there are 90° between the north and east points, 90° between the south and east, 90° between the south and west, and 90° between the north and west. These four quarters of the horizon, are again subdivided by mariners into 8 equal parts of 11° 15′ each, which they call points of the compass; so that the whole horizon, or mariners' compass, which represents it, is thereby divided into 32 equal parts or points; the names of which the student may learn in the space of half an hour, when his inclination disposes him to it.

The *Meridian* is that great circle in the heavens, which passes through the Zenith and Nadir and north and south points of the horizon and cutting it at right angles. When the Sun's centre in his apparent diurnal motion round the Earth, arrives at this circle, it is then apparent noon or mid-day: and when the centre of any celestial object transits the meridian, it is said to *culminate;* because at that time it has the greatest altitude above the horizon.*

As the Earth by its diurnal motion round its axis, moves from west to east, therefore an observer on its surface being carried along with it, will have his horizon and meridian also carried along with him the same way, namely, from west to east; and as he cannot perceive his own motion, he will see the celestial bodies apparently moving in a contrary direction, from east to west; and those bodies that were before under his horizon and hid from his sight, will begin to appear and *rise* above the east side of his horizon, and continue to rise higher and higher till they come to the meridian and culminate, after which they will gradually fall lower and lower till they come to the west side of the horizon, when they will *set* under it and disappear. Therefore those celestial bodies, that at any time are situated west of the Sun, culminate before him, and generally rise and set before him, (though not always,) and those that are east of him, culminate after him, and generally rise and set after him; (though not always, as shall be hereafter explained, when we come to treat of the fixed stars.)

THEORY OF THE INFERIOR PLANETS

MERCURY AND VENUS. PLATE 2. FIG. 2.

Let S be the Sun, ABCDEFGHIKLM, the orbit of Venus round the Sun, and let T, be the Earth. An observer in the Sun will see Venus move once round him from west to east, in the space of 224*d*.

* This is exactly true with respect to the fixed stars; but with all erratic bodies such as the Moon; the declination often changes quicker than the altitude, when near the culminating point: hence the body is a little to the East or West of the meridian when its altitude is the greatest.

17*h.* of our time nearly, and the Earth in 365*d.* 6*h.* nearly; and as Venus moves more quickly than the Earth, he will perceive, that in the course of 580 days nearly, she will have made one complete revolution round him more than the Earth: an observer, therefore, situated on the Earth, not being sensible of his own motion, will perceive Venus to have performed one complete revolution round the Sun in the space of 580 days, during which period the Earth is relatively at rest in regard to Venus: and during that time she must have been seen by him in all positions and situations in her orbit in respect to the Sun. When Venus is at the point A, in her orbit, an observer at the Earth, T, will see her and the Sun together in conjunction, in the heavens, at the star R: and this is called her *inferior conjunction:* and when Venus arrives at the point, G, in her orbit, he will also see her and the Sun together in conjunction in the heavens at the star R, and this is called her *superior conjunction:* that is, Venus, at her inferior conjunction with the Sun, is in the lowest and nearest point of her orbit to the Earth, and is below the Sun, between him and the Earth: but in her superior conjunction she is in the highest point of her orbit and most distant from the Earth, being above the Sun. As Venus moves in her orbit from the point A, or inferior conjunction to the points, B, C, D, she appears to the observer at T, to move from the Sun to the westward, and will be seen in the heavens, at the stars, Q, P, O; and as she advances on in her orbit to the points, E, F, G; she will appear to move back again eastward to the Sun, in the heavens, returning from the point, O, to the points, P, Q, R, when she will be in her superior conjunction, at R: as she proceeds on, to the points H, I, K, she will then be seen eastward of the Sun, and to have moved to the stars, U, V, W; and lastly, as she travels on, to the points, L, M, A, she appears to return back again in the heavens westward from the point, W, to the points, V, U, R, when she is in her inferior conjunction again. That arc in the heavens which shews the distance between the centres of the Sun and Venus, to an observer at the Earth, is called her Elongation. And in general, the Elongation of any planet, is that arc which appears in the heavens, to an observer on the Earth, between the Sun's centre and that of the planet; or it is the number of degrees that the planet appears, to us, east or west of the Sun. When Venus is at the points, D and K, of her orbit, she has her greatest Elongation, which is about 48°; and at all other parts of her orbit it is less; and at both conjunctions, it is evident she has no Elongation from the Sun.

Hence it follows, that while Venus moves from her inferior conjunction to her superior, she appears in the heavens west of the Sun, and therefore rises, culminates, and sets before him, and of consequence, is then the Morning Star: but from her superior conjunction to her inferior, she appears in the heavens east of the Sun, and consequently rises, culminates, and sets after him, and is then the Evening Star. Hence also as Venus moves through the lower part of her orbit KAD, her apparent motion in the heavens in respect to the Sun is westward from W, to O; that is, in respect to the Sun she moves westerly, from her greatest elongation east of the Sun, to her greatest elongation west of the Sun; and during this period in its middle she passes her inferior conjunction; and performs this her apparent westward motion in the heavens in the space of 140 days nearly: again, as Venus moves through the upper part of her orbit DGK, her apparent motion in respect to the Sun, in the heavens, is from west to east, from O, to W; that is, from her greatest west elongation to her greatest east elongation: in the middle of this period she passes her superior conjunction; and she requires no less than 440 days to complete this part of her apparent course.

We have supposed the Earth at rest in the foregoing illustration, because the phenomena in respect to the Sun would have been exactly the same, by assuming the relative motion of Venus from the Earth, as if we had taken the real motions of both. But there are other phenomena of Venus to be considered, which require us to bring in the absolute motions of the Earth and Venus round the Sun, and not their *relative* motions.

An observer in the Sun will see the Earth moving round him uniformly from west to east, and which it really does; and as he cannot judge by immediate sight, how immensely more distant the fixed stars are from him than the Earth is; he will therefore see the Earth moving uniformly through the fixed stars from west to east, and completing one whole circumference round the Sun in the space of 365*d.* 6*h.* nearly of our time. But to us, who are situated on the Earth and cannot perceive our own motion, the Sun will appear to move round the Earth from west to east in the same time and in the same circle (called the Ecliptic,) among the fixed stars, that the Earth really moved in; but with this difference, however, that the Sun will always be seen from the Earth in the opposite point of the heavens, in which the Earth is seen from the Sun: So that when he appears to us in the constellation Aries, the Earth seen from the Sun is in the opposite constellation Libra: and so on according to the order of the twelve signs.

The Sun's apparent motion from west to east, at a mean, is 58′ 5″ nearly, every day; and when Venus arrives at a certain point, suppose M, in the lower part of her orbit, and her motion in that part of her orbit being then westerly in respect to the Sun; and

if her daily motion then be 58′ 5″; in that case she will appear to us *stationary;* that is, at *rest* in the heavens among the fixed stars. From M, to the correspondent point, B, of her orbit, she passes by A, her inferior conjunction, and during this period her motion westward in the heavens, in respect to the Sun, is greater than the Sun's motion eastward in the ecliptic, wherefore the planet will appear to us, during this time, to move westward in the heavens among the fixed stars: This motion we call *retrograde*, or *in antecedentia:* hence the *retrograde* motion of an inferior planet is greatest at the time of the inferior conjunction. Again, when Venus is arrived at the point B, her motion westward, in respect to the Sun being the same as at M, and equal to the Sun's apparent motion eastward, in the ecliptic; therefore she appears again stationary to us in the heavens. But in moving through every other part in the upper part of her orbit, her motion is *direct* or *in consequentia;* that is, she appears to us to be moving from west to east faster and sometimes slower than the Sun; and when she is in her superior conjunction, her motion *in consequentia* is the greatest. Lastly, when Venus has her greatest Elongation, either east or west from the Sun, her motion is *in consequentia*, and is exactly equal to the Sun's uniform motion *in consequentia*; so that at the times of both her Elongations she appears to us to travel *directly* eastward, at the same rate the Sun always moves that way.

What we have said on the theory of Venus is equally applicable to Mercury, only with this difference, that his periods are shorter; his greatest Elongation from the Sun is only 28°; he is 57 days east of the Sun, and 57 days west of him, and consequently 57 days from one of his superior conjunctions to his next inferior conjunction with the Sun; and 39 days from his greatest east Elongation, to his greatest west; during which time he passes through the lower part of his orbit next the Earth, and which part, as in that of Venus, is convex to the Earth; and in passing through the upper concave part of his orbit to the Earth, he requires 75 days. The times he appears to us in the heavens, to be *stationary*, *direct*, or *retrograde*, are also similar and happen in similar parts of his orbit with those of Venus, but of shorter duration. In the practical part of our work, in which we shall shew how to calculate the exact places of the planets, at all times, we shall also shew how to determine the exact times of the two conjunctions of the inferior planets, with the Sun; and the exact times they are stationary, direct, or retrograde.

Mr. Ferguson is egregiously mistaken, in the times when Mercury and Venus appear to us, in the heavens, to be *stationary*, *direct*, or *retrograde*. He asserts, that the time when these planets have their greatest elongation from the Sun, is also the time that they are *stationary;* but from our preceding illustration, it is not so. When these planets have their greatest elongation from the Sun, their apparent motion in the heavens is *direct*, and exactly equal to the Sun's direct motion easterly. He also says, that from the greatest east elongation to their next greatest west elongation, their motion is retrograde during this period; and, lastly, that from their greatest west elongation to their next greatest east elongation, their motion is direct. But here he goes astray; from our illustration aforegoing, an inferior planet appears in the heavens, *stationary* between the time of its greatest east elongation and its inferior conjunction, and again *stationary* in the time between its inferior conjunction and its greatest west elongation; and during the intermediate time only its motion is *retrograde*, but in all other cases its motion is *direct*. Mr. Ferguson's editors, Messrs. Mackay, and Brewster, seem to have fallen into this, as well as several other oversights of his; as they have not taken care to correct them; which as editors, they should have done.

When Venus is at the points K, and D, of her orbit, she has her greatest elongation, and one half of her illuminated hemisphere is turned to the Earth, and then she appears *dichotomized;* and between these points and her inferior conjunction she is seen *falcated;* but at the inferior conjunction she is quite dark, because her enlightened hemisphere is altogether turned away from the Earth, and her dark one towards it: In the upper part of her orbit between the same points K, and D, she appears *gibbous;* and at her superior conjunction, she is *orbicular*, or quite full; but for a considerable space on each side of that conjunction, she is so much involved in the Sun's rays, that we are unable to see her. When she is in her upper conjunction she is farthest from us, and consequently she appears least in magnitude; but when she is in her inferior conjunction, she is then nearest the Earth, and could we see her at that time, she would appear about 37 times larger, than in the other or upper conjunction. Near the time of her shewing herself dichotomized, or half full to us, she reflects the greatest quantity of light to the Earth: The quantity of light which she reflects to us, will be always in the direct proportion of the area of that portion of her enlightened disc turned to us, divided by the square of her distance from the Earth. The young tyro, if he be acquainted with solving problems, *de maximis et minimis*, in Fluxions can find exactly the two opposite points in the orbit of Venus, when she reflects the greatest quantity of light to the earth. We shall therefore leave the solution of the problem for his future amusement.

The foregoing Theory applies equally to Mercury as to Venus; but the quantity of light, reflected from Mercury to us, is much less than that from Venus; besides Mercury can only be seen for a few days by us, on either side of the Sun; in the other parts of his orbit we lose sight of him in the Sun's beams.

THEORY OF THE SUPERIOR PLANETS AS SEEN FROM THE EARTH.

From a proper attention to the foregoing theory of the inferior planets, we shall be able briefly to establish that of the superior ones. For the Earth is a superior planet to Mercury or Venus, in the same way that Mars, Jupiter, and the Georgium Sidus are to the Earth. Let us therefore suppose ourselves to be removed from the Earth to Venus; and there observe the several motions of the Earth as they would appear then to us. Now it is evident that the whole phenomena will nearly be the same, only exhibited in opposite parts of the heavens. For, when Venus appears *stationary* in the heavens to an observer on the Earth, suppose in the constellation Aries, the Earth will appear *stationary* in the heavens to an observer in Venus, but seen in the opposite constellation Libra: Again, when Venus appears to us to run *direct* or *retrograde* in the heavens; the Earth will also appear, to an observer in Venus, to move *direct* or *retrograde*, in the opposite part of the heavens, and in each case will run over the same number of degrees exactly. Lastly, when Venus is seen by us in her inferior conjunction with the Sun, the Earth will be seen, by an observer in Venus, directly opposite to the Sun, in the heavens, because Venus at that time is between the Sun and the Earth: But when Venus is seen by us in her superior conjunction with the Sun; the Earth will be seen by an observer in Venus, in conjunction then with the Sun; because at that time the Sun is between Venus and the Earth.

The application of the preceding theory, to the phenomena of the superior planets as seen from the Earth, is easy: For the Earth is a superior planet to Venus; and therefore exhibits the same phenomena to her, that the superior planets Mars, Jupiter, &c. do to the Earth. The superior planets, therefore, appear to us, in the heavens, to be sometimes *stationary*, at other times to move *retrograde* or *in antecedentia*, and again *direct* or *in consequentia*. They have both *conjunctions* and *oppositions* to the Sun, alternately: But the inferior planets have no oppositions to him; but they have two conjunctions with him, an *inferior* and a *superior*. The superior planets are nearest the Earth at the time of their opposition to the Sun, and farthest from us at the time of their conjunction with him; the difference being equal to the diameter of the Earth's orbit, which is about 190400000 miles: Hence, at the time of their opposition, their apparent magnitude to us is the greatest. In the planet Mars, the difference is very considerable; for at the time of his opposition to the Sun, he appears 25 times larger to us than at the time of his conjunction: admitting that the Sun's rays allowed us to see him then. And this is the reason, that Mars, about the time of his opposition to the Sun, is often mistaken for a Comet, by the unskilful in astronomy. In Jupiter, the difference is not so considerable, in Saturn, &c. still less. The *ratio* of the apparent magnitude of a planet to us, will be always directly, as the square of its diameter, divided by the square of its distance from the Earth. Because the orbit of the Earth is within the orbit of the superior planets; hence these planets, at the times of their conjunctions and oppositions, turn their whole illuminated side to the Earth, as well as to the Sun; and also in the other parts of their orbits, they appear to us, to shew a full orb nearly always: Mars, indeed, in his quartile aspects with the Sun, that is, when he appears 90° from the Sun, shews himself a little gibbous, much in the same manner as the Moon appears about 4 days before or after the full. In the other planets, higher up in the system, the difference is insensible.

The time between a conjunction or opposition of Mars with the Sun, to the same again, is at a mean, about 782 days: but the exact times of the oppositions, conjunctions, and other phenomena of the planets, will be best understood, when we come to calculate them, in our practical part of the work, to which we refer. It may not be improper here, to inform the student, that all calculations relative to the planets, are made from astronomical tables, which shew the *Heliocentric* places of the planets, (from the Greek, signifying the Sun, and a centre,) that is, the places in which they appear, in the heavens, when seen from the Sun's centre. And their *Geocentric* places, (from the Greek, signifying, the Earth, and the centre,) that is, the places, in which they appear in the heavens, when viewed from the Earth's centre, are afterwards found from their heliocentric places, by plane Trigonometry.

CHAP. III.

Of the fixed Stars.

IT seems unnecessary to repeat in this place, what we have said in a general manner, relative to the fixed Stars, in our first chapter; we now propose to

take a more particular view of these heavenly bodies. 1st, then, the fixed stars appear of a sensible magnitude to the bare eye. Mr. Ferguson says the reason of which is; "because the retina is affected not only by the rays of light which are emitted directly from them, but by many thousand more, which falling upon our eyelids, and upon the aerial particles about us, are reflected into our eyes so strongly, as to excite vibrations not only in those points of the retina where the real images of the stars are formed, but also in all other points at some distance round about. This makes us imagine the stars to be much bigger than they appear, if we saw them only by the few rays which come directly from them, so as to enter our eyes without being intermixed with others." We confess ourselves at some loss to comprehend Mr. Ferguson's meaning here, as it seems to us very obscure indeed. That the fixed stars appear immensely larger to the bare eye, than their magnitude and great distance from us will admit of, is certainly fact; but what the true cause of this optical deception is remains as unaccounted for, after we have maturely considered Mr. Ferguson's reasons for it, as it did to us before we took them into consideration. For our own part, we shall venture no obscure conjectures on the subject, but admit the fact; seeing the best telescopes, with the highest magnifying powers, that have as yet been directed to the fixed stars, instead of making them look larger have an opposite effect, and make them appear less; in short our best telescopes reduce the size of the brightest (or largest apparently) of the fixed stars, to mere lucid points. Their distance from us must therefore be immensely greater than human conception can embrace.

Besides the apparent diurnal motion, common to celestial bodies round the Earth, from east to west, which is caused by the Earth's revolution on its axis from west to east in the same time; the fixed stars have another annual apparent motion from east to west round the Earth; they appear to make one revolution round the Earth, in the space of a year, more than the Sun does. The Sun appears to make $365\frac{1}{4}$ revolutions nearly, round the Earth, in the course of a year; but the fixed stars appear to make $366\frac{1}{4}$ revolutions round us in that time: so that the fixed stars seem to move with a quicker motion daily round the Earth than the Sun does: and in the course of a year they gain upon the Sun one entire revolution, advancing daily upon him 3 minutes and 57 seconds of time; and hence the fixed stars *rise*, *culminate*, and *set*, *3m. 57s.* nearly, sooner every day, than on the day before. This apparent annual motion of the fixed stars is caused by the Earth's annual motion from west to east round the Sun; which *real* motion of the Earth round the Sun, produces the Sun's *apparent* annual motion round the Earth from west to east. Let us suppose the Sun and a fixed star to culminate, that is, appear on our meridian, at the same instant of time; on the next day, our meridian which revolves round with the Earth from west to east, will move up to that fixed star almost exactly in the time that the Earth takes to perform one complete revolution, on its axis; but the Sun moving during the time the Earth took to make the revolution will have advanced eastward in the ecliptic $58'.5''$ nearly; so that the meridian, having passed the star, must continue to move eastward, till it arrives at the Sun, which will require about $3'\ 57''$ longer every day; and in the compass of a whole year, the Earth will have made one entire revolution on its axis more, in respect to the fixed stars, than to the Sun: and hence this apparent annual revolution of the fixed stars round the Earth, is obvious. And from what has been shewn, it appears, that the Earth does not require 24 hours to revolve once round on its axis in respect to the fixed stars, or to absolute space; but in respect to the Sun, it does require that time. Wherefore, in the space of 24 hours, the Earth not only makes one complete revolution on its axis, but also $\frac{1}{365}$ part nearly of another; which in the course of a year becomes one entire revolution more; so that the Earth truly revolves $366\frac{1}{4}$ times nearly on its axis in the course of a year; although, in respect to the Sun, in that time, there are only $365\frac{1}{4}$ days nearly.

The fixed stars have a third small apparent motion eastward, caused by the *precession* or rather *recession* of the equinoctial points in the heavens. By this motion the fixed stars appear to move uniformly eastward, either in the Ecliptic or parallel to it, at the rate of about $50\frac{3}{10}''$ every year, or one degree in $71\frac{1}{2}$ years; so that in the space of something less than 26000 years, one entire revolution is completed: and this period is usually called the *great year*. By this apparent motion the stars called the constellation Aries, which about 2000 years ago, were near the vernal equinox, are now advanced to that part of the heavens, which was occupied then by the constellation Taurus; and Taurus is now where Gemini was then; and so on in succession; each constellation having advanced about 30°, or a whole Sign, in the space of 2145 years. The fixed stars therefore do not appear to us now to occupy the same space in the heavens, which they did a little more than 2000 years ago, when the ancients formed them into constellations by a whole sign or 30°. This apparent motion of the fixed stars is produced by the joint actions of the Sun and Moon on the spheroidal figure of the Earth, as shall be illustrated, when we come to speak of the physical causes of the motions of the planets, and also in our theory of the Earth.

In the first chapter we informed the student, that astronomers have divided the fixed stars into seven classes or magnitudes; and also into certain contiguous collections or parcels called *constellations;* and to these constellations the ancients gave the names of men, women, birds, beasts, and fishes, &c. and also exhibited *drawings* or *pictures* of the constellations accordingly: but we must inform the student, that not one of the constellations has the least similitude to that to which the ancients pictured it. The ancient names of the constellations are, however, still retained by modern astronomers, to avoid confusion and disagreement, as much as possible, between ancient and modern astronomy

As the Moon and planets appear sometimes on the north and sometimes on the south side of the Ecliptic, but never declining more than 8 or 9 degrees from it; (except Ceres, Pallas, and Juno;) therefore that zone in the heavens extending about 9 or 10 degrees on each side of the ecliptic, and in which the Sun, Moon, and planets are always seen by us, was called by the ancients the *Zodiac;* (from the Greek, signifying a living animal,) because they divided the several constellations of fixed stars within this zone and a little on each side of it, into twelve equal parts called signs, the greater part of which they supposed to have a resemblance to living creatures, and consequently they absurdly named them accordingly. Before we proceed to describe the several constellations it is proper to inform the student, that modern astronomers, after the example of Bayer, have named the stars in each constellation, by the letters in the Greek alphabet; to the largest star in each constellation they affix the first letter, α; to the second or next largest, β: to the third, γ; and so on: so that the letter α, affixed to a star, does not indicate that star to be of the first magnitude, except the constellation, to which the star belongs, have a star of the first magnitude in it, and then in that case, α is its characteristic letter; and so of others.

There is a remarkable tract round the heavens, called *Via lactea*, or the *Milky Way*, from its peculiar whiteness: this tract extends into both hemispheres; in some parts it is of considerable breadth, in other parts it is double; and upon the whole it appears like a vast irregular zone or girdle surrounding the heavens. The whiteness of the Milky Way is now known to be caused by an infinite number of telescopic stars every where scattered through it. There are also in the southern hemisphere certain white spaces, called *Nubes Magellanicæ* or *Magellanic Clouds:* these spaces have such an exact appearance of white flying clouds, that persons unacquainted with astronomy are deceived and believe, that they are absolutely clouds. No doubt but these clouds contain an immensity of stars, but exceedingly small. We know persons who have been at the Cape of Good Hope, and viewed them with tolerably good telescopes, but their magnifying power was too small to shew any of the Nebulæ; their telescopes, however, had sufficient power to shew many thousands of stars in the Milky Way, in the Nebulæ of Orion, &c: but none in the Magellanic Clouds: and hence we conclude that the distance of the stars which compose these clouds, is, in a manner incommensurate to that of any of the telescopic ones.

ASTRONOMICAL DEFINITIONS.

1. A Sphere or Globe is a perfectly round body, having every part of its surface equally distant from a point within it, called the *centre:* a sphere is conceived to be generated by the revolution of a semicircle round the diameter, which remains at rest, till the motion end where it began. And that fixed diameter, which remained at rest, is called the *axis* of the sphere, the extremities of the axis, or two opposite points in the surface, are called the two *poles.*

2. Circles on the surface of a sphere, are either *major* or *minor.* A *major* circle is that which divides the surface of the sphere into two *equal parts* or hemispheres, and its plane passes through the centre of the sphere, and divides the solid into two equal parts; but a *minor circle* is always parallel to some major circle, and divides the surface of the sphere into two *unequal parts,* and its plane also divides the solid into two unequal parts.

3. The *poles* of a major circle are the two points in the surface of the sphere equally distant from that major circle, and every where 90° from it: and the poles of a major circle are also called the poles of all the minor circles parallel to it.

4. Major circles, which are perpendicular to another major circle, are called *secondaries* to it: these secondaries all pass through its poles, and their planes cut its plane at right angles.

5. The *Terrestrial Equator* or Equinoctial line, is that imaginary major circle on the surface of the Earth, which divides it into the northern and southern hemispheres, and is every where 90° from the north and south poles of the World, which are also its poles.

6. The *Celestial Equator* is that major circle in the heavens, which is directly over the terrestrial equator, and its poles are directly over the terrestrial poles, or north and south poles of the World. The terrestrial and celestial equators are in one and the same plane, and have one common centre, namely the centre of the Earth.

7. *Meridians* are secondaries to the equator: Or, *terrestrial meridians* are major circles on the surface of the Earth, running north and south, passing through the north and south poles of the world, and cutting the terrestrial equator perpendicularly: and the *celestial meridians* are directly over the terrestrial, and pass through the celestial poles, and cut the celestial equator at right angles. There are 90° of every meridian between the equator and each pole. The terrestrial meridians are called circles of *terrestrial Longitude;* and the celestial meridians are called circles of *celestial right ascension.* The meridians are infinite in number, on both spheres.

8. Minor circles on the Earth parallel to the terrestrial equator are called *Circles of Terrestrial Latitude,* or *Parallels of Latitude:* and those minor circles, in the heavens, parallel to the celestial equator, are called *Parallels of Declination.* They are infinite in number on both spheres: of the infinite number of these minor circles parallel to the equator, there are four belonging to each sphere specially noticed; namely, the two *tropics* and two *polar circles.* The *Tropics* (from the Greek, signifying, to turn) are two circles parallel to the equator, each being $23\frac{1}{2}°$ nearly from it. That on the north side of the equator is called the *Tropic of Cancer,* and the other on the south side, the *Tropic of Capricorn.* The two celestial tropics are the north and south limits of the Sun's apparent course in the heavens; astronomers having found by observation that the Sun never declines more than $23\frac{1}{2}$ degrees nearly from the equator. The polar circles are $23\frac{1}{2}$ degrees nearly from each pole; or $66\frac{1}{2}$ degrees from the equator. The *northern polar circle* is called the *arctic* circle (a word derived from the Greek, signifying a Bear) because it passes through one of the constellations in the heavens called the greater Bear; and the southern polar circle, the *antarctic circle,* (from the Greek, signifying, opposite and a Bear,) because it is opposite to the arctic circle. The terrestrial tropics and polar circles, are the same number of degrees from the terrestrial equator, that the celestial ones are from the celestial equator. These four minor circles divide the Earth into five parts called Zones.

9. The *Ecliptic** is a major circle in the heavens, inclined to the equator in an angle of $23\frac{1}{2}$ degrees nearly, and cutting the equator in two opposite points called the *Equinoctial points;* and the two points of the ecliptic equidistant or 90° from these, being the greatest distance the ecliptic is from the equator, are called solstitial points; and these four points are called the four cardinal points of the ecliptic. The equinoctial points are called the first points of Aries and Libra, and the solstitial points, the first points of Cancer and Capricorn. The Ecliptic is divided into 12 equal parts called signs; 6 of which are on the north side of the equator, and 6 on the south: the names and characteristic marks of the 12 signs of the ecliptic, are the same with the 12 constellations of the zodiac, which are shewn in the last chapter, which see. When the Sun passes the first point of Aries he enters into the northern hemisphere, and when he passes the first point of Libra, he gets into the southern hemisphere.

10. The *Declination*† of a celestial object is that arc of its meridian contained between its centre and equator; or it is the distance in degrees that any celestial object is north or south of the equator, and can never exceed 90 degrees.

11. The *Colures* (derived from two Greek words,

* Ecliptic (derived from the Greek, signifying to faint or swoon away), because when the Moon comes to or near this circle, at the time of the new or full Moon, an eclipse of the Sun or Moon must happen; and hence this circle was called *Ecliptieus Circulus,* that is, the Circle of Eclipses.

† In the course of a year the Sun crosses the equator twice; viz. about the 20th of March, and 22nd of September, at which times he is in the first points of Aries and Libra, and has no declination, being then in the celestial equator: three months afterwards, viz. about the 21st of June, and 21st of December, he enters the first points of Cancer and Capricorn, and is then in the tropic of Cancer or Capricorn, when he has his greatest north or south declination, which is a little less than $23\frac{1}{2}$ degrees: (the exact quantity will be shewn in the practical part of the work.) But the Moon crosses the equator twice in every *periodical month,* (or 27 days, 7 hours, and 43 minutes nearly,) at which times she is on the equator and has no declination, and about 7 days before and after these times she has her greatest north or south declination, which varies from 18° to 29° nearly, being at one time $5\frac{1}{2}$ degrees less than the Sun's greatest declination, and at another time $5\frac{1}{2}$ degrees greater than it; according to the place of the Moon's ascending Node. In March, 1811, her greatest declination was a *minimum* for her period, which is about $18\frac{1}{3}$ years nearly; and was then only a little more than 18° north or south; but 9 years afterwards, namely in 1820, her greatest declination will be a *maximum,* for her course, viz. 29° nearly, north or south. The planets also cross the equator twice during their period, and have likewise their greatest north and south declination, which is also generally something more or less than the Sun's greatest declination, according to the place of their ascending node, and the inclination of their orbits to the plane of the Ecliptic. But the fixed stars keep almost always on the same side of the equator, and at the same distance nearly from it; so that their declination is almost a constant quantity, changing only a few seconds of a degree annually.

The Moon also changes her right ascension very rapidly, running through 360° or 24 hours of time, during her periodical month; so that she makes as much right ascension in a month as the Sun does in a year. She also runs down the whole circumference or 360° of celestial Longitude in her periodical month. But the planets only complete a whole circumference of celestial Longitude and 24 hours of right ascension, during their respective periods. And the fixed stars change their Longitude only about 50 seconds generally of a degree annually, and their right ascensions commonly from 3 to 4 seconds of time annually.

signifying, maimed, and the tail,) are two meridians passing through the four cardinal points of the ecliptic, and cutting one another at right angles in the celestial poles of the equator. That which passes through the equinoctial points, or first points of Aries and Libra, is called the *Equinoctial Colure;* and the other, which passes through the solstitial points, or first points of Cancer and Capricorn, passing through the celestial poles of the ecliptic and equator, and cutting both circles at right angles, is called the *Solstitial Colure.* These two circles are of considerable use in practical astronomy, and the sphere is commonly projected on the plane of one or other of them.

12. *Terrestrial Latitude*, or the Latitude of a place on the Earth, is an arc of the meridian of the place intercepted between the place and the equator; or it is the nearest distance in degrees, of a place from the equator: or the Latitude of a place is an arc of the celestial meridian contained between the zenith of the place and the celestial equator. Latitude is called *north* or *south*, according as the place is situated north or south of the equator, and it can never exceed 90 degrees. Note, the greater the latitude the colder the climate of the place.

13. The *Horizon* is that major circle in the heavens, which divides the visible from the invisible hemisphere, and is every where 90 degrees from the zenith and nadir. A more full illustration of this definition is given in the beginning of the second chapter, which see.

14. *Terrestrial Longitude*, or the Longitude of a place on the Earth, is an arc of the equator intercepted between the meridian of the place and the *first meridian;* or it is the number of degrees, reckoned on the equator, between the first meridian and the meridian of the place, which is equal to the angle at the pole contained between these two circles. The *First Meridian* we assume to be the meridian of the Royal Observatory at Greenwich, and reckon the Longitude east and west from it till it end at 180 degrees. The French and other nations, however, commonly reckon their Longitude from the capital of their country; but this can involve no difficulty in Astronomy or Geography: for, if Paris be in three degrees east longitude from Greenwich, then Greenwich is in three degrees west longitude from Paris; so that the longitudes can easily be changed from the meridian of one place to that of another. The longitude is sometimes reckoned in hours, minutes, and seconds, by allowing 15 degrees to an hour, and proportionally; for if a place be 15° east of the meridian of Greenwich, there is one hour of difference in their respective times, for when it is 12 o'clock at Greenwich it is 1 o'clock at the other, and when it is 1 o'clock at Greenwich it is 2 o'clock at the other, and so on: but if the place is situated in west longitude the case is reversed: that is, having the Greenwich time given to find the hour at any other place having its longitude from the first meridian given. Rule, Turn the given longitude into time, by allowing 15° to an hour, and proportionally: then add this time, if the longitude be east, but subtract if west from the Greenwich time, and you have the time at the proposed place. The converse rule is equally manifest and easy.

15. Secondaries to the Ecliptic, that is, those major circles in the heavens which pass through the poles of the ecliptic and cut it at right angles, are called circles of *Celestial Latitude:* they are infinite in number, and one of them is specially noticed in astronomy, namely, the *Solstitial Colure*, which is also a meridian, or a circle of celestial right ascension.

16. *Celestial Longitude*, that is, the Longitude of a celestial object, is an arc of the Ecliptic contained between the first point of Aries, and a circle of celestial latitude passing through its centre; or it is the number of degrees of the Ecliptic between the first point of Aries and that secondary to the ecliptic, which passes through the centre of a celestial object, reckoned from west to east according to the order and succession of the 12 signs; and it can never exceed 12 signs, or 360°.

17. *Celestial Latitude*, that is, the Latitude of a celestial object, is an arc of a secondary to the ecliptic contained between the centre of that object and the ecliptic; or it is the nearest distance in degrees between its centre and the ecliptic: and it is called north or south according as the object is north or south of the ecliptic; it can never exceed 90°. The Sun is supposed to move always in the ecliptic, and therefore to have no Latitude; but this is not strictly the case; for his centre is sometimes moved a small quantity out of it, north or south, by the disturbing forces of the planets; and although we shall shew how to calculate the exact quantity of his Latitude, yet its *maximum* is so inconsiderable, that it is safely neglected in all astronomical calculations.

18. Minor circles parallel to the ecliptic are called *parallels of celestial Longitude:* they are infinite in number. From the foregoing Definitions the student may perceive; that celestial Latitude and Longitude respect the Ecliptic, in the same manner that celestial Declination and Right Ascension do the Equator, or terrestrial Latitude and Longitude do the terrestrial equator.

19. *The Obliquity of the Ecliptic*, which is equal to the Sun's greatest declination, is the angle of inclination between the Ecliptic and Equator: the obliquity of the ecliptic is not a constant quantity, as some astronomical writers suppose, but is a variable

one, and increases and decreases several seconds of a degree during the Moon's period of 18⅔ years, and even increases and decreases a small quantity during the same year; the causes of which shall be shewn when we come to treat of the physical causes of the motions of the planets: besides, astronomical observations shew, that there is a slow and gradual diminution of about half a second of a degree annually, in the obliquity of the ecliptic.

20. *Azimuth circles*, or vertical circles, are secondaries to the horizon: they pass through the Zenith and Nadir and cut the horizon at right angles: they are infinite in number, whereof two are specially noticed, namely, the *meridian* and *prime vertical*. The meridian is that azimuth circle which passes through the north and south points of the horizon; as was illustrated before, in chap. 2, which see. It is called meridian, (from the Latin, *meridies*, noon,) because when the Sun's centre by his apparent diurnal motion arrives at it, it is the noon or mid-day. The *Prime Vertical* is that azimuth circle which passes through the east and west points of the horizon, and cuts the meridian at right angles in the Zenith and Nadir.

21. The *Azimuth* of a celestial object, is an arc of the horizon contained between the azimuth circle, which passes through its centre, and the north or south points of the horizon; or it is the angle in the zenith contained between the meridian and the azimuth circle passing through the centre of the object: and is east or west, according as the object is situated east or west of the meridian, at the time of observation. Azimuth may also be defined the number of degrees, or points of the mariner's compass, that the Sun or other object bears from the north or south points of the horizon.

22. The *Amplitude* of a celestial object, is an arc of the horizon contained between the centre of the object at rising or setting; and the north or south points of the horizon, or it is the number of degrees, or points of the mariner's compass that it rises or sets from the north or south points of the horizon. Amplitude is often reckoned by mariners from the east at Sun rising, and from the west at Sun setting.

23. The *Altitude* of a celestial object is an arc of an azimuth contained between the centre of the object and the horizon; and the *zenith distance* is its altitude taken from 90°. The altitude of a celestial object increases from the time it rises till it culminates or comes to the meridian, when it is the greatest, and hence the meridian altitude is the greatest altitude.*

24. The altitude of the celestial pole, at any given place, is equal to the latitude of that place. For if an observer be on the terrestrial equator, both the celestial poles will appear in his horizon, and therefore they will have no altitude; but if he move one degree directly north on a meridian, then his zenith and horizon will move along with him north one degree, and his horizon will get one degree below the north pole: that is, the north pole will seem to rise one degree above his horizon, and his zenith, which was before in the celestial equator, will now be one degree north of the celestial equator; that is, the equator will seem to be depressed one degree below his zenith; and every degree he continues to move on northward he will raise the north pole exactly the same quantity, and depress the equator from his zenith just so much; but the distance of the zenith of a place from the celestial equator is the latitude of a place, (by Def. 12.) therefore the elevation or altitude of the pole above the horizon, which has been shewn to be always equal to it, is also equal to the latitude. Astronomers therefore commonly say, the elevation or altitude of the pole, instead of the latitude.

25. Astronomers commonly reckon three positions of the celestial sphere; namely, a *Right*, *Oblique*, and *Parallel*. *A Right sphere* is that in which the celestial bodies appear to rise or set perpendicularly to the horizon: this position belongs to an observer situated on the terrestrial equator. *An Oblique sphere* is that in which the celestial bodies appear to rise and set obliquely or slant-wise to the horizon: this position belongs to an observer situated any where between the equator and the poles; thus, the inhabitants of London have an oblique sphere. *A Parallel sphere* is that in which the celestial bodies seem to move parallel to the horizon; this position belongs only to the north and south poles of the world.

26. *Almicanthers*, or parallels of altitude, are minor circles parallel to the horizon; those minor circles parallel to the horizon, but below it, are called *Parallels of Depression*: these circles are infinite in number, of which there is one specially noticed, called the *Crepusculum* or circle of *twilight*: astronomers fix the crepusculum 18° below the horizon, and suppose that when the sun arrives in the morning within 18° of the horizon, that the twilight then commences; and also suppose that the twilight continues in the evening until the Sun is 18° below the horizon: this conclusion they deduce from observing the small fixed stars of the 6th magnitude, which cannot be seen till the Sun is 18° below the horizon, some of his rays still continuing to illuminate the upper regions of the atmosphere, which keeps those stars invisible till he has passed the crepusculum: but the unassisted eye can perceive no traces of twilight, if the Sun be more than 15° below the horizon.

27. In a *right sphere* the celestial equator is in the zenith, and the horizon is a meridian in which are two celestial poles; and all the heavenly bodies on

* See note, page 69.

the east side of the horizon appear to rise perpendicularly to it.

28. *Right Ascension* of a celestial object, is, therefore, an arc of the celestial equator contained between the first point of Aries and the meridian passing through the centre of that object, reckoned from west to east according to the order and succession of the 12 signs, till it end at 360°, or 24 hours in time: or it is the number of degrees reckoned from west to east, of the equator, between the first point of Aries and that point of the equator which rises with the object in a right sphere; which is equal to the number of hours that the object rises after the first point of Aries has risen in a right sphere.

29. *Oblique Ascension*, or descension, of a celestial object, is an arc of the equator, reckoned from west to east, contained between the first point of Aries, and that point of the equator which *rises* or *sets*, with the object in an oblique sphere.

30. *Ascensional Difference*, is the difference between the *right* and *oblique* ascensions of a celestial object; or it is the difference between 6 hours and half the time the object is above the horizon. Note, half the time any celestial object is above the horizon, is called its *Semidiurnal* or *Seminocturnal* arc.

31. *Refraction*, or the error of altitude, is a correction to be *subtracted* from all observed altitudes of the celestial bodies, to reduce them to the *true:* the rays of light, upon entering our atmosphere obliquely, are bent out of their rectilinear direction, towards the eye of the observer, by which the altitude of the celestial bodies appears greater than it really is. When the object is in the horizon the refraction is the greatest, and it decreases as the object ascends above the horizon, till it arrives at the zenith, when the refraction is nothing.

32. *The Parallax* of a celestial object, is also an error in its observed altitude, which is to be *added* to the observed altitude to reduce it to the *true:* The parallax is the difference between the altitude of the centre of an object when seen from the Earth's centre and from its surface: the parallax is greater when the object is in the horizon, and it gradually decreases from the horizon to the zenith, when it becomes nothing. The nearer an object is to the Earth, the greater the parallax; the Moon being the nearest of the celestial bodies to the Earth, her parallax is therefore, much greater than that of any other of the heavenly bodies; her horizontal parallax being 57 minutes of a degree, at a mean; but the Sun's horizontal parallax is no more than $8\frac{1}{2}$ seconds nearly of a degree; and the fixed stars have no parallax, or at least, it is inconceivably small.

ILLUSTRATION. PLATE 2. FIG. 3.

Let ATB, be the Earth, C, its centre; M, the Moon in the sensible horizon, AME; CD, the plane of the rational horizon parallel to AE; DGL, a quadrant of a vertical circle in the heavens: now an observer at C, the Earth's centre, sees the Moon's centre at F, in the heavens; but an observer on the surface at A, sees her at E, lower by the arc FE, or rather FD, for the additional part ED, vanishes, and is nothing in the heavens, being the difference between the rational and sensible horizon; (see chap. 2,) FD, therefore is the horizontal parallax of the Moon: but the arc, FD, is the measure of the angle FCD, and the angle ASC, is equal to FCD, for they are alternate angles; therefore the angle ASC, is equal to the Moon's horizontal parallax, and it is the angle under which AC, the semidiameter of the Earth appears, seen from the Moon's centre: again the horizontal parallax is the greatest; and as the Moon's altitude increases the angle at the Moon's centre decreases, till she arrive at the zenith, when it vanishes; for the arc EG is less than FD, and KI is less than EG, and so on; but these arcs are the measures of the angles at the Moon's centre under which the Earth's semidiameter is seen: but CS is to AC as the radius is to the sine of the angle ASC; and the same straight line SC, to wit the semidiameter of the Moon's orbit, remaining radius, the sines of the several angles at the Moon are always to each other as the sines of the Moon's several distances from the zenith, that is, as the cosines of the altitudes; but in very small arcs, the arcs and their sines are proportional: whence the Moon's parallax always decreases in the ratio of the cosine of her altitude. And hence we have this analogy, viz. as radius is to the cosine of the Moon's altitude; so is her horizontal parallax to her parallax in altitude.

General Deductions resulting from the foregoing Definitions, in illustration thereof.

1. In *a right sphere*, the whole concave face of the heavens is seen in the space of 24 hours nearly. The Sun is always 12 hours (exclusive of the effects of refraction) above the horizon, and 12 hours below it; and hence an inhabitant of the equator has a constant equality of day and night. The Moon at a mean is 12 hours and 24 minutes above the horizon and as long below it. The fixed stars about 2 minutes less than 12 hours above, and the same time below the horizon. The inferior planets at the times of their greatest elongation, exactly 12 hours above and below, being the same then as the Sun at other times, a few minutes more or less. And the superior planets, except Mars, differing but little from the fixed stars; Mars, indeed, sometimes may be 4 or 5 minutes more or less than 12 hours above or below the horizon.

2. In *an oblique sphere*, a portion of the concave face of the heavens is always above the horizon and visible, and an equal portion of the heavens in the opposite hemisphere is always below the horizon: in the former portion the fixed stars therein never set, but appear to move round the elevated pole, being at one time above it, and at another below it, in the space of 24 hours: but the fixed stars, in the latter, are always invisible. The magnitude of the spaces corresponds to the elevation of the pole, that is, to the latitude of the place. Thus the latitude of London is 51° 31′ north; consequently, that portion of the northern hemisphere extending from the north pole 51° 31′ southward towards the equator, is always above the horizon, and the fixed stars contained therein constantly visible, that is, they never rise nor set to an inhabitant of London. And an equal portion of the southern hemisphere extending from the south pole northwards 51° 31′ is never seen at London. And in the intermediate space, which is equal to twice 38° 29′, the complement of the latitude; that is, 76° 58′, the equator being in the middle of it; all the celestial bodies contained in it will be seen at London to rise and set: those on the equator keeping nearly 12 hours above the horizon, and as long below; those that have north declination will continue more than 12 hours above the horizon, and the remainder to 24 hours below it; but, on the contrary, those that have south declination will continue less than 12 hours above the horizon, and the remainder to 24 hours below it. Hence the greater the declination when north the longer does a heavenly body continue above the horizon at London, or at any other place in north latitude, and *vice versa*.

3. When the latitude of a place and declination of a celestial object are both equal, and both north or both south; then that object is in the zenith of the place once in the 24 hours: thus the bright star in the head of the Dragon, called *a draconis*, 51° 31′ north declination, passes over head, once every 24 hours, to the inhabitants of London; it is vertical at midnight on the 21st of June, and at noon on the 21st of December, and 3*m*. 57*s*. sooner daily. The Sun keeping between the tropics always, he is therefore directly over head, at noon, to those places on the Earth of which the latitude is equal and of the same name with his declination at the time. In the polar regions of the Earth, between the polar circles and the poles, the Sun being then in that portion of the heavens which is always above the horizon, will appear for several days visible without setting: thus at North Cape, Norway, in latitude 72° north, which is just 18° from the pole, the Sun never sets during the time his declination north exceeds 18°, that is, from the 12th of May to the 1st of August, which is a period of 81 days; and for the same space of time during the winter season he is below the horizon and never rises; but the effects of refraction will add about 4 days more to the length of the continual day-light, and deduct four days from that of the continual night: the length of the continual day is therefore 85 natural days, and that of the night 77.

4. In *a parallel sphere*, that is, at either pole of the world, one half of the heavens is continually above the horizon, which is the celestial equator, and the other is always below it and invisible; so that an observer at the north pole of the world, can only see the northern hemisphere, but no part of the southern, and *vice versa*. All the fixed stars appear to move parallel to the horizon; those above it in the visible hemisphere never *setting*, and those below it in the other hemisphere never *rising*. The Sun appears for the space of half a year visible, daily moving round in a spiral manner nearly parallel to the horizon, but gradually ascending for three months, till his altitude is $23\frac{1}{2}$ degrees nearly; then gradually descending for the space of three months, till he comes to the horizon and disappears: and during the other half year is invisible; so that, at the poles of the world, there is half a year of continual day-light, and half a year more, twilight or darkness: but the effects of refraction add about three natural days to the former and deduct the same length of time from the latter.

5. In a right sphere, that is, at the equator, the twilight is always nearly the same, and of shorter continuance than at any other part of the world; if the *crepusculum* be allowed to be no more than 15° below the horizon, then its length is one hour nearly, but if we suppose 18° for its depression, then the twilight is 1*h*. 12*m*. long. In *an oblique sphere* it is of longer duration, and the greater the latitude the longer is the twilight; thus, at London its length is from 2 hours to $3\frac{1}{2}$ hours; the exact quantity we shall shew how to calculate in the practical part of the work; it is shorter in winter than in summer, but it is not shorter when the days are at the shortest, but almost 2 months before and after that time. (We shall hereafter investigate the solution of the problem, to find the exact time when the twilight is the shortest in an oblique sphere.) At the poles the twilight continues 54 natural days before and after the totally dark night, which lasts only 85 natural days. When the complement of the latitude, minus 18°, is less than the Sun's declination, the latitude and declination being both north or both south, then there is no termination of the twilight, but it continues throughout the whole night: thus, the latitude of London is 51° 31′ north, and its complement 38° 29′, from which 18° degrees being deducted, the remainder, 20° 29′, is the Sun's declination north, an-

wering to the 23d of May and the 22d of August, when the continual twilight begins and ends; so that at London we have two months of the summer in which there is no perfect night.

Illustration of the Change of Seasons.

As the vicissitudes of day and night are produced by the *diurnal* revolution of the Earth round its axis from west to east; (of which, it is to be hoped, the student, by this time, has a clear and perfect conception;) so in like manner are the several various seasons of the year, spring, summer, autumn, and winter, produced by the *annual* revolution of the Earth round the Sun. So that by these two motions of the Earth, namely, the *rotatory* and *projectile*, are produced the changes of day and night, summer and winter, and most other natural and celestial phenomena affecting our world that come within the cognizance of man.

Now, although we, as inhabitants of the Earth, partake of all its motions in common with it, yet we are not conscious of it: we seem neither to feel it nor to know it by the senses; yet we really do move, our senses being herein deceived: we think that we are at rest on the Earth, and that the Sun and Moon, and all the other celestial bodies are moving round us; but it is not so, it is we that move, and they are at rest (except what relates to their own proper motion). But the phenomena are evidently the same to us, whether the motion be in the Earth itself or in the celestial bodies. And we must first premise, that on whatever point of the Earth's surface we are situated, we always think that we are on the highest or uppermost point thereof, that the point over-head, to wit, our zenith, is the highest point in the heavens, and that the opposite point, directly under-foot, that is, our nadir, is the lowest. This is a consequence of the principles of Gravitation, or, what is the same thing, the attraction of Gravitation, by which all bodies are drawn directly towards the Earth's centre; and that *force* or *power* by which the Earth and body mutually act and draw each other is called the *specific gravity* or *weight* of the body. By this power, therefore, all bodies in free space, and unsupported and at perfect liberty, will fall directly towards the Earth's centre, and rest on the surface which opposes them in their descent; but the gravity or weight still continues. Hence also we think that we stand *upright* always, as we really do, in respect to the plane of our own *sensible horizon;* at the same time *obliquely* to the horizon of any other place, except to that of the opposite point of the Earth, or our antipodes; whose sensible horizon is parallel to ours, and they have the same common *rational horizon* with us. Their zenith is our nadir, and *vice versa;* they think they are uppermost and on the highest point of the Earth, and that we are directly under them; as we in like manner suppose of them. And this great and important fact results from the attraction of gravitation.

An observer in the Sun will see the Earth revolving round him from west to east, in that great circle in the heavens which we call the Ecliptic, and passing through the 12 constellations, ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, and ♓, in the course of one year of our time; but we on the Earth not being sensible of our own motion, think that it is the Sun that moves through the ecliptic, and completes his revolution round the Earth from west to east in the space of a year. Wherefore the Sun apparently describes the same circle, to wit, the ecliptic, in the heavens to an observer on the Earth, that the Earth describes to an observer in the Sun; but with this difference, however, that they are seen in opposite points of the heavens respectively; that is, if the Earth be in the first point of Aries as seen from the Sun, then the Sun is seen by us in the first point of Libra; and as the Earth advances towards Taurus the Sun appears to advance towards Scorpio; and so on according to the order and succession of the 12 signs of the ecliptic.

These things being premised, it is evident, that the phenomena are exactly the same, whether we take the *real motion* of the Earth, or *apparent motion* of the Sun, as our *data* or *hypothesis:* but as it is much more convenient, in astronomy, and better adapted to our common conceptions of things, to use the *apparent* rather than the *real*, we shall, therefore, now consider the theory of the Earth and the various phenomena resulting from the apparent motions of the Sun, as if they were *real.*

The Earth being a sphere or globe, (or at least not differing sensibly from one;) hence one half of it will be always illuminated by the Sun's rays, and the other half, being deprived of his light, will be involved in darkness. That great circle on the Earth's surface which divides the illuminated hemisphere from the dark one, is called the *terminator of light and darkness;* and it is always 90° distant from that point of the Earth over which the Sun is at any time vertical. The Sun apparently moving every day round the Earth from east to west, carries the terminator of light and darkness along with him; and if he were exactly at a given time in the celestial equator, then he is exactly also vertical or directly over the terrestrial equator; and hence the terminator of light and darkness at that time passes through two poles

of the world, and were he to continue for one natural day moving round the Earth and keeping his centre always in the celestial equator, then the whole Earth would, in turn, in the course of that day be 12 hours in the illuminated hemisphere, and 12 hours in the dark one. And although for the space of one day, this hypothesis may be admitted, as the difference between it and the real fact is insensible; yet in a strictly mathematical sense it is not true; for the Sun does not tarry a second of time on the equator; as his centre crosses that line in an instant, and therefore he immediately carries the terminator of light and darkness away from the poles of the world, and involves the one in darkness and the other in his rays for the space of six months.

ILLUSTRATION. PLATE 3. FIG. 1.

Let EQ represent the celestial equator; *dc*, the ecliptic marked with the 12 signs, ♈, ♉, ♊, &c.; N and S, the north and south poles in the heavens; NS, the equinoctial colure, which in this projection is also the axis of the equator; ENQS, the solstitial colure; *abc*, the celestial tropic of Cancer, touching the ecliptic in the point ♋; *def*, the celestial tropic of Capricorn, touching the ecliptic in the point ♑: through the equinoctial points, ♈ or ♎, draw *xk* at right angles to the ecliptic; then the points *x* and *k* are the poles of the ecliptic, which are 23½ degrees from the poles of the equator, which is equal to the obliquity of the ecliptic; and about the north and south poles of the equator draw the two minor circles passing through the poles of the ecliptic *x* and *k*, which are the arctic and antarctic celestial circles.

Now when the Sun is in the equinoctial point ♈, he has no declination, and there is then an equality of day and night in all parts of the world, as has been said above; but as he moves easterly in the ecliptic towards ♉, he leaves the equator and ascends northerly, and then has north declination: let us suppose him to move on till he come to ☉; then through the points N, ☉, S, draw the meridian, which will be the meridian at that time; the arc of the ecliptic ♈☉, is his longitude then, which suppose to be 1 sign, 22°, or 52°, and in that case, we say his longitude is 22° in Taurus; the arc ☉B, is his declination north, which will be found (as shall be hereafter taught) to be 18° 17′ north; and the arc ♈B, is his right ascension from the first point of Aries, and which will be found to be 49° 35′ nearly, or 3*h*. 18*m*. 20*s*. in time; and the angle ☉♈B, is the obliquity of the ecliptic, which in this case we suppose to be 23° 27′ 43″, as it was on the 1st of May, 1810. And because the Sun's Declination is then 18° 17′ North, the Terminator of light and darkness will extend as far beyond the North pole of the World towards the equator as the point T is from N, which in this case is 18° 17′, consequently a zone extending 18° 17′ from the North pole of the World, 18° 17′ towards the equator will be then in the enlightened hemisphere, and have the constant light of the Sun: and an equal portion about the south pole of the World extending as far as the point R is from S, which is also 18° 17′, must be at that time involved in constant darkness.

Again when the Sun arrives at the solstitial point, ♋, his Longitude ♈♋, is then 3 signs or 90°, and he has then his greatest Declination North, which is equal to the Obliquity of the Ecliptic, that is, to the arc, ♋Q of the solstitial colure, or 23° 27′ 43″, as we have supposed: his Right Ascension then, or the arc ♈Q, is exactly equal to his Longitude, which is 90°, or 6 hours in time: the Sun then having his greatest North Declination, consequently illuminates the whole space between the arctic circle and the north pole of the world, which space is called by Geographers the *North Frigid Zone;* and the correspondent space between the antarctic circle and the south pole of the world or south frigid zone is then involved in darkness: (see our Deductions in this Chapter.) When the Sun has his greatest North Declination, he is then on the celestial tropic of Cancer, and because the Ecliptic and Tropic touch each other in the first point of ♋, and for some time seem to coincide; hence the Sun's Declination at that time has the least increase or decrease in a given time, and therefore the length of the day seems to remain the same for nearly a week; the ancients therefore thought that the Sun stood still or at rest when he arrived at the tropic of Cancer or that of Capricorn, and hence they called those periods the times of the *Solstices*, (from the Latin *Sol* the Sun, and *sto*, to stand.) Having illustrated the phenomena resulting from the Sun's passage through one quarter of the Ecliptic, the phenomena in the other three quarters are obvious: for the Sun having passed the solstitial point ♋, his North Declination begins to decrease and continues decreasing till he arrives at the equinoctial point ♎, when it is nothing; and afterwards in his passage from ♎, to ♑, he has South Declination increasing till he gets to ♑, when he has his greatest South Declination; and, lastly from ♑ to ♈, his South Declination decreases till he arrives at the first point of Aries, when he is again on the Equator and has no Declination. And hence it appears, that the Sun's apparent motion in the Ecliptic is that which produces all our variety of Seasons, Spring, Summer, Autumn, and Winter; and this *apparent* motion of the Sun in the Ecliptic is caused by the Earth's *real* motion round the Sun, and the axis of the Earth being inclined to the plane

of its orbit, in an angle of $23\frac{1}{2}$ degrees nearly, which we call the Obliquity of the Ecliptic.*

It may be necessary to mention, that Astronomers suppose the Spring Quarter to commence with us and all other inhabitants of the northern hemisphere, at the instant the Sun enters the first point of Aries, or equinoctial point, which is about the 20th of March, and there is then an equality of day and night in every part of the world; the beginning of the Summer Quarter, they also fix at the instant he enters the first point of Capricorn, or solstitial point Capricorn, which is about the 21st of June, when we, and all other inhabitants of the northern hemisphere, have our longest day, and those of the southern hemisphere their shortest; and again the beginning of the Autumnal or harvest Quarter they also fix for the time he enters the second equinoctial point Libra, when there is an equality of day and night again in every part of the world, which is about the 22nd or 23rd of September; and lastly the commencement of Winter Quarter they fix for the instant he enters the solstitial point Capricorn, when we and all other inhabitants of the northern hemisphere have our shortest day, and those of the southern their longest, which happens about the 21st of December. But we must remark, that all these are reversed to the inhabitants of the other hemisphere: our Summer is their Winter, and our Spring their Autumn, &c. Moreover, some think, that the beginning of each Quarter should have rather been fixed about 46 days sooner, when they would have been regulated by the length of the day rather than by the Sun's entrance into the four cardinal points of the Ecliptic.

* Almost all our astronomical writers fail in their accounting for the change of Seasons on our Earth, by their making use of an absurd Hypothesis: they suppose, that the Earth is carried round the Sun in such a manner, so as to have its *axis parallel* at any time, to what it was at any other time. Mr. Emerson's words are; "and during the Earth's motion in its orbit, the *axis always* remains in a parallel position, or pointing to the same star." Mr. Ferguson says; "the axis keeping *always parallel* to itself; by this is meant, that if a line be supposed to be drawn parallel to the Earth's axis in any part of its orbit, the axis keeps parallel to that line in every other part of its orbit." And he farther illustrates his meaning by a geometrical diagram. Now the fact is, that the proposition is fallacious; for the axis of the Earth *does not* always remain in a parallel position, but is always changing its position to any line supposed to be drawn parallel to its former position; and in the space of half the period of the Recession of the Equinoxes, it deviates from its former position, in an angle equal to twice the Obliquity of the Ecliptic, which amounts to no less than 47 degrees nearly: and yet the cause of the change of Seasons is in no measure affected thereby. The *true* and *real* cause of the change of Seasons, is that the angle of inclination of the Earth's axis to the plane of its orbit remains always nearly the same. The axis of the Earth has a conical motion round the axis of the Ecliptic, and it completes its revolution in the space of twenty six thousand years nearly; yet if it completed its period in a thousandth part of that time, to wit in twenty six years; the change of seasons would still continue as they are now, provided the angle of inclination of the Earth's axis to the plane of its orbit remained always of the same magnitude: Such a quick motion of the Earth's axis round the axis of the Ecliptic as we have now supposed could only produce a similarly quick motion of the Equinoctial points; and cause the *Syderial* year to exceed or fall short of the *tropical*, by a thousand times their present difference: but in respect to the change of seasons, this rapid change in the position of the axis, can not have the least effect; notwithstanding in the space of half the time of this supposed rotation, viz. in 13 years, the axis will be removed in an angle of 47 degrees from its former position. And hence this hypothesis of the Earth's axis always remaining in a parallel position, and pointing to the same star, is fallacious, useless, and absurd; and is by no means the cause of the change of seasons on our Earth. The angle of inclination of the Earth's axis to the plane of its orbit; or what is the same thing, the Obliquity of the Ecliptic, is *alone*, the true cause of the change of seasons, and all the other phenomena, relating thereto. Strange indeed! that so many eminent Mathematicians should be perfectly correct, in accounting for the various phenomena and seasons of all the other planets, but fail in that of their own. They all grant, that when the axis of a planet is at right angles to the plane of its orbit, that there can be no change of seasons in that planet; but when the axis stands obliquely to the plane of a planet's orbit, there must be the same change of seasons in that planet, whether its axis keeps always parallel to itself or not, provided the angle of inclination remains the same. The axis of the Earth may be perpendicular to the plane of the Ecliptic; and carried round in that position exactly parallel to itself; and then there can be no change of seasons. There is also an extreme case, where there may be no change of seasons, when the axis of a planet is inclined to its orbit; namely when the conical motion before mentioned is completed in the time of one revolution about the Sun. In order to produce a change of seasons in all cases, it is necessary that the axis should be inclined to the orbit: and that the motion of the poles of the world, about the poles of the Ecliptic, should not be completed in so short a period as one year.

TABULAR VIEW OF THE SOLAR SYSTEM.

Names of the Planets.	Mean Diameters in English miles.	Mean Distances from the Sun, in English miles.	Mean apparent Diameters, as seen form the Earth.	Mean Diameters, as seen from the Sun.	Densities, that of water being 1.	Proportional quantities of Matter.	Diurnal rotations round their axes.	Inclinations of axis to their own orbits.	Inclinations of orbits to the Ecliptic in 1780.
							d. h. m. s.		
The Sun.	883240		32′ 1.5″		$1\frac{2}{5}$	333928	25 14 8 0	82° 44′ 0″	
Mercury.	3224	36841395	10	16″	$9\frac{1}{6}$	0.1654	0 24 5 28		7° 0′ 0″
Venus.	8648	68841803	58	30	$5\frac{11}{15}$	0.8899	0 23 20 59		3 23 35
Earth.	7970	95173128		17		1.	1	66 32 0	0 0 0
Moon.	2180	95173128	31 8	5	$4\frac{1}{2}$	0.025	29 17 44 3	88 17 0	5 9 3
Mars.	5150	145014584	27	10	$3\frac{2}{7}$	0.0875	0 24 39 22	59 22 0	1 51 0
Vesta.		225000000	0.5						
Juno.		252000000	3						
Ceres.		260000000	1						
Pallas.		265000000	1						
Jupiter.	94150	495011179	39	37	$1\frac{1}{4\frac{1}{4}}$	312.1	0 9 55 37	90 nearly.	1 18 56
Saturn.	78000	907749068	18	16	$\frac{1}{3\frac{3}{5}}$	97.76	0 10 16 2	60 probably	2 29 50
Herschel.	35120	1827667455	3 54	4	$\frac{99}{1000}$	16.84			0 46 20

Names and Characters of the Planets.	Tropical Revolutions.	Siderial Revolutions	Place of Aphelion in January, 1800.	Motion of Aphelion in 100 years.	Motion of Nodes in 100 years.	Longitude of ascending node in 1750.	Excentricities the mean distance being 100000.	Greatest equation of centre.
	d. h. m. s.	d. h. m. s.	s. °. ′. ″.	°. ′. ″.	°. ′. ″.	s. °. ′. ″.		°. ′. ″.
Sun, ☉								
Mercury, ☿	87 23 14 32.7	87 23 15 34.6	8 14 20 50	1 33 45	1 12 10	1 15 20 43	7955.4	23 40 0
Venus, ♀	224 16 41 27.5	224 16 49 10.6	10 7 59 1	1 21 0	0 51 40	2 14 26 18	498	0 47 20
Earth, ⊕ or ⊖	365 5 48 48	365 6 9 12	9 8 40 12	0 19 35			1681.395	1 55 30.9
Mars, ♂	686 22 18 27.4	686 23 30 35.6	5 2 24 4	1 51 40	0 46 40	1 17 38 38	14183.7	10 40 40
Vesta, ⚶								
Juno, ⚵								
Ceres, ⚳								
Pallas, ⚴								
Jupiter, ♃	4330 14 39 2	4332 14 27 10.8	6 11 8 20	1 34 33	0 59 30	3 7 55 32	25013.3	5 30 38
Saturn, ♄	10746 19 16 15.5	10759 1 51 11.2	8 29 4 11	1 50 7	0 55 30	3 21 32 22	53640.42	6 26 42
Uranus, ♅	30637 4 0 0	30737 18 0 0	11 16 30 31	1 29 2	1 44 35	2 12 47 in 1788	90804	5 27 16

Explanation of Characters or Signs used in this Work.

$+$, *plus*, or *more*, is the sign of addition, and signifies, that the quantity following it is to be added to that preceding it. Thus, $7+3$ signifies, that 3 is to be added to 7; and 10 is called their *sum*.

$-$, *minus*, or *less*, is the sign of subtraction, and signifies, that the quantity following it is to be subtracted from that preceding it. Thus, $10-3$, signifies, that 3 is to be subtracted from 10, and 7 is called their *difference*.

$\times$, *multiplicandum*, or *multiplied by*, or *into*, is the sign of multiplication; and signifies, that the quantities between which it is placed, are to be multiplied together. Thus 7×3, signifies, that 7 is to be multiplied by 3, or 3 by 7; and 21 is called their *product* or *rectangle*.

$\div$, *dividendum*, or *divided by*, is the sign of division; and signifies, that the quantity preceding it is to be divided by that following it. Thus, $21 \div 7$ signifies that 21 is to be divided by 7, and 3 is called the quotient. This sign, however, is but seldom used; the usual manner of expressing the quotient is by placing the dividend over the divisor, in the manner of a vulgar fraction, thus, $\frac{21}{7}$.

$=$, *equalitas*, or *equality*, is the sign of equality; and signifies, that the quantities between which it is placed are equal to one another. Thus, $7+3=10$; read, 7 plus 3, equal to 10. Also $10-7=3$, read, 10 minus 7, equal to 3. Likewise, $7 \times 3 = 21$, read, 7 multiplied by 3, equal to 21. And $\frac{21}{7}=3$, read, 21 divided by 7, equal to 3.

$:$, *ad* or *ut*, is to; $::$, *sic*, so is; } are the signs of proportion.

Thus, as $6:2::9:3$, read, as 6 is to 2, so is 9 to 3.

$\surd$, *radix*, or *root*, is called the radical sign. This sign, without any number placed near the angle, always implies the *square root*; but if any other root is to be denoted, then the index of that root is to be put before the radical sign. Thus, $\surd 49$ is read the square root of 49: $\sqrt[3]{64}$ is read the cube root of 64: $\sqrt[4]{16}$ is read the biquadrate or 4th root of 16, &c. But roots are frequently denoted by fractional numbers, thus, the square root by $\frac{1}{2}$, the cube root by $\frac{1}{3}$, the 4th root by $\frac{1}{4}$, &c. Thus, $\surd 49$ may be written $49^{\frac{1}{2}}$; $\sqrt[3]{64}$ may be written $64^{\frac{1}{3}}$; $\sqrt[4]{16}$ may be written $16^{\frac{1}{4}}$, &c. Note, $\surd 49$ or $49^{\frac{1}{2}}=7$; $\sqrt[3]{64}$ or $64^{\frac{1}{3}}=4$; $\sqrt[4]{16}$ or $16^{\frac{1}{4}}=2$, &c.

$\square$, the square of; or $\overset{2}{\frown}$, signifies the same thing. Thus $\square 7$, or $7\square$, or 7^2, signifies the square of 7; that is, $7^2=49$: also, 4^3 signifies the cube of 4; that is, $4^3=64$: likewise 2^4 signifies the 4th power of 2; that is, $2^4=16$, &c. Lastly $32^{\frac{4}{5}}$, signifies the 4th power of the 5th root of 32; or what is the same thing, the 5th root of the 4th power of 32; wherefore $32^{\frac{4}{5}}=16$, &c.

———, *vinculum*, or *chain*, is used to bind the terms of a compound quantity together; when the whole compound quantity has to be multiplied by another quantity; or when it has to be raised to any power. Thus, $\overline{3+4} \times 5 = 35$; also $\overline{3+4}^2 = 49$, &c.

In modern books the Parenthesis is used instead of the Vinculum; thus, $(3+4)^2=49$; $(3+4) \times 5 = 35$.

(.) full point, this is sometimes used for $\times$, the sign of multiplication. Thus $\overline{3+7} . \overline{5-2}$, is the same as $\overline{3+7} \times \overline{5-2} = 30$.

Also, when quantities are represented by single letters, then the sign of multiplication is commonly omitted, and the letters written after one another like letters in a word. Thus, $a \times b$, or $a . b$, is written ab, which signifies, the product of a multiplied by b; likewise adb is the same as $a \times d \times b$; and aa is written a^2, or the square of a; also aaa is written a^3, or the cube of a; moreover, $3a^2b^3$ signifies, $3 \times a^2 \times b^3$, that is, 3 times the square of a multiplied by the cube of b; and $4a^3b^2c$, is the same as $4 \times a^3 \times b^2 \times c$, or 4 times the cube of a multiplied by the square of b, and that product by c.

Suppose $a=2$, $b=3$, $c=4$, and $d=5$. Then, ab, or $a \times b$, or $a . b = 2 \times 3 = 6$; and $adb = 2 \times 5 \times 3 = 30$; also, aa, or $a^2 = a \times a = 2 \times 2 = 4$; likewise, $a^3 = 2 \times 2 \times 2 = 8$; moreover, $3a^2b^3 = 3 \times a \times a \times b \times b \times b = 3 \times 2 \times 2 \times 3 \times 3 \times 3 = 324$; and $4a^3b^2c = 4 \times a \times a \times a \times b \times b \times c = 4 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 1152$; again, $ab+ac = 2 \times 3 + 2 \times 4 = 14$; also $2ac - ab = 2 \times 2 \times 4 - 2 \times 3 = 10$; and $\frac{a^2b - ac}{c} = \frac{2 \times 2 \times 3 - 2 \times 4}{4} = 1$, &c.

THE

ELEMENTS OF GEOMETRY.

CHAP. I.

INTRODUCTION.

GEOMETRY is the science of magnitude, of which there are three kinds; namely, *Lines, Superfices, and Solids;* the word is derived from the Greek, (signifying, the Earth, and to measure,) and at first signified no more than to measure the Earth; but it is now taken in a general sense, and signifies that Science, which exhibits and demonstrates the nature and properties of Lines, Superfices, and Solids; and lays the foundation of applying these properties of magnitude, to all those Sciences, in which, Lines, Superfices, and Solids are concerned. It is, therefore, intimately connected with Astronomy; and he, who has the laudable desire of becoming a proficient in Astronomy, must previously understand the fundamental Propositions of Geometry, and learn *plane* and *spherical* Trigonometry. And then he is fully qualified to proceed in the study of this sublime science.

A Line is that magnitude which has *Length* only, without breadth; it has, therefore, only one Dimension, to wit, *Length:* its extremities or bounds are points. A Superfices is a magnitude, which has Length and Breadth, but no depth or thickness; it has therefore two Dimensions, to wit, *Length* and *Breadth:* its extremities or bounds are lines. And lastly, a Solid is a magnitude, which has Length, Breadth, and Depth: it has, therefore three Dimensions, namely *Length, Breadth,* and *Depth* or Thickness: its extremities or bounds are Superfices or Surfaces.—Hence, points bound lines, lines bound Superfices, and Superfices bound Solids.

Geometricians conceive magnitude to be generated or described by motion; thus, if a point move, it describes a *line;* if a line move laterally, that is, sideways (but not endways) it describes a *superfices:* and, if a superfices move laterally, (but not edgeways,) it describes a *solid.* And beyond this we can conceive no other magnitude generated; for if a solid move, it can only generate another solid, such as the plane superfices of a section of it would describe.

GEOMETRICAL DEFINITIONS. PLATE 4.

1. A *Point* is that which hath no parts or magnitude. (1)
2. A *Line* is length without breadth. (2)
3. The ends of a line are points.
4. A *right* line is that which lies evenly between its extreme points. (3)
5. A *Superfices* is that which hath length and breadth, but no depth.
6. The extremities of a Superfices are lines.

(1) A point must be conceived to have no dimensions, it is only a mark in or an extremity of magnitude; it is without length, breadth, or depth, and constitutes no part of that magnitude, in which it is a mark or an extremity. It only separates the parts of a line, or is an extremity of it.

(2) A line must be conceived to have only one dimension, namely, Length only, but no breadth; it either separates a superfices into parts, or is an extremity of it, but constitutes no part of it.

(3) A right line is the shortest distance between two points; or it is such, that if the eye be placed in the direction of the line, then the point, or extremity next the eye, will hide the whole line. But a *curve,* or crooked line is not the shortest distance between its extreme points, but lyeth in a bending manner, either regularly or irregularly between them. If it bend regularly, such as any part of the circumference of a circle does, then it is called a *regular curve.* As AB. is a right line; CD, is a curve line. (Fig. 1.)

7. A *plane Superfices* is that which lies evenly between its extreme lines; or it is that, in which, any two points being taken, the right line joining these two points, lies wholly in that Superfices. (4)

8 A *plane Angle* is the inclination of two right lines, which meet one another in the same plane, but are not in the same direction. (5)

9. When a right line, standing on another right line, makes the two angles on each side of it equal to one another, then each of the equal angles is called a *right angle;* and the line, which stands upon the other, is called a *perpendicular* to that on which it stands. Thus, if the two adjacent angles ABC, and DBC, (Fig. 4.) be equal to one another, then they are right angles.

10. An *acute Angle* is that which is less than a right angle. Thus, the angle B, (Fig. 2.) is an acute angle.

11. An *obtuse Angle* is that which is greater than a right angle. Thus, the angle A, (Fig. 5.) is an obtuse angle.

12. A *Circle* is a plane figure contained by one bending line called the Circumference, and is such, that all right lines drawn from the centre or middle point to the circumference are equal to one another. A circle is described by the motion of a right line upon a plane, one end or extremity of the line being at rest or fixed and the right line being carried round that fixed point, describes the circle; and the circumference is described by the other extreme point which is carried round: the describing line is called the *Radius* or *Semidiameter* of the circle, and is equal to the distance between the points of a pair of Compasses when used to describe a circle. Hence, all Radii (Radiuses) of the same Circle are equal to one another.

13. The Diameter of a circle is a right line drawn through the centre and terminated both ways by the circumference; and it divides the circle into two equal parts called *Semicircles.* Thus, (Fig. 6.) ADB, is a circle; CD, is the Radius or describing line; ADB, the circumference; AB, the diameter, which is equal to twice the Radius; and it is evident, that it divides the circle into two equal parts.

14. The Circumference of every circle is supposed to be divided into 360 equal parts called *degrees;* (marked, °;) each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts called *seconds;* and the seconds again divided into 60 equal parts called *thirds,* &c.: but commonly into tenths, which are the smallest divisions or parts of the circumference used in Astronomical calculations. Note, any part of the circumference of a circle, is called an *arch* or *arc.*

15. A plane angle is measured by an arc of a circle described about the angular point as a centre; and that arc of the circle contained between the two right lines, which contain the angle, is called the measure of the angle. Thus if the arc AB contains 50°, then the angle D, is an angle of 50° (Fig. 7.)

16. A *right angle* is an angle of 90°; an *acute angle* is any angle less than 90°; and an *obtuse angle* is any one greater than 90°. What an arc wants of a quadrant or 90°, is called the *complement* of that arc; thus, 40° is the *complement* of 50°: and what an arc wants of a semicircle or 180°, is called the *supplement* of that arc; as 60° is the supplement of 120°. So in like manner, what an acute angle wants of a right angle, is the complement of that angle; and what any angle wants of two right angles is the supplement of that angle.

Let ADB, (Fig. 8.) be a semicircle, and C, its centre; and let the whole arc ADB, which is 180°, be bisected (*i. e.* divided into two equal parts) in the point D; then each of the parts, AD, and DB, is a quadrant or 90°: join DC, which may be produced at pleasure; then it is manifest that the right line DC leans to neither side, but makes the adjacent angles DCA, DCB, equal to one another, and so each of them is a right angle and contains 90°.

(4) A plane superfices, or as it is commonly called, a *plane,* is the least surface that can be bounded by the same lines; it lies perfectly flat, and has neither raised nor sunken parts in it. Or it is such that if the eye be placed in a continuation of it, then the line or extremity next the eye will hide the whole plane.

(5.) An angle is the opening between two right lines, and is neither increased nor made less by lengthening or shortening the lines that contain it, but by opening them wider, or closing them towards each other. The point in which the lines meet is called the angular point.—Some writers make three kinds of angles, viz. *right-lined, curve-lined,* and *mixed:* a right-lined or plane angle is contained by right lines; a curve-lined by curve lines; and a mixed angle by a right line and a curve: but the last two kinds are useless and absurd. Thus, ABC. is a plane angle, and B, the angular point. (Fig. 2.)

When an angle is denoted by three letters, the letter in the middle stands at the angular point; but an angle may be denoted by a single letter, provided there be but one angle at that point, and then the letter at the angular point will denote it: but when several angles are at the same point, then three letters must be used to denote any one of them. Thus, the angle contained by the right lines AB and BC is called the angle ABC, or CBA, or the angle B. (Fig. 2.) But there being more angles than one at the point P, (Fig. 3.) three letters must be used to denote any one of them. Thus, the angle contained between DP and SP, is called the angle DPS or SPD; that contained by SP and LP, is called the angle SPL, LPS; and the whole angle contained between DP and LP, is called the angle DPL, or LPD. Sometimes an angle is denoted by a small letter within the angle placed near the angular point. Thus the angle ABC, (Fig. 2.) may be called the angle *x.* Note, any Superfices or Solid, which is bounded or limited by extremities, is called a *Figure.*

17. A *plane Triangle*, is a plane figure bounded by three right lines: and every triangle has seven essential parts, namely, three sides, three angles, and an area or space within it. If the three sides be equal to one another, it is called an *equilateral* triangle; if two sides be equal, it is an *isosceles* or *equicrural* triangle; and if all the sides be unequal, it is a *scalene* triangle. Again, if it have one right angle, it is called a *right angled* triangle; if it have one obtuse angle, it is an *obtuse angled* triangle; but if it have its three angles acute, it is an *acute angled* triangle. Thus, (Fig. 9) X, is an equilateral triangle; Y, (Fig. 10.) an isosceles one; Z, (Fig. 11.) a scalene one; W, (Fig. 12.) a right angled triangle; S, (Fig. 13.) an obtuse angled one; and, T, (Fig. 14.) an acute angled one. Note, every plane triangle has at least two acute angles, and the third angle, may be right, obtuse, or acute; which makes the three kinds of triangles, in respect to the angles.

18. *Parallel right lines* are such as lie in the same plane, and being infinitely produced both ways never meet. (Fig. 15.) That is, parallel right lines always keep the same distance from each other; the perpendicular distance between them being always the same.

19. A Quadrilateral or four sided figure, is that which is bounded by four right lines.

20. Of four sided figures, that is a *Square*, which hath all its sides equal, and all its angles right angles. As A, (Fig. 16.)

21. That an *Oblong* or *Rectangle*, which has its opposite sides equal, and all its angles right angles. As, B. (Fig. 17.)

22. That a *Rhombus* which hath all its sides equal; but it has two opposite angles *acute* and equal, and the other two *obtuse* and equal. As, C. (Fig. 18.)

23. That a Rhomboides, which only hath its opposite sides and angles equal; two of the angles being acute, and two obtuse. As D. (Fig. 19.)

24. A *Parallelogram* is a Quadrilateral figure whose opposite sides are parallel: and it is evident that, a Square, an Oblong, a Rhombus, and a Rhomboides are Parallelograms; and that a right line joining any two of its opposite angles, bisects the Parallelogram. Note, that right line is called the Diagonal.

25. A Trapezium is any quadrilateral, which is not a parallelogram. As R. (Fig. 20.)

26. A *multilateral* or many sided figure, is any figure bounded by more than four right lines; and these are also called Polygons.

AXIOMS.

1. Things that are equal to the same thing are equal to one another.

2. Things that are double of the same thing are equal to one another.

3. Things that are halves of the same thing are equal to one another.

4. Things that are Equimultiples or Submultiples of the same or equal things are equal to one another. Thus, if there be two equal magnitudes, then 5 times, 6 times, &c. the one, will be equal to 5 times, 6 times, &c. the other, and $\frac{1}{5}$, $\frac{1}{6}$, $\frac{3}{4}$, &c. the one will be equal to $\frac{1}{5}$, $\frac{1}{6}$, $\frac{3}{4}$, &c. the other.

5. If equals be added to equals, the whole will be equal; and if equals be taken from equals, the remainders will be equal.

6. But, if equals be added to unequals, the whole will be unequal; and if equals be taken from unequals, the remainders will be unequal.

7. The whole is greater than its part. And the whole is equal to all its parts.

8. If the extremities of two right lines coincide, then these two right lines mutually coincide, and are equal to one another.

9. Those magnitudes which are congruous, that is, which fill the same space, are equal to one another.

10. All right angles are equal to one another.

11. If a right line be perpendicular to one of two parallel right lines, it is also perpendicular to the other; and all the perpendiculars, between two parallel right lines, are equal to one another.

POSTULATES.

1. Grant, that a right line may be drawn from any point to another point.

2. That a right line may be produced to any length; that is, that it may be continued far enough in the same direction.

3. That a circle may be described about any centre, at any distance from that centre; that is, that a circle may be described about a given point as a centre, with any given right line as a Radius.

4. That it is possible, for a right line to bisect a plane angle; that is, that a right line may divide a right lined angle into two equal parts.

5. That a right line may be drawn from any point either in or out of a given right line, at right angles to that line.

6. That a right line may be drawn, through a given point, parallel to another right line.

7. That from the greater of two unequal right lines, a part may be cut off equal to the less.

8. That a plane angle may be made eqnal to a given plane angle.

Note. In the following propositions, the student will meet with several of the foregoing Axioms and

Postulates made use of, but not quoted; as a constant reference to them is thought unnecessary and troublesome, and would be quite puerile among Geometricians.

Any two sides of a triangle may be called, *the sides*, and the remaining third one, *the base*.

In a right angled triangle, the side opposite the right angle, is called the *Hypothenuse*, and the other two sides containing the right angle, *the Legs*, any one of which may be termed the *Base*, and the other is then called, the *Perpendicular* or Cathetus.

Propositions are divied into Problems and Theorems: in a Problem there are some things given called the *data*, and something required to be done or to be found called the quœsitum or quœsita: in a Theorem there are certain conditions supposed, called the *hypothesis*; and other matters are asserted as conclusions from that hypothesis, and these are to be demonstrated: and what is to be demonstrated is called the *thesis*. It behoves every student, to see clearly his data, quœsita, hypothesis, and thesis; in the beginning, else he blunders in his proposition.

Proposition 1. Theorem.

If there be two triangles that have two sides of the one equal to two sides of the other, each to each, and have also the angle contained by the two sides of the one, equal to the angle contained by the two sides equal to them, of the other: then these two triangles are congruous; that is, the base of the one shall be equal to the base of the other; and the remaining angles of the one equal to the remaining angles of the other, each to each, which are opposite to equal sides; and the area of the one, equal to that of the other.

Let ABC, DEF, (Fig. 21.) be two triangles, having the side AB equal to DE, AC equal to DF, and the angle A equal to D. I say that the base BC is equal to the base EF, and the angle B equal to the angle E, the angle C equal to the angle F, and the area of the triangle ABC equal to the area of the triangle DEF.

For, if the triangle ABC be laid on the triangle DEF, so that the point A be on D, and the right line AB be on DE; then the point B will coincide with E, because AB is equal to DE, (by the hypothesis;) and the right line AC will coincide with DF, because the angle A is equal to D; (by hyp.;) and the point C will coincide with F, because AC is equal DF; (by hyp.;) and because the extreme points B, and C, of the base BC, coincide with the extreme points E and F, of the base EF; wherefore the base BC coincides with the base EF, and is equal to it: (by axiom 8;) consequently the angle B coincides with E and is equal to it; C is equal to F; and the two triangles are congruous, and their areas equal to one another. Which was to be demonstrated.

Hence the three following propositions are manifest.

Prop. 2. Theorem.

If two angles of one triangle be equal to two angles of another each to each; and one side of the one equal to one side of the other, either the side between the two angles of the one, equal to the side between the two angles equal to them of the other; or a side opposite to one of the equal angles: then, these two triangles are congruous; and have their other correspondent parts equal.

Prop. 3. Theorem.

If the three sides of one triangle be equal to the three sides of another, each to each: then, these two triangles are congruous, &c.

Prop. 4. Theorem.

If the hypothenuse and one leg of a right angled triangle be equal to the hypothenuse and one leg of another, each to each: then these two triangles are congruous, &c.

Prop. 5. Theorem.

The angles at the base of an isosceles triangle are equal to one another: that is, if a triangle have two equal sides; then the two angles, opposite to them, are equal.

Let ABC (Fig. 22.) be an isosceles triangle, having the side CA equal CB: then I say, that the angle A is equal to the angle B.

Demonstration.

For, let the angle ACB be divided into two equal parts by the right line CD. (by postulate 4.) Then because CA is equal to CB, (by hyp.) and CD is a common side to each of the triangles ACD and BCD; therefore the two sides CA and CD are equal to the two sides CB and CD, each to each, and they contain equal angles; (by construction;) wherefore the two triangles are congruous; (by proposition,) consequently the angle A is equal to the angle B. Q.E.D.

Corollary.—And because the base AD is equal to the base BD. and the adjacent angles ADC and BDC are equal and therefore right angles; (by definition 9.) wherefore it is manifiest, that if a right line bisect the vertical angle of an isosceles triangle, it bisects the base, and is a perpendicular to it.

Prop. 6. THEOREM.

IF two angles of a triangle be equal to one another; then the two sides opposite to them, are also equal to one another.

This proposition is only the converse of the last, and is therefore manifest.

Corollary.—Hence, every equilateral triangle, is also equiangular; and every equiangular triangle, is also equilateral.

Prop. 7. THEOREM.

IF a right line stand upon another right line: then the two angles made thereby, are either two right angles, or added together, equal to two right angles; that is, each of them is 90°, or the two added together are equal to 180°.

DEMONSTRATION.

FOR if the two angles be equal to one another, then each of them is a right angle; (by def. 9.) and the right line is a perpendicular to the other: but let the right line BE, (Fig. 23.) stand slant-ways on AC; and let BD be drawn at right angles to AC; (by postulate 5.) then the two right angles ABD and CBD fill the same space that the obtuse angle ABE and acute angle CBE do, and are therefore equal to them; (by Ax. 9:) or what is the same thing, the angle DBE is the excess of the obtuse angle ABE above a right angle, and also what the acute angle CBE wants of a right angle; therefore the obtuse and acute angle added together make two right angles. Q. E. D. Moreover it is plain, that the semicircle ADC is the measure of both angles, and therefore both added together make 180°.

Corollary.—Hence, it is plain, that all the angles made at the same side of a right line, are equal to two right angles, or 180°; and that all the angles made about a point, are equal to four right angles, or 360°.

Prop. 8. THEOREM.

IF two right lines cut (cross) each other: then the vertical angles, that is, those that are vertically opposite to each other, are equal to one another.

DEMONSTRATION.

LET the two right lines AB, CD (Fig. 24.) cut each other in the point P: I say that the angle APC is equal to the angle BPD, and the angle APD, equal to CPB.

For the two angles APC and CPB are equal to two right angles, (by Prop. 7.) and therefore equal to the two angles CPB and BPD, which are also equal to two right angles; from these equals take away the angle CPB, which is common, and there remains the angle APC equal to the angle BPD. In the same manner we demonstrate, that the angle APD is equal to the angle CPB. Q. E. D.

Prop. 9. THEOREM.

IF a right line meet two right lines, in the same plane, and make the alternate angle equal to one another: then, these two right lines are parallel.

DEMONSTRATION.

LET the right line AB meet the two right lines AC, DB, and make the angle CAB equal to the angle DBA: then I say AC is parallel to DB. (Fig. 25.) For draw BC, from the point B, at right angles to AC; and AD from the point A, at right angles to DB (by postulate 5): then the two triangles ABD, and ACB, have two angles of the one equal to two angles of the other, each to each, namely, the angle CAB equal to DBA, (by hypothesis) and the angles at D and C equal, for they are right angles, (by construction) and the side AB, which is opposite to the right angle in each, common; therefore the two triangles ABD and ACB are congruous; and consequently AD is equal to BC; but those right lines, which have equal perpendiculars between them, are parallel, therefore AC is parallel to BD. Q. E. D.

Prop. 10. THEOREM.

IF a right line cut two right lines in the same plane, and make the exterior angle with the one, equal to the interior opposite angle, on the same side, with the other; or the two interior angles, on the same side, equal to two right angles: then, these two right lines are parallel.

DEMONSTRATION.

LET the right line AB cut the two right lines CD, EF, (Fig. 26.) and make the exterior angle x, equal to the interior angle y: then I say, that CD is parallel to EF. For because the angle x is equal to the angle y, (by hyp.) and also to r, (by prop. 8.) therefore r is

equal to y, and they are alternate angles, wherefore CD is parallel to EF. (by prop. 10.) Again, because n and y are equal to two right angles, (by hyp.) and n and r are equal to two right angles, (by prop. 7.) therefore the two angles, n and r, are equal to the two angles n and y; take away the common angle n from both, and there remains the angle r equal to y, and they are alternate angles; wherefore CD is parallel to EF. Q. E. D.

Corollary.—Right lines parallel to the same right line, are parallel to one another.

From the two foregoing propositions, the following converse one is manifest.

Prop. 11. Theorem.

If a right line cut two parallel right lines: then it makes the alternate angles equal; the exterior angle equal to the interior opposite angle, on the same side: and the two interior angles, on the same side, equal to two right angles, or 180°.

Prop. 12. Theorem.

The three angles of every plane triangle are equal to two right angles, or 180°.

Demonstration.

Let ABC (Fig. 27.) be a plane triangle; I say that the three angles x, y, and v, added together are equal to two right angles, or 180°. For, through the vertex C, draw a right line DF parallel to AB; (by post. 6:) then the angle x is equal to o, and n is equal to y, because they are alternate angles; to these equals add the angle v; then the three angles, o, v, and n, are equal to the three angles of the triangle, x, y, and v; but the three angles, o, v, and n, are equal to two right angles or 180° (by Cor. prop.7.), consequently the three angles of the triangle, which are equal to them, are also equal to two right angles, or 180°. Q.E.D.

Corollary 1.—If one side of a triangle be produced, (Fig. 28,) the exterior angle d is equal to the sum of the two angles a and b; for the two angles c and d are equal to two right angles, (by prop. 7.) and therefore equal to the three angles of the triangle, a, b, and c; take away the angle c, which is common, and there remains the angle d, equal to the two angles, a and b. That is, the exterior angle of any plane triangle is equal to the two interior remote angles. Wherefore, if the exterior angle and one of the remote angles be given, the other remote angle will be found by deducting the given one from the exterior angle.

Cor. 2.—The two acute angles of a right angled triangle make one right angle, or 90°; and if one of them be given, the other will be found, by deducting the given one from 90°.

Cor. 3.—If two angles of a triangle be given, the third angle is found, by deducting the sum of the two given angles from 180°.

Cor. 4.—If one angle of a triangle be given, the sum of the other two is found, by deducting the given angle from 180°.

Cor. 5.—An angle of an equilateral triangle, is two thirds of a right angle, or 60°.

Cor. 6.—The sum of all the angles of a Quadrilateral figure, are equal to four right angles, or 360°. This will be evident by dividing the figure into two triangles, all the angles of which make the angles of the figure. And as every multilateral figure is made up of as many triangles as the figure has sides, abating two; and the angles of these triangles constitute the angles of the figure; wherefore the sum of all the angles of any multilateral figure, are equal to twice as many right angles as the figure has sides, abating four right angles.

Cor. 7.—If the two legs of a right angled triangle be equal; then each of the acute angles is half a right angle, or 45°.

Prop. 13. Theorem.

Parallelograms upon the same base and between the same parallels, are equal to one another.

Demonstration.

Let ABDC, ABEF, be two parallelograms on the same base AB, and between the same parallels AB and CE: I say, that the area of the parallelogram ABDC is equal to the area of the parallelogram ABEF. In the case in which the points D and F concur, (Fig. 29.) it is evident that each of the parallelograms is double of the triangle ABD, and therefore they are equal to one another; but when they are separated (Fig. 30, 31.) because the opposite sides of a parallelogram are equal, therefore AB is equal to CD and also to FE, consequently CD is equal to FE, from both take FD (Fig. 30.) or to both add FD, (Fig. 31.) then the remainder or sum CF is equal to DE; and because the figures are parallelograms, AC is equal to BD, and AF to BE; therefore the two triangles ACF and BDE, have the three sides of the one equal to the three sides of the other, each to each, and are equal (by prop. 3.); now from

the whole trapezium ABEC, take away these equal triangles, and there will remain the parallelogram ABDC equal to the parallelogram ABEF. Q. E. D.

Note. A parallelogram is commonly denoted by the two letters that stand at the opposite angles. Thus, the parallelogram ABDC, may be called, the parallelogram BC.

Prop. 14. Theorem.

Triangles upon the same base and between the same parallels are equal to one another. This is evident, from the last proposition; for they are the halves of the parallelograms, standing on the same base and between the same parallels with them. And if a triangle and a parallelogram stand on the same base, and be between the same parallels; then the area of the triangle is equal to that of half the parallelogram.

Prop. 15. Theorem.

If the two adjacent angles ADC and BDC, (Fig. 32.) be two right angles, or together equal to two right angles: then will AD and DB, be in the same direction, and make but one right line. This is evident, as it is the converse proposition to prop. 7.

Prop. 16. Theorem.

In a right angled triangle, the area of the *square* of the hypothenuse is equal to the *sum* of the areas of the *squares* of the two legs; that is, to the two added together.

Demonstration.

Let ABC (Fig. 33.) be a right angled triangle having the right angle at B; and let DC, CK, and AT, be squares described upon the sides: then I say, that DC, the square of the hypothenuse, is equal to CK and AT, the squares of the two legs. For join AE, BF; and let BL be drawn through the point B parallel to AD or CF. Then because the angles CBA, CBK, are both right angles, (by hypothesis,) therefore AB and BK are in the same right line (by prop. 15.); for the same reason CB and BT are in the same right line; again because the two right angles ACF and BCE are equal: to both add the angle ACB, and then the whole angle ACE is equal to the whole angle BCF; but the two sides FC, and CB, are equal to the two sides AC and CE, each to each; therefore the area of the triangle FCB, is equal to the area of the triangle ACE (by prop. 1); and the parallelogram CL, is double of the triangle FCB (by prop. 14,) for they stand upon the same base CF, and are between the same parallels BL and CF; and the square CK is double of the triangle ACE, for they stand on the same base CE, and are between the same parallels AK and CE; but the doubles of equal things are equal; therefore, the parallelogram CL is equal to the square CK: In the same manner we demonstrate, if BD, SC, be joined, that the parallelogram AL, is equal to the square AT: wherefore the whole square DC, is equal to the two squares CK and AT. Q. E. D.

Scholium.

This noble proposition, which is commonly known by the name of the Pythagorean Theorem, from Pythagoras the discoverer, is of the greatest utility in almost every branch of mathematics. The Trigonometrical Tables could not be constructed without its aid; consequently Astronomy, Navigation, and many other branches, are indebted to it, for the solution of almost every important problem therein: even the proportion between the diameter and circumference of a circle, which is a fundamental inquiry, could not be approximated without its assistance. Euclid makes it the 47th prop. of his 1st Book of the Elements of Geometry. And we shall often hereafter have occasion to use it; when its excellence will appear.

Definitions.

1. A *Segment* of a circle is a figure contained by a right line, called the chord, and that part of the circumference cut off by the chord. If a right line pass through a circle, it divides the circle into two Segments; and if it pass through the centre, then that chord is a diameter, and the segments are semicircles; but if it do not pass through the centre, then the chord is less than the diameter, and one of the segments is less than a semicircle and the other is greater. Thus, (Fig. 34.) AB, is the chord of the arc ACB, or ADB, and the two segments are S, and T.

2. A *Sector* of a circle is a figure contained by two Radii, and the arc between their extremities. Thus, (Fig. 35,) ACB is a Sector.

3. A right line is said to *touch* a circle, when being produced, it does not cut it; that is, it just meets the circumference in one point, and being produced, goes farther from it on both sides, and is commonly termed a *Tangent*.

Prop. 17. THEOREM.

An angle, at the centre of a circle, is double to the angle at the circumference, when the same arc is the base of both angles.

DEMONSTRATION.

LET ADB be a circle (Fig. 36,) ACB an angle at the centre, and ADB an angle at the circumference: I say, that the angle ACB is double of the angle ADB. For join DC, and produce it to E: then the exterior angle BCE, of the triangle BCD, is equal to the two interior angles CBD and CDB; (by cor. 1. prop. 12;) but these two angles are equal; (by prop. 5;) because BC is equal to CD; and therefore the angle BCE which is equal to them both, is double to one of them, BDE: in like manner it may be demonstrated, that the angle ACE, is double of the angle ADE: wherefore the whole angle ACB, is double of the whole angle ADB. Q. E. D.

Corollary.—1. Hence an angle, at the circumference of a circle, is measured by half the arc on which it stands: for the angle at the centre, which is double to it, is measured by that arc; (by def. 15.) and therefore its measure is the half of that arc.

Cor.—2. An acute angle at the circumference of a circle stands on an arc less than a semicircle; an obtuse angle on one greater; and a right angle exactly on the arc of a semicircle. And that an angle in a semicircle is a right angle is easily otherwise demonstrated. Thus. Let ACB (Fig. 37,) be a semicircle, and AB the diameter, D the centre; and ACB an angle in the semicircle. I say that the angle ACB is a right angle; for join CD; then the angle DAC, is equal to the angle DCA; (by prop. 5.) because DA is equal to DC; for the same reason, the angle DBC is equal to the angle DCB; therefore the whole angle ACB is equal to both the angles at A and B; but when one angle of a triangle is equal to the other two, that angle is a right one; wherefore the angle ACB is a right angle. Q. E. D.

Prop. 18. THEOREM.

ALL the angles at the circumference of a circle, which stand on the same arc, are equal to one another; or, what is the same thing, all the angles in the same segment of a circle, are equal to one another. *Note*, this is one of the *great* properties of a circle.

This is evident, for each of them is half the angle at the centre, (by prop. 17.) and each is measured by half the arc it stands on, that is, by half the same arc.

Prop. 19. THEOREM.

THE two opposite angles of any quadrilateral figure, inscribed within a circle, (that is, when all its four angular points are in the circumference,) are equal to two right angles.

This is evident; because both angles stand upon the whole circumference, and therefore half the circumference, or 180 degrees, is the measure of both.

Prop. 20. THEOREM.

A RIGHT line drawn at right angles to the diameter of a circle, from either of its extreme points, does not cut the circumference of the circle; but being produced both ways, only *touches* the circumference in the extremity of the diameter, and is a tangent to the circle.

This is evident.

Prop. 21. THEOREM.

IF a right line touch a circle, and another right line be drawn from the point of contact, within the circle: then the angles, which that chord makes with the tangent, are equal to those made in the alternate segments of the circle.

DEMONSTRATION.

LET CGDEF, be a circle (Fig. 38,) S its centre, and AB a tangent to it, touching it in the point C, and CD a chord drawn from the point of contact and dividing the circle into two segments: then I say, that the angle CFD is equal to the angle DCB, and the angle CGD equal to the angle ACD: for draw the diameter CE, and join DE: then it is plain that the tangent AB is at right angles to the diameter CE: also the angle CDE being an angle in a semicircle, is a right angle; wherefore the other two angles DEC, and ECD, of the triangle DEC, are equal to a right angle, and are therefore equal to the right angle ECB; that is, to the two angles, ECD, and DCB; take away the angle ECD, from both; and there remains the angle DEC equal to the angle DCB; but the angle DEC is equal to the angle DFC (by prop. 18,) for they stand on the same arc; wherefore the angle DCB is equal to the angle DFC. Again, because the two opposite angles at G and F are equal to two right angles, (by prop. 19.) they are therefore equal to the two

angles DCB and DCA, which are also equal to two right angles; take away the equal angles F and DCB, and there will remain the angle DCA equal to the angle at G; but the other angles have been proved to be equal. Q. E. D.

OF PROPORTION.

Definitions.

1. Four magnitudes or numbers, are said to be in the same *ratio* or proportion ; the first to the second as the third to the fourth; when the first is the same multiple or submultiple of the second, that the third is of the fourth. Thus, 6 is to 2, as 9 is to 3; for 6 is treble of 2, and 9 is treble of 3: also, 3 is to 5, as 9 is to 15; for 3 is $\frac{3}{5}$ of 5, and 9 is $\frac{3}{5}$ of 15. These proportions are expressed for the sake of brevity, thus, as 6 : 2 :: 9 : 3; and as 3 : 5 :: 9 : 15.

2. The first term is called the *antecedent* and the second the *consequent;* so in like manner, the third is an *antecedent* and the fourth a *consequent;* that is, the quantity compared is *antecedent,* and that to which it is compared, is *consequent;* and the number of times, part or parts of a time, that the antecedent contains the consequent; that is, the quotient arising by dividing the antecedent by its consequent, is called the *ratio* or proportion.

3. When three magnitudes or numbers are in the same ratio; that is when the consequent of the first ratio is antecedent in the second; then that in the middle, is called a *mean proportional* between the first and third; thus, 3, 6, and 12: here 6 is a mean proportional between 3 and 12: for 3 : 6 :: 6 : 12: and when three quantities are proportionals; the first is said to have a *duplicate* ratio to the third, of that which it has to the second : and if four quantities be in continued proportion; then the ratio of the first to the fourth is *triplicate* of that which it has to the second, and so on.

4. *Inverse ratio,* or inversion, is when the consequent of the first ratio is taken as antecedent, and the consequent of the second as antecedent. Thus, direct, as, 6 : 2 :: 9 : 3; then inversely, as 2 : 6 :: 3 : 9, and this holds true always.

5. *Alternate ratio,* or alternation, is when the antecedent is compared to the antecedent; (as if it were a consequent;) and the consequent (as if it were an antecedent) to the consequent. This holds true always in numbers, and also in magnitudes of the same kind : but if the first ratio be between lines and the second between superfices or solids; alternation cannot be used. Thus directly, 6 : 2 :: 9 : 3; alternately, 6 : 9 :: 2 : 3.

Some properties of proportionals which are self evident, or Axioms.

In order to render this as simple as possible: suppose $6 = a$, $2 = b$, $9 = c$ and $3 = d$. Then assume four quantities in direct proportion,

1. As $a : b :: c : d$, *i. e.* 6 : 2 :: 9 : 3; here a and c, (6 and 9,) are the antecedents, and b, and d, (2 and 3,) are the consequents, and the proportion is direct.

And if four quantities be directly proportional, viz. in the same ratio; then they shall be proportionals.

2. Inversely, as $b : a :: d : c$. that is, as 2 : 6 :: 3 : 9.

3. Alternately, as $a : c :: b : d$, that is, as 6 : 9 :: 2 : 3.

4. Compounded by Addition, as

$a + b : b :: c + d : d$, that is, as 8 : 2 :: 12 : 3.

$a + b : a :: c + d : c$, that is, as 8 : 6 :: 12 : 9.

5. Divided by Subtraction, as

$a - b : b :: c - d : d$, that is, as 4 : 2 :: 6 : 3.

$a - b : a :: c - d : c$, that is, as 4 : 6 :: 6 : 9.

6. Compounded by Multiplication, or equimultiples, as

$a \times b : b :: c \times b : d$, that is, as 12 : 2 :: 18 : 3.

$a \times b : a :: c \times b : c$, that is, as 12 : 6 :: 18 : 9.

And by Division : but there must be equimultiples correspondently taken, or sub-multiples, as

$a : b \times b :: c : d \times b$, that is, as 6 : 4 :: 9 : 6.

$b : b \times b :: d : d \times b$, that is, as 2 : 4 :: 9 : 18, &c.

7. In correspondent powers or roots, as

$a^2 : b^2 :: c^2 : d^2$, that is, as 36 : 4 :: 81 : 9.

$a^3 : b^3 :: c^3 : d^3$, that is, as 216 : 8 :: 729 : 27.

Note. The first and fourth terms of a proportion are called the *extremes,* and the second and third, or middle terms, the *means.*

8. If four quantities be proportionals, then the *rectangle* or *product* of the two *extremes* is equal to the rectangle of the two *means.* Thus, as in the foregoing, assume, $a : b :: c : d$, 6 : 2 :: 9 : 3; then, $a \times d = b \times c$; that is, $6 \times 3 = 2 \times 9$, both being $= 18$; the reason is evident, because a is the same multiple of b that c is of d (by the hypothesis), and consequently the products or multiplications must be equal. Q.E.D. Also, if the rectangle of the extremes be equal, then the four quantities are proportionals. This is evident from the former part.

9. If three quantities be proportionals: then the product or rectangle of the *two extremes* is equal to the *square* of the *mean* or middle term.

Suppose, $6 = a$, $12 = b$, and $c = 24$.

Let us assume, as $a : b :: b : c$, i. e. 6 : 12 :: 12 : 24; then $a \times c = b^2$; that is, $6 \times 24 = 12^2$, both being equal to 144 : the reason is the same as in the last.

10. *Geometrical proportion* or progression is when the *ratio* of the first to the second is the same as

the ratio of the second to the third, and the third to the fourth, and the fourth to the fifth, and so on; and the first and last are the *extremes*, and all the others *means*. Thus, 2, 6, 18, 54, 162, &c. are called geometrical proportionals: For 2 : 6 :: 6 : 18 :: 18 : 54 :: 54 : 162, &c. And 6 is called a geometrical mean between 2 and 18; and 18 is a geometrical mean between 6 and 54, &c. Also the *square root* of the product of two numbers is the geometrical mean between them: so $\sqrt{2\times18}=6$, which is the geometrical mean between 2 and 18.

11. *Arithmetical proportion* or progression is when the difference of the first and second is the same as the difference of the second and third, and the difference of the third and fourth, and fifth and sixth, and so on: 3, 5, 7, 9, 11, &c. are arithmetical proportionals: Also half the sum of two quantities is their arithmetical mean. Thus, $\frac{3+7}{2}=5$, which is the arithmetical mean between 3 and 7.

Definition.—Similar right-lined figures are those which have their angles equal, each to each, and the sides about the equal angles proportionals.

Prop. 22. Theorem.

If one side of a triangle be bisected, and a right line be drawn through the point of bisection (or middle point) parallel to the base of the triangle, within the same; then I say, that that right line bisects the other side of the triangle, and is equal to half the base of the triangle. Moreover if one side of a triangle be divided into any number of equal parts, and right lines be drawn parallel to the base, within the triangle, through the points of section: then I say that the other side of the triangle is also divided by these parallels into the same number of equal parts; and each parallel is the same part or parts of the base, that the correspondent part or parts of the sides are of the sides.

Demonstration.

Let ACB be a triangle (Fig. 39.), and let AC be bisected in D, and DE drawn through D be parallel to AB: then I say that CE is equal to EB, and DE equal to the half of AB; for let DF be drawn parallel to CB: then the two triangles, CDE and DAF, have a side, CD, of the one equal to, DA, a side of the other (by the hypothesis), and the angles adjacent to these equal sides, equal, each to each, by reason of the parallels; whence these two triangles are congruous; consequently DF is equal to CE, and AF is equal to DE; but DF is equal to EB, and DE to FB, for they are the opposite sides of a parallelogram; wherefore CE is equal to EB, and DE to AF: and so DE is equal to the half of AC. Q.E.D.

Again, let AC be divided into three equal parts in the points F and H (Fig. 40.); and right lines be drawn parallel to the base AB, through F and H; and also a parallel to the side CB: then we demonstrate as before, that CE, ED, and BD, are equal to one another; and also AK, KI, and IB, are equal to one another; and universally, whatever part CF is of CA, the same part is CE of CB, and FE of AB: also, whatever parts CH is of CA, the same parts is CD of CB, and HD of AB. Q.E.D.

Prop. 23. Theorem.

If a triangle be cut off from another triangle by a right line parallel to one of the sides; then the triangle cut off is similar to the whole triangle; that is, these two triangles are equiangular to each other, and the sides about the equal angles are proportionals, and the sides opposite to the equal angles are homologous and have the same ratio.

Demonstration.

Let the right line DE, (Fig. 41.) be drawn parallel to AB, one side of the triangle CAB: then I say that the triangle CDE, is similar to the triangle CAB. For it is manifest, that the two interior angles CDE and CED, are equal to the angles A and B; and the angle C is common to both triangles; wherefore the triangles are equiangular: and again, because DE is parallel to AB, whatever part or parts CD is of CA, the same is CE of CB and DE of AB; (by prop. 22:) wherefore as CD is to CA, so is CE to CB, and so is DE to AB (by defi. 1. of proportion); also as CD : DA :: CE : EB. &c. Q.E.D.

Prop. 24. Theorem.

If two triangles he equiangular to each other; they shall be similar, and have the sides about the equal angles proportionals, and the sides opposite to the equal angles homologous, that is, of the same ratio.

Demonstration.

Let CAB, (Fig. 41.) FGH, (Fig. 42.) be two equiangular triangles, having the angle C equal to F, A to G, and B to H; I say, that they are similar: For make CD equal to FG and CE equal FH, and join DE: then because the two sides CD and CE are equal to FG and FH, each to each, and the angle C, is equal to the angle F; therefore the triangles, CDE and FGH are congruous; (by prop. 1.) consequently the base DE is equal to GH, and the angle CDE equal to G, and CED equal to H; wherefore DE is parallel to AB; and hence (by the last proposition)

as CD : CA :: CE : CB, *i. e.* FG : CA :: EH : CB; also, as CD : CA :: DE : AB; *i. e.* FG : CA :: GH : AB: And by alternation, inversion, &c. the sides about the equal angles are proportionals, and the triangles are similar, and the sides opposite to the equal angles are homologous, and have the same ratio. Q.E.D.

Prop. 25. THEOREM.

IN a right angled triangle, the perpendicular, let fall from the right angle upon the hypothenuse, divides the triangle into two triangles, similar to the whole and to one another; and the perpendicular is a mean proportional between the segments of the hypothenuse; and each leg of the whole triangle is a mean proportional between the hypothenuse and that segment of it adjacent to that leg.

DEMONSTRATION.

LET ABC be a right angled triangle, (Fig. 43.) having the right angle at B; and let BD be drawn at right angles to AC: then I say that the triangles ABD, CBD, are similar to the whole triangle ABC, and also to one another: and that DB, is a mean proportional between AD and DC; also that AB, is a mean proportional between AC and AD; and that BC, is a mean proportional between AC and CD. For because the angles ADB and ABC, are equal, for they are right angles, and the angle A, is common to the two triangles ADB and ABC, therefore the remaining angle ABD, is equal to the remaining angle C; therefore the triangle ABD is equiangular, and consequently similar to the whole triangle ABC: in the same manner we demonstrate that the triangle CBD is similar to the whole triangle ABC; and because they are both similar to the whole, it is evident, that they are similar to one another. Again, in the similar triangles, as, AD : DB :: DB : DC, and so DB is a mean proportional between AD and DC; also, as AC : AB :: AB : AD; and as, AC : BC :: BC : DC. Q.E.D.

Prop. 26. THEOREM.

IF from any point in the circumference of a semicircle a right line be drawn perpendicularly to the diameter; then the square of that perpendicular, is equal to the rectangle (or product) of the segments of the diameter.

DEMONSTRATION.

LET the right line BD, (Fig. 44.) be drawn from the point B, in the circumference at right angles to the diameter AC: I say that the square of BD is equal to the rectangle (or product) of AD and DC; for join AB and CB: then the angle ABC, is a right angle, because it is an angle in a semicircle; and DB is a mean proportional between AD and DC; (by the last prop.) consequently AD $\times$ DC $=$ DB^2. (by 9th, of proportion.) Q.E.D.

SCHOLIUM.

THIS is called the greatest or chief property of the circle: and if the semidiameter or radius be called, r; and AD, be called, x; and DB, be called, y; then $\overline{CD = 2r - x}$; AD $\times$ CD $=$ DB^2, is expressed, $2r - x \times x = y^2$, or $2rx - x^2 = y^2$, which is called the equation of the circle: and almost every thing relative to a circle is determined and regulated thereby.

Prop. 27. THEOREM.

IF two right lines in a circle cut each other; then the rectangle of the segments of the one, is equal to the rectangle of the segments of the other.

DEMONSTRATION.

LET ADBF, (Fig 45.) be a circle, and the two right lines in it AB and DF, cut each other in the point C; then I say that AC $\times$ CB $=$ DC $\times$ CF; for join AD, and BF; then the angle B, is equal to D; and A is equal to F, (by prop. 18.) where the triangles ACD, and FCB, are equiangular and similar; whence, as AC : FC :: CD : CB, consequently AC $\times$ CB $=$ DC $\times$ CF. (by prop. 8th of proportion,) Q.E.D.

Prop. 28. THEOREM.

IF any point be assumed without a circle, and from that point right lines be drawn, cutting the circle, and terminated by the concave circumference; then the rectangle of the whole secant line, and its external part, between the assumed point and the convex circumference, is equal to the rectangle of any other and its external part.

DEMONSTRATION.

LET P be any point taken without the circle ABDC, (Fig. 46.) and let the right lines PA, PB, be drawn from it cutting the circle in C and D, and terminated at A and B: then, I say, that PA $\times$ PC $=$ PB $\times$ PD. For join AD and BC; then the two triangles APD and BPC, are similar, because the angle A is equal to B, for they stand on the same arc, CB, and

the angle P is common to both triangles; whence, as AP : DP :: BP : CP; consequently PA × PC = BP × DP. Q.E.D.

Coroliary.—From hence it is manifest, that PB × PD = PF^2, that is, to the square of the tangent.

Prop. 29. THEOREM.

IN every parallelogram, the parallelograms that stand about the diagonal, are similar to the whole and to one another, and their complements are equal to one another.

DEMONSTRATION.

LET AC, (Fig. 47.) be a parallelogram, whose diagonal is DB, HK, and LI, parallelograms about the diagonal, and AG and GC, are called the *complements:* I say that HK and LI are similar to AC, and to one another; and that the area of AG is equal to the area of GC. For it is evident, that the whole triangle ABC, is equal to the whole triangle BCD; and from these equals, take away the two triangles HBG and LGD, and the two equal to them, BKG and DGI, and there remains the complement AG equal to the complement GC. Again because HG is parallel to AD, therefore the triangle BHG is similar to BAD; whence, as BH : HG :: BA : AD; and for the same reason, as DL : LG :: DA : AB. And in like manner we demonstrate, that the sides about the other equal angles are proportionals; wherefore HK and LI are similar to AC; and consequently they are similar to one another. Q.E.D.

Prop. 30. THEOREM.

SIMILAR Parallelograms, which have one angle common to both, are about the same diagonal. This is evident, being but the converse of the last; for when they are about the same diagonal, they are similar, (by the last prop.) and consequently, conversely, when they are similar, they stand about the same common diagonal. Q.E.D.

Prop. 31. THEOREM.

IF a right line bisect the vertical angle of a triangle and cut the base; then the segments of the base shall have the same ratio to each other, which the sides adjacent to the segments have.

DEMONSTRATION.

LET ABC (Fig. 48.) be a triangle; and let the vertical angle ACB be bisected by CD: then I say, that as AD : DB :: AC : BC; for produce AC to E, and make CE equal to CB, and join BE: then because CE is equal CB, the angle EBC is equal to the angle E; but the exterior angle ACB is equal to both these angles, and DCB, which is half of it, (by the hypothesis) is equal to one of them CBE; and they are alternate angles; wherefore CD is parallel to BE: whence, as AD : DB :: AC : CE; but CB is equal to CE; therefore, as AD : DB :: AC : BC. Q.E.D.

Prop. 32. THEOREM.

THE chord of an arc of 60° is equal to the radius of that circle. Or in other words; the side of a regular hexagon inscribed in a circle, is equal to the semidiameter of the circle.

DEMONSTRATION.

LET ABCDEF, (Fig. 49.) be a circle divided into six equal parts of 60° each; and let the chords AB, BC, be drawn: then I say that each chord is equal to the radius. For let G be the centre, and join AG and BG: then, because AG is equal to BG, the angle A, is equal to B; but the angle G is an angle of 60° (by the hypothesis), for it stands on an arc of 60°; wherefore the angles A and B are each of them 60°; consequently the triangle ABG is equiangular, and therefore equilateral; wherefore AB is equal to AG or BG, the radius. Q.E.D.

Prop. 33. THEOREM.

IN all circles their circumferences have the same ratio or proportion to one another, that their diameters or radii have. And all the aliquot parts, or submultiples of the circumferences, have the same ratio as the diameters or radii, or the aliquot and similar parts of these. This is evident.

OF SOLIDS. DEFINITIONS.

1. A solid is that which hath length, breadth, and depth or thickness: (see the introduction to this chap.) a solid is bounded by superficies or surfaces.

2. A right line is said to be at right angles or perpendicular to a plane, when it makes right angles with all the right lines drawn upon the plane, from the point in which it meets the plane. To make this familiar; suppose a plumb-line to pass through a floor perfectly smooth and level, then right lines drawn on the floor to the plumb-line will make right angles with it, and the plumb-line is perpendicular to the floor: also the Earth's axis is perpendicular to the plane of the equator and to the planes of all the parallels of latitude. Hence, if a right line be perpendicular to a plane; then all the planes which pass through that right line, are assumed to be perpendi-

cular to that plane; so the planes of all the meridians, are assumed to be perpendicular to the plane of the equator, because they all pass through the axis.

3. If two right lines meet in a point in a plane, and one of them stand perpendicularly to the plane but the other obliquely; and if another plane pass through both these right lines; then the acute angle contained between the common section of the two planes, and the oblique line is called the inclination of that oblique line to the plane first mentioned. And if a third plane be conceived to stand at right angles to the oblique line, and pass through the point where these two lines meet, the *inclination* of this plane to the first mentioned is the acute angle contained between the two right lines. Thus, the axis of the Earth is at right angles to the plane of the equator, and the axis of the ecliptic stands obliquely to it, but at right angles to the plane of the ecliptic; the plane of the solstitial colure passes through both axes and cuts the planes at right angles; and the acute angle contained between that radius of the sphere which is the common section of the planes of the equator and solstitial colure, and the Earth's axis, is called its inclination, or obliquity to the plane of the equator; and the angle contained between the Earth's axis and the axis of the ecliptic is called the inclination or obliquity of the ecliptic to the equator, and it is measured by that arc of the solstitial colure contained between the ecliptic and equator, and is equal to the Sun's greatest declination; and the inclination of the Earth's axis to the plane of the ecliptic is its complement.

4. A *solid angle* is contained by three or more plane angles set upon a plane, and meeting together in a point above that plane; and if the angles be unequal, it is necessary that the greatest of them shall be less than the sum of all the rest.

5. A *cube* is a solid contained by six equal squares. To be familiar: dice are cubes.

6. A *parallelopipedon* is a solid contained by six parallelograms, of which every two opposite ones are equal and similar. If they be oblongs or rectangles, it is called a right angled parallelopipedon. To be familiar: chests, boxes, &c. are parallelopipedons.

7. A *prism* is a solid contained by several plane figures, of which there are two opposite which are equal and similar, and all the others are parallelograms. If the two opposite and equal figures be triangles it is called a triangular prism; if quadrilaterals, a quadrilateral prism; if pentagons, a pentagonal prism; &c. Hence, every cube is a parallelopipedon, but every parallelopipedon is not a cube; and every parallelopipedon is a prism, but every prism is not a parallelopipedon.

8. A *pyramid* is a solid contained by plane triangles set up on a plane and meeting together in a point called the vertex. If the base upon which the triangles stand be a triangle, it is called a triangular pyramid; if it be a square, it is called a square pyramid, &c. To be familiar: any pillar which has flat sides, and ending in a point, is a pyramid.

9. A *cylinder* is a solid generated by the revolution of a rectangle round one of its sides, which remains at rest till the motion end where it began; and that fixed line which remained at rest is called the *axis* of the cylinder. To be familiar: drums, round rulers, gardeners' rolling-stones, &c. are cylinders. Both the ends or bases are circles, and the round surperficies is called the *curve surface.*

10. A *right cone* is a solid generated by the revolution of a right angled triangle about one of the legs, which remains at rest till the motion end where it began; and that fixed line which remained at rest is called the *axis* of the cone. To be familiar: sugar-loaves, &c. are cones. The base of a cone is a circle, and the round superficies ending in a point is called the *curve surface.* A cone is a round pyramid.

11. A *sphere* or *globe* is a solid generated by the revolution of a semicircle round the diameter, remaining at rest till the motion end where it began. And that fixed diameter, which remained at rest, is called the *axis;* the point in the middle, the *centre;* and every right line drawn through the centre, and terminated both ways by the surface, is a diameter of the sphere; and all the diameters are equal to the axis and to one another. Balls, bullets, &c. are globes.

12. A *spheroid* (from the Greek, signifying a sphere and form or shape) is a solid generated by the revolution of a semi-ellipsis round one of the diameters, which remains at rest till the motion end where it began. And that fixed diameter which remained at rest is called the *axis* of the spheroid. If the axis be the transverse diameter of the ellipsis, then it is called a *prolate* spheroid; but if it be the conjugate diameter, it is called an *oblate* spheroid. To be familiar; an egg, if it were similar at both ends, would be a prolate spheroid; and turnips, oranges, &c. are oblate spheroids.

Of the MENSURATION *of* MAGNITUDES, *applicable to* ASTRONOMY.

DEFINITIONS.

1. EVERY magnitude is measured by another magnitude of the same kind: and the measuring magnitude must be the most simple and regular of its kind; and such as is clearly comprehended, and allowed by all, to be the proper measuring magnitude.

2. A *line* is measured by a *right line* of a given length; as 1 foot, 1 yard, 1 mile, &c. And it is supposed, that an accessible right line can be measured by the student, without any previous in-

structions; that is, that he can ascertain, by means of a measuring instrument, how many feet, yards, poles, &c. are in a right line between any two points, having no obstacle between them, to prevent the application of the measuring instrument. But when obstacles intervene between the points; then we call that an inaccessible right line; such as the distance over a lake, the distance from London to Madrid, the distance between the centres of the Earth and Moon, &c. And to measure these inaccessible right lines requires the assistance of plane Trigonometry, which shall be taught hereafter.—As no right line can conveniently nor perfectly be applied to a *curve line;* whence the mensuration of all kinds of curve lines, as well as that of inaccessible right lines, is obliged to call in the aid of mathematical science to ascertain their lengths. The proportion between the diameter and circumference of a circle, has been ascertained to great accuracy, by mathematicians; but the length of other curves is involved in deep mathematical researches; and is called by mathematical gentlemen, the *rectification of curve lines.* But fortunately the circle is the principal curve used in astronomy, and the mensuration of arcs of a circle, that are inaccessible, is the business of spherical trigonometry, which shall be taught, with that of plane, hereafter.

3. A *superficies* is measured by a given *square;* as a square foot, a square yard, a square mile, &c. And the *number* of such squares as that superficies or surface contains, are called the *area* or superficial content of that superficies. Thus, suppose an oblong or rectangle, to be 7 feet long and 3 feet broad, then it is plain, that it contains 21 square feet. And the number of these squares, which is found by multiplying the length by the breadth, namely, $7 \times 3 = 21$ square feet, is called the *area* or content of that oblong. But the area of most figures and surfaces cannot be ascertained without the aid of mathematical science; and in such as chiefly relate to astronomy, we are now about to instruct the student.

4. A *solid* is measured by a given *cube;* as a cubic foot, a cubic yard, a cubic mile, &c. And the *number* of such *cubes* as that solid contains, is called the *solidity* or solid content of that solid. Thus suppose a square parallelopipedon, (or chest,) to be 4 feet long, 2 feet broad, and 2 feet deep; then it is plain that it contains 16 cubic feet, viz. $4 \times 2 \times 2 = 16$, which is found by multiplying the length, breadth, and depth together: the solidity of most solids cannot be found without the aid of mathematical rules, which shall be given in this chapter.

Of the CIRCLE. Problem 1.

Given the diameter of a circle to find the circumference. Rule: say,

As 7 is to 22, } so is the diameter to the circumference. But more accurately still, as 1 is to 3.14159, &c.*
or as 113 is to 355, }
or as 1 is to 3.1416, }

Example 1.—Suppose the diameter of a circle to be 42 miles: required the circumference?

As 7 : 22 : : 42 : 132 miles, }
As 113 : 355 : : 42 : 131.946 + miles, } the circumference required.
As 1 : 3.1416 : : 42 : 131.9472 — miles, }

Example 2.—The diameter of the Earth's equator is 7970 English miles: required the circumference?

As 1 : 3.1415926 : : 7970 : 25038.493022 = the circumference, or $25038\frac{1}{2}$ miles nearly.

Problem 2.

Given the circumference of a circle to find the diameter. Rule: say,

As { 22 : 7 / 355 : 113 / 3.1416 : 1 } : : the circumference to the diameter.

* To find the exact proportion between the diameter and circumference of a circle, was a problem that engaged the attention of the ancient mathematicians, for many ages. Archimedes discovered, that if the diameter be 7, the circumference will be a small quantity less than 22. Metius discovered, that if it be 113, the circumference is something more than 355; but the quantity of the excess or defect, neither of them was able to determine.

By the introduction, however, of the ten Arabic characters, 0, 1, 2, 3, &c. into arithmetic, and the invention of decimal fractions, and the application of arithmetical calculations to geometry: the solution of the problem has been effected to a degree of exactness, far beyond human conception. Van Cullen carried the proportion to 36 places of decimal figures all perfectly true; which are said to be engraved on his *tomb,* in the church-yard of Leyden; as if he had set bounds to succeeding mathematicians: but since the invention of Fluxions (by Sir Isaac Newton), and Infinite Series, Mr. Sharp, in a much more easy way, determined it to 72 places; and Mr. Machin, since that, to more than 100 places, all true. Van Cullen's numbers are more than sufficiently true, to calculate the number of grains, each less than a grain of mustard seed, that would fill a globe, which should circumscribe the orbit of Georgium Sidus, and not err a thousandth part of a grain from the truth: but Mr. Sharp's numbers are sufficiently true to calculate the number of grains contained in a globe that should circumscribe thousands of the fixed stars: and Mr. Machin's numbers carry the matter to a degree of accuracy inconceivably farther.

The numbers are thus expressed. If the diameter of a circle be 1; the circumference, is
3.14159265358979323846264338327950 28, &c. but six or seven decimal figures are generally supposed to be sufficiently accurate for the nicest calculations in astronomy. Thus, as, 1 : 3.1415926 : : the diameter : the circumference. And for most other purposes; as 1 : 3.1416.

Example 1.—Given the circumference of a circle 132 feet, required the diameter of that circle?

As 22 : 7 : : 132 : 42 feet,
As 355 : 113 : : 132 : 42.017 feet,
As 3.1416 : 1 : : 132 : 42.014 feet,
} the diameter required.

Example 2.—Given the circumference of the Earth's equator, 25038½ English miles, required the diameter?

As 3.1415926 : 1 : : 25038.5 : 7970 miles nearly.

Problem 3.

Having the radius and degrees of an arc given, to find the length of that arc. Rule:

Find the whole circumference of the circle by problem 1; then say, as 360° is to the given degrees so is the whole circumference to the length of the arc required.

Example.—Required the length of an arc of 60°, the radius being 100 feet? Here the diameter is 200 feet.

As 1 : 3.1416 : : 200 : 628.32 = the whole circumference.

Then, as 360° : 60° : : 628.32 : 104.72 feet, the answer.

Hence, from the three foregoing rules, if the radius be given, and the length of the arc; we can find the degrees in that arc: and if the degrees of an arc, and the length of the arc be given, we can find the radius, and, consequently, the whole circumference, &c. And the exact proportion between the diameter and the circumference of a circle is the *datum* from which almost all problems relative to a circle are solved.

Mensuration *of* Superficies, *or* Surfaces, *or* Magnitudes *of* two Dimensions.

Problem 1.

Given the length and breadth of a right angled parallelogram, that is, a square or an oblong; to find the area or content. Rule: multiply the length by the breadth, and the product is the area or content. The reason is evident.

Example 1.—Required the area of a square, which is 20 miles in the side; that is 20 miles long and 20 broad?

Multiply 20
By 20

Product, 400 square miles, the area or content.

Example 2.—Required the area of an oblong or rectangle, whose length is 25000 miles, and breadth 4000 miles.

Multiply 25000
By 4000

Product, 100000000 square miles the area.

Problem 2.

Given the length and the perpendicular breadth of an oblique angled parallelogram, that is, a rhombus or a rhomboides, to find the area. Rule: multiply the length by the perpendicular breadth, and the product is the area or content.

The reason of this rule depends upon prop. 13 of this, viz. that parallelograms standing on the same base and between the same parallels are equal to one another: Let AE (fig. 29 and 30.) be an oblique angled parallelogram; and BC a right angled one, both standing on the same base, AB, and between the same parallels: they are therefore equal (by prop. 13). But the area of BC is found by multiplying AB by AC or BD, that is, the length by the perpendicular breadth of the oblique angled parallelogram. Q.E.D.

Examples to this problem are unnecessary; the operations being the same as in the last.

Problem 3.

Given the base and perpendicular height of a triangle to find its area or content. Rule: multiply the base by the perpendicular height and half the product is the area. The reason of this rule is the same as that of the last: for every triangle is equal to half the parallelogram which stands on the same base and between the same parallels with itself.

Examples in this, as in the last problem, are unnecessary.

Problem 4.

Given one of the equal sides of a regular polygon, of a given number of sides, and the perpendicular from the centre to the middle of each side, to find the area of the polygon. Rule: multiply the given side by the perpendicular, and take half the product for the area of that triangle; then multiply that area by the number of sides, and this gives the whole area of the polygon. Otherwise, multiply half the sum of all the sides by the perpendicular, and the product is the area. The reason is manifest from the last.

Example.—Given AB = 100 feet (fig. 49.), one side of a regular hexagon, and perpendicular GO = 86.6 feet: required the area?

100 × 6 = 600 = the sum of the six sides.
Multiply half the sum, 300
By the perpendicular, 86.6

Product, 2580 square feet the area required.

Problem 5.

Given the diameter and circumference of a circle, to find the area. Rule: multiply half the circumference by half the diameter, and the product is the area required. The demonstration of this rule is deduced from that for finding the area of a regular polygon: for a circle is a polygon of an infinite number of sides, and the circumference is the sum of all the sides, and the semidiameter or radius is the perpendicular height of all the infinite number of equal triangles, whose bases being infinitely small end at last in the circumference, which is equal to the sum of them all; wherefore half their sum, that is, half the circumference multiplied by the perpendicular, that is, by the radius, produces the area of the circle. Q.E.D.

Example 1.—Given the diameter = 100 miles, and circumference = 314.16 miles, required the area of the circle.

Multiply half the circumference 157.08
by half the diameter 50

Product, 7854 square miles the area required.

Example 2.—Given the diameter = 1, and circumference = 3.1416, required the area of that circle.

half circum. = 1.5708
half diam. = .5

Product, .7854 = area of a circle whose diameter is 1; which is $\frac{1}{4}$ of Van Cullen's number, if carried far enough.

Problem 6.

Given the diameter of a circle, to find the area. Rule: First find the circumference, (by problem 1.) and then the area (by problem 5.) Or multiply the square of the diameter by .7854, and the product is the area required.

Example 1.—Given the diameter of a circle = 100 miles, it is required to find the area of that circle.

As, 1 : 3.1416 :: 100 : 314.16 = Circumference.
Then, half the circum = 157.08
Multiply by half the diam = 50

The product = area = 7854 Square miles.

Otherwise,
100 × 100 = 10000 = the square of the diameter.
.7854

7854.0000 = area

Example 2.—Given the diameter of the equator = 7970 miles; it is required to find the area of that circle.

As, 1 : 3.1415926 :: 7970 : 25038.493022 = the circumference.
Then half the circumference = 12519.246511
Multiply by half the Diam. 3985

Product = area of the equator = 49889197.346335 Square miles.

Otherwise,
7970 × 7970 = 63520900 = the square of the diam.
Multiply by .78539815

Area 49889197.346335 Square miles.

Hence the area of the plane of the Earth's equator, is 49889197.346335 square miles; and because the whole surface of a sphere, is equal to 4 times the area of any of its great circles, (as shall be demonstrated immediately) wherefore if this number be multiplied by 4 the product 199556789 omitting the fraction, is the number of square miles on the Earth's surface; if it were a perfect sphere, but the true figure of the Earth, being that of an oblate spheroid; this number must be diminished in the proportion of the equatorial diameter to the axis nearly; that is, as 230 to 229; the whole surface of the Earth is therefore nearly 198689151 square miles.*

Proposition 34. Theorem.

The *areas* of all *similar plane figures*, whether they be rectilinear, curvilinear, or mixed, are directly proportional to the squares of their correspondent homologous lines: and the surfaces of all similar solids are also directly proportional to the squares of their correspondent homologous lines. But here it is to be specially noticed, that all similar plane figures and similar solids are those which have the same number of correspondent lines, and these directly proportional, each to each, and all their correspondent angles equal, each to each.

Demonstration.

In squares the proposition is manifest, because the squares of the sides are their areas. In similar oblongs or rectangles, their areas are found by multiplying their lengths by their breadths; but their lengths and breadths are proportional (by the hypothesis); but the product of two quantities proportional to two other quantities, is to the product of the other two, as the square of one of the former two, is to the square of its correspondent one of the

* In reducing the surface of the sphere to that of its inscribed spheroid, we have supposed, that their surfaces are as their axes nearly; but this is not mathematically true: at the same time, the figure of the Earth differing so little from that of a perfect sphere, that the result is sufficiently true for our purpose. Those gentlemen who choose to find the surface of a spheroid with mathematical accuracy, are referred to Hutton's Mensuration, page 300, second edition.

latter. Thus suppose one oblong to be 5 feet long and 2 broad; and another to be 15 long and 6 broad; that is, as 5 : 2 : : 15 : 6; then 5 × 2 = 10, the area of the former; and 15 × 6 = 90 the area of the latter: then these two areas, viz. 10 and 90, are as the square of 5 to the square of 15, that is, as 25 to 225; or as the square of 2 to the square of 6, that is, as 4 to 36: For the area of the greater is 9 times the area of the less, and so is each greater square 9 times its correspondent one. In circles also, because their areas are found by multiplying half their circumferences by half their diameters; and their circumferences being directly as their diameters; hence these products, that is, the *areas* of the circles, are directly proportional to the squares of their diameters, or squares of their circumferences, or squares of their aliquot parts or similar submultiples. And the same thing is true, of similar triangles, polygons, ellipses, parabolas, &c. and of the surfaces of all similar solids. *Note.*—This universal property of similar superficies, is generally termed, the *duplicate ratio* of the homologous sides or lines.

Corollary.—Hence, the diameters of the planets being known, we can find their surfaces; or compare them with the surface of our Earth; by saying, as the square of the Earth's diameter is to the square of the diameter of a planet, so is the Earth's surface to the planet's surface. Also, the apparent discs or semicircular faces of the planets, are to each other as the squares of their apparent diameters. Thus, the diameter of Mars at the time of his opposition to the Sun appears 5 times larger than at the time of his conjunction with him; hence his disc appears 25 times larger in the one case than in the other.

Prop. 35. THEOREM.

THE convex surface of a sphere is equal to the curve surface of its circumscribing cylinder; that is, the surface of a sphere, is equal to the area of a circle whose diameter is double to that of the sphere: or four times the area of the plane of one of its great circles.

DEMONSTRATION.

LET ALB (Fig. 50.) be a semicircle, and AHGB an oblong described about it; and let both revolve round AB; then the oblong generates a cylinder, and the semicircle, a sphere, inscribed within the cylinder. Draw CE parallel to AH or BG, and KF, infinitely near to CE; and through O, draw OP parallel to AB or HG; and join OD: then because the radius of a circle is perpendicular to the tangent, it is also perpendicular to the curve at the point of contact; therefore the angle DOS is a right angle, and is equal to the right angle COP: take away the common angle DOP from both, and there remains the angle POS equal to the angle DOC; wherefore the two right angled triangles DOC and POS are equiangular and similar, and have the sides opposite to the equal angles proportionals; whence, as DO, the hypothenuse of the one, is to OS, the hypothenuse of the other; so is CO, a leg of the one, to OP, the correspondent leg of the other; consequently, by multiplying extremes and means, DO × OP = CO × OS; but DO is equal to DL, that is, to CE; and OP is equal to EF; therefore CE × EF = CO × OS. Again, because the whole surface of the sphere is composed of an infinite number of surfaces, generated by their respective portions of the generating semicircle; and the curve surface of the cylinder, in like manner, of an infinite number of surfaces generated by the respective portions of the revolving line HG; and the surface described by OS, will be to that described by EF, always, as OS × OC, is to CE × EF; but it has been demonstrated that CE × EF is equal to OC × OS; consequently the surface of the sphere described by OS, is equal to the surface of the cylinder described by EF; wherefore the surface of the whole sphere described by the semicircumference ALB, is equal to the whole curve surface of the cylinder described by HG. And it is evident, that the curve surface of a cylinder is found by multiplying the circumference of either of its bases by its length or axis; but the points H, G, or L, in the revolution, describe the circumference of the base, which is equal to a great circle of the sphere. Hence the circumference of a great circle of the sphere multiplied by the diameter or axis, is equal to the surface of the sphere; but the whole diameter multipled by the whole circumference of a circle, produces 4 times the area of that circle. Whence the convex surface of a sphere is equal, &c. Q.E.D.

Corollary.—Hence it is manifest, that the surface of a sphere is equal to two thirds of the whole surface of its circumscribing cylinder; because the curve surface of the cylinder, which is equal to the surface of the sphere, is double to both the surfaces or areas of both bases; and consequently the curve surface and the areas of both bases added together, are to the surface of the sphere, as 3 is to 2.

Note.—We shall give rules for finding the areas of ellipses, and parabolas, when we treat on Conic Sections.

MENSURATION *of* SOLIDS, *or* MAGNITUDES *of* THREE DIMENSIONS.

PROBLEM 1.

GIVEN the length, breadth, and depth of a right angled parallelopipedon, (of which the cube is one species,) to find its solidity; that is, to find the

number of cubes it contains. Rule: multiply the length, breadth, and depth into each other; and the product is the solidity or content.

Example 1.—What is the solidity of a cube, which is 20 feet in the side; that is, how many cubic feet are contained in a room in the form of a cube, which is 20 feet long, 20 feet broad, and 20 feet high or deep?

Multiply the length	20
by the breadth,	20
	400
and by the depth	20
Product,	8000 = cubic feet, the answer, or solidity.

Example 2.—The length of a right angled parallelopipedon is 100 inches, the breadth 5 inches, and the depth 3 inches; how many cubic inches does it contain.

Multiply length	100
by the breadth	5
	500
and by the depth,	3
Product	1500 = cubic inches, the solidity or answer.

Problem 2.

Given the base-diameter and axis of a cylinder, to find the solidity. Rule: Multiply the area of the base by the axis; that is, multiply the square of the base-diameter by .7854, which gives the area of the base; and multiply by the axis, or, multiply the square of the base-diameter by the axis, and then multiply by .7854: any of these methods gives the solidity of the cylinder.

Example.—The base-diameter of a cylinder is 40 inches, and the length or axis 50 inches; how many cubic inches does that cylinder contain?

Multiply base-diam.	40
by base diam.	40
Square base diam.	1600
Multiply by	.7854
	1256.64$\phi\phi$
Multiply by the axis	50
Solidity,	62839.$\phi\phi$ cubic inches.

Otherwise,

	40
	40
	1600
	50
	80000
	.7854
Solidity,	62832.$\phi\phi\phi\phi$ Answer.

Problem 3.

Given the diameter of a sphere or globe, to find the solidity.—Rule: Multiply the cube of the diameter by $\frac{2}{3}$ of .7854, i. e. .5236, which is equal to $\frac{1}{6}$ of 3.1416, or more correctly to $\frac{1}{6}$ of Van Cullen's number. *Note*, Every sphere and spheroid is $\frac{2}{3}$ of its circumscribing cylinder: as shall be demonstrated immediately.

Example 1.—The diameter of a sphere is 12 inches; what is its solidity or content?

	12
	12
	144
	12
Cube of the diam.	1728
Multiply by	.5236
Solidity,	904.7808 cubic inches. Answer.

Example 2.—The diameter of the Earth's equator is 7970 English miles; and allowing it to be a perfect sphere; how many cubic miles constitute its solidity?

$7970 \times 7970 \times 7970 \times .523598775596 = 265077935233 =$ cubic miles (omitting the fraction), which is the number of cubic miles in the solidity of the Earth, allowing it to be a perfect sphere; but the axis being 34 miles less than the equatorial diameter; this number must be reduced, in the proportion of 230 to 229. Consequently the whole solidity of our Earth, taking the true spheroidal figure, is 263925422471 cubic miles. And to render the result of this problem correct to the nearest unit, we have made use of 12 places of decimal figures, taken from the correct ratio of the diameter and circumference; and also used the proportion between the equatorial diameter and axis, which we have deduced from a comparison of our own calculations, with those that seemed to stand the test of mathematical accuracy and latest observations: The solidity, therefore, of the Earth, is truely 263925422471 cubic or solid miles; and we presume that our number is more accurate than any ever given before. The numbers which we have used are generally correct to the nearest unit; the solidity may be depended on, except what results from the inequalities of the surface; but the aggregate of all these would not add $\frac{1}{10}$ of a mile to the Earth's diameter.

Problem 4.

To find the solidity of a Prism. Rule: multiply the area of a section of the prism perpendicular to the sides of the parallelograms, by the length of any of these sides, and the product is the solidity. The reason is manifest, and requires no example.

Problem 5.

To find the solidity of a pyramid or cone. Rule: multiply the area of the base by $\frac{1}{3}$ part of the perpendicular altitude: that is, in a right cone, multiply

the area of its circular base, by $\frac{1}{3}$ part of its axis; or $\frac{1}{3}$ of the area of the base, by the whole axis.

The reason is, because every triangular prism is constituted of three equal pyramids, which have the same base and same perpendicular altitude that the prism has, and consequently each pyramid is equal to $\frac{1}{3}$ part of the prism: and every prism, let it have as many sides as it will, is made up of a number of triangular prisms; and every pyramid is in like manner constituted of a number of triangular pyramids, and the sum of the whole pyramids, is equal to $\frac{1}{3}$ part of the whole prism. And a cone is a pyramid of an infinite number of sides; in the same manner, that a cylinder is a perpendicular prism of an infinite number of sides. Hence every right cone is $\frac{1}{3}$ part of its circumscribing cylinder, and every sphere is $\frac{2}{3}$ of it; whence, the cylinder, sphere, and cone are in the proportion of 3, 2, and 1. Which Archemides also has demonstrated.

Proposition 36. THEOREM.

THE solidity of a sphere is equal to two thirds the solidity of its circumscribing cylinder: and every spheroid is two thirds of its circumscribing cylinder.

DEMONSTRATION.

FOR a sphere is made up of an infinite number of square pyramids whose bases are contained by four arcs of great circles, and the radius of the sphere is the common perpendicular height of all the pyramids; and the surface of the sphere is the sum of all the areas of the bases; but the solidity of a pyramid is found by multiplying $\frac{1}{3}$ part the area of the base, by the perpendicular altitude; and hence $\frac{1}{3}$ part the surface of the whole sphere multiplied by the radius or semidiameter produces the solidity of the whole sphere; or what is the same thing, $\frac{1}{6}$ part of the whole surface, multiplied by the diameter; but $\frac{1}{6}$ part of the surface is found by multiplying the square of the diameter by $\frac{1}{6}$ of 3.1416 or .5236; wherefore the whole solidity is produced, by multiplying the cube of the diameter by .5236; but the solidity of the circumscribing cylinder is found by multiplying the cube of the diameter or axis by .7854; and these two numbers are to each other as 3 to 2; whence, the sphere is equal to two thirds of its circumscribing cylinder. Q.E.D. The spheroid shall be demonstrated, when we treat of the ellipsis.

Prop. 37.

ALL similar solids are to each other as the cubes of their homologous lines or sides. That is, all spheres are to each other as the cubes of their diameters, or cubes of their circumferences, or cubes of their radii, &c. And in like manner, all similar cylinders, cones, prisms, pyramids, spheroids, &c. are to each other as the cubes of their homologous, sides or lines. This is evident from what we said of similar superficies; because the solidity is produced by the multiplying of three proportional lines together, in each; and these products are to each other as the cube of any one of them in the first multiplication, is to the cube of its correspondent one in the second.

Note.—This general property of all similar solids is commonly termed, the *triplicate ratio* of their homologous sides or lines. Hence the solidity of all the celestial bodies can be found and compared with that of our Earth.

Of CONIC SECTIONS, as applicable to Astronomy.

GENERAL DEFINITIONS.

1. IF a right cone be cut by a plane passing through the vertex and cutting the plane of its circular base: then the plane of that section is an *isosceles triangle.* This is evident.

2. If it be cut by a plane parallel to its base; the plane of that section is a *circle;* and the diameter of that circle is to the diameter of the base, as the axis of the cone cut off is to the axis of the whole cone. This is also evident.

3. If it be cut by a plane cutting the axis obliquely, and passing through both sides of the cone; then the plane of that section, is called an *ellipsis;* (or vulgarly an oval;) and is the same as the figure produced, by *cutting a cylinder* obliquely.

4. If it be cut by a plane parallel to one of the slant sides; then the plane of that section is called a *parabola.*

5. Lastly; if it be cut by a plane, which is not parallel to the opposite slant side of the cone; and this plane being produced, do not cut the opposite slant side, but always goes farther from it: then the plane of that section is called a *hyperbola.* This curve is something similar to the parabola in its figure; but much more complex in its properties.

SCHOLIUM.

TWO of these five conic sections, viz. the triangle and circle, have been treated of, in our Elements of Geometry; and their chief properties, as applicable to Astronomy, pointed out, and demonstrated. The other three, viz. the ellipsis, parabola, and hyperbola,

have now to be considered; and the business of *conic sections* is to exhibit such properties or theorems, and solve such problems as relate to these three sections. But the hyperbola being a curve that has *scarcely any connection* with Astronomy; we shall therefore be brief on it. The *ellipsis* is the principal curve used in Astronomy; because the orbits of all the planets and comets are ellipses; hence it concerns the young astronomer to be conversant with every thing relative to an *ellipsis*. The parabola is chiefly used to abridge calculations relative to the comets; as the properties of this curve are more simple than those of the ellipsis; and a small part of the curve of an ellipsis, very excentric, differs not sensibly from a similar part of a parabola. A considerable part of our conic sections must therefore be allotted to the parabola. We have to observe, that we have chosen the *description* of the curve, in planes, in preference to the section of the cone, as the properties are more easily deduced therefrom; and we shall finish in each case, by proving that the curve generated by motion is exactly the same as that produced by cutting the cone.

Of the ELLIPSIS.

Definitions.

1. Let any right line AB (Fig. 1.) be taken in a plane; and let the two points F and P be taken in AB, at equal distances from each end, and from the centre C; and suppose a *thread* equal in length to AB, to have its two ends fixed, (by small loops about two pins,) at F and P, and the thread being perfectly stretched by the point of a pen or pencil, as G; if the pen be carried quite round till the motion end where it began, then the figure described on this plane, is called an ellipsis;* the curve described by the pen or pencil, is the periphery or circumference of the ellipsis; the two fixed points F, and P, are called the foci or focuses; the right line AB, is called the *transverse* diameter or greater axis; the right line DE, passing through the centre, at right angles to AB, is called the *conjugate* diameter or less axis; and the distance CF, or CP, of either focus from the centre, is called the *excentricity*. Hence it is evident, that the sum of any two right lines drawn from the foci to any point D, or G, in the curve, is always equal to the transverse diameter; because their sum is always equal to the thread used in the description, which is, by supposition, equal to it; also the two right lines FD, and PD, are each of them equal to the semi-transverse diameter AC or BC.

2. Any right line as TS, (Fig. 2.) drawn through the centre and terminated both ways by the curve is called a diameter, and it divides the ellipsis into two equal parts: Hence, the transverse diameter AB is the greatest, and the conjugate diameter DE the least. Also, the extremities of a diameter are called its vertexes.

3. A right line touching an ellipsis at the vertex of a diameter, and which being produced both ways, does not cut it, but always goes farther off it, is called a tangent at that vertex. And hence it is evident that the tangents at the vertexes of the transverse and conjugate diameters, are at right angles to these diameters; and that all right lines drawn parallel to these tangents within the ellipsis are also at right angles to them, and bisected by them. Also that tangents to the ellipsis at the vertexes of all other diameters, stand obliquely to these diameters, as do all right lines drawn parallel to the tangents within the ellipsis: but the diameter bisects all these parallels, as shall be demonstrated hereafter.

4. Those right lines drawn parallel to the tangent, and bisected by the diameter, are called ordinates to that diameter. Thus GH, KL, (Fig. 2.) are called ordinates to the transverse diameter; YS, is an ordinate to the conjugate diameter DE; and UI, is an ordinate to the diameter TS. But it generally occurs, that we call the semiordinates, by the name of *ordinates;* because most properties are deduced from the semiordinates.

5. That right line or ordinate to the transverse diameter which passes through either of the foci, is called the *parameter* or *latus-rectum* of the ellipsis; thus let P, (Fig. 2.) be one of the foci, then GH, drawn through it, at right angles to AB; is the parameter or *latus rectum* of the ellipsis.

6. If any point be taken in a diameter; then each part of the diameter between the point and the vertexes is called an *Abscissa;* and the ordinate passing through that point, is called the ordinate of those Abscissæ. Thus AR, and RB (Fig. 2.) are Abscissæ, and KL their ordinate; AP, and PB, are Abscissæ, and GH their ordinate.

Proposition A. Theorem.

The rectangle or product of the sum of two right lines, and the difference of those right lines, is equal to the difference of their squares.

Demonstration.

Let AB, and BC, be two right lines of which BC is the greater, and (Fig. 3.) placed in the same right line with AB; and make BD equal to AB; then AC is their sum, and DC is their difference. I say that the rectangle or product of AC and DC, is equal to the square of AB taken from the square of BC. For let BH be the square of BC, and join CF, and through D and A draw DG and AK parallel to BF or CH, and through O draw KE parallel to

* It will be shewn in prop. 10, that the figure thus formed is the same as that produced by cutting a cone in the manner described in definition 8.

AC or FH; then LG and DE being about the diagonal CF, are squares, and LG is the square of BD; and the rectangles BE and EG taken together are equal to the differences of the squares LG, BH; but BE and EG are also equal to the rectangle AE, now AE is the rectangle of AC the sum, and CE, that is, DC, the difference of AB and BC, wherefore, the rectangle or product of two right lines, is equal to the difference of their squares. Q.E.D.

Corollary.—Hence it is manifest, that the square of the whole line CB is equal to the sum of the squares LG, DE of both the parts BD, DC, together with twice the rectangle BO, or OH, of both the parts; and also, that the square of the whole line is equal to four times the square of half the line.

Proposition B. THEOREM.

IF a right line be divided into two parts, the square of one of the parts added to the square of the whole line, is equal to twice the rectangle of the whole line and that part, together with the square of the other part.

DEMONSTRATION.

LET the right line AC (Fig. 4.) be divided into two parts in the point B; I say that the sum of the squares of AC and AB, is equal to twice the rectangle of AC and AB, together with the square of BC: for on AC describe the square CH, and construct the figure, as in the last; then CE and BH, are twice the rectangle of AB and AC; but they are greater than CE and ED, by BE, which is equal to the square of BA; add FD, which is the square of BC, to both, and then twice the rectangle of AC and AB, together with the square of BC, make up the whole square CH, and the square BE; that is, twice the rectangle of AC and BA together with the square of BC, are equal to the sum of the squares of AC and AB. Q.E.D.

Proposition 1. THEOREM.

THE rectangle of the focal distances from either focus, to the vertexes of the transverse diameter, is equal to the square of the semiconjugate; and also equal to the rectangle of the semitransverse and semi-*latus-rectum*. That is, the square of the semiconjugate is equal to the rectangle of the focal distances, and also equal to the rectangle of the semitransverse and semi-*latus-rectum*.

DEMONSTRATION.

LET ADBL, (Fig. 5.) be an ellipsis, AB, and DL, its transverse and conjugate diameters; F and P, its foci; and HK its latus rectum: then, I say, that the rectangle of AF and FB, is equal to the square of CD; and also equal to the rectangle of AC and HF: for join PH and DF; then it is evident, that FB is the sum of AC and FC, and AF is their difference; but the rectangle of the sum and difference is equal to the difference of the squares of those lines; (by prop. A); wherefore the rectangle of FB and AF, is equal to the difference of the squares of AC, and FC; but FD is equal to AC; (by def. 1;) whence the rectangle of FB and AF, is equal to the difference of the squares of FD and FC; but the square of CD is equal to the difference of the squares of FD and FC; (by prop. 16, Geo.) consequently the rectangle of AF and FB is equal to the square of CD. Q.E.D. Again, because AB is equal to FH and HP; (by def. 1;) wherefore the squares of AB, and FH, added together, are equal to twice the rectangle of AB and FH together with the square of HP, the other part, added; (by prop. B;) but the square of HP is equal to the squares of HF and FP; (by prop. 16. Geom.) consequently the squares of AB and FH, are equal to twice the rectangle of AB and FH, together with the squares of FH and FP; take away the square of FH, which is common; and there remains the square of AB equal to twice the rectangle of AB and FH together with the square of FP; but the square of AB is equal to four times the square of AC; and twice the rectangle of AB and FH is equal to four times the rectangle of AC and FH; and the square of FP is equal to four times the square of FC; wherefore, four times the square of AC is equal to four times the rectangle of AC and FH together with four times the square of FC; and consequently the square of AC is equal to the rectangle of AC and FH together with the square of FC; but AC is equal to FD, (by def. 1;) whence the square of FD is equal to the rectangle of AC and FH, together with the square of FC; but the square of FD is equal to the squares of FC and DC; (by prop. 16, Geom.;) consequently the rectangle of AC and FH, together with the square of FC, are equal to the squares of FC and DC; take away the square of FC, which is common, and there remains the rectangle of AC and FH, equal to the square of DC: but the rectangle of AF and FB has also been demonstrated to be equal to the square of CD. Q.E.D.

Corollary.—Hence it follows that the semiconjugate is a *mean proportional* between the focal distances; and also a mean proportional between the semitransverse and the semi-*latus-rectum*: also that the whole conjugate diameter is a mean proportional between the whole transverse diameter and the whole latus rectum; and consequently, that the latus rectum is a *third proportional* to the transverse and conjugate diameters.

Note.—That any ordinate which is a third proportional to any oblique diameter and its conjugate parallel to the tangent, is also called a *latus rectum*

to that oblique diameter: but such oblique diameters and *latera recta*, are scarcely ever used in Astronomical Theories.

LEMMA.

HALF the difference of two quantities, added to half their sum, gives the greater; and half their difference taken from half their sum, leaves the less. And conversely; that quantity, which added to half the sum of two, gives the greater, or subtracted from half their sum, leaves the less, is equal to half the difference of the two quantities.

ILLUSTRATION.

LET AB and BC (Fig. 6), be two unequal quantities, of which AB is the greater, and BC the less; and let AD or DC be half their sum; also take AE equal to BC; then it is evident, that EB is their difference, and that ED or DB is half their difference, and that the greater AB is made up of the half sum AD and half difference DB; and that the less BC is the remainder left, when the half difference DB is taken from the half sum DC.

Prop. C.

IF a right line be divided into two parts; four times the rectangle of the whole line and one of the parts, together with the square of the other part, are equal to the square of the line, which is the sum of the whole line and the first mentioned part.

DEMONSTRATION.

LET the right line AB (Fig. 7) be divided into two parts in the point C, and produced till BD is equal to BC: I say that the square of AD is equal to four times the rectangle of AB and CB, together with the square of AC: for on AD describe the square DF; and construct the double figure. (as in prop. A.) Then it is evident that X and Y are the rectangle of AB and BC, and that the whole square is equal to Z and four times that rectangle; and Z is the square of AC; wherefore, four times the rectangle of AB and CB together with the square of AC, are equal to the square of AD, which is the sum of AB and CB. Q.E.D.

Prop. 2.

IF any point be taken in the curve of the ellipsis, and an ordinate be let fall from that point perpendicularly to the transverse diameter; and also a right line be drawn from each focus to the assumed point; then, as the semitransverse diameter is to the excentricity, so is the distance of the perpendicular from the center to half the difference of those two right lines drawn from the foci to the said assumed point: moreover, four times the rectangle of the semitransverse diameter, and half the difference of those lines, is equal to the difference of their squares.

DEMONSTRATION.

LET the point G, (Fig. 5.) be assumed in the periphery of the ellipsis, and let GF and GP be drawn to it from the foci, and GO at right angles to the transverse diameter: then I say, that as AC the semitransverse is to FC the excentricity, so is OG the distance of the perpendicular from the centre, to half the difference of GF and GP, or the difference between the semitransverse and either of them. Moreover, I say, that four times the rectangle of the semitransverse and the said half difference, are equal to the difference of the squares of GF and GP. For produce GF the greater to T; and about the centre G, with the radius GP the less, describe the semicircle IPT: then FT is equal to the sum of GF and GP, and FI is their difference: bisect FT in Z; then FZ, half their sum, is equal to the semitransverse AC; (by def. 1.) and ZG is half their difference and equal to the half of FI; because ZG added to FZ gives the greater FG, and subtracted from ZT leaves GP the less; also, CO is half the difference of FO and OP, because when added to FC their half sum, it gives FO the greater, and when taken from PC it leaves PO the less; but FS is the difference of FO and OP; therefore CO is equal to the half of FS: and because the two right lines FT and FP are drawn from the point F, without the circle cutting it in the points I and S, and terminated at T and P; the rectangle of FT and FI, is equal to the rectangle of FP and FS: (by prop. 28. Geom.) and consequently the rectangles of their halves are also equal; that is the rectangle of FZ and ZG is equal to the rectangle of FC and CO; whence, (by 8. of propor.) as FZ : FC :: CO : ZG; but FZ is equal to the semitransverse, FC is the excentricity, CO is the distance of the perpendicular from the centre, and ZG is the difference between the semitransverse and FG or GP, or it is half the difference between FG and GP; hence, as the semitransverse is to the excentricity, so is the distance of the perpendicular from the centre, to half the difference of the two right lines from the assumed point to the foci, which is equal to the excess of the greater above the semitransverse, or the excess of the semitransverse above the less. Q.E.D. Again, because the right line ZT, which is equal to FZ, is divided into two parts in the point G; four times the rectangle of TZ and GZ, together with the square of GT are equal to the square of a line which is their sum, that is to the square of FG, but GT is equal to GP, and ZT is equal to the semitransverse and ZG is half the difference of FG, GP; wherefore the square of FG is equal to the

square GP, together with four times the rectangle of the semitransverse and half the difference of those lines; that is, four times the rectangle of the semitransverse, and half the difference of FG and GP, are equal to the difference of their squares; or the rectangle of the whole transverse, and the whole difference are equal to the difference of their squares. Q.E.D.

Astronomical Application. SCHOLIUM.

WE have informed the reader in a former part of the work, that the orbits of the planets and returning comets, are ellipses, and that the Sun is placed in or near one of the foci. Now the semitransverse diameter is called the *mean distance;* because, if we suppose the Sun to be in the focus F, (Fig. 5.) then the planet being at B, is farther from the Sun than when at A, by FP; but when it is at D, then FD, which is equal to AC, is its *mean distance;* because it is then farther from the Sun than when at A, by FC, the excentricity; but so much nearer to him than when at B, by the excentricity. When the planet is at A, it is in the *perihelion;* when at D, the extremity of the conjugate diameter, it is at its *mean distance;* and when at B, it is in its *aphelion:* in any part of the orbit between A and D, as at H, its distance is less than the mean distance; but in any part between D and B, as at G, its distance is greater than its mean distance. Then from G, let Gx be drawn parallel to AB; and Gx, which is equal to CO, the distance of the perpendicular GO from the centre, is the nearest distance of the planet from the conjugate diameter: but, (by the preceding proposition,) as the semitransverse is to the excentricity, so is CO, or its equal Gx, to the difference between the semitransverse and either of the lines GF or GP: that is, as the mean distance of a planet from the Sun is to the excentricity of its orbit, so is its nearest distance from the conjugate diameter to the difference between its mean distance and its present distance from the Sun: and this is evidently universal in every point of the orbit. Hence, if any three of these four proportionals be given, the fourth can be found by the common rule of proportion. Moreover; if the planet be in the point N, so that ND be equal to DG, then it is plain, that its distance, FN, from the Sun, which is equal to PG, is as much less than the mean distance as FG is greater than it: and in all points of the orbit, equally distant from the extremity of the conjugate or less axis, the difference between the mean distance and each of the correspondent distances is the same; and four times the rectangle of the mean distance, and half the difference of those correspondent distances are equal to the difference of their squares.

Prop. 3. THEOREM.

IF any ordinate be drawn to the transverse diameter: then it shall be, as the square of the transverse is to the square of the conjugate, so is the rectangle of the abscissæ of that ordinate to the square of the semiordinate; or as the square of the semitransverse is to the square of the semiconjugate, so is the rectangle of the abscissæ to the square of their semiordinate. And, as the transverse is to the latus rectum, or, as the semitransverse is to the semi latus rectum, so is the rectangle of the abscissæ to the square of their semiordinate.

DEMONSTRATION.

THE same construction remaining as in the last: (Fig. 5.) I say, that as the square of AC is to the square of CD, so is the rectangle of AO and OB to the square of GO. For, because, as BC : FC :: OC : GZ; (by the last;) whence by addition, as BC : BC + FC or BF :: OC : OC + GZ; (by 4 of proportion); and alternately as BC : OC :: BF : OC :: GZ; and again by addition, as BC : BC + OC + BF : BF + OC + GZ; but BC + OC is equal to AO; and BF + OC + GZ, is equal to FG + FO; whence, as BC : AO :: BF : FG + FO. Again, because, AC : FC :: OC : GZ; hence by subtraction, AC : AC — FC, or AF :: OC : OC — GZ; and alternately AC : OC :: AF : OC — GZ; and again by subtraction AC : AC — OC, or BO :: AF : FA — OC + GZ; but FA — OC + GZ, is equal to FG — FO; whence, as AC : BO :: AF : FG — FO; but we had, as BC : AO :: BF : GF + FO; wherefore by multiplication, as AC × BC : BO × AO :: AF × BF : $\overline{FG + FO} \times \overline{FG - FO}$; but AC × BC, is the square of the semitransverse; and AF × BF, is equal to the square of the semiconjugate, (by prop. 1. of conics); also, $\overline{FG + FO} \times \overline{FG - FO}$, or the rectangle of the sum and difference of FG and FO, is equal to the difference of their squares; (by prop. A;) but the square of GO, is equal to the difference of the squares of FG and FO; (by prop. 16. Geom.) moreover BO × AO, is the rectangle of the abscissæ: whence, as the square of the semitransverse, is to the square of the semiconjugate, or as the square of the whole transverse, is to the square of the whole conjugate, so is the rectangle of the abscissæ to the square of their correspondent semiordinate. Q.E.D. Again, because the rectangle of the transverse and latus rectum is equal to the square of the conjugate; (by prop. 1. of conics;) whence, as the square of the transverse is to the rectangle of the transverse and latus rectum, so is the rectangle of the abscissæ to the square of the semiordinate; but the square of the transverse is to the rectangle of the transvere and latus rectum, as

the transverse is to the latus rectum (by division); consequently, the transverse is to the latus-rectum, as the rectangle of the abscissæ is to the square of their semiordinate. Q.E.D.

Corollary 1.—Hence, as the rectangle of any two abscissæ is to the square of their semiordinate, so is the rectangle of any other two abscissæ to the square of their semiordinate; because they have all the same ratio of the transverse to the latus rectum.

Corollary 2.—Much in the same manner, it may be demonstrated, that if the conjugate diameter be divided into two abscissæ by an ordinate parallel to the travsverse, that the square of the conjugate is to the square of the transverse, as the rectangle of the abscissæ is to the square of their semiordinate.

Scholium.

Let the semitransverse AC (Fig. 5.) $= t$, the semiconjugate CD $= c$, the abscissæ BO $= x$, and the semiordinate OG $= y$; then AO the other abscissæ $= 2t - x$: hence, (by this prop.) as $t^2 : c^2 :: \overline{2t - x} \times x : y^2$; that is, $t^2 : c^2 :: 2tx - x^2 : y^2$; and therefore, $\frac{c^2}{t^2} \times \overline{2tx - x^2} = y^2$: and this is called the equation of the ellipsis; but, $2tx - x^2 = z^2$, is the equation of the circle, (see scholium, prop. 26. Geom:) and hence the equation of the ellipsis differs only from the equation of the circle, in having the quantity $\frac{c^2}{t^2}$; that is, the quotient of the square of the conjugate divided by the transverse, for a factor, in the value of the square of the semiordinate: now let $\frac{c^2}{t^2} = n^2$; and we have $n^2 \times \overline{2tx - x^2} = y^2$; but $2tx - x^2 = z^2$; whence $n^2 \times \overline{2tx - x^2} = n^2z^2 = y^2$; therefore $nz = y$; consequently, as $1 : n :: z : y$; that is, as unit is to the quotient arising by dividing the conjugate by the transverse, so is the semiordinate in the circle to the semiordinate in the ellipsis; both having the same abscissæ common.

Astronomical Application.

The same things being supposed, as in scholium prop. 2; and the planet at G, (Fig. 5.); then Gx which is equal to CO, is the perpendicular distance of the planet from the less axis, and if AC the mean distance be added; then AO, their sum, is one abscissæ, and BO their difference is the other; and because we have demonstrated, that as the semitransverse is to the semi-latus-rectum, so is the rectangle of the abscissæ to the square of their semiordinate; wherefore, as the mean distance of a planet from the Sun is to the semi-latus-rectum of the orbit, so is the rectangle of the sum of the mean distance and perpendicular distance of the planet from the less axis, and difference of the same, to the square of the perpendicular distance of the planet from the greater axis.

Prop. 4. Theorem.

If an ellipsis and two circles have the same centre, and the transverse be the diameter of the greater circle, and the conjugate that of the less; and if an ordinate be drawn to the transverse dividing it into abscissæ; and also an ordinate be drawn to the conjugate, dividing it into abscissæ: then the ordinate of the circumscribing circle shall have the same ratio to the ordinate of the ellipsis, (both having the same abscissæ common,) that the transverse has to the conjugate; and the ordinate of the inscribed circle shall have the same ratio to the ordinate of the ellipsis that the conjugate has to the transverse: moreover, the area of the circumscribing circle has the same ratio to the area of the ellipsis that the transverse has to the conjugate; and the area of the inscribed circle has the same ratio to the area of the ellipsis that the conjugate has to the transverse.

Demonstration.

Let ADBL, (Fig. 8.) be an ellipsis, C its centre; ANBM a circle about it, and XDKL another within it; draw the ordinates IH and EU, to the transverse and conjugate: I say, that as AB is to DL so is IH to EG; and, as DL is to AB so is WY to EU: for, because the rectangle of AF and FB is equal to the square of FI (by prop. 26 Geom.); and the square of AB is to the square DL as the rectangle of AF and FB is to the square of EF; (by prop. 3 of conics;) therefore, the square of AB is to the square of DL, as the square of FL is to the square of FE; consequently, AB is to DL, as FI is FE, or as their doubles IH, to EG: and in the same manner we demonstrate, that DL is to AB as WY is to EU. Q.E.D. Again, I say, that the area of the circumscribing circle is to the area of the ellipsis as AB is to DL: for, draw TR infinitely near and parallel to IH; then it is evident, that the area of the circle is made up of an infinite number of the small spaces, one of which is IHRT, the area of which is found by multiplying HI by OF; and the area of the ellipsis, in like manner, is made up of an infinite number of small spaces, one of which is EGSP, the area of which is found by multiplying EG by FO; and consequently the areas of these two spaces are in the ratio of HI to EG; and therefore the sum of all the spaces, which constitute the area of the circle, is to the sum of all those which constitute the area of the ellipsis, as HI to EG; but HI is to EG as the transverse is to the conjugate;

whence the area of the circumscribing circle is to the area of the ellipsis, as the transverse is to the conjugate: and in like manner it may be demonstrated, that the area of the inscribed circle is to the area of the ellipsis, as the conjugate diameter is to the transverse. Q.E.D.

Corollary 1.—Hence it is evident that the ellipsis is a *mean proportional* between the circumscribing and inscribed circles.

Cor. 2.—Because the area of a circle is found by multiplying the square of the diameter by .7854, (see problem 6 of Geom.) wherefore the area of the ellipsis is found by multiplying the rectangle or product of the transverse and conjugate diameters by the same number .7854; for these areas are to each other as the transverse is to the conjugate.

Cor. 3.—Hence the areas of ellipses are to each other as the rectangles of their respective diameters. And because the square of the conjugate is equal to the rectangle of the transverse and latus rectum; (by prop. 1. of conics) hence the conjugate is equal to the square root of the transverse multiplied by the square root of the latus rectum; hence the areas of ellipses are to each other, as the rectangles of their transverse diameters multiplied into the product of the square roots of their respective transverse diameters and latera recta; that is, as the square roots of the cubes of their transverse diameters into the square roots of their latera recta.

Cor. 4.—If F be the focus; (Fig. 8;) then the ordinate IH, which passes through F, and is the latus rectum GE produced to the circumference of the circle, is equal to the conjugate. For the rectangle of AF and FB is equal to the square of FI; (by prop. 26. Geom.) and also equal to the square of the semi conjugate CD (by prop. 1. of conics); hence the square of FI is equal to the square of CD; therefore FI is equal to CD; and the whole ordinate IH is equal to the whole conjugate DL. Q.E.D.

Cor. 5.—If any semiordinate of the circle FI, (Fig. 8.) be so cut in the point E, that FI is to FE, as an ordinate in the circle is to an ordinate in the ellipsis; then the point E is in the periphery of the ellipsis; or if E be in the periphery of the ellipsis, then the point I is in the circumference of the circle.

Prop. 5. Theorem.

If a right line *touch* the periphery of an ellipsis in any point between the vertexes of its two principal diameters, then it is evident that the tangent and transverse diameter being produced will meet in a point; and if an ordinate to the transvere be drawn from the point of contact, and produced till it meet the circumference of the circumscribing circle in a point: then I say, that the right line joining this point and the point of section of the transverse and tangent to the ellipsis, shall touch the circle, and be a tangent to it in that point.

Demonstration.

Let the right line GD, (Fig. 9.) touch the ellipsis in the point G, and let it be produced till it meet the transverse AB produced in D; also from the point G, draw the ordinate GF, which produce till it meet the circumference of the circumscribing circle in H, and join HD: then I say, that HD touches the circle in the point H: for if not it must cut it, if produced, in H and in some other point I; then through I draw the ordinate IS parallel to HF, and cutting the tangent and periphery of the ellipsis in the points T and P; then by reason of the similar triangles, as HF : GF .. IS : TS; (by prop. 24, Geom.) and because the ordinate IS of the circle is cut in the point T, so that IS : TS :: HF : GF; hence the point T *is* in the periphery of the ellipsis; (by cor. 5 of the last;) but it *is not* by the hypothesis, which is impossible; consequently it is impossible for the right line DH to cut the circle; and therefore it only touches it, and is a tangent to it in the point H. Q.E.D.

Corollary 1.—Hence, conversely, if a right line, HD, touch a circle and be produced till it meet the transverse, AB, of the inscribed ellipsis produced in D; and if an ordinate HF be drawn from the point of contact H, and cutting the ellipsis in G; then the right line GD, joining the points G and D, is a tangent to the ellipsis in the point G.

Cor. 2.—Hence right lines drawn tangents to an ellipsis from the vertexes of a diameter are parallel to one another; for the tangents, at the extremities of the transverse or conjugate diameters, are at right angles to these diameters, and therefore parallel to one another; and when they are at the extremities of an oblique diameter, they are at opposite points of the periphery, and make the alternative angles with that oblique diameter equal, and consequently they are parallel.

Prop. 6. Theorem.

If from any point in the periphery of an ellipsis, two right lines be drawn to the foci, and from the same point a right line be drawn bisecting the outward angle contained between one of these lines and the other produced: then I say, that line is a tangent to the ellipsis.

Demonstration.

Let the point O, (Fig. 9,) be taken in the periphery of the ellipsis, and let the two right lines ON, OM

be drawn to the foci N and M, and produce NO to L, and let the right line ROK, drawn through the point O, bisect the outward angle MOL: I say that RK is a tangent to the ellipsis: for, make OL equal to OM, and join LM cutting KR in K: then because the angles MOK and LOK are equal, by the hypothesis, and the two sides MO, OK are equal to the two sides LO, OK, each to each; therefore the triangles are congruous (by prop. 1, Geom.); consequently MK is equal to LK, and the angles at K equal, and therefore right angles: now take any point, R in KO, produced either way, and join NR MR, and LR; then because MK is equal to LK, and KR is common to the two triangles, and they contain right angles; wherefore MR is equal to LR, and NR to both; then, MR and NR are equal to LR and NR; but LR and NR are greater than LN, because two sides of a triangle are greater than the third; wherefore MR and NR are also greater than LN; but LN is equal to MO and ON; whence MR and NR are likewise greater than MO and ON, that is, greater than the transverse diameter, and consequently the point R is beyond the periphery of the ellipsis and in that of a greater one; and in the same manner we demonstrate that every other point in the right line KR is beyond the periphery of the ellipsis, except the point O; wherefore the right line KO is a tangent to the ellipsis in the point O. Q.E.D.

Corollary 1.—Hence the right line drawn from the foci to the point of contact, make equal angles with the tangent: for the angle KOL is equal to the angle RON, because they are vertically opposite; and KOL is equal to KOM; whence RON is equal to KOM. Q.E.D.

Cor. 2.—Hence from this proposition and the 5th we have two easy methods of drawing a tangent to any point in the periphery of an ellipsis, having the foci or transverse given.

Cor. 3.—If a right line touch an ellipsis, and if right lines be drawn from the foci to the point of contact, these right lines are in the same ratio with the perpendiculars to the tangent, drawn from the foci; for the triangle MOK is similar to NOZ, because the angle MOK is equal to the angle NOZ, and the angles at K and Z are right angles; whence, as MO : MK .: NO : NZ, and alternately, as MO : NO :: MK : NZ. Q.E.D.

Prop. 7. Theorem.

If a right line touch an ellipsis, and a diameter be drawn from the points of contact; it bisects all its ordinates.

Demonstration.

Let the right line DR, (Fig. 12.) touch the ellipsis ADBV, in the point D; let SP be drawn parallel to DR; and draw the diameter DV, cutting SP in N: then I say SP is bisected in N: for through SD and P draw the ordinates to the axis major SG, DF, and PE, and produce them till they meet the circumference of the circumscribing circle in IK, and O; also produce the transverse AB till it meet the tangent DR produced in R; and produce SP till it meet AR in L; and join IL and KR; then KR is a tangent to the circle in the point K; (by prop. 5 of conics,) and the right line joining I and L passes through the point O; for because OE is parallel to IG; whence, as IG : SG :: OE : PE, wherefore the point O is in the circumference of the circle (by cor. 5. prop. 4, of conics); IL also cuts KC the radius in some point M; join NM: then because SL is parallel to DR, and OE to KF, the triangles PEL and DFR are similar; whence, as DF : PE :: FR : EL; but DF : PE :: KF : OE (by prop. 5, of conics); hence by equality, KF : OF :: FR : EL; and consequently the triangles KFR and OEL are similar, and therefore equiangular; therefore the angle KRF is equal to OLE, and consequently KR is parallel to OL or ML (by prop. 10. Geom.), again because NL is parallel to DR, and ML to KR; therefore as CD : CN :: CR : CL; and as KC : CM :: CR : CL; whence, CD : CN :: CK : CM, consequently MN is parallel to KF; and because KR is a tangent to the circle, therefore the angle CKR is a right angle, and ML is parallel to KR, consequently the angle CML is a right angle; and because the radius cuts the chord IO at right angles, it bisects it; wherefore IM is equal to MO; and since MN has been demonstrated to be parallel to KF, that is, to IS; wherefore, as IM : MO :: SN : NP; but IM is equal to MO, consequently SN is equal to NP; and therefore the ordinate SP is bisected by the diameters DV, in N. Q.E.D.

Corollary.—Hence, if a right line touch an ellipsis, and a right line be drawn from the point of contact, and bisecting any ordinate parallel to the tangent; then that right line is a diameter, and passes through the centre: and if a right line in an ellipsis be bisected by a diameter, then that right line is an ordinate and parallel to the tangent at the vertex of that diameter.

Scholium.

We might add many more Theorems on the properties of the ellipsis, but the foregoing 7 being all that are generally useful in Astronomy, we shall therefore trouble our reader with no more; but refer him, (if led by a laudable desire of becoming a proficient in the whole theory of conic sections,) to Simpson, Hamilton, Emerson, Jacks, Stone, &c. who have written largely on the subject.

Prop. 8. PROBLEM.

GIVEN two right lines M and N (Fig. 13.), equal to the transverse and conjugate diameters, to construct the ellipsis. (Suppose M = 46 feet, yards, miles, &c.; and N = 30.)

CONSTRUCTION.

DRAW two right lines AB and DH at right angles to each other, and intersecting in the point C; make AC and CB, each equal to the half of M (23), and CD and CH, each equal to the half of N (15); then with the radius HF, equal to the semitransverse AC, and centre H, describe the arcs E and F; then the points F and E are the foci: for join HF and HE; and it is evident (by Def. 1,), because HF and HE are each equal to the semitransverse. Now having the foci, F and E, and transverse AB, the ellipsis can be described (as in Def. 1.), which let be ADBH. Q.E.F.

Prop. 9. PROBLEM.

THE transverse and conjugate diameters of an ellipsis being given, and the ellipsis, in consequence thereof, to draw a tangent to any point in the periphery.

SOLUTION.

GIVEN the ellipsis ADBH (Fig. 13.), and P a point in the periphery thereof; it is required to draw a right line which shall touch the ellipsis in the point P. As the ellipsis is given; find the foci F and E; describe the circumscribing circle AGB; then through the given point P, draw the ordinate PI, which produce till it meet the circumferenc of the circle in T; join CT, and draw TK at right angles to CT, and produce it till it meet the transverse produced in K; join PK, and PK is the tangent required: (by cor. prop.) QE.I.

Prop. 10. THEOREM.

IF a cone be cut by a plane passing through both its sides, the section will be an ellipsis.

Let the cone DVE (Fig. 16.) be cut by the plane TBA*b* meeting the sides VD, VE, the section TBA*b* will be an ellipsis.

For if a plane, VDE, pass through the axis of the cone perpendicular to TBA*b*, and let their common section be AT; also let GPQ be a section perpendicular to the axis of the cone, or parallel to the base, which is therefore a circle, and let its common section with VDE, and TBA, be GQ and PN*p*; moreover, let MBK be a section parallel to the base passing through the middle point, C, of TA. Now, because the plane VDE is perpendicular to both the planes TBA, GPQ, their common section PN*p* is perpendicular to VDE, and therefore perpendicular to AT and GQ which are in that plane. For the same reason BC*b* is perpendicular to AT and MK. Therefore PN and BC are ordinates to the diameters GQ, and MK, of the circles GPQ*p*, and MBK*b*, and consequently the rectangle of GN and NQ is equal to the square of PN, and the rectangle of MC and CK is equal to the square of BC. But, from the similar triangles TGN, TMC, and ANQ, ACK, TN : GN :: TC or AC : MC, and NA : NQ :: CA : CK. Therefore, by compounding, the rectangle of TN and NA is to the rectangle of GN and NQ as the square of AC is to the rectangle of MC and CK; that is, the rectangle of TN, NA is to the square of PN as the square of AC is to the square of BC. But this, by Prop. 3, is the property of an ellipsis whose semi-transverse axis is CA or CT, semi-conjugate axis BC, abscissæ AN, NT, and semi-ordinate PN. Consequently, the curve TBA*b* is an ellipsis.

Of the PARABOLA.

DEFINITIONS.

1. LET AB (Fig. 10.) be a right line, and F a point out of it, and ACD a right angle (or an instrument commonly called a square), which let be placed in the same plane in which the right line and point are, in such a manner that one side of it, AC, shall coincide with AB, and let the other side, CD, coincide with the point F; and at D, the extremity of CD, let there be fixed one end of a thread equal in length to CD or SM, and the other extremity of the thread fixed at F; and let part of the thread, GD, be brought close to the side, CD, by the point of a pen or pencil; then let the square be moved from S towards A, keeping its side, AC, always coincident with AB, and, in the meantime, the thread being always kept extended by means of the point of the pen or pencil: then, by this motion, the point will describe on the plane a curve called a *Parabola**, one half of which will be described by the square moving from S towards A, and the other half by turning the square and moving from S towards B. Thus the curve LVN described by such a motion, is called a *Parabola*.

2. The right line, AB, is called the *Directrix*; the point F the *Focus*; any right line at right angles to the directrix is called *a Diameter*; and that which passes through the focus is called the *Axis* or prin-

* It is demonstrated in Prop. 14, that the curve generated in this manner is the same as that formed by cutting a cone as mentioned in Definition 4.

cipal diameter, thus VM is the axis, or principal diameter, and the point V, where it meets the curve, is its *Vertex*, and it is called the principal vertex, or vertex of the parabola; also KP is a diameter, and K its vertex.

3. A right line, which is terminated both ways by the curve, and bisected by a diameter, is called an *Ordinate* to that diameter. Thus, IH, GK, and LN are ordinates to the axis or principal diameter, and they cut it at right angles; also, if HR be bisected by OP in Z, then HR is an ordinate to OP; and it is evident, that HR cuts OP obliquely. Hence the axis or principal diameter is cut perpendicularly by its ordinates, but all other diameters are cut obliquely by their ordinates.

4. That part of a diameter intercepted between its vertex and ordinate, is called *the Abscissa* of that ordinate. Thus, VF is the abscissa of IH, VE the abscissa of GK, and VM the abscissa of LN; also KZ is the abscissa of HR.

5. The ordinate IH, which passes through the focus, and cuts the axis at right angles, and is evidently equal to four times VF or VS, is called the *latus rectum* or *parameter* of the axis or of the parabola; also any ordinate which is four times that segment of a diameter intercepted between its vertex and directrix, is called the latus rectum of that diameter.

6. A right line which touches the curve in a point, and being produced does not cut it, is called a *tangent* to the parabola.

Axioms.

1. The distance of the focus from the vertex of the parabola is equal to the distance of the vertex from the directrix; that is, $EV = VS$, Fig. 10.

2. The distance of any point in the curve from the focus is equal to the sum of the abscissæ of the latus rectum and ordinate to the axis drawn from that point; that is, $FG = VF + VE$. For GF is equal to GC, each being equal to that part of the thread which is disengaged from the side of the square; and GC is equal to SE; therefore FG is equal to SE; and SE is equal to EV and VS; that is, to VE and VF, Fig. 10.

3. If the distance of any point from the focus of a parabola be equal to the perpendicular distance of that point from the directrix, or, what is the same thing, equal to the sum of the distance of the focus from the vertex and abscissæ of an ordinate to the axis drawn from the assumed point; then that point is in the curve of the parabola. Thus, if $FC = VF + VE$, then the point G is in the curve of the parabola.

4. If the distance of any point from the focus of a parabola be greater than the perpendicular distance of that point from the directrix; then that point is on the outside, or beyond the curve of the parabola; but if it be less, then that point is within the parabola, between the curve and the axis.

5. Every right line drawn through the focus of a parabola, except the axis, if produced far enough, will cut the curve in two points, one of which is between the extremity of the latus rectum and the vertex, and the other at a distance below the extremity of the latus rectum; for when produced till it meet the curve, that part of it from the focus to the curve is equal to the perpendicular distance of that point on the curve from the directrix; and this distance can never be infinite so long as the said right line makes any acute angle, how small soever, with the axis.

6. Any right line drawn parallel to the axis, or at right angles to the directrix, can cut the curve but in one point only.

Prop. 11. Theorem.

If any point be taken in the curve of a parabola, and if two right lines be drawn from that point, one to the focus, and another parallel to the axis, or at right angles to the directrix; then if the angle contained by these two right lines be bisected: I say that the right line bisecting the angle, is a tangent to the parabola in the assumed point.

Demonstration.

Let the point P (Fig. 11.) be taken in the curve of the parabola VIH, whose axis is VH, focus F, and directrix AB; and let PF be drawn to the focus, and PC parallel to VH, or at right angles to AB; and also, let PD bisect the angle CPE: then I say, that PD is a *tangent* to the parabola. For take any other point G in DP produced, and join GF and GC, and draw GA through the point G, parallel to BH, and join CF cutting PD in O: Then because the angle CPO is equal to the angle FPO, (by the hypothesis,) and PC is equal to PF (by axiom 3,); and PO is common to the two triangles CPO, PFO: therefore CO is equal to OF, and the angle COP equal to FOP (by prop. 1. of Geom.): again, because CO is equal to OF and OG is common to the two triangles COG and FOG, and they contain equal angles; consequently GC is equal to GF; but GC, the hypothenuse of the triangle AGC, is greater than the leg GA; whence GF is also greater than GA; and so the point G is on the outside of the parabola (by axiom 4.); and in the same manner we demonstrate, that if any other point be taken in PD between P and the directrix, that this point is also on the outside of the parabola: whence there is no

point in PD except P, which meets the curve; and consequently PD is a tangent to the parabola in the point P. Q.E.D.

Corollary.—Hence we have an easy method of drawing a tangent to any point in the curve of a parabola, if the axis and focus be given.

Prop. 12. THEOREM.

IF to any point in the curve of a parabola, a tangent be drawn, and produced till it meet the axis produced; and also from the same point an ordinate be drawn to the axis cutting it at right angles: then that part of the axis produced between the vertex and the point in which it meets the tangent is equal to the abscissa of that ordinate.

DEMONSTRATION.

LET VPA (Fig. 14.) be a parabola; VB its axis, F the focus, V the vertex, P a point in the curve, PC a tangent to the parabola in P, and produced till it meet the axis produced in C, PD an ordinate to the axis let fall perpendicularly to it from P: I say that DV is equal to VC: for through P draw PE parallel to VB, and join PF. Then because PC is a tangent, the angle EPC is equal to the angle FPC (by prop. 11.); but the angle EPC is equal to the angle PCF, because they are alternate angles; wherefore the angle FPC is equal to the angle PCF; and consequently FP is equal to FC; but PF is equal to VD and VF (by axiom 2.); and therefore FC, that is, VF and VC, is equal to VD and VF; take VF, which is common, from both, and there remains VD equal to VC. Q.E.D.

Corollary.—Hence, if the vertex and axis of a parabola be given, we have another easy method of drawing a tangent to any point in the curve. For if P be a point in the curve, draw the perpendicular PD to the axis, and produce the axis till VC be equal to VD, and join PC, which is the tangent required.

Prop. 13. THEOREM.

THE rectangle of the latus-rectum and any abscissa, is equal to the square of the semiordinate of that abscissa; and the squares of the semiordinates have the same ratio to each other, which their abscissæ have.

DEMONSTRATION.

LET APV (Fig. 14.), be a semiparabola, F the focus, V the vertex, EG the directrix, IF the semilatus-rectum = FG or twice VF, PD and AB semiordinates to the abscissæ VD and VB: I say, that the rectangle of the latus-rectum and VD is equal to the square of PD; and that the rectangle of the latus-rectum and VB is equal to the square of AB; and also, that as VD is to VB so is the square of PD to the square of AB: for, join FP; then FP is equal to EP or GD; and because VD is divided into two parts in the point F, four times the rectangle of VD and VF together with the square of FD are equal to the square of their sum, which is DG (by prop. C.); and DG is equal to PF; therefore the square of PF is equal to four times the rectangle of VD and VF together with the square of FD; but the square of PF is equal to the squares of PD and DF (by prop. 16. Geom.); whence, the squares of PD and DF are equal to four times the rectangle of VD and VF, together with the square of FD; take away the square of FD, which is common, and there remains the square of PD equal to four times the rectangle of VD and VF; but the latus-rectum is equal to four times VF (by Def. 5.); wherefore the rectangle of the latus-rectum and VD, is equal to the square of PD. And in the same manner we demonstrate, that the rectangle of the latus-rectum and VB is equal to the square of AB. Moreover, the rectangle of the latus-rectum and VD is to the rectangle of the latus-rectum and VB, as VD is to VB (by division); consequently, as VD is to VB, so is the square of PD to the square of AB. Q.E.D.

SCHOLIUM.

THIS Theorem is called the great property of the parabola; by the assistance of which almost every thing relating to the parabola is determined and regulated. Call the ordinate PD = y, the abscissa VD = x, and the latus-rectum = r; then, (by the foregoing Theorem,) $r \times x = yy$, or $rx = y^2$, and this is called the equation of the parabola.

Prop. 14. THEOREM.

IF a cone be cut by a plane which is parallel to one of its sides, the section will be a parabola.

Let the cone DVE (Fig. 15.) be cut by the plane SAW parallel to the side VD: the section SAW is a parabola.

For let a plane VDE pass through the axis of the cone perpendicular to the plane SAW, and let AT be their common section; also let GPQp be a section of the cone parallel to the base, and consequently a circle, and let its common sections with SAW and VDE be Pp and GQ. Because the plane VDE is perpendicular to the planes SAW, and GPQp, therefore their common section, Pp, is perpendicular to VDE, and consequently to the lines AT, and GQ, which are in VDE. Therefore PN is an ordinate to the diameter GQ of the circle GPQp, and the rectangle of GN and NQ is equal to the square of PN; but from the similar triangles ANQ, VBA, AN: NQ :: VB: BA and :: BA: L, taking L a third proportional to VB and BA: therefore the rectangle of AN and L is equal to the rectangle of NQ and BA

$=$ the rectangle of NQ and GN $=$ the square of PN. But this by Prop. 13. is the property of a parabola of which the abscissa is AN, the latus rectum L, and the semi-ordinate PN. Hence the curve SPApW is a parabola.

We could add many more Theorems of the properties of the parabola; but because any more than those few that we have given scarce ever occur, either in the theory or practice of astronomy, we chuse therefore to enlarge no farther on the subject of Conic Sections. At the same time the reader may, if he think proper, pursue the study, and read the several authors, we formerly mentioned.

General Equations *of the* five Sections *of a* Cone.

Let $x =$ an abscissa, and $y =$ its correspondent semiordinate. Then we have, for the

Equation of the Triangle,

$rx = y$; where r is the quotient of the perpendicular divided by the base;

Equation of the Circle,

$2rx - x^2 = y^2$; where r is the radius of the circle;

Equation of the Ellipsis,

$r \times \overline{2tx - x^2} = y^2$; where r is the quotient of the square of the conjugate diameter, divided by the square of the transverse, and t is the semitransverse;

Equation of the Parabola,

$rx = y^2$; where r is the latus rectum, or the quotient of the square of any given ordinate, divided by its given abscissa;

Equation of the Hyperbola,

$r \times \overline{2tx + x^2} = y^2$; where r and t are the same as in the ellipses.

And here we may observe, that of these five sections, only two are returning ones; that is, the circle and ellipsis have returning curves, which if continued on, return to the same point from whence the motion began. For it is evident, in the equation of the circle, $2rx - x^2 = y^2$; that when $x = 2r$, the diameter, then the equation is $x^2 - x^2 = y^2$; that is, $y = 0$; so that when x, the abscissa, is equal to the diameter, the ordinate vanishes, and is nothing; and by continuing the motion, the other semicircle is described. Also in the equation of the ellipsis, $r \times \overline{2tx - x^2} = y^2$; when $x = 2t$, the transverse, then the equation is $r \times \overline{x^2 - x^2} = y^2$; that is, $y = 0$; wherefore when x, the abscissa is equal to the transverse, then the ordinate vanishes, and is nothing; and as in the circle, by continuing the motion the curve returns to the first point from whence the motion began.

But in the Equation $\begin{cases} rx = y \\ rx = y^2 \\ r \times \overline{2tx + x^2} = y^2 \end{cases}$ of the $\begin{cases} \text{Triangle.} \\ \text{Parabola.} \\ \text{Hyperbola.} \end{cases}$

It is evident, that as x, the abscissa, increases, y, the ordinate, increases; and when x is infinite, then y is infinite; and at no time can y vanish and be equal to 0, but in the nascent state when $x = 0$.

Astronomical Scholium.

In determining the equation of the curve, commonly called the *trajectory*, or comet's orbit; and comparing it with those of the conic sections; then it is evident, that if the equation corresponds to that of a circle or an ellipsis; then that is a returning trajectory, and the comet necessarily revolves round the Sun: but if the equation be similar to that of any of the other three sections; then the trajectory is not a returning one; and the comet does not revolve round the Sun; and having passed its perihelion, it can never return again to our system, but fly off into infinite space; and probably in its flight will gravitate towards other suns and systems; and may then have the equation of its trajectory such as to be a returning one, to some Sun, round which it must continue eternally to move; or be otherwise changed by the *Omnipotent Fiat.*

This is the most difficult, scientific, and sublime part of Astronomy; as it involves a considerable knowledge of Geometry, Conic Sections, Plane and Spherical Trigonometry, combined with a clear conception and sound reasoning faculty: the student, therefore, is exhorted to apply his most serious contemplations to this sublime part of the science, if he be desirous of making good progress therein, which, we lament to say, falls to the lot of very few, very few indeed!

PLANE TRIGONOMETRY.

Definitions.

1. PLANE Trigonometry* is the ascertaining the magnitude of the inaccessible sides and angles of a plane triangle, by comparing them with those sides

* The word is derived from the Greek, (τριγωνος, a triangle, and μετρεω, to measure,) which seems to have been the cause of most of our trigonometrical writers giving an absurd definition of this science. Trigonometry is generally defined by

and angles of the triangle which are accessible and can be measured. And we again repeat, what we have formerly mentioned; i. e. that every triangle has seven essential parts, viz. three sides, three angles, and an area; and that if any three of these seven (except the three angles of a plane triangle), be accessible and be measured; then, by plane trigonometry and superficial mensuration, the other four inaccessible parts can be determined to the same degree of certainty and accuracy, that the accessible parts have been measured. But in trigonometry, the seventh essential, to wit, the area, is generally left unnoticed: and trigonometry, therefore, is confined to the determination of the inaccessible sides and angles only of a triangle. To accomplish the object of trigonometry, mathematicians have calculated the lengths of certain right lines belonging to arcs of a circle, by comparing them with the radius of that circle; and which radius they commonly supposed to be divided into 1.0000000, or 1.0000000000 equal parts; and to these right lines, whose lengths they have calculated in the equal parts of the radius and disposed into tables, they have given the names of Sines, Tangents, Secants, Cosines, Cotangents, and Cosecants, &c. Now, by plane Trigonometry we ascertain the lengths of all inaccessible right lines in Astronomy; such as the distance of the Moon from the Earth; the distance of the Sun and planets from it; and the angular distance of the celestial bodies from each other, &c. Hence the importance of this branch of science to an astronomer is obvious. And here it is necessary to remark, that one accessible line at least must be actually and accurately measured; and this for Astronomical purposes, is a line on the surface of the Earth; as the length of a degree, or $\frac{1}{360}$ part of the circumference; by which the whole circumference and consequently the diameter or radius of the Earth is determined; and by comparing the diameter of the Earth with other inaccessible right lines, by the assistance of plane trigonometry, the exact lengths of these inaccessible right lines are obtained, &c.

2. The *Sine* of an arc is a right line drawn from one end of the arc perpendicularly to the diameter drawn to the other end of the arc; or it is half the chord of double the arc.

3. The *Tangent* of an arc is a right line drawn perpendicularly from one end of the arc to the diameter drawn to that end or extremity, and terminated by the diameter drawn to the other end of the arc, and produced till it meet the tangent.

4. The *Secant* of an arc is that right line drawn from the centre to one end of the arc, and produced till it meet the tangent drawn from the other end of the arc.

5. The Cosine, Cotangent, and Cosecant of an arc, are the Sine, Tangent, and Secant of the complement of that arc.

6. The Sine, Tangent, Secant, Cosine, Cotangent, and Cosecant, of an arc, are also called, the Sine, Tangent, &c. of the angle which that arc measures.

Illustration of the foregoing Definitions.

Let AGBH (Fig. 1.), be the circumference of a circle, OB an arc of it; C the centre, and join CB, CO; from the point B draw BP at right angles to CB, and produce it till it meet CO produced in P; through the points O and C, draw OD, and CG, each parallel to BP; also through the points O, and G, draw OI, and GF, each parallel to CB; and produce OD to H. Then, because OD and GC are each parallel to BP, which is perpendicular to the diameter AB; therefore, OD, and GC, are each perpendicular to it; and the semicircle AGB is bisected in G, consequently each of the arcs AG, and GB = 90°. We demonstrate, in like manner, that OI, and GF, are each perpendicular to CG. And because the two arcs BO, and OG, make 90°; therefore they are complements to each other; but BO, measures the angle BCO, and OG measures the angle OCG; wherefore the angles BCO, and OCG, are also complements to one another. Moreover, OD is the sine of the arc OB, and OI is the sine of the arc OG (by Def. 2.); wherefore IO is the cosine of the arc OB (by Def. 5.); consequently, OD is the sine, and OI the cosine of the angle BCO. In the same manner we shew, that BP is the tangent of the arc BO (by Def. 3.), and of the angle BCO; and that GF is the cotangent of the arc BO, and of the angle BCO; for it is the tangent of their complement. Also CP is the secant, and CF the cosecant of the arc BO, or of the angle BCO (by Def. 4.); Moreover it is evident, that OD, the sine of OB, is half the chord OH, which is the chord of twice OB;

these gentlemen; *the art or science of measuring the sides and angles of triangles.* But this definition is absolutely the *converse* of the correct one; because trigonometry *is not* the science of measuring the sides and angles of triangles, which can be actually measured; for it is the science which shews how to find the magnitude of the sides and angles of triangles, which (in many instances) *cannot* be actually measured.

Trigonometry has nothing to do with the actual admeasurement of the parts of a triangle; it is built on a rational hypothesis; viz. that a sufficient number of the parts of a triangle have been actually measured and which parts are taken as *data;* and the business of trigonometry is to determine the other parts of the triangle, which are supposed inaccessible, and probably cannot be actually measured; and these parts constitute the *quæsita:* and by trigonometry these required parts can be found to the same degree of exactness to which the several parts of the data were measured.

and that CD = OI, is the cosine of OB, or of the angle BCO; and that IC = OD, is the cosine of the arc OG, or of the angle OCG; and likewise that BP is the cotangent, and CP the cosecant of the arc OG, or of the angle OCG.

The arc AGO, (as we formerly defined in our Geometry,) is called the supplement of OB; and OD, which is the sine of OB, is also the sine of the arc AGO, and also the sine of the obtuse angle ACO; because it is a right line drawn from O, one end of the arc, perpendicularly to the diameter drawn to A, the other end of the arc: so that the sine, tangent, and secant of any arc less than 90°, or of an angle less than a right angle, are also the sine, tangent, and secant of the supplement of that arc, or what it wants of 180°; and likewise of the supplement of that angle to two right angles: but the cosine, cotangent, and cosecant of an arc less than 90°, or of an acute angle, are the sine, tangent, and secant of the excess of the supplementary arc or angle, above 90° or a right angle.

The sine OD of the arc OB, divides the diameter in the point D, into two segments, called *versed-sines;* thus BD is the versed-sine of the arc OB; and AD, is the versed-sine of the arc AGO; and they are also the versed-sines of the angles which those arcs measure. And the versed-sine of an arc less than 90°, is the excess of the radius above the cosine of that arc; but the versed-sine of the supplementary arc is the sum of the radius and cosine. Thus BD = CB — CD, and AD = AC + CD.

Scholium.

The square of the radius CO, is equal to the sum of the squares of OD, and CD (by prop. 16, Geom.), the sine and cosine: the square of the secant is equal to the sum of the squares of the radius and tangent; that is, the square of CP is equal to the squares of CB and BP; and the square of CF is equal to the squares of CG and GF; and lastly, the rectangle of the versed-sines of an arc and of its supplement is equal to the square of the sine of that arc; that is, the rectangle of AD and DB is equal to the square of OD (by prop. 26 of Geom.).

From the foregoing definitions and the illustration, it is manifest that the sines of arcs increase as the arcs increase, till the arc become 90°, and then the sine is the greatest, and is equal to the radius of the circle; thus GC the radius, is the sine of the arc AG or BG, or of either of the right angles ACG, or BCG, or of 90°; that is, the radius is the sine of 90°, or of a right angle; but the sines of all other arcs or angles, are less than the radius.

It is also evident that the tangents and secants of arcs increase as the arcs increase; but not similarly to the sines; for as the arc approaches towards a quadrant or 90° the quantity of increase becomes greater, till the arc become 90°, when it is infinite; so that the tangent and secant of 90° are infinite; for then they are parallel right lines and can never meet; whereas the quantity of increase in the sines of arcs becomes less and less as the arc approaches towards a quadrant; and when it is 90°, the increase vanishes and becomes nothing. Also it is manifest, that the secant in its nascent state is equal to the radius; that is, the secant of 0°, is equal to the radius. And the tangent of 45° is equal to the radius; because in that case, the radius, tangent, and secant, constitute an isoceles triangle, and the radius and tangent are the two equal legs, being each opposite to equal angles of 45° each. Moreover the sine of 30° is equal to half the radius; because it is half the chord of 60°, which is equal to the radius. (by prop. 32. Geom.)

The sine, tangent, &c. of half an arc, are called the semi-sine, semi-tangent, &c. of the whole arc; but they are not the halves of the sine, tangent, &c. of the whole arc. Thus, the sine, tangent, &c. of 30°, are the semi-sine, semi-tangent, &c. of 60°. Hence the semi-tangent of 90°; that is the tangent of 45°, is equal to the radius.

Problem A.

Given the two legs of a right angled triangle, to find the hypothenuse. *Solution.*—Rule. Add the squares of the two given legs together; and extract the square root of the sum; which square root will be the hypothenuse required. The reason is manifest, from prop. 16. of Geometry.

Example.

In the triangle ABC, right-angled at B (Fig. 3.); there are given the base AB = 408, and perpendicular BC = 306, feet, yards, miles, &c. It is required to find the hypothenuse AC, in the same measure.

$408 \times 408 = 166464 =$ the square of AB.
$306 \times 306 = 93636 =$ the square of BC.

Sum $260100 =$ the sum of their squares, $= AC^2$.
$\sqrt{260100} = 510 = AC$. required.

Problem B.

Given the hypothenuse and one leg of a right angled triangle, to find the other leg. *Solution.*—Rule: Subtract the square of the given leg from the square of the hypothenuse; and extract the square root of the remainder; which square root will be the leg required. The reason is evident, as in the other.

Note.—If the product of the sum and difference of the hypothenuse and given leg be taken, it is the same as the difference of their squares (by prop. A of Conics).

Example.

In the triangle ABC, right angled at B (Fig. 3.), there are given the hypothenuse AC = 510, and leg AB = 408: it is required to find the other leg BC?

$510 \times 510 = 260100 = AC^2$
$408 \times 408 = 166464 = AB^2$

Difference, $93636 = BC^2$

$\sqrt{93636} = 306 = BC$, required.

Otherwise,

AC = 510
AB = 408

Sum = 918

Diff. 102

$\sqrt{918 \times 102} = \sqrt{93636} = 306 = BC$

Corollary.—From the two foregoing problems it is manifest, that if any two sides of a right angled triangle be actually measured, or supposed to be of a given magnitude, then the remaining third side can be found without the trouble of measuring it.

LEMMA 1.

The secant of an arc or of an angle is equal to the square root of the sum of the squares of the radius and tangent of that arc, or of that angle.

The cosine of an arc, or of an angle is equal to the square root of the difference of the squares of the radius and sine of that arc or of that angle, or to the square root of the product of the sum and difference of the radius and sine of that arc or angle. This lemma is evident, because the two triangles CBP and ODC (Fig. 1.), are right angled at B and D; and CP the hypothenuse, is the secant of the arc OB or angle OCB, and BP the leg is the tangent of the same arc or angle, and CB, the other leg, is the radius. Also, OD and CD, the sine and cosine of the arc OB, are the legs, and CO the radius, is the hypothenuse.

LEMMA 2.

The triangles CDO, CBP, COI, and CFG, are equiangular, and therefore similar. Whence,

1. As the cosine of an arc or of an angle is to the sine of the same, so is radius to the tangent of the same, *i. e.* as CD : OD :: CB : BP.

2. As the cosine of an arc or of an angle is to radius, so is radius to the secant of the same, *i. e.* as CD : CO :. CB : CP.

3. As the tangent of an arc or of an angle is to radius, so is radius to the cotangent of the same, *i. e.* as BP : CB :: CG : GF.

4. As the sine of an arc or of an angle is to the cosine of the same, so is radius to the cotangent of the same, *i. e.* as CI : IO : : CG : GF.

5. As the sine of an arc or of an angle is to radius, so is radius to the cosecant of the same, *i. e.* as DO : CO : : CG : CF.

6. As the semi-tangent of an arc or of an angle is to radius, so is the sine of the same to the versed-sine of the supplement of the same. Let AOB (Fig. 2.), be a semicircle; BO an arc, and OD its sine; and join AO, and erect the perpendicular CS, and about the center A describe the arc CL, and let fall the perpendicular LT; then the angle OCB is double to the angle A, therefore the arc CL is equal to one half of OB, because they have equal radii; consequently, LT the sine, SC the tangent, AS the secant, and AT the cosine, are the semi-sine, semi-tangent, &c. of OB; and by similar triangles, as SC : CA : : OD : DA.

7. As the semi-sine of an arc or of an angle is to the semi-cosine of the same, so is the sine of the same to the versed-sine of the supplement of the same, as LT : TA : : OD : DA.

SCHOLIUM.

Several other properties of the sines, tangents, &c. of arcs or angles might be given; and these may be varied by inversion, alternation, &c. And it is manifest, that if the radius be supposed to consist of a determinate number of equal parts, and if the sine of an arc or of an angle be given in the same equal parts, then the tangent, secant, cosine, &c. of the same arc or angle can be found by the application of the foregoing lemmas. And by the assistance of these properties, and other geometrical deductions and arithmetical calculations, mathematicians have computed, in the equal parts of the radius, the lengths of the sines, tangents, secants, and versed-sines of all arcs of the quadrant; and which, being tabulated, form what is called a table of *natural* sines, tangents, secants, and versed-sines; and by taking the logarithms of these *natural numbers*, another table is formed called a *logarithmic* table of sines, tangents, &c. or a table of *artificial* sines, tangents, &c. And as all arithmetical operations are generally performed much more easily by logarithms than by their respective *natural numbers*; the logarithmic table therefore is for the most part that used in trigonometrical operations. And hence it seems necessary, as an introduction to trigonometry, to explain the nature and use of logarithms. But Astronomy being the ultimate object of the present work, the reader is not to expect any more from us concerniug these valuable numbers, than what is *brief* and *applicable*. Those who are desirous of gaining a perfect account of the origin, nature, history, construction, use, &c. of logarithms, are referred to Dr. Hutton's excellent book, on the subject, in which they will find the most

ample information. Hutton's logarithmic tables and Taylor's, are the most correct we know of.

Of LOGARITHMS.

Logarithms are numbers calculated by mathematicians, to abridge tedious and difficult calculations in arithmetic. Every number has a logarithm, and to distinguish the number from its logarithm, they call the number itself, the *natural number*, (for what reason we know not,) and that number, which corresponds to it, and which answers the purpose intended, *i. e.* to shorten the arithmetical calculation, they call the logarithm of that number.

Our present tables of logarithms are calculated on the hypothesis, that the logarithm of unit, or 1, is 0; and the logarithm of 10 is unit or 1; and consequently, the logarithm of 1, with two noughts, or 100 is 2; the logarithm of 1000, 3; &c. that is, that the logarithm of unit with any number of cyphers annexed, is equal to the number of cyphers annexed to unit. Thus

Numbers.	*Logarithms.*
1	0.0000000
10	1.0000000
100	2.0000000
&c.	

Hence it is manifest, that all numbers between unit and 10, have their logarithms between 0, and 1; that is some fraction less than 1; as the logarithm of 2 is 0.30103, &c. the logarithm of 4 is 0.60206, &c. between 0 and 1: also the logarithms of all numbers between 10 and 100 are greater than 1, but less than 2; that is, 1 and some fraction of unit: thus the logarithm of 32, is 1.50515, &c. and the logarithm of 64, is 1.80618, &c. So in like manner, the logarithms of all numbers between 100 and 1000 are greater than 2 but less than 3, *i. e.* 2 and some fraction: thus the logarithm of 320, is 2.50515, &c. And hence it appears, that the logarithm of every number, (except 1, 10, 100, 1000, &c.) consists of two parts, to wit, a whole number and a fraction; and the whole number is for distinction's sake called the *index* of the logarithm, and is less by unit than the number of places or figures in the natural number: but if the natural number consist of a whole number and a fraction annexed, then the index of the logarithm is the same as that which the whole number should have without respect to the fractional part of the natural number: so that the index of the logarithm of 408, or 408.5 or, 408.75, &c. is 2: but the fractional part of the logarithms of these natural numbers is different in each; for the fractional part of the logarithm of any natural number, has respect to every figure in it, as well to those of the decimal fraction, as to those of the whole number. Thus,

Numbers.	*Logarithms.*
408.	2.6106602
408.5	2.6111921
408.75	2.6114578
&c.	

Hence the index of the logarithm of any natural number is always known, as it is evidently less by one, than the number of figures in the integral part of the natural number; that is, if the integral part of the number have 5 figures in it, then the index of its logarithm is 4; if it have 11 figures in it, the index is 10, &c. But the fractional part of the logarithm of any natural number, (except 1, or 10, or 100, or 1000, &c.) is determined by an exceedingly tedious and difficult arithmetical calculation: and hence tables, containing the fractional parts of many thousands of natural numbers, have been constructed by several eminent mathematicians, which tables are of the utmost importance in trigonometry, and in almost every other part of the mathematics. And because the index of the logarithm of any given natural number is always known, hence the fractional part of the logarithm is commonly called the logarithm, and is only given in tables of logarithms, and the index omitted, and left to the operator to affix.

In Hutton's or Taylor's tables, which are those we use, the logarithms are carried to 7 decimal places or figures; and these are sufficiently accurate to determine any natural number less then 10 millions, to the nearest unit.

The logarithms of all natural numbers, (except those of 1, or 10, or 100, or 1000, &c.) are incommensurate to their natural numbers; that is, they are of that nature as never to terminate; in the same manner that the square root of 2, 3, 5, &c. is 1 and a decimal fraction, which can never terminate; and the more figures to which the root is carried, it is still manifestly the more accurate: so in like manner, the more figures to which logarithmic tables are carried, they are of consequence the more accurate. The diameter or the radius and circumference of a circle are also incommensurate to each other; but the *proportion*, or *ratio* between the radius and circumference of a circle, has been carried to a degree of *exactness*, many millions of millions of times beyond that of any other incommensurable ratio we know of. See the note to problem 1. in geometry.

If any whole number be divided by 10, or 100, or 1000, &c. then the logarithms of the whole number and the several quotients are alike, differing only in their indexes.

			Numbers	*Logarithms*
Thus,			40875	4.6114578
40875	divided by	10 =	4087.5	3.6114578
		100 =	408.75	2.6114578
		1000 =	40.875	1.6114578
		10000 =	4.0875	0.6114578

If the logarithms of two natural nnmbers be added together the sum is the logarithm of their product; and if the logarithm of one natural number be subtracted from that of another, the remainder is the logarithm of the quotient of the latter divided by the former: also, if the logarithm of any natural number be multiplied by 2, the product is the logarithm of its square; if multiplied by 3, it gives the logarithm of its cube, or third power; if by 4, it gives the logarithm of its fourth power, &c. Moreover, if the logarithm of any natural number be divided by 2, the quotient is the logarithm of its square root; if by 3, it gives the logarithm of its cube root, &c. Again, because addition of logarithms answers to multiplication of natural numbers, and subtraction to division; if the logarithm of the first terms of a proportion be subtracted from the sum of the logarithms of the second and third, the remainder is the logarithm of the fourth proportional; and hence we have this

General Rule to find a Fourth Proportional.

ADD the logarithms of the second and third terms together, and from their sum subtract the logarithm of the first; the remainder is the logarithm of the fourth proportional sought. This rule will be amply exemplified in our Trigonometry.

Note. In Hutton's and Taylor's Tables the reader will find perspicuous rules for finding the logarithm of any given natural number; and, on the contrary, for finding the natural number answering to any given logarithm; also, for finding the logarithmic sine, tangent, &c, of any given arc; and, on the contrary, for finding the arc answering to any logarithmic sine, tangent, &c. In referring him to these books we doubt not that on inspection of the tables and a few minutes' attentive reading he will become perfectly acquainted with the mode of using the tables.

Introductory Rules, &c.

1. IF one of the acute angles of a plane right angled triangle be given, subtract it from 90°, and the remainder is the other acute angle.

2. If two angles of a plane oblique angled triangle be given, add them together and subtract their sum from 180°, the remainder is the third angle.

3. If one angle of any plane triangle be given, subtract it from 180°, and the remainder is the sum of the other two angles.

These rules are evidect from prop. 12, of Geom.

4. When the sine, tangent, secant, &c. of a given angle greater than 90°, are wanted, subtract the given angle from 180°, and take the sine, tangent, &c. of the remainder. This rule follows from our illustration of the trigonometrical definitions.

5. In every triangle the greater side is opposite to the greater angle, and *vice versâ*.

6. In an obtuse angled triangle, if a perpendicular be drawn from the obtuse angle to the opposite side, then that perpendicular falls within the triangle, and divides it into two right angled triangles; but if it fall from an acute angle upon the opposite side produced, then there will be two right angled triangles, one of which is on the outside of the first triangle, and the other is made up of both.

Proposition A. THEOREM.

1. IN any right angled plane triangle; as radius is to the hypothenuse; so is the sine of any one of the acute angles, to its opposite leg: and as the hypothenuse is to radius, so is any one of the legs, to the sine of its opposite angle. Also, as radius is to any one of the legs; so is the tangent of the angle opposite to the other leg, to the other leg; and so is the secant of that angle to the hypothenuse: and as any one of the legs is to radius, so is the other leg, to the tangent of its opposite angle; and so is the hypothenuse, to the secant of that angle*.

2. And in any plane triangle, as the sine of any one of the angles is to its opposite side; so is the sine of any other angle to its opposite side: and, as any one of the sides is to the sine of its opposite angle; so is any other side, to the sine of its opposite angle†.

Note.—When a side is to be found, begin the proportion, with the correspondent side in the tabular similar triangle, which will be radius, sine, tangent, &c. as the case requires, of a given angle: but when an angle is to be found, begin the proportion with a side, and the fourth proportional will be the sine, tangent, &c. of the required angle. By the application of the foregoing Theorem, all the several cases in right angled trigonometry, and the two first cases of oblique angled, are solved.

* By varying these proportions, by inversion, alternation, and equality; every proportion possible in right angled trigonometry, will be obtained.

† We have thought it best not to separate the rules from the examples in the solution of the cases by inserting the demonstrations of the propositions, and have therefore given these demonstrations altogether at the end of the plane trigonometry.

Right Angled PLANE TRIGONOMETRY.

Case 1.

Given the hypothenuse and angles of a right angled plane triangle, to find the two legs.

Example.—In the right angled plane triangle ABC (Fig. 4.) there are given the hypothenuse AC = 555 feet, yards, miles, &c.; and the angle A = 33° 8′, and consequently the angle C = 36° 52′: the legs AB and BC are required.

Solution.

1. To find AB, one of the legs.

			Logarithms.
As radius, or sine of	B	90°. 00′.	10.0000000
Is to the hypothenuse	AC	555.0	2.7442930
So is the sine of the angle	C	36° 52′	9.7781186
To the Leg	AB	332.975	2.5224116

Or,

As secant A : AC :: radius : AB.

Or,

As secant C : AC :: tangent C : AB.

2. To find CB, the other leg.

			Logarithms.
As radius, or sine of	B	90°. 0′	10.0000000
Is to the hypothenuse	AC	555.0	2.7442930
So is sine angle	A	53° 8′	9.9031084
To the leg	CB	444.0188	2.6474014

Or,

As secant A : AC :: tangent A : CB.

Or,

As secant C : AC :: radius : CB.

We have left the operations of the last four analogies for the exercise of the student; and he will find that they bring the same results. Moreover, in the two operations above, he is to observe, that the logarithms of the second and third terms are added together, and the logarithm of the first term subtracted from their sum, in order to obtain the logarithm of the fourth term sought: according to the foregoing general rule.

He may also observe that the logarithm of the radius in almost all trigonometrical tables is 10, and consequently the radius itself in natural numbers is 10 thousand millions of equal parts; and the sines, tangents, &c. in the tables, are calculated to that radius.

Case 2.

Given one of the legs and the angles of a right angled plane triangle to find the hypothenuse and the other leg.

Example.—In the right angled plane triangle ABC (Fig. 4.) there are given, the leg AB = 333 miles, &c.; and the angle A = 53° 8′; and consequently, the angle C = 36° 52′: it is required to find the hypothenuse AC, and the other leg, BC.

Solution.

1. To find AC, the hypothenuse.

As the sine of the angle	C	36° 52′	9.7781186
Is to the leg	AB	333.0	2.5224442
So is the radius or sine of	B	90° 0′	10.0000000
To the hypothenuse	AC	555.0416	2.7443256

Or,

As Radius : AB :: Secant A : AC.

Or,

As Tangent C : AB :: Secant C : AC.

2. To find BC, the other leg.

As the Sine of the angle	C	36° 52′	9.7781186
Is to the leg	AB	333.0	2.5224442
So is the sine of the angle	A	53° 8′	9.9031084
To the leg	BC	444.0521	2.6474340

Or,

As radius : AB :: tangent A : BC.

As tangent C : AB :: radius : BC.

Case 3.

Given the hypothenuse and one of the legs of a rignt angled plane triangle, to find the two acute angles and the other leg.

Example.—In the right angled plane triangle ABC, (Fig. 4.) there are given the hypothenuse AC = 555, and the leg AB = 333; it is required to find the angles A and C and the leg BC.

1. To find the angles A and C.

As the hypothenuse	AC	555.0	2.7442930
Is to radius or the sine of	B	90° 0′	10.0000000
So is the leg	AB	333.0	2.5224442
To the sine of the angle C		36° 52′ 11.6″	9.7781512
From		90° 0′ 0″	
Subtract the angle C =		36 52 11.6	
Remains the angle A =		53 7 48.4	

Or,

As AB : radius :: AC : secant A.

And the angle A subtracted from 90°, leaves the angle C.

2. To find the leg BC, the angles being first found.

As radius, or the sine of	B	90° 0′	10.0000000
Is to the hypothenuse	AC	555.0	2.7442930
So is the sine of the angle A		53° 7′ 48.4″	9.9030900
To the leg	BC	444.	2.6473830

Or,

As radius : AB : : tangent A : BC;

Or,

As secant A : AC : : tangent A : BC;

Or,

As secant C : AC : : radius : BC;

Or,

As tangent C : AB : : radius : BC;

Or,

As sine C : AB : : sine A : BC.

Hence it appears that there are only two analogies for finding the angles, but there are six several analogies for finding the third side, besides the method given in problem B.

Case 4.

Given the two legs of a right angled plane triangle, to find the two acute angles, and the hypothenuse.

Example.—In the right angled plane triangle ABC (Fig. 4.), there are given the two legs, AB = 333, and BC = 444, to find the angles A and C and the hypothenuse AC.

Solution.

1. To find the angles A and C.

As the leg	AB	333.0	2.5224442
Is to radius			10.0000000
So is the leg	BC	444.0	2.6473830
To tangent of the angle A		53° 7′ 48.4″	10.1249388
From		90° 0′ 0′	
Subtract the angle A =		53 7 48.4	
Remains the angle C =		36 52 11.6	

Or,

As BC : radius : : AB : tangent C. And the angle C subtracted from 90° leaves the angle A.

2. To find the hypothenuse AC, the angles being first found.

As radius			10.0000000
Is to the leg	AB	333.0	2.5224442
So is the secant angle A		53° 7′ 48.4″	10.2218488
To the hypothenuse AC		555	2.7442930

Or,

As tangent A : BC : : secant A : AC;

Or,

As radius : BC : : secant C : AC;

Or,

As tangent C : AB : : secant C : AC;

Or,

As sine A : BC : : radius : AC;

Or,

As Sine C : AB : : radius : AC.

And as in the 3d case so it appears in this, that there are only two analogies by which the angles can be found, but six several ways for finding the hypothenuse or third side, besides the method given in problem A.

Scholium.

The student will observe, that we have been strict in the determination of the required parts: we have carried out the required sides to four decimal places, or to the ten thousandth part of an unit; and the required angles, to the tenth part of a second of a degree. For common purposes the nearest unit is sufficient, without paying any regard to the fractional part, *e. g.* If the problem belong to navagation, the nearest mile in a side, or the nearest minute in an angle is all the degree of accuracy generally expected in that branch (except in lunar observations), but in astronomy the case is otherwise; because astronomers are often obliged to find great magnitudes by small ones, and a very small inaccuracy in the less, may produce a very considerable error in the greater: and hence we recommend to the student's attention, to acquire a perfect knowledge of finding the proportional parts of small remainders, belonging to natural numbers, arcs, or logarithms.

The rule called *Practice* in our books of arithmetic, is often serviceable in such cases, and saves many figures in these delicate operations. In Taylor's tables, the logarithmic sines and tangents are computed to every second of the quadrant, but the logarithmic secants are omitted. Upon the whole, an expertness in the common rules of addition, subtraction, multiplication, and division, is of the highest importance in all astronomical calculations.

Oblique angled PLANE TRIGONOMETRY.

Case 1.

Given one side of a plane triangle, and the three angles, to find the other two sides.

Example.—In the plane triangle ABC (Fig. 5.); there are given the side AB = 434 miles, the angle A = 53° 8′, the angle B = 67° 23′, and consequently the angle C = 59° 29′: it is required to find the sides AC and BC.

Solution.

1. To find the side AC.

As the sine of the angle	C	59° 29′	9.9352459
Is to the side	AB	434.0	2.6374897
So is the sine of the angle	B	67 23	9.9652480
To the side	AC	465.0416	2.6674918

2. To find the side BC.

As the sine of the angle	C	59° 29′	9.9352459
Is to the side	AB	434.0	2.6374897
So is the sine of the angle	A	53° 8′	9.9031084
To the side	BC	403.0437	2.6053522

Scholium.

This case is of the utmost importance in astronomy, for it appears that if a base line AB, and the two angles A and B at its extremities, be actually measured, or otherwise be accurately determined; then the lengths of the two inaccessible lines AC and BC can be found by this case, and to the same degree of accuracy that the base and angles at the base were measured or previously determined. And thus the distance of the Moon from the Earth and other inaccessible distances are determined, as shall be more fully illustrated hereafter.

Case 2.

Given two sides of a plane triangle, and an angle opposite to one of them, to find the other two angles and the third side. But it is necessary previously to ascertain, as a part of the *data*, whether the angle opposite to the other given side be accute or obtuse, otherwise the problem admits of two different solutions.

Example.—In the plane triangle ABC (Fig. 6.) there are given, the side AB = 434, the side AC = 465, and the angle C (opposite to AB) = 59° 29′. It is required to find the angles A and B and third side BC, the angle B opposite to AC, being previously known to be acute.

Solution.

1. To find the angle B, previously known to be acute.

As the side	AB	434.0	2.6374897
Is to the sine of the angle	C	59° 29′	9.9352459
So is the side	AC	465.0	2.6674530
To the sine of the angle	B	67° 22′ 15.8″	9.9652092

Angle C = 59° 29′ 0″
Angle B = 67 22 15.8

C + B = 126 51 15.8

From 180° 0′ 0″
Sub. C + B = 126 51 15.8

Rem. angle A = 53 8 44.2

This is called the ambiguous case, because, when the length of the side AB is less than the length of AC, but greater than that of a perpendicular from A upon the third side BC, the extremity of AB must meet the line CBB in two points B, B; and unless it be known whether the angle at B is acute or obtuse, this angle may be situated at either of those points. This ambiguity, however, although existing in the abstract theory of triangles, is not the occasion of any doubt when trigonometry is applied to astronomical purposes, since a careful attention to the nature and condition of the problem under consideration will always obviate the ambiguity.

2. To find the third side BC, the angles being first found.

As the sine of the angle	C	59° 29′	9.9352459
Is to the side	AB	434.0	2.6374897
So is the sine of the angle	A	53° 8′ 44.2″	9.9031781
To the side	BC	403.1085	2.6054219

Note.—If the conditions of the problem required, that the angle B were to be obtuse, then the suppliment of 67° 22′ 15.8″, *viz.* 112° 37′ 44.2′ is to be taken for the angle B. Hence

Angle C =	59° 29′ 0″		180° 0′ 0″
Angle B =	112 37 44.2		172 6 44.2
C + B =	172 6 44.2	Angle A =	7 53 15.8

Then, as sine C : AB : : sine A = 7° 53′ 15.8″ : BC, the operation is the same as before.

Prop. B. Theorem.

1. In any plane triangle, as the sum of any two sides of it is to their difference, so is the tangent of half the sum of the two angles opposite to those sides, to the tangent of half the difference of these angles.

2. And as the base or third side of the triangle is to the sum of those two sides, so is the difference of those sides, to the difference of the segments of the base, made by a perpendicular to the base, let fall from the angle opposite to the base.

Note.—By the assistance of this proposition the two following cases are solved.

Case 3.

Given two sides of a plane triangle, and the angle contained between them, to find the other two angles and the third side.

Example.—In the plane triangle ABC (Fig. 5.), there are given, the side AC = 465, the side AB = 434, and the contained angle A = 53° 8′. It is required to find the angles B and C, and the third side BC.

Solution.

AC =	465		180° 0′
AB =	434	Angle A =	53 8
Sum sides =	899	Sum	2)126 52
Diff. sides =	31	½ sum angles =	63 26

Half sum angles B and C = 63° 26′ 0″
Half diff. angles B and C = 3 56 41.7

Greater angle B = 67 22 41.7

Less angle C = 59 29 18.3

1. To find half the difference of the angles B and C.

As the sum sides	AC and AB 899	2.9537597
Is to the difference of AC and AB	31	1.4913617
So is tangent ½ sum	B and C 63.26	10.3009994
To tangent ½ the diff. B and C	3° 56′ 41.7″	8.8386014

Then half the difference of the angles added to half their sum gives the greater angle B, but subtracted leaves the less C.

2. To find the third side BC, the angles being first found.

As the sine of the angle C	59° 29′ 18.3″	9.9352687
Is to the side AB	434.0	2.6374897
So is the sine of the angle A	53° 8′	9.9031084
To the side BC	403.0226	2.6053294

Or, as sine B : AC : : sine A : BC.

Note.—See the other method of Solution to this Case, in the next Scholium.

SCHOLIUM.

IN astronomical calculations, we have generally the *logarithms* of two sides, and their contained angle given; to find the other two angles, and the third side; which is done by this rule.

Subtract the logarithm of the less side, from that of the greater, increasing its index by 10; look out the remainder as the logarithmic tangent of an angle: take 45° from that angle; then add the logarithmic tangent of the remainder to the logarithmic tangent of half the sum of the two required angles, this sum (abating 10 from the index) will be the logarithmic tangent of half the difference of the two remaining angles of the triangle; lastly, add the half difference to the half sum, and you have the greater angle; subtract it, and you have the less.

We shall here solve the example to case 3d, by this rule, supposing that instead of the real values of AC and AB being given, we have only the logarithms of those sides given, viz. log. AC = 2.6674530, log. AB = 2.6374897, also the contained angle A = 53° 8′, as before.

		180°	0′ 0″
Angle A =		53	8 0
	2)	126	52 0
Half sum of angles B and C =		63	26 0
Half diff. of the angles B and C =		3	56 41.7
Greater angle B =		67	22 41.7
Less angle C =		59	29 18.3

Log. greater side AC + 10		12.6674530
Log. less side AB		2.6374897
Log. tan.	46° 58′ 29.8″	10.0299633
Subtract	45 0 0	
Log. tan.	1 58 29.8	8.5376043
Log. tan.	63 26 0	10.3009994
Log. tan. the half diff. of the angles B & C	3 56 41.7	8.8386037

Then the third side BC is found as before.

This rule will be again exemplified in our calculations of the geocentric places of the planets.

CASE 4.

GIVEN the three sides of a plane triangle to find the three angles.

Example.—In the plane triangle ABC, (Fig. 7.) there are given, the side AB = 434, the side AC = 465, and the side BC = 403; it is required to find the angles A, B, and C.

Solution.—Let fall the perpendicular CD, which divides the base AB into two segments, *viz.* AD the greater, and DB the less.

Side AC = 465	1. To find the difference of the segments of the base.			
Side BC = 403				
—	As the base AB		434.0	2.6374897
Sum sides = 868	Is to the sum of the sides AC + BC		868.0	2.9385197
Diff. sides = 62	So is the diff. of the sides AC — BC		62.0	1.7923917
Base AB = 434				
—				
½ the base = 217	To the diff. of the segment of the base		124.0	2.0934217
½ diff. seg. 62				
—				
sum = AD = 279				
—				
diff. = DB = 155				

Note.—The fourth proportional in this analogy may be found more easily by the common rule of proportion without the application of logarithms. Thus As 434 : 868 : : 62 : 124.0

Then in the two right angled triangles ADC and BDC, there are given the hypothenuses AC and BC, and the legs AD and DB, to find the angles A and B, whence,

1. To find the angle A, by Case 3 of right angled Trigonometry.

As the leg	AD	279.0	2.4456202
Is to radius			10.0000000
So is the hypothenuse AC		465.0	2.6674530
To the secant of the angle A		53° 7′ 48.4″	10.2218488

Or,

As AC : radius : : AD : sine angle ACD, or cosine of A.

2. To find the angle B;

As BD : radius : : BC : secant B.

Or,

As BC : radius : : BD , cosine of B.

Then the sum of the angles A and B taken from 180° leaves the angle C.

In the following Scholium, we have given another method of finding the angles, without the assistance of a perpendicular.

SCHOLIUM.

THIS case can be solved without letting fall a perpendicular by either of the following rules.

1. To the* arithmetical complements of the logarithms of the two sides containing the given angle, add the logarithms of the excesses of the semi-perimeter of the triangle above these sides; half the sum of these four logarithms is the log. sine of half the angle sought: or, 2. To the arithmetical complements of the logarithms of the two sides containing the given angle, add the logarithm of the semi-perimeter, and the logarithm of the excess of the semi-perimeter above the side opposite the angle sought; then half the sum of these four logarithms will be the log. cosine of half the required angle. *Note:* If the angle sought should be very obtuse, so that its half approaches nearly to 90°, the second of these rules is the most proper to use, because the differences of the cosines of angles nearly equal to a quadrant are much larger than the differences of their sines, and therefore greater accuracy is ensured by finding the angle from a cosine: if, on the other hand, the angle sought be very acute, then the 1st rule should be used, because the sines of very small angles have greater differences than their cosines: between these extremes either rule may be used indifferently.

Example.—Suppose the three sides to be the same as in the 4th case aforegoing, and the angle A sought.

RULE 1st.

AC =	465	arith. comp. log.	465	7.3325470
AB =	434	arith. comp. log.	434	7.3625103
BC =	403	log.	186	2.2695129
		log.	217	2.3364597
	2)1302			2)19.3010299
semi-perim.	651	651		
sub. AC,	465	AB, 434		
remainder	186	217	log. sin. ½ angle A = 26° 33′ 54.2″	9.6505149
			2	
		angle A,	53 7 48.4	

RULE 2d.

AC =	465	arith. comp. log.	465	7.3325470
AB =	434	arith. comp. log.	434	7.3625103
BC =	403	log.	651	2.8135810
		log.	248	2.3944517
	2)1302			2)19.9030900
	651			
sub. BC =	403			9.9515450
remainder =	248	log. cos. ½ angle A,	26° 33′ 54″2	
			2	
		angle A,	53 7 48.4	

* The arithmetical complement of a logarithm is the number which will compleat it to 10.0000000, and is performed practically, by taking every figure from 9, except the last significant figure in the right hand, which must be taken from 10.

A few Problems shewing the use of Plane Trigonometry in Astronomy.

1. GIVEN the semi-diameter of the Earth = 3985 miles, and the Moon's horizontal parallax = 57′ 0″: the distance of the Moon's centre from that of the Earth is required.

SOLUTION.

LET T be the centre of the Earth (Fig. 8.), and L that of the Moon; AT the semidiameter of the Earth = 3985 miles; when the Moon is in the sensible horizon the angle LAT is a right angle; also the angle L is the Moon's horizontal parallax, which we have now supposed to be 57′ 0″; and hence, in the right angled triangle LTA, there are given the leg AT and the angle L to find the hypothenuse TL the distance of the centres: which by Case 2. of right angled trigonometry is,

As the sine of the angle	L	0 ° 57 ′ 0 ″	8.2195811
Is to	AT	3985.0	3.6004283
So is radius or sine of		90 ° 0 ′	10.0000000
To the distance	TL	240351.7	5.3808472

This is the mean distance of the Earth's centre from that of the Moon, *viz.* 240352 miles; but the Moon's orbit being an ellipsis, she is therefore considerably nearer the Earth at one time, and more distant at another than the *mean*, and consequently her horizontal parallax is a variable quantity, changing from 53′ to 62′ nearly; in order therefore to solve the problem with accuracy for a given time, we must first calculate the Moon's horizontal parallax to the tenth part of a second at the time proposed, (and which we shall shew how to do hereafter), and then proceed as above, and the true distance will be obtained to the same degree of accuracy with the *data*.

2. In the same manner do we find the distance of the Sun's centre from that of the Earth: but the Sun's horizontal parallax is an exceedingly small angle, being at a mean no more than 8″.65; which was determined by the last transit of Venus in 1769: hence for the Sun's distance.

As the sine of	0 ° 0 ′ 8.65 ″		5.6219140
Is to	3985.0		3.6004283
So is radius			10.0000000
To the distance	95173128		7.9785143

And hence it appears, that the mean distance of the centre of the Sun from that of our Earth is 95173128 English miles; and if 1600241 miles the excentricity, be added and subtracted; we have

96773369 for the aphelion distance, and 93572887 the perihelion distance.

Note.—Having obtained the mean distance of the Earth from the Sun; then the mean distances of the other planets from him, are found by this analogy, *viz.* As the square of the time of the Earth's periodical revolution round the Sun is to the square of the time of that of another planet; so is the cube of the Earth's mean distance, to the cube of that of the other planet: the cube root of which is the mean distance of that planet from the sun.

3. Given the distance of Venus from the Sun = 68891486 miles, and that of the Earth 95173128, and the angle of elongation of Venus from the Sun = 30° 0′; it is required at that time to find the absolute distance between the centres of Venus and the Earth; Venus being east of the Sun, and having passed her greatest elongation.

Solution.

Let S (Fig. 9.) be the Sun's centre, T that of the Earth, and V that of Venus; TA a portion of the Earth's orbit, BVC a portion of the orbit of Venus, which by the problem lies east of the Sun: then it is evident the angle of elongation T is the same for two points C and V; *viz.* for C, before the greatest elongation which is at I, and for V afterwards; and consequently, by the problem, V is the place of the planet: from S let SD be drawn at right angles to VC, and the point D bisects VC. Then in the right angled triangle STD there are given the hypothenuse TS = 95173128, and the angle T = 30° 0′, to find the legs TD and SD, which by Case 1. of Right Angled Trigonometry is,

1. To find SD.

As radius - - - -	10.0000000
Is to TS, 95173128 - -	7.9785144
So is sine angle T, 30° 0′ -	9.6989700
To SD, 47586565 - - -	7.6774844

2. To find TD.

As radius - - - -	10.0000000
Is to TS, 95173128 - -	7.9785144
So is sine angle TSD, 60° 0′ -	9.9375306
To TD, 82422356 - -	7.9160450

Then in the right angled triangle SDV; there are given the hypothenuse SV = 68891486, and leg SD = 47586565, to find the leg VD; which is best performed by problem B, thus:

SV = 68891486		
SD = 47586565		
Sum = 116478051	log.	8.0662441
Diff. = 21304921	log.	7.3284799
VD = 49815211	log. 2)	15.3947240
TD = 82422356	log.	7.6973620
VD = 49815211		

TD—VD=TV= 32607145 = distance after passing the greatest elongation.

TD+VD=TC=132237567 = distance before the greatest elongation.

We have thought it the better way to solve this problem by letting fall a perpendicular, whereas the student may see, that although this is an application of the 2d. or ambiguous case of oblique trigonometry, yet the ambiguity is removed by the conditions of the question. For it is evident that there are two points in the orbit on the east side, and two points on the west side, which have the same angle of elongation; and that the planet when east of the Sun, has the greater distance from the Earth before it arrives at the point when it has its greatest elongation; and the less distance afterwards; and *vice versâ*, when it is west of the Sun. Again, as VS : radius : : VD : sine of the angle VSD = 46° 18′ 38.7″: then the angle TSD—VSD = the angle TSV = 13° 41′ 21.2″.

In the same manner, the absolute distance of any superior planet from the Earth at a given time is calculated; but the planet's distance from the Sun will be the greater side of the triangle in this case; and the Earth's distance the less; the other parts of the calculation are similar; only that the angle of elongation may be right, acute, or obtuse; a circumstance that can never happen in respect to an inferior planet.

4. Given the distance of Venus from the Sun = 68891486, and that of the Earth = 95173128, and the contained angle at the Sun between these Radii Vectores (called the Anomaly of Commutation) 13° 41′ 35″; to find the absolute distance of Venus from the Earth; and the angle of elongation of Venus from the Sun.

Solution.

In the triangle TVS (Fig. 9.), there are given the side SV = 68891486, the side ST = 95173128, and the contained angle TSV = 13° 41′ 21.2″, to find the angles T and TVS, and the third side TV: which by Case 3 is thus, by method 2d. See the Scholium to Case 3.

		180° 0′ 00″
Angle TSV =		13 41 21.2
		2)166 18 38.8
Half sum angles =		83 9 19.4

Greater side ST		95173128	log.	17.9785144
Less side SV		68891486	log.	7.8381655
Arc	54° 6′ 3.4″	- Tangent		10.1403489
Minus	45 0 0			
Rem.	9 6 3.4	- Tangent		9.2046380
Half sum	83 9 19.4	- Tangent		10.9207010
Half diff.	53 9 19.6	- Tangent		10.1253390

Angle T = 29 59 59.8

Then, as sine angle T : VS : : sine angle TSV : VT = 32607137, the distance of Venus from the Earth.

5. If the distance of the Earth from the Sun be 95000000 miles, and the distance of the Moon from the Earth 240000 miles, and that the angle contained between those distances at the Earth's centre is 89° 6′; what is the distance of the Moon from the Sun at that moment?

SOLUTION.

IN the oblique angled plane triangle MES (Fig. 18) there are given EM the distance of the Moon from the Earth; ES the distance between the Sun and Earth and the angle MES to find MS the distance of the Sun and Moon.

95000000 = ES	From - -	180° 0′
240000 EM	Take - -	89 6
95240000 = Sum		2)90 54
94760000 = Difference	Half sum ∠s. -	45 27

As 9524	Co. Ar. -	6.0211806
Is to 9476 - -	-	3.9766251
So is Tang. 45° 27′	-	10.0068222
To Tang. 45° 18′ 19″		10.0046279
45 27 0		
90 45 19 = ∠SME		
89 14 41 = its supplement		
As sine 89° 14′ 41″	Co. Ar.	0.0000377
Is to 95 millions -	- -	7.9777236
So is sine 89° 6′	- - -	9.9999464
To MS = 94996533	- -	7.9777077

The less angle, or that at the Sun is 8′ 41″, but this angle being small it cannot be used in this common rule, as 7 places in the logarithms will not bring out a true answer.

6. I took the altitude of a cloud = 47° 10′ when it bore SE from me; and when its bearing was SE½E, the altitude was 41° 17′: what was the direction of the wind?

SOLUTION.

It is shewn in the second edition of Whiting's Select Exercises, page 29, that with the ratio of two sides of a plane triangle and other data, the angles may be found. Now, as the cloud moves on in a direction parallel to the horizon, its height may be taken as the radius at the times of the two observations, and the natural cotangents of the two altitudes and the difference of the azimuths will form a triangle similar to that on the horizontal plane.

Let WE represent the west and east points of the horizon, OA the south-east, OB the SE½E point. If A be the point over which the cloud was passing when the first altitude was taken, and B that over which it was when the second altitude was taken, then OA and OB are in the ratio of the natural cotangents; O being the place of observation.

Cot. 41° 17′	1.1389441	From -	180° 0′ 0″
Cot. 47 10	.9270914	Take -	5 37 30
Sum - -	2.0660355		2)174 22 30
Difference	.2118527	Half sum ∠s	87 11 15

As 20660355	Co. Ar.	2.6848625
Is to 2118527		6.3250285
So is Tang 87° 11′ 15″		11.3084325
To Tang. 64 20 11		10.3183235
22 51 4 = ∠OBA = DOB		

Then ∠ BOA = 5° 37′ 30″ + DOB = 22° 51′ 4″ = 28° 28′ 34″ = the ∠ AOD. Then 45° = AOE − 28° 28′ 34″ = DOE = COW, the angle the direction of the wind makes with the W. point towards the N. = 16° 31′ 26″, or WNW¼W, 1° 22′ 26″ W.

7. The diameters of the celestial bodies are found by comparing their horizontal parallaxes with the angle under which their semidiameter appears when seen from the Earth's centre. And as the horizontal parallax of any celestial object is the angle under which the semidiameter of the Earth is seen, by an observer at the centre of that object at the time proposed; hence, the ratio of the semidiameter of the Earth to that of the celestial body is as the sine of its horizontal parallax to the sine of the angle under which its semidiameter is observed: but the sines of small arcs or angles, have nearly the same ratio that the angles themselves have; consequently, as

the horizontal parallax of a celestial body is to the semidiameter of the Earth, so is the angle under which its semidiameter is seen from the Earth's centre, to its semidiameter nearly. But care must be taken that the horizontal parallax be exactly fitted to the time of observation; and the horizontal parallax of a celestial body, and the angle under which its semidiameter appears at any time, have always the same ratio to each other, nearly in respect to the Sun and Moon; there is indeed some difference, which therefore must be noticed; in planets it is nothing.

Example.—The Moon's semidiameter, corrected for the augmentation, was observed to subtend an angle of 15′ 4″, when her horizontal parallax was calculated to be 55′ 17.4″; what is her absolute diameter; the diameter of the Earth being 7970 English miles.

As the sine of	55′ 17.4″	log.	8.2063540
Is to -	7970	-	3.9014583
So is the sine of	15′ 4″	log.	7.6417419
To - -	2171.9	-	3.3368462

Hence the Moon's diameter as determined by this operation is 2172 English miles; and by taking a mean of the results of many observations; her correct diameter appears to be 2170½ English miles. And in the same manner do we find the diameter of the Sun, Moon, and the Planets.

We might have added several other astronomical problems to exercise the student in plane trigonometry; but we have deemed it unnecessary in this place, as we should thereby only retard his advance to the study of Spherical Trigonometry, which follows next in order; besides he will find in our future calculations abundance of problems involving all the several cases of right and oblique angled plane trigonometry.

Demonstration *of Prop.* A.

Let ABC (Fig. 10.) be a right angled triangle, having the right angle at B; and let DEF and PKI (Fig. 11 and 12.), be two other triangles equiangular and similar to it, having the angles F and I equal to C; and D and P equal to A; also let the hypothenuse DF of the one, and the leg IK, (answering to BC) of the other, be each equal to the natural radius of the tables, then it is evident that FE and DE are the sines in the tables of the arcs FH and DG, or of the angles D and F, that is, of A and C; it is also manifest that KP is the tangent of the arc KO, or of the angle I, that is, of C; and that IP is the secant of the angle C: but the three triangles are similar, by the hypothesis; whence, as DF : AC : : FE : CB, and as DF : AC : : DE : AB; and by inversion, as AC : DF : : CB : FE, and as AC : DF : : AB : DE, also as, IK : BC : : KP : AB, and as IK : BC : : IP : CA; and inversely, as BC : IK : : AB : KP, and as BC : IK : : CA : IP, and these analogies expressed in words, form the 1st part of the theorem.

Next, let ABC (Fig. 13.), be any oblique angled triangle, and from C, let CD be drawn at right angles to AB; then, as radius is to AC, so is the sine of the angle A to the perpendicular CD, by the 1st part; whence the rectangle of the radius, and perpendicular is equal to the rectangle of AC and sine of the angle A; for the same reason, the rectangle of the radius and perpendicular is equal to the rectangle of BC and sine of the angle B; consequently the rectangle of AC, and sine of the angle A, is equal to the rectangle of BC, and sine of the angle B; wherefore, these four magnitudes are proportionals; that is, as the sine of the angle B, is to AC, so is the sine of the angle A, to BC; which is the second part of the theorem.

Demonstration *of Prop.* B.

Let ABC (Fig. 14.) be a plane triangle, and let AB be regarded as the base, and AC, BC, the two sides; produce AC, the greater side, to G; and about C as a center, with the distance CB, describe the semicircle FLBG; join FB and BG, and through F draw FP parallel to AB, and let fall the perpendicular CD upon AB. Then, it is evident, that AG is the sum of the sides AC and CB, and that AF is their difference. Also, the angle BCG is equal to the sum of the two angles A and ABC (by Cor. Prop. 12. Geom.) but the angle BFG is equal to half the angle BCG (by Prop. 17. Geom.) consequently the angle BFC is equal to half the sum of A and ABC; also, because FP is parallel to AB the angle PFG is equal to A; hence the angle BFP, which is the excess of the half sum above the less, is half the difference of A and ABC. Upon FB take F*b* equal to the radius in the tables, and produce FP, FG, to meet *bg* drawn parallel to BG in *p* and *g*. Now the angle FBG is a right angle, because it is an angle in a semicircle, consequently F*bg* is a right angle, and therefore *bg* and *bp* are the tangents of the angles *b*F*g* and *b*F*p* to the radius of the tables, that is, they are the tangents of the half sum, and half difference of the angles A, and ABC; and since FP is parallel to AB, therefore as AG : AF : : BG : BP; but because of the parallels BG, *bg*, we have BG : BP : : *bg* : *bp*; therefore, by equality, as AG : AF : : *bg* : *bp*, that is, as the sum of AC and BC, is to the difference of AC and BC; so is the tangent of half the sum of the angles A and ABC, to the tangent of half the difference of A and ABC. Which is the 1st part of the proposition to be demonstrated.

Again, because the perpendicular CD bisects the chord BL in D, therefore AL is the difference be-

tween the segments AD and DB; and since the point A is without the circle FLBG, therefore the rectangle of AG and AF is equal to the rectangle of AB and AL (by Prop. 28. Geom.), and consequently these four magnitudes are proportionals; that is, as AB : AG : : AF : AL, or as the base, is to the sum of the sides; so is the difference of the sides, to the difference of the segments of the base made by a perpendicular let fall from its opposite angle. Which is the 2d part of the proposition to be demonstrated.

DEMONSTRATION *of the Rule in the Scholium to Case 3d.*

LET ABC (Fig. 15.) be any plane triangle, and *abc* (Fig. 16.) a right angled triangle, such that the sides *ab* and *bc*, containing the right angle, are equal respectively to the sides AB, and BC, of the triangle ABC. Then, by Prop. B, AB + BC : AB — BC : : tan. $\frac{1}{2}$ (C + A) : tan $\frac{1}{2}$ (C — A); and *ab* + *bc* : *ab* — *bc* : : tan. $\frac{1}{2}$ (*c* + *a*) : tan. $\frac{1}{2}$ (*c* — *a*); but (from the hypothesis) AB + BC is equal to *ab* + *bc*, and AB — BC is equal to *ab* — *bc*; therefore, by equality, tan. $\frac{1}{2}$(*c*+*a*) : tan. $\frac{1}{2}$(*c*—*a*) : : tan. $\frac{1}{2}$(C+A) : $\frac{1}{2}$(C — A). Now $\frac{1}{2}$(*c* + *a*) is equal to 45° and the tangent of 45° is equal to the radius; also *c* — *a* = 2*c* — (*c* + *a*) and $\frac{1}{2}$ (*c* — *a*) = *c* — $\frac{1}{2}$ (*c* + *a*) = *c* — 45°; consequently radius : tan. (*c* — 45°) : : tan. $\frac{1}{2}$ (C + A) : tan. $\frac{1}{2}$ (C — A). Farther, in the right angled triangle *abc* (by Prop. A), *bc* : *ab* : : radius : tangent of the angle *c*, that is, BC : AB : : radius : tan. of *c*; from whence the log. tan. of *c* will be known by adding the log. radius, *viz.* 10, to the log. of the greater side AB, and subtracting the log. of the less BC; which answers to the first part of the rule: and *c* being thus found, *c* — 45° is known, and the proportion, radius : tan. (*c* — 45°) : tan. $\frac{1}{2}$ (C + A) : tan. $\frac{1}{2}$ (C—A) will be found to correspond with the latter part of the rule.

DEMONSTRATION *of the 1st Rule in the Scholium to Case 4th.*

LET ABC (Fig. 17.) be any plane triangle; then the rectangle contained by any two of its sides AC, CB, is to the rectangle contained by the excesses of the semi-perimeter of the triangle above those sides; as the square of the radius is to the square of the sine of half the angle contained by those sides. For produce the shorter side CB so that CD may be equal to CA, join AD, and bisect the angle ACB by the line CF, and draw BG and BE respectively parallel to CF and AD. Then it is obvious that CF is perpendicular to, and bisects, AD; therefore by prop. CD or CA : FD : : rad. : sin $\frac{1}{2}$ C, and CB : EB or FG : : rad. : sin $\frac{1}{2}$C, wherefore by compounding these analogies AC × CB : FD × FG : : rad^2 : sin^2 $\frac{1}{2}$ C. Now by Prop. 16. Geom. AB^2 = AG^2 + GB^2, and BD^2 = DG^2 + GB^2, hence AB^2 — BD^2 = AG^2 — DG^2, and therefore by Prop. A. Conic Sections, (AB + BD) . (AB — BD) = (AG + DG) . (AG — DG) = 2FD × 2FG, consequently FD × FG = $\frac{1}{2}$(AB + BD) . $\frac{1}{2}$(AB — BD); but $\frac{1}{2}$(AB + BD) = $\frac{1}{2}$(AB + CD — CB) = $\frac{1}{2}$(AB + AC — CB) = $\frac{1}{2}$(AB + AC + CB) — CB; = semi-perim. — CB; and $\frac{1}{2}$(AB — BD) = $\frac{1}{2}$[AB — (CD — CB)] = $\frac{1}{2}$[AB — (AC — CB)] = $\frac{1}{2}$(AB — AC + CB) = $\frac{1}{2}$(AB + AC + CB) — AC = semi-perim. — AC: therefore AC × CB : (semi-perim. —AC) . (semi-perim. — CB) : : rad.^{2} : sin.^{2} $\frac{1}{2}$C which was to be demonstrated. In the practical rule we have directed the operator to add the arithmetical complements of the two sides containing the angle sought, to the logarithms of the excesses of the semi-perimeter above these sides, and to take half the sum of the four logarithms for the sine of half the angle required. Now the sum of the logarithms of AC and CB is in fact a quantity to be *subtracted*, because the product of these two sides forms the 1st term of the proportion, and therefore, when instead of this, we *add* the arithmetical complements of these sides, we get 20.0000000 too much, but by omitting the logarithm of the 3d term of the proportion, *viz.* rad.^{2} which is also 20.0000000, the excess of 20.0000000 by using the arithmetical complements is exactly compensated, and therefore the half sum of the four logarithms gives the log. sine of $\frac{1}{2}$ C, as stated in the rule.

DEMONSTRATION *of the 2d Rule in the Scholium to Case 4th.*

LET ABC (Fig. 17.) be any plane triangle, then the rectangle contained by any two of its sides AC, CB, is to the rectangle contained by the semi-perimeter, and the excess of the semi-perimeter above the third side AB: as the square of the radius, to the square of the cosine of half the angle contained by those two sides. From the demonstration of the last rule, AC × CB : ED × FG : : rad.^{2} : sin.^{2} $\frac{1}{2}$ C, that is, from what is there shewn AC × CB : $\frac{1}{2}$[AB + (AC — CB)] . $\frac{1}{2}$[AB — (AC — CB)] : : rad.^{2} : sin.^{2} $\frac{1}{2}$ C, hence by prop. A. of the Conic Sections AC × CB : $\frac{1}{4}$(AB^2 — AC^2 + 2AC × CB — CB^2) : : rad^2 : sin.^{2} $\frac{1}{2}$ C and by division of proportion AC × CB : $\frac{1}{4}$ (AC^2 + 2AC × CB + CB^2 — AB^2) : : rad.^{2} : rad.^{2} — sin.^{2} $\frac{1}{2}$ C. But rad.^{2} — sin.^{2} $\frac{1}{2}$ C = cos.^{2} $\frac{1}{2}$ C, and $\frac{1}{4}$ (AC^2 + 2AC × CB + CB^2 — AB^2) = $\frac{1}{4}$[(AC + CB)^2 — AB^2] = (Prop. A. conic sections), $\frac{1}{4}$ [AC + CB + AB). (AC + CB — AB)] = $\frac{1}{2}$ (AC + CB + AB). $\frac{1}{2}$ (AC + CB —AB) = (semi-perim). (semi-perim — AB). Therefore AC × CB : (semi-perim). (semi-perim. — AB) : : rad.^{2} : cos.^{2} $\frac{1}{2}$ C, which was to be demonstrated, and this proportion will be found equivalent to the second of the practical rules in the aforesaid Scholium.

SPHERICAL TRIGONOMETRY.

Definitions.

1. As plane trigonometry is that science which teaches the method of ascertaining the magnitude of the inaccessible sides and angles of plane triangles, by comparing them with a sufficient number of sides and angles that are accessible and have actually been measured, or otherwise determined; so in like manner, *Spherical Trigonometry* teaches the method of ascertaining the magnitude of the sides and angles of spherical triangles, which are inaccessible (or supposed to be so), by comparing them with a sufficient number of sides and angles whose magnitudes have been actually ascertained by astronomical observations, or otherwise determined and given.

Note.---The great circles of the sphere have been already defined in our theory of the Earth, which see.

2. A *Spherical Angle* is the angle formed by the ultimate directions of two arcs of great circles at their intersection, or it is the angle contained by the two tangents to those great circles and which cut each other in the angular point, each tangent being in the plane of that great circle to which it is a tangent; a spherical angle is hence equal to the angle of inclination of the planes of the two great circles which constitute it; and the measure of a spherical angle is that arc of a great circle which is contained between the two great circles, and one of whose poles is the angular point. Thus the angle that two meridians make at the poles is a spherical angle, and it is measured by that arc of the equator intercepted between these meridians.

3. A spherical triangle is a figure on the surface of the sphere, made by the arcs of three great circles.

4. A right angled spherical triangle, is that which has one of its angles, a right angle, or 90°. An oblique angled spherical triangle is that which has all its angles oblique, or which has no angle 90°. A quadrantal spherical triangle, is that which has one side 90°.

Axioms.

1. The greater side of any spherical triangle is opposite to the greater angle; and *vice versâ*.

2. Equal sides are opposite to equal angles; and *vice versâ*. Also, in an isosceles spherical triangle, if a great circle bisect the angle contained by the equal sides, it bisects the base and cuts it at right angles; and if it bisect the base, it cuts it at right angles and bisects the angle contained by the equal sides. The reason is evident, from plane geometry.

3. Any side of a spherical triangle is less than a semicircle, or 180°.

4. The sum of the three angles of any spherical triangle is greater than two right angles, or 180°; but less than six right angles, or 540°.

Scholium.

Most of the writers on spherical trigonometry have dwelt much at length upon the ambiguities which occur in solving some of the cases, but however desirable it may be that the nature and limits of these ambiguities should be clearly ascertained in works that treat in the abstract of spherical trigonometry as a science, it will not be necessary to enlarge upon them here, since our intention is to dwell upon spherical triangles only as connected with astronomy, and in doing this we shall not find ourselves embarressed by any such ambiguities: the nature and conditions of the problem will always remove every doubt as to the quæsita. For instance, if in calculating the Sun's declination, and right ascension, the data be the longitude of the Sun and the obliquity of the ecliptic; since we know the quarter of the ecliptic that the Sun is then in, we know whether the declination is north or south, and whether the arc of right ascension obtained by the calculation is to be reckoned directly, from the first point of Aries, eastward, or in the contrary order, westward; and in like manner, whether it is to be reckoned from the first point of Libra, eastward, or westward.

Introductory Rules and Directions.

If the radius, which is always given, be one of four proportionals; and if the order of the proportionals be known, then if any two of the other three terms be given, the other third term can be found by calculation.

Thus, let the radius, and the quantities A, B, and C, be the four terms of a proportion, and this their direct order in the analogy, where A, B, and C, may represent either the sines, or the tangents, &c. of arcs, and suppose,

1. A and B given; to find C.

Say, As rad. : A :: B : C, directly; whence $C = \frac{A \times B}{\text{Rad.}}$

2. A and C given; to find B.

Say, As A : rad. :: C : B, inversely; $B = \frac{\text{Rad.} \times C}{A}$

3. B and C given; to find A.

Say, As B : radius : : C : A alternately and inversely, $A = \frac{\text{Rad.} \times C}{B}$

4. If the first, of four proportionals, be the *cotangent* of an arc or of an angle, and the second term be the radius; make the radius the first term, and the *tangent* of that arc or angle the second. Or, if the first, of four proportionals, be the *tangent* of an arc or of an angle, and the second term be the radius; make the radius the first term, and the cotangent of that

arc or angle the second. This change, in the analogy, renders the operation more easy, because it makes the radius the first term. And the reason is, because the radius, is to the tangent of an arc or of an angle, as the cotangent of that arc or angle, is to the radius; and *vice versâ*. See lemma 2. of plane trigonometry.

5. If the first, of four proportionals, be the *cosine* of an arc or of an angle, and the second term be the radius; make the radius the first term, and the *secant* of that arc or angle the second. Or, if the first, of four proportionals, be the sine of an arc or of an angle, and the second term be the radius; make radius the first term, and the *cosecant* of that arc or angle the second. This change in the analogy, like the other, renders the operation easier, because it makes the radius the first term.

* Propsition A. *Theorem.*

In any right angled spherical triangle, the relations of the sides and angles expressed in the six following analogies, always obtain:

1. As radius is to the sine of the hypothenuse, so is the sine of any one of the oblique angles, to the sine of its opposite leg.

2. As radius is to the cosine of either of the legs, so is the sine of the oblique angle adjacent to that leg, to the cosine of the angle opposite to it.

3. As radius is to the cosine of either of the legs, so is the cosine of the other leg, to the cosine of the hypothenuse.

4. As radius is to the tangent of either of the oblique angles, so is the sine of the leg adjacent to that angle, to the tangent of the leg opposite to it.

5. As radius is to the tangent of either of the oblique angles, so is the cosine of the hypothenuse, to the cotangent of the other oblique angle.

6. As radius is to the tangent of the hypothenuse, so is the cosine of either of the oblique angles to the tangent of its adjacent leg.

And by the assistance of these six analogies all the cases of right angled spherical trigonometry are solved; for there are always two terms of some one of these proportions given, besides the radius, and from thence, the remaining, or unknown term, may be found by one of the first three introductory rules.

Lord Napier, the illustrious inventor of logarithms, discovered a very neat and concise rule which unites within itself all the particular analogies above given for the solution of right angled spherical triangles, and forms perhaps the most remarkable "compendium of results" to be met with in mathematical science. This rule has obtained the name of *Napier's Catholic*, or *Universal Proposition;* but, before introducing it to the notice of the trignometrical student, it will be necessary to make some observations upon what Napier has called the "*five circular parts.*" These circular parts are the two legs of a right angled spherical triangle, the complements of the two oblique angles, and the complement of the hypothenuse: the right angle being of a constant magnitude, is not taken into consideration, and does not from a *part.*

Thus, in the spherical triangle ABC (Fig. 1.) right angled at B; the circular parts are, the leg AB, the leg BC, the complement of the angle C, the complement of the hypothenuse AC, and the complement of the angle A; and in every case of right angled spherical trigonometry proposed for solution, there are always *three* of these five circular parts involved, *viz.* two that are given, and one which is sought; and of these three parts, that which lies between the other two is called the middle part;† whether it be one of the two *given* parts, or the part *sought.* So in the triangle ABC, if we suppose the complement of the angle A, and the two legs AB, BC, to be the three parts concerned, some two of which must be given, and the third sought; then the leg AB, because it is situated in the middle between the complement of A and the leg BC, is the middle part, whether it be given or sought; and the complement of the angle A, and the leg BC, are called adjacent parts, because they are not separated from the middle part by the intervention of any other part, for, as before observed, the right angle at B is not considered a part. Again, if we suppose the leg AB, the complement of the hypotenuse AC, and the complement of the angle C, to be the three parts concerned; then the leg AB is the middle part, whether it be given or required; and the other two parts, namely, the complement of the hypothenuse AC, and the complement of the angle C, which are separated from the middle part by the intervention of a part on each side, are called opposite parts. These things premised we proceed to

Napier's Catholic Rule.

The rectangle under the radius and the sine of the middle part is equal to the rectangle under the tangents of the adjacent parts, and also to the rectangle under the cosines of the opposite parts.

The manner of applying this rule so as to find the proper analogy for the solution of any case of right

* The demonstration of this proposition, and of propositions B, C, D, &c. are given at the end of the spherical trigonometry.

† Some writers on trigonometry have erred, and bewildered learners of the science, by saying, that *any* one of the circular parts may be *assumed* the middle part; but in this they are entirely mistaken; for the *middle part cannot be assumed;* because of the three parts concerned, one of them is determinately, and of neccessity, the middle part, and that one may sometimes be the part sought, and sometimes one of the two given parts.

angled spherical triangles will be easily understood from one or two instances.

Let us suppose that in the right angled spherical triangle ABC (Fig. 1.) there are given the leg AB, and its adjacent angle A, to find the other oblique angle C: here, the complement of the angle C is the middle part, and the leg AB, and the complement of the angle A, are opposite parts: therefore, by our rule, radius × cos. C = cos. AB × sin. A*, and by resolving this equation into an analogy according to art. 8. of the proportion (page 93.) we have radius : cos. AB : : sin. A : cos. C, which is equivalent to the 2d analogy of proposition A; and hence cos. C may be found by the 1st introductory rule.

Again, in the same triangle, let it be supposed that the leg AB, and its adjacent angle A, are given, to find the hypothenuse AC; then the complement of the angle A is the middle part, and the leg AB, and the complement of the hypothenuse AC, are adjacent parts; wherefore, radius × cos. A = tang. AB × cotang. AC; now by converting this into an analogy, we obtain cotang. AC : radius : : cos. A : tang. AB, and by changing the 1st term into radius by the 4th introductory rule, radius : tang. AC : : cos. A : tang. AB, which agrees with the 6th analogy of proposition A, from whence tang. AC may be found by the 3d introductory rule.

If we apply Napier's Catholic Rule to the solution in succession of the six cases of right angled spherical trigonometry that here follow, changing, where necessary, the 1st term of the resulting proportion from a tangent or a cotangent into radius by means of the 4th introductory rule, we shall arrive at some one of the six analogies of proposition A; and as this proposition is demonstrated hereafter, the truth of the Catholic proposition is at the same time established.

In the arithmetical solutions of the cases, we shall take the proportions from prop. A, leaving, in most of the cases, the method of deducing them from the rule of the circular parts as a suitable exercise for the student, we have also to observe that wherever the 1st term of the proportion as obtained from prop. A would be a cotangent or tangent, we have changed it into the radius, but when the first term is a sine or cosine, we have not used the 5th rule, because it brings *secants* into the analogy; and although the application of rule 5th renders the operation somewhat easier; yet as Taylor's and many other tables have no secants, we have only given the analogies, which find the required parts by sines and tangents, without the use of secants. But the operator may, if he chuse, use that rule, provided he calculate by Hutton's Tables, or any other set of tables that have secants; or he may find the log. secant of an arc by subtracting its log. cosine from 20.

Solution of the several Cases of right angled Spherical Trigonometry.

CASE 1.

GIVEN the hypothenuse and one of the oblique angles, to find the other oblique angle and the two legs.

Example.—In the spherical triangle ABC (Fig. 1.), right angled at B; there are given, the hypothenuse AC = 70° 30′, and the angle A = 51° 31′; it is required to find the angle C, and the legs BC and BA.

SOLUTION.

1. To find the angle C, we have by the 5th, analogy of prop. A,

As radius.....	10.0000000
Is to the tang. of the angle A 51° 31′....	10.0996541
So is cos. of the hypoth. AC 70° 30′....	9.5234953
To the cotan. of the angle C 67° 13′ 20.2″	9.6231494

2. To find the leg CB opposite to the given angle A. By the 1st analogy,

As radius.....	10.0000000
Is to sine of the hypoth. AC 70° 30′....	9.9743466
So is the sine of the angle A 51° 31′....	9.8936448
To the sine of the leg CB 47° 33′ 7.0″...	9.8679914

3. To find the leg AB, adjacent to the given angle A. By the 6th analogy,

As radius.....	10.0000000
Is to tang. of the hyp. AC 70° 30′.....	10.4508513
So is cosine of the angle A 51° 31′.....	9.7939907
To tang. of the leg AB 60° 21′ 27.1″....	10.2448420

If we were to find the angle C by the Catholic proposition, we should have the complement of the hypothenuse AC for the middle part, and the complements of the angles A and C, for adjacent parts; whence radius × cos. AC = cotang. C × cotang. A, which converted into a proportion is : cotang. A : rad. : : cos. AC : cotang. C, and by taking radius for the 1st term, and tang. A for the 2d, according to rule 4th, this is changed into the same analogy with that we have used from prop. A.

* The *cosine* of C is put down, because the circular part is the *complement* of C, and the sine of this complement is the same as the cosine of C; the *sine* of A is written, because it is the *complement* of A, which is the circular part; and we are according to the rule, to take the *cosine* of this complement, which is the same as the sine of A.

Case 2.

Given the hypothenuse and one of the legs, to find the other leg and the two oblique angles.

Example.—In the spherical triangle ABC (Fig. 1.) right angled at B; there are given the hypothenuse AC = 70° 30′, and the leg AB = 60° 21′ 27.1″ It is required to find the leg BC, and the angles A and C.

Solution.

1. To find the leg CB. By inversion of the 3d analogy of prop. A,

As cosine of the leg AB 60° 21′ 27.1″	9.6942420
Is to radius.....	10.0000000
So is cos. of the hyp. AC 70° 30′ 0″....	9.5234953
To cosine of the leg BC 47° 33′ 7.0″..	9.8292533

2. To find the angle A adjacent to the given leg AB. By inversion of the 6th analogy, and changing the 1st term into radius by rule 4th,

As radius.....	10.0000000
Is to cotang. of hyp. AC 70° 30′ 0″....	9.5491487
So is tangent of leg. AB 60° 21′ 27.1″	10.2448420
To cosine of the angle A 51° 31′ 0″....	9.7939907

3. To find the angle C opposite to the given leg AB. By inversion of the 1st analogy,

As sine of the hyp. AC 70° 30′ 0″....	9.9743466
Is to radius.....	10.0000000
So is sine of the leg AB 60° 21′ 27.1″	9.9390840
To sine of angle C 67° 13′ 20.2″.....	9.9647374

The proportion for finding CB may be changed so as to have radius for its 1st term by rule 5, thus,

As radius : secant AB : : cosine AC : cosine BC.

And that for finding the angle C may be changed, by the same rule, thus,

As radius : cosecant AC : : sine AB : sine C.

Case 3.

Given the two legs, to find the two oblique angles and the hypothenuse.

Example.—In the spherical triangle ABC right angled at B (Fig. 1.), there are given the leg AB = 60° 21′ 27.1″, and the leg BC = 47° 33′ 7″: it is required to find the angles A and C, and the hypothenuse AC.

Solution.

1. To find the angle A. By the 4th analogy of prop. A, varied (see the observation at the end of this case),

As radius.....	1.00000000
Is to cotang. of the leg BC 47° 33′ 7.0″	9.9612619
So is sine leg AB 60° 21′ 27.1″.....	9.9390840
To cotang. of the angle A 51° 31′ 0″...	9.9003459

2. To find the angle C. In a similar manner,

As radius.....	10.0000000
Is to cotangent of leg AB 60° 21′ 27.1″	9.7551580
So is sine leg BC 47° 33′ 7.0″.....	9.8679913
To cotang. of the angle C 67° 13′ 20.2″	9.6231493

3. To find the hypothenuse AC. By the 3d analogy,

As radius.....	10.0000000
Is to the cosine leg AB 60° 21′ 27.1″..	9.6942420
So is cos. of the leg BC 47° 33′ 7.0″....	9.8292533
To cos. hypothenuse AC 70° 30′ 0″	9.5234953

The proportion for finding the angle A is obtained thus:

By proposition A, analogy 4th, rad. : tang. A : : sin. AB : tang. BC, and by alternation, rad. : sin. AB : : tang. A : tang. BC, but tang. A : tang. BC : : cotang. BC : cotang. A, therefore by equality, rad. : sin. AB : : cotang. BC : cotang. A, which by alternation is the same as the proportion we have used.

Case 4.

Given one of the legs and its adjacent angle, to find the other oblique angle, the other leg, and the hypothenuse.

Example.—In the spherical triangle ABC right angled at B (Fig. 1), there are given the leg AB = 60° 21′ 27.1″, the angle A, adjacent, = 51° 31′: it is required to find the angle C opposite to the given leg, and the leg BC opposite to the given angle, and the hypothenuse AC.

Solution.

1. To find the angle C. By the 2d analogy of prop. A.

As radius.....	10.0000000
Is to cos. of the leg AB 60° 21′ 27.1″	9.6942420
So is sine of the angle A 51° 31′ 0″....	9.8936448
To cos. of the angle C 67° 13′ 20.2″....	9.5878868

2. To find the leg BC opposite to the given angle. By the 4th analogy,

As radius.....	10.0000000
Is to tang. of the angle A 51° 31′ 0″....	10.0996541
So is sine of the leg AB 60° 21′ 27.1″	9.9390840
To tang. of the leg BC 47° 33′ 7.0″....	10.0387381

3. To find the hypothenuse AC, by the 6th analogy varied.

As radius		10.0000000
Is to cotang. of the leg AB	60° 21′ 27.1″	9.7551580
So is cos. of the angle A	51 31 0 ...	9.7939907
To cotang. of the hyp. AC	70 30 0 ...	9.5491487

This proportion for finding the hypothenuse is obtained thus,

From the 6th analogy of prop. A, we have rad. : tang. AC : : cos. A : tang. AB, and by alternation, rad. : cos. A : : tang. AC : tang. AB; but tang. AC : tang. AB : : cotang. AB : cotang. AC, therefore by equality, rad. : cos. A : : cotang. AB : cotang. AC, which proportion alternated, becomes the same as that above used.

Case 5.

Given one of the legs, and its opposite angle, to find the other oblique angle, the other leg, and the hypothenuse.

Example.—In the spherical triangle ABC (Fig. 1.), right angled at B; there are given the leg AB = 60° 21′ 27.1″, and the angle opposite to it, C = 67° 13′ 20.2″; it is required to find the angle A, the leg BC, and the hypothenuse AC.

Solution.

1. To find the angle A adjacent to the given leg AB, by the 2d analogy of prop. A, and inversion.

As cos. of the leg AB	60° 21′ 27.1″ ..	9.6942420
Is to radius		10.0000000
So is cos. of the angle C	67 13 20.2 . .	9.5878868
To sine of the angle A	51 31 0	9.8936448

2. To find the leg BC adjacent to the given angle C, by the 4th analogy, varied,

As radius		10.0000000
Is to tang. of the leg AB	60° 21′ 27.1″	10.2448420
So is cotang. of angle C ..	67 13 20.2 .	9.6231498
To sine of the leg BC ...	47 33 70 . .	9.8679918

3. To find the hypothenuse AC, by the 1st analogy alternated and inverted.

As sine of the angle C	67° 13′ 20.2″ .	9.9647374
Is to radius		10.0000000
So is sine of the leg AB	60 21 27.1 . .	9.9390840
To sine of the hyp. AC	70 30 0. . . .	9.9743466

The proportion for finding the angle A may be changed by rule 5th, thus,

As radius : secant AB : : cosine C : sine A.

And that for finding AC may be changed, by rule 5th, to this:

As radius : cosecant C : : sine AB : sine AC.

The proportion for determining the leg BC is found thus,

By the 4th analogy of prop A, rad. : tang. C : : sin. BC : tang. AB; but rad. : tang. C : : cotang. C : rad.; therefore by equality, cotang C : rad. : : sin. BC : tan. AB, and by alternation, rad. : tang. AB : : cotang. C : sin. BC.

Case 6.

Given the two oblique angles, to find the two legs, and the hypothenuse.

Example.—In the spherical triangle ABC (Fig. 1.), right angled at B; there are given the angle A = 51° 31′, and the angle C = 67° 13′ 20.2″: it is required to find the legs AB and BC, and the hypothenuse AC.

Solulion.

1. To find the leg AB, by alternation and inversion of the 2d analogy of prop. A.

As sine of the angle A	51° 31′ 0″	9.8936448
Is to radius		10.0000000
So is cos. of the angle C	67 13 20.2 . .	9.5878868
To cos. of the leg AB	60 21 27.1 . .	9.6942420

2. To find the leg BC, in a similar manner.

As sine of the angle C	67° 13′ 20.2″ . .	9.9647374
Is to radius		10.0000000
So is cos. of the angle A	51 31 0. . . .	9.7939907
To cos. of the leg BC	47 33 7	9.8292533

3. To find the hypothenuse AC, by the 5th analogy inverted, and changing the 1st term into radius by rule 4th.

As radius		10.0000000
Is to cotang. of the angle A	51° 31′ 0″	9.9003459
So is cotang. of the angle C	67 13 20.2	9.6231498
To cos. of the hypoth. AC	70 30 0 . .	9.5234957

The proportion for finding AB may be changed, by rule 5, thus:

As radius : cosecant A : : cosine C : cosine AB.

And the proportion for finding BC may be changed to this,

As radius : cosecant C : : cosine A : cosine BC.

The solution of the several cases of oblique angled spherical trigonometry, depends on the following propositions.

Prop. B. Theorem.

In all spherical triangles, as the sine of any side is to the sine of its opposite angle; so is the sine of any other side to the sine of its opposite angle.

Prop C. THEOREM.

IN any oblique angled spherical triangle, if a great circle be drawn perpendicularly from one of the angles to its opposite side (or to the opposite side produced, if necessary); then, 1st, the cosines of the angles at the base are proportional to the sines of the segments of the vertical angle; 2d, the cosines of the sides are proportional to the cosines of the segments of the base; 3d, the cosines of the segments of the vertical angles are inversely proportional to the tangents of the sides; 4th, the sines of the segments of the base are inversely proportional to the tangents of the angles at the base.*

Prop. D. THEOREM.

1. As the sine of half the difference of two sides of any spherical triangle is to the sine of half their sum, so is the tangent of half the difference of their opposite angles, to the cotangent of half the angle contained by these sides.

2. As the cosine of half the difference of two sides is to the cosine of half their sum, so is the tangent of half the sum of their opposite angles, to the cotangent of half the angle contained by these sides.

3. As the sine of half the difference of two angles is to the sine of half their sum, so is the tangent of half the difference of the two sides opposite to these angles to the tangent of half their interjacent side.

4. As the cosine of half the difference of two angles is to the cosine of half their sum, so is the tangent of half the sum of the two sides opposite to these angles, to the tangent of half their interjacent side.†

Prop. E. THEOREM.

As the rectangle under the sines of any two sides of a spherical triangle is to the rectangle under the sines of the excesses of the semi-perimeter above those sides; so is the square of the radius, to the square of the sine of half the contained angle.

Prop. F. THEOREM.

As the rectangle under the sines of any two sides of a spherical triangle is to the rectangle under the sines of the semi-perimeter, and the sine of the excess of the semi-perimeter above the third side; so is the square of the radius, to the square of the cosine of half the contained angle.

General Direction.

IN the solution of the several following cases of oblique angled spherical trigonometry, by the assistance of a perpendicular, it is always necessary to let the perpendicular fall from the extremity of a given side, and opposite to a given angle; and strict attention must be had whether the perpendicular falls within the triangle or on the outside of the triangle on the base produced; for if it fall within the triangle, it divides the base into two segments; but if it fall without the triangle, then the sum of the base and the external part without the triangle, is one leg of the greater triangle, and the external part is a leg of the less. But for the satisfaction of the astronomical student, we must observe, that there is no problem can occur to him in his calculations, in which he will not with a slight consideration perceive, whether the perpendicular falls within the triangle, or on the outside of it. When the three sides are given to find an angle, then the solution does not proceed by means of a perpendicular.

Solution of the several Cases of oblique angled Spherical Triangles.

CASE 1.

GIVEN two sides of a spherical triangle and an angle opposite to one of them, to find the other two angles and the third side.

Example.—IN the oblique angled spherical triangle ADC, (Fig. 2.), there are given the side AC = 70° 30′ 0″, the side CD = 58° 30′ 0″, and the angle A = 51° 31′ 0″: it is required to find the angles D and DCA, and the third side DA; the angle D being previously known to be acute.

SOLUTION.

FROM C draw the great circle CB at right angles to DA; consequently CB falls within the triangle, because the angles D and A are both acute.

1. To find the angle D by prop. B, in the triangle DCA.

As the sine of DC	58°	30′	0″	. . . .	9.9307658
Is to the sine of A	51	31	0	. . . .	9.8936448
So is the sine of AC	70	30	0	. . . .	9.9743466
To the sine of D	59	55	49.4	. . . .	9.9372256

* Since the tangents of two arcs or angles are inversely as the cotangents, the 3d analogy may be changed thus, the cosines of the segments of the vertical angles are *directly* proportional to the cotangents of the sides; and the 4th, thus, the sines of the segments of the base are *directly* proportional to the cotangents of the angles at the base.

† The analogies in this proposition having been invented by Lord Napier, are from him called Napier's analogies; they are not often used in astronomy, the problems in this science being mostly solved by letting fall a perpendicular.

2. In the right anngled triangle DCB, to find the leg DB.

As radius		10.0000000
Is to cosine of D	59° 55′ 49.4″	9.6998826
So is tangent of DC	58 30 0	10.2126807
To tangent of DB	39 16 15.2	9.9125633

DB =	39° 16′ 15.2″		180° 0′ 00″
AB =	60 21 27.1		99 37 42.3
DA =	99 37 42.3	Supple. of DA =	80 22 17.7

3. In the right angled triangle ABC, to find the leg AB.

As radius		10.0000000
Is to cosine A	51° 31′ 0″	9.7939907
So is tangent AC	70 30 0	10.4508513
To tangent AB	60 21 27.1	10.2448420

4. In the triangle DCA, to find the angle DCA by prop. B.

As the sine of DC	58° 30′ 00″	9.9307658
Is to the sine of A	51 31 00	9.8936448
So is the sine of DA	99 37 42.3	9.9938387
To the sine of DCA	115 9 26.3	9.9567177

180° 0′ 00″
64 50 33.7

Angle DCA = 115 9 26.3

SCHOLIUM.

IF the conditions of the *data* had not specially limited the angle D to be acute, the problem would have been involved in ambiguity because it appears, that another triangle APC, has the same data, the base of which AP is the difference of AB and DB, and the angle APC is the supplement of D. It is also to be remarked, that AD being greater than a quadrant, that its opposite angle ACD will always be obtuse, and *vice versâ*; provided the other two angles D and A are both acute. But as we have before observed, these ambiguities can never embarrass the astronomer.

For example, all those stars whose declination exceeds the latitude of the place (the declination and latitude having both the same name), will have the same azimuth twice on the east side of the meridian, and twice on the west side; at which times the zenith distances and hours of culmination will be very different indeed, as in the foregoing case; for if C be the pole, A the zenith, and CD the star's polar distance; then the star has the same azimuth at D and P, and AD and AP are the two zenith distances, and the two angles ACD and ACP are the horary angles before or after culmination: now, he is truely unfit for the practice of astronomy, who has not previously ascertained, whether the greater or less zenith distance and horary angle be those which are sought; and these he ascertains, by knowing whether the star have passed its greatest azimuth or not.

CASE 2.

GIVEN two angles and a side opposite to one of them, in a spherical triangle, to find the third angle and the other two sides.

Example.—In the oblique angled triangle ACD (Fig. 3.), there are given the angle A = 51° 31′, the angle D = 59° 55′ 49.4″, and the side AC = 70° 30′: it is required to find the angle ACD, and the two sides DC and DA.

SOLUTION.

THE perpendicular CB being drawn as before,

1. In the oblique angled triangle ACD, to find the side CD.

As the sine of D	59° 55′ 49.4″	9.9372256
Is to the sine of AC	70 30 00	9.9743466
So is the sine of A	51 31 00	9.8936448
To the sine of CD	58 30 00	9.9307658

2. In the right angled triangle DCB, to find the angle DCB.

As radius		10.0000000
Is to the tangent of D	59° 55′ 49.4″	10.2373430
So is the cosine of DC	58 30 00	9.7180851
To the cotang. of DCB	47 56 6	9.9554281

3. In the right angled triangle ACB, to find the angle ACB.

As radius		10.0000000
Is to the tangent of A	51° 31′ 00″	10.0996541
So is the cosine of AC	70 30 00	9.5234953
To the cotang. of ACB	67 13 20.2	9.6231494

Ang. DCB =	47° 56′ 6″		180° 0′ 0″
Ang. ACB =	67 13 20.2		115 9 26.2
Ang. DCA =	115 9 26.2	sup. DCA =	64 50 33.8

4. In the oblique angled triangle ACD, to find the side DA.

As the sine of A 51° 31′ 00″ 9.8936448
Is to the sine of DC 58 30 00 . . . . 9.9307658
So is the sine of DCA 115 9 26.2 . . . 9.9567178

To the sine of DA 99 37 42 . . . . 9.9938388

180° 0′ 0″
80 22 18

DA = 99 37 42

CASE 3.

GIVEN two sides and their contained angle, in a spherical triangle, to find the other two angles and the third side.

Example.—In the oblique angled spherical triangle ACD (Fig. 3.), there are given the side AD = 99° 37′ 42.3″, the side AC = 70° 30′ 00″, and the angle A = 51° 31′ 0″: it is required to find the angles D and DCA, and the side DC.

SOLUTION.

LET fall the perpendicular CB, from the angle DCA, but always opposite to the given angle A.

1. In the right angled triangle ABC, to find the base AB.

As radius 10.0000000
Is to the cosine of A 51° 31′ 00″ . . . 9.7939907
So is the tangent of AC 70 30 00 . . 10.4508513

To the tangent of AB 60 21 27.1 . 10.2448420

From AD = 99° 37′ 42.3″
Take AB = 60 21 27.1

Remains DB = 39 16 15.2

2. In the two right angled triangles ACB and DCB, to find DC, by prop. C. part 2d.

As the cosine of AB 60° 21′ 27.1″ . . 9.6942420
Is to the cosine of AC 70 30 00 . . . 9.5234953
So is the cosine of DB 39 16 15.2 . . 9.8888318

To the cosine of DC 58 30 00 . . . 9.7180851

And now having the three sides, and the angle A given; as sine DC : sine A : : sine AC : sine of D; and as sine of DC : sine of A : : sine of AD : sine of ACD.

CASE 4.

GIVEN two angles and the side between them of a spherical triangle, to find the other two sides and the third angle.

Example.—In the oblique angled spherical triangle ACD (Fig. 3), there are given the angle A = 51° 31′ 0″, the angle ACD = 115° 9′ 26.3″, and the side AC = 70° 30′ 0″: it is required to find the sides AD and DC, and the angle D.

SOLUTION.

Having let fall the perpendicular CB, as in the foregoing cases.

1. In the right angled triangle ACB, to find the angle ACB.

As radius 10.0000000
Is to the cosine of AC . . 70° 30′ 00″ . 9.5234953
So is the tangent of A . . 51 31 00 : 10.0996541

To the cotangent of ACB 67 13 20.2 9.6231494

From ACD . . . 115° 9′ 26.3″
Take ACB 67 13 20.2

Remains DCB = 47 56 6.1

2. In the two right angled triangles ACB and DCB, to find the side DC, by prop. C.

As cosine of DCB . 47° 56′ 6.1″ . . . 9.8260573
Is to tangent of AC 70 30 00 . . . 10.4508513
So is cosine of ACB 67 13 20.2 . . . 9.5878873

To the tangent of DC 58 30 00 . . . 10.2126813

And now having the two sides AC and DC, and the two angles A and ACD; the remaining side and angle are found thus, as sine A : sine DC : : sine ACD : sine AD; and as sine DC : sine A : : sine AC : sine D.

CASE 5.

GIVEN the three sides of a spherical triangle, to find the three angles.

Example.—In the spherical triangle ACD (Fig. 4), there are given the side AC = 70° 30′, AD = 99° 37′ 42.3″, and DC = 58° 30′. It is required to find the angles A, D, and C.

Practical Rules deduced from Props. E *and* F.

1. From half the sum of the three sides, subtract each of the sides containing the required angle. Then add together the arithmetical complements of the log. sines of the two containing sides, and the log. sines of those two remainders: half the sum of these four logarithms is the log. sine of half the angle sought, or,

2. From half the sum of the three sides, subtract the side opposite the required angle: then add together the arithmetical complements of the log. sines of the two containing sides, the log. sine of the half sum, and the log. sine of the above remainder: half the sum of these four logarithms is the log. cosine of half the angle sought.

Note.—The arithmetical complement of a log. sine is the same as the log. cosecant, rejecting 10 from the index.

SOLUTION.

To find the angle C opposite to AD.

By Rule 1st.

AC =	70° 30′ 00″		
AD =	99 37 42.3		
DC =	58 30 00		
	2)228 37 42.3		
½ sum of sides =	114 18 51.1		114 18 51.1
subtract AC =	70 30 00	subtract DC =	58 30 00
remainder =	43 48 51.1	remainder =	55 48 51.1
AC =	70° 30′ 00″	Ar. comp. log. sine	0.0256534
DC =	58 30 00	Ar. comp. log. sine	0.0692342
½ sum — AC =	43 48 51.1	log. sine	9.8403081
½ sum — DC =	55 48 51.1	log. sine	9.9176209
			2)19.8528166
½ angle C =	57 34 43.1	log. sine	9.9264083
	2		
angle C =	115 9 26.2		

By Rule 2d.

AC =	70° 30′ 00″		
AD =	99 37 42.3		
DC =	58 30 00		
	2)228 37 42.3		
½ sum of sides =	114 18 51.1		
Subtract AD =	99 37 42.3		
Remainder =	14 41 8.8		
AC =	70° 30′ 00″	ar. com. log. sine	0.0256534
DC =	58 30 00	ar. com. log. sine	0.0692342
Half sum =	114 18 51.1	log. sine	9.9596620
½ sum — AD =	14 41 8.8	log. sine	9.4040085
			2)19.4585581
½ angle C =	57 34 43.2	log. cosine	9.7292790
	2	180° 00′ 00″	
		114 18 51.1	
Angle C =	115 9 26.4		
		65 41 8.9	

And in like manner the other angles A and D may be found, but more easily when one of the angles is found as above, thus, as sine DA : sine C : : sine AC : sine D; and as sine DA : sine C : : sine DC : sine A.

REMARK.

THERE yet remains another case given by most writers on trigonometry, *viz.* when the three angles are known, to find the sides; but as this case can rarely, if ever, occur in astronomy, we have not thought it necessary to give an example of it. The solution is usually effected by means of a triangle, called the supplemental triangle, which is thus constructed.

ABC (Fig. 5.) being supposed the triangle whose angles are given, from A, B, C, as poles describe great circles forming by their intersection the spherical triangle DEF: this last is termed the supplemental triangle to ABC, because its sides EF, DF, DE, are the supplements respectively of the arcs which measure the angles A, B, C, opposite these sides, and its angles D, E, F, are the supplements of the opposite sides BC, AC, AB, in the triangle ABC. Consequently by taking the supplements of the three given angles A, B, C, we have the sides of the triangle DEF, from whence any one of its angles may be found by case 5th, and the supplement of this angle will be the corresponding side of the triangle ABC.

From the properties of the supplemental triangle, it follows, that if one side of a spherical triangle be 90°, or the triangle be a quadrantal one, then the supplemental triangle to this will be a right angled one, and therefore the rules for the solution of right angled triangles may be adapted (by the use of the supplemental triangle) to quadrantal ones.

DEMONSTRATION *of Proposition* A, *in the Spherical Trigonometry.*

LET ABC (Fig. 6.) be a spherical triangle, having the right angle ABC, let O be the centre of the sphere, and OA, OB, OC, radii drawn to the three angles of the triangle. In the plane OAC, draw CE perpendicular to OA, and in the plane OAB, draw ED perpendicular to the same line OA, and join CD. Then since OE is at right angles to both EC and ED, it is perpendicular to the plane ECD, and because the plane OAB passes through the line OE, the planes OAB, ECD, are perpendicular to each other; but the angle ABC being, by hypothesis, a right angle, the plane BOC is perpendicular to OAB: hence the planes ECD, OBC, being each perpendicular to the plane OAB, their common section, CD, will also be perpendicular to the plane OAB, and the plane triangle ECD is consequently right angled at CDE. Farther, it is obvious, that since EC, ED, which are in the planes OAC, OAB, are each perpendicular to OE, therefore the angle CED is equal to the spherical angle CAB, and also that CD, CE, are the sines of the arcs BC, AC, to the radius of the sphere OC.

Now in the triangle ECD, we have, by prop. A. plane trigonometry, radius : CE : : sin. CED : CD, that is, radius : sine of hypoth. of sph. triangle

ABC : : sin. sph. angle CAB : sin. BC which is the 1st part of the proposition to be demonstrated.

To demonstrate the 2d and 3d parts, let ABC (Fig. 7.), be a spherical triangle right angled at B; about the pole A, describe the great circle DEF, and let it meet AB, AC, and BC, produced in D, E, and F: then since the angles at B and D are right angles, it is evident that F is the pole of ABD, and also that BD, CE, are the complements of AB, AC, that CF and EF, are the complements of BC, DE, and likewise that DE is the measure of the spherical angle A, and BD the measure of F. Now in the triangle CEF right angled at E, we have by part 1st rad. : sin. CF : : sin. C : sin. EF, that is, rad. : cos. BC : : sin. C : cos. A, which is the 2d part of the proposition.

Again by part 1st, in the triangle CEF, rad. : sin. CF : : sin. F : sin. CE, and by alternation rad. : sin. F : : sin. CF : sin. CE, that is, rad. : cos. AB : : cos. BC : cos. AC which is the 3d part.

Next, in Fig. 6. draw BL perpendicular to OA, and LK perpendicular to the same line in the continuation of the plane OAC, meeting OC produced in K, and join KB, then it may be demonstrated that KB is perpendicular to the plane OAB exactly in the same manner as it was shewn that CD is perpendicular to that plane, and consequently that the triangle LKB is right angled at LBK. It is also manifest that because LK, LB, are each perpendicular to OA, therefore the angle KLB is equal to the spherical angle CAB, and likewise that BL is the sine of the arc AB to the radius OB, and that BK is the tangent of the arc BC to the same radius.

Now in the triangle LKB we have by prop. A, plane trigonometry, rad. : tang. angle KLB : : BL : BK; that is, rad. : tang. sph. angle CAB : : sine AB : tang. BC, which is the 4th part of the proposition.

Farther, in Fig. 7, we have by the part just demonstrated, rad. : tang. C : : sin. CE : tang. FE; that is, rad. : tang. C : : cos. AC : cotang. A, which is the 5th part.

Lastly, in the same Fig. and by part 4th, rad. : tang. F : : sin. FE : tang. CE, that is, rad. : cotang. AB : : cos. A : cotang. AC, or by alternation, rad. : cos. A : : cotang. AB : cotang. AC; but cotang. AB : cotang. AC : : tang. AC : tang. AB, therefore by equality, rad. : cos. A : : tang. AC : tang. AB, and by alternation, rad. : tang. AC : : cos. A : tang. AB, which is the 6th and last part of the proposition to be demonstrated.

Demonstration *of Proposition* B.

Let ACD (Fig. 8. 9.) be a spherical triangle, and from the angle C draw the arc CB of a great circle perpendicular to the opposite side AD, or this side produced. Then, in the right angled spherical triangle DBC, by prop. A, part 1st, rad. : sin. CD : : sin. D : * sin. BC; therefore, rad. × sin. BC = sin. D × sin. CD, for the same reason, in the right angled spherical triangle ABC, rad. × sin. BC = sin. A × sin. AC, consequently sin. D × sin. CD = sin. A × sin. AC, and by resolving this equation into an analogy, we have sin A : sin. CD : : sin. D : sin. AC.

Q. E. D.

Demonstration *of Proposition* C.

The same figures and construction remaining as in the last, we have by prop. A, part 2d, rad : cos. BC : : sin. DCB : cos. D, and rad. : cos. BC : : sin. ACB : cos. A, therefore, by equality, sin. DCB : cos. D : : sin. ACB : cos. A, which is the 1st part of prop. C.

Again, by prop. A, part 3d, rad. : cos. BC : : cos. DB : cos. DC, and rad. : cos. BC : : cos. AB : cos. AC, whence by equality, cos. DB : cos. DC : : cos. AB : cos. AC, which is part 2d.

Farther, by prop. A, part 6th, rad. : tang. DC : : cos. DCB : tang. BC, therefore rad. × tan. BC = tan. DC × cos. DCB; for the same reason, rad. × tan. BC = tan. AC × cos. ACB; hence, tan. DC × cos. DCB = tan. AC × cos. ACB, and this equation converted into a proportion, gives tan. DC : tan. AC : : cos. ACB : cos. DCB, which is part 3d.

Lastly, by prop. A, part 4th, rad. : tan. D : : sin. DB : tan. BC, therefore, rad. × rad. × tan. BC = tan. D × sin. DB; for the same reason, rad. × tan. BC = tan. A × sin. AB; consequently tan. D × sin. DB = tan. A × sin. AB, and by converting this equation into an analogy, we have sin. DB : sin. AB : : tan. A : tan. D, which is part 4th.

Demonstration *of Proposition* D.

Preparatory Construction, &c.—Let ACD (Fig. 10.) be any spherical triangle in which the angle CAD is the less of the two angles at the base; draw the arc of a great circle AN, so that the angle DAN may be equal to D, and produce DC to meet it in N: draw also the arc NO of a great circle perpendicular to AD; this arc will obviously bisect the angle DNA, and the base DA. Bisect the angle CAN by the arc AE, meeting NO in E, and join the arc CE; likewise from E draw the three arcs EK, EL, EM, perpendicular to the three sides of the triangle ANC. Then,

1st, The three perpendiculars EK, EL, EM, are all equal to each other. For in the right angled triangle EMA, EM is dependent on (that is, may be found from the hypothenuse) EA, and the opposite angle EAM, see prop. A, part 1st, and in the right

* In Fig. 9. sin. BDC is equal to sin. ADC because these angles are supplements to each other.

angled triangle EKA, EK is dependent upon the same hypothenuse EA, and the opposite angle EAK which is equal, by the construction, to EAM, consequently EK = EM. In a similar manner it may be shewn that EK = EL, therefore EM = EK = EL.

2d, The angle NCA is bisected by CE. For in the right angled triangle ELC the angle LCE is dependent on its opposite leg LE and the hypothenuse CE, and in the right angled triangle EMC, the angle MCE is dependent on its opposite leg ME, which has been proved equal to LE, and the same hypothenuse CE, consequently the angle LCE = MCE, that is, the whole angle NCA is bisected by CE.

3d, It is farther obvious from reasoning analogies to that applied above, that AM = AK; NL = NK; and CL = CM: also that the angle MEA = KEA; KEN = LEN; and LEC = MEC.

4th, Moreover the angle AEO is equal to the angle CEM. For,

OEA + AEK + KEN = OEM + MEC + CEL + LEN,

because the sum of the angles on each side of this equation is equal to 180°, whence by taking away KEN = LEN, there will remain

OEA + AEK = OEM + MEC + CEL

that is, OEA + AEM = OEM + 2MEC

or OEA + OEA + OEM = OEM + 2MEC

Therefore 2OEA = 2MEC, and OEA = MEC.

5th, AM is half the sum, and CM half the difference of the sides AC, DC, of the triangle ACD. For DN = AN, and LN = KN. therefore DL = AK, that is, DC + CM = AM = AC − CM, hence 2CM = AC − DC, and CM = $\frac{1}{2}$(AC − DC), wherefore AM = AC − CM = AC − $\frac{1}{2}$(AC − DC) = $\frac{1}{2}$(AC + DC). Likewise, it is manifest, that EAM is equal to half the difference, and EAO equal to half the sum of the angles CDA, CAD.

These things laid down, the triangle ACE being divided into two right angled ones by the perpendicular EM, we have by prop. C. part 4th, sin. CM : sin. AM : : tang. EAM : tang. ECM; that is, sin. $\frac{1}{2}$(AC − CD) : sin. $\frac{1}{2}$(AC + CD) : : tang. $\frac{1}{2}$(CDA − CAD) : cotang. $\frac{1}{2}$DCA; which is the 1st part of prop. D.

Again, in the right angled triangle AEO, by prop. A, part 5th, rad. : tang. EAO : : cos. AE : cotang. AEO or cotang. CEM, therefore, rad. × cotang. CEM = cos. AE × tang. EAO, and in the triangle CEM we have by the same prop. and multiplying means and extremes, rad. × cotang. CEM = cos. CE × tang. ECM, hence cos. AE × tang. EAO = cos. CE × tang. ECM, and resolving this into a proportion, cos. CE : cos. AE : : tang. EAO : tang. ECM. But by prop. C, part 2d, cos. CE : cos. AE : : cos. CM : cos. AM, therefore, by equality cos. CM : cos. AM : : tang. EAO : tang. ECM, that is, cos. $\frac{1}{2}$(AC − CD) : cos. $\frac{1}{2}$(AC + CD) : : tang $\frac{1}{2}$(CDA + CAD) : cotang. $\frac{1}{2}$DCA; which is the 2d part of the proposition.

Farther, in the right angled triangle CEM, by prop. A, part 4th, rad : (tang. CEM =) tang. AEO : : sin. EM : tang. CM, and by the same prop. in the triangle AEO, rad. : tang. AEO : : sin. EO : tang. AO; therefore, by equality, sin. EM : tang. CM : : sin. EO : tang. AO, and by alternation, sin. EM : sin. EO : : tang. CM : tang. AO; now by prop. A, part 1st, rad. : sin. AE : : sin. EAM : sin. EM : : sin. EAO : sin. EO, or by alternation, sin. EM : sin. EO : : sin. EAM : sin. EAO; therefore, again, by equality, sin. EAM : sin. EAO : : tang. CM : tan. AO, that is, sin. $\frac{1}{2}$(CDA − CAD) : sin. $\frac{1}{2}$(CDA + CAD) : : tang. $\frac{1}{2}$(AC − CD) : tang. $\frac{1}{2}$AD, which corresponds with the 3d part of the proposition.

Lastly, in the right angled triangles EAM, EAO, by prop. A, part 6th, rad. : tang. AE : : cos. EAM : tang. AM, and rad. : tang. AE : : cos. EAO : tang. AO; therefore, by equality, cos. EAM : tang. AM : : cos. EAO : tang. AO, and by alternation, cos. EAM : cos. EAO : : tang. AM : tang. AO; that is, cos. $\frac{1}{2}$(CDA − CAD) : cos. $\frac{1}{2}$(CDA + CAD) : : tang. $\frac{1}{2}$(AC + CD) : tang. $\frac{1}{2}$AD, which is the last part of the proposition.

This proposition is usually demonstrated by the combination of various analytic formulæ relating to the sines and tangents of arcs; the demonstration here given proceeds upon more elementary principles, and therefore agrees better with the general plan of our work: it is taken in substance from Delambre's Astronomy, and, as far as we are aware, is here presented to the English reader for the first time.

Demonstration *of Prop.* E.

Let ACD (Fig. 11.) be any spherical triangle on the surface of the sphere ACME, whose centre is O; on CA produced each way take the arcs CM, C*m*, each equal to CD; and the arcs AN, A*n*, each equal to AD, and through the points M, *m*, and N, *n*, let the minor circles MD*m*, ND*n*, pass, whose diameters are M*m*, N*n*, and whose planes are supposed to be respectively perpendicular to the radii of the sphere OC, OA. Also let HPL be a circle whose plane passes through the centre of the sphere at right angles to CO; from K, the intersection of CO and M*m*, draw KD, join OP, HP, PL, bisect the last two in *s* and *x*, and draw *st*, P*p*, *xy*, perpendicular to HL, and join O*s*, O*x*. Then it is manifest from this construction, that the arc A*m* = CD − AC; AM = CD + AC; NCA*m* = AN + A*m* = AD + CD − AC; *mn* = AN − A*m* = AD + AC −

CD. Also, because the planes MDm, NDn, are each perpendicular to the plane CHEL, their common section Dd is perpendicular to this plane, and consequently to the line Mm, therefore Kd, Op, are the cosines of the equal angles DKn, POL, that is, of the angle DCA, to the radii Km, OL; hence Km : dK : : OL : pO, and by composition, &c. Km : $\frac{1}{2}dm$: : OL : $\frac{1}{2}p$L = yL. Moreover, because the lines dm, dn, are respectively perpendicular to OC, OA, therefore the angle mdn = COA, whose measure is the arc CA, and in the plane triangle mdn, sin. mdn, or sin. CA : $\frac{1}{2}mn$: : sin. mnd : $\frac{1}{2}md$. Also, because the triangle OxL is right angled at x, and xy is perpendicular to OL, therefore OL : Lx : : Lx : Ly, and OL : yL : : OL^2 : Lx^2. Now, by multiplying the corresponding terms of these three proportions together, viz.

Km : $\frac{1}{2}dm$: : OL : yL
sin. CA : $\frac{1}{2}mn$: : sin. mnd : $\frac{1}{2}md$
OL : yL : : OL^2 : Lx^2

and dividing by such quantities as are common to the antecedents and consequents there arises sin. CA × Km : $\frac{1}{2}mn$: : sin. mnd × OL^2 : : Lx^2, or sin. AC × Km : sin. mnd × $\frac{1}{2}mn$: : OL^2 : Lx^2, but Km = KD = sin. CD; sin. mnd = sin. $\frac{1}{2}$ the arc NCAm = sin. $\frac{1}{2}$(AD + CD − AC) = sin. [$\frac{1}{2}$(AD + CD + AC) − AC]; $\frac{1}{2}mn$ = sin. $\frac{1}{2}$ the arc mLn = sin. $\frac{1}{2}$(AD + AC − CD) = sin. [$\frac{1}{2}$(AD + AC + CD) − CD]; and Lx = sin. LOx = sin. $\frac{1}{2}$LOP = sin. $\frac{1}{2}$DCA, whence, by substituting these values in the last proportion, it becomes sin. AC × sin. CD : sin. (semi-perim. − AC) × sin. (semi-perim. − CD) : : rad.^{2} : sin.^{2} $\frac{1}{2}$DCA. Q. E. D.

DEMONSTRATION *of Prop.* F.

RETAINING the figure and construction to the last demonstration, join MN. Then in the plane triangle MNd, the angle MNd standing upon the arc MEn, is measured by half the arc MEn, which half arc is the supplement of half the arc MCn, or of half the sum of the three sides AD, AC, DC. The arc MN is also manifestly = AC + CM − AN = AC + CD − AD. Now the lines Md, Hp, being the versed-sines of the equal angles DKM, POH, to the radii MK, HO, therefore MK : $\frac{1}{2}$Md : : HO : $\frac{1}{2}$Hp = Ht; also in the triangle MNd, sin. MdN or sin. AC : $\frac{1}{2}$MN : : sin. MNd : $\frac{1}{2}$Md; likewise, for reasons similar to those in the last demonstration, HO : Ht : : HO^2 : Hs^2, from whence, by multiplying the corresponding terms of these three proportions together, &c. as before, we obtain sin. AC × MK : $\frac{1}{2}$MN : : sin. MNd × HO^2 : Hs^2, or sin. AC × MK : sin. MNd × $\frac{1}{2}$MN : : HO^2 : Hs^2. But MK = DK = sin. CD; sin. MNd = sin. $\frac{1}{2}$(AD + AC + CD) from what is shewn above; $\frac{1}{2}$MN = sin. $\frac{1}{2}$ arc MN = sin. $\frac{1}{2}$(AC + CD − AD) = sin. [$\frac{1}{2}$(AC + CD + AD) − AD]; and Hs = sin. $\frac{1}{2}$ angle POH = cos. $\frac{1}{2}$ POL = cos. $\frac{1}{2}$ DCA; therefore, by substituting these values in the last proportion, sin. AC × sin. CD : sin. (semi-perim.) × sin. (semi-perim. − AD) : : rad.^{2} : cos.^{2} $\frac{1}{2}$DCA. Q. E. D.

LAWS of MOTION and MECHANICS applied to ASTRONOMY.

INTRODUCTION.

DEFINITIONS.

1. Motion is a continual and successive change of place.

2. Celerity (or Velocity), is the swiftness or slowness of a body in motion, by which it passes over a given space in a given time.

3. But Rest is the permanence of any body in the same place.

Hence it follows, that Motion, Velocity, and Rest, are twofold: *viz. absolute and relative.*

4. Absolute Motion is the change of absolute place; and its celerity is measured by absolute space.

5. But Absolute Rest is the permanence of a body, in the same absolute place.

6. Relative Motion is the change of relative place; the celerity of which is measured by relative space.

7. And Relative Rest is the permanence of a body in the same relative place.

Whence it is plain, that we may think ourselves to be at rest, when at the same time we are absolutely in motion. Thus, a person in a ship is relatively at rest in respect to the several parts of the ship. And were he within the ship, and she to sail evenly through the water, without being disturbed by the wind and waves; then would such a person conceive himself to be at rest, because he is relatively at rest, in regard to all the parts of the ship about him, all having the same common motion;

yet he and all the parts of the ship are absolutely in motion; and may have an absolute celerity of 10 or 12 miles per hour.

So in like manner, our Earth is carried round the Sun, with the amazing velocity of 67000 miles per hour, yet we cannot perceive our motion, because it is calm and uniform, and all the parts of the Earth having the same common motion with ourselves, are therefore to us, relatively at rest. And the same thing applies to the diurnal motion of the Earth, by which the inhabitants of London are carried round the Earth's axis, at the rate of 580 miles per hour; and yet we perceive not this motion, any more than the other; because all the parts of the Earth are relatively at rest.

8. The length of the line described by the centre of a body in motion is called the *space passed over.* And if a body move in a right line, then the length of the right line described by its centre is its space passed over: but if the body move in the circumference of a circle, or in any other curve; then the length of that right line, which is equal to the length of the curve described by the centre of the body, is correctly its space passed over.

9. Equitable or Uniform Motion is when a body moves over equal space in equal times; that is, when the celerity is always the same.

10. Accelerated Motion is that in which the velocity is continually increasing; and Retarded Motion, in which it is continually decreasing. And if, in every infinitely small and equal portion of time, equal increments of velocity be added; or equal decrements be taken away: then the motion is said to be *equably accelerated* or *retarded.*

11. Momentum (which is often called the Quantity of Motion, or sometimes the Force of Strength) is that power or force incident to a moving body, by which it exerts itself to move from its present place, and to continue to move forward in the same rectilinear direction. But an Obstacle or Impediment is that which obstructs or resists motion, and must either destroy or diminish it.

12. The Moving Force is the power of an agent to produce motion. And an Impressed Force is an action exerted on a body to change its state, either of motion or rest.

Hence the moving and impressed force are the same, *in initio;* but when a body is once in motion, then it is by some new impressed force that its Momentum, Force, or Velocity is changed. And it is evident that those moving forces are equal, which acting alike in the same direction, produce equal quantities of motion in the same body in the same time. Also those forces are contrary whose lines of direction are contrary; that is, when the forces act in the direction of the same right line, but towards the opposite extremities of that line.

13. Gravity is a force bearing downwards, by which bodies tend to the Earth in right lines. This force acts continually upon all bodies; and without intermission, exerts itself to draw all bodies towards the Earth's centre. But this principle of gravity, or the attraction of gravitation, as it is called, belongs not to the Earth alone, but to the Sun, Moon, and all the planetary bodies in our System; and no doubt, from analogy, to all the innumerable systems of worlds, contained in infinite space.

14. A centripetal force is that power by which a body continually tends to some point as a centre: and hence it follows, that gravity is a certain centripetal force.

15. But by a *centrifugal* force is meant, a force by which any body tends to recede from a centre.

And these forces are estimated by the contrary forces, which are able to keep the bodies in the same state. So if any body revolved about an immoveable centre, the force by which it endeavours to recede from the centre is the centrifugal force; but the action of the string resisting and continually drawing back the body towards the centre, by which it comes to pass, that the body is always retained in the same circle, will be as a centripetal force equal to the centrifugal one; so that one of these forces may be aptly estimated by the other. So likewise the force of gravity of any body is known by a force contrary and equal to it, by which its descent may be hindered. But that force may be either the weight of another body acting contrarily, by means of a mechanical instrument, as for example, a balance; or a centrifugal force that will arise, if that body revolve in a circle about the centre of the Earth, with some certain and determinate velocity, as the Moon does; or lastly it may be the firmness and resistance of another body, on which the pressing weight rests; as the surface of the Earth, &c.

16. The *accelerating quantity* of any force is the measure of the velocity, which that force generates in a given time. Thus, at the same distance from the Earth, all bodies, of how unequal weight soever they are, do descend with equal swiftness; and therefore their accelerating forces are equal: this is confirmed by experiment. For a guinea and a feather descend with the same velocity, when let fall in an exhausted receiver. But at unequal distances from the Earth, they are unequally accelerated; the greater the distance, the less is the accelerating force.

Axioms, *and Self-Evident Principles.*

1. Every body will continually remain in its present state, whether that be a state of rest or motion; except it be induced to change that state by means of an external agent. Therefore every *mutation* in a natural body proceeds from some extraneous

cause or agent. For every body is a listless heap of matter, and can produce no mutation or change in itself.

2. Effects are proportional to their adequate causes. And from experience and incontrovertible facts, it appears, that the causes of natural things are such, as are the *most simple*, and are sufficient to explain the phenomena. Because the *deity* always acts in the most perfect and simple way: in manifestation of his infinite wisdom.

3. If a body be put in motion, it will continue to move for ever, with the same velocity, and in the same direction, if not acted on by some extraneous agent or force. And hence all simple motion is *rectilinear* and *equable*. So if a stone be cast from the hand, it would continue to move on for ever in the same right line and with the same velocity; if it were not retarded in its progress by the resistance of the air, which would finally destroy its motion altogether; and the power of gravity, which acting from the beginning, destroys in a certain time, its projectile motion upwards, and afterwards causes it to descend.

4. Natural effects of the same kind have the same causes; as the descent of a stone and of an apple proceed from the same cause, viz. attraction of gravitation. Also the reflection of light, from a mirror, the Moon, Venus, &c. is alike produced.

5. If any body be moved in any direction, all its particles, which are relatively at rest in it, proceed together in the same direction, with the same velocity; that is, a relative plane being moved, that which is placed in it will be moved also.

6. If two equal bodies be carried along with the same velocity; then their momenta (quantities of motion or forces), will be equal. Hence,

7. If unequal bodies have the same velocity; then their momenta are in the direct ratio of their quantities of matter.

8. If two equal bodies be carried along with unequal velocities; then their momenta are in the direct ratio of their velocities.* And hence,

9. If two unequal bodies move with unequal velocities; then their momenta are in the direct ratio of the product or multiplication of their respective velocities and quantities of matter. Thus, if a body of 2*lb.* weight, move with a velocity of 20 feet in a second of time; and another body of 5*lb.* weight move with the velocity of 32 feet per second: then their respective momenta, are as, $2 \times 20 = 40$; and $5 \times 32 = 160$; that is, as 40 to 160, or as, 1 to 4. Hence also,

10. The velocity is always directly as the momentum, and inversely as the quantity of matter, that is, as the momentum divided by the quantity of matter. Thus, if the momentum be as 40, and the quantity of matter as 2; then the velocity is as, $\frac{40}{2} = 20$.

11. The quantity of matter is as the momentum directly, and velocity inversely; that is, as the momentum divided by the velocity. Thus, if the momentum be as 40, and the velocity as 20; then the quantity of matter is, as $\frac{40}{20} = 2$.

12. In equable or uniform motions. If two bodies move, then the spaces passed over by them will be as the products of their velocities and times respectively. Thus, if one body move uniformly 5 seconds, at the rate of 16 feet per second; and another continue to move 8 seconds, at the rate of 15 feet per second; then the spaces passed over, are $5 \times 16 = 80$; and $8 \times 15 = 120$ feet: that is, the spaces are, as 80 to 120, or as 2 to 3.

13. The velocity is directly as the space passed over, and inversely as the time. And the time is directly as the space passed over, and inversely as the velocity. The illustration of this proposition is the same as that of 9 and 10.

14. Because the momenta are directly as the product of the velocity and quantity of matter: and the spaces passed over as the product of the velocity and time: therefore if the velocity be the same in both cases, then the time will be to the quantity of matter, as the space passed over is to the momentum. This is easily illustrated by taking the foregoing method of comparison.

15. Hence to cause a less body to have an equal momentum with that of a greater; the velocity of the less body must be to that of the greater, in the direct ratio of the quantity of matter in the greater, to that in the less; that is, as the *weight* of the greater is to that of the less. Thus, to cause a body of 1*lb.* weight to have as great a momentum as one of 20*lb.*: the velocity of the less must be 20 times greater than that of the greater; or the body of 1*lb.* weight, must pass over 20 times the space that the other of 20*lb.* passed over in the same or equal time.

* This fundamental law or principal of moving bodies, has been doubted by some eminent philosophers, and even denied by the sceptical: and we must allow, that it is very difficult to prove the truth of it by experiment; because it is a difficult thing to ascertain exactly with what velocity a body moves at any time. To make a body move at one time, with double, treble, &c. the velocity it had at another time, is no easy matter; besides the momentum or force of a moving body is not easily determined experimentally. However, all experiments made hitherto, seem to confirm the law; and so far as it has been applied in astronomy, all the celestial phenomena agree exactly with it. The truth of the proposition is built on this principle, that if an impressed force give a body a certain velocity, it will require double that force to give it a double velocity, &c.; and that the several momenta, being the ultimate effects, must be proportional to the first causes.

Corollary.—From the foregoing principles is deduced the theory of what are called the *mechanical powers*, or machines used to raise great weights, and to overcome great resistances. And here it is necessary to premise, that *action and reaction* are equal and contrary: thus if I press the table with my hand, then is my hand pressed by the table with an equal force: or if a body in motion impinge upon or strike another body at rest; then the body in motion receives the same stroke from that at rest which it gave the other: and the reaction of the body at rest is called *resistance*. This results from what is called the *vis inertiæ* of matter, which is a certain unwillingness in matter to change its state, whether that be a state of motion or rest; or perhaps to speak more correctly, dead matter has no willingness at all to change its present state; and it is evident that the primary cause of the resistance, is the momentum of the moving body. Hence also, ships being very heavy bodies, and moving with a certain velocity, and having their momenta made up of their whole quantity of matter or weight, multiplied by their velocities, do when they strike against a rock, meet with a prodigious resistance, which too frequently dashes them to pieces.

Of the Mechanical Powers.

The number of simple mechanical powers is variously stated by the writers on natural philosophy; some making but five, others six, and others seven. However those which are most generally admitted as the mechanical powers are, the *lever*, the *axis in peritrochio*, the *inclined plane*, the *wedge*, the *pulley*, and the *screw*. Of these the *lever* is that only of which we shall treat; the other powers not being connected with astronomy.

Definition.

A *lever* is an inflexible line, moving about a fixed point called the *fulcrum* or prop. And in the theory, the line or lever is supposed to be *without weight*; and although the instruments used as levers, namely hand-spikes, crow-bars, &c. have weight in themselves, yet the theory is not affected by their weight.

Let AB (Fig. 1. plate 8.), be an inflexible right line, and let it revolve on the point C, assumed any where between its extremities; then the point C about which it revolves, is called the centre of motion, and D immediately appled to it is the fulcrum. Now it is manifest that the spaces passed through, by the extremities A and B, in the revolution about C, are in the ratio of AC to BC; that is, AC : BC :: A*a* : B*b*; and if AB be a lever at first in an horizontal direction, and made to revolve about C, into the position *ab*; and if we let fall the perpendiculars *a*F, *b*E; then the triangles C*a*F, and C*b*E, are similar; hence as AC : BC :: FC : EC; that is, the points *a* and *b*, reduced to the horizontal right line AB, fall at F, and E.

And hence the spaces described by the extremities of the lever, are directly proportional to the distances from the fulcrum or centre of gravity: and consequently the momenta of the extremities, are in the same ratio, *i. e.* if the bodies be of equal weight; but if they be of unequal weight, then their momenta are proportional to their weights (or quantities of matter), multiplied into their velocities, or distances from the fulcrum or centre of motion. Whence follows,

Theorem 1.

That when two bodies or weights are suspended at the ends or extremities of a lever, so that if the weight of one of them being multiplied into its distance from the fulcrum, be exactly equal to the product of the weight of the other body into its distance from the fulcrum; then the momenta of both bodies are equal, *i. e.* these bodies are in *equilibrio*. On this theory is established the nature of the instrument called the *steel-yard*. The most exact experiments prove, that *one pound weight*, being suspended at 20 inches from the fulcrum of a lever, will exactly sustain, in equilibrio, a weight of 20*lb.* at one inch from the fulcrum. Moreover 20lb. suspended at 5 inches from the fulcrum, will only sustain 2*lb.* at the distance of 50 inches from the fulcrum. For $20 \times 5 = 50 \times 2 = 100$, the momenta.

Now in astronomy the fulcrum or prop is manifestly the *centre of gravity*. Thus if the Earth and Moon be conceived as two bodies suspended at the extremities of a lever: and if we suppose the weight (or density) of the Earth to be 49 times greater than that of the Moon; then the fulcrum or centre of gravity between these bodies is 49 times nearer the Earth than to the Moon. And it is about this centre that both revolve round the Sun.

Theorem 2.

In any two bodies, the common *centre of gravity* divides the right line joining their centres into two parts, which are proportional to the weights or quantities of matter in the bodies; but in an inverted order; *i. e.* the centre of gravity is as much nearer to the greater as the greater exceeds the less. Thus, if one body be 20 times heavier than another; then the distance of the centre of gravity from that body is 20 times nearer to it than to the other. And as we observed of the Moon and Earth; if the Moon be only equal to $\frac{1}{49}$ part of the Earth in density and quantity of matter; then the distance between their

centres being supposed to be divided into 50 equal parts; the right line joining their centres, will have the centre of gravity of these two bodies in it 49 times nearer the Earth than to the Moon; that is, the number 50 must be divided in the ratio of 49 to 1. And this is very nearly the exact distance. The demonstration is evident, from what has been said of the lever.

Problem.

Given the densities, or rather, the quantities of matter in any number of bodies, and their respective distances from each other, to find their *common centre of gravity*.

Solution.

Find the centre of gravity between any two of the bodies, by dividing the right line joining their centres in the ratio of their quantities of matter; so that the less distance be between the centre of gravity and the greater body: then join the centre of the third body and this centre of gravity by a right line, and conceive both to be as one body in their centre of gravity: let this right line be divided in the ratio of the sum of the quantity of matter in both, to that in the third body; observing that the point of division be nearer the greater, if its quantity of matter exceed that of the sum of the other two, as in the other case. Then this point of section will be the common centre of gravity of the three bodies.

After the same manner, conceive this centre of gravity to be the centre of another body, in which the quantity of matter of the three is united; and find the centre of gravity between a fourth body, and this aggregate, &c. And thus the common centre of gravity of the whole solar system may be found, provided we have the exact proportion of the quantities of matter in the Sun and planets, with their distance from him, given: and astronomers have determined the ratio of the quantities of matter in the Sun and each planet to great accuracy; and their respective distances, with still greater accuracy.

Now it is about the common centre of gravity of the whole system, that all the planets continually revolve. This point is not then the Sun's centre; but more or less distant from his centre, according to the position of the planets to the Sun and to each other: and the Sun himself revolves round this common centre of gravity of the whole system: but the quantity of matter in the Sun being at least one million times greater than the aggregate of the quantity of matter in all the planets, including their satellites; therefore the common centre of gravity of the whole system is never far distant from the Sun's centre; and at the maximum, it is at least one million of times nearer to his centre than to the common centre of gravity of all the planets. Moreover it is the centre of gravity between the Earth and Moon, that describes the Orbis Magnus, round the common centre of gravity of the whole system, and not the centre of the Earth; as astronomers are commonly wont to say: but the centre of the Earth, not being very remote from the centre of gravity between the Moon and Earth; and the common centre of gravity of the whole system being always near the Sun's centre; hence we commonly say, that the Orbis Magnus is described by the revolution of the centre of the Earth round that of the Sun. This idea then is to be restricted, and requires some correction; and it is the most delicate part of astronomical calculation, to determine with accuracy, the quantity by which the Sun and Moon's places are affected by the various positions of these centres of gravity. But of this we shall have occasion to speak more fully hereafter.

Definition.

There are four sorts of levers. 1. When the fulcrum is between the power and the weight. 2. When the fulcrum is at one end of the lever, and the power at the other end, and the weight somewhere between the ends or extremities of the lever. 3. When the fulcrum is at one extremity, the weight at the other, and the power is applied somewhere between the extremities: and this is called the *vectis impotens* or impotent lever. 4. The *crooked lever*, which, when reduced to a right line, becomes a lever of any of the three kinds, according to circumstances.

Theorem 3.

Let AB (fig. 2, 3, 4, plate 8,) be a lever: then fig. 2, is a lever of the first sort, C is the fulcrum: now if the weight W, suspended at the end A, be to the power P, suspended at the other extremity B, exactly in the ratio of BC, to AC; then the power will sustain the weight; and if the power be ever so little increased it will raise the weight, and the equalibrium will be destroyed. Again, fig. 3, is a lever of the second kind, having the fulcrum C at one extremity, and the power P suspended at the other extremity B, and the weight W, applied to the lever at A: then as before, if W be to P, as BC, to AC; the power will sustain the weight; and if the power be ever so little increased it will raise the weight. Moreover, fig. 4, is a lever of the third kind, having the fulcrum or support at one end C; the weight W, suspended at the other end A, and the power P, applied to the lever between its extremities: then if W be to P, as BC to AC; the

power will sustain the weight, and if the power be increased it will raise the weight. The demonstration of the three cases is evident, from what has been said of the laws of motion, &c.

Corollary.—The first and second sorts of levers are those which are only useful in mechanics; because, if the lever be long enough, a small power will sustain or raise a great weight. But the third sort is disadvantageous, and is never used but in cases of necessity; such as raising a ladder against a building, &c. There is one circumstance, however, common to all the mechanical powers; namely, that whatever is gained in momentum or power, is lost in velocity: or in other words, what is gained in power is lost in time.

Of the Composition and Resolution of Forces, &c.

THEOREM 4.

LET ABCD (Fig. 5.), be a parallelogram, and if a body at A, be acted upon separately by two forces, in the directions AB and AC, which would cause the body to be carried through the right lines AB and AC in the same time. Then both forces acting at once, will cause the body to be carried through the diagonal of the parallelogram.

DEMONSTRATION.

LET the right line AC be supposed to move parallel to itself, so that the point A be always in AB, whilst the body moves uniformly in AC; then when A arrives at *b*, the body will have moved to *a*; and because the motion of the body in AB and AC is supposed equable, and completed in the same time; hence, the parts A*b*, and *bd*, are in the ratio of AB and AC; that is, of AB to BD, because BD is equal to AC; and consequently the point *a*, is in the diagonal AD; wherefore the body will always be found in the diagonal, and move uniformly in it; and consequently it will arrive at the point D at the same instant of time, it would have arrived at B or C, if one of the forces only had acted. Q.E.D.

In illustration.—Let us suppose a ship to be sailing directly south, uniformly at the rate of eight miles an hour, and at the same time that there is a current running, directly west at the rate of six miles an hour, then the ship will be found to be 8 miles to the southward, and 6 miles to the westward at the end of every hour; and consequently the ship will have moved uniformly in the diagonal or hypothenuse, between the south and west, at the rate of 10 miles an hour; for the triangle is right angled and $8^2 + 6^2 = 100 = 10^2$.

Corollary.—Hence any single force AD, denoted by the diagonal of a parallelogram, is equivalent to two forces denoted by the sides AB and BD; and consequently any single force may be resolved into two forces, an infinite number of ways. Moreover the three forces in the directions of AB, AC, and AD, are in the ratio of AB, AC, and AD, respectively.

THEOREM 5.

IF three forces A, B, C, keep oneanother in equalibrio; they will be proportional to three sides of a plane triangle, drawn parallel to their several directions, DI, CI, CD. (Fig. 6.)

DEMONSTRATION.

LET DC represent the force C, produce AD, and BD, and complete the parallelogram DICH: then the force in the direction DC, is equal to the two forces DH, DI, in the directions DH, DI. Take away DC, and putting the two forces DH, DI equal thereto, the equilibrium will still remain. Therefore DI is equal to the force A opposite to it; and DH or CI equal to its opposite force B: and as CD represents the force C; the three forces A, B, and C, have the ratio to one another, that the three sides of the triangle DI, CI, and CD, have to each other. Q. E. D.

Note.—This theorem is easily illustrated by experiment.

Corollary 1.—Hence if three forces acting against one another, keep each other in equalibrio; these forces will be respectively, as the three sides of a plane triangle drawn perpendicularly to their lines of direction; because this triangle will be similar to the other whose sides are parallel to the lines of direction.

Corollary 2.—And because the sides of plane triangles, are directly proportional to the sines of their opposite angles: hence if three forces keep each other in equalibrio; then these three forces have the ratio to each other, which the sines of the angles opposite to the sides of the triangle, expressing the ratio of the forces, have to each other.

THEOREM 6.

IF a body impinge or act against any plane surface, it exerts its force in a line perpendicular to that surface.

DEMONSTRATION.

LET the body A (Fig. 7.) moving in the direction AB, with a given velocity, impinge on the plane surface FG at the point B. Draw AC parallel to FG, and BC, perpendicular to FG; and let AB represent the force of the moving body A. Then the force AB may be resolved into two forces equivalent to it, viz. AC, and CB. But the force AC is

parallel to the plane, and therefore cannot act upon it; whence the surface FG is only acted upon by the force CB, in a direction perpendicular to it. Q. E. D.

Corollary 1.—Hence, if a body impinge upon a plane surface, at right angles to that surface; then that surface receives the whole force of the stroke: but if it strike the surface obliquely; then the magnitude of the stroke upon the surface, will be to the whole force of the body, as the sine of the angle of inclination ABF or its equal BAC, is to the radius. And the magnitude of the force lost, to that of the whole, as the cosine of the inclination is to the radius.

Corollary 2.—And hence, if the angle of inclination be 45°, or half a right angle; then the surface receives such a part of the force of the moving body, as the square root of $\frac{1}{2} = .7071$, is less than 1. Or as the side of a square is to its diagonal.

Corollary 3.—If an elastic body A impinge upon a hard or an elastic plane FG; the angle of reflection will be equal to the angle of incidence.

For if AD be parallel to FG, the motion of A, in the direction of AD, is not at all changed by the stroke: and by the elasticity of one or both bodies, the body A is reflected back to AD, in the same time it moved from A to B; let it pass to D; then AB and BD are equal, being described in the same time, by the same uniform velocity; hence the two right angled triangles, ABC, DBC, have their hypothenuses equal, and the side BC common to both; wherefore the angle CBA is equal to the angle CBD; and consequently their complements ABF, and DBG are equal. Q. E. D.

Corollary 4.—Hence, the particles of light are perfectly elastic bodies; for all observations made on light falling on a plane surface, confirm the fact, that the angle of reflection is always equal to the angle of incidence. And this property of the particles of light is of the utmost importance in practical astronomy, navagation, optics, &c.

Of the Perpendicular Descent of Heavy Bodies.

Theorem 7.

The velocities of bodies falling freely by their own weight, from a state of rest, are directly proportional to the times of falling.*

Demonstration.

For because the force of gravity is the same in all places near the Earth's surface, and this is the force by which bodies descend. Therefore the falling body is urged by a force which acts constantly and equally; and if we suppose the time in which the body is falling, to be divided into equal and very small parts; and that gravity acting in the first small part of time, gives the body a certain velocity; then if gravity ceased to act, the body would continue to move on constantly in a right line towards the Earth's centre with that velocity: but since gravity acts constantly without intermission, the same force of gravity will communicate another equal degree of velocity in the second small part of time; and consequently the body will now have a double degree of velocity: and in like manner, in the third part of time, it will receive another equal increment of velocity, and then have a treble degree of velocity; and so on. And hence the velocity will increase continually with the time: that is, the velocity will be always proportional to the time of falling. Q. E. D.

Corollary.—Hence the motion of bodies falling freely by their own weight, from a state of rest, is a motion equally accelerated. And the motion of bodies thrown upwards is a motion equally retarded; for the force of gravity acting constantly and equally against the vertical motion, in equal times will equally diminish it, even till all the motion upwards is quite destroyed: and gravity will afterwards cause the body to descend; and at equal distances from the vertex, the velocity, in the ascent and descent, will be the same.

Theorem 8.

The space passed over by a body falling freely by its own weight from a state of rest is equal to half the product of the whole time in motion by the last acquired velocity: also the spaces passed over in any times, reckoning from the beginning of the fall, are directly proportional to the squares of those times; thus if a body fall from a state of rest 16 feet in one second of time†, in two seconds it will fall 4 times that space, *i. e.* 64 feet; in three seconds it will fall 9 times that space, *i. e.* 144 feet; and so on. That is, the spaces passed over, reckoning from the beginning of the fall, will be as the squares of the numbers, 1, 2, 3, 4, 5, &c. *viz.* 1, 4, 9, 16, 25, &c.

* This proposition is confined, and although it be physically true, yet it is not mathematically so; because the power of gravity diminishes from the Earth's surface to its centre where it is nothing: and in the hypothesis we suppose it to be always equally powerful; hence if the body fall but a small space, as a mile or two, the effect cannot be sensible: but if it were to fall many miles, as for instance one thousand, then indeed the truth of the proposition must fail. The resistance of the air also is a very sensible obstacle to bodies in motion, at or near the Earth's surface. But experiments shew, that when the resistence of the air is removed, the heaviest and lightest bodies fall with equal velocity, as in the experiment of a guinea and a feather, in an exhausted receiver.

† The space actually fallen through by a body in the 1st second of time is $16\frac{1}{12}$ feet, or 16 feet, 1 inch.

DEMONSTRATION.

For if we suppose the time to be divided into an indefinite number of infinitely small parts, or instants, as in the foregoing theorem, and m to be the velocity acquired in the 1st of these instants, then since the velocity increases as the time, the velocities at the end of the 2d, 3d, 4th, &c. instants, will be $2m$, $3m$, $4m$, &c. and as the velocity during any one instant may be considered uniform, and the spaces described by uniform motions are as the times and velocities conjointly, therefore the spaces described in the 1st, 2d, 3d, &c. instants will be $1m$, $2m$, $3m$, &c. and the sum of these will be equal to the whole space passed over; but these quantities constitute an arithmetical progression, whose 1st term is m, the last, or greatest term tm (putting t for the whole time) and the number of terms, t: now the sum of any arithmetical progression is found by multiplying the sum of the extremes by half the number of terms, therefore if s be put for the whole space passed over, then $s = \frac{1}{2} t (m + tm)$, but tm is evidently equal to the last, or greatest velocity, v, of the body, and m, the first velocity is infinitely small in respect of this, and may consequently be omitted in the equation $s = \frac{1}{2} t (m + tm)$ or $s = \frac{1}{2} t (m + v)$, which will therefore become $s = \frac{1}{2} tv$, as was to be proved. Moreover, since $v \propto t$*, and from what has been just shewn $s \propto tv$. therefore $s \propto t^2$. Q. E. D.

Corollary.—Hence the velocity acquired by a falling body, at the end of the fall, will be such, that if it continued to move uniformly with that velocity during the time of the fall, it would pass over a space equal to twice that already fallen through. For when a body moves uniformly, the space passed over is found by multiplying the time by the velocity; therefore, if, as before, the body be supposed to fall for the time, t, and v be the velocity acquired at the end of that time, then it would in an equal time move uniformly through the space tv with that velocity, which is double the space ($\frac{1}{2} tv$) passed through in the same time by the action of the accelerating force. Q. E. D.

THEOREM 9.

A projected body, whose line of direction is parallel to the horizon, describes by its motion the curve of a parabola†.

DEMONSTRATION.

LET us suppose that a heavy body, as for instance a cannon ball, is projected in the direction AD (Fig. 8.), parallel to the plane of the horizon; I say that the path of this heavy body moving in free space, is the curve of a parabola. For if gravity did not act, it would continue to move for ever in the same rectilinear direction AD; and pass over equal spaces AB, BC, CD, &c. in equal times; that is, the spaces passed over, in the direction AD, will be always as the times in which they are described; but since gravity acts equally and constantly every instant of time, it causes the body to descend; so that while the body by its projected rectilinear direction would have arrived at B, gravity, in the same time, causes it to fall to G: that is it falls through BG perpendicular to the horizontal line AD;

* The character $\propto$ is to be read, *varies as*, or, *is proportional to*; so $a \propto b$ signifies that a varies as, or is proportional to, b; that is, if a becomes A, and b becomes B, then $a : A :: b : B$.

† This proposition, though it be physically true, yet it is not mathematically so: because in our demonstration, we suppose that the right lines BG, CI, DL, &c. are all parallel to each other, and to the axis AK; but this is not exactly the case; because these right lines all tend towards the Earth's centre, in which if produced, they would meet. If, however, as in theorem 7, we suppose the distance and time of continuance to be but small; then the error is insensible; but if the magnitude of the distance and time of continuance of the motion be great; then the truth of the proposition manifestly fails: besides we suppose gravity always to act equally during the whole time of motion, but it does not; for at the Earth's surface, gravity is the most powerful, and its power diminishes inwards, from the surface towards the centre, and its power in this case, is always proportional to the distance from the centre: gravity also diminishes upwards from the Earth's surface inversely, as the square of the distance from the Earth's centre. On these several accounts, therefore, the path of a projectile cannot be a perfect parabola, even supposing the resistance of the air to be removed. There is another small deviation from mathematical accuracy, which even affects our reasoning in this note; for we have here supposed that all right lines drawn perpendicularly to the plane of the Earth's surface, or rather the horizon, meet in the centre; and this would certainly be the case, if the Earth were a perfect sphere; but it is not, for it is an oblate spheroid, as we have formerly stated; and consequently, right lines proceeding from various points, each perpendicular to the plane of the respective horizon, do not meet in the centre, but meet at various points more or less distant from it. And from all these considerations, it is obvious, that when we assert, that the path of a projectile is a parabola, the proposition is to be understood in a very limited sense; that is, both the time and space passed over must not be very considerable. There is, however, no machine with which we are acquainted, that can cast a body so distant or detain it so long above the Earth's surface as sensibly to affect the truth of the proposition. If we suppose the resistance of the air to be removed; even a cannon cannot project a ball, or a mortar cast a shell so far or sustain it so long above the surface, as to affect the certainty of the proposition, in any sensible degree.

We may here farther add, that if the body be projected in a direction *inclined* to the horizon, its path will still be in the curve of a parabola; (subject to the deviations above stated) but we have deemed it sufficient for our present purpose to take the case only of projection in a direction parallel to the horizon.

which space BG is the same as gravity would cause it to fall through, whether it had or had not the horizontal projectile direction. In like manner, when the body should have arrived at the points C, D, &c. it will be found at the points I, L, &c. the extremities of the perpendiculars CI, DL, &c. But the spaces passed over by a body falling freely by its own weight, are as the squares of the times, by the last theorem; hence BG, CI, DL, &c. or their equals AF, AH, AK, &c. are proportional to the squares of the times; but AB, AC, AD, &c. are directly as the times; or their equals, FG, HI, KL, &c. are as the times; and consequently AF is to AH, as the square of FG is to the square of HI; and AF is to AK, as the square of FG is to the square of KL, &c.: that is, the abscissæ are as the squares of their semi-ordinates, which is the characteristic property of the parabola; from what we have delivered in our conic sections. Wherefore the curve AGIL is a parabola. Q. E. D.

Theorem 10.

Gravity decreases according to the square of the distance from the centre; that is, gravity is reciprocally proportional to the square of the distance from the centre. Thus if a body weigh 16*lb.* on the surface of the Earth; then if it be carried upwards to twice the distance from the Earth's centre it will weigh but $\frac{1}{4}$ that weight; and if it be carried to three times that distance from the centre, it will weigh but $\frac{1}{9}$ of its real weight on the surface, &c. For instance let us suppose the radius of the Earth, in round numbers, to be 4000 miles; then the gravity of a body of 16*lb.* weight on the surface, is only equal $\frac{1}{4}$ of 16*lb.* or 4*lb.*, and if we suppose it to be carried upwards to the distance of 2 semi-diameters from the centre, or 8000 miles; and if to 3 semi-diameters, its gravity is only $\frac{16}{9}$*lb.*, or $1\frac{7}{9}$*lb.*, &c.*

Demonstration.

Let C be the centre of the Earth (Fig. 9.), ABD its surface; CR, CP, two rays, proceeding from the centre, and very near each other; take the distances AI, IL, LR, &c. each equal to CA the radius of the Earth; and about the centre C with the distance CI, CL, CR, &c. describe the small arcs IK, LO, RP, &c. which may be considered as right lines, as not differing sensibly from right lines; because the rays CR, CP are supposed very near each other, and AB, IK, LO, RP, &c. are small arcs of circles whose radii are nearly 4000, 8000, 12000, 16000, &c. miles. It is likewise manifest, that IK is double of AB, LO is treble of AB, RP is quadruple, &c. And if we conceive a cone of rays to proceed from the centre, having the point C for its vertex, and AB, IK, LO, RP, &c. its diameter; at the several distances from the vertex; then because the areas of circles are to each other as the sqares of their diameters; therefore the area of the base of the cone whose diameter is IK is 4 times the area of the base of the cone whose diameter is AB; in like manner, the area at L, is 9 times the area at A; that at R is 16 times the area at the surface: but the same quantity of rays, that is, of gravity, affect the base of the cone at A, the surface, as do affect the base at I; and hence the area of the base being 4 times greater there than at the surface, a given space at that distance can only receive $\frac{1}{4}$ the quantity of rays, that the same space would receive at the surface of the Earth; that is, a body is acted upon by gravity, by a force, at the Earth's surface, 4 times more powerful, than at twice the distance from the centre: for the same reason the power of gravity at the Earth's surface is 9 times greater than at the distance of 3 semidiameters from the centre; and so on. That is, universally the power of gravity is *inversely* as the square of the distance from the centre. Q. E. D.

Scholium.

Gravity is the great agent of nature, and all bodies in the universe are influenced by it. And the foregoing proposition exhibits the exact law by which gravity acts at different distances; and as the power of gravity decreases according to the square of the distance; hence in small distances its power is immense, and at great distances it becomes exceedingly small; and as its power does not decrease as the distance increases; but in a much higher proportion, namely in the ratio of the square of the

* This proposition is a fundamental one, in its various applications to practical astronomy; for by it we compare the power of gravity of the Moon and planets; and consequently their perturbating forces, by which they disturb each other's motions according to their different distances from the Sun and from each other, &c. And although the proposition be physically true, and can scarcely ever involve in it any sensible error, in its application to astronomy; yet it is not mathematically true; because in the demonstration, we suppose the whole gravity of the Earth to be collected together in the Earth's centre; but this is by no means the case: all the particles of matter of which the Earth is composed not only gravitate to each other, but likewise to all the other bodies in the universe. Thus every particle of matter in the Earth gravitates to the Moon; and the force of attraction of each particle is inversely as the square of the distance; and hence the matter immediately under the Moon gravitates much more powerfully towards her, than that at the opposite side of the Earth; and the matter in other situations must be more or less affected, by the mutual attraction of those parts of the Earth and Moon. The centre of attraction, therefore, is not exactly the centre of the Earth, but rather some point a little removed from it towards the Moon.

distance. The gravity of a body is not proportional to its magnitude, but to its quantity of matter; yet still, whether the body be heavy or light, at different distances from the centre of attraction, its gravity is always inversely proportional to the square of the distance from the centre. Sir Isaac Newton was the first who gave a perfect demonstration of this great law of nature: he conceived, that the same principle or power, which caused a stone or heavy body, when at liberty, to fall, or caused all bodies to have weight, and to exert themselves, at all times by a constant endeavour to descend towards the Earth, must be a principle or property, common to all matter in the universe; and this principle he called *attraction of gravitation.* He farther suggested that the Moon must be retained in her orbit round the Earth, by the same power of gravity; and that all the planets, in like manner, were kept in their orbits, by the same powerful agent. And by rational deductions from experimental philosophy, he established his hypothesis; and all observations made hitherto, on the motions of the celestial bodies, confirm his theory of gravitation, beyond all possibility of doubt. Indeed the Newtonian principles of gravitation are now established upon a basis as stable as the elements of geometry, or other eternal scientific truths.

Because *action* and *reaction* are always equal and contrary; hence the attraction of gravitation between the Earth and Moon is always equal and contrary; and let the quantity of the force, by which the Earth draws the Moon towards it, to retain her in her orbit, be what it may, by the very same force does the Moon act upon the Earth, and consequently urges the Earth towards her; and as they move towards each other with equal momenta, by reason of the equality of their mutual attraction of gravitation; hence the velocity of the Moon is to that of the Earth, as the quantity of matter in the Earth is to that of the Moon: and since the centrifugal force is opposed to the centripetal, these bodies do not move in a rectilinear direction towards each other, but in circular, or rather elliptical orbits round their common centre of gravity, and with velocities inversely as their quantities of matter; that is the radius vector of the Moon's orbit round the common centre of gravity, is always 49 times greater than that of the Earth, provided the quantity of matter in the Earth be 49 times that in the Moon. And in like manner, the Earth and Moon gravitate towards the Sun, and also towards all the planets; and if we suppose the whole quantity of matter in the Earth and Moon to be placed in their common centre of gravity; then this point moves towards the common centre of gravity of the whole solar system, with the same force that all the other bodies in the system do, towards that common centre of gravity of the whole system. But, as we observed of the Earth and Moon, the centripetal force is balanced by another; that is, by a centrifugal force, by which all the bodies in the system are drawn from their rectilinear motion, towards each other, namely, by their projectile force; and hence they are carried round in circular or elliptical orbits about the common centre of gravity of the whole system.

Now it is manifest, that the planets must greatly disturb each other in their respective revolutions round the Sun, or rather round the common centre of gravity of the whole system: and according to the foregoing theorem these disturbing forces, which are no other than the attraction of gravitation, must cause considerable perturbations in the motions of all the planets; as well in the secondary planets as in the primary ones: and the foregoing theorem exhibits the laws by which these perturbations are regulated. For, by that theorem, the force of gravity is always inversely as the square of the distance; and hence the nearer the bodies are to each other so much the more are their motions disturbed, by their gravitating towards each other.

The Newtonian theory of gravitation has, therefore, developed a new and a perfect system of astronomical calculus; entirely unknown to the ancients: and although this system involves in its operations many inequalities in the motions of the heavenly bodies, with which former astronomers were unacquainted, yet the Newtonian theory accounts for all these inequalities; and being aided by geometry and the other mathematical sciences, we can construct astronomical tables thereby, which shew the places of the planets for all times; and to a degree of exactness, almost beyond credibility.

The inequalities of the Moon's motions greatly perplexed the ancient astronomers; and to account for these inequalities, they were obliged to advance strange and absurd hypotheses, upon which they computed their lunar tables; yet still the results of their tables varied in many cases much from the truth and from actual observations. But the Newtonian theory of gravitation has dispelled the cloud of uncertainty, and developed the pure and simple operations of nature. To one simple property, to wit, to attraction of gravitation, common to all the bodies in the universe, and to every particle of matter in each of them, and probably imparted by the *Omnipotent Deity* in creation to all material bodies; to this wonderful agent, together with the projectile velocity communicated to each body by the same incomprehensibly *Great Being, in the Beginning,* do we owe all the various phenomena of the celestial bodies. By the projectile force the bodies are impelled eternally to move forward, and by the attraction

of gravitation they are retained in their orbits; and hence they continue to revolve in their orbits round the common centre of gravity of the whole system. And notwithstanding gravity produces considerable perturbations and changes in the motions of the planets. Yet the quantity of motion, that is, the momentum of force of the whole system, remains the same: and on account of the *vis inertiæ* of matter, this quantity of motion in the whole system must eternally remain the same: except it please the Omnipotent Deity to order it otherwise.

The Newtonian theory of gravity opens to our view, with what wonderful simplicity and wisdom the Deity has established the laws of nature. By gravity all the parts of the Earth are kept compactly together; and by its influence, the Earth assumes the spherical figure: and the like happens to all the other celestial bodies. By gravity, as we have mentioned before, all the celestial bodies are retained in their orbits; which prevents them from flying off into infinite space. In short, it is no other thing than gravity, that constitutes the weight of any body on our Earth: so that even our own bodies gravitate towards the Earth's centre: and being material heavy bodies, we are therefore kept by our weight to the Earth's surface, &c.

Theorem 11.

The areas, which a revolving body describes by radii drawn to a fixed centre of force, are always proportional to the times in which they are described; and are all in the same plane. That is, the radius vector of any planet, in its revolution round the common centre of gravity, always describes equal areas in equal times.

Demonstration.

Let S be the centre of force (Fig. 10.); and let the time be divided into very small portions. In the first portion of time let the body describe the space AB, by its projectile force; then in the second portion of time it would describe an equal space BK, in the same right line by its projectile force only acting. But since we suppose the central force to act from the beginning; therefore instead of the body being at K, at the end of the second portion of time, it is drawn towards S by the central force, and moves in the diagonal BC; and consequently the body is found at C, at the end of the second portion of time. Now the areas of the two triangles SAB and SBK are equal; because their bases AB and BK are equal, and they have the same altitude; to wit, a perpendicular drawn from S to AK (produced if need be); but the area of the triangle SCB is equal to the area of the triangle SBK; because the vanishing triangle BCK, being contained by three infinitely small right lines, can have no magnitude, nor affect the equality. Whence the triangle SBC is also equal to the triangle SAB. For the same reason, the triangle SCD, described in the third portion of time, is equal to SBC, that is to SAB. And in like manner are the triangles SDE, SEI, &c. described in the 4th, 5th, &c. portions of time, all equal to each other, and to SAB. Now let the number of triangles be increased, and their bases decreased, *in infinitum*; and the centripetal force will act continually, and the polygon ABCDEI, &c. will become a curve, and the areas will be proportional to the times of description; and consequently the radius vector of the revolving body will describe equal areas in equal times. Q. E. D.

Corollary.—1. Hence if a body revolve in the circumference of a circle, by reason of a projectile force, and a central one; then its velocity is always the same; because it describes equal sectors in equal times, but equal sectors stand on equal arcs; and hence equal arcs are described in equal times, and therefore the velocity is equable.

Corollary.—2. If a body revolve about one of the foci of an ellipsis its velocity is continually changing, and at one extremity of the greater axis (or transverse diameter) the velocity is the greatest, and the other extremity it is the least, and it is the same in opposite points of the orbit.

Corollary.—3. If a body revolve about one of the foci of an ellipsis, or any other point in any curve, as a center, then its velocity in that curve is always inversly proportional to the perpendicular drawn from the fixed point or centre, to the tangent of the curve in the given point. For, let a body revolve about the fixed point C (Fig. 11.), in the elliptical curve BDA, and let the two equal and very small triangular spaces *cba* and *cer* be described in equal times; then *ba* and *er*, are parts of the curve described in equal times: and therefore these spaces are proportional to the velocities with which they were described; from C let fall the two perpendiculars CD, CG, to the two tangents to the curve, which tangents are in the directions of the infinitely small parts of the curve *ba* and *er*; then the area of the triangular space $cba = ba \times \frac{1}{2}$ CD; and the area of $Cer = er \times \frac{1}{2}$ CG. But these areas are equal by the hypothesis, whence by putting them equal to each other, and omitting the common factor $\frac{1}{2}$, we have $ba \times CG = er \times CD$; which equation being turned into an analogy, gives $ba : er :: CG : CD$; but *ba* and *er* are as the velocities, when the velocities are inversely as the perpendiculars. Q.E.D.

Corollary.—4. The angular velocity at the center of force, is always inversely proportional to the square of the radius vector. For, let the two very small and

equal triangular spaces, *cnm* and *cop*, be described about C, in equal times; and about the centre, C, with the radius *cn*, describe the arc *nx*: then the arcs *nl*, *vx*, express the ratio of the angular velocity at the points *n* and *o*, because they are supposed to be described in equal times; and the areas of the sectors, *cnl* and *cvx* are to each other as these arcs; that is, the angular velocity at *n* is to the angular velocity at *o* as the sector *cnl* is to the sector *cvx*; but the area of *cnl* is equal to the area *coz*, by the hypothesis; (rejecting the infinitely small vanishing triangles *nlm* and *ozp*,) and the area of *coz* is to the area of *cvx* as the square of *co* is to the square of *cv*, or to the square of *cn*; and as the sectors express the ratio of the angular velocities; therefore the angular velocities are inversely proportional to the squares of the radii vectores. Q.E.D.

Scholium.

It seems necessary, in this place, to collect together, such parts of the foregoing principles of natural philosophy, as we shall have occasion to use in our subsequent astronomical demonstrations; to the end, that the student may the more conveniently refer to them, for evidence of the truths advanced. We therefore shall number these principles; and having already demonstrated them we shall now assume them as philosophical axioms.

Axioms.

1. All simple motion is rectilinear, and if a body be put in motion by a single impulse, it will continue to move for ever in the same right line with the same velocity; that is, it will pass over equal spaces in equal times; and its momentum will be proportional to the product of its velocity and quantity of matter.

2. A body being impelled by two forces, at the same time, but acting in different directions, does not move in the direction of either, but in a right line between them; and which right line is the diagonal of a parallelogram whose sides are in the direction of the two forces, and each in the ratio of its respective force. Thus, suppose a body receive an impulse capable of causing it to move 80 yards directly south in a second of time, and at the same time the body receives another impulse capable of causing the body to move directly east 60 yards in a second; then the body will move in the hypothenuse or diagonal of a parallelogram, between the south and east at the rate of 100 yards per second. For $\sqrt{80^2+60^2}=100$.

3. And hence any single force may be resolved into two or more forces, at pleasure.

4. If a body move from a state of rest, by means of a force acting constantly and equally in the same direction, then the velocity is always proportional to the time; and the whole space passed over is proportional to the square of the time. Moreover, the velocity acquired at the end of the time will be such, that a body moving uniformly with it during the whole time, would move over a space double of the other.

5. The centre of gravity between two bodies is always in a right line joining their centres, and its distance from each is inversely proportional to the quantity of matter in each. And if the bodies revolve round each other, then each body revolves round their common centre of gravity, and the greater body describes the less orbit. Also the momentum of both bodies is the same; and consequently their velocities are inversely proportional to their quantities of matter. That these principles hold good between the Earth and Moon, astronomical observations have clearly proved; and observations farther prove that the same laws of motion perfectly exist between Jupiter, Saturn, and the Georgian, and their satellites. And lastly, that the same general laws of motion take place, without the least deviation therefrom, between the Sun and all the planets moving round him. All are constantly moving round their common center of gravity.

6. The Force of gravity decreases in the same proportion that the square of the distance from the centre of force encreases; that is, the power of gravity is inversely proportional to the square of the distance. And hence the gravity of a body is directly proportional to its quantity of matter, and inversely as the square of its distance from the centre,

7. Any body revolving about a fixed centre, describes by the radius vector, equal areas in equal times; that is, the are as described by the radius vector are always proportional to the times of description.

This is a fundamental law in the motions of all the planetary bodies. Kepler was the first who discovered the existence of this universal law, but it appears that he was unable to demonstrate it. Sir Isaac Newton has since given a perfect demonstration of its absolute certainty.

8. The velocity of a body in any point of its orbit is always inversely proportional to the right line drawn perpendicularly from the centre of motion to a tangent to the curve or orbit, at the point in which the body is. And hence if a planet move in the curve of an ellipsis, its velocities in the perihelion and aphelion, are as the aphelion distance, to the perihelion distance; because these right lines are the perpendiculars to the tangents of the curve at the extremities of the major axis.

9. The angular velocity at the centre of force is always inversely proportional to the square of the radius vector.

Note.—We have taken it for granted that a body, moving first by a projectile force, in a rectilinear direction, and then constantly solicited by a central force to change its rectilinear direction, must necessarily move in a curve; but this is too evident to need any illustration.

Of Centripetal and Centrifugal Forces.

Definitions.

1. Centripetal Force is that force by which a body is impelled to a certain point as a centre. In this case, all the particles of which the body is composed are acted upon by that force.

2. Centrifugal Force is the reaction or resistance which a moving body exerts to prevent its being turned out of its direct course, and by which it endeavours to continue its rectilinear direction and uniform velocity. This force is opposite and equal to the centripetal force; because action and reaction are always equal and contrary.

For, since a body once put in motion endeavours to proceed uniformly in the same right line, it is manifest that no moving body can describe an orbit by its own simple motion, unless it is detained in that orbit by some force. For example, suppose a body revolving uniformly in the circumference of the circle ACE (fig. 12.), and when it comes to A, if the force that kept it in the orbit be taken away, it will still continue to move uniformly with its last velocity in the same direction, and consequently it will proceed in the tang. AB, and run out *in infinitum*. Therefore, that it may be detained in the circumference, it is necessary that some force should continually act, and which is equivalent to the force in A. Now the force which constantly solicits the body towards D, the centre of the circle, is what we call a centripetal force, and that force which constantly exerts itself to carry the body farther from the centre, is called the centrifugal force. Let the body by its projectile force describe the space AB in a very small portion of time, and let the central force acting for the same time, be capable of carrying the body through the infinitely small right line BC; then the body will describe the diagonal AC by these two forces jointly acting. And if we suppose the number of portions of time to be infinitely increased, the number of diagonals will also be infinite, and the figure becomes a polygon of an infinite number of sides, that is, it becomes a circle.

Now BC or *bc* may be taken as parallel to AE, because we have supposed them infinitely near to it, they are therefore perpendicular to the tang. AB. Moreover, BC or *bc* represents the centripetal and centrifugal forces; and it is evident that AB, which represents the projectile force, is infinitely greater than these; and although the centripetal or centrifugal force be infinitely less than the projectile, yet, as the projectile force is always a given finite quantity, and as the centripetal force continually acts in every infinitely small portion of time, the aggregate also becomes a given finite quantity in a given time, and therefore is sufficiently powerful to cause the body to revolve in an orbit.

In regard to the revolutions of the planets round the Sun, or the satellites about their primaries, it is gravity alone which retains them in their orbits; that is, their centripetal force is gravity: and their centrifugal force results from their projectile force, by which they resist the constant impulses of gravity with an equal and contrary force. For no body will change its present state of motion or rest without making a resistance equal to the force applied; or, as we have said before, action and reaction are always equal and contrary.

The orbits of the planets are ellipses; but since these ellipses differ but little from perfect circles, we shall, therefore, first ivestigate the laws that bodies would observe in circular orbits, and then treat of the same in ellipses, &c.

Theorem 1.

If a body revolve in the circumference of a circle, and is retained in it by means of a centripetal force tending to the centre of the circle. And if ACE (fig. 12.) be the circular orbit, whose centre, or point of attraction, is D, its diameter AE, and radius AD. Then putting R = AD, the radius F = the absolute force at the distance R; S = the space a body could descend through, by that central force, in the time t, and let $c = 3.1416$. The periodic time, or the time of one complete revolution in the circumference, will be as $ct\sqrt{\frac{2R}{s}}$ And the velocity or space it describes in the circumference in the time t, as $\sqrt{2Rs}$.

Demonstration.

Let AB be a tangent to the circle at the point A; take AC, an infinitely small arc, and draw BC parallel to AD, and CP perpendicularly to it. Now let the body descend by the central force, through the infinitely small space BC = AP, while by its projectile force it would have described the right line AB; consequently, by the two forces acting conjointly it describes the diagonal AC, or infinitely small arc AC, in the same time that BC or AP is described. But 2CR = the whole circumference of the circle. Moreover, we have demonstrated in our geometry, that the square of the chord AC, is equal to the rectangle of the diameter and versed

sin. AP, that is $AC^2 = 2R \times AP$, and $AC = \sqrt{2R \times AP}$. And because in falling bodies, the spaces passed over are proportional to the squares of the times or velocities, and consequently the square roots of the spaces are proportional to the times; hence as $\sqrt{S} : t :: \sqrt{AP} : t\sqrt{\frac{AP}{S}}$, the time of moving through AP or AC. And since the body is supposed to move uniformly in the circumference, therefore AC is to the time of its description, as the whole circumference to the time of one revolution, that is as $\sqrt{2R \times AP} : t\sqrt{\frac{AP}{S}} :: 2c\mathrm{R} : \frac{2ct\mathrm{R}}{\sqrt{2RS}} = ct\sqrt{\frac{2R}{S}}$, the whole time of one complete revolution. Q.E.D.

Also by the laws of uniform motion, as the time of describing $AC = t\sqrt{\frac{AP}{S}}$, is to $AC = \sqrt{2R \times AP} :: t : \sqrt{2RS}$, the velocity of the body, or space described in the circumference, in the time t. Q.E.D.

Corollary 1. Hence the velocity of the revolving body is equal to that which a falling body acquires in descending through half the radius AD, by the force at A uniformly continued; (and when a body revolves in a circle it is evident that the centripetal force is always the same). For as $\sqrt{S}$ (the height) : 2S (the velocity) :: $\sqrt{\frac{1}{2}R}$ (the height) : $\sqrt{2RS}$ the velocity acquired by falling through $\frac{1}{2}R$.

Cor. 2. Hence if a body revolve uniformly in a circle, by means of a given centripetal force; the arc which it describes, in a given time, is a mean proportional between the diameter of the circle and the space which the body would descend through in the same time, and with the same given force. For, 2R (the diameter) : $\sqrt{2RS} :: \sqrt{2RS} : S$, where $\sqrt{2RS}$ is the arc described and S the space descended through, in the time t.

Cor. 3. If a body revolve in any curve about a centre of force; then the velocity of the body in any point of that curve, is equal to the velocity of a body revolving in a circle whose radius is the radius of curvature. For the radius of curvature, at any given point in a curve is the radius of a circle whose circumference coincides with an indefinitely small part of the curve, at the given point, and consequently the velocity in the curve is equal to the velocity in the circle.

Theorem 2.

If several bodies revolve in circles round the same or different centres, then the periodic times will be directly proportional to the square roots of the radii, and inversely as the square roots of the centripetal forces; that is, the periodic times are directly proportional to the square roots of the quotients arising by dividing the radii by the centripetal forces, respectively.

Demonstration.

Let F = the centripetal force at A, (Fig. 12.) tending to the centre D of the circle V, = the velocity of the body, R = the radius AD, P = the periodic time. Then (by theorem 1), $P = ct\sqrt{\frac{2R}{S}}$, but S is proportional to F the force that generates it, and by substituting F for S, we have $P \propto ct\sqrt{\frac{2R}{F}}$, and since the factors, c, t, and $\sqrt{2}$, are constant given quantities, they may be rejected, hence we have P, always proportional to $\sqrt{\frac{R}{F}}$, or $P \propto \sqrt{\frac{R}{F}}$. Q.E.D.

Cor. 1. Hence the periodic times are directly proportional to the radii, and inversely as the velocities; that is, the periodic times are directly proportional to the quotients arising by dividing the radii by the velocities respectively. For (by theorem 1), $V \propto \sqrt{RS} \propto \sqrt{RF}$, therefore $V^2 \propto RF$; but $P \propto \sqrt{\frac{R}{F}}$, hence $P^2 \propto \frac{R}{F}$, therefore by multiplication we have $V^2P^2 \propto R^2$ and by extracting the square root of both sides $VP \propto R$; and $P \propto \frac{R}{V}$. Q.E.D.

Cor. 2. The periodic times are directly proportional to the velocities, and inversely as the centripetal forces. For $V^2 \propto RF$, hence $R \propto \frac{V^2}{F}$, and $\frac{R}{V} \propto \frac{V}{F}$. But $\frac{R}{V}$ is as the periodic time by (cor. 1), therefore $P \propto \frac{V}{F}$.

Theorem 3.

If several bodies revolve in circles round the same or different centres; then the velocities are directly proportional to the radii, and inversely as the periodic times; that is, the velocities are directly proportional to the quotients arising by dividing the radii by the periodic times respectively.

Demonstration.

For, using the same notation as before, we have, by theorem 1. $V \propto \sqrt{RS} \propto \sqrt{RF}$; and by cor. 2. theo. 2. $P \propto \frac{V}{F}$, or $PF \propto V$, and $F \propto \frac{V}{P}$; whence $V \propto \sqrt{RF} \propto \sqrt{R \times \frac{V}{P}}$, and $V^2 \propto \frac{RV}{P}$ and hence $V \propto \frac{R}{P}$. Q.E.D.

Cor. 1. The velocities are directly proportional to the products of the periodic times, and the centripetal forces respectively. For we have in the demonstration aforegoing $V \propto PF$.

Cor. 2. The squares of the velocities are directly proportional to the products of the radii, and the centripetal forces respectively: For we have also in the demonstration $V \propto \sqrt{RF}$, hence $V^2 \propto RF$.

THEOREM 4.

If several bodies revolve in circles round the same or different centres; the centripetal forces are directly proportional to the radii, and inversely as the squares of the periodic times; and if the centripetal force acting upon the bodies be similar to that of gravity, viz. if it be inversely proportional to the square of the distance from the centre, then the squares of the periodic times will be directly proportional to the cubes of the radii.

DEMONSTRATION.

Still retaining the same symbols as in the three foregoing propositions. Then by theo. 2. $P \propto \sqrt{\frac{R}{F}}$; hence $P^2 \propto \frac{R}{F}$, and $P^2F \propto R$, whence $F \propto \frac{R}{P^2}$ which is the 1st part of the theorem. Again, because the centripetal force is here by hypothesis, inversely as the square of the distance or radius; that is, directly proportional to the quotient arising by dividing 1, (unit) by the square of the radius, hence $F \propto \frac{1}{R^2}$ but F, it has been shewn, is universally as $\frac{R}{P^2}$, therefore $\frac{1}{R^2} \propto \frac{R}{P^2}$, and hence $P^2 \propto R^3$; that is, the squares of the periodic times are directly proportional to the cubes of the radii, or distances from the centre. Q.E.D.

THE following Lemmata will enable us to determine the circumstances attending the motion of bodies in *elliptical orbits*.

1. If AB (Fig. 13.) be any straight line inclined to the plane XZ, and if perpendiculars from every point of it, to the plane XZ, form the line *ab*, then AB will be to *ab*, as radius to the cosine of the inclination of AB to the plane XZ. For produce BA, *ba*, to meet in O, then the angle BO*b* is obviously the inclination of BA to the plane XZ, and since OB*b*, OA*a* are similar right angled triangles, therefore OB : OA : : O*b* : O*a*, and by division of proportion AB : OB : : *ab* : O*b*, whence by alternation AB : *ab* : : OB : O*b*, but by trigonometry OB : O*b* : : rad. : cos. BO*b*, therefore by equality AB : *ab* : : rad. : cos. BO*b*.

2. Let ABCD be a rectangle whose side AC is parallel to the plane XZ, and let it be projected into the figure *abcd*, by perpendiculars as before, then, it is plain, that the projection *abcd*, is also a rectangle, and that its breadth *ac* is equal to AC, but its length *ab*, is diminished, as already shewn in the ratio of radius : cosine of the angle BO*b*, therefore the area is diminished in the same ratio, that is, area of rect. ABCD : area of rect. *abcd* : : rad. : cos. BO*b*.

3. The area of *any plane figure* projected in the manner already stated will be diminished in the ratio of radius : the cosine of the angle of inclination of the plane of the figure, with the plane of projection. For, if we conceive the area of the figure divided into an indefinitely great number of small portions, or elements, by equidistant parallel lines, at right angles to the intersection of the plane of the figure with the plane of projection, each of these elements may be regarded as a rectangle whose breadth is the distance between any two adjacent parallels, and its length equal to that of either parallel (for these lengths differ by a line that may become less than any assignable one) and by Lemma 2. each of these rectangles, is to its projection, as radius, to the cosine of the angle of inclination of the plane of the figure, and that of projection, therefore the area of the whole figure, is to the area of its projection in the same ratio.

4. Now suppose a circle to be inclined to the plane XZ, and let an indefinitely great number of ordinates be drawn to that diameter of it which is parallel to the intersection of the plane of the circle, and the plane XZ then by Lem. 1. the projection of each of these ordinates will be diminished in the *constant* ratio of radius, to the cosine of the inclination of the plane of the circle, and the plane XZ, therefore it follows from the cor. to prop. 4. of the conic sections, that the circumference of the circle will be projected into an ellipsis, whose transverse diameter is to its conjugate in the ratio just stated.

5. The projection of any square circumscribing the circle will be a parallelogram. For the planes passing through opposite sides of the square are parallel, and therefore their intersections with the plane of projection will be parallel.

6. The areas of all the parallelograms circumscribing an ellipsis are equal. From what has been shewn, it follows that the ellipsis may be conceived to be the projection of some circle, and any parallelogram circumscribing it as the projection of some one of the squares that may circumscribe the circle, but the areas of all the squares drawn to circumscribe the same circle are equal, and by Lem. 3. the area of each parallelogram is in a *constant* ratio to that of its corresponding square, consequently the areas of the parallelograms are all equal to each other.

7. If instead of supposing *that* diameter of the circle projected which is parallel to the intersection of the plane of the circle and the plane of projection as in Lem. 4. we imagine any other diameter of the circle and its ordinates to be projected, the projections of these ordinates will have a constant ratio to their corresponding ordinates in the circle, for each of these latter is obviously inclined in a constant angle to the plane of projection; also the projections of the several abscissæ, are in a constant ratio to the abscissæ in the circle, but the squares of the ordinates in the circle are equal to the rectangles under

their respective abscissæ, therefore also the squares of the projections of these ordinates will be *as* the rectangles under the projections of the abscissæ. Now tangents at each extremity of the diameter of the circle are manifestly projected into tangents to the ellipsis parallel to the projections of the ordinates, these projections are therefore according to def. 4. of the ellipse, ordinates to that diameter of the ellipsis which is the projection of the diameter of the circle, and hence the squares of the ordinates to *any* diameter of the ellipsis are in a constant ratio to the rectangles under their abscissæ; but at the centre of the ellipsis this ratio becomes that of the square of one semi-diameter to the square of its conjugate: therefore the square of any diameter of an ellipsis is to the square of its conjugate, as the rectangle under the abscissæ of that diameter, to the square of the ordinate.

8. If from any point, P, in the periphery of an ellipsis (Fig. 14.) straight lines PS, PH, be drawn to the foci, and from the centre C, the semi-diameter CD, be drawn parallel to the tangent at P, then will the intercepted portion PE of the greater focal distance SP, be equal to the semi-transverse CA. Through H draw HK parallel to CD. By prop. 6 of conic sections and prop. 11. of the geometry, the angle PKH is equal to the angle PHK, and therefore the line PK is equal to PH; also because CS is equal to CH, and CE parallel to HK, therefore SE is equal to EK, consequently PE $= \frac{1}{2}$(PK + PS) $= \frac{1}{2}$(PH + PS) $=$ (by def. 1. ellipse) AC.

9. The distance of either focus, S (Fig. 14.) from any point, P, in the periphery of an ellipsis, is to its distance ST, from a tangent to that point, as the semi-diameter CD, parallel to the tangent, is to the semi-conjugate axis CB. For draw PF perpendicular to DC, then from the similar triangles SPT, EPF, SP : ST : : PE : PF, that is, by Lem. 8. SP : ST : : AC : PF. Now the rectangle contained by CD and PF is equal to one fourth of a parallelogram circumscribing the ellipsis, and touching it at the points P, D, and therefore by Lem. 6, CD . PF $=$ AC . BC, whence by resolving this into a proportion AC : PF : : CD : BC, and comparing this with the last proportion, we have by equality SP : ST : : CD : BC.

10. If P be any point in the periphery of an ellipsis (Fig. 15.) to which PT is a tangent, S one of the foci of the ellipsis, and Sp be drawn near to SP, and meeting the tangent in F, also pM perpendicular to SP, then if SpF be supposed to move towards SP, the limit of the ratio of the rectangle under the latus rectum and pF, to the square of pM, will be a ratio of equality: that is if L $=$ latus rectum, the ratio L.pF : pM^2 continually approaches to a ratio of equality, as p moves towards P.

Draw CD, and *por* parallel to PT, also the diameter PCN; let ST be perpendicular to TP, and suppose CB the semi-conjugate axis. And since pr is parallel to the tangent at P, pr is an ordinate to the diameter PN, therefore by Lem. 7. CP^2 : CD^2 : : Pr . Nr : pr^2, hence $\frac{CD^2}{CP^2} = \frac{pr^2}{Pr.\ Nr}$; also from the similar triangles Pro, PCE, PE : PC : : Po : Pr, that is, by Lem. 8. AC : PC : : Po : Pr, whence $\frac{PC}{AC} = \frac{Pr}{Po}$ therefore by multiplying the corresponding sides of this equation and the last together, we obtain $\frac{DC^2}{AC.PC} = \frac{pr^2}{Po.Nr}$ and multiplying by Nr, $\frac{CD^2.Nr}{AC.PC} = \frac{pr^2}{Po}$. Now the limit of N$r$ is obviously 2PC, therefore the limit of $\frac{CD^2.Nr}{AC.PC}$ is $\frac{2CD^2}{AC}$, and consequently the limit of $\frac{pr^2}{Po}$ is $\frac{2CD^2}{AC}$, but the angle FSP being evanescent, the limit of the ratio of Po : pF is a ratio of equality, as likewise the limit of pr : po,* wherefore also the limit of $\frac{po^2}{pF}$ is $\frac{2CD^2}{AC}$. Again, from the similar triangles SPT, *op*M, we have SP^2 : ST^2 : : po^2 : pM^2 and : : $\frac{po^2}{pF} : \frac{pM^2}{pF}$ and by Lem. 9. SP^2 : ST^2 : : CD^2 : CB^2 and : : $\frac{2CD^2}{AC} : \frac{2CB^2}{AC}$: therefore by equality $\frac{2DC^2}{AC} : \frac{2CB^2}{AC}$: : $\frac{po^2}{pF} : \frac{pM^2}{pF}$: but it has been demonstrated that the limit of $\frac{po^2}{pF}$ is $\frac{2CD^2}{AC}$, therefore the limit of $\frac{pM^2}{pF}$ is $\frac{2CB^2}{AC}$; now $\frac{2CB^2}{AC}$ is equal to L the latus rectum by cor. to prop. 1. conic sections, therefore L is the limit of $\frac{pM^2}{pF}$ and L.pF continually approaches to equality with pM^2. Q. E. D.

Of the Motion of Bodies in Elliptical Orbits.

THEOREM 5.

IF a body revolve in an ellipsis by means of a centripetal force tending to the focus, the force varies inversely as the square of the distance of the body from the focus.

* Since the angle SPC, or o Pr is a *finite* angle, it may perhaps be at first supposed by some readers that the ratio of or: pr will always be finite, and therefore that the ultimate equality of po and pr is inadmissible; but the case is not so, as may be thus shewn. Let Pr $= x$, $pr = y$, 2PC $= a$, 2CD $= b$, then from the general relation of the abscissa and ordinates to oblique diameters, see Lem. 7. $a^2 : b^2 : : ax - x^2 : y^2$ and $y^2 = \frac{b^2}{a^2}(ax - x^2)$ hence $x : y : : y : \frac{b^2}{a^2}(a - x)$, let x and y each diminish to o, then the ratio to which they approach is that of o : $\frac{b^2}{a^2} \times a$, or $o : \frac{b^2}{a}$, an infinite ratio, consequently x, or Pr, ultimately bears no assignable ratio to y, or pr. Now the ratio of or: Pr is *constant* and *finite*, because the triangle o Pr always continues similar to EPC, therefore, also or, ultimately bears no assignable ratio to pr, that is, the limit of the ratio of pr : po is a ratio of equality.

DEMONSTRATION.

LET APH (Fig. 15.) be the ellipsis, S the focus to which the body is solicited, and about which it revolves, let the body be at P, at which point let PT be a tangent, draw SF indefinitely near to SP and complete the construction as before described to this figure. Then if the centripetal force were constant, the distance pF, through which the body is drawn from the tangent in any time, t, would be as the square of the time, as follows from reasoning similar to that in theo. 8. mechanics, and if the force vary, and the time be constant it is manifest the space passed over will vary as the force, therefore, generally, pF varies as the centripetal force, and square of the time conjointly. Now by theo. 11. mech. the time is as the area of the sector SPp, therefore $p\mathrm{F} \propto$ centrip. force $\times$ sector $\mathrm{SP}p^2$ that is as centrip. force $\times \mathrm{SP}^2 \times p\mathrm{M}^2$. But by Lem. 10. $p\mathrm{F} \propto \frac{p\mathrm{M}^2}{\mathrm{L}}$, therefore $\frac{p\mathrm{M}^2}{\mathrm{L}} \propto$ centrip. force $\times \mathrm{SP}^2 \times p\mathrm{M}^2$ or dividing by $p\mathrm{M}^2$, $\frac{1}{\mathrm{L}} \propto$ centrip. force $\times \mathrm{SP}^2$ hence the centrip. force $\propto \frac{1}{\mathrm{L} \cdot \mathrm{SP}^2}$ that is, (since L is constant) $\propto \frac{1}{\mathrm{SP}^2}$. Q.E.D.

THEOREM 6.

IF the centripetal force varies inversely as the square of the distance from the focus, and several bodies revolve in different ellipses about a common focus, the areas of the sectors described by the radii vectores in the same time, are directly proportional to the square roots of the latera recta of each ellipsis respectively: and the velocity of each body at any point of its orbit, is directly proportional to the square root of the latus rectum of that orbit and reciprocally as the perpendicular from the focus to the tangent at that point.

DEMONSTRATION.

Referring again to fig. 15. let APH be one of the orbits, S the focus, and centre of force, and the same construction being made as before, also putting L for the latus rectum of this orbit, we have by Lem. 10. $\mathrm{L} \times p\mathrm{F} = p\mathrm{M}^2$, but the time being supposed given pF is as the central force, which is also by hypothesis as $\frac{1}{\mathrm{PS}^2}$, hence $\frac{\mathrm{L}}{\mathrm{PS}^2} \propto p\mathrm{M}^2$, and $\mathrm{L} \propto \mathrm{PS}^2 \times p\mathrm{M}^2$, or $\sqrt{\mathrm{L}} \propto \mathrm{PS} \times p\mathrm{M}$; but PS $\times$ pM is equal to twice the area of the sector SPp, therefore the area of the sector SPp is as $\sqrt{\mathrm{L}}$.

Again, the velocity in an indefinitely small portion of time is as the arc described, Pp; and since SPT, and PpM, are similar right angled triangles, $\mathrm{ST} : \mathrm{SP} :: p\mathrm{M} : p\mathrm{P}$, whence $\mathrm{ST} \times p\mathrm{P} = \mathrm{SP} \times p\mathrm{M}$, but $\sqrt{\mathrm{L}} \propto \mathrm{PS} \times p\mathrm{M}$ as above, consequently $\sqrt{\mathrm{L}} \propto \mathrm{ST} \times p\mathrm{P}$ and $p\mathrm{P} \propto \frac{\sqrt{\mathrm{L}}}{\mathrm{ST}}$. Q. E. D.

THEOREM 7.

THE areas of the ellipses are in the ratio compounded of the square roots of the latera recta of the orbits, and the periodic times; and the squares of the periodic times are directly proportional to the cubes of the semi-transverse axis, or mean distances.

DEMONSTRATION.

FOR let a be the area of a sector of any one ellipsis described in a given time, 1, A = the area of the whole ellipsis, L = its latus rectum, P = periodic time, and M = mean distance; then as $a : \mathrm{A} :: 1 : \frac{\mathrm{A}}{a}$ the time of describing the whole area = P, but by the last theo. $a \propto \sqrt{\mathrm{L}}$, hence $\frac{\mathrm{A}}{\sqrt{\mathrm{L}}} \propto \mathrm{P}$ and $\mathrm{A} \propto \mathrm{P}\sqrt{\mathrm{L}}$; again, let N = the semi-minor axis of the ellipsis whose semi-transverse axis is M, and because the latus rectum is a third proportional to the transverse and conjugate axes, $\mathrm{L} = \frac{2\mathrm{N}^2}{\mathrm{M}}$, hence $\mathrm{L} \propto \frac{\mathrm{N}^2}{\mathrm{M}}$; now $\mathrm{M} \times \mathrm{N} \propto \mathrm{A}$, and, as has just been shewn $\mathrm{A} \propto \mathrm{P}\sqrt{\mathrm{L}}$, therefore by substitution $\mathrm{M} \times \mathrm{N} \propto \mathrm{P}\sqrt{\frac{\mathrm{N}^2}{\mathrm{M}}}$ and by squaring, and dividing by N^2, $\mathrm{M}^2 \propto \frac{\mathrm{P}^2}{\mathrm{M}}$, whence $\mathrm{M}^3 \propto \mathrm{P}^2$, that is, the squares of the periodic times are as the cubes of the mean distances. Q. E. D.

Cor. 1. Hence $\mathrm{P} \propto \mathrm{M}^{\frac{3}{2}}$ or $\sqrt{\mathrm{M}^3}$, that is, the periodic times are directly proportional to the square roots of the cubes of the mean distances.

Cor. 2. Also $\mathrm{M} \propto \mathrm{P}^{\frac{2}{3}}$ or $\sqrt[3]{\mathrm{P}^2}$, that is, the mean distances are directly proportional to the cube roots of the squares of the periodic times.

Cor. 3. If the mean distances in two ellipses be equal, the periodic times will be equal, whatever be the ratio of the conjugate axes.

Cor. 4. Hence if one of the ellipses becomes a circle whose radius is equal to the mean distance in the ellipsis, the periodic time in the ellipsis will be the same as in the circle.

We may now take a short view of the two most remarkable and fundamental propositions that have been demonstrated relating to bodies attracted to a centre of force, and of the consequences to be drawn from them. These propositions are, 1st. The areas described by lines drawn from a body to an immoveable centre of force, about which it revolves, are proportional to the times of description; and 2ndly. If bodies revolve either in circles or ellipses, and are attracted to the centres of the circles, or to one of the foci of the ellipses, by forces which vary inversely as the square of the distance of the body from the centre of force, then the squares of the periodic times are as the cubes of the radii of the circles or as the cubes of the mean distances in

the ellipses. The sagacious and indefatigable Kepler was the first who discovered that these are the very laws which are observed by the planets in their motions round the Sun; he likewise found that the orbits of the planets were not perfect circles as had formerly been supposed, but ellipses having the Sun in one of their foci; he first conjectured this to be the case with regard to the orbit of Mars, and having *assumed* that orbit to be an ellipsis, he calculated upon this hypothesis several distances of Mars from the Sun at various times, and had the satisfaction of finding them exactly agree with the distances deduced from the observations of the astronomer Tycho Brahé. Upon succeeding so well with the orbit of Mars he applied himself to those of the other planets, and soon found that the hypothesis of the elliptical orbit corresponded likewise with their motions. Kepler having observed also that the velocities of the planets when in their apsides are inversely as their distances from the Sun, immediately saw that they must describe in *those* points of their orbits areas round the Sun proportional to the times, and from farther calculations and observations he became assured that the 1st of the two main propositions above noticed was applicable to the planetary revolutions, and that the planets describe about the Sun in *all* parts of their orbits areas proportional to the times. This great man, who must be regarded as the father of the modern astronomy, had a singular and unceasing ardour for discovering analogies and harmonies in nature, and although this passion frequently led him into visionary and erroneous conceits, we are perhaps indebted to it for the knowledge of the 2d principal law; some years after the discovery of the 1st, the fanciful imagination of our astronomer led him to compare the powers of the mean distances of the planets with those of the periodic times, when after a long course of tedious computation he was rewarded with the discovering, upon May 15th, 1618, of the proportionality of the cubes of these distances to the squares of the periodic times. With the accuracy of a mathematician he has noted down the precise day upon which he was enabled to find this analogy. Kepler, however, although he had by unwearied industry, and comparing together a great number of astronomical observations, satisfied himself of the existence of these remarkable laws respecting the planetary motions, was unable to trace them as the effect of any common principle, this was reserved for the calm and penetrating mind of Newton, who by a profound analysis discovered, and by a sublime geometry demonstrated, that these would be exactly the circumstances that would occur to bodies acted upon by a gravitating or central force varying inversely as the square of the distance, combined with a tangential or projectile force; he moreover found that the force of gravity as observed on the surface of the Earth, was precisely that which would retain our Moon in her orbit, and cause her to make her periodic revolution in the time which we observe she does; from all which he justly concluded that all the primary planets are retained in their orbits by a force which is no other than an attracting, or gravitating force towards the centre of the Sun, or nearly so, and varying inversely as the square of the distance, and that the secondary planets while they accompany their respective primaries in their revolutions round the Sun, and partake in a measure of his attractive force, are preserved in their orbits by the superior attractive force of their respective primaries. The universal gravitation of all the bodies in the universe towards each other according to the reciprocal ratio of the square of the distance, is now therefore universally acknowledged by astronomers, and while upon some occasions there have appeared slight variations in the motions, whose accordance with this principle has not been at first apparent, yet has it always been found upon more mature reflection, and as the progressive improvements in the modern, or fluxional analysis, have enabled mathematicians more accurately to compute all the circumstances relating to the attractive influence of the planets on each other, that these variations have been effects which ought to be expected to take place upon this principle, and that all the minute disturbances in the planetary motions are deducible from this great and fundamental law.

From what has been demonstrated in theorem 7, it follows that if the mean distance from the Sun, (or rather from the focus) of any one of the planets be known, then the mean distances of all the other planets may be found. For the times of their periodic revolutions are determined by observation, therefore, by proportion. As the square of the periodic time of revolution of that planet whose mean distance is known, is to the square of the time of the periodic revolution of any other planet; so is the cube of the given mean distance, to the cube of the required mean distance of that other planet; the cube root of which will be the mean distance itself; and being doubled, will be the transverse, or greater axis of the orbit of that planet. Also the *relative* distances of all the planets from the Sun are easily determined by this method, by *assuming* the mean distance of any one of them, as for instance, the Earth, to be given, as suppose equal to 100000 of any equal parts, and then proceeding by the foregoing proportion, by which we shall have the relative distances of the other planets expressed in the same equal parts.

Example.—Agreeably to accurate astronomical observations, our sidereal year, or the time of the periodic revolution of the Earth round the Sun, is 365*d.* 6*h.* 9*m.* 12*s.*; and the time of Mercury's periodic revolution, 87*d.* 23*h.* 15*m.* 35*s.* Now suppose the mean distance of the Earth from the Sun, or the semi-major axis of the Earth's orbit, to be 100000 equal parts; it is required to find the semi-major axis of the orbit of Mercury, that is, his mean distance from the Sun.

$$\left.\begin{array}{rl} \oplus & 365d.\ 6h.\ 9m.\ 12s. = 31558152 \\ \mercury & 87\ 23\ 15\ 35 = 7600535 \end{array}\right\} \text{Seconds.}$$

Then. As $\overline{31558152}^2 : \overline{7600535}^2 :: \overline{100000}^3$: the cube of the semi-major axis of Mercury's orbit; or by alternation, as $\overline{31558152}^2 : \overline{100000}^3 :: \overline{7600535}^2$: the cube of the semi-major axis of Mercury's orbit.

Now to render the arithmetical operation shorter and more convenient, it will be best to divide the second term by the first, and then to extract the cube root of the quotient; and this cube root becomes a general multiplier; and the cube root of the square of the time of any planet's periodic revolution in seconds being multiplied by this general multiplier, the product is its mean distance sought. Thus,

$$\text{Earth's assumed dist.} = \overline{100000}^3 = 1.004099,$$
$$\text{per. time in seconds} = \overline{31558152}^2$$

whose cube root is 1.001365, which is a general multiplier.

Then the cube root of the square of 7600535 = 38657.12. And 38657.12 × 1.001365 = 38709.886, or omitting the fraction, 38710 equal parts, which is the mean distance of Mercury from the Sun, the Earth's mean distance being assumed at 100000. And in the same easy manner, if we take the cube root of the square of the periodic time of revolution of any other planet, in seconds, and multiply it by the foregoing general multiplier, we shall obtain the mean relative distance of that planet from the Sun.

Having obtained the mean relative distances of the planets from the Sun, (or rather from the common centre of gravity of the whole solar system,) we immediately obtain the true distance of each from the Sun; the true distance of the Earth from him being first ascertained. Now by the last transits of Venus, the Earth's true distance appears to be 95173128 English miles; (see our plane trigonometry;) then say as 100000 is to the relative distance of Mercury 38710, so is 95173128, to 36841517 English miles, which is Mercury's mean distance from the Sun. And in the same way do we find the absolute mean distance of each of the other planets from the Sun.

THEORY OF THE ELLIPTICAL ORBIT, ON MATHEMATICAL PRINCIPLES;

With Methods for Finding the Places of the Sun and Planets, at any given Time, by Arithmetical Calculations.

DEFINITIONS.

1. As all the orbits of the planets are ellipses differing, some more and some less, from perfect circles; of course each orbit has a transverse and a conjugate diameter; or, as astronomers frequently call them, a major and a minor axis; the transverse diameter or major axis of the orbit is called the line of the *apsides;* and either extremity of the line of the apsides, is called an *apsis.*

2. Aphelion is that apsis which is farthest from the Sun. And perihelion is the other apsis, which is the nearest point of the orbit to the focus in which the Sun is supposed to remain.

3. So in like manner, in regard to the Earth, we say that the Sun is in apogee or perigee, when he is farthest from or nearest to the Earth; that is, we say that the Sun is in the apogee, when the Earth is in the aphelion; and that he is in the perigee, when the Earth is nearest to him, or in the perihelion.

4. And because the Moon's orbit about the Earth is an ellipsis; hence we likewise call the major axis of her orbit, the line of the apsides; and the farther apsis the apogee, and the nearer the perigee.

5. Let APB (Fig. 16.), be the orbit of a planet, S the focus (or place of the Sun), C the centre, A the aphelion, and P the planets place in its orbit. Then the angle PSA, is called the *true anomaly;* that is, the true anomaly of a planet is the angle at the focus, contained between its radius vector and the line of its apsides, reckoning from the aphelion. In the Comets, however, the anomaly is reckoned from the perihelion.

6. If, when the planet is in A the aphelion, the right line LS be supposed to coincide with the line of the apsides, and then to move uniformly about the focus S, so as to complete a revolution exactly in the same time that the planet completes its revolution in its orbit, and that during the time the

planet is moving in its orbit from A to P, the right line SL has revolved into the position SL. Then the angle LSA is called the *mean anomaly* of the planet: and the angle LSP, which is the difference between the *true* and *mean* anomaly, is called the *Equation of the centre*, or *prosthapheresis*.

7. If about the centre of the ellipsis C, with the radius CA, we describe the circle ANB; and suppose that a body moving uniformly in the circumference of this circle, completes its revolution exactly in the same time that the planet completes its revolution in its orbit, and if CD be drawn parallel to LS; then the angle DCA is equal to the angle LSA; wherefore the angle DCA is equal to the *mean anomaly*; and consequently the difference between DCA and PSA is equal to the *equation of the centre*. Through P draw NI at right angles to AB; and join SD, SN, and CN. Then the angle NCA is called the *excentric anomaly*.

Problem 1.

To find the excentricity of the Earth's orbit, and its form, supposing the semi-major axis to be 100000, or truly 95173128 English miles.

Solution.—*First Method by Astronomical Observation.*

As the orbit of the Earth is an ellipsis, and the centre of the Sun is nearly in one of the foci. By a good instrument (a micrometer) measure the Sun's diameter when he is in the apogee and perigee (which is at present about the 29th of June, and 29th of December); then divide the major axis 200000 in the ratio of these two measures of his diameter, and half the difference of the parts gives the excentricity sought. Moreover the greater part is the aphelion distance, and the less that of the perihelion; also the square root of the product of the two parts, is the semi minor axis.

Example.—From a mean of several observations accurately taken at the royal observatory, the Sun's diameter in the apogee is $31' 31''.0 = 1891''.0$, and in the perigee $32' 35''.6 = 1955''.6$. It is required to find the aphelion and perihelion distance, the excentricity of the orbit, and the minor axis; the major axis being supposed 200000.

Operation.

Greater diam. ⊙ =	1955″.6
Less diam. ⊙ =	1891 .0
Sum	3846 .6

As 3846″.6 : 200000 :: 1955″.6 : 1016794 = app. dis.
3846.6 : 200000 :: 1891.0 : 98320.6 = peri. dis.

2)3358.8

Half difference = 1679.4 = excen.

And $\sqrt{101679.4 \times 98320.6} = 99985.8$ = semi-minor axis by prop. 1. Conic Sections. And 199971.6 the minor axis itself.

This computation is founded upon the law in optics that the tangents of the angles under which an object appears are reciprocally as the distances of the object from the eye, and since in very small angles the ratio of the arcs which measure the angles may be taken for that of the tangents, the distances of the Sun at the two times of observation will be in the inverse ratio of the observed angles. Hence, if this method could be depended on; we have obtained the principal elements of the terrestrial orbit by it: and we have to remark, that we have found the ratio of the two axes and the excentricity by it, sufficiently near to the truth, to shew, that the terrestrial orbit differs but little from a perfect circle.

This method is simple and expeditious; but the results obtained by it are by no means sufficiently exact, to be taken as elements for the construction of astronomical tables: the reason is, because the diameter of the Sun is too small to find the diameters of the Earth's orbit by it: and moreover, the spheroidal figure of the Sun, must affect the measure of his diameter in a small degree, because we may not measure the very same diameter in both cases, and if so, then some small error must in consequence arise in the calculation: the refraction also must considerably affect the observation, for the lower limb of the Sun has a greater Elevation by refraction than the upper limb has; and the only way we can do in this case is always to measure his diameter horizontally: lastly no micrometer or instrument perhaps ever was made that will measure an angle to the tenth part of 1″ of a degree; and this exactness even were we sure it was correct, would not answer accurate astronomical purposes.

2d. Method.

Find the Sun's motion in right ascension for the space of 1, 2, or 3 days when he is in his apogee and perigee, and reduce the two quantities of right ascension (both being for the same space of time) into motion of longitude, by spherical trigonometry. Then the square roots of these two quantities of longitudes are in the ratio of aphelion and perihelion distances; because the angular velocities in the aphelion and perihelion are in the inverse ratio of the

squares of the aphelion and perihelion distances; and consequently the square roots of the velocities (that is, of the measures of the angles described in the same time), are inversely proportional to the aphelion and perihelion distances. And having the ratio of the aphelion and perihelion distances, the minor axis and excentricity will be found as before.

And here it is to be noted, that the Sun's motion in right ascension, at both times, must be obtained from a good astronomical clock shewing siderial time, accompanied by a transit instrument, at an observatory.

Example.—By taking a mean of several observations made at the royal observatory, of the Sun's motion in right ascension, on the days of his being in the apogee and perigee, and reducing the same into motion of longitude, we have found that in a solar day, mean time, he moved 61′ 10″.08 in perigee, and 57′ 11″.76 in apogee. From hence it is required to find the excentricity and minor axis of the orbit; the major axis being taken at 200000?

In peri. 61′ 10″.08 = 3670″.08, its sq. root is, 60.58118
apogee 57. 11.76 = 3431.76, its sq. root is, 58.58123

Sum, 119.16241

as 119.16241 : 200000 : : 60.58118 : 101678.3 = aph. dis
58.58123 : 98321.6 = peri. dis

2) 3356.7

Half difference, 1678.3 excentri.

And $\sqrt{101678.3 \times 98321.6} = 99985.9 =$ the semi minor axis, and 199971.8 = the minor axis itself.

This method, like the former one, obtains results not exact enough for the construction of astronomical tables; because the Sun's motion for 1 day is too small to compare it with that for half a year; and if we take the motion for 2 or 3 days, in apogee and perigee, then we necessarily make use of numbers too distant from the maximum and minimum in the apsides, which are the ratios alone that we have occasion for: the results, however, in both methods, are of considerable importance, as affording assistance in more correct methods for obtaining the elements of the orbit.

3d. Method.

Let the greatest equation of the centre be determined, by taking the difference between the Sun's mean longitude and his true longitude for three or four days, at the time the equation is the greatest; then the equation must be corrected for the lunar equation, the aggregate of the effects of the planetary perturbations, and the nutation of the Earth's axis &c. Now all these minor corrections, that is, the aggregate amount of them, should be applied to the Sun's true longitude with a contrary sign, to that used in reducing the mean longitude to the true; and then the difference of this and the mean longitude is the equation of the centre. And it will appear that the equation increases every day, till it arrives at the maximum, after which it begins to decrease daily; and the exact quantity of the greatest equation must lie between those for the two days, on which it is greatest; and the greater of these two must be increased by the equation of second difference (see our rules for interpolation); and then you will have the greatest equation of the centre correctly. Moreover the Sun's true longitude must be deduced from actual observations of his right ascension, taken at an observatory, as we have said in our 2d. method. Also, if a circle be described about the focus in which we suppose the centre of attraction (the Sun) is lodged, with the radius which is a *mean proportional* between the major and minor semi axis; then the area of this circle is equal to the area of the ellipsis: because the product of the two extremes is equal to the square of the mean; and the area of an ellipsis is equal to the area of a circle, when the product of the two axes is equal to the square of the diameter of the circle; and the same is evidently true in regard to their halves. Hence it is evident, that the point in the orbit in which the circle and ellipsis cut each other, is the planet's place when the equation of the centre is the greatest.

Now let APB, (Fig. 17.) be the elliptical orbit, and PLS a sector of the circle whose area is equal to that of the ellipsis; then the point P is the planet's place when the equation is the greatest. It is also manifest that in orbits nearly circular (as is the case in the planetary orbits), that the point P is nearly at the extremity of the minor axis. Now the same construction being made as in definition 5, 6, and 7, let ST be drawn perpendicularly to NC produced: and because the point P is nearly at the extremity of the minor axis, the angle NCA and its equal SCT are nearly right angles; and consequently ST is nearly equal to SC the excentricity: also the angle NSA will be nearly equal to PSA, both being nearly right angles. Now the angle NCA — NSA (PSA,) = SNC; and DCA — NCA = DCN; add these two equations together (the angle NCA being affirmative in the one and negative in the other therefore destroyed), and we have DCA — PSA = DCN + SNC; which (as NC is nearly parallel to DS,) is nearly equal to twice the angle DCN; that is, the difference between the *mean and true anomaly*, or greatest equation of the centre, is nearly equal to twice the angle DCN, or to twice the arc DN, which measures the angle DCN; now the arc DN is exactly equal to ST, which is nearly equal to SC

the excentricity, as we prove thus. Let p = the periodic time of description both in the circle and ellipsis; and t = the time in which the planet moved from A to P in the ellipsis, and also in which the body moved from A to D in the circle: then, as the area ADC : the area of the whole circle :: $t : p$; the area of the ellipsis : the area ASP : $p : t$, also* the area of the circle : the area of the ellipsis :: area ASN : area ASP; therefore the area ADC : ASP :: the area ASN : ASP; and because the areas ADC and ASN have the same ratio to the same quantity ASP, they are therefore equal to one another; take away the area ACN, which is common to both, and there remains the area DCN = SNC; but DCN = $\frac{1}{2}$DN×CN, and SCN = $\frac{1}{2}$ST×CN; hence, $\frac{1}{2}$DN×CN = $\frac{1}{2}$ST×CN; wherefore DN = ST.

Lastly, as the arc DN, which (as before shewn) is equal to half the greatest equation, is nearly equal to SC the excentricty of the orbit; we have only to find the length of that arc in parts of the radius or semi-major axis. Now as 3.141592 : .5, the radius, :: 360° : 57° 17′ 44″.8, the length of an arc equal to the radius of the circle; and as 57° 17′ 44″.8 : half the greatest equation :: 100000, the semi-major axis, : the excentricity.

Example.—From a mean of many observations, the greatest equation of the centre, in regard to the Earth's orbit, was found to be 1° 55′ 28″; the excentricity to the mean distance 100000 is required.

2)1° 55′ 28″ 57° 17′ 44″.8 = 206264″.8

Half = 0 57 44 = 3464″.0

As 206264.8 : 3464.0 :: 100000 : 1679.395 = the excentricity sought. And hence the aphelion and perihelion distances, and the minor axis, may be found as before; thus,

100000.000 mean distance, or semi-major axis.
±1679.395 excentricity.

101679.395 aphelion distance.

98320.605 perihelion distance.

$\sqrt{101679.395 \times 98320.605}$ = 99985.903 the semi-minor axis; and 199971.806 = the minor axis. Hence we find, that the minor axis is so nearly the same in the three foregoing methods, that the results of the 3d method may be relied on. But the greatest equation has a secular diminution, at present, of 17″.18 in 100 years; and hence the excentricity is also now diminishing.

SCHOLIUM.

THIS 3d method, though it is not mathematically true, yet in orbits in which the greatest equation of the centre does not exceed 3 or 4 degrees, the error in excentricity will always be less than $\frac{1}{1000000}$ part of the semi-major axis. In the orbits of the Earth and Venus, the error is quite insensible; in those of Jupiter, Saturn, and the Georgian, it is but small; in that of Mars it is something more; but the orbit of Mercury differs so much from a circle, that the excentricity found by this method must be corrected; and we shall shew how to find the correction and to apply it; immediately after we have solved Kepler's problem, which is no more than from proper data, to find the difference between the true and mean anomaly, or the equation of the centre at any given point in the orbit.

4th METHOD, *by Astronomical Observations on the Planet Mars.*

THE periodic times of the revolutions of Mars and the Earth being known: let ABCD (Fig. 18.) be the Earth's orbit; M the place of Mars; S the focus of the orbit, in which we suppose the Sun is placed. Observe the exact time when Mars is in opposition to the Sun, in which case the Earth is at A, in the right line joining the centres of the Sun and Mars, or nearly in that line. Then the place of Mars must be observed after he has completed his period (of 687 days), when he will return to the same point in his orbit M: but the Earth will only be at B in its orbit, from which point the Sun will be observed in the line BS and Mars in the line BM: at that time observe the places of the Sun and Mars; that is, their angular distance: which if it cannot be taken with an instrument is easily calculated, by having the right ascensions taken of both on the same day. Then in the plane triangle SMB, there will be given the two angles MSB and MBS, and consequently the third angle SMB; and if we assume MS = 100000 equal parts; then as sine angle MBS : sine angle SMB :: MS : SB; hence SB will be known. Again, when Mars has finished another revolution, and the Earth is at C; then the line SC may be found the same way. And in like manner when Mars has completed a third revolution, we can find SD. Then we have three right lines given in magnitude and position, drawn from the focus, to find the position and magnitude of the major and minor axis of the ellipsis that shall pass through the three given points B, C, and D. And hence the place of

* From what we have demonstrated in our Conics it follows, that the area of the elliptical segment API is to the area of the circular segment ANI as PI is to NI; and moreover the area of the triangle PSI is to the area of the triangle NSI as PI is to NI; whence, by addition, the area of the circle is to the area of the ellipsis as ASN is to the area ASP.

the aphelion and perihelion will be determined by this method.

We should now proceed to the arithmetical computations of the elements of the ellipsis; but we have to lament, that we are not at present, furnished with any astronomical observations sufficiently accurate to exemplify the method. We are therefore obliged to omit the numerical operation.

Problem 2d.

To find the place of the Sun's apogee and perigee, that is, the place of perihelion and aphelion of the Earth; and the time when the Sun is in these points.

Note.—When the Sun is in apogee, as seen from the Earth among the fixed stars in the Heavens; the Earth, as seen from the Sun, is among the same fixed stars at the time it is in the perihelion.

1st Method, *by Astronomical Observation.*

Measure the Sun's diameter by a good micrometer, about the time the Earth is at the mean distance, when his diameter increases and decreases most; and this is about the 1st of April, and 1st of October. Then half the interval, between the times when his apparent diameter was exactly the same in both observations, being added to the time of the first observation, will give the time when the Sun was in apogee.

Example.—Suppose, that on the 1st of April (1810) I found the Sun's diameter at noon, to measure $32'\ 2''.4$; and on the 1st of October following, at noon, it measured $32'\ 2''.2$; and on the 2d day at noon $32'\ 2''.7$. Required the time that the Sun was in apogee, and the *mean place* of the apogee.

1st October,	$32'\ 2''.2$	1st April,	$32'\ 2''.4$
2d October,	32 2.7	1st October,	32 2.2
daily increase,	= .5	difference,	= .2

As .5 : .2 : : 24*h*. : 9*h*. 36*m*. Hence on the 1st October, at 9*h*. 36*m*. P. M. the Sun's diameter was equal to his diameter on the 1st April, at noon. Now the interval between these is 183*d*. 9*h*. 36*m*; the half of which 91*d*. 16*h*. 48*m*. being added to 1st April, gives July 1*d*. 16*h*. 48*m*. for the time the Sun was in apogee. Or 181*d*. 16*h*. 48*m*. from the beginning of the year, which is January 1st, at noon.

And as 365*d*. 5*h*. 48*m*. 48*s*. : 181*d*. 16*h*. 48*m*. :: 360° : 179° 5′ 32″, which added to 280° 28′ 30″ the mean longitude of the Sun for the 1st January 1810 at noon, gives 99° 34′ 2″ (rejecting 360°) or 9° 34′ 2″ ♋ for the mean longitude of the aphelion.

This method is true in theory; but the observations must be repeated for a vast number of years before any astronomical accuracy can be deduced therefrom; because the daily change in the Sun's diameter is too small a quantity to find the exact time and place of the apogee therefrom. Yet in the space of a century, a conclusion may be inferred, that shall be sufficiently exact for finding the mean anomaly, so as to ascertain thereby the Sun's semidiameter and some other minor requisites for several centuries.

2d Method, *by Astronomical Observation.*

Find the Sun's motion in right ascension for the space of two or three days near to 1st April, and the 1st October; and reduce the same into motion of longitude, correcting the same for the lunar equation and the effects of the perturbations of the planets, &c. (as mentioned in method 2 of problem 1). Then interpolate the exact time on the 1st October, that the motion in longitude is exactly the same as his motion on the 1st April at noon; and half the interval of these times applied to 1st April, as in the former method, gives the true time of the apogee. The mean longitude is also found as in the former method.

An example to this method would be so like to that of the former, that we deem it unnecessary to give one. We have only to remark, that it is by this method chiefly, and by taking a mean of many results, that astronomers have ascertained, to the greatest exactness, the time and place of the Sun's being in apogee and perigee. There is a third method, which is the same as our fourth method of finding the excentricity of the orbit, by observations on the planet Mars, which see. This third method, although strictly mathematical is nevertheless attended with many uncertainties and troublesome calculations, which render it a way little practised by modern astronomers: at the same time it is useful, in corroboration of the results of the other methods.

The place of the apogee and perigee has a motion of about $0'\ 11''.8$ annually; that is, the line of the Earth's apsides is revolving about the Sun at the ratio of $0'\ 11''.8$ annually, or 0° 19′ 35″, in 100 years, easterly; and if we include the precession of the equinoxes, the motion in longitude for 100 years is 1° 43′ 35″ easterly.

Lemma.

If C be any angle, then rad.^{2} : tang.$^2\frac{1}{2}$ C :: rad. + cos. C : rad. — cos. C.

Referring to Fig. 2. plate 6, the construction of which has been already described in art. 6, page 117, and supplying the line OB, which is omitted, the triangle BDO will be similar to LTA, for they are right angled at D and T, and the angle OBD or OBA + OAB = a right angle = OAB or LAT + ALT,

therefore the angle OBA is equal to ALT, and AL : LT :: BO : BD, but since the angle OAB or LAC at the circumference, is half of OCB, at the centre, Prop. 17. Geom.: the chord OB is equal to twice the sine LT, hence, calling the angle OCB, C, the last proportion becomes rad. : sin. $\frac{1}{2}$C :: 2sin. $\frac{1}{2}$C : rad — cos. C, or $\frac{1}{2}$ rad : sin. $\frac{1}{2}$C :: sin. $\frac{1}{2}$C : rad. — cos. C.

Again, from the similar triangles AOD, ATL, AL : AT :. AO : AD; now if a perpendicular were drawn from C to AL, the distance of the intersection of this perpendicular from A would be the cosine of LAC, and equal to the half of AO, because AOB is also a right angle, and AC is equal to CB; hence AT (which is likewise the cosine of LAC) = $\frac{1}{2}$AO, and the above proportion is the same as rad. : cos. $\frac{1}{2}$C :: 2cos. $\frac{1}{2}$C : rad. + cos. C or $\frac{1}{2}$ rad. : cos. $\frac{1}{2}$C :: cos. $\frac{1}{2}$C : rad. + cos. C, therefore cos.^{2} $\frac{1}{2}$C = $\frac{1}{2}$ rad. (rad. + cos. C), and from what has been already shewn, sin.^{2} $\frac{1}{2}$C = $\frac{1}{2}$rad. (rad. — cos. C), wherefore by division $\frac{\text{sin.}^2\,\frac{1}{2}\text{C}}{\text{cos.}^2\,\frac{1}{2}\text{C}} = \frac{\text{rad.} - \text{cos. C}}{\text{rad.} + \text{cos. C}}$; but by art. 1. Lemma 2. page 117, $\frac{\text{sin.}\,\frac{1}{2}\text{C}}{\text{cos.}\,\frac{1}{2}\text{C}} = \frac{\text{tan.}\,\frac{1}{2}\text{C}}{\text{rad.}}$ therefore $\frac{\text{tan.}^2\,\frac{1}{2}\text{C}}{\text{rad.}^2} = \frac{\text{rad.} - \text{cos. C}}{\text{rad.} + \text{cos. C}}$ which is equivalent to the proportion to be demonstrated.

If radius be taken = 1, then tan.^{2} $\frac{1}{2}$C = $\frac{1 - \text{cos. C}}{1 + \text{cos. C}}$

Problem 3d.

Given the semi major-axis, the excentricity, the periodic time, and time of motion from the aphelion, of a planet; to find its *true and mean anomaly*, and its distance from the Sun; and also the equation of the centre, or the difference between the true and mean anomaly.

Solution. 1*st Method.*

Let APB (Fig. 17), be the orbit of a planet, P its place; and the same construction remaining, as in Method 3, Problem 1; it has there been demonstrated that the right line ST, is always equal to the arc DN. And because the periodic time and time in motion are given; hence, as the periodic time is to the time in which the planet moved from A to P in the ellipsis, or from A to D uniformly in the circle; so is 360°, to AD, which arc measures the angle ACD the mean anomaly; whence the mean anomaly is given. And because CS the excentricity, and AC the semi-major axis, are given; and the degrees of an arc equal to the radius AC = 57° 17′ 44″.8; hence, as AC : CS :: 57° 17′ 44″.8 : the length of CS in degrees. Now assume the arc AN, which measures the angle ACN or its equal SCT, as near to the truth as you can judge from the excentricity of the orbit and mean anomaly. Then by plane trigonometry, as radius is to sine of SCT, so are the degrees in SC, to the degrees in ST = DN: then add the degrees in DN to those assumed in AN; and if the sum be equal to the degrees in AD the mean anomaly, then the assumed angle ACN is correct: but if not, which is generally the case; repeat the operation, using the last found angle instead of the assumed one, till you obtain a result exactly agreeing with the mean anomaly; and then you have the exact quantity of the angle ACN the excentric anomaly.

Next, to find the *true anomaly*. Let v be the other focus of the ellipse, and suppose the semi-major axis AC = 1, and because by prop. 16. geometry, SP^2 = SI^2 + PI^2, and Pv^2 = vI^2 + PI^2, therefore SP^2 — Pv^2 = SI^2 — vI^2 and by prop. A. page 104 (SP + Pv) × (SP — Pv) = (SI + vI) × (SI — vI) = (because Sv is bisected in C,) 2SC × 2CI; hence, SP + Pv : 2CI :: 2SC : SP — Pv, or 2 : 2CI :: 2SC : SP — (2 — SP) that is by division 1 : CI :: SC : SP — 1; whence SP = 1 + CS × CI = 1 + CS × cos. ACN. But by the Lemma, tan.^{2} $\frac{1}{2}$ASP = $\frac{1 - \text{cos. ASP}}{1 + \text{cos. ASP}}$; also, by plane Trigonometry, SP or 1 + CS × cos. ACN : rad. (1) :: SI or CS + cos. ACN : cos. ASP which last is therefore equal to $\frac{\text{CS} + \text{cos. ACN}}{1 + \text{CS} \times \text{cos. ACN}}$. Hence by substitution, tan.^{2} $\frac{1}{2}$ASP is equal to $1 - \frac{\text{CS} + \text{cos. ACN}}{1 + \text{CS} \times \text{cos. ACN}}$ divided by $1 + \frac{\text{CS} + \text{cos. ACN}}{1 + \text{CS} \times \text{cos. ACN}}$ which quotient is $\frac{1 + \text{CS} \times \text{cos. ACN} - \text{CS} - \text{cos. ACN}}{1 + \text{CS} \times \text{cos. ACN} + \text{CS} + \text{cos. ACN}} = \frac{1 - \text{CS} + \text{cos. ACN}\,(\text{CS} - 1)}{1 + \text{CS} + \text{cos. ACN}\,(\text{CS} + 1)} = \frac{\text{BS} - \text{cos. ACN} \times \text{BS}}{\text{AS} + \text{cos. ACN} \times \text{AS}} = \frac{1 - \text{cos. ACN}}{1 + \text{cos. ACN}} \times \frac{\text{BS}}{\text{AS}}$ = by the Lemma tan.^{2} $\frac{1}{2}$ACN × $\frac{\text{BS}}{\text{AS}}$, consequently $\sqrt{\text{AS}}$: $\sqrt{\text{BS}}$ :: tan. $\frac{1}{2}$ACN : tan. $\frac{1}{2}$ASP, that is, *as the square root of the aphelion distance, is to the square root of the perihelion distance; so is the tangent of half the excentric anomaly, to the tangent of half the true anomaly.*

The distance from the Sun, or radius vector is obtained from the expression for it in the aforegoing investigation, viz. SP = 1 + CS × cos. ACN or more generally SP = AC + CS × cos. ACN, but if the angle ACN be between 90° and 270°, that is, if the planet be more than 3 and less than 9 signs from the aphelion, then cos. ACN is negative, and SP = AC — CS × cos. ACN, so that, *the radius vector is equal to the semi-major axis increased or*

diminished by the product of the excentrictiy and cosine of the excentric anomaly according as the planet is in the 1st and 4th, or 2d and 3d quadrants from aphelion.

Lastly. Having the true and mean anomaly, subtract the less from the greater, and the remainder is the *equation of the centre.*

Corollary.—As the body which is supposed to move uniformly in the circle ADB and to set out with the planet at A, coincides with it again at B, and as at A the planet moves with its least angular velocity (cor. 3. theo. 11. page 150) therefore from A to B or in the first six signs of anomaly, the angle ACD will be greater than ASP, or the mean anomaly will be greater than the true, consequently the equation of the centre must be subtracted from the mean anomaly to obtain the true: and *vice versâ* in the other half of the orbit.

Scholium.

This is called Kepler's problem, because he first proposed it; but it appears, that he knew of no way to come at a direct solution, but solved it by tedious operations and comparisons of results. Now although the preceding method of solution is by approximation, yet it is both easy and expeditious, and may be carried to any degree of accuracy: it is therefore that which is used by modern astronomers in computing the equation of the centre. We shall, nevertheless, give another method, which is direct and strictly mathematical; because it is important and satisfactory, to compare the results of different methods. But it will be necessary first to exemplify our 1st Method.

Example 1st. *In the Earth's Orbit.*

Given the major axis of the Earth's orbit = 100000; excentricity = 1681.3 periodic time or anomalistic year = 365*d.* 6*h.* 14*m.* 2*s*, and time of motion from the aphelion = 32 days. Required the *true and mean anomaly*, the equation of the centre, and the Earth's distance from the Sun: and supposing the true longitude of the apogee to be 3*s.* 9° 30′ 00″, the Sun's true longitude is also required, exclusive of the minor equations.

Operation.

As 365*d.* 6*h.* 14*m.* 2*s.* : 32*d.* :: 360° : 31° 32′ 21″ = the mean anomaly. And as 100000 : 1681.3 :: 57ᵍ 17′ 44″.8 : 0ᵍ 57′ 48″ = 3468″.0, the arc of a circle equal to the excentricity. Now it is evident that the excentric anomaly is about half a degree less than the mean anomaly; whence let us assume the excentric anomaly = 31ᵍ 0′ 0″; and the operation will stand thus,

Assumed excent. an. AN	31	0	0	log. sine	9.7118893
Arc = excentricity			3468″	log.	3.5400791
ST = DN	0	29	46 = 1786″	log.	3.2519184
Add AN	31	0	0		
1st approx. to AD	31	29	46		
Mean anomaly	31	32	21		
1st error, too small +	0	2	35		
	31	0	0		
AN once corrected	31	2	35	log. sine	9.7123820
Arc = excentricity			3468″	log.	3.5400791
ST = DN	0	29	48 = 1788″	log.	3.2524611
Add AN	31	2	35		
2d approx. to AD	31	32	23		
Mean anomaly	31	32	21		
2d error, too great —	0	0	2		
	31	2	35		
AN twice corrected	31	2	33	log. sine	9.7123750
Arc = excentricity			3468″	log.	3.5400791
ST = DN	0	29	48 = 1788″	log.	3.2524541
Add AN	31	2	33 = excentric anomaly		
3d approx. to AD	31	32	21 = mean anomaly exactly.		

Semi. major axis	100000.0		
Excentricity ±	1681.3		
Aphel. dist.	101681.3	half its log.	2.5036205
Perihel. dist.	98318.7	half its log.	2.4963181

Then as $\sqrt{101681.3}$		log.	2.5036205
Is to $\sqrt{98318.7}$		log.	2.4963181
So is tan. ½ exc. an.	15 31 16.5	log.	9.4436134
To tan. ½ true an.	15 16 28.5	log.	9.4363110
	2		
True anomaly	30 32 57		
Mean anomaly	31 32 21		
Equation of the centre	0 59 24		

Excentricity	1681.3	log.	3.2256452
Excentric anomaly	31° 2′ 33″	log. cos.	9.9328719
	1440.5		3.1585171
Add sem. maj. axis	100000.0		
Rad. vector SP	101440.5		

Mean long. of apogee	99°	30′	0″				
True anomaly	30	32	57				
Sun's true long.	130	2	57 =	4	10	2	57

Hence the mean anomaly is 31° 32′ 21″, true anomaly 30° 32′ 57″, equation of the centre 59′ 24″, radius vector, or the Earth's relative distance from the Sun, 101440.5, and the Sun's longitude (exclusive of the minor equations) 4*s*. 10° 2′ 57″.

2d Method.—*Of finding the Excentiric Anomaly.*

This method was first given by Dr. John Keill, and depends upon the first two terms of a series, the rule deduced from which is direct and very concise, and the results obtained by it extremely accurate. We are obliged to omit the investigation of theformula, because it involves a considerable knowledge of fluxions and infinite series, which are subjects that we do not suppose those for whom our work is principally designed to be much acquainted with.

Let s and c represent the sine and cosine of the mean anomaly to radius 1, e the excentricity, $a =$ the semi-major axis, and $r = 206264.8$, the length of an arc, expressed in seconds, which is equal to the radius, then the arc DN is equal to

$$r\left[\frac{es}{a + ec} - \tfrac{1}{2}\cdot\left(\frac{es}{a + ec}\right)^3 - \&c.\right]$$

or, which is better suited to computation,

$$\mathrm{DN} = \frac{esr}{a + ec} - \frac{1}{2r^2}\cdot\left(\frac{esr}{a + ec}\right)^3 - \&c.$$

Hence this practical rule,

Add the log. of the excentricity to the log. cosine of the mean anomaly, find the corresponding natural number, and to it add the semi-major axis, subtract the log. of this sum from the sum of the log. of the excentricity, the log. sine of the mean anomaly, and the constant log .5.3144251 ; the remainder is the log. of the arc DN in *seconds*, or of the difference between the mean and excentric anomaly. This takes in only the 1st term of the series, which will be sufficiently correct for orbits of small excentricity such as that of the Earth, but if the excentricity be considerable, the arc DN must be corrected by means of the 2d term ; thus,

Call the last found logarithm, A, then from three times A subtract the constant log. 10.9298802, the remainder is the log. of a natural number, which is to be subtracted from DN in order to correct it.

Note. 5.3144251 is the log. of 206264.8 $= r$, and 10.9298802 is the log of $2r^2$ or 2 the log. 5.3144251 increased by 0.3010300, the log. of 2.

Operation.—*Taking the same example as before.*

Mean anom. 31° 32′ 21″	log. cos. 9.9305837	log. sin. 9.7185692	
Excen. . . . 1681.3	. . log. . . . 3.2256452	Excen.	3.2256452
Nat. num. . 1433	3.1562289	con. log.	5.3144251
S. ma. ax. 100000			8.2586395
101433 subtract its logarithm			5.0061792
nat. number 1788″ = 29′ 48″ = DN			3.2524603
			[A]

From the mean anomaly	31° 32′ 21″
Subtract arc DN	0 29 48
remains excen. anom. . .	31 2 33

the same as by the 1st. method.

The computation of the true anomaly, &c. would be the same as before.

We shall now calculate the correction of DN by the 2d part of the Rule, in order to see whether it be necessary to notice it.

A	3.2524603
	3
	9.7573809
Subtract const. log.	10.9298802
Nat. num. .0672″ —	2.8275007

Hence, in the terrestrial orbit, this corection may safely be omitted, for in the present example it is less than $\frac{1}{11}$th of a second, and the maximum will never amount to $\frac{1}{12}$th of a second. It may likewise be neglected in the orbit of Venus ; but in the orbit of Mars it may amount to one minute, and therefore should be attended to in calculating the equation of the centre for that planet, and in like manner for Jupiter, Saturn, Uranus, and the Moon. The excentric anomaly of Mercury is best calculated by the 1st Method.

If we have several excentric anomalies to calculate for the *same* planet, as for instance, the *Earth*, we may add the constant log. 5.3144251 to 3.2256452 the log. of the excentricity and the sum, viz. 8.5400703 will be a constant log. to be used in finding DN for that planet.

Example 2d.—*In the Orbit of Mars.*

Given the relative semi-major axis of the orbit of Mars 100000, the excentricity 9308.8, and the mean anomaly 45° ; to find the true anomaly, &c.

Operation.

Mean anom. 45° 0′ 0″	log. cos. 9.8494850	log. sin.	9.8494850
Excentricity 9308.8 . .	log. . . . 3.9688937	. . . . .	3.9688937
Nat. num. 6582.3	3.8183787	con. log.	5.3144251
S. maj. axis 100000			9.1328038
106582.3 subtract its logarithm			5.0276851
1st. value of DN, 12738.″5			4.1051187
			3
			12.3153561
subtract const. . .			10.9298802
correction 24.″3 . . .			1.3854759

Hence the corrected value of DN is 12738.5 — 24″3 = 12714″2. = 3° 31′ 54″.2, which subtracted

from 45°, the mean anomaly leaves 41° 28′ 5″.8 = the excentric anomaly.—The rest are found as before.

Scholium.

Hence, by the assistance of the foregoing problem, we are enabled to construct tables of the equation of the centre, for all the planets whose orbits are known, that is, where the relative magnitude of the semi-major axis and the excentricity of the orbits are known. For we have only to find the *true anomaly* answering to 1°, 2°, 3°, &c. to 180°, of *mean anomaly*, by one of the methods shewn in the problem; and the differences give the equation of the centre for each degree of mean anomaly respectively. And in orbits, such as those of the Earth and Moon, we can extend the tables, by interpolation, for every 10′ of mean anomaly; as in our solar and lunar tables; but in regard to the planetary tables this extension is deemed unnecessary, because the proportional part of the difference (between the next greater and next less degree of mean anomaly to that given), for the odd minutes and seconds of the given mean anomaly, is generally true enough, and if greater accuracy be required, the proportional part may be corrected by the equation of second difference.

Hence, also by this problem, we can calculate the exact relative magnitude of the radius vector or its logarithm, for every degree of mean anomaly for each planet; and consequently construct tables thereof, and extend the tables if we please for every 10′, by interpolation. And if the computer choose, he may also find the radius vector by either of the following analogies, viz. As the sine of half the true anomaly is to the sine of half the excentric anomaly, so is the square root of the perihelion distance, to the square root of the radius vector, or distance from the Sun. Or as the sine of the true anomaly is to the sine of the excentric anomaly, so is the semi-minor axis to the radius vector. The computer has therefore three different ways of finding the radius vector, so that he may either use that given in the problem or one of the foregoing, as he may think most convenient.

Problem 4th.

To find the Periodic Time of the Revolutions of the Planets round the Sun.

Solution by Astronomical Observations.

1st Example.—Of the Earth.

By comparing the *apparent* diurnal motion of the Sun and fixed stars round the Earth attentively, it will be found, that the fixed stars make one revolution round the Earth in a year more than the Sun does; so that 366 revolutions nearly, of the fixed stars are completed in the same time that the Sun makes but 365. And hence we discover, that the Sun has an apparent easterly motion among the fixed stars, which he completes in the space of a year. Now this *apparent* annual motion of the Sun among the stars is caused by the *real* motion of the Earth round the Sun, from west to east, in the same space of a year. And it is manifest that in whatever point of the heavens the Sun is seen from the Earth, that the Earth is in the opposite point in the heavens, as seen from the Sun; and consequently these two points are 180° distant. And because it is impossible to observe those stars which are near the Sun, by reason of his effulgence, we may, as it will answer the same purpose, observe any fixed star opposite to him; and this star will be on the meridian at midnight; then reckon the number of days from that time till the same star comes again next year to the meridian exactly at midnight; and this number of days gives the periodic time of the Earth's revolution round the Sun. But observations shew, that if any star be on the meridian at midnight, or any other particular hour in the night, that the same star will not culminate exactly at the same hour in an even number of days, but it will culminate a little after that hour on one night, and a little before that hour on the following night. Thus the star Spica Virginis, March 15th, 1814, passed the meridian by observation at $12h.\ 1m.\ 46s.$ and on March the 16th, at $11h.\ 58.\ 7s.$ hence the star was in opposition to the Sun between these times of the 15th and 16th March; wherefore, to find the time of opposition we proceed thus:

	h.	*m.*	*s.*
From	12	1	46
Subtract	11	58	7
Difference in 24*h.*	0	3	39

	h.	*m.*	*s.*
From	12	1	46
Subt.	12	0	0
	0	1	46

	m.	*s.*		*m.*	*s.*		*h.*		*h.*	*m.*	*s.*
As	3	39	:	1	46	: :	24	:	11	36	59
Add									12	1	46
Time of opposition 15th M.									23	38	45

By proceeding in a similar manner from observations taken of the same star on the 15th and 16th of March, 1813, we find that the opposition took place on the 15th day at $17h.\ 29m.\ 28s.$ Hence the difference $6h.\ 9m.\ 17s.$ being added to 365 days gives $365d.\ 6h.\ 9m.\ 17s.$ the time of the periodic revolution of the Earth round the Sun, or as it is commonly called, *the Sidereal Year*.

Scholium.

The foregoing method is true in theory, but the period of time is too short to produce an accurate conclusion. For it is manifest, that a period of 100

years must determine the matter 100 times more accurately than that of one year; for if we suppose the error resulting from one year to be 100 seconds, it follows that two observations of equal accuracy, taken 100 years apart must reduce the error to one second. And hence the more remote the two observations are from each other, so much the more accurate will be our conclusion. Now we have accurate observations taken 100 years ago by Dr. Halley, and by comparing them with those taken by Dr. Maskelyne and Mr. Pond 100 years afterwards, we have found that the Earth has made 100 revolutions round the Sun during an interval of 36525*d*. 15*h*. 31*m*. 55*s*. which being divided by 100 gives 365*d*. 6*h*. 9*m*. 19*s*, for the length of the sidereal year.

We have to observe that any other situation or position of the Sun and a star, besides that of opposition, will answer the purpose equally well, provided the Sun and star have exactly the same difference of right ascension for both observations. Moreover, because the change in the Sun's right ascension is for the most part unequal during the space of twenty-four hours, hence the best time for taking these observations is about the times of the equinoxes and solstices, when his diurnal motion in right ascension is nearly equable for one day.

Of the Tropical Year.

Observations shew, that the equinoctial points have a small retrograde motion by which means the Sun apparently moves from an equinoctial or solstitial point to the same again, in less time than from a fixed star to the same again, by about 20*m*. 24*s*, and this space of time which consists of 365*d*. 5*h*. 48*m*. 48*s*. viz. the interval of time from the Sun's centre passing an equinoctial or solstitial point till his coming to the same again, is called a *Tropical Year*, which is the same as the *common year*.

And to find the length of the tropical year, we proceed thus; take the Sun's meridian altitude on the day before he passes the equator, and on the following day when he has passed it, correct these altitudes for the refraction and Sun's semidiameter, and from both thus corrected, subtract the complement of the latitude of the observatory, the remainders will be the Sun's declination at noon for each day respectively; and having his declination for the noon of both days, the time of the equinox will be found thus:

By observation 20th March, 1814, the Sun's declination at noon was found to be 17′ 23″ south, and the 21st, it was 6′ 19″ north, wherefore,

	′	″	
20th	17	23	S
21st	6	19	N
Change in 24*h*. =	23	42	

As 23′ 42″ : 17′ 23″ : : 24*h*. : 17*h*. 36*m*. 12*s*. hence the time of the equinox was March, 20*d*. 17*h*. 36*m*. 12*s*. Now by observation, and a like process, we find that the time of the preceding equinox 1813, was March 20*d*. 11*h*. 47*m*. 22*s*. whence the difference 5*h*. 48*m*. 50*s*. being added to 365*d*. gives 365*d*. 5*h*. 48*m*. 50*s*. for the length of the tropical year.

By taking an interval of 100 years, we have found the length of the tropical year to be 365*d*. 5*h*. 48*m*. 48$\frac{3}{4}$*s*.

2*d Example.—Of Venus or Mercury.*

The best way of finding the time of the periodic revolution of an inferior planet, is by ascertaining the exact time between two successive conjunctions, of the same kind, of the planet with the Sun. And because the dark side of the planet is turned towards the Earth, at the time of an inferior conjunction, the planet is therefore at that time invisible to us, (except in a transit), and at the time of either conjunction it is rendered invisible by the immense effulgence of the Sun's rays; wherefore let the planet's elongation be observed as near to the time of conjunction as possible, and note the exact time when the elongation east of the Sun is equal to the same west of him; then half the interval between these two times being added to the time of the former, gives the time of the conjunction. But we must carefully see that both observations are correspondent ones, and they should be taken as distant from the time of the greatest elongation as possible, in order to render the change in elongation sufficiently sensible at those times.

But it is manifest that the exact time of an inferior conjunction of Mercury or Venus is best determined by their transits over the Sun's disc; and hence, if the interval of time between two transits be divided by the number of conjunctive periods between them, the quotient will be the length of time between two nearest conjunctions. Now by taking a mean of many times of the inferior conjunctions of Venus and the Sun, ascertained from her transits over the Sun's disc, and in the common way, it appears, that the space of time from one conjunction to another is 583*d*. 22*h*. 0*m*. 47*s*. Hence, *to find the periodic time of the revolution of Venus round the Sun; divide the product of this number and the periodic time of the Earth's revolution, by their sum, the quotient is the periodic time of Venus.* Thus*,

* Investigation of the Rule. Let, e = the Earth's periodic time, v = that of Venus; t = the time between two conjunctions; and $c = 360°$. Then the daily motion of the Earth in its orbit $= \frac{c}{e}$; and that of Venus $= \frac{c}{v}$; and their difference $\frac{c}{v} - \frac{c}{e}$, or $\frac{ce - cv}{ev}$ = the daily motion of Venus from

	d.	h.	m.	s.	s.
From ☿ to ☿	583	22	0	47	= 50450447
Earth	365	6	9	12	= 31558152
Sum					82008599

$\frac{50450447 \times 31558152}{82008599} = 19414096.7s. = 224d.\ 16h.\ 48m.\ 16.7s.$ = the periodic time of Venus round the Sun.

In the same manner do we find the periodic time of Mercury.

3d Example.—*Of Mars, or any other Superior Planet.*

The periodic time of the revolution of a superior planet round the Sun is found from knowing the exact time between two successive oppositions of the Sun and planet; and the time of a superior *planet's opposition* is found by observation in the same manner nearly as we have shewn for that of a fixed star. But to proceed with accuracy: the right ascension, at the time of culmination should be exactly determined by means of the transit instrument and astronomical clock; and likewise the declination, from the observed meridian altitude at the same time. Those observations should be continued for some time, so that you may have the right ascension and declination of the planet, for 2 or 3 days before the opposition, and as many days after it. Then from having the right ascension and declination given, compute the planet's longitude and latitude for each time respectively; and also the Sun's longitude for the same times. Now by comparing the longitude of the Sun and planet for each correspondent time, we can easily determine the time of the opposition, namely, when their difference of longitude = 180°, which will be the time of the ecliptic opposition: and this must be farther corrected; that is, we must reduce it to the orbit opposition, which takes place when the point opposite to the Sun (the Earth's centre) and the planet are equally distant from the node. Now this correction is easily found, by means of any two latitudes and their respective difference of longitude being given; from which the place of the node will be determined.

Now having found the exact time between two successive oppositions of the Sun and planet, we find its *periodic time*, thus: *Divide the product of this time, and the periodic time of the Earth by their difference, and the quotient is the periodic time of the planet sought.* As we have investigated in the note.

Scholium.

The periodic times of the planets, determined by the foregoing methods, are called their *mean times;* because they are results found from taking a *mean* of many times: and it is evident, that the more we have, so much the more accurate are the results. It is also plain, that the more distant similar astronomical observations are from one another, so much the more exactly do we find *mean results.* Now, if the observations of Hipparchus, which were made about 140 years before the Christian æra, and those of Ptolemy, which were made about, A. D. 90, could be depended on; then by comparing them with those of our own time; we should have an interval of 18 and 19 hundred years, from which *mean results* of great accuracy could be obtained. But astronomers of the present age have to lament, that the observations of these noble ancient astronomers fall far short of modern accuracy, when brought to the test of comparison with those of our time, and hence these observations of remote date are now useless to us, except in the date and magnitude of some solar and lunar eclipses, which in some cases are of utility in the lunar theory.

The mean periodic times of the planets are the most essential part of the *data*, in the construction of astronomical tables, for finding the places of the planets. For it is evident that if we divide 360°, by the number of days and decimal of a day, in the periodic time of any planet round the Sun; the quotient is the *mean daily motion* of that planet in degrees round the Sun: and by multiplying the *daily motion* by 2, 3, 4, 5, &c. we have the mean motion for *days.* Or what is the same thing, by the constant addition of the motion of 1 day, we get the motion of any number of days during the time of our common year; and moreover, by multiplying the *daily motion* of any planet by the number of days in a common year, viz. 365, and by 366 for a bissextile year we get the mean motion for each year: wherefore, by adding the mean motion for a common year 3 times successively, and then that for a bissex-

the Earth; also the Earth's motion in the time of 1 revolution of Venus, $= \frac{cv}{c}$: wherefore as $\frac{ce - cv}{ev} : 1d :: \frac{cv}{e} : \frac{v2}{e - v} =$ the time after 1 revolution of Venus till the conjunction; whence $\frac{v2}{e - v} + v = t$; and $ev + tv = te$; therefore $v = \frac{te}{e + t}$, which is the rule given. Also, $e = \frac{vt}{t - v}$, which finds the periodic time of a superior planet: or in words, divide the product of the Earth's periodic time, and the time between two oppositions, by their difference; and the quotient is the periodic time of that superior planet. Moreover, $t = \frac{ev}{e - v}$, which is the time from opposition or conjunction to the same again, of the Sun and any planet whose periodic time is given. Where v = the periodic time of the *interior* planet, and e = that of the *exterior.*

tile next; and so on by a constant addition, we get the mean motion for any number of years, as far as we choose to carry our tables. And in the same easy manner do we obtain the mean motion for centuries, &c.

But it will be necessary, for the sake of accuracy, that the degrees, or decimal of a degree, expressing the *daily mean motion* of a planet, should have at least 10 places in the decimal: or if we choose to express the same in sexagesimals; then the daily motion should be carried to minutes, seconds, thirds, fourths, and fifths; and in like manner the degrees expressing the yearly motion must be carried to *fifths* at least; because whatever error may exist in the last figure of the least denomination, it is necessarily increased by the continual addition of multiplication thereof.

We have also to remark, that the *mean motion* of the Earth, in longitude, is not deduced from the *sidereal*, but from the *tropical* year; by which means the effect of the retrograde motion of the equinoctial points is removed. So in like manner we reckon the longitude of all the planets from the equinoctial point ♈; and hence their mean motion in longitude must also be determined from their mean *tropical revolutions*, and not from their *sidereal* or periodic revolutions round the Sun, in order to remove the effect of the recession of the equinoctial point ♈. Now the *recession* of the equinoctial points, or as it is commonly called the *procession* of the *equinoctial* points, because the fixed stars appear to advance in longitude by it, is 50″.25, during a tropical year of 365*d*. 5*h*. 48*m*. 48*s*. And hence it becomes necessary to find the length of a tropical revolution for each of the planets; and by a tropical revolution, we mean the time taken by a planet, revolving round the Sun, from an equinoctial or solstitial point to the same again; and this revolution is evidently performed in less time than the periodic or sidereal revolution, by reason of the contrary motion of the equinoctial point; by which means it moves towards the planet and shortens the time of revolution. Wherefore we have given the time of the *sidereal* revolution, and the annual procession of the equinoxes, to find the time of the tropical revolution. The operation is simple and easy; thus,

1st Example.—*For Venus.*

As 365*d*. 5*h*. 48*m*. 48*s*. : 224*d*. 16*h*. 48*m*. 17*s*. : : 50″.25 : 30″.91 = the motion of the equinoxes, during the time of a sidereal revolution of Venus: then,

As 360° + 30″.91 : 360° : : 224*d*. 16*h*. 48*m*. 17*s*. : 224*d*. 16*h*. 40*m*. 34*s*. = the tropical revolution of Venus.

2d Example.—*For Mars.*

As 365*d*. 5*h*. 48*m*. 48*s*. : 686*d*. 23*h*. 30*m*. 35*s*. : : 50″.25 : 94″.51 the motion of the equinoxes, during the time of a sidereal revolution of Mars.

As 360° + 94″.51 : 360° : : 686*d*. 23*h*. 30*m*. 35*s*. : 686*d*. 22*h*. 18*m*. 27*s*. = the tropical revolution of Mars.

In the same manner we find the time of the tropical revolution of all the other planets; by having the sidereal revolution and annual procession given. Moreover these two analogies may be much contracted; thus, In the first, take the nearest hour only, and reject the minutes and seconds in the first and second terms; and the fourth term will still be found truely to the nearest $\frac{1}{100}$ part of a second. Also in the second analogy, find the excess of the *sidereal* above the *tropical* revolution; and subtract it from the sidereal: thus, in the two foregoing examples,

♀ As 360° + 30″.91 : 30″.91 : : 224*d*. 17*h*. : 463*s*. = 7*m*. 43*s*.
♂ As 360° + 94″.51 : 94″.51 : : 686*d*. 23½*h*. 4328*s*. = 1*h*. 12*m*. 8*s*

	d.	*h.*	*m.*	*s.*	*d.*	*h.*	*m.*	*s.*
	224	16	48	17	686	23	30	35
Subtract			7	43		1	12	8
Time of tropical rev. same as before,	224	16	40	34	686	22	18	27

Now to find the mean motion in longitude, for days, months, years, &c. we use the tropical revolutions; thus, for the Sun's (Earth's) mean motion. As 365*d*. 5*h*. 48*m*. 48*s*. : 1*d*. : : 360° : 0.98564727213, or in sexagesimals, 59′. 8″. 19‴. 48⁗. 38⁗′.8 the Sun's daily mean motion in longitude. Hence we have for,

Days	☉ motion in lon. *s.*	°	′	″	‴
1	0	0	59	8	19.8
2	0	1	58	16	39.6
3	0	2	57	24	59.4
4	0	3	56	33	19.2
&c.			&c.		

And 0°.98564727213 × 365 = 11*s* 29° 45′ 40″ 30‴.9 = the Sun's mean motion for 365 days; by adding the motion of 1 day to this, and rejecting 12 signs, we have 0*s*. 0° 44′ 48″ 50‴.7, for his motion in 366 days, or bissextile year. Wherefore, by adding the motion 3 times successively for *common* years, and once for the bissextile, we obtain the Sun's mean motion for any number of years, as far as we choose to carry our tables. Moreover $\frac{1}{24}$ part of the *daily* motion, is the *hourly*; and $\frac{1}{60}$ of this, that for 1 minute, &c.

But before constructing tables, the Sun's *mean longitude* from the *mean equinox* must be accurately determined by astronomical observations, for a particular *meridian and point of time*. Now by taking a mean of several hundred observations taken at the royal observatories of Greenwich and Paris, and allowing 20″ for the *aberration*, the Sun's *mean longitude from the mean equinox* January 1st, at noon at Greenwich, 1800, was 9*s* 10° 53′ 29″.8. Then to this add the motion for a common year of 365 days,

3 times successively, and then that for 366 days, and so on; and we shall construct the table of the Sun's mean longitude for any number of years to come, as far as we choose to continue the tables: and by subtraction we are enabled to suit the tables for past time. And in like manner if we apply the motion for any number of centuries, to the mean longitude for 1800, we have the mean longitude for the beginning of that century, &c. The whole of which is evident without farther illustration. And by these means were constructed the tables of mean solar motion, which we have given for the several epochs therein.

These solar tables seem to agree to great exactness with the observations made for many years at Greenwich by our late excellent astronomer royal, Doctor Maskelyne.

In the construction of tables of the *heliocentric mean longitude* of the planets, it is necessary first of all, that the heliocentric longitude of the planet from the *mean equinox* for a particular meridian and point of time, should be accurately settled by astronomical observations. Now the exact time of the inferior conjunction of the Sun, and an inferior planet, or the opposition of a superior planet and the Sun, is the instant when the Earth and planet, seen from the Sun, have the same longitude: and hence if we add or subtract 180°, or 6*s*, to or from the Sun's mean longitude at the instant of conjunction of an inferior planet, or opposition of a superior one, we have the planet's mean longitude at that time; and consequently by means of the planet's mean motion, we are enabled to settle its mean longitude for any particular instant or epoch of time. The mean motion in longitude is determined from the time of the tropical revolution; thus,

♀ As 224*d*.16*h*.40*m*.34*s*.: 1*d*.::360°: 1°.60217298832
= the daily mean motion of Venus in longitude
= 1° 36′ 7″ 49‴ 21⁗ 55⁗′.7. In sexagesimals.

♂ As 686*d*.22*h*.18*m*.27*s*.: 1*d*.::360°: 0°.52407126338
= the daily mean motion of Mars in longitude
= 0° 31′ 26″ 39‴ 23⁗ 34⁗′.4. In sexagesimals.

And 1°.60217298832 × 365 = 7*s* 14° 47′ 35″ 18‴ 23⁗ 50.5⁗′; for Venus in a common year.

And 0°.52407126338 × 365 = 6*s* 11° 17′ 9″ 38‴ 23⁗ 3⁗′; for Mars in a common year. And by adding the motion for one day we have for,

♀ 7*s* 16° 23′ 43″ 7‴ 45⁗ 46⁗′.2
♂ 6 11 48 36 17 46 37.4 } in bissex. years.

Note.—12 Signs, or 360° are always to be rejected, as in the case of Venus, &c.

Now from a mean of several observations, deduced from the mean heliocentric longitude of Venus at her inferior conjunctions and Mars in opposition, we find that the mean heliocentric longitude reckoned from the mean equinox for the noon of 1st of January 1800, mean time for these planets, is as follows.

Venus 4*s*. 26° 45′ 46″.0
Mars 7 23 45 48.5

And by adding the mean motion for common years 3 times successively, and then that for the bisextile, and so on; and by subtracting for past times; we construct the tables of mean motion for as many years as we choose, &c. All which is evident without farther illustration. And in like manner do we construct the tables of the mean heliocentric longitude of all the other planets.

The equation of the centre being applied to the *mean longitude* gives the true heliocentric longitude from which find the geocentric longitude; and the nutation of the Earth's axis, or the equation of the equinoctial points, and the aberration being applied to the geocentric longitude of Mercury and Venus, gives their true longitude from the true equinox: the perturbations or disturbing forces of the Earth and the other planets on these two inferior planets, being generally found to be too small to be attended to. But in regard to the Earth, and the other planets higher up in the system, the case is quite otherwise; for their disturbing forces on each other are very sensible, and on this account several more equations must be introduced, in order to correct for the perturbations of their mean motions. Besides, the Earth is affected in its motion round the Sun by the Moon, and on this account we must attend to a lunar equation, in respect to the Sun's (Earth's) motion.

Problem 5.

To find the place of the nodes, and inclination of the orbits of the planets to the plane of the ecliptic.

Solution. *By Astronomical Observations.*

Take several observations of the planet's right ascension and declination, and from these compute its longitude and latitude for each (as we shall shew in our practical astronomy); and these will be the planet's geocentric longitude and latitude; and find the Sun's longitude for the same times, and by adding or subtracting 180°, we have the Earth's heliocentric longitude. Take the difference between the Sun's longitude and that of the planet, and you have the angle of elongation.

Let S be the Sun (Fig. 19.), T the Earth, and P the planet; N the node, NU a portion of the ecliptic; draw PU perpendicular to the ecliptic, and join SU and TU: then the angle PTU is the geocentric latitude of the planet, and PSU its heliocentric latitude; ST is the radius vector of the Earth (the Sun's distance); PS the radius vector of the planet; and SU the *curtate* distance; and the angle PTS is the elongation of the planet. Then in the plane triangle PTS, we have the two sides ST and SP and the angle PTS given, to find the angle SPT; hence, as

SP : ST : : sine PTS : sine angle SPT, which is therefore given, and consequently the other angle PST is given, which is the difference between the heliocentric longitudes of the Earth and planet; whence the heliocentric longitude of the planet is also given. But this is not perfectly exact; yet when the latitude is small, the error is not sensible; and it is at that time that we want the heliocentric longitude; to wit, when the planet is near the node. And because PU is by construction perpendicular to the plane of the ecliptic, it is therefore perpendicular to UT and US. Now if UP be made the radius, then UT and US are the tangents of the angles TPU and SPU, namely the cotangents of the geocentric and heliocentric latitudes: but in the plane triangle UST, as UT : US : : sine UST : sine UTS; that is, as cotangent geocentric latitude is to cotangent heliocentric latitude: but the cotangents of two arcs are inversely proportional to their tangents: hence, as sine UTS : sine UST : : tangent geocentric latitude : tangent heliocentric latitude; that is, as the sine of the elongation is to the sine of the difference of the heliocentric longitudes of the Earth and planet, so is the tangent of the geocentric latitude to the tangent of the heliocentric latitude, which is therefore given. And to find SU, the planet's *curtate* distance, say, as radius is to the cosine (PSU) heliocentric latitude, so is (SP) the radius vector to (SU) the curtate distance.

Having thus computed the heliocentric latitude, from the actual observations, for the space of two or three days before and after the planet passed the node, we find the exact time and consequently its longitude when it passed the node, thus, suppose the heliocentric longitude at 10 o'clock mean time to be 7*s* 8° 30′ 40″, and latitude 4′ 30″ S, on one night; and on the next night at the same hour, the longitude was 7*s* 10° 10′ 20″ and latitude 5′ 40″ N.

	longitude.	latitude.
Hence	7*s* 8° 30′ 40″	4 30 S.
	7 10 10 20	5 40 N.
Diff in 24 hours =	1 39 40	10 10

As 10′ 10″ : 4′ 30″ : : 24*h*. : 10*h*. 37*m*. 22*s*. = the time the planet passed the node after the 1st observation.

As 24*h*. : 10*h*. 37*m*. 22*s*. : : 1° 39′ 40″ : 44′ 7″ = the change in longitude after the 1st observation till the planet passed the node.

1st long.	7*s* 8° 30′ 40″	time of 1st observation	10*h* 0*m* 00*s*
Add	44 7	Add	10 37 22
long. node	7 9 14 47	time the planet passed the node	20 37 22

If a superior planet pass the node nearly at the time of opposition; then the place of the node may be found as accurately from the *geocentric* latitude as from the *heliocentric* latitude.

To find the Inclination of the Orbit to the Plane of the Ecliptic.

Take the change in longitude from the time of the 1st observation till the planet comes to the node as the *base* of a right angle plane triangle, and the heliocentric latitude at the time of the 1st observation as the perpendicular of the triangle, and the *angle* opposite to the perpendicular will be the *inclination* of the orbit. Thus, as above, the change, or difference of longitude = 44′ 7″ = 2647″, latitude = 4′ 30″ = 270″. Then as 2647 : 270 : : radius : tangent 5° 49′ 27″ = the inclination sought. But this method will not be sufficiently accurate, if the difference of longitude of the planet and node be more than 1 degree, so that when this is the case we must take a spherical triangle (as the figure really is), instead of a plane triangle; and then the more distant the planet is from the node, so much the more accurately shall we find the angle of the inclination. Hence from the same *data*; by spherics, as sine of the planet's distance from the node, is to radius, so is the tangent of the latitude, to the tangent of the inclination.

The nodes of the planets have a motion, which is found by observation, by comparing their places at two different times; and it is evident, that the greater the interval of time between the two observations, so much the more accurately will the motion be determined. See our tabular view of the planets, in which we have given the principal elements of the orbits of all the planets, which have been settled with astronomical accuracy.

General Scholium.

We have now briefly developed to the student, a general view of the solar system, and of the various motions of the planets, and the inequalities with which their motions are affected. The several causes of which inequalities are the power of gravity, the projectile force first impressed, the *elliptical orbit* which must necessarily result from the composition of these two forces, and the spheroidal figure of the Earth and the other planets which revolve quickly round on their axes. So that the very figure of the solid body of the Earth produces apparent inequalities in the celestial phenomena. Moreover we have to add to these the perturbations in the planetary motions produced by the planets' gravitating towards each other. And finally, a new and wonderful apparent inequality, produced by the motion of the particles of light and that of our Earth round the Sun, called the *aberration* of light. And this last discovery of the aberration, must of itself, without the aid of all other indisputable arguments, demonstrate satisfactorily, the motion of the Earth round the Sun.

We are now led to see to what a number of necessary considerations modern astronomers must attend; the greater part of which were unknown to the ancients: and hence their conclusions must have differed greatly from ours and from the truth. They had not even just notions of the figure and magnitude of the Earth: and though some few among them, imagined that the Earth revolved on its axis, and round the Sun at the same time; yet the power which retained it, and all the planets in their orbits, viz. *gravity*, was unknown to them. They supposed all motion to be *circular;* but modern astronomers have discovered, that there is not one planet whose orbit is a perfect circle, but an ellipsis with the Sun placed in, or very near to one of the foci of the ellipsis. How very different now are the speculations of students in astronomy from those of former times? True, indeed, they have a much greater number of objects to keep in view; but they have the light of mathematical truths and certainty to direct them. Conjecture and hypothesis, so much used by the ancients, have given place to accurate reasoning and true philosophy aided by correct and delicate observations: so that upon the whole, the sublime science of astronomy is now arrived at a state of perfection, of which ancient astronomers could have no conception.

PRACTICAL ASTRONOMY.

INTRODUCTION.

PRACTICAL Astronomy is divided into two parts; of these, one belongs to the *observing Astronomer*, tde other to the *mathematical:* the former is the business of the observer, furnished with proper astronomical instruments at an observatory constructed for the purpose; the latter is the work of the mathematician: that is, the one furnishes the *data*, the other discovers the *quæsita;* the observer provides the *materials* for the building, the other constructs the *edifice:* and although these are two distinct astronomical offices, yet they ought to be united in the observing astronomer; for no man is qualified to be an observer, who is not a good mathematician; but on the other hand, there are many profound mathematicians, who can make the best use of astronomical observations, when once furnished with them, yet know very little of the necessary instruments, and the best mode of using them, at an observatory. We here proceed to the second part, making use of such astronomical tables, as have been constructed, from the best observations.

The two Fundamental Problems of Solar Astronomy.

To find the Sun's *true Longitude*, and the *true obliquity of the Ecliptic*, at any given instant of time.

Example.—Required the Sun's true longitude, and the obliquity of the ecliptic, for the 1st day of January 1820, for the apparent noon at Greenwich; that is, for 0*h*. 3*m*. 36*s*. of mean time.

PROBLEM 1. *Precepts for the Longitude.*

1. From Table 1, take out the epochs, with all the arguments for the given year, and place them in a horizontal line, as in the example.

2. From Table IV, take out the mean motions of the same quantities for the day of the month, and place them under their correspondent ones.

3. From Table V, take out the mean motion of the hours, minutes, and seconds past noon, of the several quantities, and place them under their correspondent ones.

But the time from the mean noon, in this example, being the equation of time, and which is always a small quantity, and here only 3*m*. 36*s*. the better way is to take the motion for the minutes and seconds, and apply them to the secular equation, and then place this quantity with its proper sign, under the mean longitude for the year and day.

The proportional part of the secular variation of the mean longitude and perigee for each year from 1800 to 1900 is put down in Table 1. The secular equation of the longitude for 1820 is there found to be 0″.8 which is added to the motion for 3*m*.36*s* (the time from mean noon) in the 2*d* column of the Example, and the sum, 9″.7 is placed under the mean longitude. The secular equation of the perigee for 1820 is found also in Table 1. to be 1″, which, in the Example is included in 9*s* 9° 49′ 42″ the place of the perigee. Then adding up all the respective quantities

gives the mean longitude, place of the perigee, &c. for the given time.

But if we were computing the Sun's place for any year out of the 19th century, we must then find the proper proportional part of the secular variation of the longitude and perigee for the given year, by Table 3.

4. With the arguments M 416, and A 503, enter table VI, and take out the correction of A 28, and add it to A.

5. From the mean longitude subtract the place of the perigee, and you have the mean anomaly.

6. With the mean anomaly 0*s*. 0° 13′ 58″, enter table VII, and take out the equation of the centre and the secular variation.

The equation for 0*s*. 0° 10′ is 11*s*. 29° 59′ 35″.6, and the change for 10′ is 20″.6; then as 10′ : 20″.6 :: 3′ 58″ : 8″.2, which is the proportional part for the odd minutes and seconds, and which, according to its sign is to be added; consequently 11*s*. 29° 59′ 43″.8 is the equation of the centre for 1810. The secular variation is between 0″.05 and 0″.10, or 0″.07 for 100 years, therefore for 10 years it is 0″.0, which place under the equation of the centre.

7. Find the lunar equation in table X and its continuation. Thus, with the argument A 531, we have 6″.1 for the 1st part; with the first part 6″.1 and mean anomal 0*s*.0°, we have 0″.1 for the second part; with the arguments, M 416 and A 531 we have 0″.3, the third part: then the sum of these three parts, 6″.5, diminished by the constant quantity 0″.5, gives 6″.0, which is the lunar equation to be put under the equation of the centre.

8. With the arguments B 278 and C 911, enter table XI, and take out the effect of the perturbating force of Venus, 6″.9, and place it under the lunar equation.

9. With the arguments B 278 and D 280, enter table XII, and take out the perturbation of Mars 7″.5, and write it under the last.

10. With the arguments B 278 and E 914, enter table XIII, and take out the perturbation of Jupiter, 4″.9, and set it under the others.

11. With the arguments B 278 and F 22, enter table XIV, and take out the perturbation of Saturn, 0″.9, and write it under the others.

12. Add the aggregate of these seven equations to the mean longitude, and you have the *true longitude* from the *mean equinox*.

But care must be taken, if the sign of the secular variation be —, not to add it with the rest, but to subtract it from their sum.

13. With the argument N 982 enter table VIII, and take out the lunar nutation, — 2″.0; and with the day of the month take out the solar nutation, + 0″.3; these two added together according to their signs, give — 1″.7 for the whole nutation, which being applied to the true longitude from the mean equinox, gives 9*s*. 10° 3′ 48″.5, the Sun's true longitude from the true equinox.

PROBLEM 2. *For the Obliquity of the Ecliptic.*

1. THE mean obliquity of the ecliptic for the beginning of 1800 was 23° 27′ 57″, and it diminishes half a second or 0″.5 annually, wherefore 0″.5 × 20 = 10″.0, the diminution at the beginning of 1820.

2. With the argument N 982, enter table VIII, and take out the lunar or node equation, + 9″.6, and in the continuation of the same table for the day of the month take out the solar equation, — 0″.4, these three quantities added together, according to their signs, give — 0″.8, which being applied to the mean obliquity, makes 23° 27′ 56″.2, *the true obliquity* for the 1st January, 1820.

SCHOLIUM.

WE have assumed the secular diminution of the obliquity to be 50″.0, or 0″.5 for a year; Mr. Vince puts it at 52″.1, or 0″.521 for a year; Dr. Maskelyne, in the Nautical Ephemeris for 1810, reduced it to 43″.4, or 0″.434 for a year; but it appears that he afterwards augmented it again to 50″.0, which we have taken as the most correct for the present time. The Doctor said, in 1810, that he conceived the secular diminution to be a *variable quantity*; but that all the reasonings of the French astronomers to prove it to be *periodical*, and that the obliquity would again increase, appeared to him unsatisfactory and inconclusive.

We have omitted a small equation of the longitude, called the variable part of the aberration; the maximum of this equation is no more than $\frac{17}{60}$ of a second of a degree; and consequently it is too small at any time to be taken into consideration, in calculating the true longitude. Indeed astronomers are well contented if they are certain that the result of their calculation gives the Sun's true longitude to the nearest second. The *mean aberration* of light from the Sun is 20″; and this quantity is contained in the several epochs, and therefore the calculator is thereby relieved from the consideration of any aberration affecting his calculations for the Sun's true longitude.

The equation of time which is used in this example to reduce apparent noon to mean time may be found by the first method of Prob. 4, which depends only upon the Sun's mean longitude for the given day; and it will be sufficiently exact for finding the true longitude if we neglect the small effects produced by the disturbing forces of the planets, and take the equation to the nearest second without these.

Calculation of the Sun's true Longitude for the apparent Noon at Greenwich, for the following Days of January 1820; and the Obliquity of the Ecliptic for the 1st.

	Years and Days.	☉ Longitude.	Perigee.	M	A	B	C	D	E	F	N
		s. ° ′ ″	s. ° ′ ″	416	503	278	911	280	914	22	982
Sec. Eq. + 0.8	1820,	9 10 3 30.5	9 9 49 42	0	0	0	0	0	0	0	0
Eq. T. 3′ 7.4	Day 1.	0 0 0 0	0 0 0 0	416	503	278	911	280	914	22	982
36″ 1.5	S.E.&E.T.	. . . . + 9.7	9 9 49 42	Correct	28						
+ 9.7	M.Long.	9 10 3 40.2	9 10 03 40		531						
Add Sum Equat.		0 0 0 10.0	0 0 13 58	☾ Nut. — 2″.0							
☉ True Long. M. Equin.		9 10 3 50.2	Mean Anomaly.	☉ Nut. + 0 .3							
Nutation ☾ and ☉		. . . . — 1.7	☾ or N + 9.6	Nut. — 1 .7							
			☉ Jan. 1 — 0.4								
☉ True Long. True Equin.		9 10 3 48.5	Sec. Eq. — 10.0	23 27 57.0							
			Sum — 0.8	Sum — 0.8							
☉ Semidiameter		. . . 16 17.8	Obliquity Eclip.	23 27 56.2							

1st Day.

Equat. Centre									11	29	59 43.8
Sec. Variation											0.0
Lunar Equat.											6.9
B & C											6.9
B & D											7.5
B & E											4.9
B & F											0.9
Sum Equat.									0	0	0 10.0

Sec. Eq. + 0.8	1820,	9 10 3 30.5	9 9 49 42	416	503	278	911	280	914	22	982
Eq. T. 4′ 9.9	Day 2.	0 0 59 8.3	0 0 0 0	36	34	3	4	1	0	0	0
5″ 0.2	S.E.&E.T.	. . . + 10.9	9 9 49 42	452	537	281	915	281	914	22	982
+ 10.9	M. Long.	9 11 2 49.7	9 11 2 49	Correct	25						
Add Sum Equat.		0 0 2 10.3	0 1 13 7		562						
☉ True Long. M. Equin.		9 11 5 0.0	Mean Anomaly.	☾ Nut. — 2.0							
Nutation ☾ and ☉		. . . . — 1.7		☉ Nut. + 0.3							
				Nut. — 1.7							
☉ True Long. True Equin.		9 11 4 58.3									
☉ Semidiameter		. . . 16 17.8									

2d Day.

Equat. Centre									0	0	1 45.5
Sec. Variation											0.0
Lunar Equat.											4.5
B & C											6.9
B & D											7.5
B & E											5.0
B & F											0.9
Sum Equat.									0	0	2 10.3

Sec. Eq. + 0.8	1820,	9 10 3 30.5	9 9 49 42	416	503	278	911	280	914	22	982
Eq. T. 4′ 9.9	Day 3.	0 1 58 16.7	0 0 0 0	73	68	5	9	3	0	0	0
33″ 1.4	S.E.&E.T.	. . . + 12.1	9 9 49 42	489	571	283	920	283	914	22	982
+ 12.1	M. Long.	9 12 1 59.3	9 12 1 59	Correct	22						
Add Sum Equat.		0 0 4 10.8	0 2 12 17		593						
☉ True Long. M. Equin.		9 12 6 10.1	Mean Anomaly.	☾ Nut. — 2.0							
Nutation ☾ and ☉		. . . — 1.7		☉ Nut. + 0.3							
				Nut. — 1.7							
☉ True Long. True Equin.		9 12 6 8.4									
☉ Semidiameter		. . . 16 17.8									

3d Day.

Equat. Centre									0	0	3 47.2
Sec. Variation											0.0
Lunar Equat.											3.2
B & C											6.9
B & D											7.5
B & E											5.1
B & F											0.9
Sum Equat.									0	0	4 10.8

Sec. Eq. + 0.8	1820,	9 10 3 30.5	9 9 49 42	416	503	278	911	280	914	22	982
Eq. T. 5′ 12.3	Day 4.	0 2 57 25.0	0 0 0 1	109	101	8	13	4	1	0	0
1″ 0.0	S.E.&E.T.	. . . + 13.1	9 9 49 43	525	604	286	924	284	915	22	982
+ 13.1	M. Long.	9 13 1 8.6	9 13 1 8	Correct	17						
Add Sum Equat.		0 0 6 11.5	0 3 11 25		621						
☉ True Long. M. Equin.		9 13 7 20.1	Mean Anomaly.	☾ Nut. — 2.0							
Nutation ☾ and ☉		. . . . — 1.6		☉ Nut. + 0.4							
				Nut. — 1.6							
☉ True Long. True Equin.		9 13 7 18.5									
☉ Semidiameter		. . . 16 17.8									

4th Day.

Equat. Centre									0	0	5 48.7
Sec. Variation											— 0.1
Lunar Equat.											2.3
B & C											6.9
B & D											7.4
B & E											5.4
B & F											0.9
Sum Equat.									0	0	6 11.5

SPHERICAL TRIGONOMETRY *applied in the Solution of Astronomical Problems.*

Problem 3.

Given the obliquity of the ecliptic and the Sun's true longitude, to find the Sun's right ascension and declination.

1st Example.

The obliquity of the ecliptic, on the 1st of January, 1820, is 23° 27′ 56″.2; and the Sun's true longitude at the time of the apparent noon at Greenwich, on the same day, is 9*s* 10° 3′ 48″: it is required to find the Sun's right ascension and declination, at the same instant of time.

Solution.

Preparatory Directions.

If the Sun's longitude be less than 3 signs or 90°; then the first point of Aries is the nearest equinoctial point; if it be more than 3 signs but less than 6 signs or 180°, then the first point of Libra is the nearest equinoctial point; and his longitude must be subtracted from 6 signs or 180°, and the remainder is his longitude west from the nearest equinoctial point Libra: if it exceed 6 signs, but less than 9 signs or 270°; then 6 signs or 180° is to be deducted from it, and the remainder is his longitude east from the nearest equinoctial point Libra: and lastly, if it exceed 9 signs, then it must be subtracted from 12 signs or 360°, and the remainder is his longitude west from the nearest equinoctial point Aries.

Also, if the longitude be less than 6 signs, the declination is north; but if it be more than 6 signs, it is south.

	s	°	′	″	
Wherefore, from	12	0	0	0	
Subtract,	9	10	3	48	the long. east from ♈.
Remains	2	19	56	12	= 79° 56′ 12″, the longitude west from ♈.

Let the figure NQSE (Fig. 1.), be the celestial sphere, projected stereographically on the plane of the solstitial colure; N the north pole, S the south pole; EQ the celestial equator; ♑ ♈ ♎ the ecliptic; ⊙ the Sun's place in the ecliptic; and NB⊙S the Sun's meridian.

Then in the right angled spherical triangle ♈B⊙ right angled at B; there are given the angle ⊙♈B, the obliquity of the ecliptic = 23° 27′ 56″.2, and the hypothenuse ♈⊙, the Sun's longitude = 79° 56′ 12″; to find the leg ♈B, the Sun's right ascension from Aries west; and the leg ⊙B, the Sun's declination south.

1. To find ♈B the Sun's right ascension, west.

By Case 1st, of right angled Spherical Trigonometry.

As radius				10.0000000
Is to the tang. of the long.	79°	56′	12″	10.7508825
So is the cos. of the obliq.	23	27	56.2	9.9625110
To tang. of the right asc.	79	3	1.6	10.7133935

2. To find the declination ⊙B, south.

By the same Case.

As radius				10.0000000
Is to the sine of the long.	79°	56′	12″	9.9932665
So is the sine of the obliq.	23	27	56.2	9.6000997
To the sine of the decli.	23	5	0.6	9.5933662

From 360° 00′ 0.0″
Take 79 3 1.6 west.

Remains 280 56 58.4 east = 18*h* 43*m* 47.9*s* the the Sun's true right ascension, and his declination is 23° 5′ 0.6″ south.

Examples 2, 3, and 4.

The Sun's true longitude on the 2d January, 1820, is 9*s* 11° 4′ 58″; on the 3d, it is 9*s* 12° 6′ 8″; and on the 4th, it is 9*s* 13° 7′ 18″; and the obliquity remains the same, viz. 23° 27′ 56″.2: the right ascension and declination, for these three days respectively, are required.

2d Day.

	s	°	′	″			
	12	0	0	0			
	9	11	4	58			
	2	18	55	2	= 78° 55′ 2	log. tang.	10.7080098
					23 27 56.2	log. cos.	9.9625110
					77 56 46	log. tang.	10.6705208
					360 00 00		
					282 3 14	= 18*h* 48*m* 12.9*s* = *R*.	
					78° 55′ 2″	log. sine	9.9918241
					23 27 56.2	log. sine	9.6000997
Dec. South					23 0 9.2	log. sine	9.5919238

Abridged Operation.

3d Day.

s	°	′	″
12	0	0	0
9	12	6	8

2 17 53 52 =	77 53 52	log. tang.		10.6687307
	23 27 56.2	log. cos.		9.9625110
	76 50 35.7	log. tang.		10.6312417
	360 00 00			
	h m s			
	283 9 24.3 = 18 52 37.6 = *R*.			
	° ′ ″			
	77 53 52	log. sine.		9.9902390
	23 27 56.2	log. sine		9.6000997
Dec. South	22 54 50.3	log. sine		9.5903387

4th Day.

s	°	′	″
12	0	0	0
9	13	7	18

2 16 52 42 =	76 52 42	log. tang.		10.6324466
	23 27 56.2	log. cos.		9.9625110
	75 44 30.1	log. tang.		10.5949576
	360 00 00			
	h m s			
	284 15 29.9 = 18 57 2 = *R*.			
	° ′ ″			
	76 52 42	log. sine		9.9885100
	23 27 56.2	log. sine		9.6000997
Dec. South	22 49 4.0	log. sine		9.5886097

Problem 4.

Given the Sun's true right ascension from the true equinoctial point, his mean longitude, and the nutation in longitude, to find the equation of time.

Example.—Required the equation of time for apparent noon at Greenwich on the 1st day of January, 1820.

First Method of Solution,

In which the Mean Longitude only is used.

With the Sun's mean longitude for the given *day*, viz. 9*s* 10° 3′ 30″ enter table XX, and take out the tabular value of the equation of time, which (including the proportional part for 3′ 30″) is found to be + 3′ 37″.7; take out also from the same table, the secular vsriation — 15″.55 and find a proportional part of it for the interval between the year 1810, and the given time, which in this example will be just $\frac{1}{10}$th or — 1.5, apply this to the tabular equation + 3′ 37″.7 and yon have + 3′ 36″.2 for an approximate value of the equation of time. But if the given time had not been apparent noon, then a proportional part also of the variation of the equation in one day (viz. of 28″.9) must have been taken for the time after apparent noon and applied to 3′ 36″.2, the equation already found. With the arguments A. B. C. &c. (See Ex. Prob. 1.) enter table XXI and also with the argument 500 + 2 B + N in the same line as you enter with the argument N. and take out the effects arising from the disturbing forces of the planets; apply these, with the constant quantity — 3″.1 to the approximate equation last found, and you have the corrected equation of time.

	s.	°	′	″
Sun's mean longitude, January 1st...	9	10	3	30

For 9*s* 10° 0′ 0″ the equation is.....	+ 3 36
For 0 0 3 30 the prop. part is	1.7
	+ 3 37.7
Prop. part of sec. variation	— 1.5
Prop. part for time after noon	0.0
Approx. equat.....	+ 3 36.2
For B. 278 and C. 911	+ 0.6
B. 278 D. 280	+ 0.5
B. 278 E. 914	+ 0.4
B. 278 F. 22	+ 0.1
A. 531	+ 0.4
N. 982	+ 0.1
500 + 2 B + N = 38	+ 0.1
Constant	— 3.1
Correct. equat. for app. noon of Jan. 1st, 1820	+ 3 35.3

A similar process would

give for app. noon of	Jan. 2d	+ 4 3.7
.....	Jan. 3d	+ 4 31.7
.....	Jan. 4th	+ 4 59.6

Second Method of Solution.

Reduce the nutation in longitude into right ascension, by multiplying it by the natural cosine of the obliquity of the ecliptic, 0.917, or, which is equivalent to this, subtract $\frac{1}{12}$th of the nutation in longitude from itself, and you have the nutation in right ascension; then apply the nutation in right ascension (with a contrary sign to the nutation in longitude) to the true right acsension from the true equinox, and you have the true right ascension from the mean equinox; the difference between which and the mean longitude, being reduced to time, is the true equation of time.

Example.—On the 1st, 2d, 3d, and 4th, days of January, 1820, at the time of the apparent noon at Greenwich, the nutation in longitude being — 1″.7, the Sun's true right ascension from the true equinox, and his mean longitude from the mean equinox were as follow; the true equation of time for these several instants of time is required.

	1st Day.	2d Day.	3d Day.	4th Day.
	s. ° ′ ″	*s.* ° ′ ″	*s.* ° ′ ″	*s.* ° ′ ″
True ☌ from true equinox.	9 10 56 58.4	9 12 3 14	9 13 9 24.3	9 14 15 29.9
$\frac{11}{12}$ of the nutation	+ 1.6	+ 1.6	+ 1.6	+ 1.6
True ☌ from mean equinox.	9 10 57 0	9 12 3 15.6	9 13 9 25.9	9 14 15 31.5
Mean longitude	9 10 3 40.2	9 11 2 49.7	9 12 1 59.3	9 13 1 8.6
Difference in degrees, &c. . .	0 0 53 19.8	0 1 0 25.9	0 1 7 26.6	0 1 14 22.9
Equation of time	0*h.* 3*m.* 33.3*s.*	0*h.* 4*m.* 1.7*s.*	0*h.* 4*m.* 29.8*s.*	0*h.* 4*m.* 57.5*s.*

In the foregoing problems we have shewn the method of calculating the obliquity of the ecliptic, the Sun's longitude, right ascension, and declination, and the equation of time for the first four days of January, 1820, and have thus enabled the astronomical student to compute any one of the 2d monthly pages of the nautical ephemeris. With regard to the obliquity of the ecliptic, that may be taken as constant during the whole month, because the change in that period is too small to affect the right ascension, declination, or the equation of time. The results of these computations are here collected together.

Oblq.	1820, Jany.	Sun's Longitude.	Right Ascension.	Declination.	Equat. of Time.
		s. ° ′ ″	*h.* *m.* *s.*	° ′ ″	*m.* *s.*
23 27 56.2	1*st*,	9 10 3 48	18 43 47.9	23 5 1	+ 3 33.3
	2*d*,	9 11 4 58	18 48 12.9	23 0 9	4 1.7
	3*d*,	9 12 6 8	18 52 37.6	22 54 50	4 29.8
	4*th*,	9 13 7 18	18 57 2.0	22 49 4	4 57.5

Scholium.

The 1st method in problem 4, of computing the equation of time is the same as that given by Mr. Vince in the third volume of his astronomy; we have however only introduced it in order to find the equation of time to be used in computing the Sun's true longitude by problem 1*, after this is obtained we conceive the most correct way of finding the equation of time is by the 2*d method*, which was first given by Dr. Maskelyne. The tables of equations for the perturbations of the planets used by Mr. Vince in his method appear to be only the equations of longitude reduced to time, but in this he is a little incorrect, because it is the right ascension answering to the aggregate of all the equations of longitude which affects the equation of time, and this corresponding portion of the equator or right ascension may be either equal to, greater, or less than that aggregate. There are also other circumstances attending Mr. Vince's method that we think cause still farther inaccuracy; it may nevertheless be used with advantage in finding an approximate equation of time to be used in problem 1, and should the equation afterwards found by the 2d method, differ much from this, we may use the last found equation as the approximate one, and with this repeat the calculation of the Sun's longitude, which will thus be obtained with great accuracy.

Problem 5.

Given the Sun's declination, and the obliquity of the ecliptic, to find his longitude and right ascension from the true and mean equinoxes.

Example.—On the 1st day of January, 1820, suppose the Sun's declination to have been observed at the instant he transited the meridian, to be 23° 5′ 0″.6 south, at which time the obliquity of the ecliptic was 23° 27′ 56″.2, and the equation of time + 3*m.* 33.3*s.* what were his true longitude, and true right ascension from the true and mean equinoxes.

* If the computer be in possession of the former numbers of the nautical ephemeris, he may assume the equation of time the same as it is found 4 or 8 years previous to the time for which he is calculating, as the error by so doing will be quite inconsiderable, and thus save himself the trouble of finding the equation by the 1st method.

SOLUTION.

THE same construction remaining as in problem 3 (Fig. 1), then in the spherical triangle B♈⊙, right angled at B, there are given the angle B♈⊙, the obliquity = 23° 27′ 56″.2, and the leg. B⊙, the declination south = 23° 5′ 0″.6, to find the hypothenuse ♈⊙, the true longitude west from the first point of Aries, the true equinoctial point, and the leg ♈B, the true right ascension west from the same point.

1. To find the hypothenuse ♈⊙, the longitude west.
By case 5th of right angled spherical trig.

As the sine of obliquity	23° 27′ 56″.2	9.6000997
Is to radius		10.0000000
So is the sine declination	23 5 0.6	9.5933662
To the sine of longitude	79 56 12.0	9.9932665

2. To find the leg ♈B, the right ascension west.
By the same case.

As radius		10.0000000
Is to the tan. of the declin.	23° 5′ 0″.6	9.6296094
So is the cotan. of the obliq.	23 27 56 .2	10.3624113
To the sine right ascension	79 3 1.6	9.9920207

From	360° 00′ 00″	
Subtract the longitude.	79 56 12.0	west.
Remains the longitude	280 3 48.0	east.

True long. in signs, degrees, &c. from the true equinox.	9s 10° 3′ 48″
Nutation of longitude	+ 1.7
True long. from the mean equinox	9 10 3 49.7

From	360° 00′ 00″	
Subtract the right ascension	79 3 1.6	west.
Remains the right ascension	280 56 58.4	east.

True right ascension in time from the true equinox	h. m. s. 18 43 47.9
Nutation in right ascension	+ 1.6
True right ascension from the mean equinox	18 43 49.5

SCHOLIUM.

IF we now subtract the aggregate of the equation of time, reduced into motion, with its proper sign, the secular equation of longitude, the equation of the centre, and its secular variation, the lunar equation, and the several equations produced by the perturbations of the planets; (as in the calculations for the true longitude), from the true longitude from the mean equinox; then the remainder will be the true mean epoch for the longitude, for the noon of the 1st of January, 1820, mean time. Thus,

Equation of time	+ 3 33.3	= 0′ 8″.8
Secular equation	+	= 0 0.8
Sum of the several equations	+	= 0 10.0
		+ 0 19.6

	s ° ′ ″
From	9 10 3 49.7
Take	0 0 0 19.6
Remains the mean epoch of longitude for the 1st of January, 1820.	9 10 3 30.1

And thus by the assistance of this problem we are enabled to construct tables of the epochs of the mean longitude of the Sun; but inorder to this, the exact quantity of the obliquity of the ecliptic is a primary and fundamental datum, the determination of which is the business of the observing astronomer; if the Sun's centre passed over the meridian of the observatory at the very moment he was in either of the solstitial points of the ecliptic, and if his altitude, above the horizon of the observatory, that instant, were accurately taken and truly corrected for the effects of refraction, then the difference between the correct altitude of his centre, and the complement of the latitude of the observatory, gives the obliquity of the ecliptic at that time. But the fact is, that the Sun can scarcely ever transit the meridian at the moment of the solstice, and hence astronomers must compute the quantity that the Sun's altitude is less than the altitude of the summer solstitial point, or greater than the winter one, by knowing his distance from it; and because the change in the Sun's meridian altitude, during a natural day, on either side of the solstice, is very little, consequently only a small error in the altitude of the solstitial point can be produced by the error in the approximate time of the solstice. By a mean of the results of a great number of observations taken for this purpose, at the royal observatories of Greenwich and Paris, &c. the obliquity of the ecliptic has been determined to very great exactness, a secular diminution of it has also been discovered amounting to 50″ in 100 years, and besides a periodic increase and decrease of 9″.6 during the Moon's period of 18½ years, as likewise another very samll annual increase and decrease.

There is another method of determining the obliquity of the ecliptic, and which was that used by ancient astronomors; this requires the altitude of

the Sun's centre to be taken on the days of the summer and winter solstices; then half the difference of those two altitudes is the obliquity; this is certainly equally accurate with the former method; and as it makes no use of the latitude of the place of observation, it was the very best way the ancients could adopt: modern astronomers have to lament that the ancients were in all probability, not provided with accurate astronomical instruments, otherwise the secular diminution of the obliquity of the ecliptic, might now be settled to the greatest precision. Aristarchus, 280 years before Christ, makes the obliquity 23° 51′, and taking it at a mean in 1800 to be 23° 28′; the difference is 23′, the quantity of the diminution in 2080 years; hence 23′ divided by 2080 gives 0″.663 for a year, or 66″.3 for 100 years; which, if we allow for the badness of the instruments that Aristarchus might make use of, differs not materially from 50″, the present rate of diminution for 100 years: besides, within these few years past, the rate of diminution seems to decrease; yet so very little, that in 100 years the diminution will probably not amount to 1″.

The Sun's declination, which also constituted one of our *data*, in this last problem, must also be determined at an observatory; now the meridian altitude of the Sun's centre being accurately taken, and corrected for the effects of refraction; the difference between this altitude and the complement of the latitude of the observatory, gives the declination at that instant: and hence the obliquity of the ecliptic and the Sun's declination for the same instant of time, being accurately determined by observation, furnish *data* to the astronomer, for computing the epochs of the Sun's mean longitude, in his solar tables. We have also to observe, that at, or near, the times of the equinoxes is the best time for taking the observation for determining the Sun's declination; because the change in declination is then the greatest; at the equinoxes, the ratios of the increments of the declination, right ascension, and longitude, are as the sine, cosine of the obliquity, and the radius; that is, as 40, 92, and 100 nearly; but in every other point of the ecliptic the ratio of the increment of declination, to those of the right ascension and longitude, is less, and at the solstices it vanishes and becomes nothing. Now let us suppose that the astronomer has taken 10 observations near each equinox, that is, 20 during the year, for the declination; and for each of these with the obliquity of the ecliptic, he has calculated (by the assistance of this last problem), the correspondent longitude and right ascension, then by taking a mean of the results of these 20 observations, he arrives at greater accuracy; and by repeating these observations during a series of years, he still attains a higher degree of accuracy. The solar tables given in the present work were constructed by M. Delambre, the mean longitudes of which were computed from the mean results of nearly 2000 observations taken at the observatories of Greenwich and Paris.

Problem 6.

Given the obliquity of the ecliptic and the Sun's right ascension, to find his longitude and declination.

Example.—On the 1st day of January, 1820, at the time of the apparent noon at Greenwich, the obliquity of the ecliptic was found to be 23° 27′ 56.″2, and the Sun's right ascension was observed by the transit instrument, to be 18*h*. 43*m*. 47.9*s* mean solar time from the true equinox; at which time the equation of time was + 3*m*. 33.3*s*; it is required to find the Sun's declination, and his true longitude from the true and mean equinox, at that instant of time.

Solution.

The same construction remaining, as in problem 3 (Fig. 1); then in the spherical triangle ⊙ϒB, right angled at B; there are given the angle ⊙ϒB = 23° 27′ 56.″2, the obliquity, and the leg ϒB = 79° 3′ 1″.6, the Sun's true right ascension west from the first point of Aries, the true equinoctial point; to find the hypothenuse ϒ⊙, the Sun's true longitude from the same point, and the leg ⊙B, his declination south.

	h	*m*	*s*	°	′	″	
Take ☌ =	18	43	47.9 =	280	56	58.4,	east.
From				360	00	00	
Remains ☌, ϒB =				79	3	1.6,	west.

1. To find ϒ⊙ the true longitude, west.

By Case 4th, of right angled Spherical Trigonometry.

As radius		10.0000000
Is to cotang. right ascen.	79° 3′ 1″.6	9.2866064
So is cosine of the obliq.	23 27 56.2	9.9625110
To the cotang. of the lon.	79 56 12	9.2491174

2. To find ⊙B the true declination, south.

By the same Case.

As radius		10.0000000
Is to the tangent obliquity,	23° 27′ 56″.2	9.6375887
So is the sine right ascen.	79 3 1.6	9.9920207
To the tangent declination,	23 5 0.6	9.6296094

From	360° 00′ 00″
Take long.	79 56 12, west.
Remains lon. from the true equinox.	280 3 48, east = 9 10 3 48, east
Nutation, ...	+ 1.7
Longitude, east from the mean equinox.	9 10 3 49.7,

And, if the mean motion for the equation of time, and the aggregate sum of the several other equations of longitude be applied to the true longitude east from the mean equinox, as in the last problem; then we have the epoch of the Sun's mean longitude for 1st day of January, 1820, as before.

If, in the same right angled spherical triangle ♈⊙B; any other two of these four parts; viz. the obliquity, longitude, right ascension, and declination, be given; then the other two parts can be found; in which the student may exercise himself at his leisure.

There is a fifth part belonging to the same triangle, namely, the angle ♈⊙B, which we have hitherto left unnoticed; this is called the vertical or the ecliptic angle, and it is contained between the Sun's meridian and the ecliptic; it is equal to the complement of the obliquity when the Sun is at either equinox, ♈ or ♎, and is then the least; and it is 90° when he is at either solstice, ♋ or ♑, at which times it is the greatest. This angle becomes useful in the calculation of solar eclipses. If the obliquity and the Sun's longitude be the *data*, as in problem 3, then the quantity of this angle is found by this analogy, viz. radius : tangent obliquity : : cosine longitude : cotangent vertical angle. See Case 1st, of right angled Spherical Trigonometry.

Having now solved the 6 primary problems of solar astronomy, and sufficiently exemplified them, we shall proceed to the solution of those called *topical* or *local*; these respect an observer situated on a particular or given point on the surface of our Earth; for there are various phenomena of the Sun, which differ widely from each other, at different places on the Earth; *e. g.* the length of the day and night, except at the time of the equinoxes, is very different to the inhabitants of London, and to those under the equator, where there is a constant equality of day and night: also to the inhabitants of the equator, all the celestial bodies appear to rise and set perpendicularly to the horizon; but to those of London obliquely, &c. &c.

The requisite *data* now to be made use of, are the Sun's declination (and which we have shewn how to find in the preceding problems, for any given instant of time), and the latitude and longitude of the proposed place, for which we are to compute the phenomena; and these are given in most of our books on geography, navigation, &c. besides, we shall shew hereafter how the latitude and longitude are astronomically determined with accuracy.

All our former calculations were made for the *Sun's centre*, but in most topical problems we must have respect to his limb, that is, the bounding circle of his disk, as well as to his centre. Now his apparent semidiameter, that is, the angle under which his semidiameter is seen at the Earth, is not a constant but a variable quantity; at a mean it is about 16′ 1″, and it increases and decreases about 17″, making a difference of 34″ nearly, between the greatest and least apparent semidiameters; this is caused by the elliptical figure of the Earth's orbit; when the Earth is in the *perihelion*, which is about the latter end of December, then the Sun's semidiameter appears the greatest, or 16′ 17″.8; and about the latter end of June, when the Earth is in the *aphelion*, it is no more than 15′ 45″.5, when it is the least. The magnitude of the Sun's apparent semidiameter at any given time, therefore, becomes a necessary part of the *data*, in most of the following problems; whence this,

Auxiliary Problem.

To find the Sun's apparent semidiameter, at any given time.

Solution.

With the Sun's mean anomaly, found as in prob. 1, enter table XVI, and you have his apparent semidiameter. Thus, for the 1st, 2d, 3d, and 4th days of January, 1820, his apparent semidiameter is 16′ 17″.8, which is inserted under his true longitude in the calculations for those days. *Note.*—The refraction is given in table XVII; see definition 31, p. 78.

Problem 7.

Given the latitude and longitude of a place, and the Sun's declination, to find his amplitude and ascensional difference, and the true time of his *rising* and *setting*. See definitions 22, 30, pages 77, 78.

Example.—On the 1st day of January, 1820, the Sun's declination for the apparent noon at Greenwich, being 23° 5′ 1″ south, decreasing at the rate of 11″.0 per hour; his semidiameter 16′ 18″, the equation of time + 3*m*. 33.3*s*, increasing at the rate of 1.2*s* per hour. What is his true amplitude, ascensional difference, and time of his rising and setting in the morning and evening at London in latitude 51° 31′, north longitude 0° 6′ west from Greenwich? The horizontal refraction, minus the Sun's horizontal parallax, being taken at a mean, 33′.

SOLUTION.

LET HZON (Fig. 2.), be a stereographic projection of the celestial sphere on the plane of the meridian of London, Z the zenith, N the nadir, P the north pole, S the south pole, EQ the equator, HO the horizon, M⊙N the Sun's parallel of declination south, cutting the horizon in the point ⊙; through ⊙, draw the Sun's meridian P⊙S cutting the equator in B; then the meridian NCZ is the *prime vertical*, and C is the east or west point of the horizon; also the meridian PCS passing through C (and which, according to this projection is projected on the axis PS), is the *six o'clock hour circle*, H is the south point of the horizon, O the north point of it; and ⊙ is the Sun in the horizon, at the time of his rising or setting.

The arc of the horizon H⊙ is the amplitude of the Sun, at rising or setting from the south; and the arc ⊙ C, the complement of H ⊙ is his amplitude from the east or west at rising or setting, which in this case is south; also, the arc OP or its equal EZ is the latitude of London; and the arc EH or its equal PZ is the complement of the latitude, but the arc EH is the measure of the spherical angle ⊙CB, which therefore is the complement of the latitude: moreover the point B is that point of the equator, which corresponds with the Sun at rising or setting; likewise the arc EB of the equator, being reduced to time, gives the hours, minutes, &c. from noon, of the Sun's rising or setting; and the arc BC, the complement of EB, called the *ascensional difference*, being reduced to time, gives the hours, minutes, &c. from six o'clock in the morning or evening, of the Sun rising or setting, and which in this case is after six in the morning, but before six in the evening: and lastly, the arc ⊙B of the Sun's meridian (being the distance of his parallel of declination from the equator), is the Sun's declination south.

Then in the spherical triangle CB⊙ right angled at B; there are given the angle BC⊙ = 38° 29′, the complement of the latitude 51° 31′, and the leg B⊙ = 23° 5′ 45″ for the morning, and 23° 4′ 17″ for the evening; to find the hypothenuse C⊙ the amplitude from the east southerly in the morning, and from the west southerly in the evening; and also to find the leg BC the time the Sun rises after six in the morning, or the time he sets before six in the evening. Assume four hours to be the time from noon of the Sun's rising and setting.

Then, 11″.0 × 4 = 44″ = the change in declination nearly; and 1.2*s* × 4 = 4.8*s* the change in the equation of time; hence,

The declination at noon,	23° 5′	1″
	±	44
Declination at rising =	23 5	45
Declination at setting =	23 4	17

	m	*s*
Equation of time at noon =	3	33.3
	∓	4.8
Equation at rising =	3	28.5
Equation at setting =	3	38.1

Note.—The longitude of London is too small in this case to affect the declination and the equation of time.

1. To find the amplitude from the east in the morning.

By Case 5th of right angled spherical trigonometry.

As the cosine latitude 51° 31′ 0″	9.7939907
Is to radius	10.0000000
So is sine declination 23 5 45	9.5935853
To sine amplitude 39 4 38	9.7995946

To find the amplitude in the evening.

As cosine latitude 51° 31′ 0″	9.7939907
Is to radius	10.0000000
So is sine declination 23 4 17	9.5931506
To sine amplitude 39 1 51	9.7991599

2. To find the ascensional difference in the morning.

By the same Case.

As radius	10.0000000
Is to tangent declination 23° 5′ 45″ . . .	9.6298683
So is tangent latitude 51 31 0 . . .	10.0996541
To sine ascen. difference 32 26 30	9.7295224

To find the ascensional difference in the evening.

As radius	10.0000000
Is to tangent declination 23° 4′ 17″ . . .	9.6293546
So is tangent latitude 51 31 0 . . .	10.0996541
To sine ascen. difference 32 23 55	9.7290087

Here the amplitude or arc of the horizon contained between the Sun's centre and the true east point is 39° 4′ 38″ southerly at rising, and 39° 1′ 52″ from the west, southerly, at setting; and the ascensional difference at Sun rise is 32° 26′ 30″ = 2*h*. 9*m*. 46*s*.; and at Sun set, it is 32° 23′ 55″ = 2*h*. 9*m*. 35.6*s*. But these results require correction, for on account of the horizontal refraction, the Sun's centre will appear to be in the horizon in the morning sooner than it really is, and therefore both the amplitude and ascensional difference must (in the present example) be decreased by a certain quantity depending upon the refraction, and the oblique angle which the Sun's meridian when he is in the horizon makes with the horizon; that is, the angle C⊙B; this

angle is found by right angled spherical trigonometry; thus,

1. To find the angle C⊙B, in the morning.

As cosine declination 23° 5′ 45″	9.9637170
Is to radius	10.0000000
So is sine latitude 51 31 00	9.8936448
To sine angle C⊙B 58 19 13	9.9299278

2. To find the angle C⊙B, in the evening.

Cosine declination 23° 4′ 17″	9.9637960
Radius	10.0000000
Sine latitude 51 31 00	9.8936448
Sine angle C⊙B 58 18 12	9.9298488

Horizontal refraction, minus hor. par. = 33′ 0″ = 198″0, then

1st, To find the correction of the amplitude.

To the logarithm of the seconds of the refraction minus the parallax, add the log. sine of the angle C⊙B and the log. secant of it; reject 20 from the index, and you have the logarithm of the seconds of the correction to be added or subtracted as the case requires.

2d. To find the correction of the ascensional difference.

To the logarithm of the seconds of the refraction minus the parallax, add the log. secant of the declination and the log. secant of the angle C⊙B; reject 20 from the index, and you have the logarithm of the seconds of the correction to be added, or subtracted as the case requires.

Seconds of refraction = 1980″ log.	3.2966652
angle 58° 19′ 13 log. sine	9.9299278
angle 58 19 13 log. secant	10.2796997
Corr. amplitude 3208″ log.	3.5062927
53′ 28″	
Refraction 1980″ log.	3.2966652
Declination 23° 5′ 45″ log. secant . .	10.0362830
Angle 58 19 13 log. secant . .	10.2796997
Cor. as. differ. 4099″ log.	3.6126479
1° 8′ 19″	

Amplitude in the morning	39° 4′ 38″
Correction	— 53 28
True amplitude from the east	38 11 10

Note.—If the latitude and declination have the same name, add the corrections; but if they have contrary names (as in the present case), subtract. Also when the declination and latitude have the same name; then the ascensional difference added to 6 hours gives the setting, and subtracted from 6 hours, the rising; and *vice versâ*.*

Ascensional difference in the morning	32°	26′	30″
Correction	— 1	8	19
True ascensional difference	31	18	11

	h	*m*	*s*
True ascensional difference in time . . .	2	5	12.7
Add	6	0	0
True apparent time of Sun rise	8	5	12.7
Equation of time	+	3	28.5
True mean time of Sun rise, as should be shewn by a chronometer	8	8	41.2

Scholium.

By proceeding in the same manner, the correct amplitude, and time of the Sun's setting for the evening are obtained; but the corrections for the morning may be used for the evening, the difference being insensible in so short a space of time. The reader will perceive, that the foregoing calculations are made for the instant of the Sun's centre appearing or disappearing in the visible horizon; but if the first moment of his upper limb appearing or disappearing be required; then take the sum of the refraction and the Sun's semidiameter reduced to seconds, and proceed to find the corrections with this sum as was before done with the refraction only, and you will obtain the true amplitude, and the true moment that the first point of the Sun's disk appears above, or the last point sinks below the horizon.

It will be observed that we have assumed a time before and after the noon of the day for which the preceding calculation is made in order to obtain the

* *Demonstration of the Rules for correcting the Amplitude and Ascensional Difference.*

In the small triangle ⊙*bd* (Fig. 2), right angled at *b*, (which in this case may be taken as a plane triangle) $bd = s$, represents the horizontal refraction — the horizontal parallax, = 1980″; ⊙$b = x$ the change in amplitude, and ⊙d the change on the parallel of declination, and its correspondent arc on the equator $Be = y$, the change in ascensional difference. Let a and $a' =$ sine and cosine of the angle C⊙B, or its equal ⊙db; $d' =$ cosine declination, and $r =$ the radius: then, by plane trigonometry, as $a' : s :: a : \frac{sa}{a'} = x$. Again, as $a' : s :: r : \frac{rs}{a'} =$ ⊙d; but $d' : r ::$ ⊙$d : Be = y$, that is, as $d' : r :: \frac{rs}{a'} : \frac{r^2 s}{a'd'} = y$; so that x, the change in amplitude, is equal to $\frac{sa}{a'}$, and y, the change in ascensional difference, equal to $\frac{r^2 s}{a'd'}$; and these values of x and y, put into words, and the subtraction of the log. cosines changed into the addition of the log. secants, rejecting 20 from the indices, give the rules.

Sun's declination at the time of his rising and setting nearly; this time we supposed to be 4 hours from noon, and although it appears, by the calculation, that the Sun will rise on the 1st day of January, at London, at 8*h*. 5*m*. 12.7*s*. A. M.; yet this differs so little from 4 hours, from the apparent noon, that the error in declination can produce no sensible error in the result; but should there happen to be a very large difference between the assumed time, and that found by the calculation; then it may become necessary to repeat the operation, with the declination more strictly adjusted to the time.

This problem has been solved, as it usually is, by the application of right angled spherical trigonometry, but it may be performed in a more direct manner by oblique angled trigonometry as we have then no occasion to apply any corrections: see problem 10, and scholium. It may perhaps be thought by some that the solution has been here given with tedious and unnecessary exactness, although we can scarcely anticipate such an objection from any one entitled to the name of an astronomer: it is true that for the purpose of common almanacs it might be sufficient to approximate the true time of the Sun's rising or setting to a few minutes, but still it is desirable that the computer should be in possession of the means of making the calculation with as much accuracy as can be attained, because he can then much better estimate the amount of error incurred by less accurate methods, and judge whether it be of any importance.

The method usually followed by calculators of common almanacs, for finding the time of the Sun's rising and setting is to take the declination at noon, and then with that declination and the latitude, they find the corresponding ascensional difference, which being added to and subtracted from six hours, gives the time of rising and setting nearly; and from the time of rising they subtract three or four minutes and add the same to the time of setting (for the effect of refraction), and the results they tabulate in their calendar, as the *true* times of the rising and setting of the Sun. This method (if the proper correction for the effects of refraction be applied), might do very well about the time of the solstices, when the diurnal change in the Sun's declination is small; but at other times, particularly at the time of the equinoxes, when the diurnal change in the Sun's declination is considerable, it falls far short of astronomical accuracy, and the error increases also, as the latitude increases. The computer may for the sake of exercise, find the difference between the Sun's true rising and setting, calculating strictly by this problem and the incorrect common way, by calculating for a few days, about the latter end of April, for the latitude of Archangel in 64° 34′ north, or even for London in 51° 31′ north.

In the same right angled spherical triangle CB⊙, if any two of the five parts, namely, the co-latitude, declination, ascensional difference, amplitude, or the angle with the horizon C⊙B, be given, then the other three can be found.

Problem 8.

Given the latitude and longitude of a place and the Sun's declination, to find his altitude and azimuth at the hour of six A.M. or P.M.

Note.—The latitude and declination must both have the same name, otherwise the Sun is depressed below the horizon at the hour of six.

Example.—At London in latitude 51° 31′ north, longitude 0° 6′ west, on the 21st day of June, 1820, the Sun's declination for the apparent noon at Greenwich being 23° 27′ 56″ north, and the hourly change of declination for that day 0″.0, the Sun being in the tropic of Cancer. What are his altitude and azimuth at the hour of six apparent time?

Solution.

Let the figure be projected as in the last problem, and *m*⊙*n* (Fig. 3), be the Sun's parallel of declination north, cutting the six o'clock hour circle SCP in the point ⊙, and through the points Z,⊙,N, draw the vertical or azimuth circle Z⊙N; then in the spherical triangle CB⊙ right angled at B, there are given the angle ⊙CB = 51° 31′ the latitude, and the hypothenuse C⊙ = 23° 27′ 56″ the declination; to find the leg BC, which measures the angle BZC the azimuth, north from the east A.M., or north from the west P.M.; and the leg B⊙ the altitude.

1. To find the azimuth from the east or west.
By case 1st of right angled sph. trig.

As radius		10.0000000
is to tangent declination	23° 27′ 56″	9.6375875
so is the cosine latitude	51 31 00	9.7939907
To tangent azimuth	15 7 00	9.4315782

2. To find the altitude.
By the same case.

As radius		10.0000000
is to sine declination	23° 27′ 56″	9.6000987
so is sine latitude	51 31 00	9.8936448
To sine altitude	18 9 43	9.4937435

Hence the Sun's azimuth, at six o'clock in the morning or six o'clock in the evening, on the 21st of June, or longest day in the year 1820, at London, is 15° 7′ 0″ from the east towards the north in the morning, and the same from the west towards the north in the evening; or its complement 74° 53′ 0″, is the Sun's true azimuth reckoned from the north easterly or westerly; also his true altitude is 18° 9′ 43″ above the horizon, to which if we add the difference between his refraction and parallax in altitude = 2′ 44″ then we have 18° 12′ 27″ for the true apparent altitude of the Sun's centre at six o'clock A.M. or P.M.

Note.—If the longitude, and the daily change in declination had been considerable, we must then have proportioned the declination to the hour of six at the given place before proceeding to find the azimuth and altitude.

In the same triangle B⊙C, contained between the Sun's azimuth circle and the six o'clock hour circle, is found thus, as radius : tangent latitude : : cosine declination : cotangent angle B⊙C. And if any other two of the five parts of the triangle CB⊙ be given, the other three can be found by right angled spherical trigonometry.

Problem 9.

Given the latitude and longitude of a place, and the Sun's declination, to find his altitude and the hour of the day when he is due east or west, that is, when his centre is in the prime vertical.

Example.—At London in latitude 51° 31′ north, longitude 0° 6′ west, on the 21st day of June, 1820, the Sun's declination for the apparent noon at Greenwich being 23° 27′ 56″ north, and the hourly change of declination 0″0, the Sun being that day in the tropic of Cancer. What are his altitude and the hour of the day A.M. and P.M, when he is due east or west.

Solution.

The same construction being made as in the last problem; through the point ⊙ (Fig. 3.) in which the parallel of declination $m⊙n$ cuts the prime vertical ZCN, draw the Sun's meridian PTS; then in the spherical triangle ⊙TC right angled at T, there are given the angle ⊙CT = 51° 31′, the latitude, and the leg ⊙T = 23° 27′ 56″, the declination; to find the hypothenuse ⊙C, the altitude, and the leg TC which measures the angle TPC the hour from 6, AM or PM.

1. To find the altitude.

By case 5 of right angled spherical trig.

As sine latitude	51°	31′	00″	9.8936448
Is to radius				10.0000000
So is sine declination	23	27	56	9.6000987
To sine altitude	30	34	36	9.7064539

	°	′	″
True altitude Sun's centre	30	34	36
Diff. refraction and parallax	+	1	20
True app. alt. Sun's centre	30	35	56

2. To find the hour from six.

By the same case.

As radius				10.0000000
Is to tangent declination	23°	27′	56″	9.6375875
So is cotangent latitude	51	31	00	9.9003459
To sine hour from six	20	11	14	9.5379334

	h.	*m.*	*s.*
Add to and subtract from	6	0	0
Time from six in degrees 20° 11′ 14″ =	1	20	45
True app. time A.M.	7	20	45
True app. time P.M.	4	39	15

In the same triangle, the angle C⊙T contained between the Sun's meridian and the prime vertical is found thus, as cosine declination : radius : : cosine latitude : sine angle C⊙T. And if any other two of the five parts of the triangle CT⊙ be given, the other three can be found by some of the cases of right angled spherical trigonometry.

Problem 10.

Given the latitude and longitude of a place, and the year and day of the month, to find the time of the beginning and end of twilight, and consequently the length or duration of twilight.

Example.—On the 2d day of January, 1820, what is the time of the Sun's setting, and the end of twilight, at London, in latitude 51° 31′ north, longitude 0° 6′ west.

Note.—The crepusculum is supposed to begin or end when the Sun is 18° below the horizon.

Preparatory corrections of the Sun's declination, &c.

Assume 4*h*, P. M. for the time of Sun set, and 6*h*, P. M. for the end of twilight.

	Declination.			Eq. time.		
				h.		
1820, Jan. 2d at noon	23°	0′	9″ +	4*m.*	1.7*s.*	4 = ⅓
3d	22	54	50	4	29.8	6 = ¼
Daily difference	0	5	19	0	28.1	

			6)5	19			4)5	19
			0	53			1	20
Equa. 2d diff			—	2	equa. 2d diff.		—	3
Cor. of dec. for 4 hours			0	51	cor. of dec. for 6 hours		1	17
$\frac{1}{6}$ of 28.1 sec. = 4.7					$\frac{1}{4}$ of 28.1 sec. = 7.0			
2d day, dec. at noon	23°	0′	9″			23°	0′	9″
Cor. for 4 hours	0	0	51		cor. for 6 hours	0	1	17
Dec. at 4 P. M.	22	59	18		dec. at 6 P. M.	22	58	52
Add	90	0	0		Add	90	0	0
Pol. dis. at 4 P. M.	112	59	18		pol. dis. at 6 P. M.	112	58	52
Equa. of time	+ 4m 1.7s					+ 4m 1.7s		
Cor. for 4 hours		4.7			cor. for 6 hours		7.0	
Eq. time at 4 P. M.	+ 4m 6.4s				eq. time at 6 P. M.	+ 4m 8.7s		
To	90°	0′	0″			90°	0′	0″
Add ⊙s re. — hor. par.	0	33	0		add for crepusculum	18	0	0
Zen. dis. at 4 hours for setting.	90	33	0		Zen. dis. at 6 hours for end of twilight	108	0	0

SOLUTION.

THE same construction being made as in the former problems; let $x\odot y$ (Fig. 4), be a parallel of depression 33′ below the horizon; and $b\odot a$ another parallel of depression 18° below the horizon, which is therefore the crepusculum circle; then,

1. ZP = 38° 29′ the co. lat.; P⊙ = 112° 59′ 18″ the polar distance for setting; and Z⊙ = 90° 33′, the zenith distance for setting. Hence in the spherical triangle ⊙ZP the three sides are given to find the angle ⊙PZ, the hour angle at setting. The operation by rule 2d, case 5th, of oblique angled spherical trigonometry will be thus,

Co. lat.	38°	29′	00″	ar. com. log. sine	0.2060093
Polar dist.	112	59	18	ar. com. log. sine	0.0359364
Zenith dist.	93	33	00		
	2)242	01	18		
Half sum	121	00	39	log. sine	9.9330162
Sub. zen. dis.	90	33	00		
Remainder	30	27	39	log. sine	9.7049645
					2)19.8799264
Half hour ang.	29°	26′	16″	log. cosine ...	9.9399632
			2		
				h m s	
Hour angle	58	52	32	= 3 55 30.1	ap. time setting
Equation of time				+ 4 6.4	
Sun sets true mean time				3 59 36.5	

2. To find the angle ⊙PZ the hour at the end of twilight.

Co. lat.	38°	29′	00″	ar. com. log. sine	0.2060093
Polar dis.	112	58	52	ar. com. log. sine	0.0359132
Zenith dis.	108	0	00		
	2)259	27	52		
Half sum	129	43	56	log. sine	9.8859490
Sub. zen. dis.	108	00	0		
Remainder	21	43	56	log. sine	9.5685176
					2)19.6963891
Half hour ang.	45	10	11	log. cosine ...	9.8481945
			2		
				h m s	
Hour angle	90	20	22	= 6 1 21.4	ap. time; end of twilight.
Equation of time				+ 4 8.7	
End of twilight true me. time				6 5 30.1	
Sun sets true mean time				3 59 36.5	
Duration of twilight				2 5 53.6	

SCHOLIUM.

THIS method of finding the true apparent or mean time of the Sun's setting or rising, and the beginning and end of twilight is direct and accurate; the correction applied to the rising and setting, found by the ascensional difference, as in problem 7, may be a small quantity from the truth; because the small triangle used in finding the correction for the effects of the refraction and parallax is taken as a plane triangle, which is not strictly the case; but still the difference between that method and this will probably never amount to more than a few seconds in time: besides, the uncertainty of the quantity of the horizontal refraction at the given time, must often occasion a small difference. The refraction is a variable quantity, and depends on the *density* and *temperature* of the atmosphere at the proposed time and place, and as we know not exactly what these may be at London, &c. at the time of the Sun's rising or setting on the 2d day of January, 1820; hence, we can do no more than apply the mean refraction, which may be erroneous when the object is in the horizon by more than 1′ of a degree; but when the altitude of the object is more than 4°, then the mean refraction can differ but a few seconds from the exact quantity at the time and place.

We may here farther observe, that by computing the angle ⊙ZP, and subtracting 90° from it, we might obtain the Sun's true amplitude at setting.

AXIOMS *for correcting Astronomical Observations.*

EVERY *fixed star* has no horizontal parallax, and therefore no parallax in altitude; nor has a fixed star any magnitude in diameter.

All the celestial bodies appear to have a greater altitude above the rational horizon than they really have, by means of the refraction. And consequently the quantity of the refraction is to be subtracted from all *observed altitudes.*

The Sun, Moon, and Planets, have each a horizontal parallax, and consequently a parallax in altitude; and as the parallax in altitude makes the object appear lower; that is, its apparent altitude above the rational horizon is less than its true altitude (the one being observed from the Earth's surface, and the other from its centre); hence, the quantity of the parallax is to be added to all observed altitudes of those celestial bodies which have parallax.

The parallax in altitude decreases in the ratio of the cosine of the altitude; that is, as radius is to the horizontal parallax; so is the cosine of the altitude to the parallax in altitude: this is demonstrated immediately after the definitions, page 78.

The semidiameter of the Moon appears larger to an observer on the Earth's surface, than to one at its centre, except when she is in the horizon; and as the altitude of the Moon increases, her semidiameter appears to increase, so that when the Moon is in the horizon, her semidiameter is the least, and when she is in the zenith it is the greatest; and the quantity by which her semidiameter at any time exceeds her horizontal semidiameter is called the *augmentation* of her semidiameter.

The ratio of the augmentation is as the sine of the altitude; that is, as radius is to the sine of the altitude; so is the zenith augmentation, to the augmentation for that altitude. Table XXXV shews the augmentation of the Moon's semidiameter, which is always to be added to the horizontal semidiameter to obtain the true apparent semidiameter for that altitude.

There is also an augmentation for altitude in the semidiameter of the Sun and planets, but its quantity is too small to be taken into consideration in astronomical observations. Wherefore, to the observed altitude above the rational horizon of the Sun or Moon's lower limb, add their correct semidiameter, and you have the apparent altitude of their centres; then add their parallax in altitude, and subtract the refraction and you have the true altitude of their centres. The same rule applies to the planets, the apparent semidiameters of which increase and decrease inversely as their distances from the Earth.

Note.—For altitudes taken at Sea above the visible horizon, a correction for the height of the eye above the surface of the water is to be subtracted.

PROBLEM 11.

GIVEN the latitude and longitude of a place, the observed altitude of the Sun's lower limb, and the year and day of the month, and also the approximate time by a watch not sufficiently regulated; to find the true apparent time of observation, and the Sun's azimuth.

EXAMPLE 1.

AT Philadelphia, in latitude 39° 57′ north, longitude 75° 10′ west, on the 2d day of January, 1820, at 3 o'clock P. M. by the watch not regulated; the altitude of the Sun's lower limb, above the true sensible horizon, was found to be 14° 10′. What were his true azimuth, and the true hour of the day, apparent and mean time?

Preparatory corrections of the declination, altitude, and equation of time.

	h.	m.	s.	
Time per watch	3	0	0	P. M.
Long. 75° 10′ W. in time	5	0	40	
Appro. time at Greenwich	8	0	40	P. M.

	°	′	″
☉s decl. Jan. 2d, at noon	23	0	9
3d,	22	54	50
Diff.	0	5	19

As 24*h.* : 5′ 19″ : : 8*h.* 0*m.* 40*s.* : 1′ 46″
Equa. 2d diff subtract 3

Correction for 8*h.* 0*m.* 40*s.* — 1 43

	°	′	″	
Decl. 2d day at noon at Greenwich	23	0	9	
Decl. for approx. time at Greenwich	22	58	26	S.
Add	90	0	0	
Polar dist.	112	58	26	

	°	′	″
Observed alt. Sun's lower limb	14	10	00
Sun's semidiam. 2d, Jan. 1820	+	16	18
Apparent altitude Sun's centre	14	26	18
Subtract refraction	—	3	25
	14	22	53
Add the parallax in alt.	+		8
Sun's true alt.	14	23	1
Subtract from	90	0	0
Sun's true zenith distance	75	36	59

Equa. time Jan. 2d,	4m	1.7s
3d,	4	29.8
	0	28.1

24h. : 28s.1 : : 8h. 0m. 40s. : 9s.4

To . . .	4	1.7
Add . . .	0	9.4
Equa. time + . . .	4	11.1

SOLUTION.

LET HZON (Fig. 5), be a stereographic projection of the sphere on the plane of the meridian of Philadelphia; P the north pole, S the south pole, Z the zenith, N the nadir, HO the horizon, EQ the equator, NCZ the prime vertical, PCS the 6 o'clock hour circle, H the south point of the horizon, O the north point, and C the east or west point of it: draw the parallel of declination $m\odot n$, at the distance of 22° 58′ 26″ the Sun's declination, south of the equator; and the almacanter or parallel of altitude $b\odot a$, at the distance of 14° 23′ 1″ above the horizon; and through the point ⊙, in which the parallels cut each other, draw the meridian P⊙S, and the azimuth or vertical circle Z⊙N.

Then in the oblique angled spherical triangle **PZ⊙**; there are given the side **ZP** = 50° 3′ the complement of the latitude, the side P⊙ = 112° 58′ 26″ the polar distance, and the side ⊙Z = 75° 36′ 59″ the Sun's zenith distance; to find the angle PZ⊙ the true azimuth from the north and the angle **ZP⊙** the hour from noon. Hence by case 5 oblique angled spherical trigonometry,

1. To find the angle ZP⊙, the true apparent time for noon.

	°	′	″	
Colat.	50	3	00	ar. com. log. sine 0.1154283
Polar dist. . .	112	58	26	ar. com. log. sine 0.0358900
Zen. dist. . .	75	36	59	
	2)238	38	25	
Half sum . .	119	19	12.5	log. sine . . . 9.9404653
Subt. zen. dist.	75	36	59.0	
Remainder .	43	42	13.5	log. sine . . . 9.8394339
				2)19.9312175
½ hour angle =	22°	30′	7″	log. cosine 9.9656087
	2			
				h m s
Hour angle =	45	00	14	= 3 0 00 = true ap. time
Equa. of time				+ 4 11
True mean time at Philadelphia				3 4 11 P. M.

2. To find the angle PZ⊙, the true azimuth from the north.

	°	′	″	
Colat.	50	3	00	ar. com. log. sine 0.1154283
Zen. dist. . .	75	36	59	ar. com. log. sine 0.0138312
Polar dist. . .	112	58	26	
	2)238	38	25	
Half sum . .	119	19	12.5	log. sine . . . 9.9404653
Sub. polar dis.	112	58	26.0	
Remainder .	6	20	46.5	log. sine . . . 9.0435061
				2)19.1132309
Half azim.	68	53	2	log. cosine . . . 9.5566154
			2	
	137	46	4	= true azim. from the north.
	180	00	0	
	42	13	56	true azim. from the south.

Hence it appears that the watch was correct with true apparent time, and therefore shewed the true time agreeably with the Sun; but it was too slow for true mean time (which every chronometer ought to shew), by the equation of time, viz. 4m. 11s. And the Sun's azimuth is south 42° 13′ 56″ westerly, or S. W. ¼ S nearly; which is his true bearing at the time of observation.

SCHOLIUM.

HAVING found any one of the three angles of the triangle, the other two angles may be found by case 1st of oblique angled spherical trigonometry. Thus, the angle ZP⊙ being found by the first operation to be 45° 0′ 0″; then, as sine side Z⊙ : sine ZP⊙ : : sine P⊙ : sine PZ⊙ the azimuth; also, as sine Z⊙ : sine ZP⊙ : : sine ZP : sine P⊙Z, which angle is useful in the calculation of solar eclipses. But when only one of the three angles is required, then that angle is found independently of the others, as we have found the azimuth and hour angles in the foregoing solution independently of each other.

We have here strictly calculated the Sun's declination for the approximate time at Greenwich when the observation was taken; this is necessary if we wish to obtain great accuracy, and in this case the equation of second difference should be attended to, if it becomes sensible, as it generally does in the Sun's declination, except at or near the time of the equinoxes. The manner of finding the equation of second difference will be explained hereafter.

This is one of the most important problems in topical astronomy, either on land or sea; because

by it the true apparent time at any place is determined, the difference between which time and the true apparent time at Greenwich, either obtained by help of a chronometer or by lunar observations, gives the longitude of the place in time, and consequently in degrees, minutes, &c. The student will therefore do well to practise himself in the solution of this problem, till he become expert in the operation. The best time in the forenoon or afternoon, for taking the observation, is when the change in the Sun's altitude is the quickest, and this is when the Sun is in the prime vertical; but during half the year nearly the Sun is below the horizon when he is in the prime vertical of an oblique sphere, and consequently the thing is impracticable then. We must therefore take his altitude at the most convenient time practicable, but his distance from the meridian should, if possible, never be less than two hours, and his altitude, if possible, not less than 4 degrees, because his change in altitude, in high latitudes, when he comes near the meridian, is too slow to ascertain the true time with sufficient exactness, and in altitudes less than 4 degrees, the uncertainty of the exact quantity of the refraction may produce a considerable error in the time.

Example 2.

At Buenos Ayres, in latitude 34° 35′ south, longitude 58° 30′ west, on the 2d day of January, 1820, at 4*h*. 6*m*. P.M. by the watch not regulated, the altitude of the Sun's lower limb, above the true sensible horizon, was observed to be 35° 26′. What was the true hour of the day apparent and mean time?

	h.	*m.*	*s.*	
Time per watch	4	6	0	P.M.
Long. 58° 30′ W. in time +	3	54	0	
Approx. time at Greenwich	8	0	0	P.M.

Hence the declination of the Sun is the same as in example 1. . . . 22 58 26 S.
Subtract from 90 00 00

Polar dist. 67 1 34

	°	′	″
Observed alt. Sun's lower limb	35	26	00
Sun's semidiam. . . . . +		16	18
App. alt. Sun's centre .	35	42	18
Refraction . . . . . —		1	19
	35	40	18
Sun's Parallax in alt. . . +			7
True alt. Sun's centre .	35	41	06
Subtract from	90	00	00
Zen. dist.	54	18	54

Zen. dist.	54	18	54	
Colat.	55	25	00	ar. com. log. sine 0.0844411
Polar dist.	67	1	34	ar. com. log. sine 0.0358900
2)176	45	28		
Half sum	88	22	44	log. sine . . . 9.9998261
Subtract zen. dist. . . .	54	18	54	
Remainder	34	3	50	log. sine . . . 9.7482787
				2)19.8684359
				log cosine . . . 9.9342179

Half hour angle . . .	30	44	45
			2
Hour angle	61	29	30

True app. time	4*h*.	5*m*.	58*s*.	P.M.
Equat. time . . . . . +	4	11		
True mean time . . .	4	10	9	P.M.

	h.	*m.*	*s.*	
True apparent time	4	5	58	P.M.
Time per watch	4	6	00	P.M.
Watch fast for app. time . . .	0	0	2	
True mean time	4	10	9	P.M.
Time per watch	4	6	0	P.M.
Watch slow for true mean time	0	4	9	

Hence it appears that the watch was two seconds too fast for true apparent time as shewn by the Sun, and four minutes nine seconds too slow for true mean time.

In the first example the declination was added to 90°, and in this it has been subtracted from 90°, to obtain the polar distance; the reason is because the polar distance of a celestial object is the distance of its centre from the *elevated* pole; or, more scientifically, the polar distance of a celestial object is that arc of its meridian contained between its centre and the pole of the hemisphere in which the proposed place is situated. And hence this rule: if the latitude and declination have the same name, subtract the declination from 90°, but if they have contrary names, add the declination to 90°, and you have the polar distance.

We have omitted finding the azimuth, leaving it for the student's exercise, as also the projection of a figure for the example.

Example 3.

At Lima, in latitude 12° 1′ south, longitude 76° 49′ west, on the 2d day of January, 1820, at 2*h*. 52*m*.

50*s*. P.M. by the watch not regulated, the altitude of the Sun's lower limb, above the true sensible horizon, was observed to be 46° 10′. What was his true azimuth, and the true hour of the day apparent and mean time?

Solution.

In the oblique angled spherical triangle Z⊙S (Fig. 6.) there are given,

ZS = 77° 59′, the co. latitude.
S⊙ = 67° 1′ 34″, the polar dist.
Z⊙ = 43° 34′ 30″ the zenith dist.

1. To find the angle ZS⊙, the hour from noon.

	h. m. s.	
Time per watch	2 52 50	
Long. 76° 49′ W. in time . . +	5 7 16	
Approx. time at Greenwich . .	8 00 06	P. M.
Hence the Sun's declination is the same as in example 1 and 2. .	22 58 26	S.
Subtract from	90 00 00	
Polar dist.	67 1 34	

Ob. alt. ⊙'s lower limb	46 10 00	
Sun's semidiam. . . . +	16 18	
App. alt. Sun's centre	46 26 18	
Refraction subtract . . —	0 54	
	46 25 24	
Sun's parallax in alt. . +	6	
True alt. Sun's centre .	46 25 30	
Subtract from	90 0 0	
Zen. dist.	43 34 30	
Colat.	77 59 00	ar. com. log. sine 0.0096225
Polar dist.	67 1 34	ar. com. log. sine 0.0358900
	2)188 35 4	
Half sum	94 17 32	log. sine . . . 9.9987802
Subt. Zen. dist.	43 34 30	
Remainder	50 43 2	log. sine . . . 9.8887581
		2)19.9330508
Half hour angle . . .	22 12 29	log. cosine . . . 9.9665254
	2	
Hour angle	44 24 58	

	h. m. s.	
True app. time	2 57 40	P. M.
Equation of time . . . +	4 11	
True mean time . . .	3 1 51	P. M.

	h. m. s.	
True apparent time	2 57 40	P. M.
Time per watch	2 52 50	
Watch slow for apparent time . .	4 50	
True mean time	3 1 51	P. M.
Time per watch	2 52 50	
Watch slow for mean time	9 1	

Observe, that if the time found by the operation should differ considerably from that shewn by the watch, then the operation must be repeated, using this time instead of that shewn by the watch, to which apply the longitude in time as before, and then you have a more correct time for finding the true declination at the time of observation. But in the foregoing example, this is unnecessary; because the change in the Sun's declination for 4*m*. 50*s*, the error of the watch, is too small to affect the result sensibly.

2. In the same triangle to find the angle SZ⊙, the Sun's azimuth from the South.

Colat. . . .	77 59 00	ar. com. log. sine 0.0096225
Zen. dis. . .	43 34 30	ar. com. log. sine 0.1615895
Polar dist. .	67 1 34	
	2)188 35 4	
Half sum .	94 17 32	log. sine . . . 9.9987802
Subt. pol. dis.	67 1 34	
Remainder .	27 15 58	log. sine . . . 9.6609829
		2)19.8309751
Half azim.	34 35 49	log. cosine. . . 9.9154875
	2	
Azimuth . .	69 11 38	from the south.

Scholium.

The student may perceive that the Sun has the same azimuth at two several times, both in the forenoon and afternoon, in the present case; that is, if the latitude of the place be less than the Sun's declination, and both be North, or both South, then the Sun is twice on the same azimuth circle in the forenoon, and twice on the same azimuth circle in the afternoon. To render this matter evident; let us take an instance of a star, whose declination is greater than the latitude of London. Suppose the figure NHZO (Fig. 7.), to be a projection of the sphere on the plane of the meridian of London; and *m*, 1, 2,

3, *n*, to be the parallel of declination described by the Star in the Great Bear called β Ursæ Majoris, whose declination is 57° 22′ 25″ at the beginning of the year 1815: then it is evident that when the Star is on the meridian above the pole P, that the arc *Zm* is the excess of the Star's declination above EZ the latitude of London, which in this case is 5° 51′ 25″, and its complement 84° 8′ 35″ is the greatest altitude the star can have above the horizon of London; and *On*, which is the excess of the Star's declination above the complement of the latitude of London = 18° 53′ 25″, is the least altitude; and the Star never sets. Now when the Star is at either of the points, *m*, or *n*, its azimuth is the same, viz. due north; if we suppose it to move from *m*, it descends towards the west southerly, and when it arrives at 1, its azimuth is the angle BZO, or the arc BO; and this angle continues to increase till the Star arrives at 2, when the angle is the greatest, and then the azimuth circle just touches the parallel of declination in the point 2, or D; and the arc OF is the greatest azimuth: again, as the star continues to revolve round the pole its azimuth circle revolves with it, and when it arrives at the point 3, its azimuth is the same as at the point 1: and when it has completed its semi-revolution and arrives at the point *n*, its altitude is then the least, and its azimuth is due north; in compleating its other semi-revolution it ascends towards the east southerly, and the phenomena at the correspondent points of its parallel of declination are exactly the same. And hence every celestial body whose declination is greater than the latitude, and of the same name with it, has, at two several times on the east side of the meridian, and at two several times on the west side, the same azimuth: and those celestial bodies whose polar distance is less than the latitude never set, provided both have the same name; but if of contrary names, they never appear above the horizon, and lastly, all the celestial bodies whose declinations are less than the complement of the latitude, both rise and set.

From what has been said we may easily find the time when the Sun in the afternoon of the 2d of January 1820, had the greatest azimuth from the south at Lima. Through the point D (Fig. 6), in which the parallel of declination touches the azimuth circle, draw the meridian SDAP: then it is evident that the angle SDZ is a right angle; SD = S*m* is the Sun's polar distance = 67° 1′ 34″, in the foregoing example; SZ = 77° 59′ 00″ the colat. Hence, to find the angle DSZ, the hour from noon,

By Case 2, right angled Sph. Trig.

As radius		10.0000000
Is to tang. lat.	12° 1′ 00″	9.3280953
So is cotang. dec.	22 58 26	10.3726986
To cos. hour from noon	59 51 38	9.7007939
	h. m. s.	
	= 3 59 26.5 P. M.	

For the greatest azim. = angle SZD.

As cos. lat.	12° 1′ 0″0	9.9903775
Is to radius		10.0000000
So is cos. dec.	22 58 26	9.9641100
To sine greatest azim. from south	70 16 22	9.9737325

Here then it appears that the Sun's greatest azimuth from the south, was at 3*h*. 59*m*. 26.5*s*. P. M., at which time his azimuth was S 70° 16′ 22″ west, which is 1° 4′ 44″ greater than his azimuth at 2*h*. 57*m*. 40*s*. found aforegoing; and still greater than the complement of his amplitude at rising or setting: so that in all places on the Earth between the tropics, when the Sun transits the meridian between the zenith and the adjacent or nearest celestial tropic, the Sun from his rising, during a portion of the morning, in his ascent, inclines towards the depressed pole, till his motion that way becomes stationary and ceases: after which his motion is round the other way about the elevated pole; and a similar retrogradation takes place in describing the semi-diurnal arc in the afternoon. As the Sun has the same azimuth twice in the forenoon, and twice in the afternoon, we shall now proceed to find the time when he had the other altitude in the afternoon, and the quantity of that altitude, as also the altitude at the time of his greatest azimuth. Bisect the angle ⊙S*n* by the arc S*a*; then S*a* bisects the arc ⊙*an*, and is at right angles to it: and in the spherical triangle ZS*a* right angled at *a*; there are given ZS = 77° 59′, the co. latitude, the angle S*Za* the azimuth at the high altitude, found to be = 69° 11′ 38″; to find B*a*, the mean between the two altitudes and the hour angle ZS*a*.

1. To find the angle ZS*a* or the hour angle in the middle between the greater and less altitude.

By Case 1st. right angled Sph. Trig.

As radius		10.0000000
Is to sine lat.	12° 1′ 0″0	9.3184728
So is tang. azim.	69 11 38	10.4202319
To cotang. hour ang.	61 16 53	9.7387047

	h.	m.	s.	
Middle time	4	5	7.5	P. M.
			2	
Double mid. time . . .	8	10	15	
Sub. time gr. alt.	2	57	40	
Time less alt.	5	12	35	P. M.

2. To find the mean altitude B*a*, between the greater and less altitude.

As radius		10.0000000
Is to cotang. lat. . . .	12° 1′ 00″	10.6719047
So is cos. azim. . . .	69 11 38	9.5504811
To cotang. alt. . . .	30 55 57	10.2223858
	2	
Double	61 51 54	
Sub. gr. alt.	46 25 30	
Less altitude	15 26 24	

3. To find the altitude D*b* at the time of the greatest azimuth.

By Case 2d. right angled Sph. Trig.

As sine decl. . . .	22°58′ 26″	9.5914114
Is to radius		10.0000000
So is sine lat. . . .	12 1 00	9.3184728
To sine alt. gr. azim.	32 14 10	9.7270614

After the same manner we may find by this problem the time and azimuth at any given place; and when the place is situated in the torrid zone, and the Sun's declination exceeds the latitude, and both have the same name, then the various phenomena are determined as in this last example.

In the foregoing solution the declination of the Sun was corrected for the time of the greater altitude, and it was supposed to continue the same for the interval of time between the two altitudes; this, however, not being strictly the case, the arc S*a* does not accurately bisect ⊙*an*; therefore, if extreme accuracy be desired, we may find, in the manner already shewn, the time (nearly) when the Sun has the less altitude, then correct the declination for this time, the complement of which will be the true value of the arc S*n*, and as SZ is known = the colat. and the angle SZ*n* = the azim. the side *Zn* may be computed by oblique angled trig. the complement of which will be the altitude required.

In the same spherical triangle PZ⊙ (Fig. 5.), if any other three parts be given, then the other three can be found by oblique angled sph. trig. But should the Sun's azimuth constitute one part of the data, and his altitude be one of the parts required, and the latitude and declination be circumstanced as in the last example; then it is a necessary condition in the problem, to know whether it be the greater or less altitude that is sought; because in that case the Sun has the same azimuth at two several times, and two several altitudes, both in the forenoon and afternoon; as has been, we hope, clearly illustrated.

Problem 12.

Given the apparent altitudes of the Sun and Moon, or of a fixed star and the Moon, their apparent distance, the longitude, by account, of the place at which the observations are taken, and the time by a well regulated chronometer, to find the true distance of the bodies, and from thence the true longitude.

Preparatory Remarks, &c.

This is one of the most important problems in navigation, and as its solution depends upon a variety of circumstances intimately allied to Practical Astronomy, it seems proper to give it a place in this work. The distance of the Moon from the Sun, or from a known fixed star, being ascertained, and the corresponding instant of time noted at the place of observation, the observer finds by reference to the Nautical Ephemeris what the time is at Greenwich Observatory when the Moon is at that distance from the Sun, or from the known star, and therefore by taking the difference of these times and reducing it to degrees at the rate of 15 to an hour, he determines the difference of longitude. If the apparent, and true places of the Sun and Moon were the same, and their centres could be observed, it would only be necessary accurately to measure, with a proper instrument, the arc of distance between their centres, at any given instant; but since the apparent places often differ considerably from the true places, and the centres of the objects *cannot* be observed, it hence becomes necessary to take the arc of distance between their limbs, and likewise to measure the apparent altitudes of their limbs, in order that from thence, by making the necessary allowances, the true altitudes of their centres may be deduced, and their true distance computed.

The several corrections to be applied to observed altitudes of the heavenly bodies in order to obtain their true altitudes, have been specified in the axioms at p. 186; in Problem 11, immediately following those axioms, the allowances for semi-diameter, refraction, and parallax, have been made *separately*, which is, perhaps, the best method of impressing the mind of a learner with the effects produced by each of these causes; it is, however, customary in Nautical Astronomy, to combine the allowances for

refraction and parallax, and to tabulate the *difference*, between them, which difference is then called the *correction* of the Sun's or Moon's altitude. The Sun's correction for various apparent altitudes is given in Table 17, Part 1st, and for any altitude not inserted in the table, a proportional part must be taken between the corrections for the two adjacent altitudes; as the fixed stars have no sensible parallax, the correction of a fixed star in altitude is the same as its refraction; these corrections adapted to a state of mean refraction of the atmosphere are inserted in Table 17, Part 2d. The Moon's horizontal parallax being considerable, and likewise a variable quantity, it requires a long table to exhibit her correction in altitude—such a table is given in works treating expressly on the longitude, &c. but as the operation for computing the parallax in altitude from the given horizontal parallax (by the Rule at p. 78) is very short, we shall take this method, and dispense with a table for her correction in altitude. The apparent altitude of the Sun always exceeds the true altitude; for the refraction elevates the Sun, more than the parallax depresses him, so that the true place of the Sun, or of a fixed star, is always below the apparent place: the Moon, on the contrary, is depressed more by parallax that she is raised by refraction, which causes her true place to be always above the apparent place.

To the 2d part of Table 17, there is annexed the allowance to be made for the depression, or dip of the horizon, at different heights of the eye as far as 100 feet; this allowance is of course subtractive from observed altitudes. The augmentation of the Moon's semi-diameter is given in Table 35 of the Lunar Tables, and is entered with a double argument, viz. the horizontal semi-diameter, and the apparent altitude.

There are some other minute corrections of the altitudes which might be made, if extreme accuracy were aimed at, such as a correction of the Moon's parallax on account of the spheroidal figure of the Earth, an allowance for the diminution of the vertical semi-diameters of the Sun and Moon by refraction, a correction for refraction according to the actual state of the atmosphere as indicated by the barometer and thermometer (See Table 17, Part 3d.) and not according to the *mean* astronomical refraction, but as these corrections frequently counteract each other, and we cannot yet expect to have arrived at so perfect a solution of this problem as to be free from greater errors than those we should thus seek to avoid, it seems unnecessary to notice them.

Having then by the proper tables, &c. found the true altitudes of the centres of the Sun or star, and of the Moon, the computation of the true distance of the centres of the bodies is thus effected by spherical trigonometry.

Let *s* (Fig. 8.) be the apparent, and S the true place of the centre of the Sun or Star, both on the same vertical circle ZS, *m* the apparent, and M the true place of the Moon's centre, on the same vertical *Zm*, *ms* the apparent distance of the centres, then there are given $Zs = 90° -$ app. alt. of Sun or star $ZS = 90° -$ true alt. of do., $Zm = 90° -$ app. alt. of Moon's centre, $ZM = 90° -$ true alt. do. and *ms* the apparent distance of the centres; hence in the spherical triangle *Zsm*, there are given all the sides, from which the angle Z is found, and then with this angle and the containing sides ZS, ZM, the true distance SM is determined.

Example 1.

Being at sea on January 1st, 1820, in longitude 16° 30′ W. by account, at 11*h*. 0*m*. 0*s*. per watch well regulated; I took the altitude of Aldebaran 32° 56′ 26″, and that of the Moon's lower limb, 15° 17′ 7″, and at the same time the distance between the star and the Moon's farthest limb was 52° 1′ 59″, the height of the eye being 20 feet above the water. Required the true longitude.

	h.	*m.*	*s.*
Time at ship	11	0	0
Long. W. in time	1	6	0
Reduced time, or time at Greenwich	12	6	0

	°	′	″
Observed dist. ☽'s farthest limb from Aldebaran	52	1	59
☽'s horiz. sem. diam. at red. time by naut. alm. 14 49			
Augm. ☽'s sem. diam. Tab. 35. 4		14	53
App. dis. of centres	51	47	6
✳'s obs. alt.	32	56	26
Dip for 20 feet		4	16
✳'s app. alt.	32	52	10
✳'s *corr:* or refraction, Tab. 17, p. 2d.		1	28
✳'s true alt.	32	50	42
Obs. alt. ☽'s lower limb	15	17	7
Dip.		4	16
	15	12	51
Sem. diam. and augm.		14	53
App. alt. ☽'s centre	15	27	44
☽'s parall. in alt. as found below 52′ 19″			
☽'s refraction 3 27			
☽'s correction		48	52
True alt. ☽'s centre	16	16	36

☽'s hor. parall. at red. time by naut. alm. 54′ 17″ = 3257″ log.	3.5128178
App. alt. centre 15° 27′ 44″ log. cos.	9.9839898
☽'s parall. in alt. 52′ 19″ = 3139 log.	3.4968076

In the spherical triangle Zsm, there are now given,

Zs = 57° 7′ 50″ the ✳'s app. zen. dist.
Zm = 74 32 16 the app. zen. dist. of ☽'s centre,
Sm = 51 47 6 the app. dist. of the centres,

to find the angle at the zenith sZm: by Case 5th of oblique angled sph. trig.

° ′ ″	
57 7 50	ar. com. log. sine 0.0757675
74 32 16	ar. com. log. sine 0.0160102
51 47 6	
2)183 27 12	
91 43 36	log. sine 9.9998028
51 47 6	
39 56 30	log. sine 9.8075400
	2)19.8991205
27 4 58.4	log. cos. 9.9495602
2	
54 9 56.8 = ∠ at zenith.	

Next in the triangle **ZSM**, there are known

ZS = 57° 9′ 18″, the ✳'s true zen. dist.
ZM = 73 43 24 the true zen dis. of ☽'s cen.

and ∠SZM = 54 9 57, the angle at the zenith

to find SM, the true distance: letting fall the perpendicular Sp, we have by Case 1st of right ang. sph. trig.

As radius	10.0000000
Is to cos. SZM, 54° 9′ 57″	9.7674833
So is tang. ZS, 57 9 18	10.1900579
To tang. Zp, 42 12 13	9.9575412
From 73 43 24	
Take 42 12 13	
Remains pM, 31 31 11	

Then by Prop. C, sph. trig.

° ′ ″	
As cos. Zp, 42 12 13	9.8696788
Is to cos. pM, 31 31 11	9.9306741
So is cos. ZS, 57 9 18	9.7342942
To cos. SM, 51 22 48.4 true dist . . .	9.7952895

Now upon reference to the Nautical Almanac it appears that on January 1st, at 12h. or midnight, the distance of Aldebaran from the Moon's centre is 51° 19′ 33″, and at 15h. it is 52° 47′ 20″, whence by proportion we find the time which corresponds to the distance 51° 22′ 48″ and the true longitude as follows,

52° 47′ 20″	51°22′ 48″
51 19 33	51 19 33
As 1 27 47 :	0 3 15 : : 3h. : 6m. 39.8s.

h. m. s.	
12 0 0	
12 6 39.8	
11 0 0	
1 6 39.8 = 16° 39′ 57″ the true long. W.	

Note 1.—As the zenith distance of either body is (in this Problem) always less than 90°, the perpendicular Sp, will fall within or without the triangle, according as the angle at the zenith is less or greater than 90°, and when it falls without, pM will be equal to the *sum* of ZM and Zp.

Note 2.—If the reduced time be much different from 0h. or 12h. we must then proportionate for the change of the Moon's horizontal parallax and semi-diameter in the interval between the reduced time, and the nearest noon or midnight in the N. Alm.

Example 2.

Being at sea on January 20th, 1820, in longitude 78° 57′ E. by account at 5h. 30m. 48s. per watch well regulated; I took the altitude of the Sun's lower limb 17° 50′ 48″, and that of the Moon's lower limb 44° 36′ 31″, and at the same moment the distance of their nearest limbs was 65° 56′ 14″, the height of the eye being 20 feet above the water. What was the true longitude?

	h. m. s.	° ′ ″
Time at ship	5 30 48	
Long. E. in time	5 15 48	
Reduced time	15	
Obs. dist. ⊙ and ☽'s nearest limbs . .		65 56 14
☽'s hor. sem. diam. at red. time 16′ 10″ Augm. Tab. 35. 12		16 22
⊙'s sem. diam. at red. time		16 17
App. dist. of centres		66 28 53
Obs. alt. ⊙'s lower limb		17 50 48
Dip. for 20 feet . . . . . 4′ 16″ Sem. diam. 16 17		12 1
App. alt. ⊙'s centre		18 2 49
⊙'s correct. Tab. 17, part 1st.		2 48
True alt. ⊙'s centre		18 0 1

Obs. alt. ☽'s lower limb			44° 36′ 31″
Dip.		4′ 16″	
Sem. diam. and augm.		16 22	12 6
App. alt. ☽'s centre			44 48 37
☽'s parall. in alt. See below		42′ 1″	
☽'s refraction		58	
☽'s correction			41 3
True alt ☽'s centre			45 29 40

☽'s hor. parall. at red. time by N. Alm.	59′ 14″ = 3554″	log. 3.5507174
App. alt. cent.	44° 48′ 37″	log. cos. 9.8509183
☽'s parall. in alt.	42′ 1″ = 2521″	log. 3.4016357

In the spherical triangle Zsm, we hence have,
Zs = 71°57′ 11″ the app. zen. dist. of ⊙'s centre,
Zm = 45 11 23 the app. zen. dist. of ☽'s centre,
Sm = 66 28 53 the app. dist. of the centres,
to find the angle at the zenith:

71° 57′ 11″	ar. com. log. sine	0.0219095
45 11 23	ar. com. log. sine	0.1490817
66 28 53		
2)183 37 27		
91 48 43.5	log. sine	9.9997827
66 28 53		
25 19 50.5	log. sine	9.6312835
	2)19.8020574	
37 13 48.4	log. cos.	9.9010287
2		
74 27 36.8 = ang. at zenith.		

Next, in the triangle ZSM, there are given
ZS = 71°59′ 59″ true zen. dist. of ⊙'s centre
ZM = 44 30 20 true zen. dist. of ☽'s centre
aug. SZM = 74 27 36.8 the ang. at the zenith,
to determine the true distance, SM:

As radius		10.0000000
Is to cos. SZM,	74°27′ 36.8	9.4279846
So is tang. ZS,	71 59 59	10.4882168
To tang. Zp,	39 30 226	9.9162014

From	44°30′ 20″
Take	39 30 22.6
Remains pM	4 59 57.4

Then as cos. Zp,	39 30′ 22″.6	9.8873668
Is to cos. pM,	4 59 57.4	9.9983447
So is cos. ZS,	71 59 59	9.4899888
To cos. ZM,	66 29 4.8 the true dist.	9.6009667

Dist. of centres at 3h.	67° 58′ 38″	66° 29′ 4.8	computed dist.
Dist. of centres at noon	66 20 46	66 20 46	
Diff.	1 37 52	0 8 18.8	

3h. : 15m. 17.4s. :: 1h. : 5m. 0.6s.
3 30 48
5 15 30.6

= 78° 52′ 39″ the true long. E.

Scholium.

Such then is the solution of the problem by the ordinary and direct rules of spherical trigonometry, and persons accustomed to trigonometrical computation will find no difficulty in performing it. That valuable class of men, however, who have most occasion for the practice of this problem, have seldom an opportunity of acquiring so much knowledge of mathematical principles as to render them thorough masters of this method, besides which, they have generally complained of its tedious length. To render farther assistance, therefore, to the mariner, various scientific men have devised more easy and expeditious rules for finding the true distance between the bodies; of these, some give the *corrected distance*, others determine the correction to be applied to the observed, to give the true distance. Most of these rules still proceed entirely upon the *basis* of spherical trigonometry, and by means of subsidiary tables, adapted to particular parts of the calculation, contract the process, while a few of the inventers have successfully employed the more profound investigations of the fluxional, or differential calculus, in order to ascertain the variation produced in the distance by parallax, &c. A short account of the principal of these methods is given by the learned Dr. Brinkley, Andrews' professor of Astronomy, in the University of Dublin, in his paper on reducing the observed distance of the Moon from the Sun, or a fixed Star, to the true distance published in the 11th volume of the Transactions of the Royal Irish Academy, he observes, that "of the two classes into which the solution of this problem may be divided, the one in which the correction of the distance is given, and not the corrected distance itself, is

necessarily embarrassed by a distinction of cases, although otherwise concise, as only requiring tables to a few places of decimals. In the other class are found solutions which do not require any distinction of cases; among such solutions, therefore, it should seem that those are to be selected which can be most readily adapted to practice by precludihg the necessity, or reducing the labour of computing proportional parts, which is the great inconvenience of methods of this class.

Notwithstanding, however, the inconvenience arising from the distinction of cases, some very eminent mathematicians have exerted themselves in investigating the correction of the distance. The first method by Dr. Maskelyne, the methods by Mr. Lyons, Mr. Witchell, and Mr. Cavendish, are of this class, as are also several investigated with his usual ingenuity by M. Delambre*, one lately given by M. Legendre† and one by Mr. Bowditch, of which an account was given by M. Delambre‡.

The practical conveniences in none of these methods seem to counterbalance the inconveniences and comparing them with the solutions in which the corrected distance is obtained, several of the latter appear better adapted for general use.

An improvement of Mr. Dunthorne's solution, given in the second edition of the Requisite Tables, seems to be the first in which the distinction of cases was avoided, and which was, at the same time, sufficiently concise to be preferred to the solutions of the other class: yet, in this method, the computation of the parts for seconds was very troublesome. This rule was farther improved by Dr. Maskelyne, in his most useful preface to Taylor's Logarithmic tables, and those tables, extending to seconds, obviated the inconvenience from proportional parts. But Taylor's tables cannot be very generally diffused, and besides they require too particular attention in using them to be held in that estimation with seamen which is their due.

Mr. Mendoza [Rios], to whose exertions Nautical Astronomy is much indebted, has given in a valuable paper in the Phil. Trans. for 1797, forty different formulas for computing directly the true distance. Of these, that which he selected as best adapted to practice, affords a very concise and plain method of computation, requiring only the addition of five versed sines. It may be doubted, however, whether it is preferable to Mr. Dunthorne's first method, improved by the substitution of versed sines for cosines, as was done by Dr. Mackay§. Mr. Mendoza's method requires an extensive table for an auxiliary angle (equivalent to the auxiliary table in Mr. Dunthorne's method, and in those derived from it), and the formation of five different arguments, and also to practise it with convenience, a compleat table of versed sines, for at least the semicircle. Mr. Mendoza, anxious to improve still farther the solution of the problem, published his very extensive tables‖, by which he reduced the method to be equivalent to taking out and adding together three numbers, and not requiring the formation of arguments. His ingenuity and perseverance in forming, computing, and publishing his tables, are deserving of the greatest praise. But on several accounts those tables will not supersede the use of methods in which shorter tables are employed, although these methods should be somewhat longer in practice, provided they be equally plain. To many persons the necessary expense of the volume will be an objection; many will consider its bulk inconvenient; and many, disliking such extensive tables with double arguments, will even prefer the former method which those tables were intended to facilitate.

The method pointed out in this paper is closely connected with Mr. Dunthorne's own method, and still more closely with that method improved by Dr. Mackay. It has no distinction of cases, and requires only short tables with single arguments". "If it be thought somewhat longer in practice than that by the extensive tables of Mr. Mendoza, it is not less plain."

To the above we have to add that the late Mr. George Sanderson, gave some very distinct and concise rules for performing the problem, in the Ladies' Diary for 1787, which may also be found in Leybourn's edition of the Diaries, Vol. 3, p. 173.

As we entirely coincide with Dr. Brinkley, in the preference to be given to those methods which find the *corrected distance*, and not the *correction of the distance*, we shall now present the reader with Dr. Brinkley's method, Mr. Sanderson's, and one of Dr. Mackay's, which appear to be three of the best and shortest, and require only the assistance of tables given in this work, and the usual tables of natural and logarithmic sines, &c.

Mr. Sanderson's Method.

To the apparent distance of the Moon and Sun, or Moon and Star, add the difference of their apparent altitudes, and take half the sum: also from the apparent distance subtract the difference of the apparent altitudes, and take half the remainder.

Add together the log. sines of this half sum, of this half remainder, of the true zenith distances,

* Conn des tems, Ann: 12, and 14. † Mem. Nat. Inst. ‡ Conn des tems, 1808. § The theory and practice of finding the longitude, &c. ‖ A complete collection of tables for Navigation and Nautical Astronomy."

and the arithmetical complements of those of the apparent zenith distances (or their log. cosecants); and take half the sum.

From this half sum of the six logs. subtract the log. sine of half the difference of the true altitudes, and the remainder is the log. tangent of an arc; the log. sine of which subtracted from the said half sum of the six logs., leaves the log. sine of half the true distance*.

EXAMPLE 1st *repeated.*

REFERRING to what has been already done for the corrected altitudes, &c. we have,

	°	′	″
☉'s app. alt.	32	52	10
☽'s app. alt.	15	27	44
diff. app. alt.	17	24	26

	°	′	″		°	′	″
App. dist. of centres	51	47	6		51	47	6
	17	24	26		17	24	26
	2)69	11	32		2)34	22	40
½ sum	34	35	46	½ diff.	17	11	20

	°	′	″
☉'s true alt.	32	50	42
☽'s true alt.	16	16	36
	2)16	34	6
½ diff.	8	17	3

	°	′	″		
	34	35	46	log. sine	9.7541861
	17	11	20	log. sine	9.4705910
☉'s tr. zen. dist.	57	9	18	log. sine	9.9243521
☽'s ditto . .	73	43	24	log. sine	9.9822348
☉'s ap. zen. dist.	57	7	50	ar. com. log. sine	0.0757675
☽'s ditto . .	74	32	16	ar. com. log. sine	0.0160102
					2)39.2231417
					19.6115708
	8	17	3	log. sine	9.1586120
Log. tan.	70	35	14.4		10.4529588
	70	35	14.4	log. sine	9.9745804
Log. sine	25	41	23.9		9.6369904
			2		
True dist.	51	22	47.8		

differing only 0.6″ from the former.

* This method has been adopted, with slight modification, by Mr. Bonnycastle in his Trigonometry, 2d Edit. p. 260.

It is evident that the computer may, if he please, use the cosines of the altitudes, instead of the sines of the zenith distances, but the above mode of conducting the operation is, we think, the best for the practical navigator.

Dr. Mackay's 1st method, at length,† without using his auxiliary table, entitled 'logarithmic difference'.

FROM the natural versed sine of the apparent distance, subtract the natural versed sine of the difference of the apparent altitudes, and find the common logarithm of the remainder; to this add the log. secants of the apparent altitudes, and log. cosines of the true altitudes, and reject the tens in the index: add the natural number answering to this sum to the natural versed sine of the difference of the true altitudes, and the sum will be the natural versed sine of the true distance.

Note.—As the versed sines in most books of tables only extend to a quadrant, it is necessary to remark that if A be the excess of an arc above 90° then nat. vers. sine of 90° + A = 1 + sin. A, and if a be the excess of a versed sine above rad. or 1, then the arc corresponding to the versed sine $1 + a$ is 90° + arc corresponding to a, taking a as a sine. In the appendix to the 3d Edit. of the Requisite Tables there is a table very commodiously arranged containing the versed sines to every second from 0° to 120°.

EXAMPLE 2d. *repeated.*

	°	′	″		°	′	″
☽'s app. alt.	44	48	37	☽'s true alt.	45	29	40
☉'s app. alt.	18	2	49	☉'s true alt.	18	0	1
Diff.	26	45	48	Diff.	27	29	39

App. dist.	66	29	0	nat. v. sin.	6009842
Diff. app. alt.	26	45	48	nat. v. sin.	1071258
					4938584 its log. 6.6936024

☽'s app. alt.	44	48	37	log. sec.	10.1490817
☉'s app. alt.	18	2	49	log. sec.	10.0219095
☽'s true alt.	45	29	40	log. cos.	9.8457046
☉'s true alt.	18	0	1	log. cos.	9.9782056
					4880943 . . 6.6885038
Diff. true alt.	27	29	39	nat. v. sine	1129422
	66	29	11.8	nat. v. sin.	6010365
Subtract sec. added			7		

True distance 66 29 4.8 as before

† This is the method which has been selected by Mr. Keith in his Trigonometry, 2d Edit. p. 299, except that Mr. Keith substitutes rad. ∓ cos. for vers. sine.

When the distance is given in degrees, minutes, and seconds, and the operation is performed by the common tables, extending only to minutes, it will facilitate the work, to take the nearest minute in the distance, as has been done here, and, after the operation is finished, increase or diminish the computed distance by the seconds, as the case may require.

Dr. Brinkley's Method.

HAVING found the apparent distance of the centres of the Moon and Star, or Sun, and likewise the apparent and true altitudes, in the manner already shewn in the examples, pages 192, 193; find also the parallax answering to the complement of the Moon's apparent altitude.

Subtract the versed sine of the difference of the apparent altitudes, from the versed sine of the apparent distance, and call the remainder R.

When a Star is observed, take out, with the proper arguments, the numbers in tab. A, B, C, E, pages 198, 199, and add them together; when the Sun is observed, add together the numbers from tab. A, B, D, E; the log. of this sum added to the log. of R, rejecting 6 from the index, is the log. of a number, which taken from R, and the remainder added to the versed sine of the difference of the true altitudes, gives the versed sine of the true distance.

Note.—The versed sines may be taken as whole numbers to the radius 1000000, and in finding the log. of R, the two last figures may be considered as cyphers. The argument to be used in taking out the numbers from tables A, B, &c. is placed at the head of each.

Computation of Example 1st.

☽'s hor. parall, 54′ 17″ = 3257″ log. . . 3.51282
App. alt. cent. 15° 27′ 44″ . . . log. sine 9.42586

14′ 28″ = 868″ 2.93868

☽'s app. alt. 32° 52′ 10″ ☽'s true alt. 32° 50′ 42″
☉'s app. alt. 15 27 44 ☉'s true alt. 16 16 36

Diff. . . . . 17 24 26 Diff. 16 34 6

App. dist. . . . 51° 47′ 6″
☽'s correct . . 48′ 52″

Tab. A . . . 3515 }
135 }
Tab. B 102
Tab. C 0
Tab. E 6

3758 its log. 3.57496

v. sine 51° 47′ 381363
v. sine 17° 24′ 26″ 045797

R . . . . . 335566 its log. 5.52569

1261 3.10065

334305
v. sine . . 16 34 6 . . . 041519

v. sine . . 51 22 41.8 . . 375824

Reser. sec. 6

True dist. 51 22 47.8

Computation of Example 2d.

☽'s hor. parall. 59′ 14″ = 3554″ log. . . 3.55072
App. alt. cent. 44° 48′ 37″ . . . log. sine 9.84804

41′ 45″ = 2505″ 3.39876

☽'s app. alt. 44°48′ 37″ ☽'s true alt. 45°29′ 40″
☉'s app. alt. 18 2 49 ☉'s true alt. 18 0 1

Diff. . . . 26 45 48 Diff. 27 29 39

App. dist. . . . 66° 28′ 53″
☽'s correct. . . 41 3

Tab. A . . . 11370 }
218 }
Tab. B . . . 71
Tab. D . . . 19
Tab. E . . . 0

11678 its log. 4.06737

v. sine 66° 29′ 600984
v. sine 26° 45′ 48″ 107126

R . . . . . 493858 its log. 5.69355

5767 . . . 3.76092

488091
v. sine . . 27 29 39 . . . 112942

v. sine . . 66 29 11 . . . 601033
Exc. of sec. 7

True dist. 66 29 4

This last method of computing the true distance is considered by some nautical gentlemen the easiest and most expeditious of any. If the versed sines, as arranged in the appendix to the 3d edit. of the Requisite Tables, be used, all the proportional parts may be taken out by inspection, and even with the common tables, the operation may be soon performed.

Should the computed longitude differ considerably from that by account, it will be proper to find the reduced time answering to this computed longitude, and take from the Naut. Alm. the semi-diam. and parallax for that time, and with these compute the true distance again, which will thus be obtained still more accurately, and from thence the true longitude

To those who may wish for a solution to this problem by construction, we would recommend a very concise and neat projection for clearing the lunar distance from parallax and refraction, invented by Dr. Kelly, and given in his Spherics and Nautical Astronomy, 4th edit. p. 189.

TABLE A.

Arg. parall. answering to comp. of ☽'s app. alt.					
Sec. of arg	Arg. ′ & ″	Minutes of arg.	Sec. of arg.	Arg. ′ & ″	Minutes of arg.
5	1	....	155	32	8752
10	2	25	160	33	9043
15	3	316	165	34	9333
19	4	606	169	35	9624
24	5	897	174	36	9915
29	6	1188	179	37	10206
34	7	1479	184	38	10497
39	8	1770	189	39	10788
44	9	2061	194	40	11079
48	10	2352	199	41	11370
53	11	2643	204	42	11661
58	12	2934	209	43	11952
63	13	3224	213	44	12242
68	14	3515	218	45	12533
73	15	3806	223	46	12824
78	16	4097	228	47	13115
83	17	4388	233	48	13046
88	18	4679	238	49	13697
93	19	4970	242	50	13988
97	20	5261	247	51	14279
102	21	5552	252	52	14570
107	22	5843	257	53	14861
112	23	6133	262	54	15151
116	24	6424	267	55	15442
121	25	6715	272	56	15733
126	26	7006	276	57	16024
131	27	7297	281	58	16315
135	28	7588	286	59	16606
140	29	7879	291	60	16896
145	30	8170		61	17187
150	31	8461		62	17478

TABLE B.

Arg. correct. of ☽'s app. alt.			
Arg. min.		Arg. min.	
1	0	29	35
2	0	30	38
3	0	31	40
4	1	32	43
5	1	33	46
6	1	34	49
7	2	35	52
8	2	36	55
9	3	37	58
10	4	38	61
11	5	39	64
12	6	40	68
13	7	41	71
14	8	42	74
15	10	43	78
16	11	44	82
17	12	45	86
18	13	46	90
19	15	47	94
20	17	48	98
21	18	49	102
22	19	50	106
23	21	51	110
24	23	52	114
25	26	53	118
26	28	54	123
27	30	55	128
28	32	56	131

TABLE C.

Arg. ✳'s app. alt.		
Arg. ′ ″		
3 0		65
15		57
30		51
3 45		46
4 0		42
30		35
5 0		30
6 0		23
7 0		18
8 0		15
9 0		13
10 0		11
15 0		6
20 0		4
25 0		2
30 0		0
90 0		0

TABLE D.

Arg. ⊙'s app. alt.	
Arg. ° ,	
3 0	67
15	59
30	53
3 45	48
4 0	45
30	38
5 0	33
6 0	27
7 0	22
8 0	20
9 0	19
10 0	18
15 0	17
20 0	19
25 0	20
30 0	21
35 0	24
40 0	27
50 0	33
60 0	37
70 0	41
80 0	42
90 0	43

TABLE E.

Arg. ☾'s app. alt.	
Arg. ° ,	
3 0	55
15	48
30	43
3 45	39
4 0	36
30	30
5 0	26
6 0	20
7 0	16
8 0	19
9 0	11
10 0	10
15 0	6
20 0	4
25 0	2
30 0	0
90 0	0

Lemmata to the Investigations of the three preceding Rules.

Lemma 1.—IF P and Q denote any two circular arcs, or angles, to rad. 1, cos. Q — cos. P = 2 sin. $\frac{1}{2}$(P — Q) sin. $\frac{1}{2}$(P + Q).

Let AB, AC, AD (Fig. 9, plate 9), be three equidifferent arcs, whose common difference is the arc BC = the arc CD, and let AD = P, AB = Q. From the centre O, draw the radii OA, OC; the latter of which will be perpendicular to the chord DB, and will bisect it: draw DF, CH, BI, at right angles to OA, also BK parallel to OA. The triangles DKB and OCH being similar OC : CH :: BD = 2 DE : BK, whence OC × BK = 2 DE × CH, Now CH = sin. $\frac{1}{2}$(AD + AB) = sin. $\frac{1}{2}$(P + Q) 2 DE = 2 sin. $\frac{1}{2}$(AD — AB) = 2 sin. $\frac{1}{2}$(P — Q) BK = FI = OI — OF = cos. AB — cos. AD = cos. Q — cos. P, and OC = rad. = 1; therefore by substituting these values in the equation OC × BK = 2 DE × CH, there is obtained cos. Q — cos. P = 2 sin. $\frac{1}{2}$(P — Q) sin. $\frac{1}{2}$(P + Q).

Lemma 2.—P and Q still denoting two unequal angles, or arcs, of which P is the greater, sin. (P+Q) sin. (P — Q) = sin.^{2}P — sin.^{2}Q.

LET A (Fig. 10.) be the greater of the two angles A, C, the line AC being a common side to each, and let the other sides be produced to meet in B, thereby forming the triangle ABC. Draw BD perpendicular to AC, take Da = DA, and join Ba.

Then will aC = DC — DA,
Ba = BA,
$\angle$ BaD = BAD,
$\angle$ BaC = supp. BAC, and conseq.
sin. BaC = sin. BAC;

also sin. ABC = sin. supp. ABC = sin. (A + C)
likewise $\angle$ aBC = A — C.

In trian. ABC, by prop. A, plane trigonometry,
sin. ABC or sin. (A + C : sin. A :: AC : BC,
and in trian. aBC,
sin. aBC or sin. (A — C) : sin. a = sin. A :: aC : BC
∴ sin. (A + C) sin. (A — C) : sin.^{2}A :: AC × aC : BC^2;
but by prop. B. plane trigonometry,
AC : BC + BA :: BC — BA : aC,
whence AC × aC = BC^2 — BA^2,
and ∴ sin. (A + C) sin. (A — C) :: sin.^{2}A :: BC^2 — BA^2 : BC^2 . . . . . (1).

Again, BC : BA :: sin. A : sin. C,
or, BC^2 : BA^2 :: sin.^{2}A : sin.^{2}C,
and, by div. BC^2 — BA^2 : BC^2 :: sin.^{2}A — sin.^{2}C : sin.^{2}A . . . . . (2),
whence by comparing the analogies 1, 2, there results,
sin. (A + C) sin. (A — C) : sin.^{2}A :: sin.^{2} A — sin.^{2}C : sin.^{2}A,
consequently sin. (A + C) sin. (A — C) = sin.^{2}A — sin.^{2}C;
or, writing P and Q for A and C,
sin. (P + Q) sin. (P — Q) = sin.^{2}P — sin.^{2}Q.

Lemma 3.—If a and b be any two sides of a spherical triangle, of which a is the greater, c the third side, and C the angle opposite that side, or the angle contained by a and b, then, to rad. 1,

$$\text{vers. sin. C} = \frac{\text{cos. } (a - b) - \text{cos. } c}{\text{sin. } a \text{ sin. } b}$$

Because $\frac{1}{2}(a + b + c) - a = \frac{1}{2}(b + c - a)$
and $\frac{1}{2}(a + b + c) - b = \frac{1}{2}(a + c - b)$
therefore by prop. E, Sph. Trigonometry,
sin. a sin. b : sin. $\frac{1}{2}(b + c - a)$ sin. $\frac{1}{2}(a + c - b)$ :: rad.^{2} (1) : sin.^{2} $\frac{1}{2}$C

$$\text{hence sin.}^2 \tfrac{1}{2}\text{C} = \frac{\text{sin. } \frac{1}{2}(b + c - a) \text{ sin. } \frac{1}{2}(a + c - b)}{\text{sin. } a \text{ sin. } b}$$

But from the Dem. of the Lemma at page 162,
sin.^{2}$\frac{1}{2}$C = $\frac{1}{2}$(1 — cos. C) = $\frac{1}{2}$ vers. sin. C;
and if we put $b + c - a$ = P — Q, and $a + c - b$ = P + Q, we shall find P = c, and Q = a — b, whence by Lemma 1. sin. $\frac{1}{2}(b + c - a)$ sin. $\frac{1}{2}(a + c - b)$ = $\frac{1}{2}$[cos. (a — b) — cos. c], and by

substituting these values in the equation derived from prop. E, aforesaid, and multiplying by 2, it becomes

$$\text{vers. sin. C} = \frac{\text{cos. } (a - b) - \text{cos. } c}{\text{sin. } a \text{ sin. } b}$$

Investigation of Mr. Sanderson's Rule.

Let h = apparent altitude of the Sun or Star,
h' = apparent altitude of the Moon,
H = true altitude of the Sun or Star,
H′ = true altitude of the Moon,
d = half the apparent distance,
D = half the true distance,
a = half the diff. of the app. altitudes,
A = half the diff. of the true altitudes.

Then, referring to Fig. 8, plate 9, we have in the triangle Zsm,

$\frac{1}{2}(sm + Zs + Zm) - Zm = \frac{1}{2}(sm + Zs - Zm) = d + a$,

$\frac{1}{2}(sm + Zs + Zm) - Zs = \frac{1}{2}(sm + Zm - Zs) = d - a$,

and the sines of the zenith distances being equal to the cosines of the altitudes, therefore by prop. E, Sph. Trigonometry.

cos. h cos. h' : sin. $(d + a)$ sin. $(d - a)$ :: rad.^{2}(1) : sin.$^2\frac{1}{2}$Z :

$$\text{whence sin.}^2\tfrac{1}{2}\text{Z} = \frac{\text{sin. } (d + a) \text{ sin. } (d - a)}{\text{cos. } h \text{ cos. } h'}$$

In a similar manner we find from the triangle ZSM

$$\text{sin.}^2\tfrac{1}{2}\text{Z} = \frac{\text{sin. } (\text{D} + \text{A}) \text{ sin. } (\text{D} - \text{A})}{\text{cos. H cos. H}'}$$

and by equating these two values of sin.$^2\frac{1}{2}$Z, there results,

sin. (D + A) sin. (D − A) = sin. $(d + a)$ sin. $(d - a)$. $\frac{\text{cos. H cos. H}'}{\text{cos. } h \text{ cos. } h'}$

Put the 2d. side of this last equation = K^2, then since by Lemma 2, sin. (D + A) sin. (D − A) = sin.^{2}D − sin.^{2}A, therefore $\text{K} = \sqrt{(\text{sin.}^2\text{D} - \text{sin.}^2\text{A})}$, and $\frac{\text{K}}{\text{sin. A}} = \sqrt{\left(\frac{\text{sin.}^2\text{D}}{\text{sin.}^2\text{A}} - 1\right)}$. But, $\sqrt{\left(\frac{\text{sin.}^2\text{D}}{\text{sin.}^2\text{A}} - 1\right)}$ expresses the tangent of an arc whose secant is $\frac{\text{sin. D}}{\text{sin. A}}$ to rad. 1, whence $\frac{\text{sin. D}}{\text{sin. A}}$: $\sqrt{\left(\frac{\text{sin.}^2\text{D}}{\text{sin.}^2\text{A}} - 1\right)}$ :: 1 : $\sqrt{\frac{\text{sin.}^2\text{D} - \text{sin.}^2\text{A}}{\text{sin.}^2\text{D}}}$ = sine of that arc, = suppose, L ; now $\sqrt{(\text{sin.}^2\text{D} - \text{sin.}^2\text{A})} \div \sqrt{\frac{\text{sin.}^2\text{D} - \text{sin.}^2\text{A}}{\text{sin.}^2\text{D}}}$ is equal to sin. D, that is K ÷ L = sin. D : from which it appears, that to determine log. sin. D, we must find the arc whose log. tang. is log. K − log. sin. A ; next look out the sine of this arc, L, and then log. K − log. L will be equal to log. sin. D. If the Rule be now compared with this investigation, it will appear that by taking the half sum of the six logarithms therein mentioned, the log. of K is found, after which log. sin. D, or log. sine of true distance, is obtained in the manner above shewn.

Investigation of Dr. Mackay's Rule.

Let h = apparent altitude of the Sun or Star,
h' = apparent altitude of the Moon,
H = true altitude of the Sun or Star,
H′ = true altitude of the Moon,
d = the apparent distance,
D = true distance,
a = diff. of app. altitudes,
A = diff. of true altitudes.

Then in the triangle Zsm (Fig. 8, plate 9.), we have by Lemma 3,

$$\text{vers. sin. Z} = \frac{\text{cos. } a - \text{cos } d}{\text{cos. } h \text{ cos. } h'}$$

and in the triangle ZSM, by the same Lemma,

$$\text{vers. sin. Z} = \frac{\text{cos. A} - \text{cos. D}}{\text{cos. H cos. H}'}$$

$$\therefore \frac{\text{cos. A} - \text{cos. D}}{\text{cos. H cos. H}'} = \frac{\text{cos. } a - \text{cos. } d}{\text{cos. } h \text{ cos. } h'}$$

and by reduction cos. D = cos. A − $\frac{\text{cos. H cos. H}'}{\text{cos. } h \text{ cos. } h'}$ (cos. a − cos. d);

or by subtracting each side of the equation from unity,

1 − cos. D = 1 − cos. A + $\frac{\text{cos. H cos. H}'}{\text{cos. } h \text{ cos. } h'}$ (cos. a − cos. d),

that is, vers. sin. D = vers. sin. A + $\frac{\text{cos. H cos. H}'}{\text{cos. } h \text{ cos. } h'}$ (vers. sin. d − vers. sin. a), which gives the Rule.

Investigation of Dr. Brinkley's Rule.

The following theorem will be required in the analysis of this Rule.

If y be any function of x, and constant quantities, (x being supposed to flow uniformly) and if c be the increment of x, so that x becomes $x + c$, y will become

$$y + \frac{\dot{y}}{\dot{x}} \cdot \frac{c}{1} + \frac{\ddot{y}}{\dot{x}^2} \cdot \frac{c^2}{1.2} + \frac{\dot{\dot{y}}}{\dot{x}^3} \cdot \frac{c^3}{1.2.3} + \text{\&c.}$$

This formula was given by Dr. Brook Taylor in his Method. Increment. and is hence called Taylor's theorem. Our readers will also find it demonstrated in Lyons' Fluxions, art. 130, or in Maclaurin's Fluxions, art. 751.

Retaining the symbols used in the investigation of Dr. Mackay's Rule, we have, as is there shewn,

$$\text{cos. D} = \text{cos. A} - \frac{\text{cos. H cos. H}'}{\text{cos. } h \text{ cos. } h'} (\text{cos. } a - \text{cos. } d) \quad (1)$$

Put p = horizontal parallax } of Sun.
r = refraction of star or }
p' = horizontal parallax } of Moon.
r' = refraction }

and because the true alt. of an object = app. alt. + par. in alt. — refr. and par. in alt. = hor. par. × cos. app. alt. (See p. 78) therefore,

cos. H = cos. $(h + p \cos. h - r)$, and cos. H' = cos. $(h' + p \cos. h' - r')$

These values of cos. H and cos. H' must now be expanded in series, by means of Taylor's theorem: taking that of cos. H, we have,

$x = h$,
$c = p \cos. h - r$,
$y =$ (function of x =) cos. h.

whence $\dot{x} = \dot{h}$; $\dot{x}^2 = \dot{h}^2$; &c.
$\dot{y} = -\dot{h} \sin. h$; $\ddot{y} = -\dot{h}^2 \cos. h$; &c.

$\therefore \quad y = \cos. h$,

$$\frac{\dot{y}}{\dot{x}} \cdot \frac{c}{1} = -\sin. h\ (p \cos. h - r),$$

$$\frac{\ddot{y}}{\dot{x}^2} \cdot \frac{c^2}{1.2} = -\tfrac{1}{2} \cos. h\ (p \cos. h - r)^2, \text{ &c.}$$

and consequently,

cos. H = cos. h — sin. h $(p \cos. h - r)$ $-\frac{1}{2}$ cos. h $(p \cos. h - r)^2$ + &c.

or $\frac{\text{cos. H}}{\text{cos. } h}$ = 1 — tang. h $(p \cos. h - r)$ — $\frac{1}{2}(p \cos. h - r)^2$ + &c.

= 1 — $(p \sin. h - r \tan. h)$ — $\frac{1}{2}(p \cos. h - r)^2$ + &c.

But since p and r, are supposed to be reckoned in seconds, p sin. h — r tan. h, and p cos. h — r must each be multiplied by the length of the arc of 1″ in order to have their values in parts of the radius, therefore because the sine of 1″ may be taken for the length of the arc of 1″,

$\frac{\text{cos. H}}{\text{cos. } h}$ = 1 — sin. 1″ $(p \sin. h - r \tan. h)$ — $\frac{1}{2}$sin.^{2} 1″ $(p \cos. h - r)^2$ + &c.

Exactly in a similar manner, it is found that,

$\frac{\text{cos. H'}}{\text{cos. } h'}$ = 1 — sin. 1″ $(p' \sin. h' - r' \tan. h')$ — $\frac{1}{2}$sin.^{2} 1″ $(p' \cos. h' - r')^2$ + &c.

whence by multiplication, and neglecting some small quantities*

$$\frac{\text{cos. H cos. H'}}{\text{cos. } h \text{ cos. } h'} = 1 - \sin. 1'' \left\{ \begin{matrix} p' \sin. h' - r' \tan. h' \\ p \sin. h - r \tan. h \end{matrix} \right\} - \tfrac{1}{2} \sin.^2 1'' \left\{ \begin{matrix} (p' \cos. h' - r')^2 \\ + r^2 \end{matrix} \right\}$$

Now because the refraction varies nearly as the cotangent of the altitude, and at 45° is equal to 57″½ in its mean quantity, let

r = 57,5 cot. h — e†, r' = 57,5 cot. h' — e', then,

$$\frac{\text{cos. H cos. H'}}{\text{cos. } h \text{ cos. } h'} = 1 - \sin. 1'' \left\{ \begin{matrix} p \sin. h + p' \sin. h' - 115'' \\ e \tan. h + e' \tan. h' \end{matrix} \right\} - \tfrac{1}{2} \sin.^2 1'' \left\{ \begin{matrix} (p' \cos. h' - r')^2 \\ + r^2 \end{matrix} \right\}$$

or by a different arrangement of the terms,

$$\frac{\text{cos. H cos. H'}}{\text{cos. } h \text{ cos. } h'} = 1 - \sin. 1'' \ (p' \sin. h' - 115'') - \tfrac{1}{2} \sin.^2 1'' \ (p' \cos. h' - r')^2 - \sin. 1'' \ (e \tan. h + p \sin. h) - \tfrac{1}{2} r^2 \sin.^2 1'' - \sin. 1'' \ e' \tan. h' \quad (2)$$

this latter quantity is what is found by inspection from the small tables A, B, &c.

Table A gives sin. 1″ $(p' \sin. h' - 115'')$, of which the argument is p' sin. h', or the parallax answering to the complement of the Moon's altitude.

Table B shews $\frac{1}{2}$ sin.^{2} 1″ $(p' \cos. h' - r')^2$, the argument being $p' \cos. h' - r'$, viz. the correction of the Moon's altitude.

When a Star is observed $p = o$, and therefore $p \sin. h = o$; in this case, table C gives sin. 1″ e tan. h + $\frac{1}{2} r^2$ sin.^{2} 1″, the argument of which is h, the Star's apparent altitude, when the Sun is observed, table D gives sin. 1″ $(e \tan. h + p \sin. h)$ + $\frac{1}{2} r^2$ sin.^{2} 1″, the argument being h, or the Sun's apparent altitude.

Lastly, Table E shews sin. 1″ e' tan. h', the argument being h', the apparent altitude of the Moon.

Let α, β, γ, δ, represent the quantities given by these tables, and then we have from equations 1, 2,

cos. D = cos. A — $(1 - \alpha - \beta - \gamma - \delta)$ (cos. a — cos. d),

or subtracting each side from unity,

1 — cos. D = 1 — cos. A + $(1 - \alpha - \beta - \gamma - \delta)$ (cos. a — cos. d)

that is,

vers. sin. D = vers. sin. A + $(1 - \alpha - \beta - \gamma - \delta)$ (vers. sin. d — vers. sin. a)

or vers. sin. D = vers. sin. A + vers. sin. d — vers. sin. a — $(\alpha + \beta + \gamma + \delta)$ (vers. sin. d — vers. sin. a)

from whence the rule is derived.

These investigations have been given agreeably to the wishes of some of our subscribers, who have expressed a desire that the demonstrations of all the Rules made use of in the work should be inserted.

* The quantities omitted in the three initial terms of the series are, sin.^{2} 1″ $(pp' \sin. h \sin. h' - pr' \sin. h \tan. h' - p'r \sin. h' \tan. h. + rr' \tan. h \tan. h')$ — $\frac{1}{2}$ sin.^{2} 1″ $(p^2 \cos.^2 h - 2pr \cos. h)$: those which involve p, will, in the case of a Star vanish; and in that of the Sun, their amount will be very minute, on account of the opposite signs, and the small parallax of the Sun; the remaining quantities, sin.^{2} 1″ $(rr' \tan. h \tan. h' - p'r \sin. h' \tan. h)$ are very small, and tend to counteract each other.

† The quantity e, here introduced, is itself dependent on the refraction: Dr. Bradley's rule for computing the refraction is r = 57″ cot. $(h + 3r)$, so that according to this rule, e would express the excess of 57″ cot. h, above 57″ cot. $(h + 3r)$.

The demonstration of the 2d method of computing the excentric anomaly given at p. 165, is here added.

Put as before,

s = sine of the mean anomaly ACD } to rad. 1.
c = cosine of, do. }

a = AC, or CD, the semi-major axis.

e = SC, the excentricity.

also let z = length of an arc, to rad. 1, containing the same number of degrees, &c. as DN.

Then by Simpson's Fluxions, Vol. 2. page 501.

$$\text{sin. } z = z - \frac{z^3}{2.3} + \frac{z^5}{2.3.4.5} - \text{\&c.}$$

$$\text{cos. } z = 1 - \frac{z^2}{2} + \frac{z^4}{2.3.4} - \text{\&c.}$$

and because the sine of the difference of two arcs, or angles, is equal to the difference of the products of their sines and cosines, taken reciprocally, that is, sin. (P — Q) = sin. P cos. Q — sin. Q cos. P,* we hence deduce sin. (ACD — NCD) or

$$\text{sin. ACN} = s - cz - \frac{sz^2}{2} + \frac{cz^3}{2.3} + \text{\&c.}$$

Now by Trig. rad. : sin. ACN or SCT :: SC (e) : ST, = DN, as is proved at p. 161.

$$\text{therefore DN} = a\dot{z} = es - cez - \frac{esz^2}{2} + \frac{cez^3}{2.3} + \text{\&c.}$$

and by transposition,

$$es = (a + ce)\, z + \frac{esz^2}{2} - \frac{cez^3}{2.3} - \text{\&c.}$$

Let $es = y$, $a + ce = f$, $\frac{es}{2} = g$, $\frac{ce}{2.3} = h$, then will

$$y = fz + gz^2 - hz^3 - \text{\&c.}$$

and by reverting this series, we obtain,

$$z = \frac{y}{f} - \frac{g}{f^3}.y^2 + \frac{2g^2 + fh}{f^5}\, y^3 - \text{\&c.}$$

or, by restoring the values of f, g, &c.

$$z = \frac{es}{a + ce} = \frac{(es)^3}{2(a+ce)^3} + \frac{ce\,(es)^3}{6(a+ce)^4}\ \text{\&c.}$$

But to express this value of z in seconds, we must say, as rad. 1, is to this series, so are 206264″.8, the seconds of an arc equal in length to the radius, to a fourth quantity, consequently by multiplying the series by 206264.8 (r) we shall have z, or the angle NCD, expressed in seconds, equal to,

$$r\left[\frac{es}{a + ce} - \frac{(es)^3}{2(a+ce)^3} - \frac{ce(es)^3}{6(a+ce)^4}\ \text{\&c.}\right]$$

the first two terms of which expression are those given in the Rule.

If the angle ACD be greater than 90°, and less than 270′, its cosine will be negative, and the series will become,

$$r\left[\frac{es}{a - ce} - \frac{(es)^3}{2(a - ce)^3} - \frac{ce(es)^2}{6\,(a - ce)^4}\ \text{\&c.}\right.$$

We now proceed to the method of finding the time of passing the meridian, rising and setting of the other celestial objects for any given day and place, and likewise their latitudes and longitudes, &c. for which purposes the right ascension and declination of the objects at the given time are the requisite parts of the data: with regard to the fixed stars the computation is simple and easy, but for the planets more difficult, and still more so for the Moon.

Problem 13.

Given the latitude and longitude of a place, and the year, and day of the month, to find the time of rising, culminating, and setting, of a known fixed star, and its amplitude.

Example.—At what time does the star Aldebaran, or Bull's eye, rise, culminate, and set at Greenwich, on the 2d day of January, 1820, the right ascension of the star being at that time 4*h*. 25*m*. 36*s*. and its declination 16° 8′ 20.″4 North; the horizontal refraction being 33′.

Solution.

To find the time of the Star's culminating. Subtract the Sun's right ascension for the noon of the given day, from the star's right ascension (increased by 24 hours if need be) the remainder is the approximate time of the star's culminating; next, subtract the Sun's right ascension for the noon of the given day, from his right ascension for the noon of the following day, which will give the change in his right ascension in 24 hours; then add this change to 24 hours, and say, as this sum is to 24*h*. so is the approximate time, to the true time of culmination.

	h.	*m.*	*s.*	
✳'s right asc. + 24*h*. . . .	28	25	36	
☉'s right asc. Jan. 2d 1820, at noon,	18	48	12.9	See p. 177
Approx. time of culm. . .	9	37	23.1	
☉'s right asc. Jan. 2d. . . .	18	48	12.9	
. . . . . Jan. 3d. . . .	18	52	37.6	
Change in 24 hours		4	24.7	

As 24*h*. 4*m*. 24.7*s*. : 24*h*. : : 9*h*. 37*m*. 23.1*s*. : 9*h*. 35*m*. 37.3*s*. the true apparent time of culmination.

* This may be easily shewn from fig. 9, plate 9: the former construction of this Fig. remaining, draw EG, EN, parallel to CH, OA, respectively, and let EN meet IB produced in N. Then the triangles OCH, OEG, being similar, OC : CH :: OE : EG or NI, ∴ OC . NI = CH . OE; and the triangles OCH, BEN, being likewise similar, OC : OH :: BE : BN : and OC . BN = BE . OH, whence by subtraction OC . NI — OC . BN, or OC . BI = CH . OE — BE . OH; but CH, BE, are the sines of the arcs AC, BC, which arcs call P, and Q; and OH, OE, are the cosines of the same arcs; BI is the sine of their difference AB, and OC is the rad, (1). Consequently sin. (P — Q) = sin. P cos. Q — sin. Q cos. P.

Next, to find the time of rising and setting. In fig. 4, plate 9, let Z be the zenith, P, the elevated pole, ✳, the star, 33′ below the horizon HO, and which in consequence of the refraction will appear just in the horizon; then in the triangle ZP✳, there are given ZP = 38° 31′ 20″, the colatitude, P✳ = 73° 51′ 39″.6 the star's polar distance, and Z✳ = 90° 33′, its zenith distance, to find the measure of the angle ✳PZ, which will be equal to the star's *semi-arc of apparition ✳*p*.

	°	′	″		
Colat. . .	38	31	20	ar. com. log. sine	0.2056388
Pol. dist.	73	51	39.6	ar. com. log. sine	0.0174618
Zen. dist.	90	33	0		
	2)202	55	59.6		
	101	27	59.8	. . . log. sine	9.9912441
	90	33	0		
	10	54	59.8	. . . log. sine	9.2773344
					2)19.4916791
	56	9	10.1	. . . log. cosine	9.7458395
			2		
∠ZP✳	112	18	20.2	= sem. arc of appar.	

This arch converted into time is 7*h*. 29*m*. 13.3*s*. and by subtracting this from, and adding it to 9*h*. 35*m*. 37.3*s*. we have.

	h.	*m*.	*s*.
approx. time of ✳'s rising	2	6	24
approx. time of ✳'s setting	17	4	50.6

Then as 24*h*. : 4*m*. 24.7*s*. : : 2*h*. 6*m*. 24*s*. : 23.2*s*. the change in the Sun's right ascension between Jan. 2d at noon, and the time of the star's rising.

And as 24*h*. : 4*m*. 24.7*s*. : : 17*h*. 4*m*. 50.6*s*. : 3*m*. 8.4*s*. the change in the Sun's right ascension between Jan. 2d at noon, and the time of the star's setting.

	h.	*m*.	*s*.
⊙'s right asc. at noon, Jan. 2d	18	48	12.9
			23.2
⊙'s right asc. at ✳'s rising, ...	18	48	36.1
✳'s right asc. + 24 hours, ...	28	25	36.
Difference, ...	9	36	59.9
Sem. arc of app. subt.	7	29	13.3
True app. time of ✳'s rising, ...	2	7	46.6

⊙'s right asc. at noon, Jan. 2	18	48	12.9
		3	8.4
⊙'s right asc. at ✳'s setting, ...	18	51	21.3
✳'s right asc. + 24 hours, ...	28	25	36
Difference, ...	9	34	14.7
Sem. arc of app. add.	7	29	13.3
True app. time of ✳'s setting, ...	17	3	28

To find the *mean* times of rising, culminating, and setting, we must apply the true equation of time at each of these instants, which is thus found.

	m.	*s*.
Eq. time at noon, Jan. 2d. . .	4	1.7
3d. . .	4	29.8
Change in 24 hours, ...		28.1

	h.	*m*.	*s*.		*s*.
As 24*h*. : 28.1*s*. : :	2	7	46.6	:	2.5
	9	35	37.3	:	11.2
	17	3	28	:	20.0

	m.	*s*.		*s*.		*m*.	*s*.	
Then	4	1.7	+	2.5	=	4	4.2	eq. time at ✳'s rising
	4	1.7	+	11.2	=	4	12.9	. . . . . culm.
	4	1.7	+	20.0	=	4	21.7	. . . . . setting

And hence the star Aldebaran, on the 2d day of January, 1820, at Greenwich, rises at 2*h*. 7*m*. 46.6*s*. P. M. app. time, or 2*h*. 11*m*. 50.8*s*. mean time; culminates at 9*h*. 35*m*. 37.3*s*. P. M. app. time, or app. time, or 9*h*. 39*m*. 50.2*s*. mean time; and sets at 17*h*. 3*m*. 28*s*. app. time, or 17*h*. 7*m*. 49.7*s*. mean time.

Next, in the same triangle ZP✳, the angle PZ✳ subtracted from 90° will leave the star's amplitude. As the angle ZP✳ is already known, the angle PZ✳ may either be found from the relation of the sines of the sides to the sines of their opposite angles, or by prop. F. Sph. Trig. used in the first part of this problem; we shall take the latter method.

* By the *semi-arc of apparition*, we mean half the arc described by the star while it is above the horizon of the given place; this is by some writers called the semi-diurnal, and by others, the semi-nocturnal arc, but neither of these terms seems quite appropriate, because the arc may be sometimes described in the day-time, or while the Sun is above the horizon; at others, while he is below it; or again, part of the arc may be described while the Sun is above, and part while he is below the horizon: and thus, according to circumstances, it is a diurnal, or a nocturnal arc; or part one, and part the other.

Colat.	38 31 20	ar. com. log. sine	0.2056388
Zen. dist.	90 33 0	ar. com. log. sine	0.0000200
Pol. dist.	73 51 39.6		
	2)202 55 59.6		
	101 27 59.8 . . .	log. sine	9.9912441
	73 51 39.6		
	27 36 20.2 . . .	log. sine	9.6659399
			2)19.8628428
	31 21 30.5 . . .	log. cos.	9.9314214
	2		
	62 43 1 = ∠PZ✳		
	90 0 0		
	27 16 59		

Hence the star rises 27° 16′ 59″ to the northward of the east, and sets the same quantity to the northward of west.

The times of rising, culminating, and setting of this star, have been calculated with scrupulous exactness, merely to instruct the student, by an example, in the correct way of solving the problem; in general, it is sufficiently exact, to add and subtract the semi-arc of apparition to, and from, the time of culmination, to obtain the times of rising and setting: and the semi-arc of apparition being found for any fixed star at a given place, may be considered as a constant quantity for several years; thus, 7*h*. 29*m*. 13.3*s*. the semi-arc of apparition of Aldebaran, at Greenwich, for the 2d day of January, 1820, may, without sensible error, be used for 10 years before and after 1820. Moreover, all the fixed stars which rise and set at a given place, do rise, culminate, and set, sooner every day, than on the preceding day, by the daily increase of the Sun's right ascension, which, in this example is 4*m*. 24.7*s*. but, at a mean, it is no more than 3*m*. 56.6*s*. hence, if the daily change of the Sun's right ascension be subtracted from the apparent time of the star's rising, culminating, and setting, for any day, we shall have the same nearly for the next following day. And hence also, if the given place have any considerable longitude east or west of Greenwich, we may find the time of any star's culminating, &c. at that place, by first finding the time of culmination at Greenwich, and then allowing at the rate of 10 seconds of time for every 15 degrees of longitude, which must be *subtracted* from the time at Greenwich, to give the time of culmination for places in *west* longitude, and *added* for places in *east* longitude. For 3*m*. 56.6*s*. the mean change of the Sun's right ascension in 24 hours, divided by 24, gives 10 seconds nearly, for the change in one hour, or 15° of longitude.

Having given the solutions of some of the principal problems connected with solar and stellar astronomy, in respect to topical phenomena, we now recur to some other problems that are general, and in which our Earth is to be considered as a point, or nearly so: in the solution of these problems the data we use, are the astronomical observations made by Gentlemen properly qualified for the business at Observatories, and provided with good astronomical instruments designed for the purpose; and from these problems it will appear how the catalogues of the right ascension, declination, latitude, and longitude of the fixed stars, are constructed.

Problem 14.

Given the obliquity of the Ecliptic, and the right ascension and declination of a fixed star; to find its latitude and longitude.

Example.—At the Royal Observatory of Greenwich, on the 2d of January, 1820, at 9*h*. 35*m*. 37.3*s*. true app. solar time, the star Aldebaran was observed to transit the meridian; at which time its declination was observed to be 16° 8′ 20″.4 north; the Sun's true right ascension at that instant was, 18*h*. 49*m*. 58.7*s*. and the obliquity of the ecliptic 23° 27′ 56″.2: it is required to find the true longitude of the star, from the true and mean equinox; and likewise its latitude.

Solution.

To the Sun's right ascension, ...	18 49 58.7
Add time of culmination, ...	9 35 37.3
	28 25 36
Subtract, ...	24 0 0
Star's $\mathcal{R}$ from true equinox, ...	4 25 36
Right ascension in degrees, &c. . . .	66°24′ 0″

Let EPQN (Fig. 11.), be a stereographic projection of the sphere on the plane of the solstitial colure, EQ the equator, ♍♈♎ the ecliptic, *m*✳*n* the parallel of the star's declination 16° 8′ 20.4″ north of the equator, and ♈D the star's right ascension = 66° 24′ from the equinoctial point Aries. Through D draw the star's meridian NDP, cutting the parallel of declination in the point ✳, the place of the star; also through the point ✳, and the poles of the ecliptic, draw the star's circle of celestial longitude A✳B.

Then in the oblique spherical triangle B✳P, there are given the side BP = the obliquity of the ecliptic,

the side P$\star$ = the star's polar distance, and the angle BP$\star$, = 90° + star's right ascension, to find the angle PB$\star$, the complement of the star's longitude from Υ, and the side B$\star$ = 90° + its latitude. Hence, letting fall the perpendicular arc $\star$G upon the solstitial colure, we have in the spherical triangle PG$\star$, right angled at G; given the hypothenuse P$\star$ = 73° 51′ 39″.6, the star's polar distance, and the angle $\star$PG or the arc QD = 23° 36′, the complement of the star's right ascension, or its distance from the solstitial colure; to find the leg PG: therefore,

1. For the leg PG, or 1st auxiliary arc.
By case 1st, right ang. sph. trig.

As radius				10.0000000
Is to tan. P$\star$, or cot. dec.	16°	8′	20.″4	10.5385424
So is cos. $\star$PG, or sine $\curlywedge$ from, ♈	66	24	. . .	9.9620674
To tan. 1st aux. arc PG,	72	28	29	10.5006098
Add obliq. BP = . . .	23	27	56.2	
2d aux. arc. GB = . . .	95	56	25.2	

2. For the angle $\star$BP or arc CΩ the comp. of the star's long. from Υ, by prop. C of oblique spherics.

As sine 2d aux. arc, . .	95°	56′	25″.2	9.9976617
Is to sine 1st aux. arc,	72	28	29	9.9793590
So is tan. GP$\star$, or cot. right asc. from Υ,	66	24	0	9.6403714
To tan. GB$\star$ or cot. long. from Υ,	67	16	23.3	9.6220687

3. For the side B$\star$, the latitude of the star + 90°.

As cos. 1st aux. arc, . . .	72°	28′	29″	9.4787490
Is to cos. 2d aux. arc, . .	95	56	25.2	9.0149098
So is cos. P$\star$, or sine dec.,	16	8	20.4	9.4439957
To cos. B$\star$, or sine lat. . .	5	28	55.3	8.9801565

Hence the longitude of Aldebaran on the 2d day of January, 1820, from the true equinox, is

67° 16′ 23″.3 = 2*s*. 7° 16′ 23″.3

Equa. equin. points	+	1.7		
True long. from mean equinox,	67	16	25.0	= 2*s*. 7° 16′ 25″ from Υ

Latitude, 5° 28′ 55″.3 South.

The student should carefully attend to the varieties contained in this problem, viz.: with regard to the quarter of the ecliptic in which the Sun is at the time of observation and the title of declination whether north or south.

Scholium.

Till of late years, it was generally supposed that the fixed stars always had the same latitude; that is, that the plane of the Earth's orbit being extended to the heavens, marked out a circle called the ecliptic, which circle was immoveable; because it was supposed that the Sun's centre was the fixed point about which the Earth revolved, and that the plane of the orbit necessarily passed through that centre, and because it was also conjectured that the fixed stars always continued in the same point, as to absolute space; hence, if both these hypotheses were just; then the latitude of any star would be invariable. But the fact is, that, neither hypothesis is correct; for, in the first place, the Earth does not revolve round the Sun's centre, but round the common centre of gravity of the whole system, about which centre the Sun himself and all the planets and comets belonging to our system are constantly revolving; and this common centre has also a motion as to absolute space, according to the different distances and positions of the Sun, and the bodies belonging to the system; the orbit of the Earth, therefore, extended to the heavens is not an immoveable circle, but a curve inclining at one time to the northward and another to the southward, and consequently the Sun's apparent path in the heavens, which is similar to that of the Earth, is not a perfect circle. Astronomers therefore imagine a circle to divide these irregularities equally, which may be called the mean ecliptic, and the quantity of the small angle contained between a right line drawn from the Earth's centre to that of the Sun and the plane of the mean ecliptic is called the Sun's latitude; this angle is generally too small to affect astronomical calculations, and therefore we have taken no notice of it: and again, secondly, from the latest and best astronomical observations, it appears that the fixed stars have each a proper motion in regard to absolute space; so that they actually change their latitudes in respect to the *mean* ecliptic, and also change their apparent distances, in respect to one another. Dr. Herschel's observations seem to prove, that our system is approaching at present, towards a point or centre in the constellation Hercules, by which motion the distances of the stars from each other in that constellation seem gradually to increase, and the distances of those stars remote from the centre decrease; and hence the latitude of the stars are affected by this motion; the longitudes of the fixed

stars are also affected by this motion; so that upon the whole the longitudes of the fixed stars are affected in a fourfold way; 1st, by the procession of the equinoxes, by which they increase their longitude about 50″.3 every year; 2d, by the nutation of the Earth's axis, by which they increase and decrease their longitude during the Moon's period (of 18¼ years) about 9″.6; 3d, by the before mentioned motion, which may be called their own proper motion. Our late astronomer royal, Dr. Maskelyne, has taken great pains to ascertain the quantity of this motion in most of the principal fixed stars visible at Greenwich. The 4th cause of variation in the places of the stars is the aberration of light, which affects both their latitudes and longitudes.

Problem 15.

Given the latitude and longitude of a fixed star, and the obliquity of the ecliptic; to find the star's right ascension and declination.

Example.—The latitude of the star Aldebaran was 5° 28′ 55.″3 south, and its longitude 67° 16′ 23″.3 from the true equinox, on the 2d day of January, 1820, at the instant it passed the meridian of Greenwich, at which time the obliquity of the ecliptic was 23° 27′ 56″.2. What were its right ascension and declination at that instant?

Solution.

This problem being the converse of the last; the same figure and construction may remain, and then in the spherical triangle B✳G, right angled at G; there are given the hypothenuse B✳ = 95° 28′ 55″.3, the star's latitude + 90°, and the angle ✳BG = 22° 43′ 36″.7 the complement of the star's longitude from ♈; to find the leg BG, the 1st auxiliary arc.

1. For BG, the 1st auxiliary arc.

By case 1st, right angled spherical trigonometry.

As radius		10.0000000
Is to tan. B✳, or cot. lat.	5°28′ 55″.3	11.0178533
So is cos. ✳BG, or sine lon.	67 16 23.3	9.9648991
To tang. 1st aux. arc BG	95 56 25.2	10.9827524
Subtract obliquity BP	23 27 56.2	
2nd aux. arc PG . . .	72 28 29	

2. For the declination; by prop. C.

As cos. 1st aux. arc.	95°56′ 25″.2	9.0149098
Is to cos. 2d aux. arc.	72 28 29	9.4787490
So is cos. B✳, or sin. lat.	5 28 55.3	8.9801558
To cos. P✳, or sine dec.	16 6 20.3	9.4439950

3. For the right ascension, or the angle GP✳, its complement.

As sine 2d aux. arc . .	72°28′ 29″	9.9793590
Is to sine 1st aux. arc . .	95 56 25.2	9.9976617
So is tan. GB✳, or cot. long.	67 16 23.3	9.6220689
To tan. GP✳, or cot. right asc.	66 24 0	9.6403716

Right asc. in time 4*h*. 25*m*. 36*s*. from true equinox.
$\frac{1}{11}$ nuta., 1″.7 in time 0.1

True right asc. 4 25 36.1 from mean equinox.

By the help of this problem we calculate the Moon's right ascension and declination for any given time; her true latitude and longitude being previously calculated for that time. And also the right ascension and declination of the planets; their geocentric latitude and longitude, for the given time being previously calculated likewise. And if the right ascension of the Sun and Moon be strictly computed for the noon and midnight of every day; then we can by interpolation find the time of her culmination or passage over the meridian: and in like manner for the planets, which will be exemplified when we come to shew how these are calculated for the nautical almanac.

Scholium.

There are some cases in which the foregoing methods of solving the 14th and 15th problems may err a few seconds of a degree. The rules used by the computors of the nautical almanac, and invented by Dr. Maskelyne, are therefore to be preferred, as the results by these rules will be true to a second, or a very little more, in all cases, by only taking out the logarithms for the nearest second, and the nearest second answering to the logarithms, without proportioning for the differences of the logarithms, or for the odd parts of a second.

Dr. Maskelyne's Solution to Problem 14.

Put log. tan. dec. + 10 — log. sin. right asc. = log. tan. A, which call north, or south, as the declination is. Call O the obliquity of the ecliptic in the first six signs of right asc. south; and in the last six signs north. Then, if A and O have the same name take their sum, otherwise their difference = B, of the same name as the greater. Then, for the longitude, if A be less than 45°,

ar. com. log. cos. A + log. cos. B + log. tan. right asc. — 10 . . . = log. tan. long.

if A be greater than 45°,
log. tan. A + ar. com. log. sin. A + log. cos. B + log. tan. right asc. — 20 } = log. tan. long.
which is of the same kind as the right asc. unless B be more than 90°; in which case the quantity found of the same kind as right asc. subtracted from 12 signs will give the longitude.

For the latitude,
if long. nearer to 90° or 270°, than to 0° or 180°,
log. sin. long. + log. tan. B — 10 = log. tan. lat.
if long. nearer to 0° or 180°, than to 90° or 270°,
log. tan. long. + log. cos. long. + log. tan. B — 20 . . . . . } = log. tan. lat.
which is of the same name as B.

Application to the same Example where A *is north, and* B, *south.*

Declin.	16° 8′ 20″	log. tan. + 10	19.4614544
Right asc.	66 24 0	log. sin.	9.9620674
arc A	17 31 31 N	log. tan.	9.4993870
Obliq.	23 27 56 S		
Arc B	5 56 25 S	log. cos.	9.9976617
Arc A	17 31 31	ar. com. log. cos. .	0.0206410
Right asc.	66 24 0	log. tan.	10.3596286
Long.	67 16 23	log. tan.	10.3779313
Long.	67 16 23	log. sin.	9.9648988
Arc. B	5 56 25	log. tan.	9.0172441
Lat.	5 28 55 S.	log. tan.	8.9821429

Hence the longitude and latitude are the same as before found. The arc A being less than 45°, and the longitude nearer to 90°, than to 0°, therefore the first two rules are used.

Dr. Maskelyne's Solution to Problem 15.

Put log. tan. lat + 10 — log. sin. long. = log. tan. A, which call north, or south, as the latitude is. Call O the obliquity of the ecliptic in the first six signs of longitude, north, and in the last six signs south. Then if A and O have the same name, take their sum, otherwise their difference, = B of the same name as the greater. Then for the right ascension,
if A be less than 45°,
ar. com. log. cos. A + log. cos. B + log. tan. long. — 10 . . . } = log. tan. right asc.
if A be greater than 45°
log. tan. A + ar. com. log. sin. A + log. cos. B + log. tan. long. — 20 } = log. tan. right as.
which is of the same kind as long. unless B be more than 90°; in which case the quantity found of the same kind as long. subtracted from 360° will give the right ascension.

For the declination,
If right asc. nearer to 90° or 270°, than to 0° or 90°,
log. sin. right asc. + log. tan. B — 10 = log. tan. decl.
If right asc. nearer to 0° or 180°, than to 90° or 270°,
log. tan. right asc. + log. cos. right asc. + log. tan. B — 20 } = log. tan. decl.
which is of the same name as B.

Application to the same Example, where A *is South, and* B *North.*

Lat.	5°28′ 55″	log. tan. + 10	18.9821401
Long.	67 16 23	log. sin.	9.9648988
Arc. A	5 56 25 S	log. tan.	9.0172413
Obliq.	23 27 56 N.		
Arc. B	17 31 31 N.	log. cos. . . .	9.9793590
Arc A	5 56 25	ar. com. log. cos. .	0.0023383
Long.	67 16 23	log. tan.	10.3779293
Right asc.	66 24 0	log. tan.	10.3596266
Right asc.	66 24 0	log. sin.	9.9620674
Arc. B	17 31 31	log. tan.	9.4993900
Declin.	16 8 20 N.	log. tan. . . .	9.4614574

Here the first two rules are used, for the same reason as in the former example.

The investigation of the foregoing rules may be thus given. First, for the latitude and longitude, having given the right ascension and declination.

Produce the arc G✳ (fig. 11.), and because it was drawn perpendicular to the great circle EPQN, it will pass through ♈, the pole of that circle. Now in the spherical triangle ♈✳D, right angled at D, there are given ♈D, the star's right ascension, and ✳D its declination, whence by the 4th analogy of prop. A. page 130, as sin. ♈D : tan. ✳D : : rad. : tan. ✳♈D; [*a*] then C♈D (the obliq.) — ✳♈D = ✳♈C, and in the triangles ♈✳D, ♈✳C, right angled at D and C, we have by the 6th analogy of prop. A,

cos. ✳♈D : rad. : : tan. ♈D : tan. ✳♈
and rad. : cos. ✳♈C : : tan. ✳♈ : tan. ♈C.
therefore by compounding these two proportions,
cos. ✳♈D : cos. ✳♈C : : tan. ♈D : tan. ♈C [*b*]

Now the angles ✳♈D, ✳♈C, are those called A and B respectively in Dr. Maskelyne's rule; hence by putting the proportions *a*, *b*, into logarithms, &c. the 1st rule for the longitude is obtained, viz. log. tan. dec. + 10 — log. sin. right asc. = log. tan. A, and ar. com. log. cos. A + log. cos. B + log. tan. right asc. — 10 = log. tan. ♈C, the longitude.

The 2d rule is thus derived. Since (art. 1. lemma 2. page 117.) cos. A : sin. A : : rad. : tan. A, therefore rad. ÷ cos. A = tan. A ÷ sin. A, that is, in logarithms, 10 — log. cos. A = log. tan. A — log. sin. A, or ar. com. log. cos. A = log. tan. A + ar. com. log. sin. A — 10, whence by substitution, log. tan. A + ar. com. log. sin. A + log. cos. B + log. tan. right asc. — 20 = log, tan. long. The reason of using this latter expression for the longitude when A is more than 45°, is because the tangents of arcs greater than 45° increase very rapidly, and therefore A cannot sometimes be accurately found from its tangent, by proportional parts, and thus a small error may occur in taking out the cosine of A in the 2d part of the operation, as well as on account of the large differences of the cosines above 45°, but the differences of the sines of arcs greater than 45° being small, and the log. tan. of A being *accurately* known from the 1st part of the operation, the ar. com. of log. cos. of A is hence very correctly found, by adding the ar. com. of its log. sine to its log. tangent.

The rules for the latitude are obtained from the right angled triangle ♈✳C, in which there are known ♈C, the longitude, and the angle ✳♈C = B, whence by the 4th analogy of prop. A, rad. : tan. ✳♈C : : sin. ♈C : tan. ✳C, that is, in logarithms, log. sin. long. + log. tan. B — 10 = log. tan. lat. which is the 1st rule. Farther, since by the lemma before referred to, sin. long. = (tan. long. × cos. long.) ÷ rad :, therefore log. sin. long. = log. tan. long. + log. cos. long. — 10, which substituted in the 1st rule, gives log. tan. long. + log. cos. long. + log. tan. B — 20 = log. tan. lat, the 2d rule. This is to be preferred, when the longitude is nearer to 0° or 180°, than to 90° or 270° for reasons similar to those given for using the 2d rule for the longitude.

Secondly, For the right ascension and declination, having given the latitude and longitude.

Referring to the same figure as before; in the triangle ♈✳C, right angled at C, there are given ♈C, the star's longitude, and ✳C its latitude, whence by the 4th analogy of prop. A, sin. ♈C : tan. ✳C : : rad. : tan. ✳♈C [*c*] then C♈D — ✳♈C = ✳♈D. Next, in the triangles ♈✳C, ♈✳D, right angled at C and D, we have by the 6th analogy of prop. A,

cos. ✳♈C : rad. : : tan. ♈C tan. ✳♈

and rad. : cos. ✳♈D : : tan. ✳♈ : tan. ♈D

therefore by compounding, cos. ✳♈C : cos. ✳♈D : : tan. ♈C : tan. ♈D [*d*] : here the angles ✳♈C and ✳♈D, are those called A and B, in the rules, and by putting down the logarithmic value of the 4th terms of the proportions *c*, *d*, we have log. tan. lat. + 10 — log. sin. long. = log. tan. A, and ar. com. log. cos. A + log. cos. B + log. tan. long. — 10 = log. tan. right asc. which is the 1st rule or when A is greater than 45°, substituting as before, log. tan. A + ar. com. log. sin A — 10 for ar. com. log. cos. A, there arises log. tan. A + ar. com. log. sin. A + log. cos. B + log. tan. long. — 20 = log. tan. right asc. which is the 2d rule.

The declination is found from the triangle ♈✳D, in which there are given, ♈D the right ascension, and the angle ✳♈D = B, and therefore by the 4th analogy of prop. A, rad. : tan. ✳♈D : : sin. ♈D : tan. ✳D or in logarithms, log. sin. right asc. + log. tan. B — 10 = log. tan. dec. which is rule 1. Also because by the lemma aforesaid, sin. right asc. = (tan. right asc. × cos. right asc.) ÷ rad. therefore log. sin. right asc. = log. tan. right asc. + log. cos. right asc. — 10, and consequently by substitution, log. tan. right asc. + log. cos. right asc. + log. tan. B — 10 = log. tan. dec. which is rule 2.

Problem 16.

Given the Sun's longitude, and the latitude and longitude of the Moon, or any other celestial body, to find their distance.

Note.—To find the difference between the Sun's longitude, and that of the body. Subtract the longitude of the Sun, from the longitude of the Moon or other celestial body increased by 12 signs, if need be; the remainder, if less than 6 signs, is the difference of longitude, and the celestial object is east of the Sun; but if the remainder be more than 6 signs, then subtract it from 12 signs, and this remainder is the difference of longitude, and the object is west of the Sun.

Example.—The Sun's longitude being 7*s* 8° 25′ 38″, and the Moon's longitude 9*s* 2° 41′ 55″, and latitude 5° 11′ 30″ north or south; it is required to find the distance between their centres in the arc of a great circle?

	s	°	′	″	
From the Moon's long.	9	2	41	55	
Subtract the Sun's long.	7	8	25	38	
Difference long. . . .	1	24	16	17,	☽ east ☉.
Diff. long. in degrees &c.		54	16	17	

The Sun's centre is supposed to be exactly in the ecliptic; because his latitude at all times, is too small to affect the distance. Wherefore the difference of longitude is one leg, and the latitude of the object is the other, of a right angled spherical triangle; and the distance sought is the hypothenuse of the same triangle, hence

To find the distance ☉ and ☽.

By case 3, right angled spherical trigonometry.

As radius	10.0000000
Is to cos. diff. long. 54°16′ 17″ . . .	9.7663732
So is cos. ☽'s lat. . 5 11 30	9.9982147
To cos. distance, . 54 26 25	9.7645879

And in this manner is the distance between the Sun and Moon calculated for every noon and midnight throughout the year, and then their distance for every 3 hours of intermediate time is determined by interpolation; as will be illustrated in its proper place.

PROBLEM 17.

GIVEN the latitudes and longitudes of two celestial objects to find their distance.

Note.—Subtract the less longitude from the greater, the remainder is the difference of longitude.

Example.—The Moon's longitude being 0*s.* 4° 44′ 6″ and her latitude 0° 25′ 13″ south, when the longitude of the star Aldebaran was 2*s.* 7° 8′ 50″, and its latitude 5° 28′ 48″ south; it is required to find their distance.

SOLUTION.

From, ...	2*s.* 7° 8′ 50″
Subtract, ...	0 4 44 6
Diff. long.	2 2 24 44
Diff. long. in degrees, &c.	62 24 44

In the oblique angled spherical triangle P✳☽, (fig. 12); let P be the south pole of the ecliptic, ☽ the place of the Moon, and ✳ that of the star; then the angle P = 62° 24′ 44″, the diff. longitude; the side P☽ = 89° 34′ 47″, the complement of the Moon's latitude; and P✳ = 84° 31′ 12″, the complement of the star's latitude, are given; to find the side ☽✳, the distance. Let fall the perpendicular arc ✳D: then in the spherical triangle P✳D right angled at D.

1. To find the leg PD, or 1st auxiliary arc;

By case 1, of right angled spherical trigonometry.

As radius, ...	10.0000000
Is to cotang. star's lat. 5°28′ 48″ . .	11.0180150
So is cosine diff. long. 62 24 44 . . .	9.6656813
To tang. 1st aux. arc, 78 17 46.4 . .	10.6836963
Subtract from, . . . 89 34 47.0	
2d aux. arc D☽ . . 11 17 0.6	

2d. To find ☽✳, the distance;

By prop. C, oblique spherical trigonometry.

As cos. 1st aux. arc. 78°17′ 46″.4 . . .	9.3071790
Is to cos. 2d aux. arc 11 17 0.6 . . .	9.9915234
So is cos. star's pol. dist. or sin. lat. 5 28 48.0 . . .	8.9799956
To cos. distance, . . 62 30 16	9.6643400

And in the same manner, if the right ascensions and declinations of two celestial objects be given, their distance may be found; also if the latitudes and longitudes of two places on the Earth be given, their distance on the arc of a great circle is found in the same manner.

By the last two problems, the Moon's distance from the Sun, and from stars east and west of her, may be computed for every third hour throughout the year; but when her distance from the Sun is less than 40°, she is too near him for observing the distance. The distances in pages VIII, IX, X, and XI of each month in the Nautical Almanac, are strictly calculated in this way for every noon and midnight, and for every third hour of intermediate time, they are determined by means of interpolations; as will be explained in a future problem.

PROBLEM 18.

GIVEN the distance of an unknown comet, or star, from two known stars, to find its place in the heavens: that is, there are given the right ascensions and declinations of two stars, and the distance of a third star from each; to find the right ascension and declination of that third star; but it is necessary to know whether the unknown star lie to the northward, or southward, of the great circle passing through the two known stars; for without this datum, the problem is ambiguous.

Example.—Suppose the right ascension of the star A, (fig. 13.) to be 4° 44′ 6″, and its declination 0° 25′ 13″ north; the right ascension of the star B, 67° 8° 50′, and its declination 5° 28′ 48″ north; also the distance of the star C from A, 50° 30′, and from B, 32° 30′: it is required to find the right ascension and declination of the star C, which is supposed to lie to the northward of the great circle passing through A and B.

SOLUTION.

IN the oblique angled spherical triangle ABP, let P be the north pole of the Equator: then the arcs AP and BP are the polar distances of the two given stars, the arc AB is their distance, and the angle APB their difference of right ascension:

Right asc. of star B, ...	67° 8′ 50″
——————— A, ...	4 44 6
Diff. right asc. or ang. APB, . .	62 24 44

Then the side AP = 89° 34′ 47″, and BP = 84° 31′ 12″; and as the two sides and the contained angle of the triangle APB are the same as the two sides and the contained angle of the triangle ☾ P ✳ in the last problem (fig. 12.), therefore the third side AB is equal to ☾ ✳, which was there found to be 62° 30′ 16″, which supersedes the necessity of repeating the calculation for AB in the present case. The remaining part of the calculation will be as follows:

1. To find the angle PAB.

By prop. B. spherical trigonometry.

As sin. dist. AB, . . .	62°30′ 16″ . . .	9.9479465
Is to sin. diff. right asc.	62 24 44 . . .	9.9475819
So is sin. pol. dist. BP,	84 31 12 . . .	9.9980105
To sin. ang. PAB . .	84 2 23 . . .	9.9976459

2. In the sph. triangle ABC, to find the angle CAB

By prop. F,

AB . . .	62°30′ 16″	ar. com. log. sine	0.0520535
AC . . .	50 30 0	ar. com. log. sine	0.1125939
BC . . .	32 30 0		
	2145 30 16		
Half sum	72 45 8	log. sine.	9.9800177
Sub. BC . .	32 30 0		
Remainder	40 15 8	log. sine.	9.8103358
			2) 19.9550009
Half angle	18 17 3	log. cosine . . .	9.9775004
	2		
Angle CAB	36 34 6		
Sub. fr. PAB	84 2 23		
Angle PAC	47 28 17		

Let fall the perpendicular arc CD

3. To find AD, the 1st, aux. arc.

By case 1st. right ang. sph. trig.

As radius		10.0000000
Is to tang. AC . .	50°30′ 0″	10.0838955
So is cos. PAC . .	47 28 17	9.8299199
To tang. AD . . .	39 21 7	9.9138154
Subtract from AP	89 34 47	
2nd. aux. arc PD =	50 13 40	

4. To find PC, the stars, polar dist. or co.dec.

By prop. C.

As cos. 1st. aux. arc AD	39°21′ 7″ .	9.8883287
Is to cos. 2nd. aux. arc PD	50 13 40 .	9.8060016
So is cos. AC	50 30 00 .	9.8035105
To cos. PC, or sine dec. . .	31 45 6 .	9.7211834

5. To find the angle APC, or diff. right asc. of the stars A and C.

By case. 2nd, right ang. sph. trig.

As radius		10.0000000
Is to cot PC or tang. dec.	31°45′ 6″ .	9.7915917
So is tang. 2nd. aux. arc PD	50 13 40	10.0796953
To cos. angle APC	41 58 7	9.8712870
Add right asc. star A . . .	4 44 6	
Right asc. star C	46 42 13	
Or in time	3*h*. 6*m*. 48.9*s*.	

So that the right ascension of the star, is 3*h*. 6*m*. 48.9*s*, and its declination 31°45′ 6″ north; and in the same manner, if the latitudes and longitudes of the two known stars had been used in the computation, then we should have found the latitude and longitude of the star C.

Our next object must be to explain the manner of ascertaining the Moon's longitude, latitude, &c. On account of the variety of motions to which she is subjected, the computations for determining her place are necessarily more complex than any that have yet been entered upon; we have, however, endeavoured to render the precepts so explicit as to enable the most inexperienced in the use of astronomical tables to calculate the Moon's place with ease and accuracy.

As the Sun's longtitude, right ascension, &c. for Sept. 6th, 7th, and 8th, 1820, will be required in some of the problems respecting the conjunction, &c. of the Sun and Moon, the calculation of these requsites is here inserted before the problems relating to the Moon. The longitude, and obliquity of the ecliptic are found according to the precepts already given at pages 172, 173, and the right ascension and declination by prob. 3, page 175.

CALCULATION *of the Sun's true longitude, right ascension, declination, and semi-diameter, also the equation of time, and the obliquity of the ecliptic, for apparent noon at Greenwich, on the 6th, 7th, and 8th days of September,* 1820.

Sept. 6.

	Years and Days.	☉ Longitude.	Perigee.	M	A	B	C	D	E	F	N
Sec. eq. + 0.8″	1820,	s ° ′ ″ 9 10 3 30.5	s ° ′ ″ 9 9 49 42	416	503	278	911	280	914	022	982
Eq. T 1′, — 2.5	Sept. 6.	8 5 25 34.1	42	37	437	682	108	362	057	023	037
49″ — 2.0	S. E & E. T.	—3.7	9 9 50 24	453	940	960	019	642	971	045	019
—3.7	M. long.	5 15 29 0.9	5 15 29 1	cor.	27						
Add sum equat.		11 28 15 46.9	8 5 38 37		967						
☉ True long. M. equin.		5 13 44 47.8	Mean anom.								
Nutation ☾ and ☉		+2.6	☾ Nut. arg. N + 2.1								
☉ True long. T. equin.		5 13 44 50.4	☉ Nut. Sept. 6 + 0.5								
☉ Semi-diameter		15 54.5	Nut. in long. 2.6								
			Nut in R. asc. 2.4								

Equat. centre, . 11 28 15 0.1
Sec. variation, ... + 1.5
Lunar equat. 5.9
B and C 14.7
B and D 7.7
B and E 15.6
B and F 1.4
Sum equat. . . 11 28 15 46.9

From . . 6s 0° 0′ 0″
Subtract . 5 13 44 50.4

From ♎ . . . 16 15 9.6 log. tan. 9.4646742 . . . . . sin. 9.4469621
Obliq. . . . . 23 27 56.7 log. cos. 9.9625106 . . . . . sin. 9.6001021
14 58 17.1 log. tan. 9.4271848 Dec. N. 6°23′55″.2 sin. 9.0470642
180 0 0

R. as. from ♈ 165 1 42.9 in time 11*h*. 0*m*. 6.9*s*.
Nut. R. asc. — 2.4
mean R. asc. 165 1 40.5
mean long. 165 29 0.9
Equa. time . 0 27 20.4 . . . . . — 1*m*. 49.4*s*.

Obliq. Ecliptic.
☾ Nut. arg. N. . + 9″.6
☉ Nut. Sept. 6, + 0.4
S. eq. 0″.5 × 20.7 — 10.3
Sum . . . . . — 0.3
Obliq. 1800, 23° 27′ 57″.0
ob. 1820 Sep. 6, 23 27 56.7

Sept. 7.

Sec. eq. + 0.8″	1820,	s ° ′ ″ 9 10 3 30.5	s ° ′ ″ 9 9 49 42	416	503	278	911	280	914	022	982
Eq. T. 2′ — 4.9	Sept. 7.	8 6 24 42.5	42	73	471	685	113	364	58	023	037
9″, — 0.4	S. E & E. T.	— 4.5	9 9 50 24	489	974	963	024	644	972	045	019
— 4.5	M. long.	5 16 28 8.5	5 16 28 9	cor.	24						
Add sum equat.		11 28 14 58.8	8 6 37 45		998						
☉ True long. M. equin.		5 14 43 7.3	Mean anom.								
Nutation ☾ and ☉		+ 2.6									
☉ True long. T. equin.		5 14 43 9.9	Nut. in long. and R. as. the same as for Sept. 6.								
☉ Semi-diameter		15 54.8									

Equat. centre, 11 28 14 10.4
Sec. variation, ... + 1.6
Lunar equat. 7.4
B and C 14.6
B and D 7.8
B and E 15.6
B and F 1.4
Sum equat. . . 11 28 14 58.8

From . . 6s 0° 0′ 0″
Subtract . 5 14 43 9.9

From ♎ . . . 15 16 50.1 log. tan. 9.4364883 . . . . . sin. 9.4208567
Obliq. . . . . 23 27 56.7 log. cos. 9.9625106 . . . . . sin. 9.6001021
14 4 8.9 log. tan. 9.3989089 Dec. N. 6°1′26″.2 sin. 9.0209588
180 0 0

R. as. from ♈ 165 55 51.1 in time 11*h*. 3*m*. 43.4*s*.
Nut. R. asc. . . . . — 2.4
mean R. asc. 165 55 48.7
mean long. 166 28 8.5
Equa. time 0 32 19.8 . . . . . — 2*m*. 9.3*s*.

Obliq. Ecliptic
The same as for Sept. 6.

Sept. 8.

	Years and Days.	☉ Longitude.	Perigee.	M	A	B	C	D	E	F	N
		s ° ′ ″	s ° ′ ″								
Sec. eq. + 0.8″	1820.	9 10 3 30.5	9 9 49 42	416	503	278	911	280	914	022	982
Eq. T. 2′ — 4.9	Sept. 8.	8 7 23 50.8	43	109	505	687	117	365	058	023	037
Eq. T. 29″ — 1.2	S. E & E. T.	— 5.3	9 9 50 25	525	008	965	028	645	972	045	019
— 5.3	M. long.	5 17 27 16.0	5 17 27 16	cor.	20						
Add sum. equat.		11 28 14 12.2	8 7 36 51		28						
☉ True long. M. equin.		5 15 41 28.2	Mean anom.								
Nutation ☾ and ☉		+ 2.6	Nut. in long. and R. as. the same as for Sept. 6.								
☉ True long. T. equin.		5 15 41 30.8									
☉ Semi-diameter		15 55.0									

	s	°	′	″
Equat. centre,	11	28	13	22.3
Sec. variation, +				1.6
Lunar equat.				8.9
B and C				14.6
B and D				7.8
B and E				15.6
B and F				1.4
Sum. equat.	11	28	14	12.2

From . . 6s 0° 0′ 0″
Subtract . 5 15 41 30.8

Obliq. Ecliptic. The same as for Sept. 6.

From ♎ . . .	14 18 29.2	log. tan.	9.4066212	sin.	9.3929364
Obliq. . . .	23 27 56.7	log. cos.	9.9625106	sin.	9.6001021
	13 10 4.0	log. tan.	9.3691318	Dec. N. 5°38′ 51″.4 sin.	8.9930385
	180 0 0				

R. as. from ♈ 166 49 56.0 in time 11*h*. 7*m*. 19.7*s*.
Nut. R. asc. . . . — 2.4

mean R. asc. 166 49 53.6
mean long. 167 27 16

Equa. time 0 37 22.4 . . . . . — 2*m*. 29.5*s*.

Of the MOON.

Problem 1.

To calculate the Moon's longitude, latitude, horizontal parallax, and semi-diameter, for any given time at Greenwich, in the 19th century.

Example.—Required the Moon's true longitude, latitude, horizontal parallax, and semi-diameter, on the 6th day of September, 1820, at 12 hours, or midnight, apparent time at Greenwich.

Precepts for the longitude.

1. If the time given be apparent time, convert it into mean time, by applying the true equation of time at the given instant.

In this Example, the equation of time for Sept. 6th, at 12 hours app. time is 1*m*. 59.3*s*,* and is subtractive; therefore 12*h*. — 1*m*. 59.3*s*. = 11*h*. 58*m*. 0.7*s*. the corresponding mean time.

2. From table II of lunar tables, take out the epochs for the given year of the arguments of longitude, viz. from No. 1 to 31, the evection, anomaly, variation, mean longitude, and supplement of the node, and place them in horizontal lines: take out also the secular equations of these last mentioned five epochs, and place each, with its proper sign, under the corresponding epoch, and if there be any difference between the secular equation for the given year, and for that next following it, correct the secular equation by a part of this difference, proportional to the part of the year elapsed. Write down likewise at the same time the arguments of latitude for the given year titled II, V, VI, &c. to be subsequently used in finding the latitude.

Thus, looking for 1820 in the 1st column of table II, we find opposite to it, and under the titles 1, 2, 3, &c. the numbers 00064, 0508, 4682, &c. as put down in the calculation to this example.

The secular equation of the evection, variation, and mean longitude is the same both for 1820, and 1821, but for 1820 the secular equation of the anomaly is + 48″.0

* This is found by taking a mean between the equation of time for noon of the 6th of Sept. and for noon of the 7th, which have been already computed at page 211.

and for 1821 it is + 48″.7 : hence, as there are 8 months, or very little more, of the given year, 1820, elapsed, say as 12*m.* : 8*m.* : : 0″.7 (the diff. of the secular equations) : 0.46″ or 0″.5, which added to 48″.0 gives + 48″.5 for the correct sec. equation, which is placed under 4*s.* 28° 5′ 15″.9: the sec. equation of the supplement of the node is obtained in like manner.

3. From table III, take out the mean motions (both for longitude and latitude as before) for the given day of the month, and place them respectively under the former motions for the year.

Thus opposite Sept. 6, under the title Biss. (1820 being a Bissextile year) we find 68171, 1822, &c. which are placed respectively under 00064, 0508, &c.

4. From table IV, take out the motion for the given hours, writing them respectively under the corresponding motions for the year and month as before.

5. Take out, in like manner, from tables V and VI, the motion for the minutes and seconds of time.

There is nothing to add for the seconds but to the evection, anomaly, variation, and mean longitude. If we wish to be very accurate, we may take a proportional part of the mean anomaly, &c. for the fractional part of the second, if there be any. In the exam. 0″.3, 1″, and 0″.3 are put down as the nearest proportional parts of the anomaly, variation, and mean longitude, answering to 0.7*s.* of time.

6. Add up all the several columns from arguments 1 to 31, and likewise the supplement of the node or No. 32, taking care to attend to the sign of the secular equation in the latter: also in adding up argument 1, only five figures towards the right are to be retained in the sum, the circumference being suppposed to be divided into 100000 equal parts; in adding up arguments 2 to 9, inclusive, four figures only are to be retained; from arguments 10 to 20, both inclusive, three figures, and from arguments 21 to 31, two figures.

7. With arg. 1, enter table VIII, and take out the 1st equation of longitude, taking a proportional part, if the argument be not exactly found in the table.

In the exam. arg. 1 is 68371, the next less in the table is 68000, opposite to which is 22′ 31″.1 and as the differences are put down *per* 100 in the table, say as 100 : 1″.76 : : 371 : 6″.5296 or 6″.5 which added to 22′ 31″.1, gives 22′ 37″.6, the correct equation: this is placed at the top of the last, or right hand column in the calculation.

8. With arguments 2, 3, 4, 5, 6, and 7, enter table IX, and take out the equations corresponding to these arguments respectively (proportioning where necessary) and place them all under the 1st equation of longitude.

Arg. 2 in the exam. is 2654, and in table IX, we find opposite 2600, and under the title 2, 4′ 57″.0, and the diff. for 100 is 0″.9, therefore as 100 : 0″.9 : : 54 : 0″.486 or 0″.5, the proportional part, which it appears is subtractive; hence 4′ 57″.0 — 0″.5 = 4′ 56″.5, the correct equation; and thus of the others.

9. With arguments 8 to 31, inclusive enter tables X, XI, XII, XIII, XIV, and XV, and take out the equations as before, writing them under the others. Also with the supplement of the node, or No. 32, enter table XVI, and take out the corresponding equation.

Here the supplement of the node is 0*s* 6° 53′ 19″.9, or 0*s.* 7°, nearly, which gives 9″.2 for the equation.

10. Add the sum of these 32 minor equations of longitude (retaining the nearest second) to the evection, and you have the evection corrected.

The sum of these 32 equations is here 35′ 25″, which added to the evection, (regard being had to the sign of the secular equation) makes 5*s.* 21° 54′ 53″.

11. With the evection corrected enter table XVII, and take out the equation, and apply it to the amount of the 32 minor equations, then add this last result to the anomaly, which will give the corrected anomaly.

In table XVII we find 1° 42′ 23″.9 corresponding to 5*s.* 21°, and the diff. for 1° is 82″.1, hence 1° : 0° . 54 . 53″ : : 82″.1 : 75″.1 or 1′ 15″.1, which subtracted from 1° 42′ 23″.9 leaves 1° 41′ 8″.8, this added to 35′ 25″.1, makes 2° 16′ 33″.9 which last added to the anomaly gives 5*s.* 20° 4′ 28″.6, the corrected anomaly.

12. With the corrected anomaly enter table XVIII, and take out the equation of the centre, add this to the sum of the former equations, and then add the amount to the variation, and you have the variation corrected.

The equation of the centre is put down in the table to every 10 minutes of arg. and after proportionating it is here found to be 8° 1′ 4″.8, this added to the preceding equations makes 10° 17′ 38″.7, or 10° 17′ 39″ which is carried to the column of the variation.

13. With the variation corrected enter table XIX, and take out the equation, add this, as before, to the sum of all the preceding equations, and then add the last amount to the mean longitude, and you have the mean longitude corrected.

Here entering table XIX with the corrected variation 11*s.* 22° 6′ 22″, we find 28′ 29″.1 for the equation, this added to the sum of the former equations, makes 10° 46′ 7″.8, which is then added to the mean longitude.

14. Add the mean longitude corrected, to the supplement of the node, and you have the argument of reduction; with this arg. enter table XX, and take out the equation, and place it under the corrected longitude.

In the ex. the arg. of reduction is 5*s.* 14° 42′ 2.″7, and the reduction 10′ 29.″8.

15. From table VII take out the lunar and solar nutations: the argument for the first of these is the supplement of the node, and for the latter the day

of the month; place the amount of these, with its proper sign, under the reduction, and the sum of the mean longitude corrected, the reduction, and nutation will be the Moon's *true* longitude from the *true* equinox.

The arg. for the lunar nutation in this exam. is 0*s*. 6° 53′ 19.″9, or 0*s*. 7°, which gives + 2.″2; the solar nutation for Sept. 6 is + 0.″5, and their sum + 2.″7; but having already put down 2.″6 for the nutation, in the calculation for the Sun's longitude for this day, we have retained the same quantity here. The result of this computation is that the Moon's true longitude on Sept. 6th, 1820, at 12 hours app. time is 5*s*. 7° 59′ 15″.2.

Precepts for the latitude.

1. ADD the sum of all the equations of longitude, except the reduction, and nutation to arg. II, and you have it corrected; also with the same sum enter table XXIV, and take out the corresponding decimal part of the periphery, which add to all the arguments of latitude from V to XI, both inclusive, and you have them corrected.

The sum of the equations of longitude, exclusive of reduction and nutation, is here 10° 46′, which is carried to the column containing arg. II: next, entering table XXIV with 10° 46′, we find the nearest decimal part (on the left) to be 030, which is placed under arguments V to XI.

2. With argument I, which is the same as that of the reduction, enter table XXI, and take out the Moon's north polar distance.

This arg. is here 5*s*. 14° 42′ 2″.7 the nearest in the table is 5*s* 14° 40′, which gives 88° 26′ 27″.2, and by proportionating the correct distance answering to the argument is found = 88° 26′ 37″.8.

3. With argument II corrected, enter table XXII, and take out the corresponding equation, and write it under the distance, proportionating, if necessary, as before.

4. With the Moon's true longitude, enter table XXIII, and take out the equation answering thereto, writing it under the last.

5. With argument IV, which is the same as the 20th arg. of long. enter table XXV, and take out the equation, which write under the others. From the same table, take out the equations answering to arguments V, VI, VII, VIII, IX, and X corrected, disposing them under the others.

The 20th arg. of long. is here 983, and in the right hand column of the table, ascending, we find on a line with 980, and under arg. 20 long. 16″.8, which diminished by a proportional part for the 3, as the equation is decreasing, gives 16″.5 for the true equation.

6. In like manner, take out from tables XXVI and XXVII, the equations corresponding to arguments XI and XII corrected.

7. Add up the preceding 12 equations, and you have the Moon's true distance from the north pole of the ecliptic: if this distance be less than 90° its complement is the latitude *north*, but if it exceed 90°, the excess above 90°, is the latitude *south*.

To find the Moon's horizontal parallax and semi-diameter.

1. WITH the first two places of figures towards the left of arguments 1, 2, 4. 5, 6, 8, 9, 12, and 13 of longitude enter table XXVIII, and take out the corresponding equations.

Thus arg. 1 is taken as 68, and gives 0″.6. Arg. 2 being 26.54 is called 27 and gives 0″.7, &c.

2. With the corrected evection, variation, and anomaly, enter tables XXIX, XXX, and XXXI, respectively, and take out the equations to these arguments.

These three arguments in the example, are for evection 5*s*. 21° 54′ 53″, variation 11*s*. 22° 6′ 22″, and anomaly 5*s*. 20° 4′ 28″, which give 6″.3, 54″.6, and 52′ 45″, respectively.

3. Add up the 12 equations, and the sum will be the Moon's equatorial horizontal parallax. Then if great accuracy be desired, we must diminish the equatorial parallax, according to the latitude of the place, on account of the spheroidal figure of the Earth; this diminution is equal to $\frac{1}{289}$ × equat. parall. × nat. sin. of lat. or to .0043× equat. parall. × sin. lat.: but this correction is generally too small to require attention.

4. With the equatorial parallax enter table XXXII, and you have the semi-diameter.

Having found the Moon's longitude and latitude, and knowing the obliquity of the ecliptic, her right ascension and declination may be determined by problem 15, or more readily by Dr. Maskelyne's rule at page 207, which is the method we have used. The computation is disposed underneath that of the longitude, and as the rule just referred to has been already fully explained, nothing farther need be said here on the manner of finding the right ascension and declination.

PROBLEM 2.

To compute the Moon's place in any century before, or after, the 19th.

Example.—Suppose it were required to find the Moon's longitude, &c. for a given time in 1620, or 1920.

From table II, take out the epochs for 1820; and from table I, take out the epochs for — 200 years, or + 100 years, add these to the epochs for 1820, and the same will be the epochs for 1620, and 1920 respectively: the rest of the operation would be similar to the following example, and therefore requires no farther explanation.

CALCULATION *of the Moon's true longitude, &c. as before for Sept. 7th, 1820, at 12h. or midnight, apparent time at Greenwich, that is, for Sept. 7th., at 11h. 57m. 40.6s. mean time.*

	1	2	3	4	5	6	7	8	9	10	11	12
1820	00064	0508	4682	6337	4163	1545	5201	4176	8912	942	525	051
Sept. 7,	68444	2472	0045	1742	3885	3012	4425	7574	6071	539	781	616
11*h.*	125	298	477	131	154	171	26	179	11	32	14	32
57*m.*	11	26	41	11	13	15	2	15	1	3	1	3
	68644	3304	5245	8221	8215	4743	9654	1944	4995	516	321	702

	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1820.	635	686	896	526	937	633	782	869	78	52	46	87	20	64	83
Sept. 7,	543	790	631	150	515	490	530	114	90	13	32	64	29	23	46
11*h.*	16	45	14	17	19	12	1	0	0	0	5	1	2	2	3
57*m.*	1	4	1	1	2	1									
	195	525	542	694	473	136	313	983	68	65	83	52	51	89	32

	28	29	30	31	Evection.	Anomaly.	Variation.	☾'s Longitude.
					s ° ′ ″	s ° ′ ″	s ° ′ ″	s ° ′ ″
1820.	42	42	63	36	7 17 52 59	4 28 5 15.9	6 0 14 7	3 9 32 57.6
Sec. eq.	...	...	...	...	.....— 41	48.5	4	3.7
Sept. 7,	44	70	43	63	10 9 7 37	0 26 14 52.4	5 17 41 14	1 24 5 56.8
11*h.*	2	1			5 11 12	5 59 17.2	5 35 15	6 2 21.0
57*m.*	...	...	...	...	.... 26 52	31 1.8	28 57	31 17.6
40.6*s.*	...	...	...	...	19	22.1	21	22.3
sum pre. equa.	...	...	...	...	.... 36 26	2 1 57.3	8 44 5	9 25 25.1
	88	13	06	99	6 3 14 44	6 2 53 35.2	0 2 44 3	5 19 38 24.1

	32 Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
	s ° ′ ″	s ° ′								
1820.	11 23 40 47.3	8 26 0	258	260	738	739	521	041	564	652
Sec. eq.	—11.0									
Sept. 7,	0 13 14 19.2	8 28 2	503	872	060	429	959	401	409	328
11*h.*	1 27.4	5 7	16	18	13	15	16	19	15	2
57*m.*	7.5	27	1	2	1	1	1	2	1	
Sum eq.	.....	9 25	26	26	26	26	26	26	26	—
	0 6 56 30.4	6 9 1	804	178	838	210	523	489	015	982
	5 19 38 24.1									
arg I lat & of red.	5 26 34 54.5									

.....5*s*	19° 38′ 24″1
Reduction, ...	7 49.1
Nutation, ...	+ 2.6
☽'s true long.	5 19 46 15.8

Computation of ☽'s right ascension and declination.

Lat.....	0° 17′ 10″	log. tan. + 10 ...	17.6984157
Long. ..	169 46 16	log. sin.	9.2493964
Arc A, ...	1 36 39	log. tan.	8.4490193
Obliq. ...	23 27 57		
Arc. B, ..	25 4 36	log. cos.	9.9570043
	1 36 39	ar. com. log. cos. ..	0.0001717
	169 46 16	log. tan.	9.2563544
R. asc. ..	170 42 51	log. tan.	9.2135304
	170 42 51	log. cos.	9.9942712
	25 4 36	log. tan.	9.6701881
Dec. N. ..	4 19 5	log. tan.	8.8779897

Equations of N. polar distance.

Arg.	° ′ ″
I	89 29 36.9
II	10 22.5
III tr. *l.*	9.5
IV 20 *l.*	16.5
V	5.6
VI	8.5
VII	44.0
VIII	24.8
IX	33.9
X	5.7
XI	3.5
XII	18.8
N. P. dist.	89 42 50.2
lat. N.	0 17 9.8

Equations of Parallax.

Arg.	′ ″
1	0.5
2	0.4
4	0.4
5	1.1
6	0.3
8	0.4
9	3.6
12	0.5
13	1.3
evec	6.1
Var.	55.5
An.	52 42.9
eq. p.	53 53.0
S. D.	14 41.0

Equations of longitude.

1	22′	42″3
2	4	38.9
3	1	8.8
4		28.9
5		11.5
6	1	16.5
7	1	12.8
8		13.5
9	1	19.7
10		9.4
11		6.4
12		36.9
13		2.7
14		21.9
15		12.6
16		7.2
17		8.9
18		15.1
19		0.7
20		8.5
21		2.5
22		6.5
23		3.2
24		4.8
25		4.9
26		5.6
27		5.7
28		5.5
29		5.5
30		4.9
31		5.0
32		9.2
Sum ..	36	26.5
Evec.	1 25	30.8
Sum	2 1	57.3
An.	6 42	7.7
Sum	8 44	5.0
Var.	0 41	20.1
Sum	9 25	25.1

CALCULATION *of the Moon's true longitude, &c. as before, for apparent noon at Greenwich, on Sept. 8th, 1820, that is, for Sept. 7th, at 23h. 57m. 30.5s. mean astronomical time.*

	1	2	3	4	5	6	7	8	9	10	11	12
1820.	00064	0508	4682	6337	4163	1545	5201	4176	8912	942	525	051
Sept. 7.	68444	2472	0045	1742	3885	3012	4425	7574	6071	539	781	616
23*h.*	262	623	997	275	322	357	55	374	23	67	30	67
57*m.*	11	26	41	11	13	15	2	15	1	3	1	3
	68781	3629	5765	8365	8383	4929	9683	2139	5007	551	337	737

	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1820.	635	686	896	526	937	633	782	869	78	52	46	87	20	64	83
Sept. 7,	543	790	631	150	515	490	530	114	90	13	32	64	29	23	46
23*h.*	33	95	29	35	40	25	2	0	1	1	10	3	4	4	7
57*m.*	1	4	1	1	2	1									
	212	575	557	712	494	149	314	983	69	66	88	54	53	91	36

	28	29	30	31	Evection.	Anomaly.	Variation.	☽'s Longitude.
					s. ° ′ ″	*s.* ° ′ ″	*s.* ° ′ ″	*s.* ° ′ ″
1820.	42	42	63	36	7 17 52 59	4 28 5 15.9	6 0 14 7	3 9 32 57.6
Sec. eq.	...	...	...	...	..... —41	48.5	4	3.7
Sept. 7,	44	70	43	63	10 9 7 37	0 26 14 52.4	5 17 41 14	1 24 5 56.8
23*h.*	4	3			10 50 42	12 31 14.2	11 40 58	12 37 38.6
57*m.*	...	...	...	...	.... 26 52	31 1.8	28 57	31 17.6
30.5*s.*	...	...	...	...	 15	16.6	16	16.7
sum. pre. equa.	...	...	...	...	.... 36 53	1 54 36.8	7 57 20	8 45 2.3
	90	15	06	99	6 8 54 37	6 9 18 6.2	0 8 2 56	5 25 33 13.3

	☊ Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
	s. ° ′ ″	*s.* ° ′								
1820.	11 23 40 47.3	8 26 0	258	260	738	739	521	041	564	652
sec. eq.	— 11.0									
Sept. 7,	0 13 14 19.2	8 28 2	503	872	060	429	959	401	409	328
23*h.*	3 2.7	10 41	33	38	27	32	34	40	32	5
57*m.*	7.5	27	1	2	1	1	1	2	1	
Sum eq.	.....	8 45	24	24	24	24	24	24	24	—
	0 6 58 5.7	6 13 55	819	196	850	225	539	508	030	985
	5 25 33 13.3									
arg. I. lat & of red	6 2 31 19.0									

..... 5*s* 25° 33′ 13″3
Reduction, ... 6 23.7
Nutation, ... + 2.6
☽'s true long. 5 25 39 39.6

Computation of ☽'s right ascension and declination.

Lat.	0° 15′ 32″	log. tan. + 10, ..	17.6549937
Long...	175 39 40	log. sin.	8.8788407
Arc A, ...	3 25 4	log. tan.	8.7761530
Obliq. ..	23 27 57		
Arc. B, .	20 2 53	log. cos.	9.9728531
	3 25 4	ar. com. log. cos...	0.0007731
	175 39 40	log. tan.	8.8800872
R. asc. ..	175 54 57	log. tan.	8.8537134
	175 54 57	log. cos.	9.9988957
	20 2 53	log. tan.	9.5621981
Dec. N...	1 29 20	log. tan.	8.4148072

Equations of N. polar distance.

Arg.	° ′ ″
I	90 1 34.1
II	11 6.6
III tr. *l.*	8.6
IV 20 *l.*	16.5
V	6.5
VI	7.5
VII	43.1
VIII	24.9
IX	36.5
X	6.2
XI	3.1
XII	18.5
N. P. dist.	90 15 32.1
lat. S.	0 15 32.1

Equations of parallax.

Arg.	″
1	0.5
2	0.3
4	0.5
5	1.2
6	0.3
8	0.5
9	3.6
12	0.6
13	1.2
evec	6.4
Var.	54.6
An.	52 44.7
eq. p.	53 54.4
S. D.	14 41.4

Equations of longitude.

1	22′ 44″5
2	4 21.7
3	1 26.7
4	37.2
5	17.0
6	1 26.2
7	1 11.7
8	11.1
9	1 20.4
10	8.2
11	5.4
12	37.7
13	2.1
14	25.5
15	13.5
16	6.7
17	9.8
18	15.4
19	0.7
20	8.5
21	2.4
22	6.5
23	3.6
24	4.8
25	4.8
26	5.5
27	5.6
28	5.4
29	5.6
30	4.9
31	5.0
32	9.2
Sum	36 53.3
Ev.	1 17 43.5
sum	1 54 36.8
An.	6 2 43.7
sum	7 57 20.5
var.	0 47 41.8
sum	8 45 2.3

Results of the Moon's place, &c. for the 6th, 7th and 8th, of Sept. 1820.

Sept. 1820.	☽'s longitude.	Right ascension.	Latitude.	Declination.	Hor. paral.	Sem. diam.
	s ° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Midn. 6th. . .	5 7 59 15	160 10 7	1 21 40 N	9 50 45 N	53 54.5	14 41.4
Noon 7th. . .	5 13 52 54	165 28 28	0 49 40	7 6 39	53 53.1	14 41.0
Midn. 7th. . .	5 19 46 16	170 42 51	0 17 10	4 19 5	53 53.0	14 41.0
Noon 8th. . .	5 25 39 40	175 54 57	0 15 32 S.	1 29 20	53 54.4	14 41.4

We shall now shew the method of finding by interpolation the Moon's longitude, right ascension, latitude, &c. and the hourly motion in each, for any times intermediate between those for which they have been strictly calculated; as suppose for 1, 2, 3, 4 and 5 hours, P. M. of Sept. 7th.

Sept. 1820.	☽'s longitude.	1st. diff.	2d. diff,	Mean 2d diff.
	s ° ′ ″	° ′ ″	′ ″	′ ″
Midn. 6th. . .	5 7 59 15			
		+ 5 53 39		
Noon 7th. . .	5 13 52 54		— 0 17	
		+ 5 53 22		— 0 7½
Midn. 7th. . .	5 19 46 16		+ 0 2	
		+ 5 53 24		
Noon 8th. . .	5 25 39 40			

☽'s motion in 12 *hours between noon and midn. of Sept. 7th,* 5° 53′ 22″.

H	Prop. part.	Eq. 2d. diff.	Prop. part corrected.	Hor. mot.
	° ′ ″	″	° ′ ″	′ ″
1	0 29 26.8	— 0.3	0 29 26.5	29 26.7
2	0 58 53.7	— 0.5	0 58 53.2	29 26.6
3	1 28 20.5	— 0.7	1 28 19.8	29 26.7
4	1 57 47.3	— 0.8	1 57 46.5	29 26.7
5	2 27 14.2	— 1.0	2 27 13.2	

The several times for which we propose finding the Moon's longitude, right ascension, &c. being between the noon and midnight of the 7th, it is obvious that if her motion were uniform during this interval of 12 hours, then her longitude at 1*h.* 2*h.* &c. P. M. would be found by taking 1 — 12*th*, 2 — 12*th*, &c. parts of the motion in 12 hours, and applying these proportional parts to her place at noon of the 7th: but as the Moon's motion during 12 hours is scarcely ever equable, the proportional parts thus found require correction, by adding or subtracting the *equation of second difference* corresponding to each proportional part; and this equation is found as follows: write down the Moon's *true longitude* at two periods preceding, and two following the several times for which her longitude is required, (as is done in the last tablet but one) then take the first of these longitudes from the 2d, the 2d from the 3d, &c. thus forming the column of *first differences*; take also the differences of these differences, or the *second differences*, and likewise half the sum of the two second differences, which will be the mean second difference; then write down the proportional part for 1*h.* 2*h.* 3*h.*, &c. of the middle first difference (that being the difference of longitude within the period we have to interpolate) as in the second column of the last tablet; next with the mean second difference, 7″½, and 1*h.* 2*h.* 3*h.*, &c. as arguments enter table XXXVII and take out the several equations of second difference; thus 7″½ and 1*h.* give 0.3″ for the equation, 7″½ and 2*h.* give 0.5, &c. apply these equations with their proper signs to the proportional parts first found respectively, and you have the proportional parts corrected as in the 4th column of the tablet, the differences of which last will be the motion in each hour. It is to be observed that the equation of second difference is to be subtracted from the proportional part of the first difference of the motion in 12 hours, if that difference be increasing, but added, if it be decreasing; also in taking the mean second difference, we must have regard to the algebraic signs of the second differences; thus we have here, — 17″ + 2″ = — 15″, the half of which, or — 7½, is the mean second differ-

ence, and when, as in this case, the second differences have opposite signs, the equation is generally very small, and consequently the motion is equable or nearly so—it accordingly results here that the horary motion in longitude is very nearly constant for these 5 hours. The motion in right ascension is more variable, as appears from the following calculations which are carried on in the same way as those just described for the longitude.

Sept. 1820.	☽'s Right Asce.	1st. diff.	2d. diff.	Mean 2d. diff.
Midn. 6th.	° ′ ″ 160 10 7			
		+ 5 18 21		
Noon 7th.	165 28 28		— 3 58	
		+ 5 14 23		— 3 7½
Midn. 7th.	170 42 51		— 2 17	
		+ 5 12 6		
Noon 8th.	175 54 57			

Diff. of ☽'s right ascension for noon and midnight of Sept. 7th, 5° 14′ 23″

H.	Prop. part.	Eq. 2d diff.	Prop. part corrected.	Hor. mot.
1	° ′ ″ 0 26 11.9	+ 7.2	° ′ ″ 0 26 19.1	′ ″ 26 17.7
2	0 52 23.8	+ 13.0	0 52 36.8	26 16.5
3	1 18 35.7	+ 17.6	1 18 53.3	26 15.2
4	1 44 47.7	+ 20.8	1 45 8.5	26 14.0
5	2 10 59.6	+ 22.9	2 11 22.5	

The true proportional parts for the latitude and declination at 1*h.* 2*h.* 3*h.*, &c. are found by similar calculations, and then by adding or subtracting the proportional parts successively we obtain the longitude, right ascension, &c. at the several hours proposed. With regard to the Sun, his motion for so short a period as 5 hours is, in every respect so equable, that proportional parts of the first difference only, taken in the usual way, are sufficient. The preceding results for the Moon, and likewise the Sun's place at the same hours, are here tabulated for the purpose of future reference.

Table of results of requisites for the Moon and Sun.

Sept. 7, 1820.	H	Longitude.	hor.mot.	Right asc.	hor.mot.	Latitude.	hor.mot.	Declination.	hor.mot.	hor. par.	Sem. D.
		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	′ ″		
	0	163 52 54	29 26	165 28 28	26 19	0 49 40 N	2 42	7 6 39 N	13 51		
	1	164 22 20	29 27	165 54 47	26 18	0 46 58	2 42	6 52 48	13 53		
MOON.	2	164 51 47	29 27	166 21 5	26 16	0 44 16	2 42	6 38 55	13 54	53′ 53″.1	14′ 41″.0
	3	165 21 14	29 27	166 47 21	26 15	0 41 34	2 42	6 25 1	13 55	constant	constant
	4	165 50 41	29 26	167 13 36	26 14	0 38 52	2 42	6 11 6	13 56		
	5	166 20 7		167 39 50		0 36 10		5 57 10			
	0	164 43 10		165 55 51				6 1 26 N			
	1	164 45 36		165 58 6				6 0 30			
	2	164 48 2	constant	166 0 21	constant			5 59 33	constant	0′ 8″.7	15′ 54″.8
SUN.	3	164 50 28	2′ 26″	166 2 37	2′ 15″	0 0 0	0 0	5 58 37	0′ 56″	constant	constant
	4	164 52 53		166 4 52				5 57 40			
	5	164 55 19		166 7 7				5 56 44			

PROBLEM 3.

To find the time of the ecliptic conjunction, or new Moon, in Sept. 1820.

By an inspection of the longitudes of the Sun and Moon in the preceding table of results, it appears that they become equal between the hours of 1 and 2 on Sept. 7th, hence,

	°	′	″
From ⊙'s long. at 1*h*.	164	45	36
Subt. ☽'s long. at 1*h*.	164	22	20
Diff.		23	16

	′	″
☽'s hourly motion in long.	29	27
⊙'s do.	2	26
☽'s motion from ⊙	27	1

	′	″		*h.*	*m.*	*s.*
Then as	27	1	:	23	16	:: 1*h*. : 0 51 40.3
Add						1 0 0
Eclip. ☌, or new Moon, Sept. 7, at						1 51 40.3 P. M.

	h.	*m.*	*s.*		°	′	″
Also as	1	51	40.3	:: 2 26 :		2	57
Add					164	45	36.0
Long. ⊙ and ☾ at time ☌					164	47	41.7

Problem 4.

To find the time of the equatorial conjunction: that is, when the centres of the Sun and Moon are on the same meridian.

As this conjunction takes place when ☾'s *AR* = ⊙'s *AR*, we find by reference to the foregoing table that the conjunction is between the hours of 1 and 2; therefore,

	°	′	″
From ⊙'s *AR* at 1*h*.	165	58	6
Subt. ☽'s *AR* at 1*h*.	165	54	47
Diff.		3	19

	′	″
☽'s. hourly motion in *AR*	26	18
⊙'s. do.	2	15
☽'s. motion from ⊙.	24	3

	′	″		*h.*	*m.*	*s.*
As	24	3	: 3 19 ::	1*h*.	: 8*m*.	16.5*s*.
Add				1*h*.	0	0
Equatorial ☌ at				1	8	16.5

	′	″		″
And as 1*h*. : 8*m*. 16.5*s*. : :	2	15	:	18.6
Add				165° 58′ 6
AR ⊙ and ☽ at ☌				165 58 24.6

Scholium.

The Moon's phases as inserted in the Nautical Almanac are not scrupulously computed, but found to the supposed nearest minute of time on the hypothesis that the Sun and Moon's motion in longitude during 12 hours is uniform, but this is very seldom the case, and therefore when great accuracy is required, we ought to find the equation of second difference, and apply it in the manner already explained; it is true that a small error of a few minutes in the time of the Moon's phasis is generally of but little importance, but if there will be an eclipse at the time of new or full Moon, which we are computing, and our calculation of this time is intended to be made the basis of a computation of the elements of the eclipse, then our determination of the time of opposition or conjunction should be as accurate as possible, and the equation of second difference should by no means be neglected. Now at the time of new Moon found in the example to prob. 3, an eclipse of the Sun *does* occur, so that the preceding precepts and calculations answer a double purpose: they instruct the astronomical student in the method of finding the Moon's longitude, latitude, right ascension, &c. for any given instant of time, from whence we find the times of new and full Moon, and obtain the proper *data* for computing the Moon's distance from the Sun, and from fixed stars, and they likewise serve in preparation for the computation of the solar eclipse which happened* on Sept. 7th, 1820, and which is given in a subsequent part of this work.

That we may not be misunderstood, we wish to observe, that we mean not to intimate by the remark at the beginning of this Scholium that the computers of the Nautical Ephemeris *ought* to put down the Moon's phases with more accuracy: that work is intended, more especially, for the use of the mariner, and therefore great pains are bestowed upon every part of the book connected with the important problem of finding the longitude; those calculations which relate to phenomena merely topical, are justly considered by them as of little consequence, and therefore they think it unnecessary to increase their *already* very great labour, by employing much

* We are sorry that the words, "which will happen," as they stood in the M. S. will not now apply, owing to the unavoidable lapse of time between writing this work and sending it to press; but the considerate reader will not, we are persuaded, think that we ought to have undertaken the great labour of altering all those calculations merely for the sake of computing an eclipse in anticipation.

time and pains upon such problems, which are of scarcely any practical utility.

PROBLEM 5.

To find the time of the Moon's passage over the meridian, commonly called the Moon's southing, the right ascension of the Sun and Moon being known for the noon and midnight before, and after, the time of her culmination. This is best explained by an

Example.—Required the time of the Moon's passage over the meridian of Greenwich on the 7th, and 8th days of September, 1820, the right ascension of the Sun and Moon being as follows :—

Sept. 1820.	☽'s Right Asc.	differ. in 12 hours.	☉'s right asc.	diff. in 12*h.*
	° ′ ″	° ′ ″	° ′ ″	′ ″
Midn. 6th,	160 10 7	5 18 21	165 28 47	27 4
Noon 7th,	165 28 28	5 14 23	165 55 51	27 3
Midn. 7th,	170 42 51	5 12 6	166 22 54	27 2
Noon 8th,	175 54 57		166 49 56	

Between Midn. 6th, and Noon of 7th.

	°	′	″
☽'s mot. in $\mathcal{R}$ in 12*h.*	5	18	21
☉'s do.		27	4
Diff.	4	51	17
Subtr. from 12*h.* or	180		
Remainder,	175	8	43
☽'s $\mathcal{R}$ 6th at Midn. + 180,	340	10	7
☉'s do. subtract	165	28	47
	174	41	20

Then as 175° 8′ 43″ : 174° 41′ 20″ :: 12*h.* : 11*h.* 58*m.* 7*s.* the time of the Moon's passage over the meridian after midnight of the 6th.

Between Midn. of 7th, and Noon of 8th.

	°	′	″
☽'s mot. in $\mathcal{R}$ in 12*h.*	5	12	6
☉'s do.		27	2
Diff.	4	45	4
Subtr. from 12*h.* or	180		
Remainder,	175	14	56
☽'s $\mathcal{R}$ 7th, at midn. + 180,	350	42	51
☉'s do. subtract,	166	22	54
	184	19	57

Then as 175° 14′ 56″ : 184° 19′ 57″ :: 12*h.* : 12*h.* 37*m.* 19*s.* the time of the Moon's passage over the meridian after midnight of the 7th, being Sept. 8th, at 0*h.* 37*m.* 19*s.*

Note.—The Sun's right ascension at midnight of the 6th and 7th, in the foregoing tablet, is found by taking a mean of the right ascensions at the preceding and following each of these times, which have been calculated at pages 211, 212.

PROBLEM 6.

GIVEN the latitude and longitude of a place, and the year and day of the month, to find the true apparent time of the Moon's rising and setting at that place.

This problem is one which cannot be correctly solved by any direct operation, in consequence of which we are obliged to have recourse to approximating rules, which being troublesome and laborious, some authors have passed the Problem over in silence, and others have thought it sufficient to give a rule which may be pretty near the truth : it is true, that we do not think it a problem so important as to require great accuracy of solution, yet as it is gratifying to an ingenious enquirer to know how to solve it, we shall give the correct method by continual approximation, as well as another for the use of those who are satisfied with less accurate results.

SOLUTION 1*st.*

ASSUME the time* of rising or setting as near the true time as you can judge, and to this time apply

* In order to find the assumed time we may add the log. tangent of the Moon's declination at the nearest noon or midnight to the log. tangent of the latitude of the place; this sum, abating 10, is the log. sine. of the ascensional difference, nearly; which converted into time and added *to* or subtracted *from* 6 hours will give the semiarc of apparition in time; this applied to the time of southing will give the assumed time sufficiently near.

Having found the time of rising for two successive days, the assumed time may be had, by applying the difference between those two times to the last time of rising, and you have a near approximate time for the third day. Those who have an artificial globe may find the assumed time with less trouble.

the longitude of the place turned into time, by addition, if the longitude be west, but by subtraction, if east: the sum or difference is the *reduced* time, or time at Greenwich. Next calculate the Moon's declination for this reduced time, and then with the latitude of the place, and the Moon's declination, find the ascensional difference, which being added to, or subtracted from, 6 hours, will give the semi-arc of apparition, in a solar motion; increase this asc. into lunar motion (as in the following operation) and you have the Moon's semi-arc of apparition, which being subtracted from the time of her passage over the meridian, gives the time of her rising, and being added, gives the time of setting. In this method it is supposed that the Moon's horizontal parallax is exactly balanced by the refraction increased by her semi-diameter and the probable depression of the visible horizon below the true, and this is the case without sensible error for the rising or setting of the Moon's limbs.

Example.

Required the time of the Moon's setting on the evening of the 7th of September, 1820, at Greenwich, in latitude 51° 28′ 40″ north, and longitude 0° 0′.

Operation.

As it is known from the preceding calculations, that a conjunction of the Sun and Moon takes place a little before 2*h*. P. M. on this day, therefore the Moon will set nearly at the same time with the Sun: we may therefore assume 7*h*. for the time of the Moon's setting, and then for this hour at Greenwich we shall find her declination to be 5° 29′ 15″ N. from whence her ascensional difference is found by the analogy in Prob. 7, p. 180. viz:

	° ′ ″	
As radius		10.0000000
is to tang. dec.	5 29 15	8.9825827
so is tang. lat.	51 28 40	10.0990491
to sin. asc. diff.	6 55 53	9.0816318

	h.	*m*.	*s*.
which in time is	0	27	44
	6	0	0
sem. arc. of app. in sol. time	6	27	44

	h.	*m*.	*s*.
☽'s pass. over merid. 7th	11	58	7
ditto 8th	12	37	19
diff in 24 hours		39	12
subtract from	24	0	0
	23	20	48

Then as 23*h*. 20*m*. 48*s*. : 24*h*. :: 6*h*. 27*m*. 44*s*. : 6*h*. 38*m*. 35*s*. = ☽'s semi-arc, of appar. which added to 11*h*. 58*m*. 7*s*. gives 6*h*. 36*m*. 42*s*. P. M. on Sept. 7th, 1820, for the time of setting of ☽'s upper limb.

Solution 2*d*.

1. Assume the time of rising or setting as near as can be judged, and find the reduced time, corresponding thereto, as before.

2. Then compute strictly for this reduced time, the Moon's right ascension, declination, horizontal parallax, and semi-diameter; and also the Sun's right ascension.

3. Increase 90° by the horizontal refraction, and decrease it by the horizontal parallax, and this will be the zenith distance of ☽'s centre; also if the Moon's declination, and the latitude of the place, have the same name, subtract the declination from 90°; but if they have contrary names, add: the remainder, or sum, is the Moon's polar distance. Find likewise the co-latitude.

4. Then in an oblique angled spherical triangle, whose sides are the Moon's zenith and polar distances and the co-latitude, find the angle opposite the zenith distance, or the Moon's horary angle.

5. Subtract the Sun's right ascension from the Moon's right ascension (increased by 360°). or 24*h*. if necessary. Then to find the 1st approximate time of the Moon's rising, subtract the Moon's horary angle from this difference of the right ascensions, but add it for the time of setting.

6. If the approximate time thus found differs but little from the assumed time, it may be regarded as correct, but if there be much difference, the same operation must be repeated, using the first approximate time as the assumed time; and continue repeating this operation until the last approximate time, and that by which it was found shall agree, or very nearly so, and we shall thus have the time of rising or setting of the Moon's centre with great exactness.

Taking the same Example as before, and assuming the time of setting 7*h*. P. M. we have at that time,

	°	′	″	
☽'s right asc.	168	32	14	
☉'s right asc.	166	11	37	
Diff.	2	20	37	
☽'s declination	5	29	15	N.
☽'s hor. parall.		53	53	
☽'s sem. diam.		14	41	

° ′ ″	° ′ ″	° ′ ″
90 33 0	90 0 0	90 0 0
53 53	5 29 15	51 28 40
89 39 7	84 30 45	38 31 20

	° ′ ″		
Zen. dist.	89 39 7		
Co. lat.	38 31 20	ar. com. log. sine	0.2056388
Pol. dist. . . .	84 30 45	ar. com. log. sine	0.0019949
	2)212 41 12		
Half sum, .	106 20 36	log. sine	9.9820870
Sub. zen. dis.	89 39 7		
	16 41 29	log. sine, ...	9.4582095
			2)19.6479302
	48 10 59	log. cos.	9.8239651
	2		
Hour ang.	96 21 58		
Add diff. right asc.	2 20 37		
	98 42 35	= 6h. 34m. 50s. P. M. the 1st approx. time of Moon's setting.	

As this time differs nearly half an hour from the assumed time, we shall repeat the operation, using this approximate time as the assumed time. Now by proportionating, we find at 6h. 34m. 50s.

	° ′ ″
Moon's right asc.	168 21 15
Sun's do.	166 10 40
Diff.	2 10 35
Moon's declin. N.	5 35 7
	90 0 0
Pol. dist.	84 24 53

Hor. par. and semi-diam. as before.

	° ′ ″		
Zen. dist.	89 39 7		
Co. lat.	38 31 20	ar. com. log. sine	0.2056388
Pol. dist. . .	84 24 53	ar. com. log. sine	0.0020667
	2)212 35 20		
	106 17 40	log. sine, ...	9.9821955
Sub. zen. dis.	89 39 7		
	16 38 33	log. sine, ...	9.4569717
			2)19.6468727
	48 14 43	log. cos.	9.8234363
	2		
Hour ang.	96 29 26		
Add diff. right asc.	2 10 35		
	98 40 1	= 6h. 34m. 40s. P. M. time	

of setting of Moon's centre, which is the true time: for if we repeat the operation again, we shall find the same time result.

Mr. Bonnycastle, in his Trigonometry, and some other modern authors, have committed an error in solving this Problem, by making no allowance for the difference between solar and lunar motion: they subtract the hour angle in *solar* time from the time of the Moon's passage over the meridian, and then take the remainder for the time of her rising; by thus omitting to convert the hour angle(as first found in solar time) into lunar motion, a very considerable error may arise*. One of the first discoveries of the ancients in astronomy was, that the fixed stars had an apparently more rapid motion round the Earth than the Sun, so that they completed one revolution more than the Sun in a year, and that the Moon's apparent motion was slower, so that the Sun always made one revolution more than the Moon in about 29½ days. It is therefore not a little singular, that the writers alluded to should, in the present advanced state of science, have so far retrograded in astronomical accuracy, as not to make the necessary allowance for this difference between lunar and solar motion.

* When the Moon's declination and the latitude of the place are both North or both South, and her declination is nearly equal to the comp. of the latitude, the error may be about half an hour: *i. e.* when the semi-arc of apparition is near 12 hours, and the position of the Moon's orbit is such as to make a large angle with the horizon at the time of rising or setting.

Of the PLANETS.

Preparatory Problem.

To calculate the *radius vector*, that is, the Earth's distance from the Sun at any given time.

Example.—Required the radius vector, and its logarithm, for the apparent noon at Greenwich, of the 6th, 7th, and 8th, days of September, 1820.

Precepts.

1. Take out from tables I, IV, and V, the epochs with the arguments M, A, B, &c. for the given time, as in calculating the Sun's longitude by prob. 1, p. 172: find also the correction due to arg. A, and the Sun's mean anomaly as there directed.

The Sun's mean anomaly having been already computed for the several times given in the example, these results, together with the arguments M, A, &c. are brought forward from page 174.

2. With the mean anomaly enter table XVIII, and take out the approximate rad. vector; write down also the secular variation.

The Sun's mean anomaly for app. noon of Sept. 6, is 8*s*. 5° 38′ 37″; in the table we find that for 8*s* 5° the rad. vector is 1.007225, and that it decreases 263 for 1° of mean anom. therefore, as 1° : 38′ 37″ : : 263 : 169, the proportional decrease for 38′ 37″, hence, 1.007225 — 169 gives 1.007056, as put down in the computation. The sec. variation for 8*s* 5° 40′ is — 18.3, or — . 183 for a year, consequently for the time elapsed between 1810 and Sept. 6th, 1820, viz. 10.7 years, it will be 10.7 × — . 183 = — 2.

3. With the argument A, enter table XIX, and take out the *first* part of the lunar equation; also with the arguments M and A, take out, from the same table, the *second* part of the lunar equation; diminish the sum of these two parts by the constant quantity 3, and you have the whole lunar equation.

For Sept. 6th, arg. A is 967, hence 73 is the first part of the lunar equation, also M being 453, M and A give 6 for the second part, consequently 73 + 6 — 3 = 76, is the whole equation.

4. From the same table take out with the arguments B, C; B, D; B, E; and B, F; the equations on account of the perturbations of the planets, and the whole of the preceding equations applied to the radius first found, will give the true radius vector, and then its logarithm may be found from any table of logarithms.

Note.—It is usual to put down 10 for the index of the log. of the Earth's mean distance from the Sun.

1820.	⊙'s mean anom.	M	A	B	C	D	E	F	N
	s ° ′ ″								
Sept. 6,	8 5 38 37	453	967	960	019	642	971	045	019
7,	8 6 37 45	489	998	963	024	644	972	045	019
8.	8 7 36 51	525	028	965	028	645	972	045	019

	Sept. 6,	Sept. 7.	Sept. 8.
Approx. rad. vector,	1.007056	1.006795	1.006533
Sec. variation, ...	—2	—2	—2
Lunar equation, . . .	76	77	76
B and C, ...	34	34	33
B and D, ...	11	11	11
B and E, ...	39	39	39
B and F, ...	2	2	2
True rad. vector, . .	1.007216	1.006956	1.006692
Its log.	10.0031226	10.0030105	10.0028967

If the log. of the radius vector were to be calculated for every day in the month, it would abridge the operation to calculate, strictly, only for every 4th day, and to find the logarithm for the inter-

mediate days by interpolation; thus, take $\frac{1}{8}$th of the mean second difference, and this will be the equation of second difference to be applied for the middle day, and $\frac{1}{4}$ of *this* will be the equation of second difference for the other two intermediate days: observing, that if the first of the three first differences be greater than the last, then the equation of second difference must be added to the proportional part of the first difference, and *vice versâ*.

Problem 1.—*Of Mercury.*

To calculate the heliocentric longitude and latitude of Mercury, and also his geocentric longitude, latitude, right ascension and declination for a given time, and the instant of his passage over the meridian on any given day.

Example.—Required the true heliocentric longitude and latitude of Mercury, for the apparent noon at Greenwich, of the 6th, 7th, and 8th, days of September, 1820, and his geocentric longitude, latitude, right ascension, and declination for the same times, with the time of his passage over the meridian on those days.

Precepts for the longitude, &c.

1. From table I (of Mercury) take out the epochs of the mean longitude, aphelion, and node, for the given year, and place them in a horizontal line; but if the given year be beyond 1820, take out the epochs for 1820, under which, from table II, place the mean motions answering to the number of years between 1820, and the given year.

2. From table III take out the mean motions in longitude, &c. as before, for the given day of the month, and place them under the corresponding ones.

3. In like manner, write down from table IV, the motions for hours, minutes and seconds.

In the example, instead of reducing the given time (app. noon) to mean time, and taking out the motions for the hours, &c. we have subtracted the small motion corresponding to the equation of time from the epochs for mean noon.

4. Add together the quantities in the several columns, rejecting 12*s*, or any multiple thereof, if they occur, and you have the mean longitude, and the places of the aphelion and node for the given time.

5. Subtract the longitude of the aphelion from the mean longitude, and the remainder is the mean anomaly. With the mean anomaly enter table V, and take out the equation of the orbit, proportionating for the minutes and seconds, if there be any; and as the differences in this table are not very equable, if we wish for extreme accuracy, the proportional part should be corrected by the equation of second difference* taken from table XXXVII, Lunar Tables.

6. Apply the equation with its proper sign to the mean longitude, and you have the longitude on the orbit from the mean equinox.

7. From the longitude in the orbit subtract the longitude of the node, and the remainder is an argument, called the *argument of latitude.*

8. With this argument enter table VII, and take out the reduction, which apply to the longitude on the orbit, and you obtain the longitude upon the ecliptic reckoned from the mean equinox; correct this by the equation of the equinoxes in longitude, (Solar Table VIII) and the result is the true longitude of Mercury on the ecliptic from the true equinox.

The equation of the equinox is taken out with the arg. N, in the last page, and the day of the month, but it has been already found at page 211.

9. With the argument of latitude enter table VIII, and take out the heliocentric latitude, proportionating for the minutes and seconds, if any; and likewise, if it be thought necessary, correcting the proportional part by the equation of second difference* which will give the true heliocentric latitude of Mercury.

10. With the mean anomaly, take out, from table VI, the logarithm of Mercury's distance from the Sun, proportionating for the minutes, &c.

11. Enter table VII with the arg. of lat. and take out the corresponding number under the title "Sub. from Log." proportionating as before if necessary, then diminish the log. of the distance last found by this number, and the remainder is the logarithm of Mercury's *curtate* distance from the Sun.

* In the calculation to the example the equations of second difference have been neglected. The latitudes and longitudes of the planets are seldom wanted to be ascertained to very great accuracy. In the Naut. Alm. they are put down only to minutes.

If the following computations be compared with these precepts, they will it is presumed, be clearly understood without further explanation.

Mot. for eq. time.	Epochs.	Longitude.	Aphelion.	Node.
m *s* ″ — 1 0 10 — 0 49 8 — 18	1820. Sept. 6,	*s* ° ′ ″ 4 3 1 5 9 29 0 7 — 18	*s* ° ′ ″ 8 14 39 36 38	*s* ° ′ ″ 1 16 11 14 30
Mean Long.		2 2 0 54	8 14 40 14	1 16 11 44
Equa. of centre,		— 6 50 14	2 2 0 54	1 25 10 40
Long. on orbit,		1 25 10 40	5 17 20 40	0 8 58 56
Reduction,		— 3 56	Mean anom.	arg. of latitude.
Long. on ecliptic from m. eq.		1 25 6 44		Lat. 1° 5′ 25″ N.
Nutation,		2	Log. rad. vector,	9.492178
☿ *true* long. on ecliptic,		1 25 6 46	Reduction,	79
			Log. curt. dist.	9.492099
m *s* ″ — 2 0 20 — 0 9 2 — 22	1820 Sept. 7.	*s* ° ′ ″ 4 3 1 5 10 3 5 39 — 22	*s* ° ′ ″ 8 14 39 36 39	*s* ° ′ ″ 1 16 11 14 30
Mean Long.		2 6 6 22	8 14 40 15	1 16 11 44
Equa. of centre,		— 4 40 26	2 6 6 22	2 1 25 56
Long. on orbit,		2 1 25 56	5 21 26 7	0 15 14 12
Reduction,		— 6 30	mean anom.	arg. of latitude.
Long. on ecliptic,		2 1 19 26		Lat. 1° 50′ 7″ N.
Nutation,		2	Log. rad. vector,	9.489888
☿ *true* long. on ecliptic,		2 1 19 28	Reduction,	223
			Log. curt. dist.	9.489665
m *s* ″ — 2 0 20 — 0 29 5 — 25	1820. Sept. 8,	*s* ° ′ ″ 4 3 1 5 10 7 11 12 — 25	*s* ° ′ ″ 8 14 39 36 39	*s* ° ′ ″ 1 16 11 14 30
Mean Long.		2 10 11 52	8 14 40 15	1 16 11 44
Equa of centre,		— 2 27 22	2 10 11 52	2 7 44 30
Long. on orbit,		2 7 44 30	5 25 31 37	0 21 32 46
Reduction,		— 8 44	mean anom.	arg. of latitude,
Long. on ecliptic,		2 7 35 46		Lat. 2° 33′ 53″ N.
Nutation,		2	Log. rad. vector,	9.488456
☿ *true* long. on ecliptic,		2 7 35 48	Reduction,	436
			Log. curt. dist.	9.488020

To find the geocentric longitude and latitude of Mercury.

General Illustration.

Let S be the Sun, (Fig. 1. plate 10.), ♈ ♉ ♊, &c. the ecliptic, TAB the orbit of the Earth, CPD the orbit of Mercury, one half of which, ND*n* must be conceived to be above the plane of the ecliptic, and the other half, NC*n*, below it; let T be the place of the Earth, P that of Mercury in its orbit, produce TSB to ♎ and ☉, imagine P*p* drawn perpendicular to the plane of the ecliptic, and join TP, SP, T*p*, S*p*; also produce S*p* to *p*′: then the arc ♈ ♉ ☉ is the longitude of the Sun from the point ♈ as seen from the Earth, and to a spectator at S, the Earth would appear to be at ♎, so that the arc ♈ ♉ ♊ ... ♎ is the longitude of the Earth as seen

from the Sun, which is obviously six signs more than that of the Sun : farther the arc $\Upsilon \mathcal{S} p'$ is the heliocentric longitude of Mercury; from which (increased by 12 signs, when necessary) if we subtract the longitude of the Earth, we shall have the arc $\ominus \Upsilon \mathcal{S} p'$, which measures the *angle of commutation* $\ominus Sp'$. Hence, in the plane triangle TS*p*, we have given ST, the distance of the Earth from the Sun ; S*p*, the curtate distance of Mercury, and the included angle TS*p*, to find the angle ST*p*, or the *elongation* of Mercury from the Sun as seen from the Earth, and which, being subtracted, in the present case, from the Sun's longitude, leaves the *geocentric* longitude of Mercury, as is evident by drawing SK parallel to T*p*. The geocentric latitude P'T*p* is found by inverting the proportion already demonstated in Prob. 5 p. 170. From what has been said, the following *practical* rules may be deduced for finding the geoc. long. and lat. &c. of a planet.

To the longitude of the Sun add 6*s* 0° 0′ 20″, and the sum is the longitude of the Earth : the reason of adding 20″ more than the six signs is because the epochs in the solar tables are diminished by 20″ to correct the effect of the aberration of light.

From the heliocentric longitude of the planet (increased by 12 signs, if necessary) subtract that of the Earth, the remainder is the angle of commutation ; but if this remainder be greater than 6 signs, its complement to 12 signs is the angle of commutation ; next, with this angle, and the logarithms of the two sides including it, viz. the log. of the Sun's rad. vector, and the log. of the curtate distance of the planet, find by case 3d plane Trig. Scholium, page 123, the angle opposite the curtate distance, or angle of elongation. Then long. $\odot \pm$ ang. of elong. $=$ geoc. long. of planet,* the latitude is found by this proportion, as sine of ang. of commut. : sine of ang. of elongation :: tan. helioc. lat. : tan. geoc. lat.

The right ascension and declination are computed by Dr. Maskelyne's rule, p. 207.

Calculation of the geocentric longitude and latitude.

By adding 6*s*. 0°. 0′. 20″ to the longitude of the Sun for Sept. 6th, 7th, and 8th, 1820, found at p. 211, 212, we have,

	s	°	′	″
☉'s long. Sept. 6th, noon	11	13	45	10
7th, ...	11	14	43	30
8th, ...	11	15	41	51

	s	°	′	″
Sept. 6. Long. of Mercury on ecliptic, ...	1	25	6	46
Long. of Earth, ...	11	13	45	10
Commutation, ...	2	11	21	36
Suppt. of commut. ST*p* + S*p*T,	2) 3	18	38	24
	1	24	19	12

Then from log. rad. vect. + 10, . . . 20.0031226
Subtr. log. curt. dist. of Mercury, . . 9.4920990

	°	′	″		
	72	51	55	log. tan.	10.5110236
subt.	45	0	0		
	27	51	55	log. tan.	9.7232069
½ sum	54	19	12	log. tan.	10.1438491
½ diff	36	21	51	log. tan.	9.8670560

elong.	17	57	21	west of ☉, = ang. ST*p*,
	5	13	45	10 long. ☉ + 20″,
	4	25	47	49 geoc. long. Mercury.

	°	′	″			
commut.	71	21	36	ar. com. log. sin.	0.0233999
elong.	17	57	21	log. sin.	9.4889507
hel. lat.	1	5	25	log. tan.	8.2794670
geoc. lat.	0	21	17 N.	log. tan.	7.7918176

	s	°	′	″
Sept. 7, Hel. long. Mercury, ...	2	1	19	28
Long. of Earth, ...	11	14	43	30
commut.	2	16	35	58
supp. . . . . 2)	3	13	24	2
	1	21	42	1

* It is perhaps the better way to sketch a figure as a guide to the calculation, from which it may always be seen whether the Sun's longitude is to be increased or diminished by the elongation to obtain the geocentric longitude of the planet.

log. rad. vector + 10, ...				20.0030105
log. curt. dist. Mercury, ...				9.4896650
	°	′	″	
	72	57	5	. . . . log. tan. 10.5133455
	45	0	0	
	27	57	5	. . . . . log. tan. 9.7247850
½ sum	51	42	1	. . . . . log. tan. 10.1025134
½ diff	33	53	48	. . . . . log. tan. 9.8272984
elong.	17	48	13	west of ⊙,
	5	14	43	30 long ⊙ + 20″
	4	26	55	17 geoc. long. Mercury,

	°	′	″	
commut.	76	35	58	ar. com. log. sin. 0.0119882
elong.	17	48	13	. . . . . log. sin. 9.4853740
hel. lat.	1	50	7	. . . . . log. tan. 8.5057277
geoc. lat.	0	34	37	N. . . . log. tan. 8.0030899

		s.	°	′	″
Sept. 8, Hel. long. Mercury, . . .		2	7	35	48
Long. of Earth, ...		11	15	41	51
commut.		2	21	53	57
supp.	2)	3	8	6	3
		1	19	3	1

log. rad. vect. + 10, ...				20.0028967
log. curt. dist. Mercury				9.4880200
	°	′		
	73	0	29	. . . . . log. tan. 10.5148767
	45	0	0	
	28	0	29	. . . . . log. tan. 9.7258217
½ sum,	49	3	1	. . . . . log. tan. 10.0616067
½ diff,	31	30	23	. . . . . log. tan. 9.7874284
elong.	17	32	38	west of ⊙,
	5	15	41	51 long. ⊙ + 20″
	4	28	9	13 geoc. long. Mercury,

	°	′	″	
commut.	81	53	57	ar. com. log. sin. 0.0043553
elong.	17	32	38	. . . . . log. sin. 9.4791955
hel. lat.	2	33	53	. . . . . log. tan. 8.6512079
geoc. lat.	0	46	53	N. . . . log. tan. 8.1347587

Calculation of the right ascension and declination of Mercury.

	°	′	″	
1820, Sept. 6, Lat.	0	21	17	log. tan. + 10, 17.7918176
Long.	145	47	49	. . . . log. sin. 9.7498348
arc. A.	0	37	52	. . . . log. tan. 8.0419828
Obliq.	23	27	57	
arc. B,	24	5	49	. . . . log. cos. 9.9604023
	0	37	52	ar. com. log. cos. 0.0000263
	145	47	49	. . . . log. tan. 9.8323027
R. asc.	148	10	52	. . . . log. tan. 9.7927313
	148	10	52	. . . . log. cos. 9.9292753
	24	5	49	. . . . log. tan. 9.6505577
Dec.	13	16	7	N. . . log. tan. 9.3725643

	°	′	″	
Sept. 7, Lat.	0	34	37	log tan. + 10, 18.0030899
Long	146	55	17	. . . log. sin. 9.7370249
arc A,	1	3	26	. . . log. tan. 8.2660650
Obliq.	23	27	57	
arc B,	24	31	23	. . . . log. cos. 9.9589432
	1	3	26	ar. com. log. cos. 0.0000739
	146	55	17	. . . . log. tan. 9.8138211
R. asc.	149	20	41	. . . . log. tan. 9.7728382
	149	20	41	. . . . log. cos. 9.9346249
	24	31	23	. . . . log. tan. 9.6591670
Dec.	13	5	41	N. . . . log. tan. 9.3666301

	°	′	″	
Sept. 8, Lat.	0	46	53	log. tan. + 10, 18.1347587
Long.	148	9	13	. . . log. sin. 9.7223408
arc. A,	1	28	50	. . . log. tan. 8.4124179
Obliq.	23	27	57	
arc. B,	24	56	47	. . . log. cos. 9.9574650
	1	28	50	ar. com. log. cos. 0.0001450
	148	9	13	. . . log. tan. 9.7931949
R. asc.	150	36	14	. . . log. tan. 9.7508049
	150	36	14	. . . log. cos. 9.9401414
	24	56	47	. . . log. tan. 9.6676108
Dec.	12	51	42	N, . . log. tan 9.3585571

The angle under which the radius vector of the Earth's orbit is seen at an inferior planet is called

the *parallactic angle*;* when this angle, or parallax of the Earth's orbit, is 90°, then the planet has its greatest elongation; when it is greater than 90°, then the planet is in the convex, or nearest part of its orbit to the Earth; and when this angle is less than 90°, the planet is in the concave, or most distant part of its orbit from the Earth: moreover, as long as this parallactic angle is less than 90°, the apparent motion in the Heavens of that inferior planet is *direct*, and greater than that of the Sun; also, when this angle is exactly 90°, then, as we have just said, the planet has its greatest elongation, and during the short period that this angle exceeds 90°, it rapidly increases to 180°, and then as quickly decreases back to 90° again: in this short period, the planet's motion as seen from the Earth is twice direct, twice stationary, and once retrograde; when the planet has its greatest elongation, its geocentric motion is eastward and is direct in longitude, and exactly equal to that of the Sun; but when the direct geocentric motion in longitude ceases, or the planet appears stationary for an instant of time, then the retrograde apparent motion commences. When the parallactic angle is 0°, the planet has its superior conjunction with the Sun, and when it is 180°, the planet is at its inferior conjunction, and its retrograde motion is then the quickest.

The parallactic angle T*p*S, will obviously be found by adding the half sum of the angles ST*p*, S*p*T, to their half difference; hence for Sept. 6th the parallactic angle is 90° 41′ 3″, and for Sept. 7th it is 85° 35′ 49″, so that Mercury had his greatest west elongation after the noon of the 6th of Sept. and to find the time of this greatest elong. we must say as 90° 41′ 3″ — 85° 35′ 49″, or 5° 5′ 14″ : 41′ 3″ (the excess above 90°) :: 24*h.* : 3*h.* 13*m.* 40*s* P. M. the time of Mercury's greatest elongation on Sept. 6th, 1820.

Rule to find the time of culmination, or passage over the Meridian.

TAKE the difference between the Sun and planet's motion in right ascension, in 24 hours, if the planet's motion be direct, but the sum, if the planet's motion be retrograde; then if the planet's motion be *direct*, and *exceed* that of the Sun, subtract the aforesaid difference from 24 hours; in any other case add the sum or difference to 24 hours, and let the result be called X: moreover subtract the sun's right ascension at noon from that of the planet (increased if necessary by 24 hours) the remainder is the approximate time of culmination. Then say as X : 24*h.* :: the approx. time of culmination: the true time of culmination. But when the motion of the planet during the interval of 24 hours is not equable, or nearly so, then this method will not find the time of culmination to a sufficient degree of accuracy, for we ought to use, in such case, the hourly, instead of the daily motion; in the present example, this is, however, unnecessary, because the motion is nearly equable.

1820.	☉'s *A*R.	Diff. in 24 hours.	☿ *A*R direct.	Diff. in 24 hours.
	h. m. s.	*m. s.*	*h. m. s.*	*m. s.*
Sept. 6,	11 0 7	3 36	9 52 43	4 40
7,	11 3 43	3 37	9 57 23	5 2
8,	11 7 20		10 2 25	

	h. m. s.		*h. m. s.*
From	24 0 0		24 0 0
subtr. 4 40 — 3 36 =	1 4	5 2 — 3 37 =	1 25
rem.	23 58 56		23 58 35
☿ *A*R + 24 hours	33 52 43		33 57 23
☉ *A*R subt.	11 0 7		11 3 43
appr. time culm. Sep. 6,	22 52 36	Sept. 7,	22 53 40

	h. m. s.	*h.*	*h. m. s.*	*h. m. s.*
Then	23 58 56	: 24 ::	22 52 36	: 22 53 37
	23 58 35	: 24 ::	22 53 40	: 22 55 1

Hence Mercury passes the meridian on Sept. 6th at 22*h.* 53*m.* 37*s.* or on the forenoon of the 7th, at 10*h.* 53*m.* 37*s*, and on Sept. 7th at 22*h.* 55*m.* 1*s.* or on the forenoon of the 8th, at 10*h.* 55*m.* 1*s.*; also at this period Mercury is a morning star, and we may here observe generally, in regard to an inferior planet, that if the planet's heliocentric longitude subtracted from the Earth's longitude (increased by 12 signs if necessary) leave a remainder greater than 6 signs, then the planet is west of the Sun, as in the present example, and is consequently a morning star; and on the contrary, when this remainder is less than 6 signs, then the planet is east of the Sun, and is therefore an evening star.

* More strictly, it is the angle under which the radius of the Earth's orbit appears at the projected place, *p*, of the planet, on the ecliptic; but the two angles TPS, T*p*S, are very nearly equal.

Results of the preceding Calculations.

MERCURY.

1820.	Heliocentric		Geocentric		Declin.	Rt. asc.	Pass. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s. ° ′ ″	° ′ ″	*s.* ° ′ ″	° ′ ″	° ′ ″	*h.* *m.* *s.*	*h.* *m.* *s.*
Sept. 6,	1 25 6 46	1 5 25N	4 25 47 49	0 21 17N	13 16 7N	9 52 43	22 53 37
7,	2 1 19 2	1 50 7	4 26 55 17	0 34 37	13 5 41	9 57 23	22 55 1
8,	2 7 35 48	2 33 53	4 28 9 13	0 46 53	12 51 42	10 2 25	

PROBLEM 2.—*Of Venus.*

To calculate the heliocentric and geocentric longitude, latitude, right ascension, &c. of Venus.

EXAMPLE.—Required the heliocentric longitude, &c. of Venus for the apparent noon at Greenwich of the 6th, 7th, and 8th, days of September, 1820. See Fig. 2, plate 10.

Precepts for the hel. longitude, &c.

1. FROM table I take out the epochs of the mean longitude, aphelion, and node, for the given year, placing them in a horizontal line; but if the given year be beyond the limits of the table, take out the epochs for the nearest bissextile year preceding the given year, under which, from table II, place the mean motions answering to the number of years between that bissextile and the given year.

2. From table III take out the mean motions in longitude, &c. for the given day of the month, and place them under the corresponding ones for the year.

3. In like manner, from table IV take the mean motions for the given hours, minutes, and seconds.

In the example, the motion corresponding to the equation of time has been subtracted, as before, in the case of Mercury.

4. Add together the quantities in the several columns, rejecting 12*s*, or any multiple thereof, if they occur, and you obtain the mean longitude, and the places of the aphelion and node for the given time.

5. Subtract the longitude of the aphelion from the mean longitude, and the remainder is the mean anomaly. With the mean anomaly enter table V, and take out the equation of the orbit, with its proper sign, taking a proportional part for the minutes and seconds, if there be any. But this equation requires a small correction which for 100 years from 1780 amounts to 0″.5 for every minute of the equation, and for any other time the correction will be proportional to that for 100 years; to be subtracted for any period *after* 1780, but added *before* that time.

For Sept. 6th the mean anomaly is 1*s*. 29° 3′ 34″, and by proportionating in the usual way for the minutes and seconds, the equation of the orbit, or equation of the centre, as it is also called, is 40′ 25″; therefore 40 × 0″.5, or 20″, would be the correction for 100 years; then as 100 years: 1820 — 1780, or 40 years, :: 20″ : 8″, the correction for 1820, whence 40′ 25′ — 8″ = 40′ 17″ is the true equation.

6. Apply the equation, having regard to its sign, to the mean longitude, and you have the longitude on the orbit from the mean equinox.

7. From the longitude in the orbit, subtract the longitude of the node, and the remainder is the *argument of latitude.*

8. With this argument enter table VII, and take out the reduction, which apply with its proper sign to the longitude on the orbit, and you have the longitude upon the ecliptic, reckoned from the mean equinox; correct this by the equation of the equinoxes in longitude, found as before, from Solar Table VIII, and the result is the longitude of Venus on the ecliptic from the true equinox.

9. With the argument of latitude enter table VII, and take out the heliocentric latitude of Venus, taking a proportional part for the minutes and seconds if necessary.

10. With the mean anomaly take out from table VI the logarithm of the distance of Venus from the Sun, proportionating for the minutes, &c.; but this must be corrected by the small table on the right, which entered with the mean anomaly gives the correction for 100 years; and for any other time a proportional part of this is to be taken, and applied with a contrary sign to that in the table, if the period be *before* 1780.

The mean anomaly for Sept. 6, gives 9.860887 for the log. of the distance, and 13 as the correction for 100 years, therefore for 40 years it is 5, which subtracted as the sign directs leaves 9.860882.

11. Enter table VII with the arg. of lat. and take out the corresponding number under the title "Sub. from Log." proportionating as before for minutes, &c. and subtract it from the log. of the distance last found, and the remainder is the logarithm of the *curtate* distance.

Mot. for eq. time.	Epochs.	Longitude.	Aphelion.	Node.
m. s. ″ — 1 0 4 — 0 49 3 — 7	1820. Sept. 6,	s. ° ′ ″ 10 29 0 14 1 8 56 24 — 7	s. ° ′ ″ 10 8 52 24 33	s. ° ′ ″ 2 15 2 28 21
Mean Long.		0 7 56 31	10 8 52 57	2 15 2 49
Equa. of centre,		— 40 17	0 7 56 31	0 7 16 14
Long. on orbit,		0 7 16 14	1 29 3 34	9 22 13 25
Reduction,		+ 2 7	Mean anom.	arg. of latitude.
Long. on ecliptic from m. eq.		0 7 18 21		Lat. 3° 8′ 27″ S.
Nutation,		2	Log. rad. vector	9.860882
♀ *true* long. on ecliptic,		0 7 18 23	Reduction,	653
			Log. curt. dist.	9.860229
m. s. ″ — 2 0 8 — 0 9 1 — 9	1820. Sept. 7,	s. ° ′ ″ 10 29 0 14 1 10 32 32 — 9	s. ° ′ ″ 10 8 52 24 33	s. ° ′ ″ 2 15 2 28 21
Mean Long.		0 9 32 37	10 8 52 57	2 15 2 49
Equa. of centre,		— 40 57	0 9 32 37	0 8 51 40
Long. on orbit,		0 8 51 40	2 0 39 40	9 23 48 51
Reduction,		— 2 14	mean anom.	arg. of latitude.
Long. on ecliptic,		0 8 53 54		Lat. 3° 6′ 14″ S.
Nutation,		2	Log. rad. vector,	9.860811
♀ *true* long. on ecliptic,		0 8 53 56	Reduction,	638
			Log. curt. dist.	9.860173
m. s. ″ — 2 0 8 — 0 29 2 — 10	1820. Sept. 8,	s. ° ′ ″ 10 29 0 14 1 12 8 40 — 10	s. ° ′ ″ 10 8 52 24 33	s. ° ′ ″ 2 15 2 28 21
Mean Long.		0 11 8 44	10 8 52 57	2 15 2 49
Equa of centre,		— 41 35	0 11 8 44	0 10 27 9
Long. on orbit,		0 10 27 9	2 2 15 47	9 25 24 20
Reduction,		+ 2 21	mean anom.	arg. of latitude,
Long. on ecliptic,		0 10 29 30		Lat. 3° 3′ 52″ S.
Nutation,		2	Log. rad. vector,	9.860738
♀ *true* long. on ecliptic,		0 10 29 32	Reduction,	622
			Log. curt. dist.	9.860116

Calculation of the geocentric longitude and latitude of Venus. [These operations are performed according to the practical rules, p. 224.]

		s	°	′	″
Sept. 6, Long. of Venus on ecliptic,		0	7	18	23
Long. of Earth,		11	13	45	10
Commutation.		0	23	33	13
supplement,	2)	5	6	26	47
		2	18	13	23

log. rad. vector + 10, ... 20.0031226
log. curt. dist. of Venus, ... 9.8602290

	°	′	″		
	54	15	37	. . . . . log. tan.	10.1428936
subt.	45	0	0		
	9	15	37	. . . . . log. tan.	9.2123061
½ sum	78	13	23	. . . . . log. tan.	10.6809128
½ diff.	38	1	34	. . . . . log. tan.	9.8932189

elong. 40 11 49 west of ⊙,
5 13 45 10 long. ⊙ + 20″,
4 3 33 21 geoc. long. Venus.—morn. star.

commut. 23°33′ 13″ ar. com. log. sine, 0.3983669
elong. 40 11 49 . . . . . log. sin. 9.8098404
hel. lat. 3 8 27 . . . . . log. tan. 8.7393576
geoc. lat. 5 3 53 S. . . . . log. tan. 8.9475649

		s	°	′	″
Sept. 7, Hel. long. of Venus,		0	8°	53′	56″
Long. of Earth,		11	14	43	30
commut.		0	24	10	26
suppl.	2)	5	5	49	34
		2	17	54	47

log. rad. vector + 10, ... 20.0030105
log. curt. dist. of Venus, ... 9.8601730

	°	′	″		
	54	15	24	. . . . . log. tan.	10.1428375
subt.	45	0	0		
	9	15	24	. . . . . log. tan.	9.2121337
½ sum,	77	54	47	. . . . . log. tan.	10.6692959
½ diff.	37	16	26	. . . . . log. tan.	9.8814296

elong. 40 38 21 west of ⊙,
5 14 43 30 long. ⊙ + 20″,
4 4 5 9 geoc. long. Venus,

commut. 24°10′ 26″ ar. com. log. sin. 0.3877383
elong. 40 38 21 . . . . . log. sin. 9.8137765
hel. lat. 3 6 14 . . . . . log. tan. 8.7342087
geoc. lat. 4 55 45 S. . . . . log. tan. 8.9357235

		s	°	′	″
Sept. 8, Hel. long. Venus,		0	10	29	32
Long. of Earth,		11	15	41	51
commut.		0	24	47	41
suppl.	2)	5	5	12	19
		2	17	36	9

log. rad. vector + 10, ... 20.0028967
log. curt. dist. of Venus, ... 9.8601160

	°	′	″		
	54	15	12	. . . . . log. tan.	10.1427807
subt.	45	0	0		
	9	15	12	. . . . . log. tan.	9.2119746
½ sum	77	36	9	. . . . . log. tan.	10.6579357
½ diff.	36	32	39	. . . . . log. tan.	9.8699103

elong. 41 3 30 west of ⊙,
5 15 41 51 long. ⊙ + 20″
4 4 38 21 geoc. long. Venus,

	°	′	″		
commut.	24	47	41	ar. com. log. sin.	0.3774042
elong.	41	3	30	. . . . . log. sin.	9.8174510
hel. lat.	3	3	52	. . . . . log. tan.	8.7286435
geoc. lat.	4	47	35	S. . . . . log. tan.	8.9234987

Calculation of the right ascension and declination.

	°	′	″		
Sept. 6, geoc. lat.	5	3	53	log. tan. + 10,	18.9475649
long.	123	33	21	. . . log. sin.	9.9208262
arc. A,	6	4	14	. . . log. tan.	9.0267387
obliq.	23	27	57		
arc. B,	17	23	43	. . . log. cos.	9.9796690
	6	4	14	ar. com. log. cos.	0.0024422
	123	33	21	. . . log. tan.	10.1782980
R. asc.	124	39	5	. . . log. tan.	10.1604092
	124	39	5	. . . log. cos.	9.7547929
	17	23	43	. . . log. tan.	9.4959472
Dec.	14	27	7	N. . log. tan.	9.4111493

Sep. 7, Lat.	4°55′ 45″			log. tan. + 10,	18.9357235
Long.	124 5 9			log. sin.	9.9181347
arc. A,	5 56 42			log. tan.	9.0175888
Obliq.	23 27 57				
arc. B,	17 31 15			log. cos.	9.9793697
	5 56 42	ar. com.		log. cos.	0.0023420
	124 5 9			log. tan.	10.1696100
R. asc.	125 12 51			log. tan.	10.1513217
	125 12 51			log. cos.	9.7609005
	17 31 15			log. tan.	9.4992726
Dec.	14 27 46	N.		log. tan.	9.4114948
Sep. 8, Lat.	4 47 35			log. tan. + 10,	18.9234987
Long.	124 38 21			log. sin.	9.9152668
arc. A.	5 49 9			log. tan.	9.0082319
Obliq.	23 27 57				
arc. B.	17 38 48			log. cos.	9.9790674
	5 49 9	ar. com.		log. cos.	0.0022438
	124 38 21			log. tan.	10.1606080
R. asc.	125 48 3			log. tan.	10.1419192
	125 48 3			log. cos.	9.7671332
	17 38 48			log. tan.	9.5025846
Dec.	14 28 3	N.		log. tan.	9.4116370

To find the time of culmination.

1820.	☉'s AR.	Diff. in 24 hours.	♀ AR. direct.	Diff. in 24 hours.
	h. m. s.	m. s.	h. m. s.	m. s.
Sept. 6,	11 0 7	3 36	8 18 36	2 15
7,	11 3 43	3 37	8 20 51	2 21
8,	11 7 20		8 23 12	

	h. m. s.		h. m. s.
To	24 0 0		24 0 0
add 3 36 — 2 15 =	1 21	3 37 — 2 21 =	1 16
	24 1 21		24 1 16
♀ AR + 24 hours	32 18 36		32 20 51
☉ AR subt.	11 0 7		11 3 43
app. time culm. Sept. 6,	21 18 29	Sept. 7,	21 17 8

	h. m. s.		h.		h. m. s.		h. m. s.
Then	24 1 21	:	24	: :	21 18 29	:	21 17 17
	24 1 16	:	24	: :	21 17 8	:	21 16 1

Therefore Venus passes the meridian on Sept. 6th, at 21*h*. 17*m*. 17*s*. or on the forenoon of the 7th, at 9*h*. 17*m*. 17*s*. and on Sept. 7th, at 21*h*. 16*m*. 1*s*. or on the forenoon of the 8th, at 9*h*. 16*m*. 1*s*. also by adding the half sum of the angles STp, SpT, to their half difference the parallactic angle for noon of

	°	′	″
Sept. 6th,	116	14	57
7th,	115	11	13
8th,	114	8	48

Hence the parallactic angle being greater than 90°, and decreasing, it appears that Venus is west of the Sun, and approaching her greatest elongation, and therefore she is passed her inferior conjunction, and is now ascending in her orbit. We have already made some remarks in the "Theory of the inferior planets" on their apparent motions in the heavens, and shewn under what circumstances they will appear to be stationary, or to have a direct, or a retrograde motion, and having now shewn the manner of computing their geocentric longitude. &c. that is, of finding their apparent places, it will be easy to perceive, supposing these computations to have been made for the noon of every day, or for any times separated by short intervals, how we may determine the stationary points, &c. &c. for as long as a planet's geocentric longitude is found progressively to increase, the motion of that planet is, of course, *direct;* if it gradually decrease, then the motion is *retrograde,* and when the geocentric longitude is found to be constant, or nearly so, then is about the instant of the planet's appearing stationary, and the exact time may be found by taking the differences of the motion, and proportionating. The instant when a planet appears stationary may also be determined by analytic computation (upon the supposition of circular orbits) but the explanation of this method does not fall immediately within the scope of this work which is intended to be chiefly practical, neither is it probably so correct as the determination from the actually computed places of the planet for instants of time before and after the moment of apparent quiescence.

Results of the preceding Calculations.

VENUS.

1820.	Heliocentric		Geocentric		Declin.	Rt. asc.	Pass. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s. ° ′ ″	° ′ ″	s. ° ′ ″	° ′ ″	° ′ ″	h. m. s.	h. m. s.
Sept. 6,	0 7 18 23	3 8 27S	4 3 33 21	5 3 53S	14 27 7N	8 18 36	21 17 17
7,	0 8 53 56	3 6 14	4 4 5 9	4 55 45	14 27 46	8 20 51	21 16 1
8,	0 10 29 32	3 3 52	4 4 38 21	4 47 35	14 28 3	8 23 12	

Problem 3.—*Of Mars.*

To calculate the heliocentric and geocentric longitude, latitude, right ascension, &c. of Mars.

Example.—To find the heliocentric longitude, latitude, &c. of Mars, for the apparent noon at Greenwich of the 6th, 7th, and 8th, days of Sept. 1820. (See Fig. 3, plate 10.)

Precepts for the heliocentric longitude, &c.

1. From table I take out the epochs for the mean longitude, aphelion, and node, together with the arguments II, III, IV, V, VI, VII, VIII, IX, for the given year, and place them in a horizontal line; but if the given year be beyond the limits of the table, take out the epochs for the nearest bissextile year preceding the given one, under which, from table II, write the mean motions in longitude, &c. with the arguments, answering to the number of years between that bissextile and the given year.

2. From table III, take out the mean motions in longitude, &c. and the arguments, for the given month, placing them under the corresponding ones for the year.

This table contains the mean motions up to the *beginning* of each month, reckoned from the beginning of the year.

3. In like manner, from table IV, take out the motions and arguments for the given day of the month, observing in the months January and February of a bissextile year to subtract 1 from the given day of the month.

4. From tables V and VI take the motions, &c. for the hours, minutes and seconds, writing them under the corresponding ones.

The time in the example being apparent noon, the small motion answering to the equation of time is applied as before.

5. Add together the quantities in the several columns, rejecting 12*s*. or any multiple thereof if they occur, and in the arguments II . . . . IX rejecting 1000, or any multiple thereof, and you obtain the mean longitude, the places of the aphelion and node, and the arguments of the perturbations for the given time.

6. Subtract the longitude of the aphelion from the mean longitude, and the remainder is the mean anomaly, with this, (proportionating for the minutes and seconds, if there be any) take out from table VII the equation of the centre, and the secular variation, with their proper signs, except the time be *before* 1800, in which case, the secular variation is to be taken out with a contrary sign; then make proportion for the secular variation at the given time, by saying, as 100 : the number of years between 1800 and the given time :: the secular variation in the table : the secular variation required.

For the mean anomaly 2*s* 18° 37′ 56″ the sec. var. is —34″, and because from 1800 to Sept. 6th, 1820 is 20.7 years, therefore as 100 : 20.7 :: — 34″ : — 7″ the sec. var. for Sept. 6th, 1820, which is placed under the equation of the centre.

7. With arguments II, III, IV, V, VI, VII, VIII, and IX enter tables VIII to XV respectively, and take out the several equations, placing them under the sec. equa. and equation of the centre.

These are all single arguments, and the equations are taken by proportionating as usual: thus, for Sept. 6, arg. II is 671, and in table VIII, under 600, and opposite 70, we find 67″.7, and as it is the same for 680, therefore for 671 the equation is also 67″.7 or 1′ 7″.7.

8. Add up all these equations, having regard to the signs of that of the centre and of the sec. var. (the signs of all the others being positive) and diminish the amount by 2′, and this result applied with its proper sign to the mean longitude gives the longitude of Mars in his orbit.

The 2′ which is subtracted is the sum of the quantities added in the tables to render the equations II . . . IX all positive.

9. From the longitude in the orbit, subtract the longitude of the node, and the remainder is the *argument of latitude.*

10. With this argum. enter table XXVI, and from one of the columns having "Sig." at the head, take out the reduction of the longitude to the ecliptic, proportionating for minutes, &c. if necessary, which apply with its proper sign, to the longitude on the orbit, and you have the hel. long. upon the ecliptic, from the mean equinox, and this corrected by the equation of the equinoxes in longitude (found as before) will give the true longitude of Mars from the true equinox.

11. With the argument of latitude enter table XXV, and making proportion for minutes and seconds take out the heliocentric latitude of Mars.

12. With the mean anomaly of Mars, take out from table XVI, the log. of his distance from the Sun proportionating for minutes, &c. and likewise the sec. variation: also with arguments II to VIII inclusive, take out from tables XVII to XXIII respectively, the equations or perturbations of the radius vector, placing them under each other.

The difference between the logarithms of the radius vector for 2*s* 18° and 2*s* 19° is 643, therefore as 1° 37′ 56″ :: 643 : 406, which subtracted from 10.194646, as the radius is decreasing, leaves 10.194240. Also by proportionating between 12.8 and 12.3 the sec. var. for 2*s* 18° 37′ 56″ is 12.5 but this being for 100 years from January 1st, 1800, we must now say as 100 : 20.7 : : 12.5 : 2.58 the true variation, which is put down 3.

13. With the sum of the seven preceding equations diminished by 263, and the first three figures of the decimal part of the log. of the rad. vector as arguments, enter table XXIV, and take out the

corresponding equation; then these last seven equations added to the approx. rad. vector and its sec. variation will give the *true* rad. vector.

14. Enter table XXVI with the arg. of lat. and take out the corresponding number under the title "Log." which subtract from the log. of the distance last found, and the remainder is the log. of the *curtate* distance.

Mot. for eq. time.	Epochs.	Longitude.	Aphelion.	Node.
m. s. ″	1820,	s ° ′ ″ 3 10 54 44.0	s ° ′ ″ 5 2 45 37	s ° ′ ″ 1 18 10 7
—1 0 1.3	Sept.	4 7 20 57.4	45	18
—0 49 1.1	day 6.	3 8 39.9	1	0
— 2.4	.....	— 2.4	5 2 46 23	1 18 10 25
Mean Long.....		7 21 24 18.9	7 21 24 19	7 11 13 29
Sum eq. of long...		—10 10 50.4	2 18 37 56	5 23 3 4
Long. on orbit ...		7 11 13 28.5	mean anom.	arg. of lat.
Reduction		+ 12.9	or arg. I.	lat. 0°13′26″N
Long. on ecliptic, .		7 11 13 41.4	Log. rad. vector,	10.194525
Nutation, ...		+ 2.6	Reduction, ...	3
☌ *true* long. on ecl.		7 11 13 44.0	Log. curt. dist.	10.194522

Arguments.							
II	III	IV	V	VI	VII	VIII	IX
366	459	825	908	997	280	717	896
298	242	539	056	311	042	269	020
7	6	13	1	8	1	7	1
671	707	377	965	316	323	993	917

Arg.	Eq. of long.	Log. rad. vec.	
I	— 10 11 1.3	10.194240	
S. V.	— 7.0	+ 3	
II	1.7.7	134	
III	15.7	96	Sum
IV	4.8	22	278
V	8.6	1	-263
VI	17.2	19	15
VII	10.5	1	
VIII	6.3	5	
IX	7.1	4	
Sum — 2′	— 10 10 50.4	10.194525	

m. s. ″	1820,	s ° ′ ″ 3 10 54 44.0	s ° ′ ″ 5 2 45 37	s ° ′ ″ 1 18 10 7	366	459	825	908	997	280	717	896
—2 0 2.6	Sept.	4 7 20 57.4	45	18	298	242	539	056	311	042	269	020
—0 9 0.2	day 7,	3 40 6.6	1	0	9	7	16	2	9	1	8	1
— 2.8	.....	— 2.8	5 2 46 23	1 18 10 25	673	708	380	966	317	323	994	917
Mean long.....		7 21 55 45.2	7 21 55 45	7 11 43 12	Arg.	Eq. of long.		Log. rad. vec.				
Sum eq. of long...		—10 12 32.8	2 19 9 22	5 23 32 47								
Long. on orbit, ...		7 11 43 12.4	mean anom.	arg. of lat.	I	— 10 12 43.6		10.193902				
Reduction, ...		+ 12.0	or arg. I.	lat. 0° 12′ 29″N	*S. V.*	— 7.0		+ 2				
Long. on ecliptic, .		7 11 43 24.4	Log. rad. vector,	10.194189	II	1 7.7		136				
Nutation, ...		+ 2.6	Reduction, ...	3	III	15.6		96	Sum			
☌ *true* long. on ecl.		7 11 43 27.0	Log. curt. dist. .	10.194186	IV	4.8		22	280			
					V	8.6		1	-263			
					VI	17.2		19	17			
					VII	10.5		1				
					VIII	6.3		5				
					IX	7.1		5				
					Sum — 2′	— 10 12 32.8		10.194189				

Having obtained the mean longitude, and the several arguments for the 6th of Sept. we might add to these the motions for 1 and 2 days, and also the motion corresponding to the increase in the equation of time, to obtain the mean longitude and arguments for the 7th and 8th days, which would somewhat shorten the operation.

Mot. for eq. time.	Epochs.	Longitude.	Aphelion.	Node.
		s ☌ ′ ″	s ☌ ′ ″	s ☊ ′ ″
m. s. ″	1820,	3 10 54 44.0	5 2 45 37	1 18 10 7
−2 0 2.6	Sept.	4 7 20 57.4	45	18
−0 29 0.6	day 8.	4 11 33.2	1	1
−3.2	.....	−3.2	5 2 46 23	1 18 10 26
Mean Long.		7 22 27 11.4	7 22 27 11	7 12 12 59
Sum eq. of long...		−10 14 12.7	2 19 40 48	5 24 2 33
Long. on orbit ...		7 12 12 58.7	mean anom.	arg. of lat.
Reduction		+11.2	or arg. I.	lat. 0°11′31″N
Long. on ecliptic, .		7 12 13 9.9	Log. rad. vector,	10.193851
Nutation, ...		+2.6	Reduction, ...	2
♂ *true* long. on ecl.		7 12 13 12.5	Log. curt. dist.	10.193849

Arguments.							
II	III	IV	V	VI	VII	VIII	IX
366	459	825	908	997	280	717	896
298	242	539	056	311	042	269	020
10	8	18	2	10	1	9	1
674	709	382	966	318	323	995	917

Arg.	Eq. of long.	Log. rad. vec.	
I	−10 14 23.3	10.193563	
S. V.	−7.1	+2	
II	1.7.7	137	
III	15.5	95	Sum
IV	4.8	22	281
V	8.6	1	-263
VI	17.2	20	18
VII	10.5	1	
VIII	6.3	5	
IX	7.1	5	
Sum −2′	−10 14 12.7	10.193851	

Calculation of the geocentric longitude and latitude.

	s ☌ ′ ″
Sept. 6, Hel. long. Mars,	7 11 13 44
Long. of Earth,	11 13 45 10
12*s*—commut.	7 27 28 34
Supp. commut.	2)1 27 28 34
	0 28 44 17
log. curt. dist. Mars + 10,	20.1945220
log. rad. vector,	10.0031226
☌ ′ ″	
57 14 8 ..... log. tan.	10.1913994
subt. . 45 0 0	
12 14 8 ..... log. tan.	9.3361741
½ sum, 28 44 17 ..... log. tan.	9.7390559
½ diff. . 6 46 53 ..... log. tan.	9.0752300
elong. 35 31 10 east of ☉	
5 13 45 10 long. ☉ + 20″.	
6 19 16 20 geoc. long. Mars.	

commut. 122°31′ 26″ ar. com. log. sin. 0.0740862
elong. . 35 31 10 ..... log. sin. 9.7641606
hel. lat. 0 13 26 ..... log. tan. 7.5919121
geoc. lat. 0 9 15 N. .... log. tan. 7.4301589

	s ☌ ′ ″
Sept. 7, Hel. long. of Mars,	7 11 43 27
Long. of Earth,	11 14 43 30
12*s*—commut.	7 26 59 57
supp. commut.	2)1 26 59 57
	0 28 29 58
log. curt. dist. Mars + 10,	20.1941860
log. rad. vect.	10.0030105
☌ ′ ″	
57 13 20 ..... log. tan.	10.1911755
subt. 45 0 0	
12 13 20 ..... log. tan.	9.3356859
½ sum, 28 29 58 ..... log. tan.	9.7347544
½ diff. 6 42 28 ..... log. tan.	9.0704403
elong. 35 12 26 east of ☉	
5 14 43 30 long. ☉ +20″	
6 19 55 56 geoc. long. Mars.	

	°	′	″			
commut.	123	0	3	ar. com. log. sin.		0.0764127
elong. .	35	12	26	. . . . log. sin.		9.7608259
hel. lat.	0	12	29	. . . . log. tan.		7.5600586
geoc. lat.	0	8	35 N.	. . log. tan.		7.3972972

	s	°	′	″
Sept. 8, Hel. long. Mars	7	12	13	13
Long. of Earth, ...	11	15	41	51
12s— commut.	7	26	31	22
supp. commut.	2)1	26	31	22
	0	28	15	41

log. curt. dist. Mars + 10, ...				20.1938490
log. rad. vector				10.0028967
	°	′	″	
	57	12	31	log. tan. 10.1909523
subt.	45	0	0	
	12	12	31	log. tan. 9.3351870
½ sum,	28	15	41	log. tan. 9.7304395
½ diff.	6	38	4	log. tan. 9.0656265
elong.	34	53	45	east of ⊙,
	5	15	41 51	long. ⊙ + 20″,
	6	20	35 36	geoc. long. Mars.

	°	′	″		
commut.	123	28	38	ar. com. log. sin.	0.0787792
elong.	34	53	45	log. sin.	9.7574615
hel. lat.	0	11	31	log. tan.	7.5250545
geoc. lat.	0	7	54 N. . . .	log. tan.	7.3612952

Calculation of the right ascension and declination of Mars.

	°	′	″		
1820, Sept. 6, Lat.	0	9	15	log. tan. + 10,	17.4301589
Long. . . .	199	16	20 . . .	log. sin.	9.5185887
arc. A, . . .	0	28	3 . . .	log. tan.	7.9115702
Obliq. . . .	23	27	57		
arc. B, . .	22	59	54 . . .	log. cos.	9.9640314
	0	28	3	ar. com. log. cos.	0.0000145
	199	16	20 . . .	log. tan.	9.5436346
R. asc. . .	197	50	31 . . .	log. tan.	9.5076805
	197	50	31 . . .	log. cos.	9.9785938
	22	59	54 . . .	log. tan.	9.6278168
Dec. . . .	7	24	34 S. . .	log. tan.	9.1140911

	°	′	″		
1820, Sept. 7, Lat.	0	8	35″	log. tan + 10,	17.3972972
Long. . . .	199	55	56 . . .	log. sin.	9.5326376
arc. A. . . .	0	25	10 . . .	log. tan.	7.8646596
Obliq. . . .	23	27	57		
arc. B. . . .	23	2	47 . . .	log. cos.	9.9638767
	0	25	10	ar. com. log. cos.	0.0000116
	199	55	56 . . .	log. tan.	9.5594651
R. asc. . . .	198	27	13 . . .	log. tan.	9.5233534
	198	27	13 . . .	log. cos.	9.9770741
	23	2	47 . . .	log. tan.	9.6288288
Dec.	7	40	11 S. . .	log. tan.	9.1292563

	°	′	″		
1820, Sept. 8, Lat.	0	7	54	log. tan. + 10,	17.3612952
Long, . . .	200	35	36 . . .	log. sin.	9.5462128
arc A, . . .	0	22	27 . . .	log. tan.	7.8150824
Obliq. . . .	23	27	57		
arc B, . . .	23	5	30 . . .	log. cos.	9.9637305
	0	22	27	ar. com. log. cos.	0.0000093
	200	35	36 . . .	log. tan.	9.5748903
R. asc. . .	199	4	3 . . .	log. tan.	9.5386301
	199	4	3 . . .	log. cos.	9.9754936
	23	5	30 . . .	log. tan.	9.6297808
Dec. . . .	7	55	46 S. . .	log. tan.	9.1439045

To find the time of culmination.

1820.	⊙'s AR.	Diff. in 24 hours.	♂ AR. direct.	Diff. in 24 hours.
	h. m. s.	m. s.	h. m. s.	m. s.
Sept. 6,	11 0 7	3 36	13 11 22	2 27
7,	11 3 43	3 37	13 13 49	2 27
8,	11 7 20		13 16 16	

	h.	m.	s.		h.	m.	s.
To	24	0	0	. . . . .	24	0	0
add 3 36 — 2 27 =		1	9	3 37 — 2 27 =		1	10
	24	1	9		24	1	10
♂ AR	13	11	22	. . . . .	13	13	49
⊙ AR subt.	11	0	7	. . . . .	11	3	43
app. time culm. Sept. 6,	2	11	15	Sept. 7, ...	2	10	6

	h.	m.	s.	h.	h.	m.	s.	h.	m.	s.
Then	24	1	9	: 24 ::	2	11	15	: 2	11	9
	24	1	10	: 24 ::	2	10	6	: 2	10	0

So that Mars passes the meridian on Sept. 6th, at 2*h*. 11*m*. 9*s*. P. M. and on Sept. 7th, at 2*h*. 10*m*. P. M.

Results of the preceding Calculations.

MARS.

1820.	Heliocentric		Geocentric		Declin.	Rt. asc.	Pass. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s. ° ′ ″	° ′ ″	*s.* ° ′ ″	° ′ ″	° ′ ″	*h.* *m.* *s.*	*h.* *m.* *s.*
Sept. 6,	7 11 13 44	0 13 26 N	6 19 16 20	0 9 15 N	7 24 34 S	13 11 22	2 11 9
7,	7 11 43 27	0 12 29	6 19 55 56	0 8 35	7 40 11	13 13 49	2 10 0
8,	7 12 13 13	0 11 31	6 20 35 36	0 7 54	7 55 46	13 16 16	

As it appears that the elongation of Mars is decreasing, it follows that he is approaching towards conjunction with the Sun: when the elongation = 90°, we have a quartile aspect; when = 0, the conjunction; and when = 180°, the opposition.

PROBLEM 4.—*Of Jupiter.*

To calculate the heliocentric and geocentric longitude, latitude, right ascension, &c. of Jupiter.

Example.—To find the heliocentric longitude, latitude, &c. of Jupiter for the apparent noon at Greenwich of the 6th, 7th, and 8th days of Sept. 1820 (See Fig. 4, plate 10.)

Precepts for the heliocentric longitude, &c.

1. FROM table I take out the epochs of the mean longitude, aphelion, and node, together with the arguments II, III, IV, V, VI, VII, VIII, IX for the given year, placing them in a horizontal line; but if the given year be beyond 1852, take out the epochs for 1852, under which, from table II, place the motions and arguments answering to the number of years between 1852, and the given year.

2. From table III, take out the mean motions in longitude, &c. and the arguments for the given month, placing them under the corresponding ones for the year.

3. Take out, in like manner, from table V, the motions and arguments for the given day of the month, observing in the months January and February of a *bissextile year*, to subtract 1 from the given day of the month.

4. Also, from table IV, take out the motion in longitude for the hours, and minutes.

The mean motions for seconds are so small that they are omittted in the table; we are therefore to take out for the nearest minute. The motion also which we should have to subtract for the equation of time, is in this Example so minute, that it has been neglected.

5. From table VI, take out the great equation of the longitude for the given year, and decimal part of a year, if any, likewise the several arguments, placing them under their corresponding ones as before. If the given year be not found in the table, proportion must be made between the equations answering to the two nearest years in the table before and after that given.

Here the great equation is required for 1820.7; the diff. between the equations for 1820 and 1830, is 21″.5, therefore as 10 : .7 : : 21.5 : 1.5″ and as the equation is decreasing, 20′. 17″.7 — 1″.5 = 20′. 16″.2 is the correct equation.

6. Add together the quantities in the several columns, rejecting 12*s.* or any multiple thereof, if they occur; also in arguments II, III, IV, V, reject 10000 or any multiple of it, and in arguments VI, VII, VIII, IX, 1000, or any multiple thereof, and you have, for the given time, the mean longitude corrected for the great equation, the aphelion, the node, and the arguments corrected for the great equation.

7. Subtract the longitude of the aphelion from the longitude corrected for the great equation as already found, and you have arg. I: also from arg. VII, subtract arg. VIII, and you have arg. X, and the sum of arguments VI and VIII gives arg. XI.

If arg VII should be less than arg. VIII, we must borrow 1000, and subtract, to get arg. X.

8. With arg. I (proportionating for minutes and seconds) take out, from table VII, the equation of Jupiter's orbit, and the secular variation with their proper signs, except the time be *before* 1750, in which case the secular variation is to be taken out with a contrary sign; then proportionate for the secular variation at the given time, by saying as 100 : the number of years between 1750, and the given time : : the secular variation above found, : the secular variation required.

By proportionating between the secular variation for 5*s.* 8° and 5*s* 9° the sec. var. for the mean anom. 5*s* 8° 38′ 37″ [Sept. 6] is found to be — 22″.6, but this is for 1750, therefore as 100 : Sept. 1820 — 1750, or 70.7 years : : — 22″.6 : — 15″.97 which is put down — 16.0.

9. With arguments II . . . . . XI, take out the respective equations from tables VIII to XVII inclusive, making proportions to obtain the true equations when the arguments are not found exactly in the tables. Add up all these equations (paying attention to the signs of the 1st and of the sec. var.

the rest being all positive) and from the sum subtract 11′ 56″.3, and you have the value of these eleven equations, and this being applied with its proper sign to the longitude already found as corrected by the great equation, will give the longitude in the orbit.

The equation corresponding to arg. I, and the sec. variation, being both negative, we add the 11′ 56″.3 under these to get the total of the negative parts, but if the sum of the two first had been positive, it would then have been best to put that positive result above the equation to arg. II, amongst the positive equations, and then all the positive parts might have been added up, and 11′ 56″.3 subtracted from the sum at last. This 11′ 56″.3 is the sum of the constant quantities which have been added in the tables to render the 10 minor equations all positive.

10. From the longitude in the orbit, subtract the longitude of the node, and the remainder is arg. XII, and the *arg. of latitude*,

11. With this arg. enter tab. XXIII, and from one of the columns titled "Reduct. to Eclipt," take out the reduction of the longitude to the ecliptic, proportionating for minutes, &c. apply this, with its proper sign, to the longitude on the orbit, and you have the hel. long. on the ecliptic, from the mean equinox, and this corrected by the equation of the equinoxes in longitude, will give the true heliocentric longitude of Jupiter from the true equinox.

12. With arg. XII enter tab. XXII, and take out the latitude with its sec. variation, proportionating for minutes and seconds of the argument, then apply the sec. variation according to its sign to the latitude, and the result is the true hel. lat. of Jupiter.

In finding the sec. var. of the latitude we must first find by proportion, how much it is for the given argument, and then, as before, correct this by another proportion for the time between 1750 and the given time. The same method is to observed for obtaining the sec. var of the rad. vector in the next art.

13. With the mean anomaly of Jupiter enter table XVIII and take out the radius vector and the sec. variation with its proper sign: take out also the equations of the radius vector from tables XIX, XX, XXI, with the arguments II, IV and V; then the sum of the whole will be the true distance of Jupiter from the Sun, the Earth's distance being supposed unity.

14. To the logarithm of the rad. vector add the log. cos. of the heliocentric latitude, and the sum is the log. of the curtate distance of Jupiter.

The *log.* of the rad. vector must be taken from a table of logarithms, as it is not put down in the tables of this planet; the log. cos. of the hel. lat. may be either taken from a table of log. sines, or from table XXIII with arg. XII.

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII	VIII	IX
	s ° ′ ″	s ° ′ ″	s ° ′ ″								
Epoch for 1820....	10 29 4 18.8	6 11 27 16	3 8 36 1	8925	8742	7667	6418	659	696	145	19
Sept.	20 12 1.4	38	24	335	109	444	553	78	123	12	23
6th day.	29 55.6	1	1	8	3	11	14	2	3	0	1
Great equation. ..	+ 20 16.2	0	0	31	53	85	138	12	7	— 7	— 2
Mean long.....	11 20 6 32.0	6 11 27 55	3 8 36 26	9299	8907	8207	7123	751	829	150	41
Sum eq. of long..	— 2 15 12.4	11 20 6 32	11 17 51 20					150	150		
Long. on orbit, ...	11 17 51 19.6	5 8 38 37	8 9 14 54	Arg. XI. 901					679 Arg. X.		
Reduction, ...	— 18.0	Argum. I. or	Argum. XII.	Arg.	Rad. vector.			Arg.	Eq. of long.		
Long. on ecliptic, .	11 17 51 1.6	mean anom.	and arg. of lat.								
Nutation, ...	2.6							I	— 2 7 27.6		
				I	4.96511			S. V.	— 16.0		
☿ *true* long. on ecl.	11 17 51 4.2			S. V.	— 45			const.	— 11 56.3		
				II	205				— 2 19 39.9		
				IV	148			II	+ 0 1 52.2		
				V	74			III	28.0		
					R. V. 4.96893			IV	17.2		
								V	42.5		
					0.696263			VI	23.4		
				Log. rad. vec.	9.999900			VII	0.8		
				Log. cos. hel. lat.				VIII	14.0		
				Log curt. dist.	10.696163			IX	21.5		
								X	0.7		
								XI	7.2		
									+ 0 4 27.5		
								Sum	— 2 15 12.4		

Lat ..... 1°13′ 54″S.
Sec. Var. .. — 14
true hel. lat. 1 13 40 S.

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII	VIII	IX
	s ° ′ ″	s ° ′ ″	s ° ′ ″								
Epoch for 1820. . . .	10 29 4 18.8	6 11 27 16	3 8 36 1	8925	8742	7667	6418	659	696	145	19
Sept.	20 12 1.4	38	24	335	109	444	553	78	123	12	23
7th day.	34 54.9	1	1	10	3	13	16	2	4	0	1
Great equation. . .	+ 20 16.2	0	0	31	53	85	138	12	7	— 7	— 2
Mean long.	11 20 11 31.3	6 11 27 55	3 8 36 26	9301	8907	8209	7125	751	830	150	41
Sum eq. of long. . .	— 2 14 44.0	11 20 11 31	11 17 56 47					150	150		
Long. on orbit, . . .	11 17 56 47.3	5 8 43 36	8 9 20 21					Arg. XI. 901		680 Arg. X.	
Reduction, ...	— 18.0	Argum. I. or	Argum. XII.	Arg.	Rad. vector.			Arg.	Eq. of long.		
Long. on ecliptic, .	11 17 56 29.3	mean anom.	and arg. of lat.								
Nutation, ...	2.6							I	— 2 6′ 59.″5		
☿ *true* long. on ecl.	11 17 56 31.9			I	4.96496			*S. V.*	— 15.9		
				S. V.	— 45			const.	— 11 56.3		
				II	205				— 2 19 11.7		
				IV	148			II	+ 0 1 52.4		
				V	74			III	28.0		
				R. V.	4.96878			IV	17.1		
								V	42.6		
								VI	23.4		
								VII	0.8		
								VIII	14.0		
								IX	21.5		
								X	0.7		
								XI	7.2		
									+ 0 4 27.7		
								Sum	— 2 14 44.0		

Lat . . . . . 1°13′ 57″S.
Sec. Var. . . — 15
true hel. lat. 1 13 42 S.

Log. rad. vec. 0.696250
Log. cos. hel. lat. 9.999900
Log. curt. dist. 10.696150

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII	VIII	IX
	s ° ′ ″	s ° ′ ″	s ° ′ ″								
Epoch for 1820. . . .	10 29 4 18.8	6 11 27 16	3 8 36 1	8925	8742	7667	6418	659	696	145	19
Sept.	20 12 1.4	38	24	335	109	444	553	78	123	12	23
8th day	39 54.1	1	1	11	4	15	18	3	4	0	1
Great equation. . . .	+ 20 16.2	0	0	31	53	85	138	12	7	— 7	— 2
Mean long.	11 20 16 30.5	6 11 27 55	3 8 36 26	9302	8908	8211	7127	752	830	150	41
Sum eq. of long. . .	— 2 14 15.6	11 20 16 31	11 18 2 15					150	150		
Long. on orbit, . . .	11 18 2 14.9	5 8 48 36	8 9 25 49					Arg. XI. 902		680 Arg. X.	
Reduction, ...	— 17.9	Argum. I. or	Argum. XII.	Arg.	Rad. vector.			Arg.	Eq. of long.		
Long. on ecliptic, .	11 18 1 57.0	mean anom.	and arg. of lat.								
Nutation, ...	2.6							I	— 2 6′ 31.″4		
☿ *true* long. on ecl.	11 18 1 59.6			I	4,96481			*S. V.*	— 15.9		
				S. V.	— 45			const	— 11 56.3		
				II	204				— 2 18 43.6		
				IV	148			II	+ 0 1 52.6		
				V	73			III	28.0		
				R. V.	4.96861			IV	17.1		
								V	42.7		
								VI	23.5		
								VII	0.8		
								VIII	14.0		
								IX	21.5		
								X	0.7		
								XI	7.1		
									+ 0 4 28.0		
								Sum	— 2 14 15.6		

Lat. . . . . 1°13′ 59″S.
Sec. Var. . . — 15
true. hel. lat. 1 13 44 S.

Log. rad. vect. 0.696235
Log. cos. hel. lat. 9.999900
Log. curt. dist. 10.696135

Calculation of the geocentric longitude and latitude.

	s	°	′	″
Sept. 6, Hel. long. Jupiter, ...	11	17	51	4
Long. of Earth, ...	11	13	45	10
Commut.	0	4	5	54
Suppl. commut.	2)5	25	54	6
	2	27	57	3

Log. curt. dist. Jupiter + 10, ...				20.6961630
Log. Earth's rad. vect.				10.0031226
	° 78	′ 32	″ 19	log. tan. . . . 10.6930404
subt. . . .	45	0	0	
	33	32	19	log. tan. . . . 9.8214186
½ sum, . .	87	57	3	log. tan. 11.4463601
½ diff. . .	86	54	37	log. tan. . . . 11.2677787
Elong. .	174	51	40	west of ⊙.
	5	13	45	10 long. ⊙ + 20″.
	11	18	53	30 geoc. long. Jupiter.

Commut.	° 4	′ 5	″ 54	ar. com. log. sin.	1.1458858
Elong. . .	174	51	40	log. sin.	8.9521642
Hel. lat. .	1	13	40	log. tan.	8.3310636
Geoc. lat.	1	32	19 S. . . .	log. tan.	8.4291136

	s	°	′	″
Sept. 7, Hel. long. Jupiter, ...	11	17	56	32
Long. of Earth, ...	11	14	43	30
Commut.	0	3	13	2
Suppl. commut.	2)5	26	46	58
	2	28	23	29

Log. curt. dist. Jupiter + 10, ...				20.6961500
Log. rad. vect.				10.0030105
	° 78	′ 32	″ 28	. . . . log. tan. 10.6931395
Subt. .	45	0	0	
	33	32	28	. . . . log. tan. 9.8214598
½ sum, .	88	23	29	. . . . log. tan. 11.5515574
½ diff. .	87	34	27	. . . . log. tan. 11.3730172
Elong.	175	57	56	west of ⊙.
	5	14	43	30 long. ⊙ + 20″.
	11	18	45	34 geoc. long. Jupiter.

Commut.	° 3	′ 13	″ 2	ar. com. log. sin.	1.2508698
Elong. .	175	57	56	log. sin.	8.8473022
Hel. lat.	1	13	42	log. tan.	8.3312601
Geoc. lat	1	32	23 S. . .	log. tan.	8.4294321

Sept. 8. Hel. long. Jupiter, ...	11s	18°	2′	0″
Long. of Earth, ...	11	15	41	51
Commut.	0	2	20	9
Suppl. commut.	2)5	27	39	51
	2	28	49	55

Log. curt. dist. Jupiter + 10, ...				20.6961350
Log. rad. vect.				10.0028967
	° 78	′ 32	″ 38	. . . . log. tan. 10.6932383
Subt. .	45	0	0	
	33	32	38	. . . . log. tan. 9.8215055
½ sum,	88	49	55	. . . . log. tan. 11.6905990
½ diff. .	88	14	19	. . . . log. tan. 11.5121045
Elong.	177	4	14	west of ⊙.
	5s	15	41	51 long. ⊙ + 20″.
	11	18	37	37 geoc. long. Jupiter.

Commut	2°	20′	9″ . .	ar. com. log. sin.	1.3898011
Elong.	177	4	14	log. sin.	8.7084734
Hel. lat.	1	13	44	log. tan.	8.3314566
Geoc. lat.	1	32	27 S.	log. tan.	8.4297311

Calculation of the right ascension and declination of Jupiter.

1820, Sept. 6, Lat.	° 1	′ 32	″ 19	log. tan. + 10,	18.4291136
Long. . . .	348	53	30	log. sin. . .	9.2848021
Arc A, . .	7	56	12	log. tan. . .	9.1443115
Obliq. . . .	23	27	57		
Arc B, . . .	31	24	9	log. cos. . .	9.9312178
	7	56	12	ar. com. log. cos	0.0041800
	348	53	30	log. tan. . .	9.2930160
R. asc. . . .	350	23	46	log. tan. . .	9.2284138
	350	23	46	log. cos. . .	9.9938702
	31	24	9	log. tan. . .	9.7856590
Dec. . . .	5	48	55 S.	log. tan. .	9.0079430

Sept. 7, Lat.	1°32′	23″	log. tan. + 10,		18.4294321
Long. . .	348	45	34	log. sin. . . .	9.2898757
Arc A, .	7	51	5	log. tan. . . .	9.1395564
Obliq. .	23	27	57		
Arc B, .	31	19	2	log. cos. . . .	9.9316117
	7	51	5	ar. com. log. cos.	0.0040904
	348	45	34	log. tan. . . .	9.2982875
R. asc. .	350	16	28	log. tan. . . .	9.2339896
	350	16	28	log. cos. . . .	9.9937131
	31	19	2	log. tan. . . .	9.7842044
Dec, . .	5	52	6 S.	log. tan. . .	9.0119071
	°	′	″		
Sept 8, lat.	1	32	27	log. tan. + 10,	18.4297311
Long. . .	348	37	37	log. sin.	9.2948988
Arc A, .	7	46	3	log. tan.	9.1348323
Obliq. . .	23	27	57		
Arc B, .	31	14	0	log. cos.	9.9319980
	7	46	3	ar. com. log. cos.	0.0040032
	348	37	37	log. tan.	9.3035115
R. asc. .	350	9	9	log. tan.	9.2395127
	350	9	9	log. cos.	9.9935537
	31	14	0	log. tan.	9.7827713
Dec. . . .	5	55	16 S.	log. tan. . . .	9.0158377

To find the time of culmination.

1820.	⊙'s *AR*.	Diff. in 24 hours.	♃ *AR*. retrog.	Diff. in 24 hours.
	h. m. s.	*m. s.*	*h. m. s.*	*m. s.*
Sept. 6,	11 0 7	3 36	23 21 35	0 29
7,	11 3 43	3 37	23 21 6	0 29
8,	11 7 20		23 20 37	

	h.	*m.*	*s.*			*h.*	*m.*	*s.*
To	24	0	0	. . . . .		24	0	0
add 3 36 + 0 29 =		4	5	3 37 + 0 29 =			4	6
	24	4	5				24	4 6
♃ *AR*	23	21	35	. . . . .		23	21	6
⊙ *AR* subt.	11	0	7	. . . . .		11	3	43
app. time culm. Sept. 6,	12	21	28	Sept. 7, ...		12	17	23

	h. m. s.	*h.*	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*
Then	24 4 5	: 24	: : 12	21	28	: 12	19	22
	24 4 6	: 24	: : 12	17	23	: 12	15	17

Hence Jupiter passes the meridian on Sept. 6th, at 12*h*. 19*m*. 22*s*. or on the morning of the 7th, at 0*h*. 19*m*. 22*s*. and on Sept. 7th, at 12*h*. 15*m*. 17*s*. or on the morning of the 8th, at 0*h*. 15*m*. 17*s*.

Results of the preceding calculations.

JUPITER.

1820.	Heliocentric.		Geocentric.		Declin.	Rt. asc.	Pas. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s ° ′ ″	° ′ ″	*s* ° ′ ″	° ′ ″	° ′ ″	*h. m. s.*	*h. m. s.*
Sept. 6,	11 17 51 4	1 13 40S.	11 18 53 30	1 32 19S.	5 48 55S.	23 21 35	12 19 22
7,	11 17 56 32	1 13 42	11 18 45 34	1 32 23	5 52 6	23 21 6	12 15 17
8,	11 18 2 0	1 13 44	11 18 37 37	1 32 27	5 55 16	23 20 37	

PROBLEM 5.—*Of Saturn.*

To calculate the heliocentric and geocentric longitude, latitude, right ascension, &c. of Saturn.

Example.—To find the heliocentric longitude, latitude, &c. of Saturn, for the apparent noon at Greenwich, of the 6th, 7th, and 8th days of September, 1820, (See fig. 5, plate 10).

Precepts for the heliocentric longitude, &c.

1. From table I, take out the epochs of the mean longitude, aphelion, and node, together with the arguments II, III, IV, V, VI, for the given year, placing them in a horizontal line; but if the year be beyond 1852, take out the epochs for 1852, under which, from table II, place the motions and arguments answering to the number of years between 1852, and the given year.

2. From table III, take the mean motions in longitude, &c. and the arguments for the given month, writing them under the corresponding ones for the year.

This table contains (as before in the cases of Mars and Jupiter), the motions to the *beginning* of each month reckoned from the beginning of the year.

3. In like manner from table V, write down the motions and arguments for the given day of the month, observing in the months January and February of a *bissextile year*, to subtract 1 from the given day of the month.

4. Also, from table IV, take out the motion in longitude for the hours and minutes.

The motions for seconds being extremely small are not put down in the table, we are therefore to take out for the nearest minute. The motion, which in this example we should have to subtract for the equation of time, being less than 0″.3, is neglected.

5. From table VI take out the great equation for the year, and decimal part of a year, if any, and likewise the corrections of the arguments, placing them under their corresponding ones. When the year is not found in the table, we must proportionate as before shewn in finding the great equation of Jupiter.

The numbers in the column titled "Change for half period" shew the variation of the great equation in the space of 459.38 years, being the half of the period of 918.76 years, in which the great equations both of Jupiter and Saturn return again the same. As the table shews the equation for an interval of 470 years extending to A. D. 2020, and as the column of variations here spoken of is only of use in finding the equation for any remote periods without the limits of the table, it might have been omitted.

6. Add together the quantities in the several columns, rejecting 12*s*. or any multiple thereof, if they occur; also in arguments II, III, IV, reject 10000, or any multiple of it, and in arguments V, VI, VII, 1000, or any multiple thereof, and you obtain the mean longitude corrected for the great equation, the places of the aphelion and node, and the arguments corrected for the great equation. Arg. VII is equal to arg, V + arg. VI — 306.

If it should happen that arg. V + arg. VI is less than 306 we must borrow 1000, and then subtract.

7. From the longitude corrected for the great equation, subtract the longitude of the aphelion, and the remainder is the mean anomaly and arg. I.

8. With arg. I, take out from table VII, the equation of Saturn's orbit, and the secular variation with their proper signs, except the time be *before* 1750, in which case the secular variation is to be taken out with a contrary sign: we are to proportionate for the minutes and seconds of the argument both in finding the equation of the orbit and the sec. var. which last is afterwards to be corrected by means of another proportion for the time between 1750 and the given time, in the same manner as was shewn for Jupiter.

9. With arguments II . . . . . VII take out the respective equations from tables VIII to XIII inclusive, making proportions to obtain the true equations when the arguments are not found exactly in the tables. Add up all these equations (paying attention to the signs of the first and of the sec. var., the rest being all positive) and from the sum subtract the constant quantity 19′ 57″.3, and you have the value of the minor equations, which applied, with its proper sign, to the longitude already found as corrected by the great equation, will give the longitude in the orbit.

10. From the longitude thus found, subtract the longitude of the node, and the remainder is arg. VIII, and the *arg. of latitude.*

11. With this arg. enter table XVII, and from one of the columns titled "Reduct. to Eclipt." take out the reduction of the longitude to the ecliptic, proportionating for minutes, &c. apply this, with its proper sign, to the longitude on the orbit, and you have the hel. long. on the ecliptic, from the mean equinox, which corrected by the equation of the equinoxes in longitude, will give the heliocentric longitude of Saturn from the true equinox.

12. Enter table XVI with arg. VIII, and take out the latitude, with its sec. var. proportionating for the minutes, &c. of the argument; then apply the sec. var. according to its sign to the latitude, and you have the true heliocentric latitude.

13. With the mean anomaly of Saturn enter table XIV and take out the rad. vector, and the sec. variation with its proper sign: correct this by the equations from table XV found with arguments II, III, IV, and you have the true distance of Saturn from the Sun, that of the Earth being supposed unity.

The secular variations in the last two arts. are to be proportionated for in the same manner as for Jupiter.

14. To the logarithm of the rad. vector, add the log. cos. of the heliocentric latitude (taken either from a table of log. sines, or from table XVII with arg. VIII), and the sum is the log. of the curtate distance of Saturn.

The tables of Saturn are in almost all respects similar to those of Jupiter, so that we presume the mode of using them will be very easily understood by those readers who have studied the precepts for Jupiter.

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII
	s ° ′ ″	s ° ′ ″	s ° ′ ″						
Epoch for 1820....	0 7 47 29.9	8 29 26 12	3 22 7 10	8925	8742	7494	145	766	arg. V, 150
Sept.	8 8 24.3	44	21	335	109	219	12	45	arg. VI, 820
6th day.	12 3.6	1	1	8	3	5	0	1	970
Great equation. ..	— 47 26.1	0	0	31	53	107	— 7	8	306
Mean long.....	0 15 20 31.7	8 29 26 57	3 22 7 32	9299	8907	7825	150	820	arg. VII, 664
Sum eq. of long...	— 6 6 47.7	0 15 20 32	0 9 13 44						
Long. on orbit, ...	0 9 13 44.0	3 15 53 35	8 17 6 12						
Reduction, ...	— 42.7	Argum. I. or	Argum. VIII.						
Long. on ecliptic, .	0 9 13 1.3	mean anom.	and arg. of lat.						
Nutation, ...	2.6								
☿ *true* long. on ecl.	0 9 13 3.9								

Arg.	Rad. vector.
I	9.39862
S. V.	— 30
II	1501
III	848
IV	2891
R. V.	9.45072

Arg.	Eq. of long.
I	— 6 ° 18 ′ 17.7 ″
const.	— 19 57.3
	— 6 38 15.0
S. V.	+ 1 17.4
II	1 29.8
III	12 39.9
IV	14 59.0
V	19.2
VI	36.7
VII	5.3
	+ 0 31 27.3
Sum	— 6 6 47.7

Lat ..... 2°26′ 8″S.
Sec. Var. .. — 11
true hel. lat. 2 25 57 S.

Log. rad. vec. 0.975465
Log. cos. hel. lat. 9.999608
Log. curt. dist. 10.975073

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII
	s ° ′ ″	s ° ′ ″	s ° ′ ″						
Epoch for 1820....	0 7 47 29.9	8 29 26 12	3 22 7 10	8925	8742	7494	145	766	arg. V, 150
Sept.	8 8 24.3	44	21	335	109	219	12	45	arg. VI, 820
7th day	14 4.2	1	1	10	3	6	0	1	970
Great equation....	— 47 26.1	0	0	31	53	107	— 7	8	306
Mean long.	0 15 22 32.3	8 29 26 57	3 22 7 32	9301	8907	7826	150	820	arg. VII, 664
Sum eq. of long. ..	— 6 6 45.4	0 15 22 32	0 9 15 47						
Long. on orbit, ...	0 9 15 46.9	3 15 55 35	8 17 8 15						
Reduction, ...	— 42.6	Argum. I. or	Argum. VIII.						
Long. on ecliptic, .	0 9 15 4.3	mean anom.	and arg. of lat.						
Nutation, ...	2.6								
☿ *true* long. on ecl.	0 9 15 6.9								

Arg.	Rad. vector.
I	9.39831
S. V.	— 31
II	1501
III	848
IV	2891
R. V.	9.45040

Arg.	Eq. of long.
I	— 6 ° 18 ′ 15.0 ″
const.	— 19 57.3
	— 6 38 12.3
S. V.	+ 1 17.4
II	1 29.8
III	12 39.9
IV	14 58.6
V	19.2
VI	36.7
VII	5.3
	+ 0 31 26.9
Sum	— 6 6 45.4

Lat. ..... 2°26′ 9″S.
Sec. Var. .. — 11
true. hel. lat. 2 25 58S.

Log. rad. vect. 0.975450
Log. cos. hel. lat. 9.999608
Log. curt. dist. 10.975058

To find the longitude, latitude, and log. curt. distance for the 8th day, it will be sufficient to take them in arithmetic progression with the longitudes, &c. for the 6th and 7th already found, as, from the slowness of Saturn's motion, the differences for several days will be equable. We might also deduce the longitude for the 7th within a few tenths of a second without going over the whole calculation; for it is evident, that the mean anomaly will be increased by the motion in longitude for 1 day, that is, by 2′ 0″.6, and this, being about the 30th part of a degree, will cause a diminution of the 30th part of 1′ 22″.4 (the tab. diff. of the equations of the orbit for 3° 15′, and 3° 16′) or of 2″.7 in the first equation of longitude, and as the other equations must all remain very nearly the same, the whole variation in the longitude will be about 2′ 0″.6 + 2″.7 = 2′ 3″.3, thus giving the longitude for the 7th day 0*s* 9° 15′ 7″.2 which is only 0″.3 more than that above computed. The effect of the change in the mean anomaly upon arg. VIII will be to increase it 2′ 3″.3, and this will cause an increase in the latitude of about the 30th part of 34″, (the tab. diff.) or 1″; accordingly we find that the latitude for the 7th is 1″ more than that for the 6th. The change caused in the rad. vector by the increase of 2′ in arg. I, will be a diminution of it by the 30th part of 928, or by 31, supposing the other equations to remain the same; the diff. between the rad. vectores for the 6th and 7th, is in the foregoing calculations 32, so that we should only have erred 1 in the 5th place of decimals by estimating it from the increase in mean anomaly.

Taking the longitude, &c. for the 8th day in arithmetic progression with those of the 6th and 7th, we have for subsequent reference,

	s	°	′	″	
Sept. 8th, ♄ true long. on ecliptic,	0	9	17	9.9	
Hel. lat.		2	25	59	S.
Log. curt. dist.					10.975043

Calculation of the geocentric longitude and latitude.

	s	°	′	″
Sept. 6, Hel. long. Saturn,	0	9	13	4
Long. of Earth,	11	13	45	10
Commut.	0	25	27	54
Supp. commut.	2)5	4	32	6
	2	17	16	3

Log. curt. dist. Saturn + 10,					20.9750730
Log. Earth's rad. vector,					10.0031226
	83	54	40	log. tan.	10.9719504
Subt.	45	0	0		
	38	54	40	log. tang.	9.9069911
½ sum,	77	16	3	log. tan.	10.6459764
½ diff.	74	21	43	log. tan.	10.5529675
Elon.	151	37	46	west of ⊙.	
5*s*	13	45	10	long. ⊙ + 20″	
0	12	7	24	geoc. long. Saturn.	

Commut.	25	27	54	ar. com. log. sin.	0.3665723
Elong.	151	37	46	log. sin.	9.6768509
Hel. lat	2	25	57	log. tan.	8.6281913
Geoc. lat.	2	41	17	S. log. tan.	8.6716145

	s	°	′	″
Sept. 7, Hel. long. Saturn,	0	9	15	7
Long. of Earth,	11	14	43	30
Commut.	0	24	31	37
Supp. commut.	2)5	5	28	23
	2	17	44	11

Log. curt. dist. Saturn + 10,					20.9750580
Log. rad. vector,					10.0030105
	83	54	45	log. tan.	10.9720475
Subt.	45	0	0		
	38	54	45	log. tan.	9.9070127
½ sum,	77	44	11	log. tan.	10.6628003
½ diff.	74	55	46	log. tan.	10.5698130
Elong.	152	39	57	west of ⊙.	
5*s*	14	43	30	long. ⊙ + 20″	
0	12	3	33	geoc. long. Saturn.	

Commut.	24	31	37	ar. com. log. sin.	0.3818252
Elong.	152	39	57	log. sin.	9.6619824
Hel. lat.	2	25	58	log. tan.	8.6282409
Geoc. lat.	2	41	26	S. log. tan.	8.6720485

	s	°	′	″
Sept. 8, Hel. long. Saturn, ...	0	9	17	10
Long. of Earth, ...	11	15	41	51
Commut.	0	23	35	19
Suppl. commut.	2)5	6	24	41
	2	18	12	20

Log. curt. dist. Saturn + 10, ... 20.9750430
Log. rad. vect. 10.0028967

	°	′	″		
	83	54	50	. . . . log. tan.	10.9721463
Subt. .	45	0	0		
	38	54	50	. . . . log. tan.	9.9070342
½ sum,	78	12	20	. . . . log. tan.	10.6802493
½ diff. .	75	29	53	. . . . log. tan.	10.5872835

Elong. 153 42 13 west of ⊙.
5s 15 41 51 long. ⊙ + 20″.
0 11 59 38 geoc. long. Saturn.

Commut 23° 35′ 19″ . . ar. com. log. sin. 0.3977589
Elong. 153 42 13 . . . . . log. sin. 9.6464181
Hel. lat. 2 25 59 . . . . . log. tan. 8.6282906
Geoc. lat. 2 41 36 S. . . . . log. tan. 8.6724676

Calculation of the right ascension and declination of Saturn.

1820, Sept. 6, Lat.	2	41	17	log. tan. + 10,	18.6716145
Long. . . .	12	7	24	log. sin. . . .	9.3222539
Arc A, . .	12	36	3	log. tan. . . .	9.3493606
Obliq. . . .	23	27	57		
Arc B, . . .	10	51	54	log. cos. . . .	9.9921442
	12	36	3	ar. com. log. cos.	0.0105886
	12	7	24	log. tan. . . .	9.3320493
R. asc. . .	12	11	51	log. tan. . . .	9.3347821
	12	11	51	log. cos. . . .	9.9900835
	10	51	54	log. tan. . . .	9.2831569
Dec. . . .	2	19	20	N. log. tan. . .	8.6080225

	°	′	″		
Sept. 7, Lat.	2	41	26	log. tan. + 10,	18.6720485
Long. . .	12	3	33	log. sin. . . .	9.3199835
Arc A, . .	12	40	37	log. tan. . . .	9.3520650
Obliq. . .	23	27	57		
Arc B, . .	10	47	20	log. cos. . . .	9.9922546
	12	40	37	ar. com. log. cos.	0.0107180
	12	3	33	log. tan. . . .	9.3296747
R. asc. . .	12	8	22	log. tan. . . .	9.3326473
	12	8	22	log. cos. . . .	9.9901784
	10	47	20	log. tan. . . .	9.2800300
Dec. . . .	2	17	41	N. log. tan. .	8.6028557

	°	′	″		
Sept. 8, Lat.	2	41	36	log. tan. + 10,	18.6724676
Long. . . .	11	59	38	log. sin. . . .	9.3176609
Arc A, . .	12	45	17	log. tan. . . .	9.3548067
Obliq. . .	23	27	57		
Arc B, . .	10	42	40	log. cos. . . .	9.9923665
	12	45	17	ar. com. log. cos.	0.0108510
	11	59	38	log. tan. . . .	9.3272467
R. asc. . .	12	4	50	log. tan. . . .	9.3304642
	12	4	50	log. cos. . . .	9.9902742
	10	42	40	log. tan. . . .	9.2768128
Dec. . . .	2	16	1	N. log. tan. .	8.5975512

To find the time of culmination, by the Rule, p. 226.

1820.	⊙'s AR.	Diff. in 24 hours.	♄'s AR. retrog.	Diff. in 24 hours.
	h. m. s.	m. s.	h. m. s.	m. s.
Sept. 6.	11 0 7	3 36	0 48 47	0 14
7,	11 3 43	3 37	0 48 33	0 14
8,	11 7 20		0 48 19	

	h.	m.	s.		h.	m.	s.
To	24	0	0	. . . . .	24	0	0
Add 3 . 36 + 0 . 14 =		3	50	3 . 37 + 0 . 14 =		3	51
	24	3	50		24	3	51
♄ AR + 24 hours, . .	24	48	47	. . . . .	24	48	33
⊙ AR subtr.	11	0	7	. . . . .	11	3	43
Appr. time culm. Sept. 6,	13	48	40	Sept. 7, ...	13	44	50

	h.	m.	s.	h.	h.	m.	s.	h.	m.	s.
Then	24	3	50	: 24	: :	13	48	40	: 13	46	28
	24	3	51	: 24	: :	13	44	50	: 13	42	38

Hence Saturn passes the meridian on Sept. 6th, at 13*h*. 46*m*. 28*s*., or on the morning of the 7th, at 1*h*. 46*m*. 28*s*. civil reckoning, and on Sept. 7th, at 13*h*. 42*m*. 38*s*., or on the morning of the 8th, at 1*h*. 42*m*. 38*s*.

Results of the preceding calculations.

SATURN.

1820.	Heliocentric.		Geocentric.		Declin.	Rt. asc.	Pas. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s ° ′ ″	° ′ ″	*s* ° ′ ″	° ′ ″	° ′ ″	*h*. *m*. *s*.	*h*. *m*. *s*.
Sept. 6,	0 9 13 4	2 25 57S.	0 12 7 24	2 41 17S.	2 19 20N	0 48 47	13 46 28
7,	0 9 15 7	2 25 58	0 12 3 33	2 41 26	2 17 41	0 48 33	13 42 38
8,	0 9 17 10	2 25 59	0 11 59 38	2 41 36	2 16 1	0 48 19	

PROBLEM 6.—*Of the Georgian.*

To calculate the heliocentric and geocentric longitude, latitude, right ascension, &c. of the Georgium sidus, or Uranus.

Example.—To find the heliocentric longitude, latitude, &c. of the Georgian, for the apparent noon at Greenwich, of the 6th, 7th, and 8th days of September, 1820, (See Fig. 6, plate 10).

Precepts for the heliocentric longitude, &c.

1. From table I take out the epochs of the mean longitude, aphelion, and node, together with the arguments II, III, IV, V, VI, VII, VIII, for the given year, placing them in a horizontal line; but if the year be beyond 1852 take out the epochs for 1852, under which, from table II, place the motions and arguments answering to the number of years between 1852 and that given.

2. From table III take the mean motion in longitude, &c. and the arguments for the given month, writing them under the corresponding ones for the year.

3. From table V take the motions and arguments for the given day of the month, observing in the months January and February of a *bissextile year*, to subtract 1 from the given day of the month.

4. Also, from table IV take out the motion in longitude for the hours and minutes.

We take out for the nearest minute as before; as to the motion for the equation of time, it can never amount to more than a few tenths of a second for this planet, and in this example has not been noticed.

5. Add together the quantities in the several columns, rejecting 12*s*. or any multiples thereof, if they occur; also in arguments IV, VI, reject 10000, and in arguments II, III, V, VII, VIII, 1000, or any multiples thereof, and you have the mean longitude, the places of the aphelion and node, and the arguments for the proposed time.

6. Subtract the longitude of the aphelion from the mean longitude, and the remainder is argument I, or the mean anomaly, with this (proportionating for the minutes and seconds, if any,) take out from table VI, the equation of the orbit, and the secular variation with their proper signs, except the time be *before* 1780, in which case, the secular variation is to be taken out with a contrary sign to that in the table: then make proportion for the secular variation at the given time by saying as 100: the number of years between 1780 and that time :: the sec. var. already found : the sec. var. required.

The mean anomaly for Sept. 6th is 9*s* 4° 44′ 17″ which by proportionating gives —10″.81 for the sec. variation; then as 100 : 1820.7 — 1780 or 40.7 years : : —10″.81 : —4″.4 the true sec. var.

7. With arguments II . . . . . VIII enter tables VII to XIII respectively, and take out the several equations, making proportion when the arguments are not found exactly in the tables; place all these equations under the sec. equa., and that of the orbit.

8. Add up all these equations, having regard to the signs of that of the centre and of the sec. var. (the signs of all the others being positive), and diminish the amount by the constant quantity 7′ 20″, and you have the value of these eight equations, which, applied with its proper sign to the mean longitude already found, gives the longitude of the Georgian in his orbit.

9. From the longitude in the orbit subtract the longitude of the node, and the remainder is arg. IX, and the *argument of latitude.*

10. With this argum. enter table XVII, and from one of the columns having "Sig." at the head, take out the reduction of the longitude to the ecliptic, which apply with its proper sign to the long. on the orbit, and you have the heliocentric longitude of the Georgian on the ecliptic from the mean equinox, and this corrected by the equation of the equinoxes in longitude gives the longitude from the true equinox.

11. With arg. IX enter table XVI, and take out the latitude, proportionating for the minutes and seconds of the argument, and you have the true hel. lat. of the Georgian.

12. With the mean anomaly enter table XIV, and take out the rad. vector, take out also the equations of the rad. vector from table XV, with the arguments II, III, IV, VII, and the sum of the whole will be the distance of the Georgian from the Sun, that of the Earth being taken as unity.

13. To the log. of the rad. vector add the log. cos. of the hel. lat. and the sum is the log. of the curtate distance of the Georgian.

The log. cos. of the hel. lat. is given in table XVII.

	Longitude.	Aphelion.	Node.	Arguments. II	III	IV	V	VI	VII	VIII
	s ° ′ ″	s ° ′ ″	s ° ′ ″							
Epoch for 1820....	8 19 27 28.2	11 17 38 25	2 12 56 12	301	547	3287	883	1370	193	439
Sept.	2 51 35.4	35	10	15	7	68	22	12	48	40
6th day.	4 14.2	1	0	0	0	2	1	0	1	1
Mean long.....	8 22 23 17.8	11 17 39 1	2 12 56 22	316	554	3357	906	1382	242	480
Sum eq. of long...	+ 5 22 42.8	8 22 23 18	8 27 46 1	Arg.	Rad. vect.		Arg.	Eq. of long.		
Long. on orbit, ...	8 27 46 0.6	9 4 44 17	6 14 49 39							
Reduction, ...	− 4.6	Argum. I. or	Argum. IX.	I	19.2918		I	+ 5 ° 17 ′ 52.9 ″		
Long. on ecliptic, .	8 27 45 56.0	mean anom.	and arg. of lat.	S.V.	0		S.V.	—	4.4	
Nutation, ...	+ 2.6		lat. 0° 11′ 50″ S.	II	14			+ 5	17	48.5
☿ *true* long. on ecl.	8 27 45 58.6			III	0		II			49.8
				IV	30		III		1	4.7
				VII	51		IV		4	38.7
				R.V.	19.3013		V			2.3
			Log. rad. vec.		1.285587		VI		3	41.9
			Log. cos. hel. lat.		9.999998		VII		1	52.4
			Log. curt. dist.		11.285585		VIII			4.5
								+ 5	30	2.8
							const.	—	7	20.0
								+ 5	22	42.8

The daily motion of the Georgian being 42″.4, the longitude on the 7th will be increased by this quantity; the mean anom. will also be increased by 42″.4, which will *diminish* the equation of the centre about 0″.6, and as all the other arguments remain very nearly the same for a few days, we may take the long. on the 7th = long on the 6th + 42″.4 − 0″.6 = 8*s* 27° 46′ 40″.4. The argum. of lat. will be increased by nearly the quantity of the daily motion in long. which will increase the lat. by about 0″.6, so that the lat on the 7th may be taken at 0° 11′ 51″ S. The increase in the mean anom. we shall find will add 0.0002 to the rad. vector, and increase its log. by 4, and as the latitude is very small the log. of the curt. dist. will only be altered by this 4, which makes it 11.285589: hence, by taking the long. &c. on the 8th in arithmetic progression with those of the 6th and 7th, we shall have, on

1820,	☿ long.	Hel. lat.	Log. curt. dist.
	s ° ′ ″		
Sept. 6,	8 27 45 59	0 11 50 S.	11.285585
7,	8 27 46 40	0 11 51	11.285589
8,	8 27 47 21	0 11 52	11.285598

Computation of the geocentric longitude and latitude.

	s ° ′ ″
Sept. 6, Hel. long. Georgian, ...	8 27 45 59
Long. of Earth, ...	11 13 45 10
12*s*. — commut.	9 14 0 49
Suppl. commut.	2)3 14 0 49
	1 22 0 24

Log. curt. dist. Georgian + 10, . . .				21.2855850
Log. Earth's rad. vector, ...				10.0031226
	°	′	″	
	87	0	46	log. tan. 11.2824624
Subtr. .	45	0	0	
	42	0	46	log. tan. 9.9546322
½ sum,	52	0	24	log. tan. 10.1072943
½ diff. .	49	4	16	log. tan. 10.0619265
Elong.	101	4	40	east of ⊙
5*s*	13	45	10	long. ⊙ + 20″
	8	24	49	50 geoc. long. Georgian.

Commut.	75	59	11	ar. com. log. sin. 0.0131216
Elong. . .	101	4	40	log. sin. . . . 9.9918316
Hel. lat.	0	11	50	log. tan. 7.5368349
Geoc. lat.	0	11	58	S. log. tan. . . . 7.5417881

Sept. 7, Hel. long. Georgian, ...				8*s* 27° 46′ 40″
Long. of Earth, ...				11 14 43 30
12*s*. — commut.				9 13 3 10
Suppl. commut.				2)3 13 3 10
				1 21 31 35
Log. curt. dist. Georgian, + 10, . . .				21.2855890
Log. Earth's rad. vect.				10.0030105
	°	′	″	
	87	0	49	log. tan. 11.2825785
Subtr.	45	0	0	
	42	0	49	log. tan. 9.9546449
½ sum,	51	31	35	log. tan. 10.0998054
½ diff.	48	34	57	log. tan. 10.0544503
Elong.	100	6	32	east of ⊙
5*s*	14	43	30	long. ⊙ + 20″
	8	24	50	2 geoc. long. Georgian.

Commut.	76°	56′	50″	ar. com. log. sin. 0.0113686
Elong.	100	6	32	log. sin. 9.9932051
Hel. lat.	0	11	51	log. tan. 7.5374462
Geoc. lat.	0	11	59	S. log. tan. . . . 7.5420199

				s ° ′ ″
Sept. 8, Hel. long. Georgian, ...				8 27 47 21
Long. of Earth, ...				11 15 41 51
12*s*.—commut.				9 12 5 30
Supp. commut.				2)3 12 5 30
				1 21 2 45
Log. curt. dist. Georgian, + 10, . . .				21.2855930
Log. Earth's rad. vector, ...				10.0028967
	°	′	″	
	87	0	52	log. tan. 11.2826963
Subt. . .	45	0	0	
	42	0	52	log. tan. 9.9546576
½ sum,	51	2	45	log. tan. 10.0923413
½ diff. .	48	5	39	log. tan. 10.0469989
Elong.	99	8	24	east of ⊙.
5*s*	15	41	51	long. ⊙ + 20″
	8	24	50	15 geoc. long. Georgian.

	°	′	″	
Commut.	77	54	30	ar. com. log. sin. 0.0097439
Elong. .	99	8	24	. . . . log. sin. 9.9944505
Hel. lat.	0	11	52	. . . . log. tan. 7.5380566
Geoc. lat.	0	11	59	S. . . . log. tan. 7.5422510

Calculation of the right ascension and declination of the Georgian.

	°	′	″	
1820, Sept. 6, lat.	0	11	58	log. tan. + 10, 17.5417881
Long. . . .	264	49	50	log. sin. . . . 9.9982299
Arc A, . .	0	12	1	log. tan. . . 7.5435582
Obliq. . .	23	27	57	
Arc B, . .	23	39	58	log. cos. . . . 9.9618481
	0	12	1	ar. com. log. cos. 0.0000027
	264	49	50	log. tan. . . . 11.0434981
R. asc. . .	264	21	32	log. tan. . . . 11.0053489
	264	21	32	log. sin. . . . 9.9978917
	23	39	58	log. tan. . . . 9.6417358
Dec. . . .	23	33	50	S. log. tan. . . 9.6396275

Sept. 7, Lat.	0°11′	59″	log. tan. + 10,		17.5420199
Long. . .	264	50	2 log. sin. . . .		9.9982322
Arc A, . .	0	12	1 log. tan. . . .		7.5437877
Obliq. . .	23	27	57		
Arc B, . .	23	39	58 log. cos. . . .		9.9618481
	0	12	1 ar. com. log. cos.		0.0000027
	264	50	2 log. tan. . . .		11.0437797
R. asc. . .	264	21	45 log. tan. . . .		11.0056305
	264	21	45 log. sin. . . .		9.9978944
	23	39	58 log. tan. . . .		9.6417358
Dec. . . .	23	33	51 S. log. tan. .		9.6396302

Sept. 8, Lat.	0°	11′	59″ log. tan. + 10,		17.5422510
Long. . .	264	50	15 log. sin. . . .		9.9982347
Arc A, . .	0	12	2 log. tan. . . .		7.5440163
Obliq. . .	23	27	57		
Arc B, . .	23	39	59 log. cos. . . .		9.9618472
	0	12	2 ar. com. log. cos.		0.0000027
	264	50	15 log. tan. . . .		11.0440851
R. asc. . .	264	21	59 log. tan. . . .		11.0059350
	264	21	59 log. sin. . . .		9.9978973
	23	39	59 log. tan. . . .		9.6417416
Dec. . . .	23	33	52 S. log. tan.. .		9.6396389

1820,	☉'s *A*.	Diff. in 24 hours.	♅'s *A*. direct.	Diff. in 24 hours.
	h. m. s.	*m. s.*	*h. m. s.*	*m. s.*
Sept. 6,	11 0 7	3 36	17 37 26	0 1
7,	11 3 43	3 37	17 37 27	0 1
8,	11 7 20		17 37 28	

	h. m. s.	*h. m. s.*
To	24 0 0	24 0 0
Add 3 . 36 — 0 . 1 =	3 35	3 . 37 — 0 . 1 = 3 36
	24 3 35	24 3 36
♅ *A*	17 37 26	17 37 27
☉ *A* subtr.	11 0 7	11 3 43
Appr. time culm.	Sept. 6, 6 37 19	Sept. 7, ... 6 33 44

	h.	*m.*	*s.*	*h.*		*h.*	*m.*	*s.*		*h.*	*m.*	*s.*
Then as	24	3	35	: 24	::	6	37	19	:	6	36	20
	24	3	36	: 24	::	6	33	44	:	6	32	45

So that the Georgian passes the meridian on Sept. 6th, at 6*h*. 36*m*. 20*s*. P. M. and on Sept. 7th, at 6*h*. 32*m*. 45*s*. P. M.

Results of the preceding calculations.

GEORGIAN.

1820.	Heliocentric		Geocentric		Declin.	Rt. asc.	Pas. Merid.
	Longitude.	Latitude.	Longitude.	Latitude.			
	s ° ′ ″	° ′ ″	*s* ° ′ ″	° ′ ″	° ′ ″	*h. m. s.*	*h. m. s.*
Sept. 6,	8 27 45 59	0 11 50S.	8 24 49 50	0 11 58S.	23 33 50	17 37 26	6 36 20
7,	8 27 46 40	0 11 51	8 24 50 2	0 11 59	23 33 51	17 37 27	6 32 45
8,	8 27 47 21	0 11 52	8 24 50 15	0 11 59	23 33 52	17 37 28	

The place of this planet has been calculated strictly according to the tables now extant, but we think it proper here to remark, that owing to the very slow motion of the Georgian round the Sun, and to the deficiency, in point of number, of correct observations, it is not to be expected that the place of this planet can as yet be very accurately assigned. The late professor Robison, who was one of the first mathematicians that computed the elements of this planet's orbit, found that there must, in all probability, be inaccuracies in the observations upon which he had founded his calculations, and was obliged to apply corrections to them, drawn by analogy from the other planets, before he could even make the planet's motion consistent with that of a body in an elliptic orbit. M. Delambre, indeed, rigorously computed the tables we have used according to the theory of Laplace from the best *data* that could *then* be procured, but we believe, that some of these consisted of observations made before the Georgian was known to be a planet, and when it was regarded only as a fixed star: of such observations there appear to have been only seventeen; namely, one by Mayer, one by Bradley, three of Flamsteed's, and twelve of Lemonnier's: these, it is known, were subjected to various errors from the instruments employed, and

other attending circumstances, yet they are valuable, on account of their extending over a far longer period of time than the observations made since the discovery of the Georgian as a planet. These latter can only embrace a period of about half a revolution, and it has not been found practicable thoroughly to reconcile them with the anterior observations. It seems to us, that, *at least*, a complete revolution round the Sun must take place before we can be at all in possession of *data* sufficiently accurate for forming tables of this planet's motion. With regard to the places of the other planets, they may probably be depended on to 15″ or 20″, and this is as much as can be expected from any calculations respecting them. The longitude of a planet is deduced from its right ascension, and this is determined from the time of its passage over the meridian of some observatory, as that of Greenwich, or Paris, &c. Yet these observations themselves, taken singly, are attended with some uncertainty, which is however much diminished by taking a mean between a great number of them. As to the geocentric latitudes, from which their heliocentric latitudes are to be ascertained, there are many anomalies involved therein, and consequently the tables may differ somewhat from the truth, and from what actual observation at the time may shew; but, by means of this observed difference, properly applied, we may correct the errors of the tables, and so bring the results of calculation and actual observation nearer to each other. Astronomers have to lament, that there is not a sufficient number of proper observatories yet established; if there were good observatories and correct observers, in well chosen parts of every country of the world, including also some of the islands in the south seas, we might look forward to a yet much greater improvement in astronomical tables as the result: but, unfortunately, as the matter now is, we must be satisfied with the result deduced from observations taken at Greenwich and Paris, which are almost the only places at which any numerous and accurate observations have as yet been taken. Tobias Mayer, indeed, in his observatory at Gottingen, did much: from his own obsevations compared with those made at Greenwich, he calculated tables on the true theory of gravitation by which the Moon's place was ascertained to a degree of accuracy never before known; since that, Mr. Mason, under the direction of the late worthy Astronomer Royal, Dr. Maskelyne, farther improved Mayer's Lunar Tables, and we have since had still farther improved tables by Messieurs Burg and Burckhardt, from the latest observations: this certainly should excite emulation among gentlemen of science, in every country, to unite in establishing observatories, and pursuing with zeal every opportunity of observing the celestial phenomena, in order to perfect our astronomical tables.

We have now gone through and explained the calculations of all the essential circumstances relating to the places of the planets, and, we believe, exemplified the same more extensively than has yet been done in any treatise of astronomy. By continuing the operation in each case for three successive days, the computer has an opportunity of judging of the accuracy of his results by means of differences. It is by differences that the computers of the Nautical Almanac, and of every other good Ephemeris, judge how far their results can be relied on, and for this purpose we must at least have three results, in order to obtain two differences, and if we wish to interpolate, it is necessary that we have four, in order to get the mean of two second differences, for the purpose of applying the equation of second difference, as has been done in the calculations of the Moon's longitude, &c.

The rising and setting of the planets are easily calculated from their declination, passage over the meridian, and the latitude of the place, taking care to correct the hour angle found from these *data* for the proportional part of the sum or difference of the daily motion of the sun and planet in right ascension, otherwise we shall fall into a similar error to that before noticed, respecting the time of rising or setting of the Moon: it will be sufficient to give one

Example.—Required the apparent time of the rising of Venus, at Greenwich, on the 7th of September, 1820, her declination being (as before computed, see p. 230), 14° 27′ 46″ N, and the daily variation in declination at that time nearly insensible; the time of her passage over the meridian 21*h*. 16*m*. 1*s*; the daily change of the Sun's right ascension being greater than that of Venus, on the 7th Sept. 1*m*. 16*s*. (as before calculated) also the latitude of Greenwich 51° 28′ 40″ N. and the horizontal refraction supposed to be 33′.

Here, as in the case of the rising or setting of the Moon, we have the three sides of an oblique angled sph. trian. given, viz. the planet's zenith and polar distance, and the colatitude, to find the angle opposite the zenith distance, or horary angle.

°	′	″	°	′	″	°	′	″
90	0	0	90	0	0	90	0	0
	33	0	14	27	46	51	28	40
90	33	0	75	32	14	38	31	20

	°	′	″		
Zen. dist. .	90	33	0		
Colat.. . .	38	31	20	ar. com. log. sine.	0.2056388
Pol. dist. .	75	32	14	ar. com. log. sine.	0.0139855
2)	204	36	34		
Half sum	102	18	17	log. sine	9.9899070
Sub.zen.dis	90	33	0		
	11	45	17	log. sine.	9.3090389
				2)	19.5185702
	54	56	9	log. cos.	9.7592851
			2		
Hour-ang.	109	52	18	in time, 7*h*. 19*m*. 29*s*.	

Now this hour-angle must be diminished, because it appears, that the Sun's motion in right ascension exceeds that of Venus at the proposed time: to find the diminution, say, as 24*h*. : 1*m*. 16*s*. : : 7*h*. 19*m*. 29*s*. : 0*m*. 23*s*. therefore, 7*h*. 19*m*. 29*s*. — 23*s*. = 7*h*. 19*m*. 6*s*. is the corrected semi-time of apparition. Hence,

	h.	*m*.	*s*.	
From time of cul.	21	16	1	
Subt.	7	19	6	
Rem. time of ris.	13	56	55	or 1*h*. 56*m*. 55*s*. A.M. 8th day, civil time.
To time of culm.	21	16	1	
add	7	19	6	
Time of setting	28	35	7	or 4*h*. 35*m*. 7*s*. P. M. [8th day.

And if the equation of time were subtracted, we should have the true mean times of the rising and setting of Venus.

Whenever it appears that the planet's apparent motion westward is quicker than that of the Sun, the proportional part is to be subtracted from the hour angle; but if slower, as is always the case in respect to the Moon, the proportional part is additive. The proportional part for the Moon's motion, from the meridian to the horizon, is sometimes very considerable, as we have said before; in regard to the planets, it is commonly small, and for the most part may be neglected, especially for the purpose of insertion in common almanacs.

If the daily change in the time of the planet's culmination be known (as it is here, having been already computed, p. 230), then, instead of using the sum or difference of the daily change in right ascension of the Sun and planet, we may say, as 24*h*. is to the daily change in the time of culm. so is the approx. hour angle, to the proportional decrease or increase of it, according as the time of culmination on the suceeeding day is earlier or later than on the preceding.

If it were required to find the time of rising and setting to the greatest accuracy, we ought to take into account the horizontal parallax of Venus, which would decrease the zenith distance; but this parallax being a quantity which varies considerably, (although at its maximum but small) would require a particular computation made for it in each case, according to the relative situations of the Sun, the Earth, and Venus: the calculation would consist in the application of a well known case of plane trigonometry, for we should have in a plane triangle formed by the three bodies, the angle at the Sun subtended by the Earth and Venus equal to the difference between their heliocentric longitudes, and the sides containing that angle, viz. their distances from the Sun, to find the third side, or distance between Venus and the Earth, and then knowing this distance, and the Earth's radius, we should have to determine the angle which this radius subtends at Venus, which would be the required parallax; it is obvious, that the parallax would be greatest at the inferior conjunction of the planet, and least at the superior; its *max.* does not exceed 30″, and its *min.* is about 6″. We may also remark, that having found the time of rising and setting nearly, the declination should then be adapted to those times, and the times then corrected, using these declinations; but these minutiæ cannot be considered as worth attending to in a problem of so little real importance as that of finding the rising or setting of the planets.

Of ECLIPSES.

General principles and observations.

1. The word eclipse is derived from the Greek ἐκλείπω, signifying to *faint* or *swoon away*: hence, sick or dying persons when swooning, or death-like faintness came over them, were said by the Greeks to fall into an eclipse; and observing that the Sun and Moon after having shone with their accustomed lustre, sometimes became wholly or partially darkened, they seem to have imagined that the Moon and Sun

were at such times in a suffering state, and by a transfer of the expression to these bodies, they spoke of them in these cases, as suffering an *eclipse*.

2. All the planets and their satellites being *opaque* bodies, and having no light but that received from the Sun, necessarily cast a dark shadow on the side opposite to the Sun, and if any one of these bodies in its revolution fall into the shadow of another, it is then deprived of the Sun's light, and is *eclipsed*; or if one of the bodies come between another and the Sun, the view of the Sun from this latter is then partly or wholly intercepted, and the Sun is then said to suffer an eclipse.

3. The bodies which we propose here to notice in reference to eclipses, are the Sun, Moon, and Earth; these, in common with the other celestial bodies, being very nearly spherical, it follows, that if the shadow projected by the Earth or Moon be cut by a plane at right angles to a line joining the centres of the Sun and either of these bodies, the section must be a circle. If either of the bodies, as the Earth, were of equal diameter with the Sun, the shadow would be cylindrical, and extend into infinite space; if the Earth's diameter were greater than that of the Sun, the shadow would be conical, and extend also into infinite space: the diameter of the section of the shadow becoming greater, at greater distances from the Sun, since what in this case may be called the *imaginary* vertex of the cone would be on the contrary side of the Sun to the Earth; but as it is known that the Sun is vastly larger than the Earth or Moon, it follows, that the shadows of the Earth and Moon are conical, and terminate always in a vertex on the same side of the Sun with the Moon or Earth.

4. When the Moon is eclipsed, or falls into the shadow of the Earth, she is actually deprived in part or wholly of her light, and the eclipse is visible to all that hemisphere of the Earth which is turned towards the Moon. If it be the Sun that appears eclipsed, he does not really lose any portion of his light, but is simply hidden by the Moon, and the Moon being far less than the Earth, her shadow is but a kind of spot, which covers a part of the Earth's enlightened hemisphere. The Moon in this case produces an effect similar to that of the clouds which often intercept the rays of the Sun at a particular place, while at a short distance from thence we perceive all the objects fully enlightened.

5. An eclipse of the Moon being caused by its entering into the Earth's shadow, must consequently happen when the Moon is in opposition to the Sun, or at full Moon; but an eclipse of the Sun, as it arises from the interposition of the Moon between the Earth and Sun, will only occur when the Moon is in conjunction with the Sun, or at the new Moon; yet there does not happen an eclipse of one of the luminaries at *every* conjunction or opposition, because the Moon does not revolve in the same plane with the Sun and Earth, that is, in the ecliptic, but in an orbit inclined to the ecliptic, in consequence of which, the shadow of the Earth often falls on one side of the Moon at the time of opposition, and the Moon is frequently not situated upon a line joining the centres of the Sun and Earth at the time of conjunction; but if the Moon be at, or sufficiently near one of the nodes of her orbit at opposition or conjunction, there will then be an eclipse of one of the luminaries, because the three bodies will be exactly, or nearly in a straight line.

6. Let S, E, M, (plate 11, fig. 1,) be the centres of the Sun, Earth, and Moon, in the same straight line at the instant of opposition; let A*a*, B*b*, be tangents to the same sides of the Sun and Earth, meeting in V; imagine the triangle SAV to revolve round SV as an axis, then the line A*a* will generate the surface of a luminous conic frustum, between the Sun and Earth, and *a*V that of an opaque cone, terminating in the vertex V; the part *a*F, *b*V, of this opaque cone is the shadow of the earth or *umbra*, and in it, the Moon, when eclipsed, is either wholly or partially involved. Let C*c*, D*d*, be tangents to contrary sides of the Sun and Earth, and consequently, intersecting in a point, I, between the Sun and Earth, and suppose these tangents to revolve as before about the axis SV, and they will generate two opposite cones having the common vertex I; the cone I*nm*, thus generated, which is indefinitely extended towards *m* and *n*, is called the penumbral cone, and the portions V*adm*, V*bcn*, the *penumbra*, or imperfect shadow: these portions must be in some measure darkened, because an eye situated at any point within them, will see only part of the Sun; thus, if a spectator were at *e*, within the penumbra V*adm*, it is evident that a tangent drawn from thence to the Earth, must touch it somewhere between *a* and *d*, and produced, would meet the Sun at some point between D and C, consequently the spectator could not see the whole of the Sun's disc, and would therefore be in a state of partial obscuration; also, as he recedes from the limit *dm* of the penumbra, and approaches *a*V, a less portion of the Sun will be visible to him: whence it appears that the penumbra V*adm* is not an uniform shadow, but that it gradually becomes darker from its limit, *dm*, until it unites with the umbra, and is equally dark with it at *a*V. The existence and the gradual darkening of the penumbra from the limit *dm* to *a*V, accounts for a phænomenon respecting eclipses of the Moon, viz.: that she does not lose her light suddenly, but that it gradually becomes more and more faint, until the obscurity arrives at its greatest intensity.

7. In the preceding article we have supposed the Moon to be placed exactly in the axis of the terrestrial shadow, and she would be so placed at every opposition, if her orbit coincided with the plane of the ecliptic; but since the Moon is sometimes on one side of this plane, and sometimes on the other, it often happens that she only enters partially into the shadow of the Earth, or just touches it with her limb, or passes without touching it. In the first case, the eclipse is *partial*; in the second, it is termed an *appulse*. A *total eclipse* is when the Moon is entirely involved in the shadow, and the eclipse is called *central*, when the centre of the Moon coincides with the axis of the conical shadow.

8. Let S, M, E, (plate 11, fig. 2,) be the centres of the Sun, Moon, and Earth, in the same straight line at the time of conjunction, when there will consequently be an eclipse of the Sun: let tangents drawn to the same sides of the Sun and Moon meet in V, and those to opposite sides intersect in I, and imagine the whole figure to revolve round SE as an axis, as before, then will *a*F *b*V be the Moon's *umbra*, in which no part of the Sun can be seen, the space comprehended between the umbra and *dm*, *cn*, is the *penumbra*; for within this space, only part of the Sun is seen, for the same reason as was given in the case of an eclipse of the Moon: now, if the umbra extend as far as the Earth, it is manifest that the space *rs*, upon which the umbra falls, will suffer a *total* eclipse; the parts *mr*, *sn*, between the boundaries of the umbra and penumbra, will have a *partial* eclipse, but to all the other parts of the Earth there will be no eclipse. The Earth being at different distances from the Moon at different times, it sometimes happens that the umbra does not reach so far as the Earth, which case is represented in fig. 3, and here the space within *sr* will suffer what is called an *annular* eclipse: for if tangents were drawn from any point between *r* and *s* to the Moon, they would evidently fall within the Sun, and therefore a portion of the Sun will appear all round the Moon, forming an *annulus* or ring: also the parts *ms*, *rn*, will have a *partial* eclipse, but all the other parts of the Earth will not suffer any eclipse. In this case there is no total eclipse any where, because the Moon's umbra does not reach the Earth.

9. The appearances of the Sun's disk during an eclipse from different parts of the Earth's surface are represented in (plate 11, figs. 6 to 9.) If we suppose, in fig. 2, tangents to be drawn from the centre of the Sun passing by the Moon, and meeting the earth in *p* and *q*, then the inhabitants at the points *p* and *q* will see the Sun as represented in fig. 6, the Moon's limb just passing through the Sun's centre, and obscuring the part shewn with a dark shade; those between *p* and *r*, or *q* and *s*, will see the Sun as in fig. 7; those between *p* and *m*, or *q* and *n*, will see him with a less part obscured as in fig. 8; those between *r* and *s* (fig. 3), will observe him as an *annulus* or ring, the Moon obscuring all the central part as in fig. 9, and if the spectator be situated at the middle point between *r* and *s*, the luminous annulus will be of an equal breadth all round; lastly, the inhabitants situated between *r* and *s*, in fig. 2, will have the Sun entirely obscured by the Moon.

We may now proceed to various computations, preparatory to the calculation of eclipses.

To determine the length of the conical shadow projected by the Earth.

10. Let S (plate 11, fig. 4,) be the centre of the Sun, E that of the Earth, AB a tangent to the Earth and Sun, forming the limit of the umbra, SEV the axis of the cone: it is proposed to determine EV, the distance from the centre of the Earth to the vertex of the cone.

From E and S, draw the tangents EK, SL, to the Sun and Earth, and SA, SK, EB, EL, to the points of contact: then the angles at A, K, L, B, will all be right angles, and EB will be parallel to SA. Let the angle KES = ⊙'s sem. diam. = D, the angle LSE = ⊙'s hor. parall. = P, the angle BVE = $\frac{1}{2}$ vert. ang. of cone = C; then putting R for the Earth's radius EL, and taking the trigonometrical rad. equal to unity, we have $EV = \frac{R}{\sin. C}$, $SE = \frac{R}{\sin. P}$ and SK or SA = SE sin. D = $\frac{R \sin. D}{\sin. P}$: but by sim. triangles SA : EB (R) :: SV : EV, therefore $EV = \frac{R.SV}{SA} = \frac{R(SE + EV)}{SA}$ that is, by substitution, &c. $\frac{1}{\sin. C} = \left[\frac{1}{\sin. P} + \frac{1}{\sin. C}\right] \div \frac{\sin. D}{\sin. P}$ whence by reduction sin. C = sin. D — sin. P. Now since D never exceeds 16′ 18″, and P is always less than 9″, the arcs which measure the angles C, D, P, may be substituted for their sines, and we shall then have C = D — P; or, *half the vertical angle of the Earth's conic umbra is equal to the difference between the Sun's semi-diameter and hor. parallax**. Having thus found the angle BVE or C, we have $EV\left(= \frac{R}{\sin. C}\right) = \frac{R}{\sin D - \sin. P} = \frac{R}{D - P}$ very nearly: but in using this

* Many writers have obtained this result by supposing a straight line drawn from A to E, and then by taking the angle AES for the Sun's semi-diam., and EAB for his horiz. parall., to which they are *nearly* equal, they immediately have (Geom. prop. 12. cor. 1,) AES = EAV + AVE, or D = P + C, and C = D — P; but this method leaves us quite in uncertainty as to the amount of the error occasioned by using these near values of the semi-diam. and parallax: it is however inconsiderable as may be proved by computing the true value of C from the equation sin. C = sin. D — sin. P.

expression, if we estimate D and P in seconds we must substitute for the trig. rad. (1), the number of seconds it contains, viz. 206264″.8, and then we shall have $EV = \frac{206264.8\,R.}{D - P}$.

11. If for D we substitute in the last expression its three values when the Sun is in perigee, at mean distance, and in apogee, we shall obtain the following values for the distance of the vertex of the umbra from the Earth's centre, in terms of the terrestrial radius.

	axis of umbra.
⊙ in perigee	212.89 radii of ⊖
at mean distance . . .	216.53
in apogee	220.16

12. The greatest distance of the Moon from the Earth does not exceed 64 radii of the Earth, which being much less than the preceding quantities, shews that the Earth's shadow always extends much farther into space than the orbit of the Moon, so that if the plane of this orbit coincided with that of the ecliptic there would be an eclipse of the Moon every month. The distance of Mars from the Earth is upwards of 12000 radii of the Earth, hence the extreme end of the shadow falls very far short of the orbit of Mars, and consequently none of the heavenly bodies can be involved in the shadow besides the Moon.

To find the semi-diameter of the Earth's umbra where it is traversed by the Moon.

13. Let *mn* (plate 11, fig. 4,) be a portion of the Moon's orbit, and join E*m*. The angle ME*m* is equal to B*m*E − BVE, but B*m*E being the angle under which the Earth's radius appears at the orbit of the Moon, is equal to the Moon's horizontal parallax, and calling this *p*, we have ME*m* = *p* − C = (art. 10) P + *p* − D, that is, *the sum of the parallaxes of the Sun and Moon diminished by the Sun's semi-diam., is equal to the angular distance of the centre of the Moon from the axis of the umbra at the instant this centre enters the umbra.*

14. In Lunar Eclipses, it is found by observation that the dimensions of the Earth's shadow is greater than the rule in the last art. gives it. Mr. Vince says, "It is owing to the atmosphere of the Earth." Mr. Woodhouse says, "This circumstance is, with great appearance of probability, accounted for, by supposing those solar rays, that, from their direction, would glance by, and rase the Earth's surface, to be stopped and absorbed by the lower strata of the atmosphere. In such a case, the conical boundary of the Earth's shadow would be formed by certain exterior rays, and be larger."

There appears to be a difference of opinion about the quantity to be added to the semi-diameter of the shadow, found by the rule, to give the true one. M. Cassini gives 20″; M. Monnier 30″; and M. de la Hire, 60″. We think Mr. Mayer's rule the best, which says, "Add one 60th of the semi-diameter of the shadow found by the rule; or add as many seconds as the semi-diameter contains minutes."

No doubt many of the particles of light are absorbed in passing through the lower regions of the atmosphere; more particularly the weaker ones; such as the blue, &c. yet a great number of the stronger, such as the red, do pass; or we could never see the celestial bodies *in*, or *near* the horizon. The true cause, probably, lies in the refractive power of the atmosphere; for such rays as would pass near to, or a little above the surface of our globe, are bent down and fall upon the uneven and rough parts of the Earth, and are reflected in a variety of directions, or absorbed; consequently they never can pass on, so as to form a boundary to the umbra. But such rays as pass at a greater distance would be bent down in a curve towards the Earth; and having passed their nearest distance from the surface, would be bent back in a similar and equal curve, and would leave the atmosphere in the same direction they entered, on account of the extreme rarity of the upper regions of the air: hence, those rays which pass at a certain distance above the surface, form the extremity of the umbra, and of course increase its dimensions.

There is also another circumstance in lunar eclipses quite different from the above. If no rays were to pass into the conical shadow, the Moon would be quite invisible during her passage through the umbra; but this is not the case: she is visible during the whole time, and appears of a dusky copper colour: and as this is not to be accounted for by the ordinary principles of refraction, we must have recourse to other causes. Do not an innumerable number of atoms, or small particles of matter of various kinds float in the atmosphere? If light be a body, will not its particles strike against those atoms, and be turned out of their rectilinear course in various directions? If so, such particles of light as the red ones, which are not absorbed in the atmosphere, may be deflected into the shadow, fall upon the Moon, and thereby cause the appearance. This seems also to be the cause of twilight.

15. If to the above expression in art. 13. we add *d*, the semi-diam. of the Moon, we have P + *p* − D + *d* or P + *p* − (D − *d*) for *the angular distance of the Moon's limb from the axis of the umbra, at the instant she enters the umbra*, which *is* therefore *equal to the sum of the parallaxes of the Sun and Moon, minus the difference of their semi-diameters.*

The rules expressed by the formula P + p — D, and P + p — (D — d) have been used from the earliest periods by astronomers: rigorously the values are p — C, and p + d — C.

16. The apparent semi-diam. of the Earth's shadow at the orbit of the Moon, according as she is in apogee, at mean distance, or in perigee, may be computed by art. 13; and as the greatest apparent semi-diam. of the Moon is not more than 16′ 56″.5, it will be found that the semi-diam. of the terrestrial shadow always considerably exceeds it, hence, every time that the Moon passes through the centre of this shadow she will be totally eclipsed; and as the semi-diam. of the Earth's shadow is sometimes nearly equal to three semi-diameters of the Moon, and the Moon in the space of an hour moves over a space nearly equal to its diameter, it follows that the Moon may be wholly within the shadow, or may be totally eclipsed about two hours.

17. The angle mIn (Fig. 1.) is called the angle of the Earth's penumbra, its half is obviously equal to SIC = CES + ECI; now, since EC is very nearly a tangent to the Sun at C, the angle CES does not sensibly differ from the Sun's apparent semi-diam. and ECI or ECc (being the angle under which the Earth's rad. appears at the surface of the Sun) is extremely nearly equal to the Sun's hor. parallax, therefore half the angle of the Earth's penumbra is equal to D + P, or to the Sun's sem. diam. + his horizontal parallax.

18. The angle mEM = mIE + EmI, the latter of which being the Moon's hor. parall., we have mEM = D + P + p, that is, the app. sem. diam. of a section of the penumbra at the Moon's orbit is equal to the semi-diam. of the Sun + the sum of the hor. parallaxes of the Sun and Moon.

19. Since the app. sem. diam. of the Moon's penumbra is D + P + p, the distance of the centre of the Moon, and of the centre of the shadow, when the Moon first enters the penumbra is D + d + P + p.

20. Let us next suppose the Moon to be situated between the Sun and Earth, as in Fig. 5. and inquire into the magnitude of her shadow, &c. Now here the Moon is circumstanced exactly as the Earth has been in the preceding arts., and therefore a computation of the length of her shadow, &c., may be carried on in the same manner as has been done for the Earth, or rather may be deduced from the results already obtained, without any new investigations. Fig. 5 is supposed to be constructed in a similar manner to Fig. 4 already described: we shall first proceed

To find the length of the shadow projected by the Moon.

21. In art. 10, we found the sine of half the vert. angle of the Earth's shadow, or sin. C, to be equal to sin. D — sin. P, where D and P were the Sun's sem. diam. and hor. parall. seen from the Earth, therefore, to have the value of C for the Moon, or the angle BVM, we must substitute for D and P the sem. diam. and parallax of the Sun as they would appear from the Moon. Let r be the radius of the Moon, T and t the distances of the Earth and Moon from the Sun respectively: now the Sun's sem. diam. as seen from the Moon will be greater than from the Earth in the ratio of T : t; hence, for D we must write $\mathrm{D} \times \frac{\mathrm{T}}{t}$; the parallax of the Sun from the Earth will be to its parallax from the Moon directly as their radii, and inversely as the distances, that is, as Rt : rT, therefore for P we have to substitute $\mathrm{P} \times \frac{r\mathrm{T}}{\mathrm{R}t}$, whence sin. BVM = sin. $\mathrm{D} \times \frac{\mathrm{T}}{t}$ — sin. $\mathrm{P} \times \frac{r\mathrm{T}}{\mathrm{R}t}$ and the length of the shadow, MV, $= r \div \left[\text{sin. } \mathrm{D} \times \frac{\mathrm{T}}{t} - \text{sin. } \mathrm{P} \times \frac{r\mathrm{T}}{\mathrm{R}t}\right]$.

22. This formula for the angle BVM may be simplified, first by substituting the arcs for their sines, which will give $\mathrm{BVM} = \mathrm{D} \times \frac{\mathrm{T}}{t} - \mathrm{P} \times \frac{r\mathrm{T}}{\mathrm{R}t}$, and then, since the hor. parallaxes of the Sun and Moon are inversely as their distances from the Earth or P : p :: T — t : T, we have p — P : p :: t : T, and $\frac{\mathrm{T}}{t} = \frac{p}{p - \mathrm{P}}$, whence, by farther substitution, $\mathrm{BVM} = \left(\mathrm{D} - \mathrm{P} \times \frac{r}{\mathrm{R}}\right) \frac{p}{p - \mathrm{P}}$; also, because the sem. diam. of the Moon is to its hor. parall. as the radius of the Moon to that of the Earth, or d : p :: r : R, therefore $\frac{r}{\mathrm{R}} = \frac{d}{p}$, and BVM is consequently equal to $\left(\mathrm{D} - \mathrm{P} \times \frac{d}{p}\right) \frac{p}{p - \mathrm{P}} = \frac{p\mathrm{D} - \mathrm{P}d}{p - \mathrm{P}}$, and MV, the length of the shadow $= r \times \frac{p - \mathrm{P}}{p\mathrm{D} - \mathrm{P}d}$, or $= 206264.8\, r \times \frac{p - \mathrm{P}}{p\mathrm{D} - \mathrm{P}d}$, if we estimate the parallaxes and sem. diameters in seconds. The angle BVM has been sometimes taken by computers equal to the Sun's semi-diam., but this manifestly supposes V to coincide with E, the centre of the Earth, which is seldom the case: it appears however that the above more correct value for BVM can never differ much from the Sun's semi-diam., for, since P is always very small, if we neglect it the expression immediately becomes $\frac{p\mathrm{D}}{p}$ or D.

23. By means of the expression for MV in the last art. we find the length of the Moon's shadow to be as follows:

	Length of shadow	☽ *'s dist.*
☉ in apogee, ☽ in perigee . .	59.73 rad. of ⊖	55.90
☉ in perigee, ☽ in apogee . .	57.76	63.86

In the first of these cases it appears that the length of the shadow is greater than the Moon's distance from the Earth, and consequently the Sun may be sometimes totally eclipsed; in the latter position of the bodies the Moon's shadow does not reach the Earth, and therefore the Sun may also be eclipsed annularly.

To find the semi-diameter of the Moon's umbra at the Earth, viz. the angle rMV Fig. 5.

24. In art. 13, the semi-diam. of the Earth's umbra at the Moon was found to be $p - C$, C being half the angle at the vertex of the dark shadow, and p the Moon's horizontal parallax, or the angle under which the Earth's radius appears at the Moon: to adapt this to the present case, we must substitute for p, the angle under which the Moon's radius appears at the Earth, in other words, the Moon's semi-diameter, d, and then we shall have the semi-diam. of the Moon's umbra, or angle rMV $= d - C =$ (art. 22,) $d - \frac{pD - Pd}{p - P} = \frac{d - D}{p - P} \cdot p$ The semi-diam. of the Moon's umbra is often taken equal to $d - D$ which value is obtained thus: the angle BrM is equal to the two interior angles rMV, rVM, therefore rMV $=$ BrM $- r$VM, but BrM is nearly equal to the Moon's sem. diam. and rVM to the Sun's semi-diam. as was remarked at the end of art. 22, therefore rMV $= d - D$ nearly. We may observe, that if in the expression $\frac{d - D}{p - P} \cdot p$, we neglect P as being very small, it becomes $d - D$.

25. If we wish to find the distance of the centres of the Moon's shadow and of the Earth, at the instant the Earth's disk touches the Moon's shadow, we have only to increase the expression in the last art. by the apparent sem. diam. of the Earth seen from the Moon, that is, by the Moon's horizontal parallax; hence the distance alluded to is $p + \frac{d - D}{p - P} \cdot p$.

26. It was shewn in art. 17, that half the angle of the Earth's penumbra was equal to the Sun's semi-diam. + his horiz. parallax, from whence it follows, supposing the Earth and Moon to change places as before, that half the angle of the Moon's penumbra, or the angle mIV (Fig. 2.) is equal to the Sun's semi-diam. seen from the Moon, + the angle under which the Moon's radius appears at the Sun, or $= D \times \frac{T}{t} + P \times \frac{rT}{Rt} = \frac{Dp + dP}{p - P}$. This angle is frequently taken equal to the Sun's semi-diam. for the point I, on account of the smallness of the Moon compared with the Sun, will be near the Moon, and therefore comparatively (owing to the great distance of the Sun,) near the Earth, so that the angle CIS, or its equal mIV, is nearly equal to the Sun's semi-diameter.

27. From arts. 17, 18, it follows, that to find the apparent semi-diam. of a section of the Moon's penumbra, we have only to increase half the angle of the penumbra by the angle under which the Moon's radius appears at the Earth, that is, by her semi-diam., therefore apparent semi-diam. of Moon's penumbra $= d + \frac{Dp + dP}{p - P} = \frac{d + D}{p - P} \cdot p$. The apparent semi-diam. of the Moon's penumbra is often taken by astronomers equal to $d + D$, which they obtain thus: the angle mMV $= m$IV $+$ ImV, the former of these is nearly equal to the Sun's semi-diam. as was shewn at the end of the last art., the latter is the Moon's semi-diam. seen from the Earth's surface, and therefore nearly equal to D.

28. To find the angular distance of the centres of the Earth and of the Moon's shadow, when the Earth first enters the penumbra, we must increase the last expression by the angle under which the Earth's radius appears at the Moon, that is, by p, her horizontal parallax, which gives $p + \frac{d + D}{p - P} \cdot p$ for that distance.

29. *Problem.*—Supposing the Moon to be in the zenith, and her semi-diam. seen from the centre of the Earth to be a maximum, in which case it is 16′ 56″.4, and the Sun's semi-diam. to be a minimum, viz. 15′ 45″.5, it is required to find the actual diameter of the Moon's umbra on the Earth's surface.

The horizontal parallax of the Moon, corresponding to the given semi-diam., is by Tab. 32, 62′ 10″, and that of the Sun, 8″.6, also the augmentation of the Moon's semi-diam. on account of being in the zenith is by Tab. 35, 18″.6, hence her apparent semi-diam. from the Earth's surface is 17′ 15″.

Then first, to find the semi-diam. in seconds, of the umbra, by art. 24.

	17′ 15″		
	15 45.5		
$d - D$	1 29.5	$= 89''.5$	1.9518230
	62 10		
	8.6		
$p - P$	62 1.4	$= 3721''.4$	3.5707064
			$\bar{2}$ 3811166
p . . .	62 10	$= 3730''$	3.5717088
ang. rME, or sem. diam. ☽'s umb.		$89''.71$	1.9528254

Now, in Fig. 5, if MT be a tangent to the Earth, TME will be equal to the Moon's parall., and in the triangle MTE right angled at E, we shall have, taking the Earth's rad. = 1, sin. TME or sin. p : TE (1) :: rad. : ME = $\frac{\text{rad.}}{\text{sin. } p}$; and in the trian. MrE, rE(1) : sin. rME :: ME : sin. MrE = ME sin. rME = $\frac{\text{rad.} \times \text{sin. semi. diam. } \mathbb{D}\text{'s umb.}}{\text{sin. } p}$, whence rEM will be known, and its double will be the measure of rs in degrees.

rad.		10.0000000
sin. sem. diam. ☽'s umb.	1′ 29″.7	6.6383616
		16.6383616
sin. p.	° ′ ″ 1 2 10	8.2572600
sin. MrE	178 37 19	8.3811016
	1 29.7	
	178 38 48.7	
	180	
rEM	1 21 11.3	
	2	
rEs	2 42 22.6	

and taking the Earth's circumference at 24862 miles, it will be as 360° : 2° 42′ 22″.6 :: 24862 : 186.88 or 187 miles nearly, for the diameter of the Moon's shadow on the Earth, and this is the greatest breadth of the portion of the Earth's surface which can ever be obscured by the Moon's dark shadow.

30. *Problem.*—Supposing the Moon in the zenith as before, and her semi-diam. a minimum = 14′ 26″.5, and augmentation 13″.4, making 14′ 39″.9, also the Sun's sem. diam. a max. = 16′ 17″.8, at which time his parall. is 8″.9, it is required to determine the actual diameter of the Moon's penumbra at the Earth's surface. The equatorial horiz. parall. of the Moon to the given sem. diam. is 53′ 0″.

Here, we must have recourse to art. 27, to find, first, the sem. diam. in seconds of the Moon's penumbra.

	14′ 39″.9	
	16 17.8	
d + D	30 57.7 = 1857″.7	3.2689756
	53 0	
	8.9	
p — P	52 51.1 = 3171″.1	3.5012099
		1.7677657
p . .	53 0 = 3180″	3.5024271
ang. mME or sem. diam. ☽'s pen.	1862″.9	3.2701928

Then by an investigation exactly similar to that in the last art., we obtain sin. MmE (see fig. 2.) = $\frac{\text{rad.} \times \text{sin. sem. diam. } \mathbb{D}\text{'s penum.}}{\text{sin. } p.}$

rad.		10.0000000
sin. sem. diam. ☽'s. pen.	31′ 2″.9	7.9557585
		17.9557585
sin. p.	53′ 0″	8.1879848
sin. MmE . . .	° ′ ″ 144 8 17.4	9.7677737
	31 2.9	
	144 39 20.3	
	180	
mEM	35 20 39.7	
	2	
mEn	70 41 19.4	

then taking 24862 miles for the Earth's circumference as before, we have, as 360° : 70° 41′ 19″.4 :: 24862 : 4881.8 the diameter of a section of the penumbral cone where it meets the Earth, and since we have supposed the Moon to be in apogee, and the Sun in perigee, this will be the greatest diameter which the penumbra can ever attain.

31. In the two preceding arts. we have supposed the Sun and Moon to be in the zenith, at the time of the greatest obscuration, when the eclipse is central and total; but the common centre of the dark shadow and penumbra in its progress over the Earth's surface necessarily moves over surfaces variously inclined to the axis of the cone: thus, when the Sun is centrally eclipsed in the zenith, then the axis of the cone and penumbra is at right angles to the plane of the horizon, and the section of the dark cone and penumbra is a circle on the plane of the horizon: but when the Sun is centrally eclipsed between the zenith and horizon, then the plane of projection, viz. the plane of the horizon has an oblique position to the axis of the cone, and the dark shadow and penumbra will not be circles on the plane of projection, but elliptical figures. For instance, suppose the Sun to be centrally and totally eclipsed when his altitude above the horizon is 30°; then the dark shadow on the plane of the sensible horizon will be an ellipsis, having its conjugate and transverse diameters in the ratio of the sine of the altitude, to the radius which in the present supposition are as 1 : 2; so that if the diameter of the dark shadow at the Earth perpendicular to the axis of the cone were 80 miles, then the shadow would extend in length 160 miles, and an elliptical space on the sensible horizon of 80 miles in breadth, and 160 in length,

would be involved in total obscurity: the space on the Earth's surface involved in darkness would be somewhat greater than this, on account of its sphericity, but unless the Sun be near the horizon the difference is inconsiderable.

To determine whether an eclipse will, or will not, take place at the time of full or new Moon.

First.—For a Lunar Eclipse.

32. From the sum of the Sun and Moon's horizontal parallaxes at the time of full Moon, subtract the Sun's semi-diameter, the remainder is the semi-diameter of the Earth's shadow at the Moon (art. 13), to this add the Moon's semi-diameter; then if the sum be *greater* than the Moon's latitude at the time, a *lunar eclipse* must happen, but if the sum be *less* than the Moon's latitude, *no eclipse* can occur; because the Moon in this case, at the time she is opposite to the Sun, will pass either above or below the Earth's shadow. When the Moon's latitude exceeds the aforesaid sum, the excess shews the distance between the shadow and the Moon's limb at the time she is passing it, and, when an eclipse occurs, the excess of the said sum above the Moon's latitude shews the parts *deficient*, or how far the Moon's nearer limb enters the shadow. If the parts deficient be exactly equal to the Moon's diameter, the eclipse will just be total, but the total obscuration of no continuance; but if the deficient parts be greater than the Moon's diameter, the eclipse will be total, with continuance; and lastly, if the parts deficient be less than the Moon's diameter, the eclipse will be only partial.

33. The different eclipses which may happen of the Moon are shewn in plate 12, fig. 1., which will also by mere inspection shew the reason of the precepts in this art. The line CL represents a section of the plane of the ecliptic, OR the section of the Moon's orbit, meeting the ecliptic in the node N, the circle SH is supposed to be a section of the Earth's umbra, taken at the distance of the Moon from the Earth, and M the Moon in her orbit. Hence, if the opposition happen as in position 1, the Moon will just pass by the shadow of the Earth without entering it, and there will be no eclipse; in this case, E*v*, the distance of the centres of the Moon and shadow, is just equal to the sum of the semi-diameters of the umbra and of the Moon. In position 2, part of the Moon will pass through the Earth's shadow, and there will be a *partial* eclipse. In position 3, the whole of the Moon passes through the Earth's shadow, and there is a *total* eclipse; and in position 4, the centre of the Moon coincides with the centre of the Earth's shadow, and there is a *total* and *central* eclipse.

Secondly.—For a Solar Eclipse.

34. Find, at the time of ecliptic conjunction or new Moon, the apparent semi-diameter of the Moon's penumbra, by art. 27, to this add the semi-diam. of the Earth's disc, which is equal to the Moon's hor. parall. then if the sum exceed the Moon's latitude, there will be an eclipse of the Sun; but if the Moon's latitude exceed the said sum, no eclipse of the Sun can happen at that conjunction: moreover, when an eclipse does happen, if the semi-diam. of the Earth's disc be greater than the Moon's latitude, the Sun will be *centrally* eclipsed to *some* part of the Earth, and if the Moon's apparent semi-diam. exceed the Sun's semi-diam. at the same time, then the Sun will be *centrally* and *totally* eclipsed to those parts of the Earth over which the dark shadow passes. Lastly, if the *difference* between the semi-diameter of the Earth's disc, and the semi-diam. of the Moon's penumbra exceed the Moon's latitude, then the whole of the penumbra will pass over the Earth's disc. The reason of these precepts will be obvious by a reference to plate 12, fig. 2, which shews the different eclipses of the Sun.

35 .Let CL represent a section of the Earth's orbit, OR the line described by the centre of the Moon's umbra and penumbra at the Earth, N the Moon's node; SF the Earth, E its centre, *pn* the section of the Moon's penumbra, *u*, that of the umbra. Then in position 1, the penumbra, *pn* just touches the Earth, without falling upon it, and therefore there will be no eclipse. In position 2, the penumbra, *pn*, falls upon the Earth, but the umbra, *u*, does not; therefore there will be a *partial* eclipse where the penumbra falls, but no total eclipse. In position 3, both the penumbra and umbra fall upon the Earth; hence, where the penumbra falls, there will be a *partial* eclipse, and where the umbra falls there will be a *total* eclipse; but the other parts of the Earth will have no eclipse.

36. The aforegoing rules are the only certain criteria by which in all cases we can ascertain whether there will or will not be an eclipse at the time of new or full Moon: many astronomical writers have enlarged upon what they call the *ecliptic limits*, that is, the Sun or Earth's distance from the Moon's node at the time of opposition or conjunction, from which they determine as to the occurrence of an eclipse or not; but, after all they have done on this subject, these limits are attended with considerable uncertainty; M. Delambre says, that when the Sun's mean distance from the node is less than 13° 33′ at time of conjunction there will certainly be an eclipse of the Sun, but none if that distance be as great as 19° 44′. For the Moon he makes the limits 7° 47′, and 13° 21′. Between the limits given in each case,

the eclipse is doubtful, and the uncertainty must then be removed by the application of the rules just given.

37. The precepts in arts. 32, 34, suppose that the times of new and full Moon are known: now since we have explained in the former part of this work, the manner of computing the various circumstances relating to the Sun and Moon usually registered in an Ephemeris, we are at liberty to refer to such an Ephemeris to ascertain the time of new or full Moon, which will of course be at the instant we find the Sun and Moon's longitude equal, or differing by six signs*. If it be the Nautical Almanac to which we refer, the parallaxes and semi-diameters of the luminaries may also be easily found at the instant of conjunction or opposition, by proportionating for those times, and then by the precepts already alluded to, we may quickly determine whether an eclipse will happen or not

38. We may farther observe, that the times of the happening of eclipses may be very nearly ascertained by means of the period called the *Chaldean Saros;* which is a period of time in which the eclipses recur in the same order: as the Moon's nodes move backward $19\frac{1}{3}$ degrees every year, they pass through every point of the ecliptic in 18 years and 225 days, and this would be the regular period of the return of eclipses, if any complete number of lunations were finished without a fraction; but this is not the case. However, in 223 mean lunations, after the Sun, Moon, and nodes have been once in conjunction, they return so nearly to the same state again, that the same node which was in conjunction with the Sun and Moon at the commencement of the first of these lunations, will only be 28′ 12″ distant from the line of conjunction of the Sun and Moon again, when the last of these lunations is completed; and in this period therefore there will be a regular return of eclipses for many ages. Hence, if we have the mean times of all eclipses for any period of 19 years, we may find the mean time of an eclipse similar to any one of these, by adding to the time of its happening 223 mean lunations, viz. 18 Julian years, 11 days, 7 hours, 43 minutes, 20 seconds, when the last day of February in leap years is 4 times included, or a day less when it occurs 5 times.

Of the calculation of eclipses.

39. By some one of the rules in arts. 32, 34, 37, 38, it may be found that there are two eclipses in the month of September, 1820: the first is an eclipse of the Sun, on Sept. 7th, the middle of which happens at Greenwich very nearly to two o'clock in the afternoon; the second is an eclipse of the Moon on Sept. 22d, the middle happening at about three quarters of an hour past six in the morning. We shall take these two eclipses as examples, and compute the various circumstances relative to them, beginning with the

Calculation of the lunar eclipse, Sept. 22d, 1820.

40. In order to calculate an eclipse of the Moon with accuracy, the first step is, to find correctly the Sun's true longitude and semi-diameter, for the noon and midnight immediately preceding, and the same for the noon and midnight immediately following the opposition of the Sun and Moon; we must likewise find the Moon's true longitude, latitude, horizontal parallax, and semi-diameter for the same times, all which may be done by the precepts and tables already given in this work, and in the present example these several requisites being so computed, will be as in the following table.

1820, Sept.		☉'s long. + 6s. or long. ☋'s shad.	☽'s longitude.	☽'s latitude.	☽'s sem. dia.	☽'s hor. pa.	☉'s sem. d.
		s. ° ′ ″	s. ° ′ ″	s. ° ′ ″	′ ″	′ ″	′ ″
21st.	noon	11 28 22 34	11 17 14 23	0 31 27 S.	16 44.4	61 19.9	15 58.4
	midn.	11 28 51 56	11 24 50 24	0 10 45 N.	16 44.9	61 21.6	15 58.6
22d.	noon	11 29 21 19	0 2 26 17	0 52 44	16 43.9	61 18.0	15 58.7
	midn.	11 29 50 42	0 10 0 48	1 33 40	16 41.7	61 9.9	15 58.8

41. These longitudes, &c. must now be interpolated for two or three hours before and after the middle of the eclipse, in order to get the horary motion of the bodies, and the time of opposition: to

* The time of new or full Moon is, more accurately, at the instant of orbit conjunction, or opposition, that is, when the Sun or Earth is at the same distance from the node as the Moon is, for then the three bodies are more nearly in a *right line* than at the ecliptic conjunction or opposition; but it does not seem necessary to attend to this nicety in the calculation of eclipses, which would make scarcely any difference in the final result: if any one, however, should be desirous of knowing the orbit opposition, he may determine it by reducing the Moon's longitude in the ecliptic to that in the orbit, for 2 or 3 hours before and after the time of full Moon, and then knowing the place of the Earth at the same times, he may, by proportionating, find the orbit opposition.

interpolate the Sun's longitude it will be sufficient to take proportional parts of the first differences, as it appears that his motion is very nearly uniform, but for the Moon's longitude and latitude, the proportional parts of the first differences must be corrected by the equation of second difference, taken from table XXXVII, lunar tables, in the manner already explained at p. 217. The interpolations being effected, give the following results:

1820, Sept. 22.	☉'s long. + 6s.	☽'s longitude.	☽'s latitude.	☽'s sem. dia.	☽'s hor. p.	☉'s sem. d.
	s. ° ′ ″	s. ° ′ ″	° ′ ″	′ ″	′ ″	
5h. A. M.	11 29 4 11	11 28 0 27	0 28 19 N.	16 44.5	61 20.7	15 58.6
6	11 29 6 37	11 28 38 26	0 31 49	16 44.4	61 20.4	15 58.6
7	11 29 9 4	11 29 16 25	0 35 19	16 44.3	61 20.1	15 58.7
8	11 29 11 31	11 29 54 24	0 38 49	16 44.2	61 19.7	15 58.7
9	11 29 13 58	0 0 32 23	0 42 18	16 44.2	61 19.3	15 58.7

42. The true time of ecliptic opposition, and the Moon's latitude, parallax, &c. at that time, are next to be determined, which may be obtained by proportionating between the results in the last table; by an inspection of this table it appears that the longitudes of the Earth's shadow and of the Moon, become equal between 6 and 7 o'clock, whence the time of opposition will be found, thus;

	s. ° ′ ″
long. ☋'s shad. at 6h. . . .	11 29 6 37
7h. . . .	11 29 9 4
hor. mot. ☋'s shadow . . .	0 0 2 27
☽'s long. in eclip. at 6h. . . .	11 28 38 26
7h. . . .	11 29 16 25
☽'s hor. mot. in eclip. . . .	0 0 37 59
	0 0 2 27
☽'s hor. mot. from ☋'s shad. .	0 0 35 32
long. ☋'s shad. at 6h. . . .	11 29 6 37
long. ☽ in ecliptic do. . . .	11 28 38 26
diff. from ecl. oppos.	0 0 28 11

Then as 35′ 32″ : 28′ 11″ :: 1h. : 47m. 35.3s., hence the true time of ecliptic opposition or of full Moon is, on Sept. 22d, at 6h. 47m. 35.3s. A. M.

The Moon's lat. at 6h. being 31′ 49″ N., and her motion in latitude from 6 to 7 hours being 3′ 30″, therefore as 1h. : 47m. 35.3s. :: 3′30″ : 2′ 46″, the Moon's change in latitude in 47m. 35.3s., which added to her lat. at 6h. because the lat. is increasing, gives 34′ 35″ N. for the Moon's lat. at the instant of ecliptic opposition.

The hor. parallax of the Moon at the same instant is found in a similar manner, to be 61′ 20″.2; her semi-diam. and likewise that of the Sun, it is obvious, may be taken the same as at 7h.

43. The inclination of the Moon's *relative orbit*, (that is, of her apparent path, as seen from the Earth) to the ecliptic must next be found. To explain this, let CM (plate 12, fig. 3.) represent the Moon's horary motion in longitude on the ecliptic, SM, the hor. motion of the Sun, or rather of the Earth's shadow, *ma*, the Moon's horary motion in latitude; draw C*c*, S*s*, perpendicular, and *mc* parallel to the ecliptic, take *sb* = *ma*, and join *ac*, and *bc*N: then *ca* is the Moon's true orbit, and *cb* her relative, or apparent orbit, for on account of the Earth's shadow moving the same way as the Moon, the Moon's motion in longitude in an hour is apparently diminished by SM, the Sun's motion in an hour, and as there is no apparent diminution of the Moon's latitude, her motion will appear to be in the line *cb*; hence *cs*, the diff. of the horary motions in longitude : *sb*, the Moon's horary motion in latitude :: radius : tang. *bcs*, the inclination of the relative orbit; this is the way in which the inclination of the rel. orbit is generally determined, but it is not strictly correct, because the Moon's path is not a right line, but an arc of a great circle, and therefore the angle *bcs* is continually varying. It is more correct to solve *bcs* as a right angled spherical triangle, using the horary motions at about the middle of the eclipse: we shall compute it both ways, from which it will appear that the difference is insensible. In the present case then there are given *cs* = 35′ 32″, and *bs* = 3′ 30″ as already found, therefore,

as *cs*, 2132″	3.3287872
is to *bs*, 210″	2.3222193
so is rad.	10.0000000
to tang. *bcs*, 5° 37′ 32″	8.9934321

or, by case 3, right ang. sph. trig.

As rad.	10.0000000
is to cot. *bs*, 3′ 30″	12.9922057
so is sin *cs*, 35′ 32″	8.0143543
to cot. *bcs* 5° 37′ 32″ as before	11.0065600

44. By a bare inspection of the figure last referred to, it appears that the apparent horary motion in the relative orbit is quicker than that in the ecliptic, and that to find it, we have only to say, as radius : sec. inclin. of relative orbit :: ☽'s hor. motion from ☉ in ecliptic : ☽'s hor. motion in rel. orbit, that is,

As rad.	10.0000000
is to sec. 5° 37′ 32″	10.0020967
so is 0° 35′ 32″ = 2132″ . . .	3.3287872
to 0° 35′ 42″ = 2142″	3.3308839

so that the Moon's appar. hor. motion on the relative orbit is 35′ 42″. The cos. of inclin. of the relative orbit : rad. always as 199 to 200 nearly, so that if we increase the horary motion in the ecliptic by its 199th part, we shall have very nearly the horary motion in the rel. orbit.

45. To explain the principles upon which the remaining part of the calculation is founded, let A*q*B (plate 12, fig. 4.) be half the section of the Earth's shadow at the distance of the Moon, made by a plane perpendicular to the axis of the shadow, C the centre of the section, ACB the ecliptic, NL the Moon's relative orbit, and suppose the circles described about *a*, *m*, *b*, to be the Moon's disc at the beginning, middle, and end of the eclipse: draw C*n* perpendicular to AB. Now since *m* is supposed to be the Moon's centre at the middle of the eclipse, C*m* will be perpendicular to NL, and in the right angled triangle C*nm* we know C*n*, the Moon's latitude at the instant of ecliptic opposition, and the angle C*nm*, being the complement of the angle which the Moon's relative orbit makes with the ecliptic; hence we can find *nm* which is called the *reduction*, and also C*m*, the nearest distance of the centres of the Moon and shadow: moreover, because the horary motion of the Moon upon its relative orbit is known, we can, by proportion, find the time of describing *mn*, the difference between which and the time of ecliptic opposition at *n*, will be the time of the middle of the eclipse at *m*. Next, in the triangle C*ma*, right angled at *m*, we know C*m*, and also C*a*, which is the sum of the semi-diameters of the Earth's shadow and the Moon, hence the side *ma*, may be found, and likewise the time of describing it from the known horary motion in the rel. orbit; then this time subtracted from the time at *m*, gives the time of the beginning, and added, gives the time of the end of the eclipse. The greatest magnitude of the eclipse is represented by *pq*, this being the greatest portion of the Moon's diameter involved in the Earth's shadow: this distance is measured in 12th parts of the diameter of the Moon called digits; to find it, we have C*m*, the difference between which and C*q*, gives *mq*, which added to *mp*, (the Moon's sem. diam.) if *m* be within the shadow, gives *pq*, but if *m* be without the shadow, the difference between *mp* and *mq*, will be *pq*. We may now proceed with the computation.

In the right ang. triangle C*mn*,

As rad.					10.0000000
is to sin. *n*C*m*, 5° 37′ 32″ . . .					8.9913448
so is C*n*, . . .	0	34	35	= 2075″	3.3170181
to red. *nm* . .	0	3	23	= 203″	2.3083629
And as rad.					10.0000000
is to cos. *n*C*m*, ...					9.9979033
so is C*n*, ...					3.3170181
to C*m*, 2065″					3.3149214

The horary motion in the relative orbit has been already found to be 35′ 42″, therefore, as 35′ 42″ : 3′ 23″ :: 1*h*. : 5*m*. 41*s*. the time of describing *mn*, then

	h.	*m*.	*s*.
From time of ecl. opp.	6	47	35
subtract		5	41
rem. time mid. of eclipse . . .	6	41	54

The semi-diam. of the Earth's shadow is found by art. 13 and 14, thus,

☽'s hor. par. at ecl. opp.	61′ 20″.2	
☉'s do.	8.8	
	61 29.0	
☉'s sem. diam. . . .	15 58.7	
	45 30.3	
add about 1-60th, or . .	45.4	
sem. diam. ⊖'s shad. .	46 15.7	
☽'s sem. diam. . . .	16 44.3	
C*a*	63 0.0	= 3780″
C*m*		= 2065
	sum	5845
	diff.	1715

$\surd[(Ca + Cm)(Ca - Cm)] = ma$

$Ca + Cm$, 5845	3.7667845
$Ca - Cm$, 1715	3.2342641
	2)7.0010486
ma, 3166	3.5005243

Then as hor. mot. rel. orb. 35′ 42″, or 2142″ : 3166″ : 1*h*. : 1*h*. 28*m*. 41*s*. the time of the Moon's describing *ma*.

	h.	*m*.	*s*.
	6	41	54
	1	28	41
Beg. of eclipse,	5	13	13
end, ...	8	10	35

To find the digits eclipsed, we have

Cq, the sem. diam. ⊖'s shad. . . .	46′	16″
Cm, ...	34	25
mq, ...	11	51
mp, ☽'s sem. diam.	16	44
pq, parts def.	28	35

Then as ☽'s diam. 33′ 28″ : 28′ 35″ :: 12 dig. : 10*d*. 15*m*. the quantity eclipsed.

46. Collecting these results, it appears that the Moon was eclipsed on Sept. 22d. 1820.

	h.	*m*.	*s*.	
Beginning of eclipse, . .	5	13	13	A.M.ap.time
middle, ...	6	41	54	[at Greenwich
end, ...	8	10	35	
ecliptic opp. or full moon at	6	47	35	

☽'s lat. at that time . .	34′ 35″ N.
☽'s sem. diam. do. . .	16 44.3
☽'s hor. parall. do. . .	61 20.2
⊙'s sem. diam. do. . .	15 58.7
⊙'s hor. parall. do. . .	8.8
☽'s hor. mot. from ⊖'s sha.	35 32
ang. of rel. orb. with ecl.	5° 37 32

Digits eclipsed, 10° 15′ from north side of ⊖'s shadow, or on ☽'s southern limb.

As the Moon sets on this day at Greenwich at about 6*h*, it appears that this eclipse was only visible for three quarters of an hour from the beginning, at Greenwich.

47. It will be observed, that in this calculation the horary motions of the Moon in longitude and latitude have been assumed equable during the time of the eclipse, this is not strictly the case, but as the motions at the time of the middle of the eclipse have been taken, the error produced is extremely small, and as the lunar tables, notwithstanding the great perfection to which they have been brought, are still subject to an error of a few seconds, it would be needless to attempt farther accuracy.

48. When the principal elements of a lunar eclipse are obtained, we may find the time of beginning and end, and the digits eclipsed by construction, but it is nearly as soon done, and far more accurately, by calculation, we shall however now add for the sake of those readers who may wish it, that which is sometimes termed the *geometrical construction* of lunar eclipses.

From any convenient scale of equal parts, take in your compasses the semi-diameter of the Earth's shadow = (in this exam.) 46′ 16″, and describe the circle H*s*TR; (plate 12, fig. 5), with the sum of the semi-diameters of the Moon and shadow = 63′ 0″, describe the outer circle PFN; through the centre C draw the right line PE to represent the ecliptic, or way of the centre of the Earth's shadow; from C draw CF at right angles to PE; at the point C make the angle FCG = 5° 37′ 32″, the Moon's visible path with the ecliptic; from the same scale of equal parts take off the Moon's latitude at the time of full Moon = 34′ 35″, and set it on the right line CG, from C to L; through L draw AB at right angles to CG; then AB represents the Moon's visible path with the ecliptic, CG its axis, and CF the axis of the ecliptic, making the same angle of 5° 37′ 32″ with each other, which AB and PE if produced towards A and P would make: next join AC and BC, and with the Moon's semi-diameter 16′ 44″ taken from the scale, describe, about the points A, L, and B, as centres, the three small circles, which represent the Moon, at the beginning, middle, and end of the eclipse: when the Moon's centre arrives at A, the eclipse begins, and her limb touches the shadow in the point O, after which it enters the shadow; as she moves from A towards L her limb enters farther into the shadow, and the instant of greatest obscuration or middle of the eclipse is when her centre is at L; lastly, as she moves from L towards B, the eclipse diminishes, and when her centre comes to B it ends, her limb then just touching the Earth's shadow in the point T.

If we now measure AL on the same scale of equal parts used in the projection, it will be found equal to 52′ 46″, which is the Moon's motion in her visible path, for half the continuance of the eclipse; to find the correspondent motion in longitude, from the points A, L, and B, draw the right lines AI, L*n*, and BK parallel to CF, or perpendicular to the ecliptic PE; and these represent the Moon's latitude, at the beginning, middle, and end of the eclipse: also through the point A draw A*a* parallel to PE: then it is manifest that A*a* = KI, is the change in motion

in longitude of the Moon's centre from that of the shadow, or the Moon from the Sun, during the continuance of the eclipse; and the half thereof, AZ or I*n*, is the motion in longitude for half the time of continuance: the point *n* therefore on the ecliptic, corresponds with the time of the middle of the eclipse, or to 6*h*. 41*m*. 54*s*., whence to find the time of beginning and end, take from the scale, the horary motion of the Moon from the Earth's shadow, viz. 35′ 32″, divide it into 60 parts (if the scale be large enough), representing minutes of time, and set 41*m*. 54*s*. to the right, and 18*m*. 6*s*. to the left of the point *n*, and you will thus have the situation of the hours VI and VII, then set the other hours backwards towards P, and forwards towards E, marking every 5 or 10 minutes, and the situation of the points I and K on this scale of time, will indicate the instants of the beginning and end of the eclipse, the same as already found by calculation. And if the Moon's diameter X*r* be divided into 12 equal parts, then X*s*, the part obscured at the time of the middle of the eclipse, will be found to contain 10¼ of those parts, or 10 digits, 15 minutes, and the remainder, *sr*, which is free from shadow, 1¾, or 1 digit, 45 minutes.

49. The magnitude and duration of a lunar eclipse are the same in every part of the Earth, or in every part of space where the whole is visible: it is to the inhabitants of the Moon alone that any difference can arise; on that half of the Moon's surface which is always turned towards the Earth, the eclipse is total to all the parts involved in the shadow, and to the other half, no eclipse can ever happen. With respect to those inhabitants of the Earth to whom a lunar eclipse is visible, those towards the east will see the several phenomena sooner, and those towards the west later, by a difference of one hour for every 15° difference of longitude between the places.

Of Solar Eclipses.

50. In the computations relative to lunar eclipses, it is not required to ascertain whether any assigned spot on the Moon's surface will be covered by the Earth's shadow, or how long the obscuration will continue: in the ingress the instant of contact is merely determined, and no inquiry made as to the position of the point of contact relatively to any fixed point of the Moon's equator; the lunar latitude and longitude of the point of contact is a matter quite unimportant to an observer on the Earth's surface. To an observer however at the Moon, the case would be different: to such an one, the eclipse does not necessarily begin when the Earth's shadow comes in contact with the Moon's disk, but when it begins to obscure his station. Now an observer of a solar eclipse on the Earth's surface, is in the same predicament as such a fictitious observer at the Moon—he is desirous of knowing when, and how long the shadow of the Moon will obscure his station on the Earth, or of some point having an assigned latitude and longitude. In consequence of this, the computation of solar eclipses (when referred to an assigned point of the Earth's surface) is far more laborious than that of lunar eclipses, for the appearance of the eclipse changes with the position of the observer, and his position relatively both to the Moon, and Sun, is changing every instant in consequence of the diurnal motion of the Earth: moreover the Moon on account of her proximity to the Earth has a very considerable parallax, or, in other words, would appear in a very different part of the heavens when viewed from the Earth's centre, than when seen from certain points on its surface, the effect of which is to modify very materially that which would be the appearance of an eclipse from the Earth's centre, and it is very possible on account of the Moon's parallax, for the Sun to be eclipsed to a spectator on one part of the Earth, and to be entirely visible at the same instant on another part, as will appear from the next art.

51. To render this apparent, let us suppose T to be the centre of the Earth (plate 12, fig. 6.), O a point on its surface, whose zenith is on the radius TO continued to the sphere of the Moon, or of the fixed stars. Let S be the centre of the Sun, the angular distance of which from the zenith will be ZTS from the Earth's centre, and ZOS from the observer at O; then if the Moon is in the same vertical circle with the Sun, and O, L, S, are in the same straight line, the observer will see the centre of the Moon (L), on the centre of the Sun, and if we suppose the apparent diameters of the two bodies to be equal (which they always are *nearly*) their limbs will just cover each other, being situated in the direction of the lines O*a*, O*b*, and the eclipse will be total; but if at the same time the Sun were viewed from the Earth's centre, and consequently on the lines T*a*, T*b*, its whole surface would be visible, and observers situated at A and B would have no eclipse; the Sun would be in their zenith, and the Moon quite clear of him: to observers at D and E, the angle between the limbs of the Sun and Moon would be still greater.

52. As the lunar tables give the place of the Moon seen from the Earth's *centre*, an auxiliary calculation becomes necessary when we wish to ascertain her longitude and latitude as viewed from any point of the Earth's surface, in other words when we have to determine her parallax, that is, as the term imports, her change of place.—The difference between her longitude and latitude as seen from the centre of the Earth, and by an observer on its surface, is called

her parallax in longitude and latitude respectively. The investigation of rules for the determination of these parallaxes is one of the most difficult, and their application one of the most laborious parts of practical astronomy.

53. The most simple kind of parallax, is that difference of *altitude* between a body as seen from the centre of the Earth, and in the same vertical circle on its surface; this change of altitude is proportional to the sine of the apparent zenith distance directly, and the distance of the body from the centre of the Earth inversely. There can be no doubt this was the first kind of parallax considered by astronomers, and it would naturally lead them to ascertain what would be the corresponding change in the horary angle or right ascension of the body, and in its polar distance or declination, and having found rules for this purpose, they then proceeded to determine the parallaxes in longitude and latitude, as is quite clear from the nature of the rules which have been handed down to us for this purpose. In the rules for finding the parallaxes in right ascension and declination, the altitude of the pole, or its complement, viz. the angle which the equator makes with the horizon is a main and fundamental datum, and if the body is to be referred to any other circle instead of the equinoctial, and we can find the angle which that circle makes with the horizon, it is quite evident that any rules which will ascertain the parallaxes in right ascension and declination, will equally enable us to find the parallaxes in reference to any other circle, provided we substitute for the angle between the equator and horizon, the angle which this other circle makes with the horizon: therefore to find the parallaxes in latitude and longitude at any instant, it would be necessary first to determine the angle which the ecliptic makes with the horizon at that time, or which is the same thing, the altitude of the highest point of the ecliptic above the horizon, and the situation of that point with reference to some known or fixed point, such as ♈ of the ecliptic: accordingly in all the methods which have been given for finding the parallaxes in latitude and longitude, the determination of this highest point of the ecliptic, or, as it is called, the altitude of the nonagesimal and its longitude, are preliminary steps, the rules for which are given in art. 55.

54. As the Moon is (astronomically speaking) very near the Earth, and has in consequence a considerable parallax, it becomes necessary not only to make a proper allowance for her parallax in latitude and longitude deduced from the magnitude of the Earth considered as a sphere, but also to make a proper allowance for the small variation of the figure of the Earth from a sphere, that is, to consider the Earth, as it is in fact, a spheroid. Now the observed latitude of a place is the angle measured in a vertical plane, between the sensible horizon and the elevated pole; this sensible horizon is a plane conceived to be at right angles to the plumb line at the place of observation, or at right angles to a line which is perpendicular to the Earth's surface, but as this line will not tend to the centre of the spheroid, the latitude thus found is too large, and requires to be reduced by a quantity equal to the angle which a perpendicular to the Earth's surface at the place of observation makes with the Earth's radius belonging to that point, or, as it is technically called, by the *angle of the vertical:* this reduction according to two hypotheses of the compression of the earth at the poles, is put down in Table XXXIV, Lun. tab. which only requires to be entered with the observed latitude as an argument, and opposite is found the quantity, which taken from the *observed*, leaves what is termed the *reduced latitude.*

55. *Problem.*—Given the apparent time at a place, the Sun's right ascension, the latitude of the place, and the obliquity of the ecliptic, to find the altitude and longitude of the nonagesimal.

Find the reduced lat. (art. 54, and table XXXIV.) also add the hour from noon reduced to degrees, &c. to the Sun's right ascension and the sum (rejecting 360° if it exceed this) is the right ascension of the mid-heaven.

Let a = true alt. of the pole, or reduced lat.
q = obliq. of the ecliptic,
m = ☊ mid-heaven,
N = long. of nonagesimal,
A = alt. do.

Then tan. N = cos. q tan. m + sin. q sec. m. tan. a. and sin. A = cos. m cos. a sec. N.

In the torrid zone, A may exceed 90°, in which case we may use the formula

cos. A = cos. q sin. a — sin. q cos. a sin. m,

which will not be subject to any ambiguity.

56. *Problem.*—Given the latitude of a place, the longitude and altitude of the nonagesimal, together with the longitude, latitude, and horizontal parallax of the Moon as seen from the Earth's centre; to find the Moon's parallax in longitude and latitude.

In addition to the symbols before employed,

let M = ☽'s longitude,
L = ☽'s parall. in long.
l = do. in lat.
r = ☽'s horizontal parallax,
$\triangle$ = ☽'s lat.
$\triangle'$ = comp. ☽'s lat. or ☽'s dist. from pole of ecliptic.

Then $\tan. L = \frac{\sin. r \sin. A \sec. \triangle \sin. (M - N)}{1 - \sin. r \sin. A \sec. \triangle \cos. (M - N)}$ [formula 1.]

or, $L = \text{sin. } r \text{ sin. A sec. } \triangle \frac{\text{sin (M — N)}}{\text{sin. } 1''} + (\text{sin. } r \text{ sin. A sec. } \triangle)^2 . \frac{\text{sin. 2 (M — N)}}{\text{sin. } 2''} + \&c.$ [formula 2.]

And if tan. A cos. (M — N + ½ L) sec. ½ L be put = tan. x

$$\text{tan. } l = \frac{\text{sin. } r \text{ cos. A sec. } x \text{ sin. } (\triangle' - x)}{1 - \text{sin. } r \text{ cos. A sec. } x \text{ cos. } (\triangle' - x)}$$

[formula 1.]

or $l = \text{sin. } r \text{ cos. A sec. } x \frac{\text{sin } (\triangle' - x)}{\text{sin. } 1''} + (\text{sin. } r \text{ cos. A sec. } x)^2 . \frac{\text{sin. 2 } (\triangle' - x)}{\text{sin. } 2''} + \&c.$ [formula 2.]

57. In using these formula, regard is to be had to the *algebraic signs* of the trigonometrical lines in the different quadrants of the circle, and by adhering to the customary rules for these signs, the calculation of the parallaxes by means of the above expressions will we conceive be much more easy than by most of the methods hitherto given: the best of these rules are divided into a number of cases, sometimes requiring the addition, and sometimes the subtraction of arcs, according as the Moon is in the ascending or descending signs, and again changing according as her latitude is north or south, with many other variations in certain circumstances, which not being connected by any general principle render their application not a little perplexing.

That none of our readers may have to refer to another book for the signs of the trigonometrical quantities, we here insert a table of their values, for doing which we ought perhaps to apologize to some amongst them, who from their intimate acquaintance with analytic trig. stand in no need of such a remembrancer.

		sin.	cos.	tan.	cot.	sec.	cose.
If the arc be positive and between	0° and 90°	+	+	+	+	+	+
	90 and 180	+	—	—	—	—	+
	180 and 270	—	—	+	+	—	—
	270 and 360	—	+	—	—	+	—

		sin.	cos.	tan.	cot.	sec.	cose.
If the arc be negative and between	0° and 90°	—	+	—	—	+	—
	90 and 180	—	—	+	+	—	—
	180 and 270	+	—	—	—	—	+
	270 and 360	+	+	+	+	+	+

58. The horizontal parallax given in the lunar table is the *equatorial* hor. par. or the angle under which the radius of the Earth's equator would be seen at the Moon, but the Earth being spheroidal and compressed at the poles, the Earth's radius in any lat. N. or S. of the equator is less than the equatorial radius, and consequently the horizontal par. is less: this *reduced hor. par.* as it is called, is computed by help of table XXXIII lun. tab. in which we find the radius of the Earth for every degree of lat. according to two hypotheses of compression. From this table we take out the log. of the Earth's rad. in the given lat. (proportionating for the minutes, &c.) and add this to the log. of the equat. hor. parallax, the sum is the log. of the reduced hor. parallax.

59. In computing the circumstances of a solar eclipse, it is necessary to take into account the augmentation of the Moon's semi-diameter as she approaches the zenith, the observer then being nearer to her, this augmentation is given in table XXXV, but as one of the arguments of that table is the Moon's alt. the calculation of which is rather laborious, we shall here put down a formula for the augmentation which does not depend upon the Moon's altitude. Let d = ☽'s sem. diam. as seen from the Earth's centre, and the other symbols as before, then the augmentation is

$$d \text{ sin. } l \text{ cot. } (\triangle' - x) - \tfrac{1}{2} d \text{ sin. } {}^2l$$

The term $\frac{1}{2} d \text{ sin. } {}^2l$ is generally very minute, and may often be neglected.

60. We may now proceed to the calculation of the circumstances of solar eclipses for any particular place on the Earth's surface. A variety of methods have been given for doing this, the authors of some of which have laid claim to improvement, but most astronomers are now pretty well agreed that the nonagesimal method as it is called, which is one of the oldest, is upon the whole the best, and it is this which we shall adopt, giving however to some parts of the calculation more accuracy than has been usual. The computation of an eclipse by this method is more laborious than difficult: the operation indeed will be found very uniform and simple by any one well versed in the use of trigonometrical tables.

To compute a solar eclipse for any particular place.

61. Find correctly the Sun's right ascension, true long. and semi-diam. for the noon and midnight immediately preceding, and also for the noon and midnight immediately following the conjunction, determine also the Moon's longitude, latitude, equatorial horizontal parallax, and semi-diameter for these times, after which let all these be interpolated by means of second differences for each hour of the interval during which the eclipse lasts (or for a little longer period), and also between these hours for every 10, 15, or 20 minutes, according as it is wished to attain more or less accuracy in the result. The interpolation between the hours it will generally be sufficient to find by simple proportional parts. Next find the (art. 54.) reduced lat. of the place, the reduced horizontal parallax of the Moon (art. 58) and compute (art. 55) the long. and alt. of the nonagesimal degree at the several intervals of 10′ or 15′ &c., and likewise the parallaxes of the Moon in longitude and latitude; apply the parallaxes with their proper signs to the ☽'s long. and lat., and you

will have the apparent long. and lat. The several differences between the ☉'s long. and the ☽'s app. long. will give the apparent distance in longitude of the Sun and Moon, with this and the ☽'s app. lat. considered as two sides of a right angled plane triang., find the hypothenuse, which will be the apparent distance of the centres of the bodies: determine also the sum of the apparent semi-diameters, taking into account the augmentation of the ☽'s semi-diam., found by art. 59; let all these results be arranged in a table, placing the app. distance of the centres in the adjacent column, to the sum of the apparent semi-diameters, then by comparing these two columns it will at once be seen in which of the intervals of 10′ or 15′, &c. the eclipse begins and ends, and also in which the greatest obscuration falls; for when the app. distance of the centres is just equal to the sum of the app. semi-diameters, the eclipse is beginning, the instant of greatest obscuration is when the difference between the central distance of the bodies, and the sum of the app. sem. diameters is a maximum, and the end of the eclipse is at the instant the app. distance of the centres, after having decreased to its minimum, has again increased and become equal to the sum of the app. semi-diameters.

62. In adopting this method of calculating solar eclipses, the computer has the opportunity of suiting his own inclination as to the degree of labour he chooses to encounter, by assuming the interval of time smaller or larger, but his results will be accordingly more or less accurate. The process which has been described is much the same in substance as that used by professor Mayer,* which has been much valued: indeed Mayer was an astronomer who had the happy art of simplifying some of the most difficult problems connected with his profession.

Calculation of the solar eclipse of Sept. 7, 1820, for Greenwich.

63. The Sun's longitude and right ascension, and the Moon's longitude and latitude, &c. for the several hours during which this eclipse takes place, have been already found and tabulated at p. 218, from which it appears that the horary variations of all these quantities are exceeding nearly uniform, so that their values at any intervals between the hours will be obtained by simple proportional parts without second differences: we shall take intervals of 20 min., and by proportionating, the following results will be obtained as data for the subsequent calculations.

H. M. app. time	☉'s longitude	☉'s ☊.	☊. med. cœli.	☽'s longitude	☽'s latitude.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0 0	164 43 10	165 55 51	165 55 51	163 52 54	0 49 40 N.
0 20	164 43 59	165 56 36	170 56 36	164 2 43	0 48 46
0 40	164 44 47	165 57 21	175 57 21	164 12 31	0 47 52
1 0	164 45 36	165 58 6	180 58 6	164 22 20	0 46 58
1 20	164 46 25	165 58 51	185 58 51	164 32 9	0 46 4
1 40	164 47 13	165 59 36	190 59 36	164 41 58	0 45 10
2 0	164 48 2	166 0 21	196 0 21	164 51 47	0 44 16
2 20	164 48 51	166 1 6	201 1 6	165 1 36	0 43 22
2 40	164 49 39	166 1 52	206 1 52	165 11 25	0 42 28
3 0	164 50 28	166 2 37	211 2 37	165 21 14	0 41 34
3 20	164 51 16	166 3 22	216 3 22	165 31 3	0 40 40
3 40	164 52 5	166 4 7	221 4 7	165 40 52	0 39 46
4 0	164 52 53	166 4 52	226 4 52	165 50 41	0 38 52

☽'s equat. hor. parall. 53′ 53″.1 const. during the above interval.
☉'s parall. 8″.7 . . . . . do.
☽'s semi-diam. 14′ 41″.0 . . do.
☉'s do. . . 15 54.8 . . do.

The column containing the right ascension of the mid. heaven is found as mentioned in art. 55, by adding the Sun's right ascension to the time (in degrees) from noon.

64. The observed latitude of Greenwich is 51° 28′ 40″, and upon reference to tab. XXXIV, lun. tab. it appears that the angle of the vertical for 51° is 11′ 14″.1 (taking 299 : 300 for the ratio of the Earth's axes) the difference between the angles of the vertical for 51° and 52° is 5″.3, therefore, as 1° 28′ 40″ ::

* See his *Opera Inedita*. The formulæ for the parallaxes are taken from Delambre's Astronomy.

5″.3 : 2″.5, which subtracted from 11′ 14″.1, leaves 11′ 11″.6, or 11′ 12″ for the angle of the vertical at Greenwich, and taking this from 51° 28′ 40″, gives 51° 17′ 28″ for the *reduced latitude.*

65. Next, to find the Moon's reduced hor. parall. by art. 58. The diff. between the logarithms of the radii vectores for 51 and 52° is 247 (tab. XXXIII), therefore, as 1° : 17′ 28″ :: 247 : 72, the proportional part to be subtracted from 9.9991277, hence, for Greenwich,

log. rad. vect. is	9.9991205
☽'s eq. hor. par. 53′ 53″, or 3233″, its log. is	3.5096057
red. h. par. 53′ 46″ = 3226	3.5087262

and as the Sun has also a parallax of 8″.7 or 9″ which tends to bring the bodies apparently nearer to each other, this 53′ 46″ must be diminished by 9″, leaving 53′ 37″ for the true reduced parallax or parallax of the Moon from the Sun.

The whole of the following operations for finding the nonagesimal, parallaxes in lat. and long., &c. may now be very easily understood by comparing the calculations with the formulæ in arts. 55, 56, and 59.

66. Calculation of the nonagesimal, parallaxes in lat. and long. &c., for Sept. 7, 1820, at 0h. or app. noon.

Here we have a = 51° 17′ 28″
q = 23 27 57
r = 0 53 37
m = 165 55 51
M = 163 52 54
Δ = 0 49 40
Δ′ = 89 10 20

cos. q, . . 23° 27′ 57″	9.9625103	sin.	9.6001036
tan. m, . . 165 55 51	9.3989995	sec.	10.0132269
—.2298845	9.3615098	tan. a, 51 17 28	10.0961475
—.5122452		—.5122452	9.7094780
n.tan.N, —.7421297			
N = 143° 25′ 11″			

cos. m,	165° 55′ 51″	.	9.9867731
cos. a,	51 17 28	.	9.7961327
sec. N,	143 25 11	.	10.0952722
sin. A,	49 3 38	.	9.8781780

M 163° 52′ 54″
N 143 25 11

M—N 20 27 43

sin. r, . .	0° 53′ 37″	8.1930083	
sin. A, . .	49 3 38	9.8781780	. . . . . 28.0712316
sec. Δ, . .	0 49 40	10.0000453	
sin (M--N),	20 27 43	9.5435530	cos. 9.9716954
		7.6147846	. . . . 0110389 8.0429270
		9.9951791	 9889611
tan. L, . .	0 14 19	7.6196055	

20° 27′ 43″
7 9

M — N + ½ L, 20 34 52 = y

tan. A,	49 3 38	10.0617641		90 0 0
cos. y,	20 34 52	9.9713573		0 49 40
sec. ½ L,	0 7 9	10.0000009	Δ′, .	89 10 20
				47 10 58
tan. x,	47 10 58	10.0331223	Δ′—x,	41 59 22

sin. r, . .	0° 53′ 37″	8.1930083	
cos. A, .	49 3 38	9.8164143	. . . . . 28.1771297
sec. x, . .	47 10 58	10.1677071	
sin.(Δ′--x)	41 59 22	9.8254220	cos. 9.8711455
		8.0025517	. . . 0111757 . 8.0482752
		9.9951191	. . . 9888243
tan. l, . .	0 34 58	8.0074326	

☽'s long. . . .	163° 52′ 54″	
☽'s par. long. .	+ 0 14 19	
☽'s app. long. .	164 7 13	
☉'s long. . . .	164 43 10	
Diff. app. long.	0 35 57	= L′
☽'s lat.	0 49 40	
☽'s par. lat. . . .	0 34 58	
☽'s app. lat. North	0 14 42	= l'

L', ...	2157″	3.3338501	. . . . .	3.3338501
l', ...	882	2.9454686		
tan. z,	67° 45′ 37″	10.3883815	sin.	9.9664273
				3.3674228
		2330″.4		
		* dist. cent. . .	38′ 50″.4	

d, ...	° ′ ″ 14 41 = 881″	2.9449759	$\frac{1}{2}d$, 440.5		2.6439459
sin. l, . .	34 58 . . .	8.0073729	sin.$^2 l$ . .		6.0147458
cot.$(\Delta - l)$	41 59 23 . . .	10.0457235	.045 . .		8.6586917
	9.955	0.9980723			
	.045				
	augm. 9.910				

67. In finding the longitude of the nonagesimal in the preceding calculation sin. q, cos. q, and tan. a are all positive, a and q being each less than 90°, but tan. and sec. m are negative, because m is between 90° and 180° (see the table in art. 57), therefore the products cos. q tan. m, and sin. q sec. m tan. a are both negative, and of course their sum .7421297 is negative, which being the natural tang. of N, it follows (tab. art. 57), that the longitude, N, of the nonagesimal is either between 90° and 180°, or between 270° and 360°, but it cannot be between 270° and 360°, because the distance of the nonagesimal from the *nearest* equinoctial point is always of the same kind as the distance of the right ascension of the mid-heaven from the same point. As to A, we know that it must be less than 90°. When the Moon is to the east of the nonagesimal degree, the parallax in long. is to be added to the apparent long. to get the true—if to the west, it is to be subtracted. If however we pay attention to the algebraic sign of the parallax, the rule will be always to *add* the parallax in long. to the apparent long., to get the true longitude. At 12h. all the quantities that compose the value of tan. L are positive, therefore L is positive, and consequently the 14′ 19″ is to be added to the apparent longitude. The parallax in latitude is always subtractive.

It will be unnecessary to put down the calculations at length for each interval of 20m.—we shall insert those for 1h. 2h. 3h. and the last for 3h. 40m. which will contain a sufficient variety with regard to the signs of the quantities concerned to enable the student to compute any case that may occur.

* Instead of finding the distance of the centres by Prop. 16, Geom. it is determined logarithmically thus, put log. L' — log. l' = log. tan z; then dist. centres, or hypot. = log. L — log. sin. z.

68. *Calculation of the nonagesimal, parallaxes in lat. and long. &c. for Sept.* 7, 1820, *at* 1h. P.M. *app. time.*

The values of a, q, and r, remain as before.

m =	180°	58′	6″
M =	164	22	20
Δ =	0	46	58
Δ' =	89	13	2

cos. q,	° ′ ″ 23 27 57	9.9625103	sin.		9.6001036
tan. m,	180 58 6	8.9279486	sec.		10.0000620
	+.0155044	8.1904589	tan. a,	° ′ ″ 51 17 28	10.0961475
	—.4969505			—.4969505	9.6963131

n. tan. N, —.4814461

N = 154° 17′ 30″

cos. m,	180° 58′ 6″ . .	9.9999380
cos. a,	51 17 28 . .	9.7961327
sec. N,	154 17 30 . .	10.0452685
sin. A,	43 56 42 . .	9.8413392

M	164°	22′	20″
N	154	17	30
M — N	10	4	50

sin. r,	° ′ ″ 0 53 37	8.1930083			
sin. A,	43 56 42	9.8413392	. . . . .		28.0343880
sec. Δ,	0 46 58	10.0000405			
sin (M — N)	10 4 50	9.2431190	cos.		9.9932434
		7 2775070	.0106569		8.0276314
		9.9953469	.9893431		
tan. L,	0 6 35	7.2821601			

	10°	4′	50″
		3	17
M — N + ½ L,	10	8	7 = y

tan. A,	° ′ ″ 43 56 42	9.9840029			° ′ ″ 90 0 0
cos. y,	10 8 7	9.9931694			0 46 58
sec. ½ L,	0 3 17	10.0000002			89 13 2
					43 29 42
tan. x,	43 29 42	9.9771725	$\Delta' - x$,		45 43 20

	° ′ ″			
sin. r,	0 53 37	8.1930083		
cos. A,	43 56 42	9.8573363	. . . . .	28.1897464
sec. x,	43 29 42	10.1394018		
sin. $(\Delta' - x)$	45 43 20	9.8548909	cos.	9.8439411
		8.0446373	.0108066	8.0336875
		9.9952812	.9891934	
tan. l,	0 38 31	8.0493561		

	° ′ ″	
☽'s long.	164 22 20	
☽'s par. long. . .	+ 0 6 35	
☽'s app. long. . . .	164 28 55	
⊙'s long.	164 45 36	
Diff. app. long. . .	0 16 41	$= L'$

	° ′ ″	
☽'s lat.	0 46 58	
☽'s par. lat.	0 38 31	
☽'s app. lat. *North*,	0 8 27	$= l'$

L',	1001″	3.0004341	. . . . .	3.0004341
l',	507″	2.7050080		
tan. z, 63° 8′ 17″		10.2954261	sin.	9.9504125
			1122″.1	3.0500216
		dist. cent.	18′ 42″.1	

	° ′ ″			
d,	14 41	$= 881''.2$	9449759	$\frac{1}{2}d$, 440.5 2.6439459
sin. l,	38 31		8.0493657	sin.$^2 l$, 6.0987314
cot. $(\Delta' - x)$,	45 43 20		9.9890501	.055 8.7426773
		9″.624	0.9833917	
		.055		
	augm.	9.569		

69. At 1*h*. the right ascension, m, of the mid-heaven has become greater than 180°, but is less than 270°, its tangent therefore is positive, and consequently cos. q tan. m = .0155044 is positive, but sec. m is negative (see table art. 57), and therefore sin. q sec. m tan. a = .4969505 is negative, and taking their *algebraic* sum, the amount is —.4814461, which is consequently the tan. of an arc either in the second or fourth quadrant, but it cannot be the latter by the rule in art. 67, therefore it is between 90° and 180°.

70. *Calculation of the nonagesimal, parallaxes in lat. and long. &c. for Sept. 7, 1820, at 2h. P.M. app. time.*

The values of a, q, and r, as before

		° ′ ″
m	=	196 0 21
M	=	164 51 47
Δ	=	0 44 16
Δ'	=	89 15 44

	° ′ ″				
cos. q,	23 27 57	9.9625103	sin.	9.6001036	
tan. m,	196 0 21	9.4576633	sec.	10.0171710	
+	.2631319	9.4201736	tan. a, 51 17 28	10.0961475	
—	.5169186		—.5169186	9.7134221	
n. tan. N, —	.2537867				
N =	165 45 35				

	° ′ ″	
cos. m,	196 0 21	9.9828290
cos. a,	51 17 28	9.7961327
sec. N,	165 45 35	10.0135540
sin. A,	38 19 45	9.7925157

	° ′ ″
M	164 51 47
N	165 45 35
M — N, —	0 53 48

	° ′ ″			
sin. r,	0 53 37	8.1930083		
sin. A,	38 19 45	9.7925157	. . . . .	27.9855600
sec. Δ,	0 44 16	10.0000360		
sin. (M—N),	0 53 48	8.1944907	cos. . . .	9.9999468
		6.1800507	.0096718 .	7.9855068
		9.9957791	.9903282	
tan. L,	— 0 0 32	6.1842716		

	° ′ ″
	— 0 53 48
	— 0 0 16
M — N + ½ L,	— 0 54 4 $= y$

	° ′ ″			
tan. A,	38 19 45	9.8979454		90 0 0
cos. y,	0 54 4	9.9999463		0 44 16
sec. ½ L,	0 0 16	10.0000000		89 15 44
				38 19 33
tan. x,	38 19 33	9.8978917	$\Delta' - x$,	50 56 11

	° ′ ″				
sin. r, . . .	0 53 37	8.1930083	}		
cos. A, .	38 19 45	9.8945712	}	. . . . .	28.1929883
sec. x . .	38 19 33	10.1054088	}		
sin.(Δ'—x)	50 56 11	9.8901117	cos.		9.7994666
		8.0831000	.0098278		7.9924549
		9.9957108	.9901722		
tan. l, . .	0 42 2	8.0873892			

	°	′	″
☽'s long.	164	51	47
☽'s par. long. . —	0	0	32
☽'s app. long. . .	164	51	15
☉'s long.	164	48	2
Diff. app. long. .	0	3	13 = L′

	°	′	″
☽'s lat.	0	44	16
☽'s par. lat. . . .	0	42	2
☽'s app. lat. *North*, .	0	2	14 = l'

L′, .	193″	2.2855573		2.2855573
l' .	134″	2.1271048		
tan. z,	55° 13′ 40″	10.1584525 sin.		9.9145683
		234″.96		2.3709890
		dist. cent. 3′ 55		

d.	14′ 41″ = 881″	2.9449759	½d, 440.5	2.6439459
sin. l, . .	42 2	8.0873091	sin.²l, . .	6.1746182
cot.(Δ—x),	50° 56′ 11″ . .	9.9093548	.066	8.8185641
		8″.742. 0.9416398		
		.066		
	augm. 8.676			

71. At 2*h*. the computation of N is under the same circumstances as at 1*h*. but N has become greater than M, so that M — N is negative, its sine is consequently negative, this renders the numerator of the value of tan. L negative (for sin. r, sin. A, and sec. Δ are all positive), but the denominator is positive, for cos. (M—N) is positive, M — N being a negative arc less than 90°; hence, tan. L is negative, since it is found by dividing a negative by a positive quantity, and L (since it is always far less than 90°), must be a negative arc, and must be applied as such to the apparent longitude. The arc M — N + ½L or y is found upon the principles of algebraic addition and subtraction.

72. *Calculation of the nonagesimal, parallaxes in lat. & long. &c. for Sept.* 7, 1820, *at* 3*h*. P.M. *app. time.*

The values of a, q, and r, are the same as before.

		°	′	″
m	=	211	2	37
M	=	165	21	14
Δ	=	0	41	34
Δ'	=	89	18	26

	° ′ ″				
cos. q, . .	23 27 57	9.9625103	sin.		9.6001036
tan. m, . .	211 2 37	9.7795222	sec.		10.0671332
+ .5521188		9.7420325	tan. a,	° ′ ″ 51 17 28	10.0961475
— .5799415			—.5799415		9.7633843
n. tan. N,	—.0278227				
N =	178° 24′ 23″				

	° ′ ″		
cos. m,	211 2 37		9.9328668
cos. a,	51 17 28		9.7961327
sec. N,	178 24 23		10.0001680
sin. A,	32 24 43		9.7291675

		°	′	″
M		165	21	14
N		178	24	23
M — N, —		13	3	9

	° ′ ″				
sin. r,	0 53 37	8.1930083	}		
sin. A,	32 24 43	9.7291675	}	. . . . .	27.9222075
sec. Δ,	0 41 34	10.0000317	}		
sin.(M—N)	13 3 9	9.3538081	cos.		9.9886319
		7.2760156	.0081440		7.9108394
		9.9964486	.9918560		
tan. L, —	0 6 38	7.2795670			

	°	′	″
	— 13	3	9
	— 0	3	16
M — N + ½L,	— 13	6	25 = y

tan. A,	32 24 43	9.8027184	90 0 0
cos. y,	18 6 25	9.9885360	0 41 34
sec. $\frac{1}{2}$ L,	0 3 16	10.0000002	89 18 26
			31 43 53
tan. x,	31 43 53	9.7912496 $\Delta' - x$,	57 34 33

sin. r,	0 53 37	8.1930083	
cos. A,	32 24 43	9.9264537	28.1897759
sec. x,	31 43 53	10.0703139	
sin.(Δ'--x)	57 34 33	9.9263948 cos.	9.7293129
		8.1161707	.0083002 . 7.9190888
		9.9963802	.9916998
tan. l,	0 45 18	8.1197905	

☽'s long.	165 21 14
☽'s par. long.	— 0 6 33
☽'s app. long.	165 14 41
⊙'s long.	164 50 28
Diff. app. long.	0 24 13 = L'

☽'s lat.	0 41 34
☽'s par. lat.	0 45 18
☽'s app. lat. *South*,	0 3 44 = l'

L',	1453″	3.1622656	3.1622656
l',	224″	2.3502480	
tan. x,	81° 14′ 10″	10.8120176 sin.	9.9948996
		1470.″2	3.1673660
		dist. cent.	24′ 30″.2

d,	14 41 = 881″2	9449759	$\frac{1}{2}d$, 440.5	2.6439459
sin. l,	45 18	8.1198118	sin.$^2 l$,	6.2396236
cot.($\Delta' - x$),	57 34 33	9.8029181	.076	8.8835695
		7″.374	0.8677058	
		.076		
	augm.	7.298		

At 3h. the signs of the various quantities remain under the same circumstances as at 2h.

73. *Calculation of the nonagesimal, parallaxes in lat. and long. &c. for Sept.* 7, 1820, *at* 3h. 40m. P.M. *app. time.*

The values of a, q, and r, are still as before.

m	=	221 4 7
M	=	165 40 52
Δ	=	0 39 46
Δ'	=	89 20 14

cos. q,	23 27 57	9.9625103 sin.		9.6001036
tan. m,	221 4 7	9.9402132 sec.		10.1226728
+	.7993252	9.9027235 tan. a,	51 17 28	10.0961475
—	.6590583		—.6590583	9.8189239
n. tan. N,	+ .1402669			
N =	187 59 5			

cos. m,	221 4 7	9.8773272
cos. a,	51 17 28	9.7961327
sec. N,	187 59 5	10.0042310
sin. A,	28 25 50	9.6776909

M,	165 40 52
N,	187 59 5
M — N, —	22 18 13

sin. r,	0 53 37	8.1930083		
sin. A,	28 25 50	9.6776909		27.8707283
sec. Δ,	0 39 46	10.0000291		
sin.(M—N),	22 18 13	9.5792283	cos.	9.9662290
		7.4499566	.0068700	7.8369573
		9.9970061	.9931300	
tan. L,	—0 9 45	7.4529505		

	— 22 18 13
	— 4 52
M — N + $\frac{1}{2}$ L,	— 22 23 5 = y

tan. A,	28 25 50	9.7335081		90 0 0
cos. y,	22 23 5	9.9659763		0 39 46
sec. $\frac{1}{2}$ L,	0 4 52	10.0000004	Δ',	89 20 14
				26 35 32
tan. x,	26 35 32	9.6994848	$\Delta' - x$,	62 44 42

sin. r,	0°53′ 37″	8.1930083		
cos. A,	28 25 50	9.9441839	. . . . .	28.1857502
sec. x,	26 35 32	10.0485580		
sin. $(\Delta' - x)$	62 44 42	9.9488906	cos. . . .	9.6608195
		8.1346408	.0070238	7.8465697
		9.9969388	.9929762	
tan. l,	0 47 12	8.1377020		

☽'s long. . . . 165° 40′ 52″
☽'s par. long. . — 0 9 45

☽'s app. long. . 165 31 7
⊙'s long. . . . 164 52 5

Diff. app. long. 0 39 2 = L′

☽'s lat. 0 39 46
☽'s par. lat. . . 0 47 12

☽'s app. lat. *South* 0 7 26 = l'

L′, ...	2342″	3.3695869	. . . . .	3.3695869
l', ...	446	2.6493349		
tan. z,	79° 13′ 5″	10.7202520	sin.	9.9922646
			2384″.1	3.3773223
		dist. cent. . .	39′ 44″.1	

d, ...	14′ 41″ = 881″	2.9449759	$\frac{1}{2}d$, 440.5	2.6439459		
sin. l, . .	47 12 . . .	8.1376545	sin.$^2 l$ . .	6.2753090		
cot. $(\Delta' - x)$	62 44 42 . . .	9.7119289	.083 . .	8.9192549		
		6.231	0.7945593			
		.083				
	augm.	6.148				

The management of the signs of the quantities concerned in finding the parallaxes for 3*h*. 40*m*. will be easily understood from what has been previously stated.

74. It has not been thought necessary to insert the calculations of the parallaxes, &c. at length for every 20 minutes: the part which requires most care is that relating to the signs of the trigonometrical quantities, but there is a sufficient variety in the five preceding computations to enable a student who understands these, to manage the signs correctly in any case that may occur. We have gone through the calculations of the parallaxes and distance of the centres for each 20 minutes, and the several results are here put down.

App. time at Greenwich.	⊙'s long.	☽'s long.	☽'s par. long.	☽'s. app. long.	☽'s. lat.	☽'s par. lat.	☽'s ap lat.	Dif. ap. long.	Augm. ☽'s s.d	App. dist. centres.	Sum app. sem. diam
H. M.	° ′ ″	° ′ ″	′ ″	° ′ ″	′ ″	′ ″	′ ″	′ ″	″	′ ″	′ ″
0 0	164 43 10	163 52 54	+14 19	164 7 13	49 40N.	34 58	14 42N	35 57	9.9	38 50.4	30 45.7
0 20	164 43 59	164 2 43	11 42	164 14 25	48 46	36 8	12 38	29 34	9.9	32 9.2	30 45.7
0 40	164 44 47	164 12 31	9 7	164 21 38	47 52	37 19	10 33	23 9	9.7	25 26.4	30 45.5
1 0	164 45 36	164 22 20	6 35	164 28 55	46 58	38 31	8 27	16 41	9.6	18 42.1	30 45.4
1 20	164 46 25	164 32 9	4 7	164 36 16	46 4	39 42	6 22	10 9	9.3	11 58.9	30 45.1
1 40	164 47 13	164 41 58	1 45	164 43 43	45 10	40 53	4 17	3 30	9.0	5 31.9	30 44.8
2 0	164 48 2	164 51 47	— 0 32	164 51 15	44 16	42 2	2 14	3 13	8.7	3 55.0	30 44.5
2 20	164 48 51	165 1 36	2 40	164 58 56	43 22	43 10	0 12	10 5	8.3	10 5.1	30 44.1
2 40	164 49 39	165 11 25	4 41	165 6 44	42 28	44 15	1 47S	17 5	7.8	17 10.6	30 43.6
3 0	164 50 28	165 21 14	6 33	165 14 41	41 34	45 18	3 44	24 13	7.3	24 30.2	30 43.1
3 20	164 51 16	165 31 3	8 14	165 22 49	40 40	46 17	5 37	31 33	6.7	32 2.7	30 42.5
3 40	164 52 5	165 40 52	9 45	165 31 7	39 46	47 12	7 26	39 2	6.1	39 44.1	30 41.9

75. By comparing the last two columns of the preceding table we now see that the eclipse begins to be visible at Greenwich at some time between 0*h*. 20*m*., and 0*h*. 40*m*., for at 0*h*. 20*m*. the apparent distance of the centres is greater than the sum of the apparent semi-diameters, but at 0*h*. 40*m*. it is less: the greatest obscuration is between 1*h*. 40*m*., and 2*h*. for the diff. between the distances of the centres at 1*h*. 20*m*., and 1*h*. 40*m*. is 6′ 27″, and the diff. between the distances of the centres at 2*h*. and at 2*h*. 20*m*. is 6′ 10″.1., but between 1*h*. 40*m*. and 2*h*. there is only a variation of 1′ 36″.9, from which it follows that the central distance must have decreased after 1*h*. 40*m*., and then increased before 2*h*., or that it is at its minimum between 1*h*. 40*m*., and 2*h*. 0*m*. The end of the eclipse it is manifest is between 3*h*. 0*m*., and 3*h*. 20*m*.

To find the time of the beginning of the eclipse.

dist. cent. at 0h. 20m.	32	9.2	. . . . .	32	9.2
at 0h. 40m.	25	26.4	Sun's sem.dia.	30	45.7
	6	42.8		1	23.5

Then as 6′ 42″.8 : 1′ 23″.5 :: 20m. : 4m. 9s., therefore the eclipse begins at Greenwich at 0h. 24m. 9s.

To find the instant of the greatest obscuration.

Dist. cent. at 1h. 40m.	5	31.9	. . . . .	5	31.9
at 1h. 20m.	11	58.9	at 2h. 0m. .	3	55.0
Diff. . . .	6	27.0	. . . . .	1	36.9

Then say, as 6′ 27″ : 1′ 36″.9 :: 20m. : 5m., which is the excess of time that the distance of the centres continues to diminish, above the time that it increases, in the interval between 1h. 40m. and 2h. 0m., therefore taking 5m. from 20m. half the remainder, or 7½m. is the time before 2h. when the distance of the centres is at its minimum, that is, the minimum distance is at 1h. 52½m., which is therefore the time of greatest obscuration.

To find the time of the end.

Dist. cent. at 3h. 0m.	24	30.2	. . . . .	24	30.2
at 3h. 20m.	32	2.7	Sum app. sem. diam.	30	43.1
	7	32.5	. . . . .	6	12.9

Now if we wish to allow for the change in the sum of the sem. diameters between 3h. and the end of the eclipse, we ought first to estimate nearly the time of the end, by saying, as 7′ 32″.5 : 6′ 12″.9 :: 20m. : 16½m. nearly. Then at 3h. 16½m. it will be found by proportionating, that the sum of the semi-diameters is 30′ 42″.6, and 30′ 42″.6 — 24′ 30″.2 = 6′ 12″.4, so that the proportion for finding the end correctly will be 7′ 32″.5 : 6′ 12″.4 :: 20m. : 16m. 27s., therefore, the eclipse ends at Greenwich at 3h. 16m. 27s. The change in the sum of the semi-diameters was not taken into account in finding the time of the beginning, because that change would not have amounted to .1 of a second.

76. To find the digits eclipsed at the instant of greatest obscuration: let a = interval between 1h. 40m. and greatest obscuration = 12½m., b = interval between greatest obscuration and 2h. c = diff. between dist. of centres at 1h. 40m. and 2h. = 1′ 36″.9; d = dist. of centres at 2h.; then dist. centres at greatest obscuration $= d - \frac{b^2}{a^2 - b^2} \cdot c = 3' 55'' -$ 0′ 54″.5 = 3′ 0″.5, hence, 30′ 44″.6 — 3′ 0″.5 = 27′ 44″.1 the parts deficient, then as the Sun's sem. diam. 31′ 49″.6 : 27′ 44″.1 :: 12d. : 10d . 27′.4 the digits eclipsed.

77. The app. time of the true ecliptic conjunction, and of the ecliptic visible conjunction, as also the Moon's latitude at each of these times, are easily found, thus,

☉'s long. at 1h. 40m.	164°	47′	13″	at 2h.	164°	48′ 2′
☽'s true long. at 1h. 40m.	164	41	58	at 2h.	164	51 47
	0	5	15		0	3 45

Therefore the Moon's motion from the Sun from 1h. 40m. to 2h. 0m. is 5′ 15″ + 3′ 45″ = 9′, hence, 9′ : 5′ 15″ : : 20m. : 11m. 40s. to which adding 1h. 40m., we have 1h. 51m. 40s. for the app. time of true ecliptic conjunction.

Next, for the visible conjunction.

☉'s long. at 1h. 40m.	164	47	13	at 2h.	164	48 2
☽'s app. lon. at 1h. 40m.	164	43	43	at 2h.	164	51 15
	0	3	30		0	3 13

Hence, the Moon's apparent motion from the Sun between 1h. 40m. and 2h. is 3′ 30″ + 3′ 13″ = 6′ 43″. Then as 6′ 43″ : 3′ 30″ : : 20m. : 10m. : 25s. and 1h. 40m. + 10m. 25s. = 1h. 50m. 25s. the app. time of visible conjunction.

Again as 20m. : 11m. 40s. : : (45′ 10″ — 44′ 16″ =) 54″ : 31″, whence 45′ 10″ — 31″ = 44′ 39″ N, the Moon's true lat. at the true ecliptic conjunction.

Also, as 20m. : 10m 25s. : : (4′ 17″ — 2′ 14″ =) 2′ 3″ : 1′ 4″, and 4′ 17″ — 1′ 4″ = 3′ 13″ N., the Moon's visible lat. at the time of visible conjunction.

78. We have thus found that the Sun was eclipsed on Sept. 7th 1820.

	h.	m.	s.	
Beginning of the eclipse, . .	0	24	9	P.M. time at Greenwich
Middle, ...	1	50	18	
Greatest obscuration, . . .	1	52	30	
End, ...	3	16	27	
App. time of true ecliptic conj.	1	51	40	
Do. . . . eclip. visible conj.	1	50	25	
☽'s true lat. at ecliptic conj.		44	39	N.
☽'s visi. lat. at eclip. visi. conj.		3	13	N.
Digits eclipsed, ...	10d. 27′.4 on ☉'s North limb.			

The longitude of the luminaries at the instant of true ecliptic conjunction is easily found by proportion to be 164° 47′ 41″.6.

79. The parallaxes having all been computed by the 1st formula in art. 56, we shall before quitting this subject put down the mode of calculating them by the series in the 2d formula, taking the instant of app. noon.

1st. For the parallax in longitude.

	° ′ ″		
sin. r, ...	0 53 37	8.1930083	
sin. A, ...	49 3 38	9.8781780	
sec. Δ, . . .	0 49 40	10.0000453	
sin r, sin. A, sec. Δ, . .		8.0712316	
sin. (M — N), .	20 27 43	9.5435530	
		7.6147846	
sin. 1″		4.6855749	
		2.9292097	849.59
(sin. r, sin. A, sec. Δ)^2 .		6.1424632	
sin. 2(M — N) 40° 55′ 26″		9.8162784	
		5,9587416	
sin. 2″		4,9866049	
		0.9721367	9.38
(sin. r, sin. A, sec. Δ)^3 .		4.2136948	
sin. 3(M — N), 61° 23′ 9″		9.9434275	
		4.1571223	
sin. 3″		5.1626961	
		2.9944262	0.09
			859.06 = 14′ 19″ = L as [before.

2d. For the parallax in latitude.

	° ′ ″		
sin. r, ...	0 53 37	8.1930083	
cos. A, . . .	49 3 38	9.8164143	
sect. x, ...	47 10 58	10.1677071	
sin. r, cos. A, sec. x. . .		8.1771297	
sin. ($\Delta' - x$),	41 59 22	9.8254220	
		8.0025517	
sin. 1″		4.6855749	
		3.3169768	2074.80
(sin. r, cos. A, sec. x)^2 . .		6.3542594	
sin. 2($\Delta' - x$), 83° 58′ 44″		9 9975975	
		6.3518569	
sin. 2″		4.9866049	
		1 3652520	23.18
(sin. r, cos. A, sec x)^3 .		4.5813891	
sin. 3($\Delta - x$), 125° 58′ 6″		9.9081319	
		4.4895210	
sin. 3″		5.1626961	
		1.2768249	0.19
			2098.17 = 34′ 58″ = l as [before.

80. If the differences of the apparent distances of the centres had come out very unequal, it would then have been proper to interpolate between these for smaller intervals of time, by means of second differences, previously to finding the times of the beginning, &c. of the eclipse. It may also be observed, that although logarithms to seven places have been used in finding the parallaxes, yet those of 4 or 5 are sufficient, as the final result would be very nearly the same. The phenomena of eclipses are not affected by refraction, because the Sun and Moon have the same altitude nearly, and the part of the Sun eclipsed has the same altitude with the part of the Moon eclipsing it, and hence the refractions are equal, so that it is unnecessary to take the refraction into account.

Of the construction of solar eclipses by projection.

81. The best projection for this purpose, is that in which the eye is supposed to be at the centre of the Sun, and the plane of projection to pass through the centre of the Moon at right angles, to a line joining the centres of the Sun and Moon. As, on account of the great distance of the Sun, lines drawn from any point of the Earth's surface, to the Sun's centre, may be regarded as parallel, and at right angles to the plane of projection, it follows that all circles whose planes are oblique to the plane of projection will be projected into ellipses; all circles parallel to the ecliptic will be represented by straight lines parallel to the intersection of the ecliptic, with the plane of projection, and those perpendicular to the ecliptic by circles. To construct a solar eclipse upon the above principles, it is necessary to be provided with these elements, viz.

The apparent time of true conjunction of the ☉ and ☽.
The Sun's declination at the time of conjunction.
The Moon's true latitude at the same time.
The Moon's horizontal parallax.
The Sun's horizontal parallax.
The Moon's horary motion from the Sun in longitude.
The Moon's horary motion in latitude.
The Moon's semi-diameter.
The Sun's semi-diameter.

82. Taking for an example the construction of the solar eclipse which has just been computed, we must first find the semi-diameter of the Earth's disc, that is, the angle under which the projected semi-diameter of the Earth on the plane at the Moon appears from the Earth's centre.—It is easy to perceive that this angle will be equal to the horizontal parallax of the Moon, diminished by that of the Sun. In the present case therefore, semi-diam. Earth's disc = 53′ 53″ — 9″ = 53′ 44″. The inclination of the relative orbit, and the horary motion of the Moon

from the Sun on the relative orbit must next be determined:

⊙'s long. at 1*h*.	164	45	36	at 2*h*.	164	48	2
☽'s long. at 1*h*.	164	22	20	at 2*h*.	164	51	47
	0	23	16		0	3	45

Hence, 23′ 16″ + 3′ 45″ = 27′ 1″, the Moon's hor. mot. from ⊙ on the ecliptic. Also, 46′ 58″ — 44′ 16″ = 2′ 42″, the ☽'s hor. mot. in lat.

Then as ☽'s. hor. mot. from ⊙	27′ 1″ = 1621″			3.2097830
is to ☽'s hor. mot. in lat.	2 42 = 162			2.2095150
so is rad.				10.0000000
to tan. inclin. rel. orbit		° 5	′ 42	″ 26 8.9997320
And as rad.				10.0000000
is to sec. incl. rel. orbit		5	42	26 10.0021581
so is	27′ 1″ = 1621			3.2097830
to hor. mot. ☽ from ⊙ on rel. orbit	27 9 = 1629			3.2119411

83. Now from any scale of equal parts take a radius equal to 53′ 44″, or 53′.73, the semi-diameter of the Earth's disc, and describe the semi-circle ABD, which will represent half the Earth's disc on the plane of projection, AOD representing the ecliptic, to which draw the perpendicular OB. Take BM = BN = the obliq. of the ecliptic = 23° 27′ 57″ and join MN, and the projected pole of the Earth will be situated somewhere in this line; for to a spectator at the Sun the apparent motion of the pole, P, is the same as if the Earth's axis had a conical motion about the axis OB of the ecliptic, the angle at O, the vertex of the cone, being twice 23° 27′ 57″. The length of OP will be equal to the radius of projection multiplied by the cos. of the Sun's declin. = 53′.73 × cos. 5° 59′ 41″.* = 53′.44 = 53′ 26″; therefore take this from the scale and set it from O to meet MN in P: as the Sun has north declination at the time, the pole P will be in the illuminated disc of the Earth. Next take from the scale, O*r* = OA × cos. ⊙'s declin. × sin. lat. of the place [Greenwich] that is, = OP × sin. lat. = 53′.44 × sin. 51° 28′ 40″ = 41′.81 = 41′ 48″, draw VI *r* VI perpendicular to OP, and set off *r*VI = *r*VI = OA × cos. lat. = 53′.73 × cos. 51° 28′ 40″ = 33′.46 = 33′ 27″; also make *r*XII = *r*12 = *r*VI × sin. ⊙'s. decl. = 33′.46 × sin. 5° 59′ 41″ = 3′.49 = 3′ 29″, and upon VI *r*VI and XII *r* 12 as axes describe an ellipsis, this ellipse will represent the apparent path of Greenwich over the Earth's disc, the part VI, XII, VI, will be the diurnal track of Greenwich, and VI, 12, VI, the nocturnal; but if the Sun's decln. had been South, the upper half of the ellipse would have been the diurnal, and the lower the nocturnal track. On the axis major of the ellipse, describe the semi-circumference VI *m* VI, which divide into 12 equal parts, as shewn in the fig., and through the points of division draw parallels to O*r*, intersecting the periphery of the ellipse in the points II. 10, I. 11, XI. 1, X. 2, &c.: the hours on the semi-circle VI *m* VI may be divided into halves and quarters, if upon a large scale, or even into less divisions; and perpendiculars upon VI *r* VI, from these divisions will divide the apparent track in the same proportion. Take O*n* from the same scale equal to 44′ 39″, the Moon's lat. at the time of the ecliptic conjunction, and draw K*n*L making an angle with O*n*, equal to 84° 17′ 34″, the comp. of the angle which the relative orbit makes with the ecliptic. This angle must be set off on the *left* side of O*n*, if the Moon's lat. be north or south *decreasing*, and on the *right* if it be *increasing*: in this example it falls on the left side, and KL is part of the Moon's relative orbit: the point *n* will be the place of the Moon's centre at 1*h*. 51*m*. 40*s*., that being the time of ecliptic conjunction, and from this point the division of the Moon's relative orbit into time must commence. To make this division, find first, the situation of the hour either preceding or succeeding *n*, suppose here, of that preceding, or 1*h*., by saying, as 1*h*.: 51*m*. 40*s*. :: 27′ 9″ the Moon's hor. mot. from the Sun on the rel. orbit: 23′ 23″, the interval on the scale which corresponds to 51*m*. 40*s*. Take this off the scale and set it from *n* to I, then from I with an extent equal to 27′ 9″, mark off each way the hours XII, II, III, &c. as far as may be necessary, and subdivide them into halves, quarters, &c. or into smaller divisions, if the size of the figure will admit of it. Then to find the time of the middle of the eclipse, determine by trial, with the compasses, what two corresponding times as *t* and *s* upon the Moon's orbit and the ellipse, or apparent path of the place, are nearest together, and this will be the time of the greatest obscuration, since the centres of the Sun and Moon are then at the least distance.—To ascertain the time of the beginning, take from the scale an extent equal to 30′ 35″.8, the sum of the semi-diameters of the Sun and Moon, and find two corresponding times before the greatest obscuration, as at *p* and *q*, whose distance is equal to that extent, and this gives the time of the beginning, and if with the same interval, two corresponding times *x* and *y* be

* The Sun's declin. at the time of ecliptic conjunction is found by proportionating for 1*h*. 51*m*. 40*s*. between the decl. for noon of the 7th, and noon of the 8th Sept. as already computed at pp. 211, 212.

found after the greatest obscuration, it gives the time of the end. For the eclipse must obviously begin when the centres of the Sun and Moon arrive at the distance of the sum of their semi-diameters, and end when they have receded to that distance.

The number of digits eclipsed at the greatest obscuration is thus found. From the points *s* and *t* as centres describe circles with radii equal respectively to the Sun and Moon's semi-diameters, produce the line *st* to meet the circumferences, then measure *de* from the scale, and say as *ds*, the ☉'s semi-diam. : *de* :: 6 digits : the digits eclipsed. The digits eclipsed at any other time may be found by taking the distance at that time between the centres of the bodies and applying it to the scale, then as ☉'s sem. diam. : sem. diam. ☉ + sem. diam. ☽ — dist. centres : : 6 digits : the digits eclipsed. Or the digits eclipsed at any time may be determined by making a scale of digits, of which *ds* shall contain 6, and then measuring the part eclipsed on the same scale.

The times of the greatest obscuration, beginning, and end of the eclipse, as obtained by this projection, agree very nearly with those already found by computation.

84. From the preceding construction, the position of the Moon with respect to the zenith of the Sun's disc may be found, and the apparent path of the Moon over the Sun delineated, usually called *drawing the type of an eclipse:* for, a line drawn from O to any point of the ellipse, where the spectator is, being vertical, from the principles of the projection, the supplements of the angles O*pq*, O*st*, and O*xy*, shew the angular distances about the centre of the Sun from its vertex to the centre of the Moon at the beginning, middle, and end of the eclipse: hence, let O (plate 12, fig. 2), be the centre of the Sun, H N a line parallel to the horizon, and O*z*Z perpendicular to it, *z* being the Sun's zenith ; with a radius O*Z*, equal to the sum of the semi-diameters of the Sun and Moon, describe a circle, and make the angles Z*Oq*, Z*Ot*, Z*Oy*, equal to the supplements of O*pq*, O*st*, and O*xy* in fig. 1, respectively: on the line O*t* take O*a*, equal to *st** in fig. 1, that is, equal to the distance of the Sun and Moon's centres at the time of the greatest obscuration. Then with the centres *q*, *a*, *y*, and a radius equal to the semi-diam. of the Moon, describe three circles, and they will represent the situation of the Moon at the beginning, middle, and end of the eclipse in respect to the vertex, *z*, of the Sun, and consequently in respect to the horizon ; and the circular arc *yaq* represents nearly the apparent path of the Moon's centre during the eclipse, in respect to the horizon of Greenwich.

85. In the calculation of a solar eclipse for any particular place by the nonagesimal method, the distance of the centres is obtained more directly, and with more correctness from the latitudes and longitudes of the luminaries, as derived from the tables, than by other processes which some have thought more simple, and this it appears to us is sufficient to give the computation by the nonagesimal a decided preference: as, however, the determination of the distance of the centres by altitudes and azimuths has been much valued by some computers, we shall give an example as an instance of this method, The mode of proceeding is to find the altitudes of the Sun and Moon and their azimuths at various instants of time, during the phenomena of the eclipse, then to correct the altitudes by the parallax in altitude, and with these and the differences of the azimuths to compute the central distances: in order to find the Moon's altitude and azimuth. We must first know her right ascension and declination, and if we wish to ensure accuracy, it is necessary to compute these with more exactness than is required in finding the longitude and altitude of the nonagesimal, as a small error in these does not sensibly effect the result.

2*d* EXAMPLE.—*Calculation of the solar eclipse, Nov. 19th, 1816, at about 9h. 30m. app. time for Greenwich.*

86. Here we must first find by the precepts already given, the Moon's longitude and latitude, &c. for the noon and midnight preceding and following the eclipse, as also the Sun's longitude and declination, and then interpolate these by means of differences for the two hours preceding and following the middle of the eclipse.

Moon's place for the 18th and 19th of Nov. 1816.

Nov. 1816.	☽ 's long.	☽ 's. lat.	hor. par.	sem. dia.
	s ° ′ ″	° ′ ″	′ ″	′ ″
Noon 18th,	7 13 13 58	2 4 1 N.	60 35	16 32
Midn. 18th,	7 20 38 35	1 25 16	60 27	16 30
Noon 19th,	7 28 0 29	0 45 20	60 14	16 26
Midn. 19th,	8 5 18 45	0 5 0	59 59	16 22

* O*a* is not here actually equal to *st* in fig. 1, because the type, for the sake of convenience, has been drawn to a smaller scale than the projection of the eclipse.

	1st.	2d.	mean 2d. diff.
	° ′ ″	′ ″	
	7 24 37		
Differences		— 2 43	′ ″
	7 21 54		— 3 10.5
☽ 's longitude.		— 3 38	
	7 18 16		

	1st.	2d.	mean 2d. diff.
	′ ″		
	— 38 45	′ ″	
Differences		— 1 11	′ ″
	— 39 56		— 0 47.5
☽ 's latitude.		— 0 24	
	— 40 20		

Then as $12h.$: $\left\{\begin{matrix}8h.\\9h.\\10h.\\11h.\end{matrix}\right\}$ :: 7° 21′ 54″ : $\left\{\begin{matrix}4\ 54\ 36.0\\5\ 31\ 25.5\\6\ 8\ 15.0\\6\ 45\ 4.5\end{matrix}\right\}$ the proportional parts of the 1st differences of the Moon's longitude, which are now to be corrected by the equation of 2d difference, taken from table XXXVII, lun. tab., the arguments for which are the mean second difference and the time after noon or midnight, that is, in this case 3′ 10″.5 and 8, 9, 10, and $11h.$ The equations it will be found are 21″.1, 17″.8, 13″.2, and 7″.3 and *additive.*

	s.	°	′	″
☽'s long. at midnight 18th Nov. . .	7	20	38	35
Prop. part 1st. diff. for $8h.$			4 54	36
Equa. 2d. diff.				21.1
☽'s long. at $8h.$ 19th.	7	25	33	32.1
	7	20	38	35
Prop. part 1st. diff. for $9h.$			5 31	25.5
Equa. 2d. diff.				17.8
☽'s long. at $9h.$	7	26	10	18.3
	7	20	38	35
Prop. part 1st. diff. for $10h.$			6 8	15
Equa. 2d. diff.				13.2
☽'s long. at $10h.$	7	26	47	3.2

	s.	°	′	″
	7	20	38	35
Prop. part. 1st. diff. for $11h.$		6	45	4.5
Equa. 2d. diff.				7.3
☽'s long. at $11h.$	7	27	23	46.8

Next as $12h.$: $\left\{\begin{matrix}8h.\\9h.\\10h.\\11h.\end{matrix}\right\}$ :: — 39′ 56″ $\left\{\begin{matrix}-26\,37''.3\\-29\ 57.0\\-33\ 16.6\\-36\ 36.3\end{matrix}\right\}$ the proportional parts of the 1st. differences of the Moon's latitude, the corrections of which for the equations of 2d difference are 5″.4, 4″.5, 3″.3, and 1″.8.

	°	′	″
☽'s lat. at midn. 18th Nov.	1	25	16
Prop. part 1st. diff. for $8h.$	—	26	37.3
Equa. 2d. diff.	+		5.4
☽'s lat. at $8h.$ 19th.	0	58	44.1
	1	25	16
Prop. part 1st. diff. for $9h.$	—	29	57
Equa. 2d. diff.	+		4.5
☽'s lat. at $9h.$	0	55	23.5
	1	25	16
Prop. part 1st. diff. for $10h.$	—	33	16.6
Equa. 2d. diff.	+		3.3
☽'s lat. at $10h.$	0	52	2.7
	1	25	16
Prop. part 1st. diff. for $11h.$	—	36	36.3
Equa. 2d. diff.	+		1.8
	0	48	41.5

	☉'s longitude.			☉'s right as.			☉'s. declin.		
	s.	°	′ ″	°	′	″	°	′	″
On the 18th at noon, we find by the precepts and rules formerly given	7	26	4 26	233	44	55	19	17	34 S.
On the 19th at noon, ...	7	27	5 6	234	47	28	19	31	40
Diff. in 24 hours	0	1	0 40	1	2	33	0	14	6

And to find the longitude, right ascension, and declination, at 8, 9, 10, and 11*h*. of the 18th, it is sufficient to take simple proportional parts of the difference in 24 hours. It is also unnecessary to use the equation of 2d difference for the Moon's hor. parallax and semi-diam. at 8, 9, 10 and 11*h*: the preceding results are here brought together.

Nov. 19, 1816.	Hours A. M.	Longitude.	Latitude.	Hor. parallax	Sem. diam.
		s. ° ′ ″	° ′ ″	′ ″	′ ″
Moon,	8	7 25 33 32	0 58 44	60 18	16 27
	9	7 26 10 18	0 55 23	60 17	16 27
	10	7 26 47 3	0 52 3	60 16	16 27
	11	7 27 23 47	0 48 41	60 15	16 26
		Longitude.	Right asc.	Declin.	
		s. ° ′ ″	° ′ ″	° ′ ″	
Sun,	8	7 26 54 59	234 37 3	19 29 19S.	sem. diam.
	9	7 26 57 31	234 39 39	19 29 54	16′ 13″
	10	7 27 0 3	234 42 15	19 30 29	hor. parallax
	11	7 27 2 34	234 44 51	19 31 5	0′ 9″

87. Now it appears that the ecliptic conjunction is between 10 and 11 hours, at which time,

	′	″
☽'s hor. motion is	36	44
☉'s. ditto.	2	31
Hor mot. ☽ from ☉ in longitude	34	13

	s	°	′	″
☉'s. long. at 10*h*.	7	27	0	3
☽'s. do.	7	26	47	3
Difference,	0	0	13	0

Then as 34′ 13″ : 13′ 0″ : : 1*h*. : 22*m*. 48*s*., whence the time of ecliptic conjunction is 10*h*. 22*m*. 48*s*. A.M.

88. The horary motion of the Moon in latitude between 10 and 11*h*. it appears is 3′ 22″, and 1*h*. : 22*m*. 48″. : : 3′ 22″ : 1′ 17″, which subtracted from 52′ 3″ leaves 50′ 46″ for the Moon's lat. N decreasing at 10*h*. 22*m*. 48*s*.

Next as 34′ 13″ = 2053″	3.3123889
is to 3 22 = 202	2.3053514
so is rad.	10.0000000
to tan. 5° 37′ 10″	8.9929625

which is the angle of the Moon's visible path with the ecliptic. Then to find the time of the nearest approach of the centres as seen from the Earth's centre, say,

As rad.	10.0000000
is to cos. 5° 37′ 10″	9.9979078
so is 50′ 46″ = 3046″	3.4837299
to 50′ 31″ = 3031″ nearest app. of centres	3.4816377

and as rad.	10.0000000
is to sin. 5° 37′ 10″	8.9908743
so is 0 50 31 = 3031″	3.4816377
to 0 4 57 = 2968	2.4725120

the Moon's motion in the ecliptic, or reduction: Hence as 34′ 13″ : 4′ 57″ : : 1*h*. : 8*m*. 41*s*. the reduction in time, to which, adding 10*h*. 22*m*. 48*s*., gives 10*h*. 31*m*. 29*s*. for the middle of the general eclipse, or time of nearest approach of the centres.

89. Now to find the time of the beginning and end of the eclipse for Greenwich: from the Moon's longitude and latitude for each hour, and the obliquity of the ecliptic determine her right ascension and declination by Prob. 15, page 206, and from thence the hour angle; the hour angle is found thus: take the difference between the Sun and Moon's right ascension; then if the Sun's right ascension be the greater, add the difference to the hour from noon in the afternoon, but subtract it from the hour in the forenoon, and *vice versâ*, when the Moon's right

ascension is the greater: the sum or remainder is the Moon's hour angle from noon. Next with this angle, the Moon's declination and the latitude of the place find her alt. and azimuth by Case 3, Obl. angled Sph. Trig. p. 136, the two given sides being the colat., and the polar distance, and the contained angle the hour from noon, and the parts sought, the third side, or coaltitude, and the angle opposite the polar distance or azimuth. The Sun's altitude and azimuth are also to be found in a similar manner.

Calculation of ☽'s right ascension and declination, Nov. 19th. at 8h. A.M.

	° ′ ″		
Lat. . . .	0 58 44 N.	tan. + 10, . .	18.2326530
long. .	235 33 32	sin.	9.9163002
arc A,	1 11 13	tan.	8.3163528
obliq. .	23 27 52		
B,. . .	22 16 39	cos.	9.9663101
A, . .	1 11 13	comp. cos. . . .	0.0000932
	235 33 32	tan.	10.1638223
AR. . .	233 27 52	tan.	10.1302256
	233 27 52	sin.	9.9049792
B, . .	22 16 39	tan.	9.6124354
decl. .	18 13 11 S.	tan.	9.5174146

The Moon's right ascension and declination at 9, 10, 11, and 12 hours are found by like calculations, and from thence the Moon's hour angle as follows.

	at 8*h*.	9*h*.	10*h*.	11*h*.
☽'s. decl.	18 13 11S.	18 25 6	18 36 51	18 48 32
☽'s. *AR*.	233 27 52	234 4 40	234 41 33	235 18 29
☉'s. *AR*.	234 37 3	234 39 39	234 42 15	234 44 51
diff. *AR*.	—1 9 11	—0 34 59	—0 0 42	+0 33 38
☉'s. h. ang.	60 0 0	45 0 0	30 0 0	15 0 0
☽'s. h. ang.	58 50 49	44 25 1	29 59 18	15 33 38

90. *Calculation of the Moon's alt. and azimuth, Nov. 19th, at 8h. A.M.*

	° ′ ″
	90 0 0
	18 13 11
☽'s. pol. dist. .	108 13 11

	° ′ ″		
☽'s. hour ang. .	58 50 49	cos. . .	9.7137643
pol. dist. . . .	108 13 11	tan. . .	10.4825867
1st arc.	122 28 4	tan. . .	10.1963510
colat.	38 31 20		
1st. arc	122 28 4	cos. . .	9.7298329
2d. arc	83 56 44	cos. . .	9.0231431
pol. dist. . . .	108 13 11	cos. . .	9.4950750
alt.	3 31 19	sin. . . .	8.7883852
alt.	3 31 19	cos. . .	9.9991790
pol. dist. . . .	108 13 11	sin. . . .	9.9776616
hour ang. . . .	58 50 49	sin. . . .	9.9323664
azim.	54 31 49	sin. . . .	9.9108490

In finding the altitude and azimuth, a perpendicular is supposed to be let fall from the angle opposite to the colatitude; the *first arc* is the distance between the pole and the aforesaid perpendicular, the *second arc* is the difference between the colat. and the first arc: the cosine of the 1st arc, and the cosine of the altitude are *subtractive* logarithms, which are easily taken away by adding in *mentally* the arithmetic complements, that is, by adding in the difference between the right hand *significant* figure and 10, and between each of the others and 9.

91. *Calculation of the Sun's alt. and azimuth, Nov. 19th, at 8h. A.M.*

	° ′ ″
	90 0 0
	19 29 19
☉'s. pol. dist. .	109 29 19

	° ′ ″		
☉'s hour ang. .	60 0 0	cos. . .	9.6989700
pol. dist. . . .	109 29 19	tan. . .	10.4511257
1st. arc	125 17 26	tan. . .	10.1500957
colat.	38 31 20		

	°	′	″		
1st. arc	125	17	26	cos. . .	9.7617197
2d. arc	86	46	6	cos. . .	8.7510736
pol. dist.	109	29	19	cos. . .	9.5232514
alt.	1	51	56	sin. . .	8.5126053
alt.	1	51	56	cos. . .	9.9997697
pol. dist. . . .	109	29	19	sin. . .	9.9743771
hour ang. . . .	60	0	0	sin. . .	9.9375306
azim.	54	46	12	sin. . .	9.9121380

92. The altitudes and azimuths of the bodies at 9, 10, and 11*h*., inserted in the subsequent table of results, in art. 95, are found by similar operations; but these being the altitudes from the Earth's centre, require correction for the effect of parallax in altitude. Now as it is not the *apparent* but the *true* altitude that is here given, the usual proportion for finding the parallax in altitude cannot be directly applied, viz. radius : cos. app. alt. :: sin. hor. parall. : sin. parall. in alt., but if we use the true alt. in place of the apparent in this proportion, we shall get an approximate parallax in alt., which subtracted from the true will give the apparent altitude *nearly*, and by proportionating a second time, and using this approximate app. alt., we shall obtain the apparent altitude very nearly. The parallax to be subtracted from the true to get the observed altitude, may also be found *directly* as follows:

let r = ☽'s. horizontal parallax
a = true altitude
p = parallax in altitude

then $\tan. p = \frac{\sin. r \cos. a}{1 - \sin. r \sin. a}$

93. *Calculation of ☽'s. parallax in altitude, Nov. 19th, at 8h, A.M. by approximation.*

	°	′	″		
As radius					10.0000000
is to cos. alt. . . .	3	31	19	. . . .	9.9991790
so is sin. hor. par. .	1	0	18	. . . .	8.2440212
to sin. par. in alt. .	1	0	11	. . . .	8.2432002
	3	31	19		
approx. app. alt.	2	31	8		
As radius					10.0000000
is to cos. appr. app. alt.	2	31	8	. .	9.9995802
so is sin. hor. par. . .	1	0	18	. . .	8.2440212
to sin. par. in alt. . . .	1	0	15	. . .	8.2436014
	3	31	19		
app. alt. very nearly . .	2	31	4		

The same by the direct method.

	°	′	″				
sin. *r*,	1	0	18	8.2440212	. . . . .		8.2440212
cos. *a*,	3	31	19	9.9991790	. . .	sin. *a*, ...	8.7883863
				18.2432002	. . .	0010775	7.0324075
				9.9995318	. . .	.9989225	
tan. *p*,	1	0	15	8.2436684			

94. We have thus found that the apparent altitude of the Moon's centre at 8*h*. is 2° 31′ 4″: the apparent altitude of the Sun's centre at the same time is 1° 51′ 56″ minus 9″ (the parallax) = 1° 51′ 47″; the apparent altitudes of both luminaries being now known and the difference of their azimuths, viz. (54° 46′ 12″ — 54° 31′ 49″ =) 0° 14′ 23″, we have next to find the distance of the centres; this may be conveniently done upon the principles of middle latitude sailing, thus: take half the sum of the apparent altitudes for a mean altitude, and say, as radius is to the cosine of the mean altitudes, so is the difference of the azimuths (in seconds) to the mean parallel distance of the Sun and Moon in seconds; then take this as the base of a right angled plane triangle, and the difference of the apparent altitudes as the perpendicular, and from thence find the hypothenuse, or visible distance of the centres of the Sun and Moon, which compared with the sum of the semi-diameters at the same time will shew whether any, and what portion of the Sun is obscured at that time.

	°	′	″	
☽'s. app. alt. at 8*h*.	2	31	4	
☉'s. do.	1	51	47	
diff. app. alt.	0	39	17	= 2357″
sum. app. alt.	4	22	51	
mean app. alt.	2	11	25	

diff. azimuths 14′ 23″ = 863″

As radius	10.0000000
is to cos. 2° 11′ 25″	9.9996826
so is 863″	2.9360108
to 862″	2.9356939

Then $\sqrt{[862^2 + 2357^2]} = 2510'' = 41' 50''$ the apparent distance of the centres.

☽'s. semi. diam. at 8*h*.	16′ 27″
Augm. Table XXXV.	1
☉'s. sem. diam.	16 13
	32 41

hence the eclipse has not begun at 8 hours.

95. By making calculations similar to the preceeding for 9, 10 and 11*h*. the following results are obtained.

Nov. 19th.	at 8*h*. A.M.	9*h*.	10*h*.	11*h*.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
☉'s. azim. . .	54 46 12	42 25 24	29 6 12	14 50 42
☽'s. azim. . .	54 31 49	42 24 32	29 23 24	15 31 21
diff. azim. . .	+ 0 14 23	+ 0 0 52	— 0 17 12	— 0 40 39
☉'s. true alt. .	1 51 56	8 51 33	14 18 32	17 47 47
☉'s. par. alt.	9	9	9	8
☉'s. app. alt. .	1 51 47	8 51 24	14 18 23	17 47 39
☽'s. true alt. .	3 31 19	10 4 17	15 9 31	18 24 11
☽'s. par. alt. .	1 0 15	0 59 33	0 58 28	0 57 32
☽'s. app. alt. .	2 31 4	9 4 44	14 11 3	17 26 39
☉'s. app. alt. .	1 51 47	8 51 24	14 18 23	17 47 39
diff. app. alt. .	+ 0 39 17	+ 0 3 20	— 0 7 20	— 0 21 0
sum. app. alt. .	4 22 51	17 56 8	28 29 26	35 14 18
mean app. alt.	2 11 25	8 58 4	14 14 43	17 37 9
paral. dist. . .	862″	51	1000	2325
diff. app. alt. .	2357	800	440	1260
dist. ☉ and ☽	0 41 50	0 13 22	0 18 13	0 44 4
☉'s. sem. diam.	0 16 13	0 16 13	0 16 13	0 16 13
☽'s. sem. diam.	16 27	16 27	16 27	16 26
augm.	1	3	4	5
sum. sem. diam.	0 32 41	0 32 43	0 32 44	0 32 44
dist. centres .	41 50	13 22	18 13	44 4
diff.	0 9 9	0 19 21	0 14 31	0 11 20
	parts free.	parts eclipsed	parts eclipsed	parts free

96. The *approximate* times of the beginning and end of the eclipse may now be found thus:

Dist. centres at 8*h*.	′ ″ 41 50	dist. at 10*h*. . . .	′ ″ 18 13
9*h*.	13 22	. . . . 11*h*. . . .	44 4
hor. mot. ☽ from ☉	28 28	hor. mot. ☽ from ☉	25 51

Then as 28′ 28″ : 9′ 9″ :: 1*h*. : 0*h*. 19*m*. 17*s*.
add 8*h*. 0*m*. 0*s*.
appr. time beginning . . . . 8*h*. 19*m*. 17*s*.

And as 25′ 51″ : 14′ 31″ :: 1*h*. : 0*h*. 33*m*. 42*s*.
add 10*h*. 0*m*. 0*s*.
app. time of the end . . . . . 10*h*. 33*m*. 42*s*.

97. To find the time of the vertical conjunction, we must first find the approximate time; now we have

	°	′	″
difference azim. at 9h.	+ 0	0	52
at 10h.	—	17	12
hourly diff. in azim.	0	18	4

then as 18′ 4″ : 52″ :: 1h. : 3m.; so that the time of vertical conjunction is about 9h. 3m., and as we have the exact computed azimuths for two instants (8 and 9h.) before, and two after (10 and 11h.) this time, the true time of vertical conjunction may be very exactly found by the known rule for interpolation, viz: "if the terms of a series be supposed to be at an "unit's distance from each other, and y be any term "to be interpolated, x its distance from the first term, "a, and d, d, d, &c. the first of the several orders "of differences, then $y = a + x\,{}^{d}_{1} + x\,\frac{x-1}{2}\,{}^{d}_{2} + x\,\frac{x-1}{2}\,\frac{x-2}{3}\,{}^{d}_{3} +$ &c."

Now let the Sun and Moon's azimuth at 9h. 3m., and 9h. 4m. be determined by this formula.

	⊙'s. azim.	1st. diff.	2d. diff.	3d. diff.
	° ′ ″	° ′ ″	′ ″	′ ″
at 8h.	54 46 12			
		— 12 20 48		
9h.	42 25 24		— 58 24	
		— 13 19 12		+2 6
10h.	29 6 12		— 56 18	
		— 14 15 30		
11h.	14 50 42			

hence $d = -44448''$, $d = -3504$, $d = 126$, $x = 1h.\ 3m. = 1\frac{1}{20} = \frac{21}{20}$, and substituting these values in the series,

		°	′	″
a	=	54	46	12
xd . . . . . $= \frac{21}{20}$. — 44448 =	—	12	57	50
$x\,\frac{x-1}{2}\,d$ . . . $= \frac{21}{800}$. — 3504 =	—		1	32
$x\,\frac{x-1}{2}\,\frac{x-2}{3}\,{}^{d}_{3} = -\frac{21}{48000}$. 126″ =	—			1
⊙'s. azim. at 9h. 3m.		41	46	49

In a similar manner by taking $x = 1h.\ 4m. = 1\frac{1}{15}$ or $\frac{16}{15}$ the Sun's azim. at 9h. 4m. will be found to be 41° 33′ 55″.

	☽'s azim.	1st. diff.	2d. diff.	3d. diff.
	° ′ ″	° ′ ″	′ ″	′ ″
at 8h.	54 31 49			
		— 12 7 17		
9h.	42 24 32		— 53 51	
		— 13 1 8		+2 56
10h.	29 23 24		— 50 55	
		— 13 52 3		
11h.	15 31 21			

$d = -43637''$, $d = -3231$, $d = 176$, $x = \frac{21}{20}$

		°	′	″
a	=	54	31	49
xd . . . . . $= \frac{21}{20}$. — 43637 =	—	12	43	39
$x\,\frac{x-1}{2}\,{}^{d}_{2}$ . . . $= \frac{21}{600}$ — 3231 =	—		1	25
$x\,\frac{x-1}{2}\,\frac{x-2}{3}\,{}^{d}_{3} = -\frac{300}{48000}$. 176 =	—			2
☽'s. azim. at 9h. 3m.		41	46	43

and similarly her azim. at 9h. 4m. is found = 41° 34′ 6″.

	°	′	″		°	′	″
⊙'s. azim. at 9h. 3m.	41	46	49	at 9h. 4m.	41	33	55
☽'s. do.	41	46	43	. . . .	41	34	6
diff.	0	0	6	. . . .	0	0	11

then as 11 + 6 = 17″ : 6″ :: 1m. : 21s, wherefore the true time vert. conjunction = 9h. 3m. 21s.

97. Having now obtained the approximate time of the beginning and end of the eclipse, and the time of the vertical conjunction, the operations must be repeated, and the true distances of the centres found for these times, in order to compare them with the sum of the semi-diameters, and to correct the times if found necessary.

The differences of the right ascensions and declinations of the Sun and Moon at 8, 9, 10, and 11 hours, are so nearly equal that the right ascensions and declinations at the approximate times of the beginning and end of the eclipse, and at the time of vertical conjunction, may be found by simple proportional parts. The altitudes and azimuths must be recomputed by Sph. Trig. All these calculations are exactly similar to those which have been already explained, the results therefore are only given in the following table.

Nov. 19th.	at 8h. 19m. 17s.			at 9h. 3m. 21s.			at 10h. 33m. 42s.		
	°	′	″	°	′	″	°	′	″
☽'s. declin. . . .	18	17	1	18	25	45	18	43	25
☉'s. declin. . . .	19	29	30	19	29	56	19	30	49
☽'s. *AR*.	233	39	42	234	6	44	235	2	18
☉'s. *AR*.	234	37	53	234	39	48	234	43	43
diff.	— 0	58	11	— 0	33	4	+ 0	18	35
☉'s. hour ang. .	55	10	45	44	9	45	21	34	30
☽'s. hour ang. .	54	12	34	43	36	41	21	53	5
☉'s. azim. . . .	50	53	56	41	42	23	21	11	43
☽'s. azim. . . .	50	43	31	41	42	23	21	41	28
diff. azim. . . .	+ 0	10	25	0	0	0	— 0	29	45
☉'s. true alt. . .	4	15	20	9	12	29	16	32	2
☉'s. par. alt. . .			9			9			9
☉'s. app. alt. . .	4	15	11	9	12	20	16	31	53
☽'s. true alt. . .	5	45	45	10	23	52	17	13	49
☽'s. par. alt. . .	1	0	6	0	59	29	0	57	53
☽'s. app. alt. . .	4	45	39	9	24	23	16	15	56
☉'s. app. alt. . .	4	15	11	9	12	20	16	31	53
diff. app. alt. . .	+ 0	30	28	+ 0	12	3	— 0	15	57
sum. app. alt. . .	9	0	50	18	36	43	32	47	49
mean app. alt. . .	4	30	25	9	18	21	16	23	54
parall. dist. . . .	623″			0″			1712″		
diff. app. alt. . .	1828			723			957		
app. dist. centr. .	0	32	11	0	12	3	0	32	41
☉'s. sem. diam. .	0	16	13	0	16	13	0	16	13
☽'s. sem. diam. .		16	27		16	27		16	26
augm.			1			3			5
sum sem. diam. .	0	32	41	0	32	43	0	32	44
dist. cent. . . .		32	11		12	3		32	41
difference	0	0	30	0	20	40	0	0	3

98. We have thus found that the sum of the semi-diameters at the approximate time of the beginning, exceeds the distance of the centres by 30″, which time therefore requires correction, thus: as 28′ 28″ (the difference between the distances of the centres at 8 and 9*h.*) : 30″ :: 1*h.* : 1*m.* 3*s.*

	h.	*m.*	*s.*	
From	8	19	17	
subtract		1	3	
	8	18	14	time begin. corrected.

And as 25′ 51″ (the difference between the dist. of cent. at 10 and 11*h.*) : 3″ :: 1*h.* : 7*s.*

	h.	*m.*	*s.*	
To	10	33	42	
add			7	
	10	33	49	time end. corrected.

99. To find the time of the greatest obscuration or nearest approach of the centres and the digits eclipsed.

Paral. dist. at 9*h.*			51″
at 9*h.* 3*m* 21*s.* . . .			0
change in 3*m.* 21*s.* . . .			51
diff. app. alt. at 9*h.*	′ 13	″ 20	
at 9*h.* 3*m.* 21*s.* .	12	3	= 723″
change in 3*m.* 21*s.* .	1	17	= 77

As 77 : 51 :: rad. : tan. 33° 31′ 5″ = the angle of ☾'s. visible path from ⊙ with the vertical circle; also, as rad. : sin. 33° 31′ 5″ :: 723″ (dist. cent. at vert. conj.) : 399 = 6′ 39″ = the nearest distance of the centres at the greatest obscuration; and as rad. : cos. 33° 31′ 5″ :: 399 : 333 = 5′ 33″ = the reduction in motion on the parallel of altitude.

Paral. dist. at 9*h.*	+ 51
at 10*h.*	— 1000
hor. mot. on paral.	1051

Then say, as the hor. mot. on the paral. of alt. is to the reduction, so is 1*h.* to the reduction in time, that is, as 1051″ : 333″ :: 1*h.* : 19*m.* 1*s.*

add time vert. conj. . .	9*h.*	3*m.*	21*s.*
time greatest obscur. . .	9*h.*	22*m.*	22*s.*

	′	″
sum semi-diam. at greatest obscur.	32	43
nearest dist. cent. at do.	6	39
parts deficient	26	4

As ⊙'s. diam. 32′ 26″ : 26′ 4″ :: 12 dig. : 9*d.* 38*m.* the digits eclipsed.

100. To find the distance of the point of contact from the Sun's vertex at the beginning of the eclipse; say, as sin. sum semi-diam. : sin. diff. azim. at beginning, so is cos. ☾'s. app. alt. to sin. distance; that is, as sin. 32′ 41″ : sin. 10′ 25″ :: cos. 4° 45′ : sin. 18° 31′; observe, if the Moon's app. alt. be less than the Sun's, the supplement of this arc is to be taken for the dist. of the contact from the Sun's vertex.

101. Lastly, to find the time of the equatorial conjunction, and the lat. and long. of the place where the Sun will be centrally eclipsed on the meridian, also the diameters of the penumbra and dark shadow; it appears by an inspection of the differences of the Sun and Moon's right ascensions, that the equatorial conjunction takes place a little after 10 o'clock.

	°	′	″
Diff. right asc. at 10*h.*	— 0	0	42
at 11*h.*	+ 0	33	38
change in 1 hour	0	34	20

As 34′ 20″ : 42″ :: 1*h.* : 0*h.* 1*m.* 13*s.*
add . . . 10*h.*

time equat. conj.. . . . 10*h.* 1*m.* 13*s.* A.M.

time from noon . . . . 1*h.* 58*m.* 47*s.* = 29° 41′ [45″, long. E.

102. To determine the lat. of the place where the Sun is centrally eclipsed on the meridian, it is to be observed, that the difference of the Sun and Moon's declin. at the time of the equatorial conjunction, is equal to the Moon's parallax in altitude, at the time the sun is centrally eclipsed on the meridian; hence, we find the Sun or Moon's app. alt. for that time by saying, as the Moon's horizontal parall.: diff. declin. :: rad. : cos. ⊙ or ☾'s. alt.

	°	′	″	
⊙'s. declin. at 10*h.* 1*m.* 13*s.*	19	30	30	S.
☾'s. do.	18	37	5	S.
diff. . .	0	53	25	

As 60′ 16″ : 53′ 25″ :: rad. : cos. 27° 35′ 0″ = ⊙ or ☾'s. alt.

	°	′	″
⊙'s. app. alt. on merid.	27	35	0
⊙'s. paral. alt.			8
⊙'s. true alt.	27	35	8
⊙'s. zen. dist.	62	24	52 S.
⊙'s. declin.	19	30	30 S.
Latitude, ...	42	54	22 N.

103. To find the semi-diameters of the penumbra and dark shadow.

	′	″	
☽'s. sem. diam. at eq. conj. . . .	16	27	
augm.		4	
	16	31	
⊙'s. sem. diam.	16	13	
sem. diam. penumbra	32	44	see art. 27.
sem. diam. umbra. or dark shadow	0	18	see art. 24.
☾'s. hor. par. at eq. conj.	60	16	
subtract ☾'s. hor. par. at ⊙ . .		2	
semi-diam. Earth's disc	60	14	

mean sem. diam. of Earth in English miles 3957.

As 60′ 14″ : 32′ 44″ : : 3957 : 2150*m.* = sem. diam. penumbra, and 4300 = diam.

As 60′ 14″ : 0′ 18″ : : 3957 : 20*m.* = sem. diam. dark shadow, and 40 = diam.

Also, as sin. 27° 35′ : rad. : : 40 : 86*m.* = greater diam. of dark shadow.

RESULTS.

104. *The Sun eclipsed, Nov. 19th, 1816, visible at Greenwich.*

	h.	*m.*	*s.*	
Beginning	8	18	14	A.M.
greatest obscuration	9	22	22	
vertical conjunction	9	3	21	
ecliptic conjunction	10	22	48	
end	10	33	49	

digits eclipsed 9*d.* 38*m.* on the Sun's North limb.

The Moon makes the first impression on the Sun's disk, at 18° 31′ from his vertex on the right.

The Sun is centrally eclipsed on the meridian at 10*h.* 1*m.* 13*s.* in lat. 42° 54′ 22″ N. and long. 29° 41′ 45″ East, and the Moon's dark shadow extends 86 English miles on the meridian North and South, and 40 miles East and West. The place is in the Euxine Sea, about geographical miles N.N.E. of Constantinople.

Of OCCULTATIONS.

THE calculation of an occultation of a fixed star by the Moon, is nearly the same as that of a solar eclipse, only that the star has neither semi-diameter nor parallax, and its right ascension and declination are not sensibly changed during the occultation, whereas those of the Sun, in a solar eclipse, have a small change. It will therefore not be necessary to insert the calculation of an occultation at length—the object to be aimed at, is (as in an eclipse), to find the apparent distances of the star and of the Moon's centre at several instants of time, and then by comparing these with the Moon s apparent semi-diameter for the same instants, it will be seen at which of these times the star is visible or occult, and by proportionating between the times, the instants of immersion and emersion will be determined. Before the operation is commenced, the star's mean longitude and latitude must be reduced to the time by proper allowances for the annual procession and variation, &c. If for instance, we wished to compute the occultation of β Scorpii, which happened Feb. 19th, 1816, its true longitude and latitude are to be reduced thus, from the elements given to the computers of the Nautical Almanac by Dr. Maskelyne.

	s.	°	′	″
Mean long. β Scorpii, 1st Jan. 1815,	8	0	36	23.0
Ann. procession for 1 year, ...				50.2
Do. for 50 days to 19th Feb.				7.7
Mean long.	8	0	37	20.9
Equa. equinoctial points, ...				— 17.6
Aberration, ...				0.0
✳'s true long. 19th Feb. 1816, . . .	8	0	37	3.3
Mean latitude, ...			1	1 56N
Ann. variation, ...				— 0.4
Do. for 50 days, ...				— 0.1
Mean lat.			1	1 55.5
Aberration, ...				0.3
✳'s. true lat. 19th Feb.			1	1 55.8

The star's true lat. and long. being thus found, if we proceed according to the method in art. 85, &c. we must find from the lat. and long. the star's right ascension and declination—also, as before, the Moon's long. and lat. for several assumed times during the probable period of the occultation, and thence her right ascension and declination for those times; after which the Moon and star's hour angle—the star's hour angle is found in a similar manner to the Moon's,

viz., by taking the difference of the Sun and star's right ascension from the sun's hour angle; these being determined, the altitudes and azimuths of the bodies are next to be found, the true altitude of the Moon being reduced to the observed, by the rule in art. 92, then from the differences of the app. altitudes and the differences of the azimuths, the parallel distances and the distances of the centres will be found.

It seems necessary now to instruct the student in the method of finding the periods of Eclipses, Occultations, and Transits, which is nearly the same for all. Now by the *periods* of these phenomena we mean the times when the celestial bodies producing them after a certain number of revolutions returning to their former situations, must necessarily produce the same again; in order to determine these periods, we have to premise the following auxiliary arithmetical problems.

Problem 1.

Two whole numbers, *prime* to each other, being given, to find their *least* common multiple; that is, the least number divisible by each without a remainder.

Rule.—Their product is the number sought.

Example.—What is the least number that 7 and 11 will divide without a remainder? Answer, $7 \times 11 = 77$.

After the same manner, the continual product of three or more prime numbers, is their least common multiple.

Problem 2.

Two large numbers, prime to each other, being given to find a series of other two less numbers, nearly in the ratio with the two given numbers.

Rule.—Divide the greater by the less, and the less by the remainder, and the last divisor by the last remainder, and so on till nothing remain.

Then write unit (1) over the first quotient, and these are the first antecedent and consequent. Again multiply the first antecedent by the second quotient, and add, 0, and you have the second antecedent: also multiply the first consequent by the second quotient, and add 1, and you have the second consequent.

In like manner multiply the second antecedent and consequent by the third quotient, and to the products add the first antecedent and consequent respectively, and you have the third antecedent and consequent. And in general multiply the last antecedent and consequent by the next quotient, and to the products add the antecedent and consequent immediately preceding, respectively, and you have the next antecedent and consequent; and so on till the two given numbers are finally arrived at, which proves the operation.

Example.—Suppose the ratio of the diameter and circumference of a circle to be as 10000 to 31416; it is required to find a series of less numbers nearly expressing the same ratio? now the given numbers are not prime to each other, but can be divided each by 8, which must be done, and the quotients express the same ratio.

$$\frac{10000}{31416} = \frac{1250}{3927}.$$

```
1250)3927(3
     3750
     ----
 177)1250(7
     1239
     ----
   11)177(16
      176
      ---
    1)11(11
      -
      0
      -
```

$\frac{1}{3}, \frac{7}{22}, \frac{113}{355}, \frac{1250}{3927}$ antecedents.
consequents.

$1 \times 7 + 0 = 7$, the 2d antecedent.
$3 \times 7 + 1 = 22$, the 2d consequent.
$7 \times 16 + 1 = 113$, the 3d antecedent.
$22 \times 16 + 3 = 355$, the 3d consequent.
$113 \times 11 + 7 = 1250$, the 4th antecedent.
$355 \times 11 + 22 = 3927$, the 4th consequent.

Hence, as 1 : 3, or 7 : 22, or 113 : 355: or 1250 : 3927 :: the diameter to the circumference. Where the first ratio is not so exact as the second; and the second not so exact as the third, and so on till the ratio first given is arrived at.

If the Moon's orbit were coincident with the ecliptic, then the Moon would be centrally and totally eclipsed at the time of every full Moon; and the Sun would be centrally eclipsed to some part of the torrid zone, at the time of every new Moon: but this is not the case; for the Moon's orbit is inclined to the ecliptic in an angle of 5° 10′ nearly, and consequently the orbit and ecliptic cut each other in two opposite points, called the *Moon's Nodes*. Now according to some computers, if the Sun and Moon be more than 18° 20′ from the nodes, at the time of new Moon, there can be no eclipse of the Sun; but if their distance from the nodes be less than 16° 30′, there must be a solar eclipse to some part of the Earth: and when their distance from the nodes is between 16° 30′ and 18° 20′, the matter is uncertain,

till trial is made, according to our former precepts. Again, if their distance from the nodes, at the time of full Moon, be more than 12° 6′, there can be no lunar eclipse; but if it be less than 10° 30′, the Moon must be eclipsed, and between these limits the matter is uncertain, till trial is made as before taught.

If the nodes were stationary, that is, if they remained at rest in the same sign, degree, minute, &c. of the ecliptic, the eclipses would happen at the same time every year nearly, or rather in the same months every year; so that, if the Sun passed one node any year on the 15th of May, and the other on the 15th of November, then May and November would always be the ecliptic months, or the months in which eclipses would always happen. But the nodes have a retrograde motion, and are constantly shifting their place backwards from east to west, at the mean rate of 19° 19′ 43.4″, per year; and therefore the same Node moves round and meets the Sun in the ecliptic about 19 days sooner every year, and hence the mean time of eclipses, in any year, is 19 days sooner than in the former one: so that in about 18 years and 225 days, the nodes return to their former place, and consequently in that period the eclipses circulate through all the several months of the year. And since the time of the Moon's mean synodical revolution, that is, from change to change, or from full to full is 29 days, 12 hours, 44 minutes, and 3 seconds nearly: and her periodic revolution, from one node to the same again, 27 days, 5 hours, 5 minutes, and 35 seconds nearly.

	d.	*h.*	*m.*	*s.*		*s.*
Synodic,	29	12	44	3	=	2551443
Periodic,	27	5	5	35	=	2351135

Hence by reducing the times of both revolutions into seconds, it is manifest, that 2351135 synodic revolutions are completed in the same time that 2551443 periodic revolutions are: and were the motions of the Sun, Moon, and node equable, then all would be exactly situated at the end of that period as they were at the beginning of it: and an eclipse of either luminary happening at the beginning of this period must recur at the end of it, and be exactly similar and equal in all respects. But the inequalities of the Sun and Moon's motions, may sometimes increase or decrease this period, by 10 hours nearly.

We are now to find other shorter periods, which may approach nearly to the ratio of those great and exact ones foregoing; and this is performed by the aforegoing 2d auxiliary problem. Now by proceeding agreeably to the rule, we have

$\frac{2351135}{2551443}$, or $\frac{1}{1}$, $\frac{11}{12}$, $\frac{12}{13}$, $\frac{35}{38}$, $\frac{47}{51}$, $\frac{223}{242}$, $\frac{716}{777}$, $\frac{1655}{1796}$,

&c. each approaching still nearer and nearer to the exact ratio. The numerators express the lunations or synodic revolutions, answering to the denominators, which express the number of revolutions of the Moon to the node in the same time. The 1st, 3d, 5th, &c. viz. the numerators of the odd ones, are too great; and the 2d, 4th, 6th, &c. or even ones, are too small.

To examine these numbers, omitting the small fractions, which are not exact enough, let us try $\frac{47}{51}$, which is the 5th pair. Then 29*d*. 12*h*. 44*m*. 3*s*. × 47 = 1387*d*. 22*h*. 30*m*. 21*s*; and 27*d*. 5*h*. 5*m*. 35*s*. × 51 = 1387*d*. 19*h*. 44*m*. 45*s*.; which is 2*h*. 45*m*. 36*s*. less than the former; and consequently the Moon shall have passed the node, only 1° 31′, which is far within the limits. This period, which is 3 common years, and 292*d*. 22½*h*., nearly, would bring on a return of the same eclipses for at least 10 or 12 successive periods, were it not for the inequalities of the Moon's motion; for the equation of the Moon's centre is, of itself, sufficient to throw her beyond the limits in 4 periods: but on the other hand, the equation of her centre conspiring with all her other equations, and those of the Sun, may cause it to last 17 or 18 periods.

Let us again try the 6th fraction, viz. $\frac{223}{242}$. Then 29*d*. 12*h*. 44*m*. 2.88*s* × 223 = 6585*d*. 7*h*. 42*m*. 42*s*.; and 27*d*. 5*h*. 5*m*. 35.59*s* × 242 = 6585*d*. 8*h*. 33*m*. 33*s*., the difference is 50*m*. 51*s*, which is equal to 28′ 2″, of the Moon's motion from the node: now this number is contained 39 times in the greatest limit for a solar eclipse, and 35 times in the least; and 26 times in the greatest limit for a lunar eclipse, and 23 times in the least. And if these numbers be doubled, in order to include the limits on each side of the node, we have 78 periods for the greatest, and 70 for the least limits of solar eclipses; and 52 and 46 periods for lunar ones.

This period consists of 18 years, 11 days, 7 hours, 42 minutes, and 42 seconds; supposing 4 bissextile years included; but if there be 5 leap years included, it is then 18*y*. 10*d*. 7*h*. 42*m*. 42*s*. And hence, if to the time of any solar or lunar eclipse we add 18*y*. 11*d*. 7*h*. 42*m*. 42*s*., when only 4 leap years are included; or 18*y*. 10*d*. 7*h*. 42*m*. 42*s*., when 5 leap years are included in the period, we shall have the mean time of the return of that eclipse: but this time may vary several hours, by reason of the inequalities of the Moon's motion; and moreover the magnitude and duration, beginning and end of the very next returning eclipse, may differ greatly from the former. The inequalities also of the Moon's motion may cause the number of periods to be 7 or 8 less, or more, than those stated above.

Note.—The Moon's mean daily motion from the Sun, taken from our tables, is 12° 11′ 26″ 41.8‴,

which gives the length of the synodic month, 29 days, 12 hours, 44 minutes, 2.88 seconds. And the Moon's mean daily motion from the node, 13° 13′ 45″ 40‴, which gives 27*d*. 5*h*. 5*m*. 35.59*s*, for the length of the periodic month of the Moon to the node. And by this means our periods differ considerably from those of former astronomers, who deduced their numbers from lunar tables of less accuracy. Mr. Ferguson makes the second period of 18 years, &c. 38*s*. too much; and the Moon's distance from the node 10″ too great; and Dr. Brewster 8″ too great.

The periods of occultations are determined in the same manner as those of eclipses, and therefore require no farther illustration: but we may just observe, that on account of the small apparent motion of the fixed stars, an occultation of the same fixed star will happen nearly at the time the node takes to make one revolution, viz. in 18 years and 225 days, which is nearly the period of occultations.

Of Transits.

1*st. To determine the time of the return of the transits of Mercury.*

The Earth moves round the Sun in

	d.	*h.*	*m.*	*s.*		*s.*		
	365	6	9	12	=	31558152	or as	15779076
Mercury,	87	23	15	34	=	7600534		3800267

Then per prob. 2, $\left\{\begin{matrix}3800267\\15779076\end{matrix}\right.$, as $\frac{1}{4}$, $\frac{6}{25}$, $\frac{7}{29}$, $\frac{13}{54}$, $\frac{33}{137}$, $\frac{46}{191}$, $\frac{263}{1092}$, $\frac{572}{2375}$, &c. each set of numbers approaching nearer and nearer to the given ratio: where the numerators are the number of sidereal years of 365*d*. 6*h*. 9*m*. 12*s*., and the denominators are the number of revolutions of Mercury round the Sun in the same time nearly.

Now in 1 year a transit of Mercury cannot return, in 6 it may, in 7 it often happens, in 13 still oftener, and so on till we come to 263 and 572, when it scarcely can fail once in 50 times.

2*d. To determine the time of the return of the transits of Venus.*

The Earth revolves round the Sun in

	365*d*.	6*h*.	9*m*.	12*s*.	=	31558152*s*.	or as	15779076
Venus,	224	16	49	10	=	19414150		9707075

Then per prob. 2, $\left\{\begin{matrix}9707075\\15779076\end{matrix}\right.$, as $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{13}$, $\frac{235}{382}$, $\frac{478}{777}$, $\frac{8839}{14368}$, &c. where as in the numbers for Mercury, the numerators express the number of sidereal years, and the denominators the number of revolutions made by Venus round the Sun, in the same time nearly; each set approaching nearer and nearer to truth. The first four fractions may be omitted, as not sufficiently accurate; but the fifth, viz. $\frac{8}{13}$, that is, 8 sidereal years, or 13 revolutions of Venus, will almost always discover 1 transit and sometimes 2: the sixth, viz., $\frac{235}{382}$, or 235 sidereal years, may be carried on for upwards of 1000 years, till it discover 5 or 6 transits; and the seventh and eighth, if carried on, cannot fail scarce once in 10,000 years. We are now to shew the use of these numbers in discovering the times of the transits of Mercury and Venus.

Rule.

1st. For *Mercury*.—To the year and day in which any transit of this planet actually happened, at one of its nodes, add 6, 7, 13, 33, 46, 263, &c. years, or their doubles, trebles, &c. and you have the several years and days nearly, when the next succession of transits will probably happen at the same node. Proceed in like manner, with the year and day when a transit happened at the other node, and you have the year and day when the next succession of transits will probably happen, at that node. But the day and hour may differ from 1 to 5 days, by reason of the great equation of the centre of this planet.

2d. For *Venus*.—To the year and day in which any transit of this planet was observed, add 8, 235, 478, &c. years, or any multiple of 8 not exceeding 6 or 7 times, may be added or subtracted, after the great numbers 235, 478, &c. have been applied, and you have all the years and day when the next transits of this planet may happen. And so proceed with the time when a transit happened at the other node, &c. But the day and hour may differ sometimes more than 1 whole day, by reason of the equation of the centre, although it is but small in this planet.

Now there was a transit of Mercury at the descending node, 3d May, 1786; and one at the ascending node, 5th November, 1799. Hence,

For the descending node . .	1786	1786	1786	1786	1786	1786	&c.
Add	6	7	13	33	46	263	
Probable years of transits .	1792	1793	1799	1819	1832	2049	&c., of these 1799, 1832, & 2049 succeed.

Again,							
Take the 1st successful year	1799	1799	1799	1799	1799	1799	&c.
Add	6	7	13	33	46	263	
Probable years of transits	1805	1806	1812	1832	1845	2062	&c., of these, 1832, 1845, and 2062 succeed.
Again,							
Take the next successful year	1832	1832	1832	1832	1832	1832	&c.
Add	6	7	13	33	46	263	
Probable years of transits . . .	1838	1839	1845	1865	1878	2095	&c. of these, 1845 and 2095 succeed.
Again							
Take the next successful year	1845	1845	1845	1845	1845	1845	&c.
Add.	6	7	13	33	46	263	
Probable years of transits . . .	1851	1852	1858	1878	1891	2108	&c. of these, 1878, 1891, & 2108 succeed.

Hence the years of the transits of Mercury at his descending node, during the 19th century, are 1832, 1845, 1878, and 1891, May 4th $\pm$ 4 or 5 days. Yet some of the other years may produce small transits, if strict calculation were made for each.

For the ascending node . . ,	1789	1789	1789	1789	1789	1789	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1795	1796	1802	1822	1835	2052	&c. of these, 1802, 1822, 1835 & 2052 succeed
Again,							
Take	1802	1802	1802	1802	1802	1802	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1808	1809	1815	1835	1848	2065	&c. of these 1815, 1835, 1848 & 2065 succeed
Again							
Take	1815	1815	1815	1815	1815	1815	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1821	1822	1828	1848	1861	2078	&c. of these 1822, 1848, 1861 & 2078 succeed
Again,							
Take	1822	1822	1822	1822	1822	1822	&c.
Add . , ...	6	7	13	33	46	263	
Probable years of transits . :	1828	1829	1835	1855	1868	2085	&c. of these 1835, 1868 and 2085 succeed.
Again,							
Take	1835	1835	1835	1835	1835	1835	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1841	1842	1848	1868	1881	2098	&c. of these 1848, 1868, 1881 & 2098 succeed.
Again,							
Take	1848	1848	1848	1848	1848	1848	&c.
Add	6	7	13	33	46	263	
Probable years of transits. .	1854	1855	1861	1881	1894	2111	&c. of these 1861, 1881, 1894 & 2111 succeed.

Again,							
Take	1861	1861	1861	1861	1861	1861	&c.
Add	6	7	13	33	46	263	
Probable years of transits. .	1867	1868	1874	1894	1907	2124	&c. of these, 1868, 1894, 1907 & 2124 succeed.
Again,							
Take	1868	1868	1868	1868	1868	1868	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1874	1875	1881	1901	1914	2131	&c. of these, 1881 &c. succeed.
Again,							
Take	1881	1881	1881	1881	1881	1881	&c.
Add	6	7	13	33	46	263	
Probable years of transits . .	1887	1888	1894	1914	1927	2144	&c. here, 1894 succeeds.

Hence the years of Mercury's transits at his ascending node during the 19th century, are, 1802, 1815, 1822, 1835, 1848, 1861, 1868, 1881, and 1894, November 6th $\pm$ 4 or 5 days. Yet several of the other years may produce small transits, if strict calculation be made.

Transits of Venus.

Now there was a transit of Venus at the descending node 3rd June 1769; and one at the ascending node 4th December 1639. Hence:

For the descending node	1769	1769	1769	1769	&c.
Add	8	235	478	8839	
Probable years of transits .	1777	2004	2247	10608	&c. these succeed, except the 1st, which is doubtful.
Again,					
Take	2004	2004	2004	2004	&c.
Add	8	235	478	8839	
Probable years of transits.	2012	2239	2482	10843	&c. these all succeed except 2239, which is doubtful.
Add	8	8	8	8	
Probable years of transits.	2020	2247	2490	10851	&c. these all succeed except the 1st, which is doubtful.
Add		8	8	8	
Probable years of transits.		2255	2498	10859	&c. these all succeed.
Add			235	235	
Probable years of transits.			2733	11094	&c. these all succeed.
Add			8	8	
Probable years of transits.			2741	11102	&c. these all succeed.

Hence the transits of Venus at the descending node for succeeding centuries, are in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741, &c.; and if to many of these, 8 years be successively added and subtracted, several other years will be found which upon trial may turn out to be transit years. But in these long periods, the day of the month must vary greatly from 3rd or 4th of June, generally encreasing one day for every century.

For the times at the ascending node, the operation is the same; and the several years in which transits happen at this node will be found to be in December 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, &c. and by adding and subtracting 8 years, several other years may be found, which on trial may discover other transits, besides those we have mentioned. We shall now exhibit a table of the foregoing transits, in order to bring them together at one view.

Table of transits of Mercury and Venus.

Mercury.				Venus.			
May		November		June		December	
th		th		th		th	
4	1832	8	1802	7	2004	8	1874
8	1845	11	1815	5	2012	6	1882
6	1878	4	1822	11	2247	10	2117
9	1891	7	1835	8	2255	8	2125
		9	1848	12	2490	12	2360
		11	1861	9	2498	10	2368
		4	1868	15	2733	15	2603
		7	1881	12	2741	13	2611
		10	1894		&c.		&c.

It remains to shew, how the exact time when a transit shall happen is discovered, and also how to find its magnitude, duration, &c.

First find the year and day by the foregoing method, then compute the exact time of the inferior conjunction of the planet with the Sun; and this time will be the exact time nearly of the middle of the transit as seen from the Earth's centre. And here it must be observed, that the exact time of the conjunction may differ many hours, or even 4 or 5 days, from that found by the addition of complete sidereal years, according to the foregoing rule. Because that rule supposes the Earth and planet to revolve round the Sun, with constant and equable velocities, and that the place of the node is stationary; whereas this hypothesis is not correct in any of its parts: in respect to the inequality of the Earth's motion, the error introduced thereby is but small indeed, and the error arising from the motion of the node is also small and uniform: but in regard to the motion of the planet it is quite otherwise, particularly with Mercury. And hence it will be necessary to find the approximate time of the conjunction first, and then to calculate strictly the heliocentric and geocentric longitude and latitude of the planet for four several times, equidistant from each other, and having the time of the true conjunction in the middle interval; then from these the true time of the conjunction, or middle of the transit, and the other requsites will be determined by interpolation. Then, if the geocentric latitude of the Planet be *less* than the Sun's semidiameter at the time of the conjunction, the planet *will* transit the Sun, otherwise not. And hence we are never sure that a transit must happen, till we have found by strict calculation, that the geocentric latitude of the planet at the time of the conjunction is *less* than the Sun's apparent semidiameter.

Calculation of the transit of Mercury over the Sun's disc, 11th November, 1815.

By the precepts and examples for calculating the planets' places it will be found, that the inferior conjunction of Mercury and the Sun happens between noon of the 11th and noon of the 12th days, or 14 hours nearly after the noon of the 11th day; and hence it will be necessary to compute the heliocentric and geocentric place of the planet for four several times as follows:

Results of a calculation of the heliocentric and geocentric longitude and latitude of Mercury, Nov. 11th, 1815, at the hours:			☉'s Longitude + 20″ = the Obscuration.	
1815, Nov. 11,	Heliocentric.	Geocentric.		
P.M. Hours.	Longitude. / Latitude.	Longitude. / Latitude.	☉'s Longitude	☉'s semidi.
	s. ° ′ ″ / °	s. ° ′ ″ / °	s. ° ′ ″	′ ″
App. time 6	1 16 38 11 / 0 3 42 N.	7 19 22 59 / 0 1 53 N.	7 18 30 44	16 12
12	1 18 9 12 / 0 14 52	7 19 2 58 / 0 6 53	7 18 45 50	
18	1 19 40 26 / 0 26 4	7 18 43 30 / 0 12 5	7 19 0 56	
12 noon, or 24	1 21 12 7 / 0 37 18	7 18 22 39 / 0 17 20	7 19 16 3	

Now by an inspection of the geocentric longitude of the Sun and Mercury, it appears, that they become equal between 12 and 18 hours, hence to find the true time of the conjunction, and the beginning and end of the transit, and its magnitude.

☉'s geo. long. at 12*h*.	7*s*.	18°	45′	50″
Do. 18	7	19	0	56
Change in 6 hours		+	15	6

☿'s geo. long. at 12*h*.	7*s*.	19°	2′	58″
Do. 18	7	18	43	30
Change in 6 hours		—	19	28
Do. ☉		+	15	6
Motion ☿ *a* ☉ in 6 hours			34	34

☿'s geo. lat. at 12*h*.							′ 6	″ 53	N.
Do.	18						12	5	
Change in	6						5	12	
						s.	°	′	″
☿ at 12*h*.,						7	19	2	58
⊙ do.						7	18	45	50
Difference								17	8
						h.	*h.*	*m.*	*s.*
As	′ 34	″ 34	: ′ 17	″ 8	::	6	: 2	58	26
Add							12	0	0
Approximate time ♂							14	58	26
	h.	*h.*	*m.*	*s.*		′	″	′	″
As	6	: 2	58	26	::	5	12	: 2	35
Lat. at 12*h*.	Add							6	53 N.
App. dist. of centres								9	28 N.

Now as Mercury must be near the node at the time of a transit, the angle of his apparent path with the ecliptic may be taken as equal to the inclination of his orbit to the ecliptic, which is 7°, and by diminishing the apparent distance of the centre (9′ 28″) in the ratio of radius to the cosine of 7°, we have 9′ 23″ for the length of the perpendicular from the Sun's centre to the apparent path.

Then to ⊙'s semidiam.	′ 16	″ 12	
Add and subtract	9	23	
	25	35	= 1535
	6	49	= 409

$\sqrt{(1535 \times 409)} = 792'' = 13' 12'' =$ semi-chord of the Sun's disc. in the transit.

				h.	*h.*	*m.*	*s.*	
As	′ 34	″ 34	: ′ 13	″ 12	::	6	: 2	17 28 = semiduration.
							14	58 26
Approx. time, beginning							12	40 58
Approx. time, end							17	15 54

If the highest degree of accuracy attainable be required, it will be necessary to repeat the operation, and to find the hourly motion of Mercury from the Sun (strictly, by interpolation), for the beginning, middle, and end of the transit: it will also be necessary to attend to the effect of the aberration of light, and to that of the parallax in altitude on both objects. These however are minute circumstances, which the student may pursue for his improvement and exercise. We have only further to remark, that this transit being invisible to any part of Europe, is therefore not of that importance in astronomy, which it otherwise would have been.

The transits of Venus over the Sun's disc, are calculated exactly in the same way as those of Mercury; but the transits of Venus are of much more importance in astronomy. It is by the transits of Venus, that astronomers have been able to determine the absolute magnitudes, and distances of the Sun and planets from our Earth, and from each other: the same things may be determined by the transits of Mercury, but not to a sufficient degree of accuracy. From the last two transits of Venus over the Sun's disc, in June 1761 and 1769, astronomers found the Sun's horizontal parallax to be 8.65″ nearly at a mean, and then, by plane trigonometry, the absolute mean distance of our Earth from the Sun, *equal* to 95173128 English miles. Now this being the mean result of a great number of observations made by skilful astronomers, in various parts of the Earth, we may pretty safely assert, that the Sun's mean distance from the Earth is between 95 and 96 millions of miles.

It is to be hoped, that the next transits of this planet, in December 1874 and 1882, will be carefully noticed by skilful observers well provided with good instruments, and then the Sun's horizontal parallax will be obtained to a still higher degree of accuracy. And we ardently hope, that those whom it may please God shall be in existence and employed on this great work, may meet with a clear atmosphere, and perfect success.

Before closing this part of the subject, we shall briefly instruct the pupil in the methods used by astronomers to determine the Sun's horizontal parallax, by means of the transits of Venus; and from thence, how the distances of all the planets from the Sun, and from each other, are determined.

Method of discovering the Sun's distance from the Earth by the transits of Venus.

Before the *absolute* distances of the planets from the Sun can be determined, it is necessary to ascertain their *relative* distances; that is, the ratio, or proportion, which their mean distances from him bear to each other. Kepler (we observed in a former part of this work) was the first who discovered that the figure of the orbit of a planet was an ellipsis, having the Sun in one of the foci. This he determined by a comparison of their apparent motions in the heavens with that of a circle and an ellipsis round the Sun. He afterwards discovered in the same way, that their periodic times of revolving round the Sun, compared with their mean distances from him, were all regulated by one law: namely, that the *squares of their*

periodic times were in direct ratio of the cubes of their mean distances from the Sun. This was a deep and an important discovery, but although he found that this law did take place, and that the motions of all the bodies in the planetary system were exactly regulated thereby, yet this great astronomer was unable mathematically to demonstrate its actual certainty. It was reserved for the incomparable Newton to give the matter a perfect finish. This great man demonstrated mathematically, that all bodies moving round another, and retained in their orbits by the attraction of gravitation, *must* describe ellipses; and that the squares of their periodic times are in the direct ratio of the cubes of their mean distances from the common centre of gravity of the whole system. The demonstration of this great regulating law we have given in another part of the work; it remains for us to shew, how the *proportion* of the distances of the planets from the Sun was first discovered, independently of the great law. And with a knowledge of plane trigonometry, the matter will appear easy to the student.

Let S be the Sun (plate 12, fig. 7), DBV the orbit of Venus (or Mercury), ATC a portion of the Earth's orbit: let the Earth be at T, and draw the tangent TV. Then it is evident, that when Venus is at V, she has her greatest elongation; that is, in the triangle TSV, right angled at V, the angle STV is the greatest elongation of the planet from the Sun as seen from the Earth: now astronomers have carefully observed the quantity of this angle, and, taking the mean between the greatest and the least, it has been found to be 46° 19′ 48″ nearly; that is, the mean greatest elongation is 46° 19′ 48″. And if we suppose ST, the Earth's mean distance from the Sun, to be 100000 equal parts, then, by plane trigonometry, as radius : sine angle STV, the mean greatest elongation = 46° 19′ 48″ :: ST = 100000 : SV = 72333 equal parts, which is the mean distance of Venus from the Sun. After the same manner, we find the relative mean distance of Mercury.

Again, in the same figure, let DBV be the orbit of the Earth; ATC, a portion of the orbit of Mars, or any superior planet: now, when the Earth is at V, and Mars at C, he is then seen in opposition to the Sun at S; but when he is at T, the angle SVT is a right angle, or 90°, and he has then his quartile aspect in respect to the Sun; and the Earth has its greatest elongation from the Sun, as seen from Mars. Also the arc BV, or the angle TSV, is that part of the planet's orbit, supposing it a circle, which lies between the point of opposition and the quartile aspect. And by taking a mean of many observations, astronomers have found this angle to be 48° 58′ 54″ nearly. And hence, by plane trigonometry, as radius : to secant angle TSV = 48° 58′ 54″ :: VS = 100000 : ST = 152369 equal parts, the relative mean distance of Mars from the Sun. After the same manner, we find the relative mean distance of each of the other superior planets. Whence the relative mean distances of the planets from the Sun are as follows: viz. Mercury, 38710; Venus, 72333; the Earth, 100000; Mars, 152369; Jupiter, 520279; Saturn, 953877; Georgian, 1918331. The mean distances of the four small planets, Ceres, Pallas, Juno, and Vesta, are not yet determined with sufficient accuracy; but the results of the observations already made on these newly discovered planets, make their relative mean distances as follows: viz. Ceres, 276810; Pallas, 278915; Juno, 265233; Vesta, 236815; respectively, the Earth's mean distance from the Sun being taken, as before, at 100000 equal parts.

These things being premised, let S be the Sun's centre (plate 12, fig. 8.), C that of the Earth, V that of Venus, in a right line between the Earth and Sun: A, a point on the Earth's surface. Then, the Sun and Venus being supposed in the horizon of an observer at A, from what we have formerly illustrated, the angle ASC is the Sun's horizontal parallax, and the angle AVC the horizontal parallax of Venus at the time of her inferior conjunction, which, in the present case, we also suppose to produce a transit. Now, SC being supposed to be = 100000, and SV = 72333; then CV or AV, their difference, = 27667: but AS, or its equal nearly, SC, is to AV, or its equal nearly, VC, as the sine of AVS, or AVC, is to the sine of ASC: but because the sines of very small angles have the same ratio as the angles themselves have, hence the horizontal parallax of Venus, at the time of a transit, is to the Sun's horizontal parallax at the same time, as the Sun's relative distance from the Earth is to the relative distance of Venus from the Earth, and consequently their absolute distances from the Earth are in the same ratio, which at a mean is 100000 to 27667. Hence, if the Sun's mean horizontal parallax be taken at 8.65″, as determined from the last transit of Venus in 1769, then, as 27667 : 100000 :: 8.65″ : 31.26″, the mean horizontal parallax of Venus at the time of a transit; but these results will vary a little on account of the eccentricity of the Earth's orbit. When the Sun is in perigee, the horizontal parallax is 8.79″, and in apogee, 8.51″, the mean being 8.65″: but the orbit of Venus being so nearly circular, there is no sensible change in her horizontal parallax in all transits at the same node: but to arrive at astronomical accuracy, we must compute the exact relative length of the radius vector of the Earth and Venus at the time of the transit, and then say, as the difference of the two radii is to the relative length of the radius vector of the

Earth, so is the Sun's horizontal parallax at the time of the transit, to the horizontal parallax of Venus at that time.

Let S be the Sun (plate 12, fig. 9.), T, the Earth, and V, Venus. Now, to an observer at T, the Earth's centre, Venus will be seen on the Sun's disc at S; to an observer at A, she will be seen at E; and to one at B she will be seen at F. Again, because the angle AVT, which is the horizontal parallax of Venus, is equal to the angle FVS; wherefore the whole, FVE, is equal to twice the horizontal parallax of Venus. Hence, if it were possible that two observers, situated nearly diametrically opposite, measured the exact angular distance of the centre of Venus, from the same point of the Sun's limb, at the same instant of time, then half the difference of these two angular distances, gives the angle SBF; but the angle SBF is equal to the difference of TVB, and TSB; that is, to the difference of the horizontal parallax of Venus and the Sun: but the ratio of these angles is given, viz. as 27667 to 100000 at a mean, and supposing the angle SBF to be found by observations to amount to 22.61″, then, as 72333 : 27667 :: 22.61″ : 8.65″, the Sun's mean horizontal parallax, from which, and the known radius of the Earth, the Sun's distance is found, as already shewn at page 124 of this work.

The foregoing method is in itself impracticable; because it supposes two observers to be so situated on the Earth as to be diametrically opposite to each other, and directly in that plane which passes through the Earth's centre, and coincides with the visible tract of Venus on the Sun's disc. But, although such a coincidence of circumstances cannot be attained, yet such an hypothesis was necessary *in initio*, in order to establish first principles for the student's information, from which it must appear manifest to him, that an observer at the Earth's centre, and others at different points on its surface, must see Venus at the same instant of time projected on the Sun's disc at various points. It must also appear evident to him, that the more distant the two points on the Earth's surface are from one another, and situated in the plane passing through the visible path of Venus over the Sun's disc, the better.

But because very small angles cannot be measured to that exactness which is necessary in the present case, hence astronomers have had recourse to another method, which in a great measure removes the inconvenience; and this method employs the *observed times* of the several phenomena, as seen from different points on the Earth's surface. Now by making strict calculation of the hourly motion of Venus as seen from the Earth's centre, and adding it to the Sun's hourly motion (because her motion is then retrograde), we have her hourly motion from the Sun; and which time will be found to amount to four minutes of a degree nearly, at the time of a transit; that is, at the rate of 4″ per minute of time nearly. Then, if two observers, 90° asunder on the Earth's surface, and nearly situated in the plane of the visible path of Venus over the Sun's disc, strictly observe the hour, minute, and second, that the centre of Venus exactly passed over the Sun's limb, either at the instant of ingress or egress, it is evident that the difference of these two times (allowing for the difference of longitude of the two places in time,) will be the effect of the horizontal parallax of Venus in time. Now let us suppose, that there resulted from such an observation as we assume, a difference in time of 5*m*. 45*s*., or 5.75*m*., this being multiplied by 4″, the angular motion of Venus from the Sun, gives 23.0″ for the angular effect of the horizontal parallax of Venus at the Sun, and from hence the Sun's horizontal parallax will be determined, as we have shewn aforegoing.

We have again to remark, that we have supposed the two observers to be exactly 90° apart, a circumstance almost not to be expected; yet by knowing the distance between them on the Earth's surface, we can easily find the magnitude of the chord-line within the Earth, and from hence deduce what must have been the result, had their distance been exactly 90 degrees.

It seems quite unnecessary to enter farther into all the minute circumstances belonging to the transits of Venus. Several volumes were published both before and after the last two transits in 1761 and 1769, which the reader may consult at his leisure. We shall now only observe, that the more distant the observers are from each other, so much the more accurate will be the result, and by taking a mean of many observations, a higher degree of certainty will be obtained.

Of the MOON'S Motions and her Phenomena.

Definitions.

1. As the Moon is a *secondary* or *satellite* to the Earth, the Earth's centre is assumed by astronomers as the centre of her motion round the Earth; at the same time we know, that both the Earth and Moon revolve round their common centre of gravity, each making a revolution in the same time; but we sup-

pose the Earth at rest, and therefore refer the Moon's motion about the Earth to its centre; and in respect to her mean motion about the Earth,

2. A *sidereal* revolution is a complete revolution of the Moon round the Earth's centre; or it is the time in which the Moon moves from a fixed star, till she returns to the same again, as seen from the Earth's centre, supposing her orbit to be at rest. The time of her sidereal revolution is 27*d*. 7*h*. 43*m*. 11.5259*s*.

3. A *tropical* revolution is the time in which the Moon would move from an equinoctial or solstitial point to the same again; this time is 27*d*. 7*h*. 43*m*. 4.6795*s*.

4. A *synodic* revolution is the time between two successive conjunctions or oppositions of the Sun and Moon, as seen from the Earth's centre, which time is 29*d*. 12*h*. 44*m*. 2.8283*s*.

5. An *anomalistic* revolution is the time in which the Moon moves from her apogee or perigee to the same again; this time is 27*d*. 13*h*. 18*m*. 33.9499*s*.

6. An *adnodic* revolution is the time in which the Moon moves from either node to the same again; this time is 27*d*. 5*h*. 5*m*. 35.6030*s*.

7. A *sidereal* revolution of the apogee or perigee of the Moon is the time of a complete revolution of either about the Earth's centre; this time is 8*y*. 312*d*. 11*h*. 11*m*. 39.4089*s*.

8. A *tropical* revolution of the apogee of the Moon is the time in which the apogee moves from an equinoctial or solstitial point to the same again; this time is 8*y*. 311*d*. 8*h*. 34*m*. 57.6177*s*.

9. A *sidereal* revolution of the node, in like manner as of the apogee; this time is 18*y*. 223*d*. 7*h*. 13*m*. 17.7440*s*.

10. A *tropical* revolution of the node, in like manner as of the apogee; this time is 18*y*. 228*d*. 4*h*. 52*m*. 52.0296*s*.

11. The diurnal mean motion of the Moon is found thus: as the time of her tropical revolution is to 1 day, so is 360° to her mean diurnal motion = 13° 10′ 35.02784394″.

12. Diurnal mean motion of the apogee: as the time of a tropical revolution of the apogee is to 1 day, so is 360° to the diurnal motion = 0° 6′ 41.069815195″.

13. Diurnal mean motion of the node: as the time of a tropical revolution of the node is to 1 day, so is 360° to the diurnal motion = 0° 3′ 10.638603696″.

Note.—The year taken here is supposed to contain 365 days, and the several periods are from M. de la Lande, which differ but little from those of Burg and Burkhardt.

The following problems shew how these numbers are determined.

PROBLEM 1.

To find the mean motion of the Moon's nodes, and the place of either node. Solution, by astronomical observation.

Before we can determine the mean motion of the node, it will be necessary first to know its place at two different periods of time, and the greater the interval between these two times so much the more accurate will be the result. Now, eclipses of the Moon are evidently the best means of finding the place of the node with certainty and simplicity. If two central eclipses could be observed remote from each other in point of time, and if we knew the exact time that the Moon passed the centre of the Earth's shadow in both eclipses, that is, the number of days, hours, minutes, &c. in this interval, and also the number of revolutions that the node must have made during the same, it is obvious, that the interval being divided by the number of revolutions, the quotient would be the mean time of one revolution of the node. But it may happen that both eclipses did not take place at the same node, and in this case our divisor must consist of a certain number of whole revolutions, and half of a revolution. Now, when the Moon's centre passed through the centre of the shadow, it is evident, that she was exactly opposite to the Sun, and hence the opposite point to the Sun was then the *place* of the node. Wherefore the Sun's longitude ± 180°, at that instant, gives the longitude of the node. And the Sun's longitude may be found for that time by calculation, but still more accurately and independently of calculation, by observing his right ascension and declination on the noon of the day before and of that after the eclipse, and then by interpolating the exact time. And from these we obtain the longitude, or with the obliquity of the ecliptic and right ascension, we can find the Sun's longitude, without his declination being observed.

Cassini informs us, that a central eclipse of the Moon happened at Paris, on the 16th April, 1707, the middle of which was determined to be at 13 hours, 48*s*. apparent time, at which time the Sun's longitude was 0*s*. 26° 19′ 17″; and consequently the longitude of the node was then 6*s*. 26° 19′ 17″. And as the Moon passed from north to south, therefore this was the descending node; and if we were to take the Sun's mean longitude for that time, we should also have the mean longitude of the node.

But though central eclipses happen very seldom indeed, yet we can find the place of the Moon's node, exactly in the same manner as we have shewn how to find the node of a planet, which see, problem 5, page 170. And we have two opportunities of determining the place of the node in this way, by observation in every 27 or 28 days.

By taking two eclipses of the Moon, which were

both observed with great accuracy at Greenwich, and allowing for the Moon's distance from the node at each eclipse, and dividing the interval by 3, the number of revolutions which we knew the node to have made in that interval of time elapsed between the two eclipses, we have found that our result was so nearly the same with that which we have given from M. de la Lande, that the difference is not more than 10 seconds of time in 100 years.

Hence, to construct tables of the *mean motion* of the Moon's node.

Take the multiples of the mean motion for 1 day, for every day from 1 to 365, and tabulate them, thus:

Day.	Mean motion node.			
	s.	°	′	″
1.....	0	0	3	10.6
2.....	0	0	6	21.2
3.....	0	0	9	31.9
4.....	0	0	12	42.6
&c.			&c.	

Multiply the motion for 1 day, by 365 for a common year, and by 366 for a bissextile one, and proceed in every respect as we have directed for the Sun and planets; and you will have constructed the table of the mean motion of the Moon's node. But the mean longitude of the node from the mean equinox for a particular meridian and point of time must previously be settled by accurate observations. Now, from a mean of several observations, the mean longitude of the supplement of the Moon's ascending node from the mean equinox, on the noon (mean time) of the 1st of January, 1800, at Greenwich, was 10*s.* 26° 47′ 29.7″. And hence, by adding to this the motion for 365 days 3 times successively, and then that for 366 days, &c. we can continue our table for any number of years to which we choose to carry it; and, by subtracting, in like manner, we have the same for time prior to 1800: and in the same way we form it for centuries, &c.

The motion of the Moon's nodes being retrograde, namely, from east to west, hence, to avoid subtractions in computing the place of the node, we construct the tables for the supplement to 12*s.* or 360°, by which all the parts become additive.

Problem 2.

To find the mean time of a synodic revolution of the Moon. Solution, by astronomical observations.

Notwithstanding the great number and magnitude of the inequalities with which the Moon's motions are affected, yet this problem admits of a more easy and simple solution than almost any in astronomy. For it is evident, that if two eclipses of the Moon be observed, and if we know the exact interval of time between the middle of these two eclipses, and the number of oppositions between the Sun and Moon, which took place in that interval: then, the number of days, hours, minutes, &c. in the interval, being divided by the number of oppositions, the quotient will be the time of a synodic revolution sought. It is also evident, that the more remote, in regard to time, the two eclipses are from each other, so much the more accurate will be the result.

We have taken two eclipses of the Moon, which were both observed at Greenwich, and the interval of time between the middle of both was nearly 101 years, or exactly 36883*d.* 16*h.* 54*m.* 53*s.*, during which period there were 1249 synodic revolutions: hence, by dividing this interval by the number of revolutions, we have 29*d.* 12*h.* 44*m.* 2.828*s.*, for the mean time of one synodic revolution, which agrees with that given by M. de la Lande.

By this method the ancient astronomers determined the mean time of a synodic revolution with considerable accuracy; indeed, they succeeded better in this particular case of astronomy than almost in any other. The reason is, because several of the Moon's inequalities vanish at the time of the syzygies, and they took long periods. Ptolemy mentions a total eclipse of the Moon, which was observed at Babylon, on the 19th March, 720 years before the beginning of the Christian era; the middle was at 9*h.* 30*m.* P.M. (or at 6*h.* 45*m.* 40*s.* at Greenwich,) and this eclipse he compared with those that happened in his own time nearly 800 years afterwards; by which means he found the length of a synodic revolution very nearly the same as we now make it. This is indeed one of the most accurate numbers that we have in astronomy.

Problem 3.

To find the mean time of a *sidereal* revolution, that is, the absolute time in which the Moon revolves round the Earth: the sidereal year, and the Moon's synodic revolution being supposed both to be given, or known by the preceding problems.—Solution, by calculation.

Rule.—Multiply the sidereal year (viz. the time in which the Earth revolves round the Sun,) by the Moon's synodic revolution, and divide the product by their sum; the quotient is the Moon's sidereal revolution. The time thus found, will be as accurate as the numbers by which it is found, and which numbers are the most correct that we have in astronomy.

Operation.

	d.	*h.*	*m.*	*s.*	*s.*
Sidereal year . . =	365	6	9	12 =	31558152.0
Synodic revolution =	29	12	44	2.8 =	2551442.8
Sum					34109594.8

$\frac{31558152.0 \times 2551442.8}{34109594.8} = 2360591.5259 = 27d.\ 7h.\ 43m.\ 11.5259s.$

Note.—The demonstration of the rule is the same as that we have investigated for the planets. See the note to the scholium, problem 4, page 167.

Problem 4.

Given, the sidereal revolution of the Moon, and the precession of the equinoxes; to find the Moon's tropical revolution. Solution, by calculation.

Operation.

As $365\frac{1}{4}d.$: $27\frac{1}{3}d.$ :: $50.25''.$: $3.76''.$ = the motion of the equinox during a sidereal revolution. Then add this to 360°, and say, as $360° + 3.76'' : 3.76'' :: 27d.\ 7h.\ 43m.\ 11.5259s. : 6.8464s.$; which subtracted from the sidereal revolution gives the tropical.

		d.	*h.*	*m.*	*s.*
Sidereal revolution . . .	=	27	7	43	11.5259
Subtract		0	0	0	6.8464
Tropical revolution	=	27	7	43	4.6795

Scholium.

This is a result which we could depend upon, if the motion of the equinox at a mean be $50.25''$ annually; but Mr. Vince, after using this number, changes it to $50.10''$, which alteration makes $15''$ of a difference in 100 years, in all longitudes of the Sun, Moon, and planets; and from this number it appears Burg and Burkhardt computed their tables of the mean motions of the Sun, Moon, &c.

Problem 5.

To construct the tables of the mean motion of the Moon's longitude, from the equinoctial point ♈. The solution is by calculation alone.

Take the mean tropical motion of the Moon for 1 day in longitude, viz. 13° 10′ 35.02784394″, and multiply it by 365 for a common year, and by 366 for a bissextile, and its multiples for days; and proceed in every respect as shewn before, and the tables of the Moon's mean longitude will be constructed. But first of all, her mean longitude for a particular meridian and point of time, from the first point of Aries, must be determined by actual observation. And then to the same apply her mean motion for years, days, hours, &c. and you have her mean longitude for any proposed time.

Problem 6.

To find the place of the Moon's apogee and its motion. Solution, by astronomical observations.

Observe accurately the Moon's right ascension and declination, every day it can be done, during a whole revolution, and from these compute her longitude; and by comparing the true and mean places together, we find the time when the difference is the greatest in half a revolution, and this difference is the greatest equation of the centre for that excentricity of the orbit. Do the like for the other half of the revolution, and the middle of the interval between these times will be the time nearly when the Moon was in apogee or perigee. But these observations must be repeated for many revolutions, and proper allowance made for the motion of the apsides in each interval. Moreover, when the mean and observed daily motion in longitude become equal, then their difference being nothing, the Moon's distance from the Earth (focus) is a mean proportional between her greatest and least distances from the focus; and the equation of the centre is a maximum in that semi revolution. Then, half the greatest equation, being reduced to parts of the radius, is the excentricity, as we have shewn before at page 161; and by assuming the semi-major axis, and finding the excentricity in the same parts, we have the form of the orbit for that time.

We have given the mean time of a sidereal and tropical revolution of the apogee of the Moon as deduced from several thousand observations (see definitions 7, and 8). And the *place* of the apogee is the same as the Moon's true longitude at the time she is in apogee; and hence, having found with great exactness the instant of time that the Moon is in apogee, in any revolution, we have only to compute the Moon's true longitude from the mean equinox for that time, and we have the true longitude of the apogee from the mean equinox. Then to this longitude apply the mean motion of the apogee for the years, days, hours, &c. in the interval between that time and the mean noon at Greenwich of 1800 (or any other year), and you have the first epoch of the mean longitude of the apogee; to which add the mean motion for years, as in the cases already explained, and you will have constructed the tables of the mean motion of the apogee, &c. See the tables of the Moon's mean motion.

Problem 7.

Given, the maximum and minimum, and consequently, the mean greatest equation of the Moon's centre; to find the greatest, mean, and least excentricities of her orbit.

Observations shew, that the Moon's orbit changes its form in every synodic and anomalistic revolution. And the excentricity is greatest when the apsides lie in syzygies, that is, if the Moon be in apogee or perigee at the time of new or full Moon; and it is least when they lie in quadratures; and the mean excen-

tricity is when the apsides are between the syzygies and quadratures. And this change in the excentricity is called the *evection*. Now this variation, in the form of the orbit, is caused by the Moon's gravitating to the Sun as well as to the Earth. For it is evident, that in the syzygies the Moon's velocity round the Earth can not be affected by the Sun's attraction, or disturbing force, because her motion then is at right angles to it; in the quadratures the effect of his attraction is greatest: in the first quadrature her velocity is most diminished, and in the last it is most increased. So that for the first half of the Moon's synodic revolution her velocity is constantly retarded, and in the other half it is equally accelerated. And this acceleration and retardation combined with the motion of the line of the apsides, produce the change of excentricity or evection.

From a mean of some thousand observations, the minimum of the greatest equation of the Moon's centre, is 4° 58′ 2″, the maximum 7° 39′ 2″, and the mean 6° 18′ 32″: the several excentricities for these are required. Solution, by calculation, agreeably to our method 3, problem 1.

Greatest equation centre.				Half.		
	°	′	″	°	′	″
Minimum . . .	4	58	2	2	29	1
Mean.	6	18	32	3	9	16
Maximum . . .	7	39	2	3	49	31

	Excentricity.	
° ′ ″		
2 29 1	4334.6	=least.
As 57 17 44.8 : 3 9 16 :: 100000 :	5505.4	=mean.
3 49 31	6676.2	=greatest.

The decimal part of the excentricity should be carried out to 5 figures at least, for the determination of the equation of the centre with accuracy; but we deem that unnecessary here, as our object now is merely to shew the method by which the *quæsita* are found. And we have to remark, that the table of the equation of the Moon's centre is calculated for the mean excentricity, 5505.4; and the equation called *evection*, with all the other minor equations, are for correcting the mean equation of the centre, to reduce it to the true, &c. And these numerous equations or corrections, depend upon the various positions of the Sun, Moon, apogee, and node, at the time for which the calculation is to be made; together with the elliptical figure of the orbit. And as all the corrections are first found from the mean places, hence, in some cases, these corrections must be applied, in order to approximate the true place more nearly, upon which the true equation sometimes depends. But instead of calling it place, in this case, we should rather have said *argument*, which is the name given by astronomers to that number on which an equation or correction depends, and by which it is found. And hence we see what a multitude of inequalities and corrections the Moon's mean longitude is affected with before we can reduce it to the true longitude. Burg has 24 minor equations including the evection, before her anomaly is once corrected, then the *mean equation of the centre;* next the *variation*, or 26th equation; afterwards a 27th equation to fit her longitude in her orbit exactly to the correct place of the node; and, lastly, a 28th equation, or the reduction of the longitude in the orbit to that of the ecliptic, and then you have the Moon's true longitude from the mean equinox; and to which the nutation must be applied in order to obtain her *true* longitude from the *true* equinox.—Burkhardt has 32 minor equations; then the equation called evection; next the equation of the centre, &c. as before: but Burkhardt's tables have very greatly the advantage of Burg's in method and expedition; besides Burkhardt's tables make no use of the Sun's true longitude, which Burg's tables require to be previously computed for every longitude of the Moon for which we have to calculate, and this in itself is an important improvement.

Our plan does not allow us to enter into all the minute circumstances attending the construction of lunar tables; indeed, the subject is so extensive, and in many cases so intricate, that it would require a large volume to illustrate and exemplify every part at full length; we shall, however, after the following problems, explain the physical causes of the principal lunar motions, and the inequalities attending them, in order that the student may see how greatly the Moon's motion round the Earth (focus) is affected by the disturbing force of the Sun.

Problem 8.

To find the inclination of the Moon's orbit to the plane of the ecliptic. Solution, by astronomical observations.

The inclination of the Moon's orbit is found to be changeable, the greatest being 5° 18′, and the least 5° 0′. Observe the Moon's right ascension and declination as nearly to the time when she is 90° from the node as you can, then with these compute the longitude and latitude, and her greatest latitude is the inclination sought, at which time she is 90° from the node. But as it can seldom happen that the Moon will be exactly 90° from the node when she is on the meridian, hence observations must be taken on the day before, and that after, she has had her greatest latitude, and her greatest latitude lying between the time of these observations, it must be determined by interpolation. Now, in order to find the greatest, mean, and least inclination, the observations must be repeated several hundred times, and in all the various positions of the Sun, Moon, apogee, and node. But

there is another very important circumstance to be attended to in these observations, namely, the Moon's *horizontal parallax;* because, in finding her declination by her meridian observed altitude, we must correct her altitude for the effects of parallaxand refraction. To find the horizontal parallax, let the Moon's diameter be accurately measured by a micrometer when she is on the meridian, and from her diameter deduct the augmentation answering to her altitude. Then, as 131 : 240 : : ☾'s diameter : ☾'s horizontal parallax; from which her parallax in altitude will be found.

Problem 9.

To find the Moon's horizontal parallax. Solution, by astronomical observations.

The Moon's horizontal parallax is the angle that the Earth's semidiameter subtends, seen from the Moon's centre; or what is the same thing, it is the angular distance between the true or *rational* and the *sensible* horizon at the Moon seen from the Earth's centre: now, as the Moon's distance from the Earth is changeable, hence her horizontal parallax and apparent diameter are also changeable. When the Moon is in perigee or nearest the Earth, her horizontal parallax and apparent diameter are the greatest; but when she is in apogee they are the least; and because the sines or tangents of small arcs (not exceeding 1 degree) have nearly the same ratio which the arcs themselves, or angles which they measure, have; hence the Moon's horizontal parallax, and her apparent horizontal diameter, have always nearly a constant ratio to each other, viz. as 131 to 240; so that if we know the one, we know the other. It therefore remains to shew how this ratio is found by observation.

1*st Method.*

Observe the exact time that the Moon comes to the meridian, and take the altitude of her centre, which correct for the refraction. Also calculate the Moon's declination for the instant of time she was on the meridian: then the declination added to or subtracted from the complement of the latitude, gives the true altitude; the difference between which and the observed, or apparent altitude, is the parallax in altitude. Then, as the cosine app. altitude : radius : : parallax in altitude : horizontal parallax. See page 78. This method would be good and successful, if the Moon's declination could be found independently of her latitude; but this is hardly possible, and when we use her latitude, we at the same time make use of the *quæsitum*, that is, we make use of that for which we are seeking, because the tables from which we calculate to find the Moon's latitude, could not be constructed without the Moon's horizontal parallax being previously determined and made use of in the very observations from which the tables of the Moon's latitude are constructed. This method therefore fails, and we have given it merely as a caution to the student, that he may be careful that his *quæsita* be not involved in any part of his *data*, otherwise his conclusions may be very erroneous.

2*d Method.*

Let the Moon's altitude and azimuth be strictly observed 3 or 4 hours before she comes to the meridian, or 3 or 4 hours after she has passed it. Let her right ascension be also strictly observed when she culminates, and calculate her right ascension strictly for the time her apparent altitude was observed. Take the difference of the Moon's right ascension for both times, which is the change of her right ascension during the interval, and compute the change of the Sun's right ascension for the same interval, and subtract the Sun's change from the Moon's; then subtract this remainder from the hours, minutes, &c. of the interval, and turn the remainder into motion of 15° to the hour, and proportionally; and call this the *hour angle*. Then, in the oblique angled spherical triangle PZ☉ (fig. 5, plate 9.) there will be given, the hour angle ☉PZ, the angle PZ☉, the azimuth, and the side PZ the co-latitude; to find the side Z☉, the complement of the true altitude, which will be found, by case 4, oblique angled sph. trigonometry, p. 136. Then, the difference between the true and apparent observed altitude, is the Moon's parallax in altitude, from which her horizontal parallax will be known, as we have shewn in the preceding method. And this method of finding the Moon's horizontal parallax, is perfectly free of her latitude, or any other thing in which her parallax is previously involved; so that it is strictly mathematical, and by no means laborious in the operation. There is but one objection against it, namely, that few observatories are furnished with an accurate instrument properly adjusted for taking azimuths.

Now, the Moon's diameter being measured by a micrometer, or, which is still better, found from the time of her passage over the wires of the transit instrument, and corrected for the augmentation so as to get the horizontal diameter: the Moon's horizontal diameter compared with her horizontal parallax for the same time, both deduced from actual observations, have been found always to be in the ratio of 131 to 240; or according to the mean of a great number of observations, as 131.007 to 240. But the fractional part is too small to be considered in astronomical calculations.

The Moon's diameter or semidiameter and the horizontal parallax are changeable, and their magnitude is always inversely proportional to the Moon's

distance from the Earth. By a mean of many hundred observations taken at Greenwich, the greatest mean and least, for that latitude, are as follow; but for the equatorial diameter the horizontal parallax is a little more.

	′ ″		′ ″
Greatest horizontal parallax	61 24	diameter	33 31.5
Mean	57 33		31 25.5
Least	53 42		29 20.0

But the *hour angle* can be found by observation independently of the Moon's right ascension, by having the exact solar time between the two successive transits of the Moon's centre over the meridian, between which transits the interval falls; and the exact solar time also between the nearest transit and the time when the apparent altitude was observed. Or, the same may be done by the astronomical clock and *sidereal* time. Thus, observe the exact time by the clock when the Moon passed the meridian, and the following night also (the interval lying between these times); and the time when her altitude was taken must also be strictly observed. Then, as the time between the two transits is to interval in time, so is 360° to the hour angle. But for the sake of accuracy, this should be corrected for the equation of second difference; and therefore the times of two other transits should be observed (if it can be done), one on the night immediately before the former, and the other on that after the latter; but we shall explain this by an example.

Example.

On the following days, January 1814, the Moon's centre was observed to pass the meridian of Greenwich, mean solar time,

Days.	☽ 's passage.	1st. diff.	2d. diff.	mean 2 d.
	h. m. s.	*h. m. s.*	*h. m. s.*	
15	11 50 42			
		1 2 21		*m. s.*
16	12 53 3		—0 2 0	
		1 0 21		—2 51
17	13 53 24		—0 3 43	
		0 56 38		
18	14 50 2			

and on the 17th day, at 9*h.* 53*m.* 24*s.*, her altitude and azimuth were observed. It is required to find the hour angle for that time; that is, the angle at the pole contained between the meridian of the place, and the Moon's meridian, is required.

	h.	*m.*	*s.*
☾ 's passage 17th	13	53	24
Time of observation	9	53	24
Moon's motion in time from the meridian	4	0	0

	h.	*m.*	*s.*
To	24	0	0
Add	1	0	21
Sum	25	0	21 = the time between the two transits.

As 25*h.* 0*m.* 21*s.* : 4*h.* 0*m.* 0*s.* :: 360° : 57° 35′ 11.6″

Equation of second difference . . . . + 19.0

True hour angle 57 35 30.6

Hence, with the hour angle, the azimuth, and the complement of the latitude, we find the Moon's true altitude, from which her observed apparent altitude being subtracted, the remainder is her *parallax* in altitude, from which we find her horizontal parallax. And of all methods for finding the Moon's horizontal parallax, this is the best; because it makes no use of any astronomical tables or calculations, but finds the parallax immediately from the observations themselves, independently of all auxiliaries: and the times being observed by a clock shewing sidereal time, will give the same hour angle.

Note.—To find the horizontal parallax and semidiameter of the Moon for a given time. See the precepts shewing the use of the tables, &c.

3d Method.

In a visible eclipse of the Moon mark the exact time when the horns of the Moon are in the same vertical circle, at which time take the altitude of the Moon's centre by observation, which altitude is also the apparent altitude of the centre of the Earth's shadow; compute the Sun's depression below the true or rational horizon for that instant of time, and this depression is equal to the Moon's *true* altitude for the same instant. Then the difference between the *true* and observed *apparent* altitude is the parallax in altitude; from which the horizontal parallax will be found.

This method, even if it were correct, is seldom practicable, nay, not thrice in an age: but, besides this, it is only *true* in theory, for in the practice, the error in result may be several times greater than the parallax itself. In a lunar eclipse the Moon's horns are so ill defined, that the exact time when they are both on the same vertical circle cannot be ascertained to several minutes; and therefore the altitude of the centre of the shadow may be much greater or less than the altitude of the Moon's centre, even admitting that we could take its altitude at that time with accuracy, which we cannot. This method is, therefore, on many accounts, *visionary;* and it is somewhat surprising, that so eminent an astronomer as Dr. Keill should give it almost the first place, in the methods of solving the problem. Mr. Emerson proposes the same method of solution: but Mr. Emerson was not

a practical astronomer: in theory he was generally correct, and eminently so on central forces. The Moon's horizontal parallax or her parallax in altitude, is an essential part of the *data* in the construction of astronomical, and even nautical tables, for finding her *latitude*, declination, semidiameter, &c. and consequently it should be ascertained to the greatest degree of accuracy possible.

Note.—The parallax in altitude added to the apparent altitude gives the true altitude, and *vice versâ*.

Problem 10.

Given, the true and apparent altitude of the Moon, and her horizontal diameter; to find her augmented diameter; or the augmentation alone. Solution, by calculation.

To an inhabitant on the Earth's surface, the Moon appears to increase in diameter as she rises in altitude, and when she comes to the zenith it is greatest, and when she is seen on the horizon it appears to be least; and as she is then nearly at the same distance from an observer on the surface as from the Earth's centre; therefore her diameter, when she is seen in the horizon, is the same as if seen from the centre, and this is called her horizontal diameter. As she rises above the horizon in altitude she comes nearer and nearer to the observer on the surface, and hence her diameter seems to increase; and when she arrives at the zenith she is nearer to him by the Earth's semidiameter than when she was in his horizon. Now it is evident, that her apparent diameter is always inversely proportional to her distance from the observer; but her distance from the centre is to her distance from the observer, as the cosine of her *apparent* altitude is to the cosine of her *true* altitude; that is, her horizontal diameter is to her augmented diameter, as the cosine of her true altitude is to the cosine of her apparent altitude: this is evident from plane trigonometry. For let ABT (fig. 3, plate 2.) be the Earth, C its centre, M M, &c. the Moon; then CM her distance from the centre, is to AM her distance from the surface, as the sine of MAC (= cos. app. alt.) is to the sine of MCA (= cos. true alt.). And hence this anology, as the cosine of the true altitude is to the cosine of the apparent altitude, so is the horizontal diameter to the augmented diameter; and the difference of these two diameters is the *augmentation*. Wherefore a table of the augmentation of the Moon's diameter may be constructed to answer every degree of true or apparent altitude, and to every change in the horizontal diameter within the limits of the greatest and least. And in like manner we can construct a table of the Moon's parallax in altitude, for every degree of her *true* or *apparent* altitude, and for every change of her horizontal parallax within the limits of the greatest and least, by this anology, viz. as radius is to the cosine of the apparent altitude, so is the horizontal parallax to the parallax in altitude.

Theory of the inequalities of the Moon's motion round the Earth.

Introduction.—If the Moon were acted upon by no force but the centripetal force of the Earth, she would describe a perfect ellipsis about the Earth in its focus: the major axis would remain at rest; her periodic times would all be equal; the inclination of orbit would be invariable; her nodes would remain in the same points of the ecliptic; and in short, the development of her phenomena in one revolution, would answer for all *past* and *future* revolutions. But since the Moon gravitates to the Sun as well as to the Earth, hence her motion round the Earth is disturbed by the action of the Sun: and as she is sometimes nearer to the Sun and sometimes farther from him, consequently the *disturbing* force of the Sun becomes *variable*; and by this *variable disturbing force*, all the inequalities of the Moon's motion round the Earth are produced. True, indeed, the Moon gravitates to all the planets and other bodies in the system, as well as to the Earth and Sun; but the disturbing forces of these upon the Moon are too insensible to be taken into consideration in this place: but they are considered and their effects allowed for in the Earth's motion round the Sun, and which motion is also common both to the Earth and Moon. The subject matter therefore of which we now are about to treat, is the effects of the Sun's attraction or disturbing force upon the Moon's motion round the Earth, and by which solar action, her motion is so much disturbed.

If we assume the Moon's orbit to be a circle, then, if her mean distance from the Earth be taken at 240000 miles, this will be the semidiameter of her circular orbit, and 480000 miles its diameter; hence 480000 × 3.1416, will be its circumference in miles: and since she performs her sidereal revolution in $27d.13h.18m. = 661.3h.$, omitting the odd seconds; hence if we divide the miles in the circumference, viz. 1507968, by $661.3h.$, the quotient, 2280 miles, is the Moon's velocity per hour. Again, because the power of gravity decreases according to the square of the distance from the centre; and since the Moon's mean distance from the Earth is equal to 60 semidiameters of the Earth nearly; hence the power of gravity at the Earth's surface, is to its power at the Moon as the square of 60 or 3600 is to 1: but by the power of gravity at the Earth's surface, a body falls 16 feet in the first second of time; and if it continued to fall, the spaces passed over, reckoning from the beginning of the fall, are directly proportional to the squares of the times; hence, in 60 seconds or

1 minute, the space passed over will be $60 \times 60 \times 16$ feet $= 3600 \times 16$; but at the Moon it is 3600 times less, from what we have just shewn; and consequently only just 16 feet in 1 minute; wherefore a body at the Earth's surface gravitates as much and falls as far in 1 second of time as it would do in 1 minute of time, at the distance of the Moon. And hence the Moon is brought down from its rectilinear direction in the tangent, towards the Earth's centre by her gravitating to the Earth, 16 feet in every minute of time, or $\frac{1}{3600}$ part of a mile, and consequently the space passed over by a falling body (the same central force remaining) will be $60 \times 60 \times \frac{1}{3600}$ miles in an hour $= 10\frac{10}{11}$ miles. Now we have demonstrated in cor. 1, theor. 1, of centripetal forces, that the velocity of a body moving in a circle by means of a centripetal force, is equal to that which a falling body would acquire in falling through half the radius (the same force always remaining invariable): but half the radius $=$ 120000 miles nearly. And because the velocities of falling bodies are as the square roots of the spaces passed over; hence the velocity of a body falling $10\frac{10}{11}$ miles is to that in falling 120000, as $\sqrt{10\frac{10}{11}}$ is to $\sqrt{120000}$, wherefore $2 \times 10\frac{10}{11} \times \sqrt{\frac{120000}{10\frac{10}{11}}}$ gives 2288 miles per hour, nearly, for the velocity acquired in falling through half the radius of the Moon's orbit, which is equal by the coroll. already cited, to her velocity in her orbit: but by taking the Moon's periodic sidereal revolution we before found her velocity to be 2280 miles per hour. The small difference of 8 miles between these arises from our assuming the diameters of the Earth and orbit in round numbers, and omitting the fractional parts in the other numbers. And hence we can see how perfectly the theory of centripetal forces agrees with actual observations. The Moon's mean velocity per hour in her orbit, when we find it strictly, is 2290 miles, which is exactly the same as that which a body would acquire by falling through the radius of the orbit; and hence we can find the velocity of a falling body at the Earth's surface by this means better than by actual experiment.

Problem 11.

To find the forces by which the Sun disturbs the motions of the Moon; and the different inequalities in her motion round the Earth, produced by her gravitating to the Sun as well as to the Earth.

To render the matter as simple as possible, let us, for the present, suppose the Moon to describe uniformly a circle round the Earth: and the Earth at the same time, to describe uniformly a circle about the Sun's centre. Suppose the Moon in any point L of her orbit CROQ (plate 12, fig. 10 and 11.), her motion follows the order of the signs from west to east LRQ. Let T be the Earth, placed in the centre of the Moon's orbit; S the Sun; OCS the line of the syzygies; C the point of conjunction (fig. 10.) or of opposition (fig. 11.); R the point of the quadrature following the Moon's passage through C. Draw TS, LS, to represent the distances of the Earth and Moon from the Sun. Now it is evident, that the Moon being in L, fig. 10. her gravitation towards the Sun is more, and in fig. 11. it is less than that of the Earth towards the Sun; because in the first case she is nearer to the Sun than the Earth is, and in the other she is more distant: and her gravitation towards the Sun is to that of the Earth towards him, inversely, as the squares of the distances; that is, the Moon's gravitation is to that of the Earth's directly as $\overline{TS}^2$ is to $\overline{LS}^2$. Therefore it follows in both these cases, that the Moon's central force towards the Earth is diminished. This is plain in the first case (fig. 10.), and will also be so in the second (fig. 11.), if we consider that the excess of the Earth's gravitation to the Sun diminishes as much the Moon's gravitation towards the Earth: to determine how, and in what proportion this diminution is made, in the right line LS (produced if need be), take LD to TS in the ratio of $\overline{TS}^2$ to $\overline{LS}^2$; that is, as the Moon's gravity to the Sun is to that of the Earth's towards him. And on the right line LD as a diagonal, construct the parallelogram LG DF, making the sides, LG, DF, parallel and equal to ST. Now it is evident, that the Moon's gravitation towards the Sun, represented by LD, is divided in two other forces, LG and LF, and LG being equal and parallel to ST, acts in the same direction, and with the same force as the Earth's gravitation towards the Sun; and consequently cannot cause any inequality in the Moon's motion round the Earth. But the other force, represented by LF, expresses the quantity by which the Moon by her gravity (fig. 10.) tends towards the Sun, more than the Earth does by its gravity: or (fig. 11.) the quantity by which the excess of the tendency of the Earth to the Sun diminishes the gravity of the Moon to the Earth. Now, the Earth being nearly 400 times farther from the Sun than from the Moon; hence the right line SD is very small in proportion to TS; and consequently DF may be considered as coinciding with its parallel TS, and also the point F with the point A: so that without any sensible error LA may be taken for the difference between the gravitations of the Earth and Moon to the Sun; and may express the force producing the inequalities of the Moon's motion. Let us therefore call this line the *perturbate force*.

Again, produce the line LT, if need be; and upon LA, as a diagonal, construct the right angled parallelogram LEAB; so that the force LA is resolved in two others, LE, LB; of which the one LE, being in the direction of the ray TL, tends (in both cases) to

make the Moon recede from the Earth; and, consequently, expresses the quantity by which the perturbate force diminishes the gravity of the Moon towards the Earth; the other, LB, being perpendicular to the radius TL, is a tangent to the Moon's orbit, and acting in the direction of LB, tends to diminish the Moon's velocity while she is moving in the opposite direction to LB. But there are cases when the point E being between T and L, the force LE increases the Moon's gravitation to the Earth; and other cases where the direction of the force LB is in that of the Moon's motion, and contributes to accelerate her velocity, and both these cases are evidently when the point E is within the orbit, and the other cases when it is without the orbit; and when it falls on the orbit the force vanishes.

Lemma.

If there be two numbers or quantities (as 101 and 100) which have but a small difference, then it will be as the difference of their squares, is to twice the difference of the numbers, so is the square of any one of them to the other number. Thus $\overline{101}^2 - \overline{100}^2 = 201$, the difference of the squares; and twice the difference of the numbers $= 2$; the square of 100, one of the numbers, is 10000. But 201 is to 2 as 10000 is to 100 nearly, or, strictly, to $99\frac{1}{2}\frac{0}{0}\frac{1}{1}$; and it is evident, that as the numbers approach nearer and nearer to an equality, so much more in a greater degree will this ratio approach to the truth.

But the lemma may be demonstrated generally, thus: let the two quantities be represented by $a+1$, and a; where 1 is taken as very small when compared to a; and the sum of both is $2a+1$, and their difference is 1, but the product of the sum and difference of two quantities is equal to the difference of their squares; hence we have $\overline{2a+1.}\times 1$, or $2a+1$ for the difference of their squares, and 2 is twice their difference; and by rejecting 1 from the former, as being indefinitely small, we have $2a$ for the difference of their squares; therefore $2a$ and 2, or a and 1, express the ratio nearly; and by multiplying both by a, we have a^2 and a, for the ratio, which was the thing to be demonstrated.

This lemma being premised, we proceed. Because DL : TS :: $\overline{TS}^2$: $\overline{SL}^2$ (by construction); and HL, the difference of the right lines ST and SL, being generally above 400 times less than either of them, may be rejected when we compare them: therefore the difference between DL and TS $=$ 2HL (by the lemma); consequently DS, the difference between DH and ST, $=$ 3HL. Now, because the right lines LG, FD, and ST are equal to each other; and DF coincides with SA, DS $=$ TA; therefore TA $=$ 3HL, and HL is the sine of the Moon's distance from her next quadrature, or the cosine of her distance to the line of the syzygies. Therefore the *perturbate force* of the Moon LA is one side of a plane triangle ATL, TL the radius of the Moon's orbit is the second side; and TA, which is equal to 3 times the cosine of her distance to the line of the syzygies, is the third side.

Again, produce LH to I; then LI $=$ 2LH; and draw IK perpendicularly to LT. Then the right angled triangles ILK, ATE, are similar, because AE is parallel to KI; therefore, as AT : LI :: AE or LB : IK. But AT $=$ 3LH, and LI $=$ 2LH; wherefore LB : IK :: 3 : 2; hence, LB $= 1\frac{1}{2}$ IK; but IK is the sine of the angle ITK, which is double of ILK, or of its equal CTL, the Moon's distance from the line of the syzygies. Therefore, the force LB, accelerating or retarding the Moon's motion in her orbit, is always as $1\frac{1}{2}$ times the sine of twice the Moon's distance from the line of the syzygies.

In the same triangles, (AT : LI) 3 : 2 (:: TE : LK) :: TL + LE : TL + TK; wherefore 3TL + 3TK $=$ 2TL + 2LE; and hence, LE $= \frac{1}{2}$TL $- \frac{3}{2}$TK. Now TK is the cosine of the angle KTI, or twice the Moon's distance from the line of the syzygies. Wherefore, the force LE, increasing or diminishing the Moon's central force or gravity respectively to the Earth, is directly proportional to the difference, or to the sum of half the radius, and $1\frac{1}{2}$ of the cosine of twice the Moon's distance from the line of the syzygies. Now the inequalities of the Moon's motion being found to proceed from the combination of 3 forces, it remains to find the magnitude of their effects; and to discover the points where these forces are greatest, least, or nothing. That is, where LA, LE, and LB are greatest, least, or nothing.

PROBLEM 12.

To find the limits or points, in the orbits of the Earth and Moon, where the three forces which produce the inequalities in the Moon's motions, are greatest, least, or nothing.

Solution.—If a plane triangle TLA be constructed for all the several points of the arc CR, so that TA be always equal to 3 times LH, then,

1. The perturbate force LA is greatest where the line TA is greatest: because of the constant side TL; that is, when LH becomes equal to the radius TL; and consequently when the angle RTL is a right angle, or when the point L falls in C. Wherefore the *perturbate force* LA is greatest in the syzygies, and is then expressed by 2TL; that is, by the diameter of the Moon's orbit.

For the same reason, on the contrary, LA is least when TA or LH becomes nothing; or when the point L falls in R; wherefore the perturbate force LA is least in the quadratures, and is then expressed by TL the radius. And hence the perturbate force when the Moon is in the syzygies, is double to that in the quadratures.

2. As the angle TLA may vary from 180° to 0°; wherefore the line AE, being perpendicular to TL, must fall beyond I, in respect to T, when the angle TLA is *obtuse;* on the point I, when it is a *right one;* and between T and L when it is *acute.* In the first case, LE expresses the diminution of the Moon's gravity in respect to the Earth, because it represents a force the direction of which is contrary to the point T. In the second case this diminution is nothing; and in the third, LE represents a force increasing the Moon's gravity to the Earth, its direction then tending towards the point T.

3. The force LB is nothing when the *sine* of twice its distance to the line of the syzygies is nothing, which happens in the syzygies. The force LB is greatest when the Moon is in the octants, or 45° from the syzygies, for the greatest sine is the sine of 90°, or twice 45°, which is equal to the radius. It is also seen, that when the force LB is greatest, that it is equal to $1\frac{1}{2}$ TL.

4. The force LE is greatest when the cosine of the distance from the syzygies is greatest, which happens in the syzygies. In this case LE coincides with TA; and consequently, then LE is equal to 2TL. Again, the force LE is nothing when the angle TLA is a right angle, or when the terms of its expression destroy each other; which happens when $\frac{1}{2}$TL = $1\frac{1}{2}$ TK, or when $\frac{1}{2}$TL = TK; that is, when the cosine of twice the Moon's distance from the syzygies is equal to one third the radius. Now the third of the radius = 333333, which is the cosine of 109° 28′ 16″, its half is 54° 44′ 8″: wherefore the force LE is nothing when the Moon is 54° 44′ 8″ distant from the syzygies, or 35° 15′ 52″ from the quadratures. Moreover, if the expression of the force LE be required in the quadratures, then the distance from the syzygies being 90°, its double is 180° or 0°, the cosine of which is equal to the radius TL or TR; therefore LE = $\frac{1}{2}$TL — $1\frac{1}{2}$TL = — TL; this shews that in the quadrature the direction of the force LE is in RT; and that the Moon's gravity towards the Earth is then increased by a quantity which may be represented by TL or RT, the radius.

Corollary 1.—Hence it is evident, that, by the equality of the decrease of gravity in the Moon's conjunctions and oppositions, and by the equality of its increase in the quadratures, all the inequalities happening to the Moon's motion, in her passage from one syzygy to her next quadrature, are gradually restored in her passage from the quadrature to the next syzygy. And afterwards the like inequalities begin again in the same order from this syzygy to the next quadrature, and are again restored in her passage to the following syzygy, and so on. Whence it follows, that if the orbits of the Earth and Moon were perfect circles, that all the inequalities in the Moon's motion produced by the disturbing force of the Sun, would be completed in every half synodic revolution. But the elliptical figure of both orbits, the motion of the nodes and also of the apsides, &c. produce a more numerous and complicated set of inequalities in the Moon's motion.

Cor. 2.—When the Moon is in the syzygies her *perturbate* force diminishes her *central* force in respect to the Earth, and on the contrary, increases it when in the quadratures: but the diminution in the syzygies is double the augmentation in the quadratures: and this difference contributes to the motion of the apsides.

Cor. 3.—There are 4 points in the Moon's orbit 54° 44′ 8″ from the syzygies, where her gravitation to the Sun makes no alteration in her *central* force to the Earth.

Cor. 4.—The variation in the Moon's *central* force in respect to the Earth, tends to flatten her orbit in the syzygies, and to lengthen it in the quadratures; so that the Moon's orbit, at first circular, is now become oval or elliptical, its major axis lying in the line of the quadratures, and the minor axis in that of the syzygies, the Earth being in the centre of this elliptical figure.

Cor. 5.—The angular velocity of the Moon about the Earth (focus) must decrease from the syzygies to the quadratures, and accelerate from the quadratures to the syzygies. For the semi-axes of elliptical curves being perpendicular to the tangents drawn through their extremities; and the angular velocity in curves being always inversely proportional to the right lines drawn perpendicularly from the centre of force to the tangents to the point where the moving body is found; and the semi-minor axis is equal to this perpendicular, when the tangent passes through the extremity; and this is evidently the least perpendicular: and the semi-major axis is ± the excentricity, are the greatest or least perpendicular, in which case the tangent passes through the apogee or perigee.

Cor. 6.—The quantity by which the Moon's velocity is diminished in going from the syzygy to the quadrature, or accelerated in going from the quadrature to the syzygy (produced by the force LB) must increase from the syzygy to the next octant, and decrease again from that octant to the next quadrature: and it is always in the ratio of the sine of twice the Moon's distance from the syzygy. This inequality is called the Moon's *variation:* its maximum is 34′ 18.5″.

Cor. 7.—In the syzygies and quadratures, the Moon's radius vector, that is, the right line drawn from the Moon to the Earth, will describe equal areas in equal times: in every other point of the orbit the areas are less in proportion to the times, as the Moon is nearer to the octants. For in the syzygies and quadratures the perturbate force LA is in the direction of the rays from the Moon to the Earth; con-

sequently the force compounded of the perturbate force and the Moon's gravity tends to the Earth's centre, and is then no more than a single *central* force, by which the Moon must describe in that case, areas proportional to the times. But in every other point of the orbit, the force LA is the more oblique to the radius TL, as the Moon is nearer to the octant: and therefore the central force compounded of this force and the Moon's gravity, is proportionally farther distant from T, the Earth's centre.

Scholium.

In the foregoing theory we have supposed the Moon's orbit in its original form, to be a circle; although we know that it is an ellipsis, having the Earth's centre in one of the foci nearly; nevertheless the inequalities just explained may be conceived to have the effect of causing the ellipsis to be more convex in the quadratures, and more rectilinear in the syzygies: these inequalities must therefore contribute to make the Moon recede from the Earth in the quadratures, and to approach it in the syzygies. Besides, the Moon's orbit is not very excentric, consequently the foregoing theory answers nearly as well to her ellipsis as to a circle. And because the Moon's *central* force diminishes in the syzygies and increases in the quadratures, it is evident, that it must sometimes be too great and sometimes too little, to be always inversely proportional to the square of the Moon's distance from the Earth: yet this is necessarily always so, in respect to the centre of force, but this centre of force is moveable, and sometimes is in the direction of the Earth's centre, but at other times a little distant from it.

The difference in the distances of the Earth and Moon, the excentricities of the Earth's and Moon's orbits; together with the inclination of the plane of the Moon's orbit to that of the ecliptic, are the cause of five very considerable inequalities in the Moon's motion, namely,

1st. An inequality in the time of several of the Moon's successive periodic revolutions. 2d. Of the motion of the line of the apsides. 3rd. Of a continual alteration in the excentricity of the Moon's orbit, called the *evection*. 4th. Of the retrograde motion of the nodes. And 5th. Of the alteration of the inclination of the orbit to the ecliptic. In the following theorems, we propose to illustrate each briefly, and to develop their physical causes and attendant phenomena.

THEOREM 1.

The times of the Moon's periodic revolutions are longer when the Earth is in perihelion, than when it is in aphelion.

Demonstration.—For the nearer the Earth is to the Sun, the greater is the Earth and Moon's gravity to the Sun: therefore, the greater is the diminution of the Moon's gravity to the Earth in the syzygies. And although the increase in the quadratures is also greater, yet as the diminution is nearly double of the augmentation, it is therefore evident, that the Moon's gravitation in a whole revolution to the Earth is less in perihelion than in aphelion; and consequently the Moon is farther distant from the Earth in the first case than in the second. Hence it follows, that all other circumstances being equal, the Moon's orbit is greater when the Earth is in perihelion than when it is in aphelion. Consequently, the mean distance of the Moon from the Earth is greater, and so also the square root of its cube must be greater: but the periodic times are directly proportional to this number by Kepler's law (as we have demonstrated); wherefore the periodic times of the Moon's revolutions are likewise greater when the Earth is in perihelion than when it is in aphelion. Q.E.D.

Corollary.—By a parity of reason, the time of the Moon's revolution to the apogee and node must also be longer when the Earth is in perihelion than when it is in aphelion.

THEOREM 2.

When the Moon is in the syzygies the line of her apsides advances *directly*, or according to the order of the signs from west to east; but it moves *retrograding* from east to west, when she is in the quadratures. It *advances* with the greatest velocity when it coincides with the line of the syzygies; and *retrogrades* with the greatest velocity when it coincides with the line of the quadratures: but the quantity it *advances* exceeds the quantity it *retrogrades*; and the aggregate or sum of all these differences during several revolutions, causes an entire revolution of the line of the apsides according to the order of the signs from west to east in 9 years nearly.

Demonstration.—The alternate increase and decrease of the Moon's gravitation towards the Earth must cause the *central* force to be sometimes too great, and sometimes too small, to be inversely proportional to the squares of the rays to the centre of forces: wherefore it is evident, that the Moon's orbit can not be a perfect ellipsis having the Earth in one of its foci, unless the line of the apsides were constant and immoveable, which is not the case: but the line of the apsides being supposed moveable (as it really is), the motion of that line is *direct* in the syzygies, where the *central* force is least; and *retrograde* in the quadratures, where the *central* force is greatest. And the diminution of the *central* force in the syzygies, being not only double of the augmentation in the quadratures, but also extending 54° 44′ 8″ on either side of the syzygies; while the augmentation

extends only 35° 15′ 52″ on each side of the quadratures: it follows, that the *direct* motion must exceed the *retrograde*, and therefore the apogee advances sensibly in every semi-synodic revolution; and in about 106 synodic revolutions, or a little less than 9 years, the apogee completes an entire revolution from west to east, according to the order of the signs.

Again, the Moon's orbit being, in the present case, supposed a perfect ellipsis (as it is always nearly so), and the Earth placed in one of the foci; therefore the distances TL are not every where equal; for they are less towards the perigee and greater towards the apogee. Wherefore the perturbate force LA, which in the syzygies is always = 2TL, is proportionally greater, as the Moon, in syzygies, is nearer its apogee; and on the contrary, is proportionally less when she is nearer the perigee. Let the Moon's apogeon distance = D, and her perigeon distance = d. When the line of the apsides coincides with that of the syzygies the perturbate force LA is the *greatest*, and = 2D in the apogeon syzygy; but *least*, and = 2d in the perigeon syzygy. Now were it not for the perturbate force, the *central* force of the Moon in apogee would be to her *central* force in perigee, in the *direct* proportion of d^2 to D^2; but because of the perturbate force, these central forces are to one another, as $d^2 - 2D$ to $D^2 - 2d$; and this proportion differs most from d^2 to D^2 in the inverse ratio of the square of the distances, in proportion as D^2 is greater than d^2. Now D is the greatest distance, and d the least: therefore when the line of the apsides coincides with the line of the syzygies, the Moon's central forces differ most from the inverse ratio of the square of the distances; and then the perturbate force has most effect in advancing the line of the apsides with the greatest velocity: but in the same supposition of the coincidence of the line of the apsides with that of the syzygies, the points of the Moon's mean distances from the Earth are nearly in the line of the quadratures QR: and the Moon's central force in R is to her central force in Q, as $\overline{TQ}^2 + TR$ is to $\overline{TR}^2 + TQ$. Now this analogy is less different from the inverse ratio of the squares of the distances in proportion as TQ differs less from TR; but in this case these two quantities are equal, because the points Q and R are nearly at the Moon's mean distance: therefore when the line of the apsides coincides with that of the syzygies, then the inverse ratio of the squares of the Moon's distances from the Earth is least disturbed in the quadratures, by the perturbate force LA: and consequently the line of the apsides then retrogrades with its least velocity. And in like manner we demonstrate, that when the line of the apsides coincides with that of the quadratures, it advances with least velocity when the Moon is in the syzygies, and retrogrades with the greatest velocity when she is in the quadratures.

Theorem 3.

The excentricity of the Moon's orbit varies every instant—it is *greatest* when the line of the apsides coincides with that of the syzygies, and *least* when the line of the apsides coincides with that of the quadratures: this is called the evection, its maximum in longitude is 1° 23′ 29.5″.

Demonstration.—If the Moon's central force in the syzygies were not diminished by the perturbate force LA; or if, in its diminution, it did not alter in its ratio to the distance; then the perturbate force could have no effect in changing the figure of the Moon's orbit, and consequently the excentricity would be *constant*.

But since this is not the case, for the diminution of the Moon's central force is *greater*, being farther decreased by the perturbate force, when that central force decreases in the passage from the perigee to the apogee, and *less* when the central force increases; and consequently the inverse ratio of the squares of the distances is more disturbed when the central force decreases than when it increases; hence it follows, that the Moon must depart proportionally farther from the Earth's centre in her passage from the perigee to the apogee, and approach nearer to it in passing from the apogee to the perigee; therefore the apogeon distance becomes proportionally *greater*, and the perigeon distance *less*; and consequently the excentricity, in this case, is increased. And because this inverse ratio is most disturbed when the line of the apsides coincides with that of the syzygies, it is evident that the excentricity must then be *greatest*: on the contrary, when the line of the apsides coincides with that of the quadratures, the increase of the central force being nearly the same in the Moon's passage from the perigee to the apogee, as in that from the apogee to the perigee; in that case, the inverse ratio of the squares of the distances will be *least* disturbed; and consequently the excentricity of the orbit will be *least* augmented; that is, the Moon's orbit may then be said to have returned to its original form when the excentricity is the *least*. Q.E.D.

Note.—In our tables, the *equation* of the Moon's centre is computed for the mean excentricity, and the evection is the difference between the *mean* and *true* equation.

Theorem 4.

When the Moon is in the quadratures the nodes of her orbit are always *stationary*, but commonly *retrograde* when she is in the syzygies; in the interval from one quadrature to another they are the *most* retrogradely, when the line of the nodes coincides with that of the quadratures; and *least* when it coincides with that of the syzygies. And in every revolution of the Moon the angle of inclination of the orbit to the plane of the ecliptic *increases* in the

interval from a syzygy to the next quadrature, and *decreases* from the quadrature to the syzygy. It is *greatest* in the quadrature when the line of the nodes coincides with that of the syzygies, and *least* in the syzygy when the line of the nodes coincides with that of the quadratures.

Demonstration.—If the plane of the Moon's orbit were not inclined to the plane of the ecliptic, then the parallelograms LGDF (fig. 10 and 11, plate 12) in which the different central forces acting on the Moon are divided, would be in the plane of the ecliptic, and consequently the forces could have no power to draw the Moon out of that plane: but the case is otherwise, for the inclination of her orbit to that plane being 5° nearly, those parallelograms cannot be in the plane of the ecliptic, except when the Moon is in one of the nodes; and in this case the different forces urging her then cannot alter the position of her orbit to the ecliptic; but when she is distant from either node and has latitude, those forces not acting in the plane of the ecliptic must have a tendency sometimes to draw the Moon toward the ecliptic, and at other times from it; and therefore must alter the inclination of the orbit. Wherefore if the Moon at L have latitude, no side of the parallelogram LGDF can be exactly in the plane of the ecliptic: but DF being supposed to coincide with ST, then the triangle TLA has its side TA in the plane of the ecliptic, and the other two sides TL and LA inclined to that plane. From the point A (fig. 12, plate 12.) draw AV perpendicularly to the plane of the Moon's orbit, produced if necessary, so that V may be in that plane; and construct the parallelogram LMAV about the diagonal LA, which will therefore be divided into the forces LM = AV perpendicular to the plane of the Moon's orbit, and LV coinciding with that plane. And because LM is inconsiderable in respect to LV, and the angle VLA very small on account of the little inclination of the Moon's orbit; therefore LV may be taken as = LA. Now the effects of the force LA having been illustrated in the preceding theory: consequently it only remains to consider those of the force LM, and its relation to the increase or decrease of the Moon's central force. Now to find the relation of the force LM or AV to TL, expressing the increase of the central force in the quadrature: let TP be the line of the nodes; from the point V draw VP perpendicularly to TP; and join PA: then the angle VPA is equal to the inclination of the plane of the Moon's orbit to the plane of the ecliptic. Hence, RT or TL : TA :: radius : 3 sine LTR (by prob. 11); and TA : AP :: R : sine ATP; also AP : AV or LM :: R : sine APV. Now by multiplying these three analogies term by term, and dividing the first ratio by TA × AP, we have TL : LM :: $\overline{R}^3$: 3 sine LTR × sine ATP × sine APV: that is, the increase of the central force in the quadratures is to the force acting on the plane of the Moon's orbit, as the cube of radius is to the treble sine of the Moon's distance from the quadrature multiplied by the sine of the node's distance from the syzygy, and by the sine of the Moon's latitude from the ecliptic. And hence it follows, that this force is nothing in three cases: viz. when the Moon is in quadrature, when the line of the nodes coincides with that of the syzygies, and when the Moon has no latitude; for any one of the three sines being nothing the whole product is also nothing. Again, this force is greatest when the Moon being in syzygies, is also in her limits; for then the three forces are greatest. And, lastly, this force is generally greater in proportion as the Moon is nearer the syzygy, and has a greater latitude.

The effect of the force LM evidently makes the Moon tend continually towards the plane of the ecliptic, at which the forces TL, AL terminate; and consequently the force LM not only tends to diminish the inclination of the Moon's orbit to the plane of the ecliptic, but likewise to make her arrive at, and cross the ecliptic sooner than she otherwise would have done, had no such force influenced her. Therefore it follows, that accordingly as the force LM augments, diminishes, or becomes nothing; so the inclination of the Moon's orbit diminishes, increases, or becomes greatest: also the node towards which the Moon moves, will approach more, less, or not at all. For first, let the Moon be in L, passing from the conjunction to the next quadrature in R; it is plain, that from her tendency towards M by virtue of the perturbate force, and her tendency towards N in consequence of her own proper motion, according to the order of the signs, that she must follow a mean path, in the diagonal of a parallelogram, whose sides are in the ratio of these two tendencies: her orbit must therefore take the position L*n*, and N will approach *n*, contrary to the order of the signs; and the inclination of this orbit will increase: the same thing happens in the Moon's passage from the opposition to the next quadrature. Therefore in general, in the passage from a syzygy to the following quadrature, the Moon's node moves retrogradely, and the inclination of the orbit increases. Secondly, let the Moon be in *l*, passing from the second quadrature to the conjunction; and let *lm* be the direction of the force urging the Moon towards the plane of the ecliptic: then it is evident, that because of the direction *l*C, or the Moon's proper motion according to the order of the signs, that the Moon must move in the diagonal direction *l*Q, and her orbit take the situation *dl*Q; by which the angle of inclination becomes less than *l*D*m*, and the node

moves from D to *d*, contrary to the order of the signs. Therefore the Moon's node moves retrogradely, and the inclination of the orbit decreases, in the passage from a quadrature to the following syzygy. And in general, the Moon's node never moves according to the order of the signs, and it is stationary only when the Moon is in quadrature, or in the node; in all other cases the node moves retrogradely with a velocity increasing in proportion to the Moon's proximity to the syzygy, and as her latitude is greater. Moreover the inclination of the Moon's orbit evidently changes four times in every revolution, twice increasing and twice decreasing. It is *greatest* when the line of the nodes coincides with that of the quadratures, and *least* when that line coincides with that of the syzygies. Q.E.D.

Scholium.

The Moon's perturbate force, and consequently all her inequalities are something greater towards the conjunctions than towards the oppositions. For the distance of the Moon from the Sun being less in the conjunctions than in the oppositions by the diameter of the Moon's orbit; therefore the difference of the squares of these distances in respect to the least square, is something more than twice the difference of the roots. Moreover all the relations of the forces shewn in the preceding theory are true only in proportion as their common term of comparison is exactly known. Now the relation of a greater or less increase of the Moon's gravity in the quadratures may be accurately determined by actual observation; for this augmentation is as RT the Moon's distance from the Earth: and supposing the Earth to describe a circle about the Sun, and that RT the Moon's distance from the Earth be also supposed *constant*, and TS the distance of the Earth from the Sun be increasing: then the augmentation of the Moon's gravity decreases in the ratio of the *cube* of the Earth's distance from the Sun. For, first, this augmentation decreases in proportion as R'T expressing it, lessens in respect to ST: therefore, if the variation of the Earth's distance from the Sun caused no alteration in the gravitation of the Earth and Moon to the Sun, the increase of the Moon's gravity in the quadratures would be expressed by $\frac{RT}{ST}$; but secondly, it also decreases in the ratio of the square of ST; because the Earth in receding from the Sun causes the Moon equally to recede from him when she is in the quadratures, as both are then equidistant from him; and consequently the gravitation of the Moon towards the Sun diminishes equally with that of the Earth, that is in the ratio of the square of ST: hence it follows in general, that in the quadratures the increase of the Moon's gravity to the Earth is always as $\frac{RT}{ST}$ divided by $\overline{ST}^2 = \frac{RT}{\overline{ST}^3}$: that is, as the Moon's distance from the Earth divided by the cube of the Earth's distance from the Sun. Or the apparent diameters being inversely proportional to the distances, as the cube of the Sun's diameter seen from the Earth, divided by the Moon's diameter seen also from the Earth. The ratio of the increase is therefore found by immediate observation; by measuring the diameters of both objects at the time of the Moon's quadratures.

What has been said of the Moon's inequalities, in respect to her motion round the Earth, is equally applicable to the satellites of Jupiter, Saturn, and the Georgian. But because of the great distance of these planets from the Sun; the inequalities of their satellites must be very small; and in those nearest the planet, their inequalities are only sensible on account of their proximity.

It is also evident, that this combination of the Moon's gravitation towards the Earth and the Sun, must produce the same effects, whether we suppose the Earth to describe an ellipsis about the Sun placed in one of its foci; or the Sun to describe an ellipsis about the Earth fixed in one of its foci. For these effects depend upon the Moon's position and distance from the Earth and Sun. Now whether the Earth or Sun be supposed to move, their respective situations are still the same to the Moon, and to one another.

Enumeration of the elements necessary for the construction of lunar tables, for computing the *true* longitude and latitude, horizontal parallax, semidiameter, &c. of the Moon for a given time, which elements must be accurately determined by actual observations.

1. The *mean time* of the Moon's sidereal, tropical, and synodical periodic revolutions.
2. The mean time of the sidereal and tropical periodic revolution of the Moon's apogee.
3. The same for her nodes, or rather for her ascending node.
4. An epoch of the Moon's mean longitude from the mean equinox.
5. The same for her apogee.
6. The same for her ascending node.
7. The greatest equation of the Moon's centre for the mean excentricity.
8. The same for the mean place of the apogee.
9. The same for the mean place of the node.
10. The mean excentricity of the Moon's orbit.
11. The greatest and least excentricities of the orbit.

12. The mean variation in the octants.
13. The greatest and least variations.
14. The mean inclination of the Moon's orbit.
15. The greatest and least inclination of the orbit.
16. The mean horizontal parallax of the Moon.
17. The greatest and least horizontal parallax.
18. The ratio of the horizontal parallax to the horizontal diameter.

As the lunar tables given in this work shew most of these elements by inspection, it has not been deemed necessary to give the respective quantities or numbers in this place, as a view of the tables must be much more satisfactory to the student. But besides these elements, which are as it were principal ones, there are others which give small equations, as may be seen by the tables; and although some of these equations are but small, yet they are not to be neglected, when we wish to compute the Moon's place to the highest degree of accuracy attainable by the tables. See the tables and precepts shewing their use.

Of the Moon's Librations.

Because the Moon revolves round on her axis from west to east exactly in the same time that she completes her periodic revolution in her orbit round the Earth from west to east: hence it is plain, that if her axis were perpendicular to the plane of the ecliptic; and if her motion in her orbit, round the Earth were uniform, that the Moon would always present the same side or hemisphere to the Earth: but since no one of these circumstances is exactly the case; hence it follows, that we sometimes see a small portion of the opposite hemisphere, which before was invisible, and lose sight of an equal portion of the hemisphere, which before was visible. Now this is called the Moon's *libration;* and is caused, 1st, by the uniform motion of the Moon on her axis, and the unequal motions of the Moon from west to east round the Earth. The quantity of the Moon's libration on this account, is nearly equal to the difference between the Moon's *true* and *mean* longitude; its maximum is about $7\frac{1}{2}$ degrees on either side. 2d, By the inclination of the Moon's axis to the plane of the ecliptic; for as in the former case she apparently oscillated from east to west and *e contra;* so on this account she will shew a small portion of her polar regions to the Earth, which was before invisible, and an equal portion about her other pole will become invisible. The quantity of this libration depends on the Moon's latitude, and the obliquity of her axis to the plane of the ecliptic. There are besides these, other small librations; but they are not worth notice.

In regard to the lunar theory, the subject is so extensive and complicated, that to develop each part at full length would require a very large volume: even Newton's *Principia* does not embrace the whole. There is a small work written by the late professor Mayer, called *Theoria Lunæ Juxta Systema Newtonianum*, which has great merit, and as such we recommend it to the student; this small volume is certainly *multum in parvo;* it develops the principal inequalities of the Moon's motions, and shews how to compute them, on mathematical and physical principles. But there are several small inequalities, which have been discovered since Mayer's time, and it is hoped that with these additional equations and corrections, the Moon's place will be computed from our tables far within the limits required by our board of longitude. Messrs de la Lande, La Place, and Vince have contributed greatly towards bringing the theory to perfection, and Maskelyne, Mason, Burg, and Burkhardt have in a great measure completed the work. Still some farther improvements may be added by means of more accurate observations, with which it is to be hoped the public will from time to time be furnished by whoever may hold the important and honourable office of astronomer royal.

On the nutation of the Earth's axis and precession of the signs of the Zodiac, or recession of the equinoctial points.

It has been proved by reason, experiment, and actual mensuration, that the Earth is in the form of an oblate spheroid, occasioned by its diurnal rotation about its axis. Sir Isaac Newton makes its equatorial diameter to exceed the polar by about 34 miles, other writers make it something less; so that the thickness of the redundant matter about the equator is 17 miles nearly; and this thickness decreases till it comes to the poles, where it is nothing.

This being the case, we have now to prove that this redundant matter being acted upon by the attraction of the Sun and Moon produces a nutation of the Earth's axis.

It is evident that if the Moon be placed any where over the equator, or over the poles, she can produce no effect, but if she have any declination less than 90°, the axis must be removed out of its former position.

For let A (fig. 13, plate 12.) be a portion of this redundant matter, and B another similar and equal portion, then it is plain that the portion at A, is nearer to M the Moon than that at B, and as the effect produced is inversely as the squares of the lines MA and MB, it must follow that the axis will be removed, and the mass at A will be drawn nearer to M the Moon, or the whole Earth will be moved about its centre by the excess of the Moon's

fore commonly thought too small to be attended to attraction of the mass at A more than on that at B. As the Moon is much nearer to the Earth than the Sun, her effect will be far greater than his although he contains a much greater quantity of matter; and as their positions vary as they move round the Earth, they will sometimes act in conjunction, and at other times in opposition; hence the axis of the Earth will oscillate more or less according to the various positions of those bodies. The obliquity of the ecliptic will also be more or less affected, sometimes increasing or decreasing, and perchance it may be stationary for a short time.

In order to account for the recession of the equinoctial points, we shall suppose the Sun to move round the earth in the plane of the ecliptic, as all circumstances will remain the same as if we took the real motion of the Earth into consideration; and for the better understanding the matter we shall constantly denominate that great circle which surrounds the Earth at 90° from the poles the equator, and the corresponding one in the Heavens the equinoctial. As those two great circles are constantly in the same plane, or as the equinoctial is constantly over the equator it cannot be fixed in the heavens as the ecliptic is here supposed to be; but its plane suffers the same oscillations as the poles of the world do by the nutation of the Earth's axis. These things being premised, it now remains to shew how the recession of the equinoctial points is produced.

Suppose the Sun to be in the point Aries, and to proceed on towards Libra; then, as his declination increases, his power to disturb the Earth's axis increases, till he comes to the tropic; after which his force will decrease till he comes to Libra, where that part of the nutation which is produced by the Sun vanishes. When the Moon is on that side of the equator as the Sun, and the difference of their right ascensions less than 90° they will act in conjunction, and the plane of the equator will incline towards the Sun; but when they are on the same side, and the difference of their right ascensions considerably more than 90° the plane will recede from him. If the Moon be in the opposite hemisphere and the Sun and Moon on opposite meridians, the plane will incline to the Sun; but if they are on the same meridian and in opposite hemispheres, the plane will recede from the Sun. When the two bodies act together, the plane will approach the Sun with the sum of their joint effects; but when the two luminaries act in opposition, the plane of the equinoctial recedes from the Sun with the difference of their effects; consequently the plane inclines to the Sun more than it recedes from him. If this plane were fixed the Sun would have to pass through 180° from one equinoctial point to the other, but because the plane inclines to the Sun more than it recedes from him, he must of necessity fall into it before he has arrived at the 180th degree, and the moment his centre enters the plane, he is in his node, or equinoctial point; and as much as he falls short of the 180°, so much has the point receded to the west.

Remark.—It may happen that when the Moon is in the opposite hemisphere and her declination very small, and that of the Sun considerable, the plane will incline to him. There are also small effects produced by the planets, but they are inconsiderable. There is a point between the equator and the pole, where if the Moon were placed over it, the nutation would be the greatest. The quantity of the lunar nutation as also of the precession depends much on the place of the node.

Of the aberration of light.

It appears by astronomical observations, that the particles of light do not fly with an infinite velocity, but move over a given space in a given time. The velocity with which they move was discovered by the eclipses of Jupiter's moons; by which it was found, that light was 8*m.* 7*s.* in passing over the semidiameter of the Earth's orbit. And because in this space of time, viz. 8*m.* 7*s.*, the Earth moved through 20″ of its orbit: hence the velocity of light compared to that of the Earth in its orbit, was found to be in the proportion of 10313 to 1. Now if we conceive a right angled triangle to be constructed whose legs are in the ratio of 10313 to 1, then the less *acute* angle will be equal to 20″: and this angle of 20″ is what is called the *absolute aberration of light*, occasioned by the velocity of the particles of light compared to that of the Earth in its orbit. Let S be the Sun (fig. 14, plate 12.); AB a portion of the Earth's orbit; and in the time that a particle of light passes from S to B, or over the semidiameter of the Earth's orbit, suppose the Earth to have passed over the space AB: the angle ASB = 20″ is the absolute aberration of light, in respect to the Sun, and to which the *aberration* of all the other celestial bodies must be compared.

Since the motion of the Earth in its orbit is always at right angles to the rays of light issuing from the Sun; hence if its velocity round the Sun were always uniform, then the quantity of aberration, in respect to the Sun, would always remain the same, or 20″: but because this is not exactly the case, therefore the Sun's aberration is greater when the Earth is in perihelion than when it is in aphelion; and this change in the aberration is called the variable part of the aberration; its maximum is 0.34″, and is there-

in astronomical calculations. The aberration is therefore always taken as 20″, when the Earth's motion is at right angles to the ray of light issuing from a celestial body.

Now in regard to the fixed stars, because their distance is so great that it may be assumed as infinite, we conceive the rays of light issuing from any star to be parallel between themselves, let the Earth be in whatever part of its orbit it may. Wherefore if a star be situated in the pole of the ecliptic, then the motion of the Earth in its orbit is always at right angles to the rays of light coming from that fixed star; consequently its aberration is always 20″: that is, the *apparent* place of that star is always 20″ from its *true* place, or that in which it would be seen, if the velocity of light were infinite.

Again, let us suppose a star to be situated in the plane of the ecliptic; and let ACBO (fig. 15, plate 12) be the Earth's orbit, and let the parallel rays *S, *H, *L and *B, coming from the star, pass over the Earth's orbit. Then it is manifest, that when the Earth is at C or O that its motion is perpendicular to the ray, and consequently that the aberration of the star is in either case 20″; that is, at the time of conjunction or opposition with the Sun in respect to the Earth, the aberration is 20″. Moreover as the Earth proceeds in its orbit towards D,F,B, its motion being more and more oblique to the ray; consequently the aberration diminishes; and when it arrives at B the aberration vanishes, that is, when the Sun and star are in quadratures, the aberration of the star is nothing; because the Earth is then moving in the direction of the ray. And the quantity of the aberration is always found, in this case, by saying, as radius is to the sine of the star's distance from the quadrature, so is 20″ to the aberration; or perhaps more easily, by multiplying 20″ by the natural sine of the star's distance from the quadrature with the Sun. The reason is evident. And the star being supposed to have no latitude, hence its longitude is therefore increased or decreased by the whole quantity of its aberration; but its latitude is not affected thereby.

Lastly, if we take a fixed star which is situated between the ecliptic and its pole; that is, a star which has latitude; then both the latitude and longitude of this star will be affected by the aberration. Whence, to find the aberration of a fixed star, we propose the following

Problem.

Given the latitude and longitude of a fixed star, and the Sun's longitude; to find the aberration of that star in latitude and longitude.

Rule.—To find the aberration in latitude. Subtract the star's longitude from that of the Sun, and reject 6*s.* or 180°, if the remainder exceed 6*s.* Then to the logarithm of 20″ = 1.30103, which is a constant log., add the log. sine of the remainder, and the log. sine of the star's latitude; the sum, rejecting 20 from the index, is the log. of the aberration in latitude in seconds. To find the aberration in longitude. To the constant log. add the log. cosine of the remainder, and the log. secant of the star's latitude; the sum, rejecting 20 from the index, is the log. of the aberration in longitude.

Example.

Required the aberration of ✻ α Andromedæ in latitude and longitude, on 1st June, 1814, its latitude being 25° 41′ 6″N, and longitude 0*s.* 11° 42′ 51″, and the Sun's longitude 2*s.* 10° 17′ 6″.

	s.	°	′	″				
Sun's longitude . . .	2	10	17	6				
Star's longitude . .	0	11	42	51				
Remainder	1	28	34	15	=	58	34	15
Latitude						25	41	6

For latitude.		For longitude.	
Log. sine . . .	9.93109	Log. cosine. .	9.71721
Log. sine . . .	9.63688	Log. sect. . .	10.04517
Constant log. .	1.30103	. . . . .	1.30103
Aberration 7.4″	0.86900	. . 11.6″ . . .	1.06341

Hence the aberration in latitude is 7.4″ and in longitude 11.6″. And when the remainder or difference of longitude is less than 3*s.* or more than 6*s.* and less than 9*s.*; subtract the aberration in longitude, otherwise add, and you have the longitude corrected for aberration. The same thing is to be done if the latitude of the star be south, but if north, reverse the rule.

	°	′	″		*s.*	°	′	″
Star's lat. .	25	41	6.0N	Longitude	0	11	42	51.0
Aberration		+	7.4	. . . . .		—		11.6
Correct lat.	25	41	13.4N	Cor. long.	0	11	42	39.4

Note.—The same rule is equally applicable for finding the aberration of a planet, provided it be stationary, otherwise we must farther correct it, and find the planet's distance from the Earth at the proposed time, and compare that with the Sun's distance, thus: as the Sun's distance is to the planet's distance, so is 8′ 7″, to the time in which light will pass from the planet to the Earth. Then find the planet's motion in geocentric latitude and longitude for this time, and apply these corrections to the

latitude and longitude corrected for the former part, and you will have the *true* geocentric place of the planet corrected for the effects of aberration. And now we may remark, that because a planet's latitude is commonly small; astronomer's generally omit the effect of the latitude, and only attend to the effects of the aberration in the geocentric longitude of a planet. And hence the aberration, in respect to a planet, is easily computed.

As the Earth and Moon are carried round the Sun with the same velocity; hence the aberration of light, in respect to the Sun, is the same at the Moon as at the Earth; and this circumstance removes the effects of the aberration of light in its progress from the Moon to the Earth; because the Earth and Moon are *relatively* at rest to each other, or nearly so, in respect to the Sun: and hence, what we call the Sun's aberration, is equally applicable to the Moon.

We have further to remark, that the *aberration*, is a correction only belonging to the *observed* place of a celestial object, and not its *true* place as computed by our tables. But because observations of the Sun and Moon are so frequently made for the purposes of navigation and other matters in practical astronomy; and also because the Sun's aberration is a constant quantity of 20″; astronomers have applied 20″ to the epochs of the Sun, by which means the effects of aberration in respect to solar and lunar observations are removed: this is a convenience to the observer, as it saves him the trouble of correcting his observations for aberration; that correction being already in the tables, in respect to the Sun and Moon. And hence, the solar and lunar tables do not shew the *true*, but the apparent places of the Sun and Moon, in respect to the aberration of light. As a farther convenience to the observer, all our calculations for the *data* in the nautical almanac, are made for *apparent* time. And moreover, that which we have called the *correct* place of the star, in the preceding example, is its *observed* place, that is, it is the place in which an observer would see it; and the foregoing rules only apply, as to the signs, for reducing the *true* place to that in which an observer expects to see the object. Hence, in the construction of astronomical tables, observations taken for that purpose, the signs must be reversed, in order to reduce the observed place to that of the *absolute true* place; but by the rule aforegoing, all the calculations of a fixed star are made to suit those of the Sun and Moon, to the end, that in comparing the places of celestial objects, the observer may not be embarrassed with the effects of aberration.

When the *true* or *apparent* place of an object is found, as to latitude and longitude, the same can be found as to its *true* or *apparent* place in right ascension and declination; as we have shewn in our practical astronomy; and *vice versâ*. And besides, the effects of aberration, in respect to right ascension and declination, may be easily found from those of latitude and longitude, by proportioning the horary motion in latitude and longitude, to those of declination and right ascension respectively, for the given time.

A BRIEF HISTORY OF ASTRONOMY.

THE early history of this science, like that of many other ancient discoveries, is too much disfigured by fabulous and allegorical representations, to admit of any regular or satisfactory elucidation. It is probable, however, that some knowledge of this kind must have been nearly coeval with the human race; for besides motives of mere curiosity, which are sufficient to have excited men in all ages to examine the magnificent and varying canopy of the heavens, it is evident that some parts of the science are so connected with the common concerns of life, as to render the cultivation of them indisputably necessary.

Many traces of it have accordingly been found among various nations, which show that several of the most remarkable celestial phenomena, at least, must have been observed, and a knowledge of them disseminated at a very remote period. But in what age or country the science first originated, or by whom it was gradually methodized and improved, is extremely uncertain; nothing more being known on this subject than what can be obtained from the scanty and incidental information of ancient writers, whose accounts are often too extravagant and improbable to deserve much attention.

Among other relations of this kind may be reckoned what is mentioned by Josephus in his Antiquities, who, in speaking of the progress that had been made in astronomy by Seth and his posterity, before the deluge, asserts that they engraved the principles of the science on two pillars, one of stone and the other of brick, called the pillars of Seth; and that the former of these was entire in his time. He also ascribes to the Antediluvians a knowledge of the astronomical cycle of 600 years, which Montucla (in his Histoire des Mathematiques) thinks, with much greater reason, was an invention of the Chaldæans; and that whatever information was possessed by the Jewish annalist with respect to this remarkable period, was probably obtained either from that people, or from some ancient writings which no longer exist.

But not to insist upon this and other uncertain testimonies of the ancients, it will be sufficient to observe that, notwithstanding the contrariety of opinions which have prevailed on this subject, most authors are agreed in fixing the origin of astronomy either in Chaldæa or in Egypt; both of which nations pretended to a very high antiquity, and equally claimed the honour of producing the first cultivators of this science. The Chaldæans, in particular, boasted of their temple, or prodigiously high tower, of Belus, which is thought by some to have been an astronomical observatory, and of their celebrated philosopher and astronomer Zoroaster, whom they placed 500 years before the destruction of Troy: while the Egyptians, with similar ostentation, vaunted of their colleges of priests, which were the depositaries of every species of knowledge; and of the monument of Osymandyas, in which it is said there was a golden circle of 365 cubits in circumference, and one cubit thick, divided into 365 equal parts, according to the days of the year, and containing the heliacal risings and settings of the stars for each day, &c.

It is evident, indeed, without placing much reliance upon these accounts, that both Chaldæa and Egypt were countries extremely proper for astronomical observations, being almost constantly favoured with a pure atmosphere and a serene sky; and whatever may be thought of the tower of Belus, or the circle of Osymandyas, we cannot but form a very advantageous opinion of the knowledge of the Egyptians in practical astronomy, from the position which they have given to their pyramids, whose faces are directed with great precision towards the four cardinal points of the compass. For as it is scarcely possible that a situation so exact could have been the effect of chance, we must conclude that they were acquainted with a correct method of drawing a meridian line; which is a matter of more difficulty than is usually thought; it being well known that Tycho Brahe, the most able astronomer of his time, committed an error of several minutes in tracing that of his observatory of Uraniburg.

The Chaldæans also must have made very considerable advances in this science, if we can rely upon the testimony of Simplicius, who informs us that, at the taking of Babylon by Alexander the Great, they cited a regular series of astronomical observations for 1903 years back; and that these, through the means of Callisthenes, were afterwards communicated to the Greeks by Aristotle. But it is much to be wished that the truth of these ancient observations was better established, particularly as their historian Berosus, who appears to have lived but a little before the time of Alexander, makes no mention of any astronomical monument of this people, which was more than about 480 years anterior to that period. And, indeed, the most ancient Chaldæan observations, of which any mention is made by astronomical writers, are those of three eclipses of the moon, employed by Ptolemy in his Almagest, which were made in the years 27 and 28 of the æra of Nabonassar, or 721 and 720 years before Christ.

But though Ptolemy, and perhaps Hipparchus, from whom he had probably taken them, made no use of any observations more ancient than those here mentioned, we cannot from thence conclude that the Chaldæans first began to follow the celestial motions at this period. For such as were made in much earlier times might be suspected on several accounts; and it is besides highly probable that most of those which preceded the æra of Nabonassar were not accompanied with dates sufficiently accurate to be employed by these astronomers. The Babylonian calendar, before this æra, was in great confusion, not having been properly regulated; and it is obvious that ancient observations, either of this or any similar kind, can be but of little use, except we are able to ascertain the precise time at which they were made.

Besides these eclipses mentioned by Ptolmey, nothing more now remains of the Chaldæan astronomy, except what is attributed to them by some ancient authors, with respect to certain periods of years, which they seem to have formed for the more ready computation of the places of the heavenly bodies. And though the accounts which have been given us of one of the most remarkable of these cycles, by Suidas and Pliny, are not wholly free from objections, there can be little doubt of its having been first invented by that people. This is the celebrated period called the Chaldæan Saros, which consists of 223 lunar months, or a little more than $18\frac{1}{2}$ years; and which so far agrees with the combined motions of

the Sun and Moon, as always to bring them again into nearly the same position at the end of each cycle that they had at its commencement.

Both the Chaldæans and Egyptians, are generally supposed to have possessed a very considerable knowledge of several other branches of this science besides those here mentioned; but for want of proper authorities, this can only be judged of by some just notions which they appear to have had on the system of the world, and by the agreement which has been found among several ancient measures of the circumference of the earth. The Egyptians, in particular, appear to have known, long before the Christian æra, that the year consisted of 365¼ days, and that the planets Mercury and Venus moved round the Sun. We are also well assured of the great antiquity of the science among this people, from the recent discoveries which have been made in that country during the late war; and particularly from the figure of a zodiac brought from thence by the French, which Lalande considers as extremely ancient.

But among the various nations which claim the honour of having first cultivated this science, none pretend to possess observations of greater antiquity than the Chinese. The most remarkable of these is a conjunction of five of the planets, which, according to their annals, is said to have taken place in the reign of their emperor Tchuen-hiu, about 2500 years before Christ. They also mention an eclipse of the Sun, which happened in the constellation Scorpio, about the year 2150 of the same æra; and which is said to have proved fatal to two Chinese astronomers of the names of Ho and Hi, who were condemned to death by the emperor Tchong-kang, on account of their omitting, through negligence and intoxication, to announce the precise time at which it arrived. And from these data, apparently well attested, several eminent astronomers have endeavoured to discover whether these events could have possibly happened about the time here mentioned; but the subject is attended with too many difficulties to afford any satisfactory result.

All that we know of the Chinese astronomy is from the accounts which have been given of it by the Jesuit missionaries, who are much divided in their opinions with respect to its very great antiquity; some supposing it to have flourished at an earlier period than others. F. Du Halde, however asserts, that it was cultivated by their great lawgiver Confucius; and that Tcheou-cong, the most skilful astronomer that China ever produced, lived more than 1000 years before Christ, and passed whole nights in observing the celestial bodies, and arranging them into constellations. But whatever might have been the knowledge of this people in former times, the state of astronomy is very low in that country at present, although it is cultivated at Peking by public authority, in the same manner as in most of the capital cities of Europe.

The inhabitants of Japan, Siam, and the Mogul's empire, also appear to have been acquainted with astronomy from time immemorial; and the famous observatory at Benares is a monument both of the great ingenuity of the Indians, and of their skill in that science. A knowledge of this subject is also supposed to have prevailed among the Americans; though, in their divisions of time, they made use of the solar and not of the lunar motions. The Mexicans, in particular, are said to have discovered a singular predilection for the number 13, which they used as a kind of cycle in most of their computations. And the abbé Clavigero asserts it as a remarkable fact, that having discovered the excess of a few hours in the solar above the lunar year, they made use of intercalary days to bring them to an equality, as was done by Julius Cæsar in the Roman calendar; but with this difference, that, instead of one day every four years, they interposed 13 days every 52 years, which produces the same effect.

But the most interesting account of the rise and progress of this science hitherto given, is that which is detailed by M. Bailly, in his learned and elaborate history of Ancient and Modern Astronomy; in which he endeavours to trace its origin among the Chaldæans, Egyptians, Persians, Indians, and Chinese, to a very early period. And in consequence of the researches he has made on this subject, he is led to maintain, that the knowledge common to the whole of these nations, has been derived from the same original source: namely, a most ancient and highly cultivated people of Asia, of whose memory every trace is now extinct; but who have been the parent-instructors of all around them.

M. Bailly does not pretend to fix, with certainty, the precise situation of this ancient people; but he offers several reasons for conjecturing that it must have been somewhere about the 49th or 50th degree of north latitude, in the southern regions of Siberia. Among various other coincidents, he observes, that many of the European and Asiatic nations attribute their origin to that quarter, where the civil and religious rites, common to each, were probably first formed; and what he considers as a strong astronomical support of his hypothesis is, that the observations of the stars, collected by Ptolemy, must have been made in a climate where the longest day was 16 hours, which corresponds

to the latitude here mentioned. But as that region exhibits no traces of its ever having been inhabited by a polished people, his theory, though highly ingenious, has not sufficient force to draw our assent to his conclusions.

In investigating the antiquity and progress of astronomy among the Indians, M. Bailly examines and compares four different sets of astronomical tables of the Indian philosophers, viz. that of the Siamese, explained by M. Cassini, in 1689; that brought from India by M. le Gentil of the Academy of Sciences; and two other manuscript tables found among the papers of the late M. de Lisle; which, he observes, accord together, and all refer to the meridian of Benares. From these tables it appears, that the Indian astronomy has two principal epochs, the first being founded on a conjunction of the Sun, Moon, and planets, which is said to have taken place 3102 years before Christ; and the other 1491 years before the same æra. These periods are so connected by the mean motions of the Sun, Moon, and planets, that one of them must necessarily be fictitious; and though the celebrated author above mentioned, has endeavoured to shew that the first of them must have been founded on observations, there is great reason for believing that it was rather imagined for the purpose of giving a common origin to the signs of the zodiac, and the motions of the celestial bodies.

It is true, indeed, if, parting from the epoch 1491, we ascend, by means of the Indian tables to the year 3102, before the Christian æra, we shall find a general conjunction of the Sun, Moon, and planets, as these tables suppose; but this conjunction, which is too different from the result given by the best modern tables to have ever taken place, shews that the epoch to which they refer, is not founded upon observations; and, in fact, some elements of the Indian astronomy, seem to indicate that they were determined even long before the first epoch. The equation of the Sun's centre, in particular, which they fix at 2° 10′ 32″, could not, according to the calculations of M. de la Place, have been of this magnitude but near the year 4300 before Christ: and besides this, the equations of the centre of Jupiter and Mars are so different from what they ought to have been at this epoch, that nothing can be concluded from them in favour of their high antiquity.

To conclude, the whole of these tables, and, above all, the conjunction which they suppose at the same epoch, prove, on the contrary, that they must have been constructed, or at least rectified, in much more modern times. The ancient reputation, however, of the Indians, both in this and other sciences, leaves but little doubt, that astronomy was cultivated among them at a very remote period; and of this, the remarkable accuracy with which they have assigned the mean motions of the Sun and Moon, is a sufficient proof, as such exactitude could only have been obtained from a long series of observations. This opinion has also been ably supported by Mr. Playfair, in a dissertation on the astronomy of the Bramins, published in the second volume of the Transactions of the Royal Society of Edinburgh, where he has, likewise, adduced many instances of their critical knowledge in the other mathematical sciences, employed in their precepts and calculations.

The Greeks did not begin to cultivate astronomy till a long time after the Egyptians, of whom they were the disciples; and it is extremely difficult, amidst the fables which so much abound in the earlier periods of their history, to obtain any very correct information with respect to their knowledge in this science. All that we can learn is, that they had made observations on the celestial bodies, and divided the heavens into constellations, 13 or 14 centuries before the Christian æra; this being the period, according to the opinion of the most eminent chronologer, to which we must refer the sphere of Eudoxus.

The number of their philosophical institutions, however, afford no observer of any note, till much later times; most of their ancient sects having treated astronomy as a science purely speculative, without properly attending either to facts, or their causes. But notwithstanding the reveries in which they often indulged, their knowledge began to be greatly improved by Thales the Milesian, and other Greeks who travelled into Egypt, and brought from thence the chief principles of the science. This philosopher, who died at the age of 96 in the year 548 before Christ, was the founder of the Ionian sect, and appears to have been the first who taught his countrymen the globular figure of the Earth, the obliquity of the ecliptic, and the causes of solar and lunar eclipses; which latter phenomena he is also said to have been able to predict.

Thales had for his successors Anaximander, Anaximenes, and Anaxagoras, to the first of whom is attributed the invention of the gnomon, and geographical chart; but for which he was probably indebted to the Egyptians. He is also said to have maintained that the Sun was a mass of fire as large as the Earth, which, though far below the truth, with respect to size, was an opinion, for these early times, that does its author much credit; though to him, as in the case of Galileo, the truths he had discovered were the cause of his persecution. Both himself and his children were proscribed by the Athenians, for his attempting to subject the works of the gods to immutable laws; and his life would have paid the sacrifice of his temerity, but for the care of Pericles, his friend and disciple, who got his sentence of death changed into exile.

Next after the Ionian school was that of Pythagoras, who was born at Samos, about the year 586 before the Christian æra, and who, in the celebrity he acquired, far exceeded his predecessors. Like Thales he visited Egypt, and afterwards the Brachmans of India, from whom he is supposed to have obtained many of the astronomical truths which he brought with him into Italy, to which country he was obliged to retire on account of the despotism which then prevailed at Athens. Here he first taught the true system of the world, which, many centuries after, was revived by Copernicus; but hid his doctrine from the vulgar, in imitation of the Egyptian priests who had been his instructors. It was even thought, in this school, that the planets were inhabited bodies, like the Earth; and that the stars, which are disseminated through infinite space, are suns, and the centres of other planetary systems. They also considered the comets as permanent bodies, moving round the Sun; and not as perishing meteors, formed in the atmosphere, as they were thought to be in after times.

From this time to the foundation of the school of Alexandria, the history of astronomy among the Greeks offers nothing remarkable, except some attempts of Eudoxus to explain the celestial phenomena; and the celebrated cycle of 19 years, which had been imagined by Meton, in order to conciliate the solar and lunar motions. This is the most accurate period, for a short interval of time, that could have been devised for embracing an exact number of revolutions of these two luminaries; and is so simple and useful, that, when Meton proposed it to the Greeks, assembled at the Olympic games, as the basis of their calendar, it was received with great approbation, and unanimously adopted by all their colonies.

In the school of Alexandria, we see, for the first time, a combined system of observations, made with instruments proper for measuring angles, and calculated trigonometrically. Astronomy, accordingly, took a new form, which succeeding ages have only brought to greater perfection. The position of the stars began at this time to be determined; they traced the course of the planets with greater care; and the inequalities of the solar and lunar motions became better known. It was, in short, in this celebrated school, that a new system of astronomy arose, which embraced the whole of the celestial motions; and though inferior to that of Pythagoras, and even false in theory, it afforded the means, by the numerous observations which it furnished, of detecting its own fallacy, and of enabling astronomers in later times to discover the true system of nature.

Aristyllus and Timocharis were the first observers in this rising institution. They flourished about the year 290 before Christ; and by their assiduous labours, were the means of greatly improving the science. It was from their observations of the principal zodiacal stars, that Hipparchus was led to discover the precession of the equinoxes; and Ptolemy also founded upon them his theory of the motions of the planets.

Next after these, was Aristarchus of Samos, who made the most delicate elements of the science the objects of his research. Among other things of this kind, he attempted to determine the magnitude and distance of the Sun; and though, as may be supposed, the results he obtained were considerably wide of the truth, the methods he employed to resolve these difficult problems, do great honour to his genius. He also endeavoured to revive the opinion of the Pythagorean school, with respect to the motion of the earth: but as his writings upon the subject have not been preserved, we are ignorant to what point he had advanced, by those means, in his explication of the celestial phenomena.

The celebrity of his successor Eratosthenes, arises chiefly from his attempt to measure the Earth, and his observations on the obliquity of the ecliptic. Having remarked at Syene, a well which was enlightened to its bottom by the Sun, on the day of the summer solstice, he observed the meridian height of the Sun on the same day at Alexandria; and found, that the celestial arc, contained between the two places, was the 50th part of the whole circumference; and as their distance was estimated at 500 stadia, he fixed the length of a great circle of the Earth at 250,000; but as the length of the stadium employed by this astronomer is not known, we cannot appreciate the exactness of his measurement.

Among others who cultivated and improved this science, we may also mention the celebrated Archimedes, who constructed a kind of planetarium or orrery, for representing the principal phenomena of the heavenly bodies. But of all the astronomers of antiquity, Hipparchus of Bithynia, is the one, who, by the number and precision of his observations, as well as by the important results which he derived from them, is the most entitled to our esteem. He flourished at Alexandria about the year 162 before the Christian æra; and began his astronomical labours by attempting to determine, with more exactness than had hitherto been done, the length of the tropical year, which he fixed at 365 days, 5 hours, and 55 minutes, being near $4\frac{1}{2}$ minutes too great. Like most of his predecessors, he founded his system upon an uniform circular motion of the Sun: but instead of placing the Earth in the centre of the solar orbit, he removed it to the distance of $\frac{1}{24}$th part of the radius, and fixed the apogee to the sixth degree

of Gemini. By means of these data, he formed the first solar tables of which any mention is made in the history of astronomy: and though defective and even erroneous in principle, they are a durable monument of his genius, and three centuries afterwards were respected by Ptolemy, without his presuming to alter them.

The great astronomer next considered the motions of the Moon, and endeavoured to measure the exact time of her revolution, by a comparison of ancient eclipses. He also determined the eccentricity and inclination of her orbit, as well as the motion of her nodes and apogee; and calculated all the eclipses that were to happen for 600 years to come. We are, besides, indebted to him for the important discovery of the precession of the equinoxes, which was the fruit of the long and difficult enterprise he undertook of making a catalogue of the fixed stars, with their latitudes, longitudes, and apparent magnitudes. Geography is also indebted to Hipparchus for the method of fixing the situation of places upon the Earth, by means of their latitude and longitude; in obtaining the latter of which, he appears to have been the first who employed eclipses of the Moon; and as these researches required numerous calculations, they gave birth, under his hands, to spherical trigonometry. Many of his principal works perished with the library of Alexandria; but his catalogue of the stars, and several of his observations, have been preserved by Ptolemy in his Almagest.

Between the time of Hipparchus and Ptolemy, the chief observers of any note are Agrippa, Menelaus, and Theon; the two latter of which are better known as geometricians than astronomers. We remark, however, in this interval, the reformation of the calendar by Julius Cæsar, and a more exact knowledge of the flux and reflux of the ocean. Posidonius, a celebrated stoic philosopher, who lived about eighty years before Christ, appears to have been the first who observed the relation of these phenomena with the motions of the Moon; and of which Pliny, the naturalist, has given a description remarkable for its accuracy.

Ptolemy, the worthy successor of Hipparchus, was born at Pelusium in Egypt, in the beginning of the second century of Christianity, and was the first who undertook to reform the whole of this science, by establishing it upon a new foundation. In this enterprise, the system he formed is now well known to be erroneous; but the edifice he erected lasted near 1400 years; and even at this time, though it is entirely destroyed, his Almagest, considered as the depository of ancient observations, is one of the most precious monuments of antiquity.

One of the most important discoveries of this astronomer is that of the evection of the Moon, which he has assigned with so much exactness, that M. la Place, in opposition to the opinion of other writers, thinks it sufficient to entitle him to the character of an accurate observer; and that the charge which has been made against him, of appropriating the discoveries of his predecessors, is not well founded.

It may also be remarked, that Ptolemy has rendered great services to geography, by collecting all the determinations of the latitudes and longitudes of places then known; and by his laying the foundation of the method of projections, for the construction of geographical charts, which was but little known before his time. In short, the various works which he executed, upon a variety of subjects, are strong proofs of a great and enlightened mind, and will always insure him a distinguished rank in the history of the sciences.

With the labours of this great astronomer ended the glory of the Alexandrian school, which had now subsisted for more than five centuries, with as much credit to itself as advantage to the sciences; but the successors of Hipparchus and Ptolemy, contented themselves with commenting on their works, without adding any thing remarkable to their discoveries. The knowledge of nature which had hitherto been cultivated with so much success, gave way to the desolating irruption of the Saracens, who were led by a ferocious zeal to destroy the celebrated library of Alexandria, which contained so many treasures of learning and genius. By a singular turn, however, of human affairs, this people became afterwards the protectors and cultivators of literature and science, and were then sensible, that this frantic measure had deprived them of the most precious fruits of their victories.

The caliph Almansor first introduced a taste for the sciences into his empire; and his grandson, Almamon, who ascended the throne in 813, was a great encourager and improver of astronomy. Having constructed proper instruments, he made many accurate observations; and, among others, determined the obliquity of the ecliptic to be 23° 35′. Under his auspices also, a degree of the meridian was measured, a second time, in the plains of Singar, on the borders of the Red Sea. About the same time, or at a somewhat later period, Alfraganus likewise wrote a treatise on astronomy; and hence the science began to be greatly cultivated by the Arabians; particularly by Albategnius, who gave a new and improved theory of the Sun, from which he derived results that are much valued for their accuracy; and above all, as they directly confirm the diminution of the eccentricity of the solar orbit, as since demonstrated by the theory of gravity, and by the secular equation of the Moon. His work, entitled "The Science of the Stars," is still extant, and was long

esteemed by the Arabians. But after his time, though the Saracens had many eminent astronomers, several centuries elapsed without producing any very valuable observations, excepting those of some eclipses, observed by Ibn Junis, astronomer to the caliph of Egypt, which serve to shew the acceleration of the mean motion of the Moon.

The Persians, who for a long time were of the same religion, and subjected to the same sovereigns with the Arabs, began about the middle of the eleventh century, to throw off the yoke of the caliphs; and at this period, their calendar received, by the care of their astronomer Omar Cheyam, a new form, founded upon an ingenious intercalation, which consisted in making eight bissextile years at the end of every thirty-three common years. About the same time, also, Holagu Ilecoukan, one of their sovereigns, assembled the most considerable astronomers at Maragha, where he constructed a magnificent observatory, the care of which was confided to Nassir-Eddin. But of all the princes of this nation, the one who distinguished himself the most, by his zeal for astronomy, was Ulugh Beigh, a grandson of the celebrated Tamerlane, who was a great proficient in this science. He formed, from his own observations, at Samarcand, the capital of his empire, a new catalogue of the stars, and the best tables of the Sun and planets that had been given before those of Tycho Brahe. He also determined, in 1437, with a quadrant 180 feet high, the obliquity of the ecliptic, which he found equal to $23^\circ\ 31'\ 57''$.

During this period, the greatest part of Europe was immersed in ignorance and barbarity; which would have probably continued much longer, but for the settlement of the Moors in Spain, who first introduced a taste for literature and the sciences into this part of the world. The Arabs by those means became our instructors, as the Egyptians had been formerly of the Greeks; and, by a singular fatality, the learning which they transmitted to us, has disappeared among this people, as astronomy became neglected in the temples of Egypt, and Chaldæa, in proportion to the progress which it made in the school of Alexandria.

One of the first encouragers of learning in Europe was Frederick II., who, about 1230, set about restoring some decayed universities, and founding a new one at Vienna. He also caused the works of Aristotle and Ptolemy's Almagest, to be translated into Latin: from which latter circumstance we may date the revival of astronomy in Europe. Two years after this, John of Halifax, commonly known by the name of Sacro Bosco, compiled from Ptolemy, Albategnius, Alfraganus, and other Arabic astronomers, his work "De sphæra," which continued in great estimation for more than 300 years afterwards, and was honoured with commentaries by Clavius and other learned men. Alphonsus, king of Castile, may also be reckoned as one of the most zealous encouragers and protectors of this science: though, being but ill seconded by the astronomers of that time, the tables which he published were not found to answer the great expense which attended them.

About the same period also Roger Bacon, an English monk, besides many learned works of various kinds, wrote several treatises on astronomy; after which but little progress was made in this science till the time of Purbach, Regiomontanus, and Walther, who all flourished about the end of the fifteenth century, and by their labours prepared the way for the great discoveries which followed. Regiomontanus, in particular, who was born at Koningsberg, a town of Franconia, in 1436, and whose proper name was John Muller, rendered considerable services to astronomy, not only by his observations and writings, but by his trigonometrical tables of sines and tangents, which he computed to a radius of 1,000,000 for every minute of the quadrant, and by those means greatly facilitated astronomical computations, which had now become both numerous and intricate. John Werner, who succeeded Walther as astronomer at Nuremberg, is also deserving of notice, as being the first who proposed the method of finding the longitude at sea by observing the Moon's distance from the Sun and certain fixed stars, which is now so successfully practised in the British navy.

Next after these was Nicholaus Copernicus, the celebrated restorer of the old Pythagorean system of the world, which had been now set aside ever since the time of Ptolemy. He was born at Thorn, in Polish Prussia, in 1473, and having gone through a regular course of studies at Cracow, and afterwards at Rome, he was made by the interest of his uncle, who was bishop of Wormia, a canon of Frawenberg; in which peaceful retreat, after 36 years of observations and meditations, he established his theory of the motion of the Earth, with such new and demonstrative arguments in its favour, that it has gradually prevailed from that time, and is now universally received by the learned throughout Europe.

This great man, however, had not the satisfaction of witnessing the success of his undertaking, being threatened by the persecution of religious bigots on the one side, and with an obstinate and violent opposition from those who called themselves philosophers on the other; it was not without the greatest solicitations that he could be prevailed upon to give up his papers to his friends, with permission to make them public; but from continued importunities of this kind, he at length complied, and his book, "De Revolutionibus Orbium Cœlestium," after being suppressed for many years, was at length published,

and a copy of it brought to him a few hours before his death. His disciple Rheticus, who has rendered great services to the mathematical sciences by his extensive tables of sines, tangents, and secants, to every ten seconds, was the first who adopted his ideas; but they made but little progress till towards the 17th century.

In this interval, however, the science was not wholly neglected. Nonius in particular wrote several valuable treatises on astronomy and navigation, and invented some useful instruments, more accurate than those before known; one of these being the astronomical quadrant, on which he divided the degrees into minutes, by a number of concentric circles. Apian also, in 1540, wrote a book called the "Cæsarian Astronomy," in which he shews how to observe the places of the stars and planets by the astrolabe; to resolve astronomical problems by means of certain instruments, and to predict and calculate eclipses; and at the end of his work are added observations of five comets, one of which has been supposed to be the same with that described by Hevelius; and whose return was accordingly looked for in the year 1789, but it did not appear. Gemma Frisius, who lived about this time, is likewise deserving of notice, as being the first who recommended time-keepers for finding the longitude at sea.

The history of the science, about this epoch, also offers us a great number of excellent practical astronomers; one of the most illustrious of whom was William IV. landgrave of Hesse-Cassel, who built an observatory in that city, and furnished it with a number of the best instruments that could be obtained at that time, with which he made his own observations. He also attached to himself the celebrated astronomers Rothman, and Justus Byrgius, and with their help formed a catalogue of 400 stars with their latitudes and longitudes, adapted to the beginning of the year 1593. It was also from his pressing solicitations, that Tycho Brahe, one of the greatest observers that ever existed, procured the advantages that he enjoyed under Frederick II. king of Denmark.

This excellent Danish astronomer, who was born at Knudstorp in the county of Schonen, in 1546, began to manifest his taste for this science at the early age of 14. An eclipse of the Sun which happened in 1560, first attracted his attention; and the justness of the calculation which announced this phenomenon, inspired him with a strong desire of understanding the principles upon which it was founded. But meeting with some opposition from his tutor, and a part of his family, to these pursuits, which probably served only to increase his attachment to them, he made a journey into Germany, where he formed connections, and entered into a correspondence with some of the most eminent astronomers of that country; particularly with the landgrave of Hesse, who received him in the most flattering manner, and recommended him to the notice of his sovereign. Becoming by those means better known, on his return to Denmark, Frederick II. gave him the little island of Huen, at the entrance of the Baltic, where he built an observatory, under the name of Uraniburg, and in which, during a course of twenty years, he made a great number of observations.

His tranquillity, however, in this happy retreat, was, at length, interrupted; for soon after the death of Frederick, which happened in 1596, he was deprived, through the aspersions of some envious and malevolent persons, of his pension and establishment, and was not even allowed to follow his pursuits at Copenhagen; a minister of that time, of the name of Walchendorp, having forbidden him to continue his observations. Happily, however, he found a powerful protector in the emperor Rodolphus II., who ordered him to be properly provided for at his own expense, and gave him a commodious house at Prague. After residing in this city till the year 1601, he was taken off by a sudden death, in the midst of his labours, and at an age while he was yet capable of rendering great services to astronomy.

This great man, as is well known, was the inventor of a kind of Semi-Ptolemaic system of astronomy, that was afterwards called by his name, and which he vainly endeavoured to establish instead of the Copernican or true system. But though he was not happy in this respect, he has been of great use to astronomy by his numerous observations and discoveries. Among other things, he was well acquainted with the nature of refractions, and hence he was able to determine the places of a great number of the fixed stars, with an accuracy unknown to former times. He also proved, against the opinion which then prevailed, that the comets are higher than the Moon; and from his observations on this and the rest of the planets, the theories of their motions were afterwards corrected and improved, so that for these services he will always be celebrated and esteemed by astronomers.

Tycho Brahe, in the latter part of his life, had for his disciple and assistant the celebrated Kepler, who was born in 1571, at Wiel, in the duchy of Wirtemberg, and was one of those rare characters that appear in the world only at particular times, to prepare the way for new and important discoveries. Like his master Tycho, he appears to have attached himself to the science at a very early age; and if it be the privilege of genius to change received ideas, and to announce truths which had never before been discovered, he may justly be considered as one of the

greatest men that had yet appeared. Hipparchus, Ptolemy, Tycho Brahe, and even Copernicus himself, were indebted for a great part of their knowledge to the Egyptians, Chaldæans, and Indians, who were their masters in this science; but Kepler, by his own talents and industry, has made discoveries of which no traces are to be found in the annals of antiquity.

The philosopher, the most useful to the sciences, is one who to a profound imagination unites a scrupulous judgment, and though ardently desirous to elevate himself to the cause of the phenomena, is equally apprehensive that he may be mistaken in that which he assigns to them. Kepler owed to nature the first of these advantages, and the second to Tycho Brahe, who perceived his genius, and advised him to abandon his attachment to the mysterious analogies of figures and numbers to which he was then addicted, and to attend more closely to facts and their consequences. This appears to have had its proper effect, and Tycho dying a few years afterwards, Kepler was put in possession of his collection of observations, which he employed to the most useful purposes, having founded upon them three of the most important discoveries that have ever been made in natural philosophy.

It was an opposition of Mars, which determined him to occupy himself, in preference, upon the motion of this planet; and being then strongly attached to the Ptolemaic system as modified by Tycho Brahe, as well as to the opinion which had hitherto been generally received, that all the celestial motions must be perfectly circular and uniform, he endeavoured, for a long time, to represent those of Mars according to this hypothesis. At length, however, after many trials of this kind which he has given in detail, in his treatise called "Stella Martis," he discovered that the orbit of Mars is an ellipsis of which the Sun is placed in one of the foci, and that the planet moves in it in such a manner, that the radius vector, or a line drawn from the centre of the Sun to that of the planet, describes areas proportional to the times. This law he also soon afterwards extended to all the planets; and in 1626, he published, according to this theory, his Rudolphine tables, which will be for ever memorable in astronomy, as being the first that were founded on the true laws of the planetary motions.

It is here worthy of remark, that without the speculations of the Greek mathematicians, upon the curves formed by the sections of a cone, it is highly probable that we should yet have remained ignorant of some of the most curious and important laws of nature. The ellipse being one of these curves, its lengthened figure suggested to the mind of Kepler the idea that the planet Mars, whose orbit he had found to be more oval than circular, might possibly move in it; and soon after, by means of the numerous properties which the ancient geometers had discovered of the conic sections, he assured himself of the truth of this hypothesis. The history of the sciences affords many examples of this kind of application of pure geometry, and of the advantages attending it; for every thing, in the immense chain of truths is connected; and frequently a single observation of apparently trifling consequence, has led to a more intimate knowledge of nature, of which the phenomena are the mathematical results of a small number of invariable laws.

The perception of this truth was probably what first gave rise to the mysterious analogies of the Pythagoreans; and Kepler, who had indulged himself in researches of this kind, was indebted to it for one of his most brilliant discoveries. Being persuaded that the mean distances of the planets from the Sun ought to be conformable to those analogies, he compared them, for a long time, both with the properties of the five regular bodies, and with the notes of music. At length, after seventeen years meditation and calculation, having had the idea of comparing them with the powers of the numbers by which they are expressed, he found that the squares of the times of the revolutions of the planets are to each other as the cubes of their mean distances from the Sun; and that the same law applies equally to their satellites.

Astronomy is likewise indebted to Kepler for several other discoveries; which, though not equal to the former, are still of considerable importance. He believed that it was the attraction of the Moon which caused the flux and reflux of the ocean; and he had so far an insight into the general law of gravitation, as to suspect, that the irregularities of the lunar motions were occasioned by the combined actions of the Earth and the Sun. In his work on Optics, he has also explained the mechanism of vision, which before was unknown; and in another performance, called "Steroemetria Doliorum," he has presented several views on the nature of infinites, which had considerable influence on the revolution that geometry underwent about the end of the last century.

It is afflicting to relate, that this great man, who may be considered as the founder of modern astronomy, had his last days embittered by the horrors of poverty and distress. A small pension, which was scarcely sufficient for his subsistence, was frequently withheld or unpaid; and the trouble and vexation which this occasioned him, obscured his genius, and shortened his existence. He died on the 15th of November 1631, in the fifty-ninth year of his age, leaving nothing for his wife and family, but the glory of his name, and the fame he had so justly acquired; but as these were insufficient to relieve his own

wants, they could afford but little comfort to a helpless wife, and her wretched offspring, whose indigence is said to have been such that they had not even the common necessaries of life.

In the time of Kepler, there were not wanting several other considerable proficients in astronomy. Edward Wright, an Englishman, made several good meridian observations of the Sun, with a quadrant of six feet radius, in the years 1594, 1595, and 1596, from which he improved the theory of the Sun's motion, and computed his declination more accurately than had been done before. He also published, in 1599, an excellent work, entitled, "Certain Errors in Navigation discovered and detected;" containing a new method of projecting maps and charts, which has commonly, though erroneously, been ascribed to Mercator. The science is also greatly indebted to baron Napier of Scotland, not only for his ever memorable invention of logarithms, which has so wonderfully facilitated the business of calculation, but for some excellent theorems and improvements in spherics. About this time, likewise, Bayer, a German, published his "Uranometria," or complete Celestial Atlas, containing the figures of all the constellations visible in Europe; into which he introduced the highly useful invention of marking the stars by their names, or the letters of the Greek alphabet, which renders them so easy to be referred to with distinctness and precision.

At the same time also, that Kepler, in Germany, was tracing the orbits of the planets, and settling the laws of their motions, Galileo (who was born at Pisa, in Italy, in 1564) was meditating upon the doctrine of motion in general, and investigating its principles; and from the admirable discoveries which he made in this branch of the physico-mechanical sciences, Newton and Huygens were afterwards enabled to derive the most brilliant and complete theories of all the planetary motions. About this period also, a fortunate accident produced the most marvellous instrument that human industry and sagacity could have ever hoped to discover; and which, by giving a far greater extension and precision to astronomical observations, shewed many irregularities and new phenomena, which had hitherto remained unknown.

This invention was that of the telescope, which was no sooner known to Galileo than he set himself to improve it; and the discoveries he was by those means enabled to make, were as new as they were surprising. The face of the Moon appeared full of cavities and asperities, resembling vallies and mountains. The Sun, which had generally been considered as a globe of pure fire, was observed to be sullied with a number of dark spots, which appeared on various parts of his surface. A great number of new stars were discovered in every part of the heavens; the planet Jupiter was found to be attended with four moons, which moved round him in the same manner that our Moon moves round the Earth; the phases of Venus appeared like those of the Moon, as had before been concluded by Copernicus from his theory; and, in short, most of the observations he made furnished new proofs of the truth of the Copernican system.

In publishing the discoveries which he had made with his new instrument, Galileo shewed in the most incontestible manner, the annual and diurnal motion of the Earth; which doctrine, however, was thought so alarming, that it was immediately declared heretical by a congregation of cardinals, who were assembled upon the occasion; and its venerable author, one of the most virtuous and enlightened men of his age, was obliged to abjure, upon his knees, and in the most solemn manner, a truth, which nature and his own understanding had shewn him to be incontrovertible. After this, he was condemned to perpetual imprisonment; from which, however, at the end of a year, he was enlarged, by the solicitations of the grand duke; but, that he might not withdraw himself from the power of the inquisition, he was forbidden to quit the territory of Florence, where he died in 1642; carrying with him the regrets of Europe, enlightened by his labours, and their indignation against the odious tribunal which had treated him so unworthily.

The celebrated Harriot also, who has hitherto been known only as an algebraist, made, much about the same time, similar discoveries with those of Galileo, as appears by his papers not yet printed, which are in the possession of the earl of Egremont. Mr. Horrox, likewise, a young astronomer of great merit, about this time, deserves to be mentioned, on account of his observation of the transit of Venus over the Sun's disk on the 24th of November 1639: which event he announced to his friend Mr. Crabtree; and these two together had the singular satisfaction of witnessing, for the first time, a phenomenon which had never before been witnessed by human eyes. Horrox had even formed a new theory of the Moon, which is taken notice of by Newton; but his early death, which happened in the year 1640, put a stop to his useful labours.

The discoveries of Huygens succeeded those of Kepler and Galileo; and few men have, perhaps, merited more of the sciences, by the importance and sublimity of his researches. Among other things, his happy application of the pendulum to clocks, is one of the most advantageous presents that was ever made to astronomy. He was also the first who found that the singular appearances of Saturn are produced by a ring, by which the planet is surrounded; and his assiduity in observing it, led him to the discovery

of one of its satellites. Geometry, mechanics, and optics, are also indebted to him for a great number of discoveries, and if this rare genius had had the idea of combining his theorems on centrifugal forces, with his inquiries into the developement of curves, and the laws of Kepler, he would have taken from Newton his theory of curvilinear motions, and that of gravitation.

Next to Huygens, may be mentioned Hevelius, a burgomaster of Dantzic, who rendered himself highly useful to astronomy by his numerous and immense labours; few observers having ever existed who were more indefatigable. It is to be lamented, however, that he refused to make use of instruments with telescopic sights, an invention introduced about that time by the celebrated Dr. Hook, and which gave a precision to observations unknown to former astronomers. He even contested their utility, and a warm dispute having arisen between him and Dr. Hook upon this subject, Dr. Halley, then a young man rising fast into fame and eminence, was sent to examine his instruments, which were found to be excellent of their kind. The two astronomers made several observations together, much to their satisfaction; and among them was one of an occultation of Jupiter by the Moon, by which they determined the diameter of the latter to be 30′ 33″.

About this epoch, astronomy began to be more generally cultivated and improved, in consequence of the establishment of several learned societies, which, by exciting a spirit of emulation and enterprise among their members, greatly contributed to the advancement of every branch of the mathematical and physical sciences. The chief of these were the Royal Society of London, and that of the Academy of Sciences at Paris; both of which have rendered great services to astronomy, as well by the eminent men they have produced, as by the zeal and ardour by which the science has been cultivated by them. One of the first effects produced by these establishments was the great improvement of telescopes and other instruments, which had hitherto been too much neglected for want of proper encouragement. Huygens constructed a telescope of 123 feet; with which he long observed the Moon and planets, and was the first that discovered Saturn's ring. The celebrated Cassini also employed instruments of this kind, of 200 and 300 feet focus, with which he saw the five satellites of Saturn, with his zones or belts, as well as the shadows of Jupiter's satellites passing over his body.

The length of refracting telescopes, however, was still a great inconvenience, to remedy which, as well as the great aberration of their rays, Mersennus is said first to have started the idea of making telescopes with reflectors instead of lenses, in a letter to Descartes; and in 1663, James Gregory, of Aberdeen, shewed how such an instrument might be constructed. Newton, also, after spending some time on the construction of both these sorts of telescopes, discovered the great inconvenience which arises to refractors, from the different refrangibility of the rays of light, and therefore pursuing the other kind, he presented in the year 1672, to the Royal Society, two reflectors with spherical specula, as he could not then contrive the means of giving them a parabolic figure. It is proper to observe, however, that the defects of refracting telescopes, arising from the different refrangibility of the rays of light, have since been completely obviated by the ingenious Dollond.

Towards the latter part of the seventeenth century, and the beginning of the eighteenth, practical astronomy seems rather to have languished; but at the same time, the theoretical part was carried to the highest degree of perfection, by the immortal Newton in his "Principia," and by the astronomy of David Gregory. About this time also, clock and watch-work was greatly improved by Mr. Graham, who likewise constructed the old eight feet mural arch at the Royal Observatory at Greenwich, and the zenith-sector of twenty-four feet radius, with which Dr. Bradley discovered the aberration of the fixed stars. The reflecting telescope of Gregory and Newton was also greatly improved by Mr. Hadley; but who is still better known for his admirable invention of the reflecting quadrant or sector, now called by his name, and which is universally used at sea, and in all nice observations. Mr. Bird also, about the middle of the eighteenth century, rendered great services to astronomy, by his method of constructing and dividing large astronomical instruments; which has since been carried to the greatest degree of perfection by that admirable artist Mr. John Ramsden, whose recent death will long be regretted by astronomers and men of science in general. Reflecting telescopes were likewise not less improved by Mr. Short, who also first executed the divided object-glass micrometer, which had been proposed and described by M. Louville and others.

Thus the astronomical improvements in the last century have been chiefly owing to the greater perfection of instruments, and to the establishment of regular observatories in various parts of Europe. Roemer, a celebrated Danish astronomer, first made use of a meridian telescope; and by observing the eclipses of Jupiter's satellites was led to his discovery of the motion of light, which he communicated to the Academy of Sciences at Paris, in 1675. Mr. Flamsteed was also appointed the first astsonomer royal at Greenwich, about the same time, where he observed all the celestial phenomena for more than forty-four years; and as the fruits of his labours

published a catalogue of 3000 stars, with their places, to the year 1688, as also new solar tables, and a theory of the Moon according to Horrox. Cassini, also, the first French astronomer royal, greatly distinguished himself by his numerous observations on the Sun, Moon, and planets, and by the improvements he made in the elements of their motions.

In 1719, Mr. Flamsteed was succeeded by Dr. Halley, the friend of Newton, and a man of the first eminence in all the departments of literature and science; who had been sent at the early age of twenty-one, to the island of St. Helena, to observe the southern stars, a catalogue of which he published in 1679; and a few years afterwards he gave to the public, his "Synopsis Astronomiæ Cometicæ," in which he ventured to predict the return of a comet in 1758, or 1759. He was the first who discovered the acceleration of the Moon's mean motion; and is the author of a very ingenious method for finding her parallax, by three observed places of a solar eclipse; he also shewed the use that might be made of the approaching transit of Venus, in 1761, in determining the distance of the Sun from the Earth; and recommended the method of determining the longitude by the Moon's distance from the Sun and certain fixed stars, which has since been carried into execution at the instance of the late astronomer royal. Dr. Halley also composed tables of the Sun, Moon, and planets, with which he compared the observations he made of the Moon, at Greenwich, amounting to near 1500, and noticed the differences. About this time, an attempt was made in France to measure a degree of the earth, which was the occasion of a warm dispute concerning its figure. M. Cassini concluded, from the measurement of Picart, that it was a prolate spheroid; but Newton, from the consideration of the laws of gravity, and the diurnal motion of the Earth, had determined its figure to be that of an oblate spheroid, flattened at the poles, and protuberant at the equator. To determine this point, Louis XV. ordered two degrees of the meridian to be measured, one under or near the equator; and the other as near as possible to the pole, the result of which arduous undertaking was a confirmation of Newton's investigation. Mess. Maupertuis, Clairaut, &c. were employed on the northern expedition; and Condamine, Bouguer, Don Ulloa of Spain, &c. on the southern; who all fulfilled their commissions with great credit to themselves, and advantage to the sciences, making many observations besides those immediately connected with this subject. Among others, it was found, by those who went to the south, that the attraction of the great mountains of Peru had a sensible effect on the plumb-lines of their large instruments, which afforded an experimental proof of the Newtonian doctrine of gravitation, that has since been completely verified by the observations of Dr. Maskelyne, made on the mountain Schehallien in Scotland.

According to the Newtonian theory of attraction, this principle pervades the minutest particles of matter, and the combined action of all the parts of the Earth forms the attraction of the whole. For the same reason, therefore, that a heavy body tends downwards in a perpendicular to the Earth's surface, considered as smooth and even, it must be attracted towards the centre of a neighbouring mountain, by a force proportional to the quantity of matter contained in it; and the effect of this attraction, or the accelerative force produced by it, must depend on the nearness or distance of the mountain from the gravitating body, because this force increases as the squares of the distances decrease. Upon these principles it is obvious, that the plumb-line of a quadrant, or of any other astronomical instrument, must be deflected from its proper situation, by a small quantity, towards the mountain, and the apparant altitudes and zenith distances of the stars, taken with the instrument, be altered accordingly: e. g. if the zenith distance of a star on the meridian were observed at two stations under the same meridian, one on the south side of a mountain, the other on the north; and the plumb-line of the instrument were attracted out of its vertical position by the mountain, the star must appear too much to the north, by the observation at the southern station, and too much to the south by that at the northern; and consequently the difference of the latitudes of the two stations resulting from these observations, would be greater than it really is. If then the true difference of their latitudes be determined by measuring the distance between the two stations on the ground, the excess of the difference, found by the observations of the star above that found by this measurement, must have been produced by the attraction of the mountain; and the half of it will be the effect of such attraction on the plumb-line at each observation, provided that the mountain attracts equally on both sides.

The first hint for determining the quantity of this attraction was suggested by Newton, in his Treatise of the system of the World; and the first experiment for this purpose was conducted by M. Bouguer, and M. de la Condamine, in the year 1738. Whilst they were employed in measuring three degrees of the meridian, near Quito in Peru, they endeavoured to ascertain the effect of the attraction of Chimboraco, a mountain in that neighbourhood, which, by a rough computation, they supposed to be equal to about the 2000th part of the attraction of the whole Earth. By observing the altitudes of fixed stars at two stations, one on the south side of the mountain, and the other on the north, they found the quantity of $7\frac{1}{2}''$ in

favour of the attraction of the mountain by a mean of their observations; whereas the plumb-line, according to the theory, should have declined from the true vertical line 1′ 43″. However, though the general result is favourable to the Newtonian doctrine, the experiment was performed under so many disadvantages, as not to afford the satisfaction which was to be wished; and M. Bouguer terminates his account of their observations, with expressing his hopes, that the experiment might be repeated under more favourable circumstances, either in France or in England. Bouguer, Figure de la Terre.

Nothing was afterwards done, till Dr. Maskelyne, the late astronomer royal, made a proposal to the Royal Society for this purpose, in the year 1772; and in 1774, he was deputed to make the trial, accompanied with proper assistants, and furnished with the most accurate instruments. He made choice of the mountain Schehallien, in Scotland, for the scene of his operations, the direction of which is nearly from east to west, its mean height above the surrounding valley about 2000 feet, and its highest part above the level of the sea 3550 feet. Two stations for observations were selected, one on the north, and the other on the south side of the mountain. Every circumstance that could contribute to the accuracy of the experiment was regarded; and from the observation of ten stars near the zenith, Dr. Maskelyne found the apparent difference of the altitudes of the two stations to be 54.6′; and from a measurement by triangles, formed from two bases on different sides of the mountain, he found the difference of their parallels to be 4364 feet, which, in the latitude of Schehallien, viz. 56° 40′, answers to an arch of the meridian of 43″, which is less by 11.6″ than that found by the sector. Its half, therefore, or 5.8″ is the mean effect of the attraction of the mountain. From this experiment, conducted with great assiduity and accuracy, and tending to the establishment of the Newtonian theory, Maskelyne infers, that the mountain Schehallien exerts a sensible attraction; and, therefore, that every mountain, and every particle of the Earth, is endued with the same property, in proportion to its quantity of matter. The law of the variation of this force, in the inverse ratio of the squares of the distances, is likewise confirmed by it; for if the force of the attraction of the hill had been only to that of the Earth as the matter in the hill to that of the Earth, and had not been greatly increased by the near approach to its centre, the attraction must have been wholly insensible. He infers also, that the mean density of the Earth is at least double of that at the surface; and consequently, that the density of the internal parts of the Earth is much greater than that of those near the surface; also that the whole quantity of matter in the Earth will be at least twice as great as if it were composed of matter of the same density with that at the surface; and therefore that the hopothesis of those naturalists, who suppose the Earth to be only a great hollow shell of matter, is groundless. And finally, that the sensible deflections in the plumb-lines of astronomical instruments, by the density of the superficial parts of the Earth, must cause apparent inequalities in the mensuration of degrees in the meridian. He candidly acknowledges, however, that a single experiment is not sufficient to ascertain a matter of such importance, and recommends other experiments of a similar kind to be repeated in various places, and attended with different circumstances; since Schehallien may differ in its internal constitution from other mountains, as there is no appearance of its ever having been a volcano, which is the case of many others. Phil. Trans. vol. lxv. part 2. N° 48 and 49.

On the death of Dr. Halley, in 1742, he was succeeded by Dr. Bradley, who has rendered himself highly celebrated by two of the finest discoveries that have ever been made in astronomy, the aberration of light and the nutation of the Earth's axis. Among other things, he also formed new and accurate tables of the motions of Jupiter's satellites, as well as the most correct table of refractions yet extant. Also, with a large transit instrument, and a new mural quadrant of eight feet radius, constructed by Bird, in 1750, he made an immense number of observations, for settling the places of all the stars in the British catalogue, together with near 150 places of the Moon, the greater part of which he compared with Mayer's tables.

Dr. Bradley was succeeded in 1762, in his office of astronomer royal, by Mr. Bliss, but who, being in a declining state of health, died in 1765, and was succeeded by Nevil Maskelyne, D.D. the late astronomer royal, who has rendered considerable services to this science, by his publication of the "Nautical Almanac," the "Requisite Tables," &c. and more particularly by the great assiduity and zeal he has displayed in bringing the lunar method of determining the longitude at sea into general practice.

In the mean time, many other eminent mathematicians, both of our own, and other countries, were assiduously employed in endeavouring to promote the science of astronomy. The theory of the Moon was particularly considered by Mess. Clairaut, d'Alembert, Euler, Simpson, Walmsley, and Mayer; the latter of whom computed a set of lunar tables, for which, on account of their superior accuracy, he was rewarded with a premium of 3000*l.* by the Board of Longitude, who brought them into use in the computation of the nautical ephemeris which was published by their order. Some very accurate tables of the satellites of Jupiter, were also composed from

observations by Mr. Wargentin, an excellent Swedish astronomer, and which have since been corrected by the author, so as to render them superior to any then published.

Among the French astronomers who have also contributed to the advancement of this science, we are particularly indebted to M. de la Caille for an excellent set of solar tables, in which he has made allowances for the attractions of Jupiter, Venus, and the Moon, as well as for the observations which he made at the Cape of Good Hope, in concert with the most celebrated astronomers in Europe, in order to determine the parallax of the Sun, Moon, and the planet Mars; and for adjusting the places of the stars in the southern hemisphere, which he has done with great accuracy. In Italy also the science was cultivated with great success by S. Bianchini, Boscovich, Frisi, Manfredi, Zanotti, and others; and in Germany, by Euler, Mayer, Lambert, &c.

Such was the state of astronomy when Dr. Herschel, by augmenting the powers of telescopes beyond the most sanguine expectations, opened a scene altogether unlooked for. By this indefatigable observer we are made acquainted with a new primary planet belonging to our system, called the Georgium Sidus, attended by six satellites, which he discovered on the 13th of March, 1781, and which being at twice the distance of Saturn from the Sun, has doubled the bounds formerly assigned to the solar system. We are also indebted to him for a variety of observations on several other interesting astronomical subjects; such as the discovery of two additional satellites to Saturn, of which the number is now seven,; a new method of measuring the lunar mountains; the rotation of the planets on their axes; on the parallax of the fixed stars; catalogues of double, treble, stars, &c.; of nebulæ; and of the proper motion of the Sun and solar system; the accounts of which, together with many other valuable papers, he has communicated from time to time in different parts of the Philosophical Transactions.

It is with great pleasure we observe that at no former period has this science been cultivated with more ardour than it is at present, both in this and every other country in Europe. In France, the physico-mathematical part of the science has been greatly improved and extended by the celebrated M. la Place, who, in his elaborate work, the "Mechanique Celeste," has investigated all the phenomena, which the attraction or universal gravitation of matter could produce on the forms and motions of the celestial bodies, by their mutual actions on each other. M. Lalande, the patriarch of astronomers, is also still indefatigable in his pursuits, and by the zeal he constantly manifests for the interests of this science, has greatly promoted the study of it in almost every quarter of the globe; but particularly in Germany, where M. von Zach has been equally assiduous in forwarding its improvement. In all its collateral branches also we observe a degree of activity that has never been exceeded. New admeasurements of the Earth have been undertaken both in this country and in France, which, from the great improvements of instruments, and the skill and industry of the observers, promise a greater accuracy in the results than could have been obtained by those who were formerly engaged in this undertaking. From the zeal and abilities of major Mudge, in particular, who has been employed by our government to make a trigonometrical survey of the country, we have obtained the most accurate details on this subject that have ever yet been presented to the public.

In the year 1801, Piazzi, an Italian astronomer, discovered the small planet Ceres.

In 1802, Olbers, a German astronomer, discovered Pallas, another very small planet.

In 1804, Harding, a german astronomer, discovered Juno, a small planet, but somewhat larger than the two former.

In 1807, Dr. Olbers discovered another small planet called Vesta.

These four small planets are situated between Mars and Jupiter.

Explanation of the Astronomical Tables.

Introduction.—If the planets moved uniformly in circular orbits about the Sun, in the centre of each orbit; then their angular motion, about the Sun's centre, would always be proportional to the time of description; and if their periodic time were once accurately determined, and likewise their places at a given instant of time exactly ascertained; then we could find their exact places, for any other given time past or future, by the common rule of proportion and addition or subtraction. Thus, for instance; the Earth moves round the Sun in 365*d*. 6*h*. 9*m*. 12.3*s*. and on the 1st day of January 1800, at noon at Greenwich, the heliocentric longitude of the Earth was found by observation to be 3*s*. 10° 53′ 49.8″; wherefore the heliocentric longitude of the Earth, were its motion uniform and the equinoctial points at rest,

for any other time, suppose 10,000 days afterwards, would be found by saying, as 365*d*. 6*h*. 9*m*. 12.3*s*. : 10,000*d*. :: 12*s*. (360°) : the signs, degrees, minutes, and seconds of the orbit described in 10,000 days; and this quantity, (after rejecting therefrom, every 12 signs contained in it,) being added to 3*s*. 10° 53′ 49.8″, would determine the true heliocentric longitude of the Earth at the given time. But by reason of the retrograde motion (procession) of the equinoctial points, the Earth completes its motion to the same equinoctial point in 365*d*. 5*h*. 48*m*. 48.2*s*. called the tropical or common year; wherefore this number being taken instead of the other, in the analogy, we shall then obtain the mean motion for the given time, corresponding to the Earth's return to the same equinoctial point; hence, as 365*d*. 5*h*. 48*m*. 48.2*s*. : 1*d*. :: 360° : 0° 59′ 8.33″, which is the Earth's (or Sun's) mean daily motion in the ecliptic; and by taking the multiples of the daily motion, we have the motion for any number of days we desire; and consequently we can find the Earth's mean place as seen from the Sun, that is, its mean heliocentric longitude, for any given time. And by adding to, or subtracting from this, 6 signs, we obtain the Sun's mean longitude as seen from the Earth. And by this means, the tables of the Sun's mean longitude are constructed for years, months, days, &c. past or future. And in like manner for the planets.

But because the motion of the planets in their orbits is not uniform; for sometimes it is more rapid than the mean motion, and at other times slower; hence we construct tables to shew the difference between the *mean* and *true* place of a planet, and this difference, (which commonly is made up of several parts,) being applied to the *mean* place, gives the *true* place. Now to find the difference between the *mean* and *true* place of a planet, or rather the several parts, called equations, of which it is made up, involves the most delicate operations of practical astronomy; and the construction of the tables for that purpose, requires a profound knowledge of most branches of the mathematics, united to the most correct acquaintance with the Newtonian principles of natural philosophy; and the following tables, which we are now about to explain, have been constructed upon these principles, and from a mean of several thousand observations.

On the Tables of the Sun.

These tables are partly taken from those constructed by M. de Lambre, from the observations of our late astronomer royal Dr. Maskelyne, and the theory of M. Laplace. We have made alterations in some of them, by which we have abridged the equations of the radius vector; we have also rejected some altogether, particularly those for finding the equations of time, because they are unnecessary; and have made Table I. of the Epochs commence with the year 1800, instead of 1750 as in M. de Lambre's. Moreover, we have omitted every table and matter relative to the Sun's latitude, for this reason, that the maximum of his latitude is a quantity so small that it is almost impossible to determine it by observation, or at least it was formerly so, with the old instruments made use of by Dr. Maskelyne: but we hope that our present astronomer royal, being furnished with a new and more accurate set of instruments, will furnish the public with such accurate observations as shall enable astronomers, not only to settle the maximum of the Sun's latitude, but likewise, to correct all our present solar, lunar, and planetary tables, and thereby to bring them still nearer to a state of perfection.

Of Table I.

This table contains the epoch of the mean longitude of the Sun, and the mean longitude of the perigee, both reckoned from the mean equinox: together with the epochs of the arguments of the small inequalities affecting the Sun's longitude, arising from the disturbing force of the Moon and planets. These epochs are fitted to the noon of Greenwich mean time of the first day of January for each year. They commence with the year 1800 and end with 1900, making a period of 101 years. The column M, which is the argument of the correction of A, and for one part of the lunar equation, is the Moon's mean anomaly in common numbers; the whole circumference being supposed in this case, and in the subsequent columns, to be divided into 1000 equal parts, instead of degrees; so that 10 parts answer to 3° 36′, and so in proportion. Thus, the mean anomaly of the Moon for the beginning of 1800 was 4*s*. 3° 23.6′; hence 3° 36′ : 4*s*. 3° 26.6′ :: 10 : 342, the value of M for 1800. And note, that the secular variation for each year is to be added to the epochs of the longitude and perigee for that year respectively.

The argument A is the mean longitude of the Moon *minus* that of the Sun, diminished by 22; and Table VI. is to correct the argument A, which must be taken out with a double entry. The maximum of the correction is 22, which is equal to 8° nearly, the greatest amount of the inequalities of the Sun and Moon; and to render this correction additive, every number in it has been increased by 22, and the same deducted from the several epochs; so that when the tabular correction is added to A, we have that argument reduced back again and corrected.

The argument B is the heliocentric longitude of the Earth; C, that of Venus; D, that of Mars; E, that of Jupiter; and F, that of Saturn. These arguments serve to find the effects of the disturbing forces of

the planets, upon the Sun's longitude. N is the supplement of the longitude of the Moon's node, and serves to find the effects of the Moon's disturbing force, on account of the spheroidal figure of the Earth, on the Sun's longitude, right ascension, and the obliquity of the ecliptic; or what is the same thing, N discovers the effects of the nutation of the Earth's axis, caused by the influence of the miniscus of the Earth gravitating towards the Moon. And in all these arguments, the whole circumference is supposed to be divided into 1000 equal parts.

Of Table II.

This table is to extend the epochs of Table I, for 26 centuries before the beginning of the 19th century, and for 20 centuries after. Those years with the sign — prefixed are for past centuries, and those with + for future ones. The quantities both for past and future centuries, are all *additive* to their corresponding ones for the 19th century; this we accomplish for past centuries, by taking the complements of the sexagessimal epochs, to 12 signs, and the complements of the decimal ones to 1000: and those for future times are manifestly *additive* without change. The G and I, in the column of years, signify Gregorian and Julian style. The quantities are put down for the first three centuries, before the 19th, for the Gregorian style; but for the 3d century, they are put down for both styles, and for all years before that century, they are put down for the Julian; and so they are put down for the Gregorian style, for all future time. The supplements are for extending the epochs as far as 10,000 years before or after the beginning and end of the 19th century.

The use of Table II is obvious, for we have only to *add* the numbers therein, to those of a correspondent year of the 19th century, and we have the epochs for that year; thus, suppose we wanted the epochs for 1225; then this year is 600 years before 1825; therefore we have only to add the number for — 600 years to those for 1825, and we have the epochs sought. This is too simple to require the operation.

We have now only to remark, that this Table II can be of little use in astronomy, except it be to compare ancient observations with those of the present time; in matters of chronology, this table may become important; because we are sure with all the errors of ancient and modern astronomers, taking the maximum, that in 10,000 years these tables cannot err more than a small part of one day; which is a matter of the last importance in chronology.

Of Table III.

This Table is to find the secular variation or equation of the longitude, perigee, and greatest equation of the centre, for centuries before and after the 19th; for the 19th century, the secular variation is inserted in Table I for every year, both for the longitude and perigee, and these are to be added to their respective epochs after which they stand, for that century only; but for other centuries, the secular variation must be taken from Table III, and those small numbers in Table I omitted. The secular variation of the greatest equation of the centre is too small a quantity to be noticed for the 19th century. And the secular variation or diminution of the obliquity of the ecliptic appears at present to be 50.0″ for 100 years, that is half a second annually; and as the mean obliquity for 1800, was 23° 27′ 57″, by observation, hence it is easy to find the mean obliquity for any year before or after 1800.

Of Table IV.

This Table contains the mean motion for every day in the year, and it is too simple to require any explanation.

Of Table V.

This Table contains the mean motion for hours, minutes, and seconds of time, and like the former is quite simple in its use.

Of Table VI.

The use of this Table has been explained already, in treating of Table I.

Of Table VII.

This Table gives the equation of the centre for 1810, with the secular variation, arising from the supposed uniform diminution of the eccentricity of the Earth's orbit; and Table III contains the numbers necessary for correcting this supposed uniformity, for past and future centuries. Table VII therefore contains the principal difference between the Sun's *mean* and true longitude; and the principles on which it is constructed we have formerly explained. When the equation is subtractive, its complement to 12 signs is taken instead of the equation itself, by which means the equation is additive in every part. And 45″ have been subtracted from every number in it; 45″ being the aggregate of all the numbers added to all the other small equations, to render them additive: so that by this arrangement, all the equations are rendered additive except the secular variation of the equation of the centre, and the nutation. And observe, that the secular

variation is for 100 years, beginning with 1810; and for past time the sign of the variation must be changed.

Of TABLE VIII.

This Table contains all the equations depending on the argument N, or supplement of the longitude of the Moon's ascending node. And Table VIII continued, contains the solar nutation arising from the action of the Sun on the spheroidal figure of the Earth.

Of TABLE IX.

This Table contains the mean motion of the Sun in right ascension, for sidereal hours, minutes, and seconds. This Table is only useful in the reduction of observations taken by a clock shewing sidereal time, to hours, minutes, and seconds, of mean solar time.

Of TABLE X.

This Table and its continuation contain the lunar equation in longitude. The first part depends on the argument A. Part the second is found by entering with the Sun's mean anomaly at the side, and part the first at the top. Part the third is found by the double arguments M and A. From the sum of these three equations we must subtract the constant quantity 0.5″, and the remainder is the whole lunar equation.

Of TABLE XI.

This Table contains the correction in longitude, arising from the disturbing force of Venus, found with the arguments B and C.

Of TABLE XII.

This Table gives the correction in longitude, in the same manner, for Mars, with the arguments B and D.

Of TABLE XIII.

This Table, in like manner, gives that for Jupiter, with the arguments B and E.

Of TABLE XIV.

This Table gives the small correction in longitude, arising from the disturbing force of Saturn, found with the arguments B and F.

And to render each of these five minor equations additive, each has been increased by a quantity of seconds, either equal to, or a small matter greater than its maximum; and the sum of all the quantities added being 45″, the same has been deducted from the equation of the centre. So that by this means, the epochs of the Sun's mean longitude are not changed, on account of the equations of longitude. But 20″ have been deducted from all the epochs, on account of the constant part of the abberration of light. And when we are to calculate the places of the planets, we add these 20″ and 6*s*; that is, 6*s*. 0° 0′ 20″, to the Sun's true longitude, in order to get the heliocentric longitude of the Earth.

Of TABLE XV.

This Table gives the variable part of the solar aberration, arising from the unequal velocity of the Earth's motion round the Sun. The argument is the Sun's mean anomaly. But the maximum of this correction is so small, that the computers of the Nautical Almanac never attend to it.

Of TABLE XVI.

This Table and its continuation, shew the Sun's semidiameter, horary motion in longitude, and horizontal parallax. The argument is the Sun's mean anomaly.

Of TABLE XVII.

This Table and its continuation, shew the refraction of the celestial bodies, corresponding to their apparent altitude. Part the first gives the correction of the Sun's apparent altitude, which is the mean refraction diminished by his parallax in altitude. Part the second gives the mean refraction, or the correction of the apparent altitude of a fixed star. Part the third contains the correction of the mean refraction, agreeably to the several heights of the Mercury in the thermometer and barometer, above or below the mean state, when the heat and density of the atmosphere are at their middle state, or mean degree of temperature and weight. This correction must be applied to the numbers in part first and second, when the highest degree of astronomical accuracy becomes necessary; otherwise a sensible error may arise, particularly in low altitudes. The arguments for the thermometrical part are the height of the thermometer at the top of the table and the apparent altitude at the left side, and the proper sign is at the top of each column. The argument for the barometrical part is the height of the barometer at the foot of the table, and the proper sign is at the foot of each column. These two parts of the correction are to be added together when they have both the same sign; but their difference must be taken when they have contrary signs: and the sum or difference will be the correction, retaining the sign of the greater in both cases; and this correction, being applied according to its sign to the numbers in the first or second part of the table, will give them truly corrected.

Of Table XVIII.

This Table contains the mean radius vector, in natural numbers, to the mean distance, unity (1), diminished by 0.000100, in order to render the equations of the perturbations produced by the Moon and planets, additive, with the secular variation. The Table is fitted to 1810; and the secular variation is for 100 years, as we have observed at the foot of the Table. The argument is the Sun's mean anomaly; and when the equations are added, we have the *true* radius vector in natural numbers, of which we are to take the logarithm, from any common table of logarithms, in order to prepare it for future calculations.

Of Table XIX.

This Table contains the equations of the radius vector, produced by the disturbing forces of the Moon and planets. The arguments are the same as those for the minor equations of longitude. The sum of all the numbers added to the several equations, is 100, and this is taken from the mean radius, by which means all the minor equations become additive, and consequently the deducted sum of 100 is again added, as soon as the sum of all the equations is added to the mean radius.

Scholium.

We have, in the former part of this work, given the necessary precepts for the use of the Solar Tables, and amply illustrated the same, in several examples, in calculating the Sun's place; with which, the student, we conclude, is already acquainted. We have also illustrated the theory of the construction of most of the principal tables; particularly the tables of the mean motion, the equation of the centre, the radius vector, &c. In regard to the minor equations, arising from the Earth's gravitating to the Moon and planets, they are computed from a great number of observations taken for upwards of 50 years, at Greenwich, from the theory of M. Laplace, to which we refer the student; as the limits of our work are not sufficiently extensive to enable us to enter into every minute part of the theory.

As a material alteration has now taken place in the construction and use of most astronomical tables, by which almost all the equations or corrections are rendered *additive*, it may be proper to say a few words in explanation of this change. Since all the motions of the Earth and planets (except that round their axes), are unequal, being at one time greater, and at another time less, than the mean motion; it is evident, that those tables, which shew the difference between the mean and true motion, must at one time be additive and at another time subtractive; so that, in the use of such tables formerly the operator had to *add* or *subtract* the equation or correction, as the case might require and as the table commonly directed. But in these new Solar and Lunar Tables, all the equations are rendered additive, and a very little consideration will shew how this may, in general, be effected in an easy way: for if we assume a number either exactly equal to or greater than the maximum of the equation which we intend to render additive in every part, and then to this assumed number, add all the numbers of the equation, having the sign +, and subtract all those with the sign —, the equation will be rendered additive thereby; but the assumed number must be deducted at last, either from the final sum, or by some judicious arrangement, as we have done in the calculations of the Sun's place, where the whole amount of all the small numbers added to the minor equations, is subtracted from the equation of the centre, and that equation is all rendered additive by taking the complement of the negative numbers to 12 signs; so that in the end of the calculation, the aggregate of all the numbers added, is subtracted and consequently each equation is thereby applied with its proper sign.

Explanation of the Tables of the Moon.

Of Table I, *and its continuation.*

This Table is for the purpose of finding the epochs for centuries *before* and *after* the 19th century. Thus, suppose we wanted the epochs for the year 1225; then this year being 600 years before 1825, we take out the epochs for 1825, from table II, and those for — 600, from table I, and then add both together.

Of Table II.

This Table contains the epochs of all the arguments, which find the inequalities or equations of the Moon's longitude, latitude, and place of the ascending nodes, together with the epochs of her longitude, latitude, and place of the node, for the 19th century. The epochs are fitted to the noon of the 1st of January, mean time, for each year, at Green-

wich. In all the 32 arguments of longitude, except the 32d, the *decimal* division of the whole circumference is substituted instead of the *sexagessimal*; and the like for those of latitude, except the 1st, 2d, and 3d: by this arrangement, the operations are rendered more simple than when the sexagessimal division was made use of. By an inspection of the table, the operator will at once discover the number of parts, into which the whole circumference is divided: and after the addition, if the sum be equal to or exceed a circumference, or in the sexagessimals 12*s*, then the excess only is retained.

To render all the equations of longitude *additive*, 9° 45′ 0″, have been subtracted from each of the epochs of the mean longitude; because 9° 45′ 0″ is the aggregate or sum of all the quantities added to all the several equations of longitude for this purpose. Consequently when the sum of all the equations thus increased, is added to the epochs of longitude, the quantity subtracted from them is again restored, and the longitude *equated*. In like manner the sum of all the quantities added to the eleven minor equations of latitude, is subtracted from the distance of the Moon from the north pole of the ecliptic; so that the sum of these equations thus increased, being added to the polar distance, the quantity subtracted is again added, and the polar distance *equated*. And in like manner, for the Moon's horizontal parallax, &c.

These Tables of the Moon were originally constructed by M. Burckhardt, and published at Paris, in 1812. As soon as we received a copy of them, we compared them with M. Burg's, as re-published at Cambridge, in 1808, by Mr. Vince; and the superior excellence of M. Burckhardt's in every point of view, appeared so evident, that we immediately adopted them with a slight modification. In M. Burckhardt's Tables, the epochs were fitted to the midnight of the meridian of Paris; but we have fitted the epochs of ours to the meridian of Greenwich, and to noon, agreeably to the established method of all British astronomers, and in conformity to the calculation and use of the Nautical Almanac. The difference of the results, in the Moon's longitude and latitude, between M. Burg's Tables and ours, is not very considerable, the maximum seldom exceeding 3″; but the time spent in the operation is, at least in the one case, treble that in the other. And in respect to accuracy, we have compared the results of both sets of tables with a few of the last observations taken under Dr. Maskelyne; and upon the whole, have found *our tables* to come nearer to the truth three times out of four, both in the Moon's latitude and longitude.

In our Practical Astronomy, we have given the necessary precepts for the use of the Lunar Tables, and fully exemplified the same in the calculation of the Moon's longitude, latitude, horizontal parallax, and semidiameter; to which we refer the student and computer. And we feel confident, that computers of the Moon's place will find these Lunar Tables of the greatest value to them. By these tables they are relieved from the tedious operation of finding the arguments of the Moon's longitude and latitude, by troublesome and tedious additions, subtractions, and multiplications of sexagessimals. Moreover these tables make no use of the Sun's true longitude, which was a previous and an essential part of the *data*, in the use of Mason's or Burg's Tables. The arguments, in the use of our tables, are all found by simple additions; and the proportional parts of the several equations are almost every where found by common multiplication: upon the whole, the Moon's longitude, latitude, and horizontal parallax, can be calculated, in less than half the time that must be employed in the formation of the arguments, according to Burg's Tables and method.

TABLES OF THE SUN.

TABLE I.

EPOCHS of the *Mean Longitudes* of the Sun, and of his *Perigee*, with the *Arguments*, which govern the *Inequalities*.

The EPOCHS are calculated for *January*, at Noon, Mean Time.

YEARS.	Mean Longitude of the Sun.	Sec. Vari.	Longitude of Sun's Perigee.	Sec. Vari.	M	A	B	C	D	E	F	N
	S. D. M. S.	S.	S. D. M. S.	S.								
1800 C.	9 10 53 29.8	0.3	9 9 29 3	0	341	167	280	405	646	227	342	908
1801	9 10 39 10.2	0.3	9 9 30 5	0	588	527	279	30	177	312	376	961
1802	9 10 24 50.6	0.3	9 9 31 7	0	834	887	279	654	709	396	410	15
1803	9 10 10 30.9	0.4	9 9 32 9	0	81	247	278	278	240	480	444	69
1804 B.	9 9 56 11.3	0.4	9 9 33 11	0	327	607	277	903	771	565	478	123
1805	9 10 41 0.0	0.4	9 9 34 13	0	610	1	279	532	304	649	512	176
1806	9 10 26 40.4	0.4	9 9 35 15	0	856	361	279	156	836	733	546	230
1807	9 10 12 20.7	0.5	9 9 36 17	0	103	721	278	781	367	818	580	284
1808 B.	9 9 58 1.1	0.5	9 9 37 18	0	349	81	277	405	898	902	614	337
1809	9 10 42 49.8	0.5	9 9 38 20	0	632	475	279	34	431	986	648	391
1810	9 10 28 30.2	0.6	9 9 39 22	0	878	835	279	658	963	71	682	445
1811	9 10 14 10.5	0.6	9 9 40 24	1	125	195	278	283	494	155	716	498
1812 B.	9 9 59 50.9	0.6	9 9 41 26	1	371	555	277	907	26	239	750	552
1813	9 10 44 39.6	0.6	9 9 42 28	1	654	949	279	536	559	324	784	606
1814	9 10 30 20.0	0.7	9 9 43 30	1	900	309	279	160	90	408	818	660
1815	9 10 16 0.3	0.7	9 9 44 32	1	147	669	278	785	621	492	852	713
1816 B.	9 10 1 40.7	0.7	9 9 45 34	1	394	29	277	409	153	577	886	767
1817	9 10 46 29.4	0.7	9 9 46 36	1	676	423	280	38	685	661	920	821
1818	9 10 32 9.8	0.8	9 9 47 38	1	922	783	279	663	217	745	954	875
1819	9 10 17 50.1	0.8	9 9 48 39	1	169	143	278	287	748	830	988	928
1820 B.	9 10 3 30.5	0.8	9 9 49 41	1	416	503	278	911	280	914	22	982
1821	9 10 48 19.2	0.8	9 9 50 43	1	698	898	280	540	813	998	56	36
1822	9 10 33 59.6	0.8	9 9 51 45	1	945	258	279	165	344	83	90	90
1823	9 10 19 39.9	0.9	9 9 52 47	1	191	618	278	789	876	167	124	143
1824 B.	9 10 5 20.3	0.9	9 9 53 49	1	438	978	278	414	407	252	158	197
1825	9 10 50 9.0	0.9	9 9 54 51	1	720	372	280	43	940	336	192	251
1826	9 10 35 49.4	0.9	9 9 55 52	1	967	732	279	667	471	420	226	304
1827	9 10 21 29.7	1.0	9 9 56 54	1	213	92	278	291	3	505	260	358
1828 B.	9 10 7 10.1	1.0	9 9 57 56	1	460	452	278	916	534	589	294	412
1829	9 10 51 58.8	1.0	9 9 58 58	1	742	846	280	545	67	673	328	466
1830	9 10 37 39.2	1.0	9 10 0 0	1	989	206	279	169	598	758	362	519

Add 22" to its Long.

TABLE I.

EPOCHS of the *Mean Longitudes* of the Sun, and of his *Perigee*, with the *Arguments* which govern the *Inequalities*.

YEARS.	Mean Longitude of the Sun.	Sec. Vari.	Longitude of Sun's Perigee.	Sec. Vari.	M	A	B	C	D	E	F	N
	S. D. M. S.	S.	S. D. M. S.	S.								
1831	9 10 23 19.5	1.0	9 10 1 2	2	235	566	278	793	130	842	396	573
1832 B.	9 10 8 59.9	1.1	9 10 2 4	2	482	926	278	418	661	926	430	627
1833	9 10 53 48.6	1.1	9 10 3 6	2	764	320	280	47	194	11	464	681
1834	9 10 39 29.0	1.1	9 10 4 8	2	11	680	279	671	725	95	498	734
1835	9 10 25 9.3	1.1	9 10 5 10	2	257	40	279	296	256	179	532	788
1836 B.	9 10 10 49.7	1.1	9 10 6 12	2	504	400	278	920	788	264	566	842
1837	9 10 55 38.4	1.2	9 10 7 13	2	787	794	280	549	321	348	600	895
1838	9 10 41 18.8	1.2	9 10 8 15	2	33	154	279	173	852	432	634	949
1839	9 10 26 59.1	1.2	9 10 9 17	2	279	514	279	798	383	517	668	3
1840 B.	9 10 12 39.5	1.2	9 10 10 19	2	526	874	278	422	915	601	702	56
1841	9 10 57 28.2	1.2	9 10 11 21	2	809	268	280	51	447	685	736	110
1842	9 10 43 8.6	1.3	9 10 12 23	2	55	628	279	676	979	770	770	164
1843	9 10 28 48.9	1.3	9 10 13 25	2	301	988	279	300	510	854	804	218
1844 B.	9 10 14 29.3	1.3	9 10 14 27	2	548	348	278	924	41	938	838	272
1845	9 10 59 18.0	1.3	9 10 15 29	2	831	742	280	553	574	23	872	325
1846	9 10 44 58.4	1.3	9 10 16 31	2	77	102	280	177	106	107	906	379
1847	9 10 30 38.7	1.3	9 10 17 33	2	324	462	279	802	637	191	940	433
1848 B.	9 10 16 19.1	1.4	9 10 18 35	2	570	822	278	427	168	276	974	487
1849	9 11 1 7.8	1.4	9 10 19 37	2	853	216	280	55	700	360	8	540
1850	9 10 46 48.2	1.4	9 10 20 38	2	99	576	280	680	231	444	41	594
1851	9 10 32 28.5	1.4	9 10 21 40	3	348	936	279	304	762	529	75	648
1852 B.	9 10 18 8.9	1.4	9 10 22 42	3	592	297	278	929	294	613	109	701
1853	9 11 2 57.6	1.5	9 10 23 44	3	875	691	280	557	827	697	143	755
1854	9 10 48 38.0	1.5	9 10 24 46	3	121	51	280	182	358	782	177	809
1855	9 10 34 18.3	1.5	9 10 25 48	3	368	411	279	806	889	866	211	863
1856 B.	9 10 19 58.7	1.5	9 10 26 50	3	614	771	279	430	421	950	245	916
1857	9 11 4 47.4	1.5	9 10 27 52	3	897	165	281	60	953	35	279	970
1858	9 10 50 27.8	1.6	9 10 28 54	3	144	525	280	684	485	119	313	24
1859	9 10 36 8.2	1.6	9 10 29 56	3	390	885	279	308	16	203	347	78
1860 B.	9 10 21 48.5	1.6	9 10 30 57	3	636	245	279	933	547	288	381	131
1861	9 11 6 37.2	1.6	9 10 31 59	3	919	639	281	562	80	372	415	185
1862	9 10 52 17.6	1.6	9 10 33 1	3	166	999	280	186	612	456	449	239
1863	9 10 37 58.0	1.7	9 10 34 3	3	412	359	280	810	143	541	483	292
1864 B.	9 10 23 38.3	1.7	9 10 35 5	3	659	719	279	435	674	625	517	346
1865	9 11 8 27.0	1.7	9 10 36 7	3	941	113	281	64	207	709	551	400
1866	9 10 54 7.4	1.7	9 10 37 9	3	188	473	280	688	738	794	585	453
1867	9 10 39 47.8	1.8	9 10 38 11	3	434	833	280	313	270	878	619	507
1868 B.	9 10 25 28.1	1.8	9 10 39 13	3	681	193	279	937	801	962	653	561
1869	9 11 10 16.8	1.8	9 10 40 15	3	963	587	281	566	334	47	687	615
1870	9 10 55 57.2	1.8	9 10 41 17	3	210	947	280	190	865	131	721	668

TABLE I.

Epochs of the *Mean Longitudes* of the Sun, and of his *Perigee*, with the *Arguments* which govern the *Inequalities*.

Years.	Mean Longitude of the Sun.	Sec. Vari.	Longitude of Sun's Perigee.	Sec. Vari.	M	A	B	C	D	E	F	N
	S. D. M. S.	S.	S. D. M. S.	S.								
1871	9 10 41 37.6	1.9	9 10 42 18	4	456	307	280	815	397	215	755	722
1872 B.	9 10 27 17.9	1.9	9 10 43 20	4	703	667	279	439	928	300	789	776
1873	9 11 12 6.6	1.9	9 10 44 22	4	985	61	281	68	461	384	823	830
1874	9 10 57 47.0	1.9	9 10 45 24	4	232	421	280	692	992	468	857	883
1875	9 10 43 27.4	2.0	9 10 46 26	4	478	781	280	318	524	553	891	937
1876 B.	9 10 29 7.8	2.0	9 10 47 28	4	725	141	279	941	55	637	925	991
1877	9 11 13 56.4	2.0	9 10 48 30	4	8	535	281	570	588	721	959	45
1878	9 10 59 36.8	2.0	9 10 49 32	4	254	895	281	195	119	806	993	99
1879	9 10 45 17.2	2.1	9 10 50 34	4	500	255	280	819	650	890	27	152
1880 B.	9 10 30 57.6	2.1	9 10 51 36	4	757	615	279	443	182	974	61	206
1881	9 11 15 46.3	2.1	9 10 52 38	4	30	9	281	72	715	59	95	259
1882	9 11 1 26.6	2.1	9 10 53 39	4	270	369	281	697	246	143	129	313
1883	9 10 47 7.0	2.2	9 10 54 41	4	522	729	280	321	777	227	163	366
1884 B.	9 10 32 47.4	2.2	9 10 55 43	4	769	89	279	946	309	312	197	420
1885	9 11 17 36.1	2.2	9 10 56 45	4	52	483	281	574	841	396	231	474
1886	9 11 3 16.4	2.3	9 10 57 47	4	298	843	281	199	373	480	265	528
1887	9 10 48 56.8	2.3	9 10 58 49	4	545	203	280	823	905	565	299	582
1888 B.	9 10 34 37.2	2.3	9 10 59 51	4	791	564	279	448	436	649	333	636
1889	9 11 19 25.9	2.3	9 11 0 53	4	74	958	281	77	968	733	367	689
1890	9 11 5 6.2	2.4	9 11 1 55	4	320	318	281	701	500	818	401	743
1891	9 10 50 46.6	2.4	9 11 2 57	5	567	678	280	325	31	902	435	797
1892 B.	9 10 36 27.0	2.4	9 11 3 59	5	813	38	279	950	562	986	469	850
1893	9 11 21 15.7	2.4	9 11 5 0	5	96	432	282	579	95	71	503	904
1894	9 11 6 56.0	2.5	9 11 6 2	5	342	792	281	203	626	155	537	958
1895	9 10 52 36.4	2.5	9 11 7 4	5	589	152	280	828	158	239	571	12
1896 B.	9 10 38 16.8	2.5	9 11 8 6	5	835	512	280	452	689	324	605	65
1897	9 11 23 5.5	2.5	9 11 9 8	5	118	906	282	81	222	408	639	119
1898	9 11 8 45.8	2.6	9 11 10 10	5	364	266	281	705	753	492	674	173
1899	9 10 54 26.2	2.6	9 11 11 12	5	611	626	280	330	283	577	707	226
1900 C.	9 10 40 6.6	2.6	9 11 12 14	5	857	986	280	954	816	661	741	280

TABLE II.

MOTIONS for *past* and *future Centuries*, to be added to the EPOCHS in Table I, for the 19th Century, that is, from 1801 to 1900 inclusive, in order to obtain the EPOCHS for past and future Centuries. The Sign — indicates the past Centuries, and + the future, in respect to the Nineteenth Century.

YEARS.	Mean Longitude of the Sun.	Perigee.	M	A	B	C	D	E	F	N
	S. D. M. S.	S. D. M. S.								
—2600	11 22 0 10.0	10 15 17 16	81	224	978	667	561	713	634	309
2500	11 22 45 55.0	10 17 0 27	633	77	980	220	732	147	32	681
2400	11 23 31 40.0	10 18 43 38	185	930	982	773	904	581	431	54
2300	11 24 17 25.0	10 20 26 49	737	783	984	326	75	15	830	427
2200	11 25 3 10.0	10 22 10 0	289	637	986	879	247	450	229	800
—2100	11 25 48 55.0	10 23 53 11	841	490	988	432	418	884	627	173
2000	11 26 34 40.0	10 25 36 22	393	343	990	986	589	318	26	545
1900	11 27 20 25.0	10 27 19 33	945	196	993	539	761	752	425	918
1800	11 28 6 10.0	10 29 2 44	497	49	996	93	932	186	823	291
1700	11 28 51 55.0	11 0 45 55	49	902	997	646	104	621	222	664
—1600	11 29 37 40.0	11 2 29 6	601	755	999	199	275	55	621	37
1500	0 0 23 25.0	11 4 12 17	153	608	1	753	446	489	20	409
1400	0 1 9 10.0	11 5 55 28	705	461	3	306	618	923	419	782
1300	0 1 54 55.0	11 7 38 39	257	314	5	859	789	357	818	155
1200	0 2 40 40.0	11 9 21 50	809	167	7	412	961	792	217	528
—1100	0 3 26 25.0	11 11 5 1	361	21	10	966	132	226	615	901
1000	0 4 12 10.0	11 12 48 12	913	874	12	520	303	660	14	273
900	0 4 57 55.0	11 14 31 23	465	727	14	73	475	94	413	646
800	0 5 43 40.0	11 16 14 34	17	580	16	626	646	528	811	19
700	0 6 29 25.0	11 17 57 45	569	433	18	180	818	963	210	392
— 600	0 7 15 10.0	11 19 40 56	121	286	20	733	989	397	609	765
500	0 8 0 55.0	11 21 24 7	673	139	22	287	160	831	7	137
400	0 8 46 40.0	11 23 7 18	225	993	24	840	332	265	406	510
300 J.	0 9 32 25.0	11 24 50 29	777	846	26	393	503	699	805	883
300 G.	11 29 41 1.7	11 24 50 27	414	508	999	349	489	697	804	882
— 200	0 0 26 46.7	11 26 33 38	968	362	2	902	660	132	203	255
— 100	0 0 13 23.3	11 28 16 49	484	181	1	451	830	566	601	627
+ 100	11 29 46 36.7	0 1 43 11	516	819	999	549	170	434	399	373
200	0 0 32 21.7	0 3 26 22	68	672	1	102	341	868	797	745
300	0 0 18 58.4	0 5 9 33	584	492	1	651	511	303	196	118
+ 400	0 0 5 35.0	0 6 52 44	100	311	0	200	681	737	595	491
500	11 29 52 11.7	0 8 35 55	616	130	0	749	851	171	993	863
600	0 0 37 56.7	0 10 19 6	168	983	2	303	23	605	392	236
700	0 0 24 33.3	0 12 2 17	684	802	1	851	192	39	791	609
800	0 0 11 10.0	0 13 45 28	200	622	1	400	362	474	190	981

TABLE II.—*continued.*

YEARS.	Mean Longitude of the Sun.	Perigee.	M	A	B	C	D	E	F	N
	S. D. M. S.	S. D. M. S.								
+ 900	11 29 57 46.7	0 15 28 39	716	441	0	949	532	908	588	354
1000	0 0 43 31.7	0 17 11 50	268	294	2	503	704	342	987	726
1100	0 0 30 8.4	0 18 55 1	784	113	1	52	874	776	386	99
+1200	0 0 16 45.0	0 20 38 12	300	932	1	601	44	210	784	472
1300	0 0 3 21.7	0 22 21 23	816	752	2	150	214	645	183	844
+1400	0 0 49 6.7	0 24 4 34	368	605	2	703	385	79	582	217
1500	0 0 35 43.4	0 25 47 45	884	424	2	252	555	512	980	590
1600	0 0 22 20.1	0 27 30 56	400	243	1	801	725	947	379	962
1700	0 0 8 56.8	0 29 14 7	916	63	0	350	895	380	778	334
1800	0 0 54 41.8	1 0 57 18	468	916	3	903	66	813	176	708
1900	0 0 41 18.4	1 2 40 29	984	735	2	452	236	246	574	80
2000	0 0 27 55.0	1 4 23 40	501	554	1	1	406	680	974	452

First Supplement to TABLE II. Centuries *preceding* the Nineteenth.

2000	11 14 45.0	10 25 36 20	960	938	958	934	572	316	25	544
4000	10 29 30.0	9 21 12 40	920	876	916	868	144	632	50	88
6000	10 14 15.0	8 16 49 0	880	814	874	802	716	948	75	632
8000	9 29 0.0	7 12 25 20	840	752	832	736	288	264	100	176
10000	9 13 45.0	6 8 1 40	800	690	790	670	860	580	125	720

Second Supplement to TABLE II. Centuries *after* the Nineteenth.

2000	0 0 27 55	1 4 23 40	501	554	1	1	406	680	972	452
4000	0 0 55 50	2 8 47 20	3	108	3	2	812	360	945	905
6000	0 1 23 46	3 13 11 0	504	662	4	2	218	40	917	356
8000	0 1 51 41	4 17 34 40	5	216	5	3	624	720	889	809
10000	0 2 19 36	5 21 58 20	506	770	6	4	30	400	861	262

TABLE III.

Secular Variations of the Precession, the Obliquity, and the greatest Equation of the Center, &c.

Common Years.	Correction common to every Longitude.	Differences.	Correction of the Perigee. +	Differences.	Correction of the greatest Equation of the Center.	Differences.
	M. S.	S.	M. S.	M. S.	S.	S.
— 800	+13 2.1	59.1	21 53	1 40	— 44.4	3.4
700	12 3.0	56.9	20 13	1 36	41.0	3.3
600	11 6.1	54.8	18 37	1 33	37.7	3.2
500	10 11.3	52.9	17 4	1 28	34.5	3.0
400	9 18.4	49.7	15 36	1 24	31.5	2.8
300	8 28.7	47.9	14 12	1 21	28.7	2.8
200	7 40.8	45.6	12 51	1 17	25.9	2.6
— 100	6 55.2	43.2	11 34	1 12	23.3	2.4
0	6 12.0	41.0	10 22	1 9	20.9	2.3
+ 100	5 31.0	38.5	9 13	1 5	18.6	2.2
200	4 52.5	36.2	8 8	1 0	16.4	2.1
300	4 16.3	33.9	7 8	0 57	14.3	1.9
400	3 42.4	31.6	6 11	0 53	12.4	1.8
500	3 10.8	29.1	5 18	0 48	10.6	1.6
600	2 41.7	26.7	4 30	0 45	9.0	1.5
700	2 15.0	24.4	3 45	0 40	7.5	1.3
800	1 50.6	21.9	3 5	0 37	6.2	1.3
900	1 28.7	19.5	2 28	0 33	4.9	1.1
1000	1 9.2	17.1	1 55	0 28	3.8	0.9
1100	0 52.1	14.7	1 27	0 25	2.9	0.8
1200	0 37.4	12.3	1 2	0 20	2.1	0.7
1300	0 25.1	9.9	0 42	0 17	1.4	0.5
1400	0 15.2	7.4	0 25	0 12	0.9	0.5
1500	0 7.8	4.9	0 13	0 8	0.4	0.2
1600	0 2.9	2.6	0 5	0 5	— 0.2	0.2
1700	+ 0 0.3	0.0	0 0	0 0	0.0	0.0
1800	+ 0 0.3	2.3	0 0	0 5	0.0	0.2
1900	0 2.6	4.8	0 5	0 8	— 0.2	0.2
2000	0 7.4	7.3	0 13	0 12	0.4	0.5
2100	0 14.7	9.7	0 25	0 16	0.9	0.5
2200	0 24.4	12.1	0 41	0 20	1.4	0.7
2300	0 36.5	14.6	1 1	0 25	2.1	0.8
2400	0 51.1	17.0	1 26	0 28	2.9	0.9
2500	1 8.1	19.5	1 54	0 33	3.8	1.1
2600	1 27.6	21.8	2 27	0 36	4.9	1.3
2700	1 49.4	24.4	3 3	0 41	6.2	1.3
2800	2 13.8	26.7	3 44	0 45	7.5	1.5
2900	2 40.5	29.2	4 29	0 49	9.0	1.7
3000	+ 3 9.7		5 18		10.7	

The Correction of the Longitude is common to the Sun, Moon, Planets, fixed Stars, Perigees and Nodes, and to all the Motions calculated upon the Annual Precession being supposed 50″.1.

The Correction of the Perigee is the Correction of its mean Longitude, increased by a term proportional to the Square of the Time, which we were obliged to omit in the Table of the Motion of the Perigee.

The Correction of the Equation of the Center must be multiplied by the Sine of the Anomaly, regard being had to the Sign of the Sine.

The Correction of the Obliquity is to be applied to the mean Obliquity 23° 27′ 57″ for 1800.

The Diminution is 50″ for 100 years, or 0.5″ for 1 year; and proportionally for any other Time.

TABLE IV.

Motion for Days. JANUARY.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	1	0 0 0 0.0	0.0	00	00	0	0	0	0	0	0	1	.000
2	2	0 59 8.3	0.2	36	34	3	4	1	0	0	0	2	.003
3	3	1 58 16.7	0.3	73	68	5	9	3	0	0	0	3	.006
4	4	2 57 25.0	0.5	109	101	8	13	4	1	0	0	4	.008
5	5	3 56 33.3	0.7	145	135	11	18	6	1	0	1	5	.011
6	6	4 55 41.6	0.8	181	169	14	22	7	1	0	1	6	.014
7	7	5 54 50.0	1.0	218	203	16	27	9	1	1	1	7	.017
8	8	6 53 58.3	1.2	254	236	19	31	10	2	1	1	8	.019
9	9	7 53 6.6	1.4	290	270	22	36	12	2	1	1	9	.022
10	10	8 52 15.0	1.5	327	304	25	40	13	2	1	1	10	.025
11	11	9 51 23.3	1.7	363	338	27	44	15	2	1	1	11	.028
12	12	10 50 31.6	1.9	399	371	30	49	16	2	1	2	12	.030
13	13	11 49 40.0	2.0	435	405	33	53	17	3	1	2	13	.033
14	14	12 48 48.3	2.2	472	439	36	58	19	3	1	2	14	.036
15	15	13 47 56.6	2.3	508	473	38	62	20	3	2	2	15	.039
16	16	14 47 4.9	2.5	544	506	41	67	22	3	2	2	16	.041
17	17	15 46 13.3	2.7	581	540	44	71	23	4	2	2	17	.044
18	18	16 45 21.6	2.9	617	574	47	76	25	4	2	2	18	.046
19	19	17 44 29.9	3.0	653	608	49	80	26	4	2	3	19	.049
20	20	18 43 38.3	3.2	690	641	52	85	28	4	2	3	20	.052
21	21	19 42 46.6	3.4	726	675	55	89	29	5	2	3	21	.056
22	22	20 41 54.9	3.6	762	709	58	93	31	5	2	3	22	.057
23	23	21 41 3.3	3.7	798	743	60	98	32	5	2	3	23	.060
24	24	22 40 11.6	3.9	835	777	63	102	33	5	2	3	24	.063
25	25	23 39 19.9	4.0	871	810	66	107	35	5	2	4	25	.066
26	26	24 38 28.2	4.2	907	844	68	111	36	6	2	4	26	.068
27	27	25 37 36.6	4.4	943	878	71	116	38	6	2	4	27	.071
28	28	26 36 44.9	4.6	980	912	74	120	39	6	2	4	28	.074
29	29	27 35 53.2	4.7	16	945	77	125	41	6	3	4	29	.077
30	30	28 35 1.6	4.9	52	979	79	129	42	7	3	4	30	.079
31	31	29 34 9.9	5.1	89	13	82	134	44	7	3	4	31	.082

TABLE IV.

Motion for Days. FEBRUARY.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	1	1 0 33 18.2	5.3	125	47	85	138	45	7	3	5	32	.085
2	2	1 1 32 26.6	5.4	161	81	88	142	47	7	3	5	33	.088
3	3	1 2 31 34.9	5.6	198	114	90	147	48	8	3	5	34	.091
4	4	1 3 30 43.2	5.8	234	148	93	151	49	8	3	5	35	.093
5	5	1 4 29 51.5	5.9	270	182	96	156	51	8	3	5	36	.096
6	6	1 5 28 59.9	6.1	306	215	99	160	52	8	3	5	37	.099
7	7	1 6 28 8.2	6.3	343	249	101	165	54	9	3	5	38	.102
8	8	1 7 27 16.5	6.4	379	283	104	169	55	9	3	6	39	.104
9	9	1 8 26 24.9	6.6	415	317	107	174	57	9	4	6	40	.107
10	10	1 9 25 33.2	6.8	452	351	110	178	58	9	4	6	41	.109
11	11	1 10 24 41.5	7.0	488	384	112	182	60	9	4	6	42	.112
12	12	1 11 23 49.8	7.1	524	418	115	187	61	10	4	6	43	.115
13	13	1 12 22 58.2	7.3	561	452	118	191	63	10	4	6	44	.118
14	14	1 13 22 6.5	7.5	597	486	120	196	64	10	4	6	45	.120
15	15	1 14 21 14.8	7.6	633	520	123	200	65	10	4	7	46	.123
16	16	1 15 20 23.2	7.8	669	553	126	205	67	11	4	7	47	.125
17	17	1 16 19 31.5	8.0	706	587	129	209	68	11	4	7	48	.129
18	18	1 17 18 39.8	8.1	742	621	131	214	70	11	4	7	49	.131
19	19	1 18 17 48.2	8.3	778	655	134	218	71	11	4	7	50	.134
20	20	1 19 16 56.5	8.5	815	688	137	223	73	12	5	7	51	.137
21	21	1 20 16 4.8	8.7	851	722	140	227	74	12	5	7	52	.140
22	22	1 21 15 13.1	8.8	887	756	142	231	76	12	5	8	53	.142
23	23	1 22 14 21.5	9.0	923	790	145	236	77	12	5	8	54	.145
24	24	1 23 13 29.8	9.2	960	824	148	240	79	12	5	8	55	.148
25	25	1 24 12 38.1	9.3	996	858	151	245	80	12	5	8	56	.151
26	26	1 25 11 46.5	9.5	32	892	153	249	82	13	5	8	57	.153
27	27	1 26 10 54.8	9.7	69	925	156	254	83	13	5	8	58	.156
28	28	1 27 10 3.1	9.8	105	959	159	258	84	13	5	9	59	.159
	29	1 28 9 11.4	10.0	141	993	162	263	86	14	5	9	60	.162

TABLE IV.

Motion for Days. MARCH.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S.	D.	M.	S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	1	28	9	11.4	10.0	141	993	162	263	86	14	6	9	60	.162
2	1	1	29	8	19.8	10.2	178	27	164	267	87	14	6	9	61	.164
3	2	2	0	7	28.1	10.3	214	61	167	271	89	14	6	9	62	.167
4	3	2	1	6	36.4	10.5	250	94	170	276	90	15	6	9	63	.170
5	4	2	2	5	44.8	10.7	286	128	173	280	92	15	6	9	64	.173
6	5	2	3	4	53.1	10.9	323	162	175	285	93	15	6	10	65	.175
7	6	2	4	4	1.4	11.0	359	196	178	289	95	15	6	10	66	.178
8	7	2	5	3	9.8	11.2	395	230	181	294	96	16	6	10	67	.181
9	8	2	6	2	18.1	11.4	431	263	184	298	97	16	6	10	68	.184
10	9	2	7	1	26.4	11.5	467	297	186	303	99	16	6	10	69	.186
11	10	2	8	0	34.8	11.7	504	331	189	307	100	16	6	10	70	.189
12	11	2	8	59	43.1	11.9	540	365	192	312	102	17	6	11	71	.192
13	12	2	9	58	51.4	12.0	577	399	194	316	103	17	7	11	72	.195
14	13	2	10	57	59.7	12.2	613	433	197	320	105	17	7	11	73	.197
15	14	2	11	57	8.1	12.4	649	467	200	325	106	17	7	11	74	.200
16	15	2	12	56	16.4	12.6	686	500	203	329	108	17	7	11	75	.203
17	16	2	13	55	24.7	12.7	722	534	205	334	109	18	7	11	76	.206
18	17	2	14	54	33.1	12.9	758	568	208	338	111	18	7	11	77	.208
19	18	2	15	53	41.4	13.1	794	602	211	343	112	18	7	11	78	.211
20	19	2	16	52	49.7	13.2	831	636	214	347	114	18	7	12	79	.214
21	20	2	17	51	58.1	13.4	867	670	216	352	115	19	7	12	80	.217
22	21	2	18	51	6.4	13.6	903	703	219	356	116	19	7	12	81	.219
23	22	2	19	50	14.7	13.7	940	737	222	360	118	19	8	12	82	.222
24	23	2	20	49	23.0	13.9	976	771	225	365	119	19	8	12	83	.225
25	24	2	21	48	31.4	14.1	12	805	227	369	121	20	8	12	84	.227
26	25	2	22	47	39.7	14.3	49	839	230	374	122	20	8	12	85	.230
27	26	2	23	46	48.0	14.4	85	873	233	378	124	20	8	13	86	.233
28	27	2	24	45	56.4	14.6	121	907	235	383	125	20	8	13	87	.236
29	28	2	25	45	4.7	14.8	157	940	238	387	127	21	8	13	88	.239
30	29	2	26	44	13.0	14.9	194	974	241	392	128	21	8	13	89	.241
31	30	2	27	43	21.4	15.1	230	8	244	396	129	21	8	13	90	.244
	31	2	28	42	29.7	15.3	266	42	246	401	131	21	8	13	91	.246

TABLE IV.

Motion for Days. APRIL.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	2 28 42 29.7	15.3	266	42	246	401	131	21	8	13	91	.246
2	1	2 29 41 38.0	15.4	302	76	249	405	132	21	8	13	92	.249
3	2	3 0 40 46.3	15.6	339	110	252	410	134	22	9	13	93	.252
4	3	3 1 39 54.7	15.8	375	143	255	414	135	22	9	13	94	.255
5	4	3 2 39 3.0	15.9	411	177	257	418	137	22	9	14	95	.258
6	5	3 3 38 11.3	16.1	448	211	260	423	138	22	9	14	96	.260
7	6	3 4 37 19.7	16.3	484	245	263	427	140	23	9	14	97	.263
8	7	3 5 36 28.0	16.5	520	279	266	432	141	23	9	14	98	.266
9	8	3 6 35 36.3	16.6	557	313	268	436	143	23	9	14	99	.269
10	9	3 7 34 44.7	16.8	593	347	271	441	144	23	9	15	100	.271
11	10	3 8 33 53.0	17.0	629	380	274	445	146	24	9	15	101	.274
12	11	3 9 33 1.3	17.1	665	414	277	449	147	24	9	15	102	.277
13	12	3 10 32 9.6	17.3	702	448	279	454	149	24	9	15	103	.280
14	13	3 11 31 18.0	17.5	738	482	282	458	150	24	10	15	104	.282
15	14	3 12 30 26.3	17.6	774	516	285	463	151	24	10	15	105	.285
16	15	3 13 29 34.6	17.8	811	550	288	467	153	25	10	15	106	.288
17	16	3 14 28 43.0	18.0	847	584	290	472	154	25	10	16	107	.291
18	17	3 15 27 51.3	18.2	883	618	293	476	156	25	10	16	108	.293
19	18	3 16 26 59.6	18.3	919	652	296	481	157	25	10	16	109	.296
20	19	3 17 26 8.0	18.5	956	686	298	485	159	26	10	16	110	.299
21	20	3 18 25 16.3	18.7	992	719	301	490	160	26	10	16	111	.302
22	21	3 19 24 24.6	18.8	28	753	304	494	162	26	10	16	112	.304
23	22	3 20 23 32.9	19.0	65	787	307	498	163	26	10	16	113	.307
24	23	3 21 22 41.3	19.2	101	821	309	503	165	27	10	17	114	.309
25	24	3 22 21 49.6	19.3	137	855	312	507	166	27	11	17	115	.312
26	25	3 23 20 57.9	19.5	173	889	315	512	167	27	11	17	116	.315
27	26	3 24 20 6.3	19.7	210	923	318	516	169	27	11	17	117	.318
28	27	3 25 19 14.6	19.9	246	957	320	521	170	27	11	17	118	.320
29	28	3 26 18 22.9	20.0	282	991	323	525	172	28	11	17	119	.323
30	29	3 27 17 31.2	20.2	319	25	326	530	173	28	11	18	120	.326
	30	3 28 16 39.6	20.4	355	59	329	534	175	28	11	18	121	.329

TABLE IV.

Motion for Days. MAY.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S.	D.	M.	S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	3	28	16	39.6	20.4	355	59	329	534	175	28	11	18	121	.329
2	1	3	29	15	47.9	20.5	391	92	331	538	176	28	11	18	122	.331
3	2	4	0	14	56.2	20.7	428	126	334	543	178	29	11	18	123	.334
4	3	4	1	14	4.6	20.9	464	160	337	547	179	29	11	18	124	.337
5	4	4	2	13	12.9	21.0	500	194	340	552	181	29	12	18	125	.340
6	5	4	3	12	21.2	21.2	536	228	342	556	182	29	12	18	126	.342
7	6	4	4	11	29.6	21.4	573	262	345	561	183	29	12	19	127	.345
8	7	4	5	10	37.9	21.5	609	296	348	565	185	29	12	19	128	.348
9	8	4	6	9	46.2	21.7	645	330	351	569	186	30	12	19	129	.351
10	9	4	7	8	54.5	21.9	682	363	353	574	188	30	12	19	130	.353
11	10	4	8	8	2.9	22.1	718	397	356	579	189	30	12	19	131	.356
12	11	4	9	7	11.2	22.2	754	431	359	583	191	30	12	19	132	.359
13	12	4	10	6	19.5	22.4	790	464	362	587	192	31	12	19	133	.362
14	13	4	11	5	27.9	22.6	827	499	364	592	194	31	12	20	134	.364
15	14	4	12	4	36.2	22.7	863	533	367	596	195	31	12	20	135	.367
16	15	4	13	3	44.5	22.9	899	567	370	601	196	31	13	20	136	.370
17	16	4	14	2	52.9	23.1	935	601	372	605	198	32	13	20	137	.373
18	17	4	15	2	1.2	23.2	972	635	375	610	199	32	13	20	138	.375
19	18	4	16	1	9.5	23.4	8	669	378	614	201	32	13	20	139	.378
20	19	4	17	0	17.8	23.6	44	703	381	619	202	32	13	20	140	.381
21	20	4	17	59	26.2	23.8	81	737	383	623	204	32	13	21	141	.383
22	21	4	18	58	34.5	23.9	117	771	386	627	205	33	13	21	142	.386
23	22	4	19	57	42.8	24.1	153	805	389	632	207	33	13	21	143	.389
24	23	4	20	56	51.2	24.3	190	839	392	636	208	33	13	21	144	.392
25	24	4	21	55	59.5	24.4	226	873	394	641	210	33	13	21	145	.395
26	25	4	22	55	7.8	24.6	262	906	397	645	211	34	13	21	146	.397
27	26	4	23	54	16.2	24.8	299	940	400	650	213	34	14	21	147	.400
28	27	4	24	53	24.5	24.9	335	974	403	654	214	34	14	22	148	.403
29	28	4	25	52	32.8	25.1	371	8	405	659	215	34	14	22	149	.406
30	29	4	26	51	41.1	25.3	407	42	408	663	217	34	14	22	150	.408
31	30	4	27	50	49.5	25.5	444	76	411	667	218	35	14	22	151	.411
	31	4	28	49	57.8	25.6	480	110	414	672	220	35	14	22	152	.414

TABLE IV.

Motion for Days. JUNE.

YEARS. Comm.	YEARS. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	4 28 49 57.8	25.6	480	110	414	672	220	35	14	22	152	.414
2	1	4 29 49 6.1	25.8	516	144	416	676	221	35	14	23	153	.416
3	2	5 0 48 14.5	26.0	553	178	419	681	223	35	14	23	154	.419
4	3	5 1 47 22.8	26.1	589	212	422	685	224	35	14	23	155	.422
5	4	5 2 46 31.1	26.3	625	246	425	690	226	36	14	23	156	.425
6	5	5 3 45 39.5	26.5	661	280	427	694	227	36	14	23	157	.427
7	6	5 4 44 47.8	26.6	698	314	430	699	229	36	15	23	158	.430
8	7	5 5 43 56.1	26.8	734	348	433	703	230	36	15	23	159	.433
9	8	5 6 43 4.4	27.0	770	382	435	708	231	37	15	23	160	.436
10	9	5 7 42 12.8	27.2	807	416	438	712	233	37	15	24	161	.438
11	10	5 8 41 21.1	27.3	843	450	441	717	234	37	15	24	162	.441
12	11	5 9 40 29.4	27.5	879	484	444	721	236	37	15	24	163	.444
13	12	5 10 39 37.8	27.7	915	518	446	725	237	38	15	24	164	.447
14	13	5 11 38 46.1	27.8	952	552	449	730	239	38	15	24	165	.449
15	14	5 12 37 54.4	28.0	988	586	452	734	240	38	15	24	166	.452
16	15	5 13 37 2.8	28.2	24	620	455	739	242	38	15	24	167	.455
17	16	5 14 36 11.1	28.3	61	654	457	743	244	38	15	25	168	.458
18	17	5 15 35 19.4	28.5	97	688	460	748	246	39	16	25	169	.460
19	18	5 16 34 27.7	28.7	133	722	463	752	247	39	16	25	170	.463
20	19	5 17 33 36.1	28.8	170	756	466	757	248	39	16	25	171	.465
21	20	5 18 32 44.4	29.0	206	789	468	761	249	39	16	25	172	.468
22	21	5 19 31 52.7	29.2	242	823	471	765	250	40	16	25	173	.471
23	22	5 20 31 1.1	29.4	278	857	474	770	252	40	16	25	174	.473
24	23	5 21 30 9.4	29.5	315	891	477	774	253	40	16	26	175	.476
25	24	5 22 29 17.7	29.7	351	925	479	779	255	40	16	26	176	.479
26	25	5 23 28 26.0	29.9	387	959	482	783	256	41	16	26	177	.482
27	26	5 24 27 34.4	30.0	424	993	485	788	258	41	16	26	178	.485
28	27	5 25 26 42.7	30.2	460	27	488	792	259	41	16	26	179	.487
29	28	5 26 25 51.0	30.4	496	61	490	797	261	41	17	26	180	.490
30	29	5 27 24 59.4	30.5	532	95	493	801	262	41	17	26	181	.493
	30	5 28 24 7.7	30.7	569	129	496	806	263	41	17	27	182	.496

TABLE IV.

Motion for Days. JULY.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S.	D.	M.	S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	5	28	24	7.7	30.7	569	129	496	806	263	41	17	27	182	.496
2	1	5	29	23	16.0	30.9	605	163	499	810	265	42	17	27	183	.499
3	2	6	0	22	24.4	31.1	641	197	501	814	266	42	17	27	184	.502
4	3	6	1	21	32.7	31.2	678	231	504	819	268	42	17	27	185	.504
5	4	6	2	20	41.0	31.4	714	265	507	825	269	43	17	27	186	.507
6	5	6	3	19	49.3	31.6	750	299	509	828	270	43	17	27	187	.509
7	6	6	4	18	57.7	31.7	787	333	512	832	272	43	17	27	188	.512
8	7	6	5	18	6.0	31.9	823	367	515	837	273	43	17	28	189	.515
9	8	6	6	17	14.3	32.1	859	401	518	841	275	44	18	28	190	.518
10	9	6	7	16	22.7	32.2	895	435	520	846	276	44	18	28	191	.520
11	10	6	8	15	31.0	32.4	932	469	523	850	278	44	18	28	192	.523
12	11	6	9	14	39.3	32.6	968	502	526	854	279	44	18	28	193	.526
13	12	6	10	13	47.7	32.7	4	536	529	859	281	44	18	28	194	.529
14	13	6	11	12	56.0	32.9	41	570	531	863	282	45	18	29	195	.531
15	14	6	12	12	4.3	33.1	77	604	534	868	284	45	18	29	196	.534
16	15	6	13	11	12.6	33.3	113	638	537	872	285	45	18	29	197	.537
17	16	6	14	10	21.0	33.4	149	672	540	877	287	45	18	29	198	.540
18	17	6	15	9	29.3	33.6	186	706	542	881	288	46	18	29	199	.542
19	18	6	16	8	37.6	33.8	222	740	545	886	289	46	18	29	200	.545
20	19	6	17	7	46.0	33.9	258	774	548	890	291	46	19	29	201	.548
21	20	6	18	6	54.3	34.1	295	808	550	895	293	46	19	30	202	.551
22	21	6	19	6	2.6	34.3	331	842	553	899	294	47	19	30	203	.553
23	22	6	20	5	11.0	34.4	367	876	556	903	296	47	19	30	204	.556
24	23	6	21	4	19.3	34.6	403	910	559	908	297	47	19	30	205	.559
25	24	6	22	3	27.6	34.8	440	944	561	912	298	47	19	30	206	.562
26	25	6	23	2	36.0	35.0	476	978	564	917	300	47	19	30	207	.564
27	26	6	24	1	44.3	35.1	512	12	567	921	301	48	19	30	208	.567
28	27	6	25	0	52.6	35.3	549	46	570	926	303	48	19	31	209	.570
29	28	6	26	0	1.0	35.5	585	80	572	930	304	48	19	31	210	.573
30	29	6	26	59	9.3	35.6	621	114	575	935	306	48	19	31	211	.575
31	30	6	27	58	17.6	35.8	658	148	578	939	307	49	20	31	212	.578
	31	6	28	57	26.0	36.0	694	182	580	943	309	49	20	31	213	.581

TABLE IV.

Motion for Days. AUGUST.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	6 28 57 26.0	36.0	694	182	580	943	309	49	20	31	213	.581
2	1	6 29 56 34.3	36.1	730	216	583	948	310	49	20	31	214	.583
3	2	7 0 55 42.6	36.3	766	250	586	952	312	49	20	31	215	.586
4	3	7 1 54 51.0	36.5	803	284	589	957	313	50	20	32	216	.589
5	4	7 2 53 59.3	36.7	839	317	591	961	314	50	20	32	217	.592
6	5	7 3 53 7.6	36.8	875	351	594	966	316	50	20	32	218	.594
7	6	7 4 52 16.0	37.0	912	385	597	970	317	50	20	32	219	.597
8	7	7 5 51 24.3	37.2	948	419	599	975	319	50	20	32	220	.600
9	8	7 6 50 32.6	37.3	984	453	602	979	320	51	20	32	221	.602
10	9	7 7 49 41.0	37.5	20	487	605	984	322	51	21	32	222	.605
11	10	7 8 48 49.3	37.7	57	521	608	988	323	51	21	33	223	.608
12	11	7 9 47 57.6	37.8	93	555	611	992	325	51	21	33	224	.610
13	12	7 10 47 5.9	38.0	129	589	613	997	326	52	21	33	225	.613
14	13	7 11 46 14.2	38.2	166	623	616	1	328	52	21	33	226	.616
15	14	7 12 45 22.5	38.3	202	657	619	6	329	52	21	33	227	.619
16	15	7 13 44 30.9	38.5	238	691	622	10	330	52	21	33	228	.622
17	16	7 14 43 39.2	38.7	274	725	624	15	332	53	21	34	229	.625
18	17	7 15 42 47.5	38.9	311	759	627	19	333	53	21	34	230	.627
19	18	7 16 41 55.9	39.0	347	794	630	24	335	53	21	34	231	.630
20	19	7 17 41 4.2	39.2	383	827	632	28	336	53	21	34	232	.633
21	20	7 18 40 12.5	39.4	420	861	635	34	338	53	22	34	233	.636
22	21	7 19 39 20.9	39.5	456	894	638	37	339	54	22	34	234	.638
23	22	7 20 38 29.2	39.7	492	928	641	42	341	54	22	34	235	.641
24	23	7 21 37 37.5	39.9	529	962	643	46	342	54	32	35	236	.644
25	24	7 22 36 45.8	40.0	565	996	646	51	344	54	22	35	237	.647
26	25	7 23 35 54.2	40.2	601	30	649	55	345	55	22	35	238	.649
27	26	7 24 35 2.5	40.4	637	63	652	59	346	55	22	35	239	.652
28	27	7 25 34 10.8	40.6	674	97	654	64	348	55	22	35	240	.655
29	28	7 26 33 19.2	40.7	710	131	657	68	349	55	22	35	241	.657
30	29	7 27 32 27.6	40.9	746	165	660	73	351	55	22	35	242	.660
31	30	7 28 31 35.9	41.1	783	199	663	77	352	56	22	36	243	.663
	31	7 29 30 44.2	41.2	819	233	665	81	354	56	23	36	244	.666

TABLE IV.

Motion for Days. SEPTEMBER.

Years. Comm.	Years. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	7 29 30 44.2	41.2	819	233	665	81	354	56	23	36	244	.666
2	1	8 0 29 52.5	41.4	855	268	668	86	355	56	23	36	245	.668
3	2	8 1 29 0.8	41.6	891	302	671	90	357	56	23	36	246	.671
4	3	8 2 28 9.1	41.7	928	336	674	95	358	57	23	36	247	.673
5	4	8 3 27 17.4	41.9	964	370	676	99	360	57	23	36	248	.675
6	5	8 4 26 25.8	42.1	000	403	679	104	361	57	23	36	249	.678
7	6	8 5 25 34.1	42.2	37	437	682	108	362	57	23	37	250	.681
8	7	8 6 24 42.5	42.4	73	471	685	113	364	58	23	37	251	.684
9	8	8 7 23 50.8	42.6	109	505	687	117	365	58	23	37	252	.687
10	9	8 8 22 59.1	42.8	145	539	690	122	367	58	23	37	253	.689
11	10	8 9 22 7.4	42.9	182	573	693	126	368	58	23	37	254	.692
12	11	8 10 21 15.8	43.1	218	607	695	130	370	58	24	37	255	.695
13	12	8 11 20 24.1	43.3	254	641	698	135	371	59	24	38	256	.698
14	13	8 12 19 32.4	43.4	291	674	701	139	373	59	24	38	257	.701
15	14	8 13 18 40.8	43.6	327	708	704	144	374	59	24	38	258	.703
16	15	8 14 17 49.1	43.8	363	742	706	148	376	59	24	38	259	.706
17	16	8 15 16 57.4	43.9	400	776	709	153	377	60	24	38	260	.709
18	17	8 16 16 5.8	44.1	436	810	712	157	378	60	24	38	261	.711
19	18	8 17 15 14.1	44.3	472	844	715	162	380	60	24	38	262	.714
20	19	8 18 14 22.4	44.5	508	878	717	166	381	60	24	39	263	.717
21	20	8 19 13 30.7	44.6	545	912	720	170	383	61	24	39	264	.720
22	21	8 20 12 39.1	44.8	581	946	723	175	384	61	24	39	265	.722
23	22	8 21 11 47.4	45.0	617	979	726	179	386	61	25	39	266	.725
24	23	8 22 10 55.7	45.1	654	13	728	184	387	61	25	39	267	.728
25	24	8 23 10 4.1	45.3	690	47	731	188	389	61	25	39	268	.731
26	25	8 24 9 12.4	45.5	726	81	734	193	390	62	25	39	269	.733
27	26	8 25 8 20.7	45.6	762	115	737	197	392	62	25	40	270	.736
28	27	8 26 7 29.1	45.8	799	149	739	202	393	62	25	40	271	.739
29	28	8 27 6 37.4	46.0	835	182	742	206	394	62	25	40	272	.742
30	29	8 28 5 45.7	46.2	871	216	745	211	396	63	25	40	273	.744
	30	8 29 4 54.0	46.3	908	250	748	215	397	63	25	40	274	.747

TABLE IV.

Motion for Days. OCTOBER.

YEARS. Comm.	YEARS. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	8 29 4 54.0	46.3	908	250	748	215	397	63	25	40	274	.747
2	1	9 0 4 2.4	46.5	944	284	750	219	399	63	25	40	275	.750
3	2	9 1 3 10.7	46.7	980	318	753	224	400	63	26	40	276	.753
4	3	9 2 2 19.0	46.9	16	352	756	228	402	64	26	41	277	.755
5	4	9 3 1 27.4	47.0	53	386	759	233	403	64	26	41	278	.758
6	5	9 4 0 35.7	47.2	89	420	761	237	405	64	26	41	279	.760
7	6	9 4 59 44.0	47.4	125	454	764	242	406	64	26	41	280	.763
8	7	9 5 58 52.4	47.5	162	488	767	246	408	65	26	41	281	.766
9	8	9 6 58 0.7	47.7	198	521	769	251	409	65	26	41	282	.769
10	9	9 7 57 9.0	47.9	234	555	772	255	410	65	26	41	283	.772
11	10	9 8 56 17.3	48.0	271	589	775	259	412	65	26	42	284	.775
12	11	9 9 55 25.7	48.2	307	623	778	264	413	65	26	42	285	.777
13	12	9 10 54 34.0	48.4	343	657	780	268	415	66	26	42	286	.780
14	13	9 11 53 42.4	48.5	379	691	783	273	416	66	27	42	287	.782
15	14	9 12 52 50.7	48.7	416	724	786	277	418	66	27	42	288	.785
16	15	9 13 51 59.0	48.9	452	758	789	282	419	66	27	42	289	.788
17	16	9 14 51 7.3	49.0	488	792	791	286	421	67	27	42	290	.791
18	17	9 15 50 15.7	49.2	525	826	794	291	422	67	27	43	291	.793
19	18	9 16 49 24.0	49.4	561	860	797	295	424	67	27	43	292	.796
20	19	9 17 48 32.3	49.6	597	893	800	299	425	67	27	43	293	.799
21	20	9 18 47 40.6	49.7	633	927	802	304	427	68	27	43	294	.802
22	21	9 19 46 49.0	49.9	670	961	805	308	428	68	27	43	295	.804
23	22	9 20 45 57.3	50.1	706	995	808	313	429	68	27	43	296	.807
24	23	9 21 45 5.6	50.2	742	29	811	317	431	68	27	44	297	.810
25	24	9 22 44 14.0	50.4	779	63	813	322	432	68	28	44	298	.813
26	25	9 23 43 22.3	50.6	815	96	816	326	434	69	28	44	299	.815
27	26	9 24 42 30.6	50.7	851	130	819	331	435	69	28	44	300	.818
28	27	9 25 41 39.0	50.9	887	164	822	335	437	69	28	44	301	.821
29	28	9 26 40 47.3	51.1	924	198	824	340	438	69	28	44	302	.824
30	29	9 27 39 55.6	51.2	960	232	827	344	440	70	28	44	303	.826
31	30	9 28 39 4.0	51.4	996	266	830	348	441	70	28	44	304	.829
	31	9 29 38 12.4	51.6	33	300	832	353	443	70	28	45	305	.832

TABLE IV.

Motion for Days. NOVEMBER.

YEARS. Comm.	YEARS. Bissex.	Mean Longitude of the Sun. S. D. M. S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	9 29 38 12.4	51.6	33	300	832	353	443	70	28	45	305	.832
2	1	10 0 37 20.6	51.8	69	333	835	357	444	70	28	45	306	.835
3	2	10 1 36 29.0	51.9	105	367	838	362	445	71	28	45	307	.838
4	3	10 2 35 37.3	52.1	142	401	841	366	447	71	29	45	308	.840
5	4	10 3 34 45.6	52.3	178	435	843	371	448	71	29	45	309	.843
6	5	10 4 33 54.0	52.4	214	468	846	375	450	71	29	45	310	.845
7	6	10 5 33 2.3	52.6	250	502	849	380	451	72	29	46	311	.848
8	7	10 6 32 10.6	52.8	287	536	852	384	453	72	29	46	312	.851
9	8	10 7 31 19.0	52.9	323	570	854	389	454	72	29	46	313	.854
10	9	10 8 30 27.3	53.0	359	604	857	393	456	72	29	46	314	.856
11	10	10 9 29 35.6	53.3	396	638	860	397	457	72	29	46	315	.859
12	11	10 10 28 44.0	53.5	432	671	863	402	458	73	29	46	316	.862
13	12	10 11 27 52.3	53.6	468	705	865	406	460	73	29	46	317	.865
14	13	10 12 27 0.7	53.8	504	739	868	411	461	73	29	47	318	.867
15	14	10 13 26 8.9	54.0	541	773	871	415	463	73	30	47	319	.870
16	15	10 14 25 17.2	54.1	577	807	873	420	464	74	30	47	320	.873
17	16	10 15 24 25.5	54.3	613	840	876	424	466	74	30	47	321	.876
18	17	10 16 23 33.9	54.5	650	874	879	429	467	74	30	47	322	.878
19	18	10 17 22 42.2	54.6	686	908	882	433	469	74	30	47	323	.882
20	19	10 18 21 50.5	54.8	722	942	884	438	470	75	30	47	324	.884
21	20	10 19 20 58.9	55.0	759	976	887	442	472	75	30	48	325	.887
22	21	10 20 20 7.2	55.2	795	9	890	446	473	75	30	48	326	.890
23	22	10 21 19 15.5	55.3	831	43	893	451	475	75	30	48	327	.893
24	23	10 22 18 23.9	55.5	867	77	895	455	476	76	30	48	328	.895
25	24	10 23 17 32.2	55.7	904	111	898	460	477	76	30	48	329	.898
26	25	10 24 16 40.5	55.8	940	144	901	464	479	76	31	48	330	.900
27	26	10 25 15 48.8	56.0	976	178	904	469	480	76	31	49	331	.903
28	27	10 26 14 57.2	56.2	13	212	906	473	482	77	31	49	332	.906
29	28	10 27 14 5.5	56.3	49	246	909	478	483	77	31	49	333	.909
30	29	10 28 13 13.8	56.5	85	279	912	482	484	77	31	49	334	.911
	30	10 29 12 22.2	56.7	121	313	915	486	486	77	31	49	335	.914

TABLE IV.

Motion for Days. DECEMBER.

YEARS. Comm.	YEARS. Bissex.	Mean Longitude of the Sun. S.	D.	M.	S.	Perigee. S.	M	A	B	C	D	E	F	N	Days of the Year.	Fraction of a Year.
1	0	10	29	12	22.2	56.7	121	313	915	486	486	77	31	49	335	.914
2	1	11	0	11	30.5	56.8	158	347	917	491	488	77	31	49	336	.917
3	2	11	1	10	38.8	57.0	194	381	920	495	489	78	31	49	337	.920
4	3	11	2	9	47.2	57.2	230	415	923	500	490	78	31	50	338	.922
5	4	11	3	8	55.5	57.4	267	448	926	504	492	78	31	50	339	.925
6	5	11	4	8	3.8	57.5	303	482	928	509	493	78	32	50	340	.928
7	6	11	5	7	12.1	57.7	339	515	931	513	495	79	32	50	341	.931
8	7	11	6	6	20.5	57.9	375	549	934	518	496	79	32	50	342	.933
9	8	11	7	5	28.8	58.0	412	583	937	522	498	79	32	50	343	.936
10	9	11	8	4	37.1	58.2	448	617	939	527	499	79	32	50	344	.939
11	10	11	9	3	45.5	58.4	484	651	942	531	501	80	32	51	345	.942
12	11	11	10	2	53.8	58.5	521	685	945	536	502	80	32	51	346	.944
13	12	11	11	2	2.2	58.7	557	719	947	540	504	80	32	51	347	.947
14	13	11	12	1	10.5	58.9	593	753	950	544	505	80	32	51	348	.950
15	14	11	13	0	18.8	59.0	630	787	953	548	507	81	32	51	349	.953
16	15	11	13	59	27.1	59.2	666	820	956	553	508	81	32	51	350	.955
17	16	11	14	58	35.4	59.4	702	853	958	558	509	81	33	51	351	.958
18	17	11	15	57	43.8	59.6	738	887	961	562	511	81	33	52	352	.961
19	18	11	16	56	52.1	59.7	775	921	964	567	512	81	33	52	353	.964
20	19	11	17	56	0.4	59.9	811	955	967	571	514	82	33	52	354	.966
21	20	11	18	55	8.8	60.1	847	989	969	575	515	82	33	52	355	.969
22	21	11	19	54	17.1	60.2	884	23	972	580	517	82	33	52	356	.971
23	22	11	20	53	25.4	60.4	920	56	975	584	518	82	33	52	357	.974
24	23	11	21	52	33.8	60.6	956	90	978	589	520	83	33	52	358	.977
25	24	11	22	51	42.1	60.7	992	124	980	593	521	83	33	53	359	.980
26	25	11	23	50	50.4	60.9	29	157	983	598	523	83	33	53	360	.983
27	26	11	24	49	58.7	61.1	65	191	986	602	524	83	33	53	361	.985
28	27	11	25	49	7.1	61.3	101	225	989	606	526	83	34	53	362	.988
29	28	11	26	48	15.4	61.4	138	259	991	611	527	84	34	53	363	.991
30	29	11	27	47	23.7	61.6	174	293	994	616	528	84	34	53	364	.994
31	30	11	28	46	32.1	61.8	210	326	997	620	530	84	34	54	365	.997
	31	11	29	45	40.4	61.9	246	360	0	624	531	84	34	54	366	0

TABLE V.

Motion of the Sun for Hours, Minutes and Seconds.

HOURS.					MINUTES.				SECONDS.			
	Motion of the Sun.	A	B	C		Motion of the Sun.		Motion of the Sun.		Motion of the Sun.		Motion of the Sun.
H.	M. S.				M.	M. S.	M.	M. S.	S.	S.	S.	S.
1	2 27.8	1	0	0	1	0 2.5	31	1 16.4	1	0.0	31	1.3
2	4 55.7	3	0	0	2	4.9	32	1 18.8	2	0.1	32	1.3
3	7 23.5	4	0	1	3	7.4	33	1 21.3	3	0.1	33	1.4
4	9 51.4	6	0	1	4	9.9	34	1 23.8	4	0.2	34	1.4
5	12 19.2	7	1	1	5	12.3	35	1 26.2	5	0.2	35	1.4
6	14 47.1	8	1	1	6	14.8	36	1 28.7	6	0.2	36	1.5
7	17 14.9	10	1	1	7	17.2	37	1 31.2	7	0.3	37	1.5
8	19 42.8	11	1	1	8	19.7	38	1 33.6	8	0.3	38	1.6
9	22 10.6	13	1	2	9	22.2	39	1 36.1	9	0.4	39	1.6
10	24 38.5	14	1	2	10	24.6	40	1 38.6	10	0.4	40	1.6
11	27 6.3	16	1	2	11	27.1	41	1 41.0	11	0.5	41	1.7
12	29 34.2	17	1	2	12	29.6	42	1 43.5	12	0.5	42	1.7
13	32 2.0	18	1	2	13	32.0	43	1 46.0	13	0.5	43	1.8
14	34 29.9	20	2	3	14	34.5	44	1 48.4	14	0.6	44	1.8
15	36 57.7	21	2	3	15	37.0	45	1 50.9	15	0.6	45	1.8
16	39 25.6	23	2	3	16	39.4	46	1 53.3	16	0.7	46	1.9
17	41 53.4	24	2	3	17	41.9	47	1 55.8	17	0.7	47	1.9
18	44 21.2	25	2	3	18	44.4	48	1 58.3	18	0.7	48	2.0
19	46 49.1	27	2	4	19	46.8	49	2 0.7	19	0.8	49	2.0
20	49 16.9	28	2	4	20	49.3	50	2 3.2	20	0.8	50	2.1
21	51 44.8	30	2	4	21	51.7	51	2 5.7	21	0.9	51	2.1
22	54 12.6	31	2	4	22	54.2	52	2 8.1	22	0.9	52	2.1
23	56 40.5	32	3	4	23	56.7	53	2 10.6	23	0.9	53	2.2
24	59 8.3	34	3	4	24	59.1	54	2 13.1	24	1.0	54	2.2
					25	1 1.6	55	2 15.5	25	1.0	55	2.3
					26	1 4.1	56	2 18.0	26	1.1	56	2.3
					27	1 6.5	57	2 20.5	27	1.1	57	2.3
					28	1 9.0	58	2 22.9	28	1.1	58	2.4
					29	1 11.5	59	2 25.4	29	1.2	59	2.4
					30	1 13.9	60	2 27.8	30	1.2	60	2.5

TABLE VI.

Correction additive for the Argument A.

ARGUMENT A.

M	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	0
000	22	25	27	27	25	22	19	17	17	19	22	25	27	27	25	22	19	17	17	19	22
010	23	26	28	28	26	23	20	18	18	20	23	26	28	29	27	24	20	18	18	20	22
020	24	27	29	30	28	25	21	19	19	21	24	27	29	30	28	25	22	19	19	21	24
030	25	28	30	31	29	26	23	20	20	22	25	28	30	31	29	26	24	21	20	22	25
040	26	29	31	32	30	28	24	22	21	23	26	29	32	32	31	28	24	22	21	23	26
050	27	30	32	33	32	29	26	23	22	24	27	30	32	33	32	29	26	23	22	24	27
060	27	30	33	34	33	30	27	24	23	25	27	30	33	34	34	30	27	24	24	25	27
070	28	30	34	35	34	31	28	25	25	26	28	30	34	35	35	32	28	26	25	26	28
080	29	32	35	36	35	33	30	27	26	26	29	32	35	36	36	33	30	27	26	27	29
090	30	33	36	37	36	34	31	28	27	27	30	33	36	37	37	34	31	28	27	27	30
100	31	34	37	38	37	35	32	29	28	28	31	34	37	38	38	35	32	29	28	28	31
110	31	34	37	39	38	36	33	30	29	29	31	35	37	39	39	36	33	30	29	29	31
120	32	35	38	40	39	37	34	31	30	30	32	35	38	40	39	37	34	31	30	30	32
130	33	36	38	40	40	38	35	32	30	31	33	36	39	40	40	38	35	32	30	31	33
140	33	36	39	41	41	39	36	33	31	31	33	36	39	41	41	39	36	33	31	31	33
150	34	36	39	41	41	40	37	34	32	32	34	37	40	41	42	40	37	34	32	32	34
160	34	37	40	42	42	40	38	35	33	33	34	37	40	42	42	41	38	35	33	33	34
170	35	37	40	42	43	41	38	35	33	33	35	37	40	42	43	41	39	36	34	34	35
180	35	37	40	42	43	41	39	36	34	34	35	37	40	43	43	42	39	36	34	34	35
190	35	38	40	43	43	42	40	37	35	34	35	38	41	43	43	42	40	37	35	34	35
200	35	38	41	43	44	43	40	37	35	34	35	38	41	43	44	43	40	38	35	34	35
210	36	38	41	43	44	43	41	38	36	35	36	38	41	43	44	43	41	38	36	35	36
220	36	38	40	43	44	43	41	38	36	35	36	38	40	43	44	43	41	38	36	35	36
230	36	38	40	43	44	43	41	39	36	35	36	38	40	43	44	43	41	39	37	35	36
240	36	38	40	42	44	43	41	39	37	35	36	38	40	43	44	43	42	39	37	35	36
250	36	37	40	42	43	43	42	39	37	35	36	37	40	42	44	43	42	39	37	35	36
260	36	37	40	42	43	43	42	39	37	35	36	37	40	42	43	43	42	39	37	36	36
270	35	37	39	41	43	43	42	39	37	35	35	37	39	42	43	43	42	39	37	36	35
280	35	36	39	41	42	43	41	39	37	35	35	36	39	41	43	43	41	39	37	35	35
290	35	36	38	40	42	42	41	39	37	35	35	36	38	41	42	42	41	39	37	35	35
300	35	36	38	40	41	42	41	39	37	35	35	36	38	40	42	42	41	39	37	35	35
310	34	35	37	39	41	41	41	39	36	35	34	35	37	39	41	42	41	39	37	35	34
320	34	35	36	38	40	41	40	38	36	35	34	35	36	39	40	41	40	39	36	35	34
330	33	34	36	38	39	40	40	38	36	34	33	34	36	38	40	40	40	38	36	34	33
340	33	33	35	37	39	39	39	38	36	34	33	33	35	37	39	40	39	38	36	34	33
350	33	33	34	36	38	39	38	37	35	33	33	33	34	36	38	39	39	37	35	33	33
360	32	32	33	35	37	38	38	36	35	33	32	32	33	35	37	38	38	37	35	33	32
370	31	31	32	34	36	37	37	36	34	32	31	31	32	34	36	37	37	36	34	33	31
380	31	30	31	33	35	36	36	35	34	32	31	31	32	33	35	36	36	35	34	32	31
390	30	30	31	32	34	35	35	34	33	31	30	30	31	32	34	35	35	35	33	31	30
400	29	29	30	31	33	34	34	34	32	31	29	29	30	31	33	34	34	34	32	31	29
410	29	28	29	30	31	33	33	33	32	30	29	28	29	30	32	33	33	33	32	30	29
420	28	27	28	29	30	32	32	32	31	29	28	27	28	29	31	32	32	32	31	29	28
430	27	27	27	28	29	31	31	31	30	29	27	27	27	28	29	31	31	31	30	29	27
440	27	26	26	27	28	29	30	30	29	28	27	26	26	27	28	30	30	30	29	28	27
450	26	25	25	25	27	28	29	29	29	27	26	25	25	26	27	28	29	29	29	27	26
460	25	24	24	24	26	27	28	28	28	26	25	24	24	25	26	27	28	28	28	27	25
470	25	23	23	23	24	26	27	27	27	26	25	23	23	23	25	26	27	27	27	26	25
480	24	22	22	22	23	24	26	26	26	25	24	22	22	22	23	25	26	26	26	25	24
490	23	21	21	21	22	23	24	25	25	24	23	22	21	21	22	23	25	25	25	24	23
500	22	21	20	20	21	22	23	24	24	23	22	21	20	20	21	22	23	24	24	23	22

The constant quantity 22 has been subtracted from the Epochs of A.

TABLE VI.

Correction additive for the Argument A.

ARGUMENT A.

M	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	0
500	22	21	20	20	21	22	23	24	24	23	22	21	20	20	21	22	23	24	24	23	22
510	21	20	19	19	19	21	22	23	23	22	21	20	19	19	20	21	22	23	23	23	21
520	20	19	18	18	18	19	21	22	22	22	20	19	18	18	18	20	21	22	22	22	20
530	20	18	17	17	17	18	19	21	21	21	20	18	17	17	17	18	20	21	21	21	20
540	19	17	16	16	16	17	18	20	20	20	19	17	16	16	16	17	18	20	20	20	19
550	18	17	15	15	15	16	17	18	19	19	18	17	15	15	15	16	17	18	19	19	18
560	17	16	15	14	14	15	16	17	18	18	17	16	15	14	14	15	16	17	18	18	17
570	17	15	14	13	13	14	15	16	17	17	17	15	14	13	13	13	15	16	17	17	17
580	16	15	13	12	12	13	13	15	16	16	16	15	13	12	12	12	14	15	16	17	16
590	16	14	12	11	11	11	12	14	15	16	15	14	12	11	11	11	12	14	15	16	15
600	15	13	11	10	10	10	11	13	14	15	14	13	12	10	10	10	11	13	14	15	14
610	14	13	11	9	9	9	10	12	13	14	14	13	11	10	9	9	10	12	13	14	14
620	14	12	10	9	8	9	9	11	12	13	13	12	10	9	8	8	9	11	12	13	13
630	13	12	10	8	7	8	8	10	12	13	13	12	10	8	7	7	8	10	11	13	12
640	13	11	9	7	6	7	7	9	11	12	12	11	9	7	6	6	7	9	11	12	12
650	12	11	9	7	5	6	6	8	10	11	11	11	9	7	6	5	6	8	10	11	11
660	11	10	8	6	5	5	5	7	9	11	11	10	8	6	5	5	5	7	9	11	11
670	11	10	8	6	4	4	4	6	8	10	10	10	8	6	4	4	5	6	8	10	10
680	10	9	8	6	4	4	4	5	8	9	10	9	8	6	4	3	4	6	8	9	10
690	10	9	7	5	3	3	3	5	7	9	10	9	7	5	3	3	3	5	7	9	10
700	9	9	7	5	3	3	2	4	6	8	9	9	7	5	3	2	3	4	6	8	9
710	9	9	7	5	3	3	2	3	6	8	9	9	7	5	3	2	2	4	6	8	9
720	9	9	7	5	3	3	1	3	5	7	9	9	7	5	3	1	2	3	5	8	9
730	9	8	7	5	3	2	1	2	5	7	9	9	7	5	2	1	1	3	5	7	9
740	8	8	7	5	2	2	1	2	4	7	8	8	7	5	2	1	1	2	4	7	8
750	8	8	7	5	2	1	0	2	4	7	8	9	7	5	2	1	0	2	4	7	8
760	8	9	7	5	2	1	0	1	4	6	8	9	7	5	3	1	0	2	4	6	8
770	8	9	8	5	3	1	0	1	4	6	8	9	8	5	3	1	0	1	4	6	8
780	8	9	8	6	3	1	0	1	3	6	8	9	8	6	3	1	0	1	4	6	8
790	8	9	8	6	3	1	0	1	3	6	8	9	8	6	3	1	0	1	3	6	8
800	9	10	9	6	4	2	0	1	3	6	9	10	9	7	4	2	0	1	3	6	9
810	9	10	9	7	4	2	1	1	3	6	9	10	9	7	4	2	1	1	4	6	9
820	9	10	10	8	5	3	1	1	4	6	9	10	10	8	5	2	1	2	4	7	9
830	10	11	10	8	6	3	1	2	4	7	9	11	11	8	6	3	2	2	4	7	9
840	10	11	11	9	6	4	2	2	4	7	10	11	11	9	6	4	2	2	4	7	10
850	10	12	12	10	7	4	2	2	4	7	10	12	12	10	7	4	3	3	5	7	10
860	11	13	13	11	8	5	3	3	5	8	11	13	13	11	8	5	3	3	5	8	11
870	11	13	14	12	9	6	4	4	5	8	11	13	14	12	9	6	4	4	5	8	11
880	12	14	14	13	10	7	5	4	6	9	12	14	14	13	10	7	5	4	6	9	12
890	13	15	15	14	11	8	6	5	7	9	12	15	15	14	10	8	6	5	7	10	13
900	13	16	16	15	12	10	7	6	7	10	13	16	16	15	12	9	7	6	7	10	13
910	14	17	17	16	13	10	8	7	8	11	14	17	17	16	13	10	8	7	8	11	14
920	15	17	18	17	14	11	9	8	9	12	15	17	18	17	15	11	9	8	9	12	15
930	16	18	19	18	16	13	10	9	10	12	16	18	19	18	16	13	10	9	10	12	16
940	16	19	20	20	17	14	11	10	11	13	16	19	21	20	17	14	11	10	11	13	16
950	17	20	22	21	18	15	12	11	11	14	17	20	22	21	18	15	12	11	12	14	17
960	18	21	23	22	20	16	13	12	12	15	18	21	23	22	20	16	13	12	13	15	18
970	19	22	24	23	21	18	15	13	13	16	19	22	24	24	21	18	15	13	14	16	19
980	20	23	25	25	22	19	16	14	15	17	20	23	25	25	23	19	16	14	15	17	20
990	21	24	26	26	24	20	17	15	16	18	21	24	26	26	24	21	17	16	16	18	21
0	22	25	27	27	25	22	19	17	17	19	22	25	27	27	25	22	19	17	17	19	22

The constant quantity 22 has been subtracted from the Epochs of A.

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. −	Mean Anomaly.	Equation.	Diff. +	S. V. −	Mean Anomaly.	Equation.	Diff. +	S. V. −
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
0 0 0	11 29 59 15.0	20.6	0.00	0 8 0	0 0 15 39.5	20.3	2.49	0 16 0	0 0 31 43.5	19.8	4.93
0 0 10	11 29 59 35.6	20.6	0.05	0 8 10	0 0 15 59.8	20.3	2.54	0 16 10	0 0 32 3.3	19.7	4.98
0 0 20	11 29 59 56.2	20.5	0.10	0 8 20	0 0 16 20.1	20.4	2.60	0 16 20	0 0 32 23.0	19.7	5.03
0 0 30	0 0 0 16.7	20.6	0.16	0 8 30	0 0 16 40.5	20.4	2.65	0 16 30	0 0 32 42.7	19.7	5.08
0 0 40	0 0 0 37.3	20.6	0.21	0 8 40	0 0 17 0.9	20.3	2.70	0 16 40	0 0 33 2.4	19.6	5.13
0 0 50	0 0 0 57.9	20.6	0.26	0 8 50	0 0 17 21.2	20.3	2.75	0 16 50	0 0 33 22.0	19.6	5.18
0 1 0	0 0 1 18.5	20.6	0.31	0 9 0	0 0 17 41.5	20.3	2.80	0 17 0	0 0 33 41.6	19.6	5.23
0 1 10	0 0 1 39.1	20.6	0.36	0 9 10	0 0 18 1.8	20.3	2.85	0 17 10	0 0 34 1.2	19.6	5.28
0 1 20	0 0 1 59.7	20.6	0.42	0 9 20	0 0 18 22.1	20.3	2.90	0 17 20	0 0 34 20.8	19.6	5.33
0 1 30	0 0 2 20.3	20.5	0.47	0 9 30	0 0 18 42.4	20.3	2.96	0 17 30	0 0 34 40.4	19.6	5.38
0 1 40	0 0 2 40.8	20.6	0.52	0 9 40	0 0 19 2.7	20.3	3.01	0 17 40	0 0 35 0.0	19.5	5.43
0 1 50	0 0 3 1.4	20.5	0.57	0 9 50	0 0 19 22.9	20.2	3.06	0 17 50	0 0 35 19.5	19.6	5.48
0 2 0	0 0 3 21.9	20.6	0.62	0 10 0	0 0 19 43.2	20.3	3.11	0 18 0	0 0 35 39.0	19.5	5.52
0 2 10	0 0 3 42.5	20.5	0.68	0 10 10	0 0 20 3.4	20.2	3.16	0 18 10	0 0 35 58.5	19.5	5.57
0 2 20	0 0 4 3.0	20.5	0.73	0 10 20	0 0 20 23.7	20.3	3.21	0 18 20	0 0 36 18.0	19.4	5.62
0 2 30	0 0 4 23.5	20.6	0.78	0 10 30	0 0 20 43.9	20.2	3.26	0 18 30	0 0 36 37.4	19.5	5.67
0 2 40	0 0 4 44.1	20.6	0.83	0 10 40	0 0 21 4.1	20.2	3.31	0 18 40	0 0 36 56.9	19.4	5.71
0 2 50	0 0 5 4.7	20.6	0.89	0 10 50	0 0 21 24.3	20.2	3.36	0 18 50	0 0 37 16.3	19.4	5.77
0 3 0	0 0 5 25.3	20.5	0.94	0 11 0	0 0 21 44.5	20.2	3.42	0 19 0	0 0 37 35.7	19.4	5.82
0 3 10	0 0 5 45.8	20.6	0.99	0 11 10	0 0 22 4.7	20.1	3.47	0 19 10	0 0 37 55.1	19.4	5.87
0 3 20	0 0 6 6.4	20.5	1.04	0 11 20	0 0 22 24.8	20.1	3.52	0 19 20	0 0 38 14.5	19.3	5.92
0 3 30	0 0 6 26.9	20.5	1.09	0 11 30	0 0 22 44.9	20.2	3.57	0 19 30	0 0 38 33.8	19.3	5.97
0 3 40	0 0 6 47.4	20.6	1.14	0 11 40	0 0 23 5.1	20.1	3.62	0 19 40	0 0 38 53.1	19.3	6.02
0 3 50	0 0 7 8.0	20.5	1.19	0 11 50	0 0 23 25.2	20.1	3.67	0 19 50	0 0 39 12.4	19.3	6.06
0 4 0	0 0 7 28.5	20.5	1.24	0 12 0	0 0 23 45.3	20.1	3.72	0 20 0	0 0 39 31.7	19.3	6.11
0 4 10	0 0 7 49.0	20.5	1.29	0 12 10	0 0 24 5.4	20.1	3.77	0 20 10	0 0 39 51.0	19.2	6.16
0 4 20	0 0 8 9.5	20.5	1.35	0 12 20	0 0 24 25.5	20.1	3.82	0 20 20	0 0 40 10.2	19.2	6.21
0 4 30	0 0 8 30.0	20.5	1.40	0 12 30	0 0 24 45.5	20.0	3.87	0 20 30	0 0 40 29.4	19.2	6.26
0 4 40	0 0 8 50.5	20.5	1.46	0 12 40	0 0 25 5.6	20.1	3.93	0 20 40	0 0 40 48.6	19.2	6.31
0 4 50	0 0 9 11.0	20.5	1.51	0 12 50	0 0 25 25.6	20.0	3.98	0 20 50	0 0 41 7.7	19.1	6.35
0 5 0	0 0 9 31.6	20.6	1.56	0 13 0	0 0 25 45.7	20.1	4.03	0 21 0	0 0 41 26.9	19.2	6.40
0 5 10	0 0 9 52.1	20.5	1.61	0 13 10	0 0 26 5.7	20.0	4.08	0 21 10	0 0 41 46.0	19.1	6.45
0 5 20	0 0 10 12.5	20.4	1.67	0 13 20	0 0 26 25.7	20.0	4.13	0 21 20	0 0 42 5.1	19.1	6.50
0 5 30	0 0 10 33.0	20.5	1.72	0 13 30	0 0 26 45.7	20.0	4.18	0 21 30	0 0 42 24.1	19.0	6.55
0 5 40	0 0 10 53.5	20.5	1.77	0 13 40	0 0 27 5.7	20.0	4.23	0 21 40	0 0 42 43.2	19.1	6.59
0 5 50	0 0 11 14.0	20.5	1.82	0 13 50	0 0 27 25.6	19.9	4.28	0 21 50	0 0 43 2.2	19.0	6.65
0 6 0	0 0 11 34.5	20.5	1.87	0 14 0	0 0 27 45.6	20.0	4.33	0 22 0	0 0 43 21.2	19.0	6.69
0 6 10	0 0 11 54.9	20.4	1.92	0 14 10	0 0 28 5.5	19.9	4.38	0 22 10	0 0 43 40.2	19.0	6.74
0 6 20	0 0 12 15.3	20.4	1.98	0 14 20	0 0 28 25.4	19.9	4.43	0 22 20	0 0 43 59.1	18.9	6.79
0 6 30	0 0 12 35.8	20.5	2.03	0 14 30	0 0 28 45.3	19.9	4.48	0 22 30	0 0 44 18.0	18.9	6.83
0 6 40	0 0 12 56.3	20.5	2.08	0 14 40	0 0 29 5.1	19.8	4.53	0 22 40	0 0 44 36.9	18.9	6.88
0 6 50	0 0 13 16.7	20.4	2.13	0 14 50	0 0 29 25.0	19.9	4.58	0 22 50	0 0 44 55.8	18.9	6.93
0 7 0	0 0 13 37.1	20.4	2.18	0 15 0	0 0 29 44.8	19.8	4.63	0 23 0	0 0 45 14.7	18.9	6.98
0 7 10	0 0 13 57.5	20.4	2.23	0 15 10	0 0 30 4.7	19.9	4.68	0 23 10	0 0 45 33.6	18.9	7.02
0 7 20	0 0 14 17.9	20.4	2.29	0 15 20	0 0 30 24.5	19.8	4.73	0 23 20	0 0 45 52.4	18.8	7.07
0 7 30	0 0 14 38.3	20.4	2.34	0 15 30	0 0 30 44.3	19.8	4.78	0 23 30	0 0 46 11.1	18.7	7.12
0 7 40	0 0 14 58.7	20.4	2.39	0 15 40	0 0 31 4.0	19.7	4.83	0 23 40	0 0 46 29.9	18.8	7.17
0 7 50	0 0 15 19.1	20.4	2.44	0 15 50	0 0 31 23.8	19.8	4.88	0 23 50	0 0 46 48.6	18.7	7.21
0 8 0	0 0 15 39.5		2.49	0 16 0	0 0 31 43.5	19.7	4.93	0 24 0	0 0 47 7.3	18.7	7.26

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff +	S. V. —	Mean Anomaly.	Equation.	Diff. +	S. V. —	Mean Anomaly.	Equation.	Diff. +	S. V. —
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
0 24 0	0 0 47 7.3	18.7	7.26	1 2 0	0 1 1 31.9	17.3	9.43	1 10 0	0 1 14 39.8	15.5	11.40
0 24 10	0 0 47 26.0	18.7	7.31	1 2 10	0 1 1 49.2	17.2	9.48	1 10 10	0 1 14 55.3	15.4	11.44
0 24 20	0 0 47 44.7	18.6	7.35	1 2 20	0 1 2 6.4	17.2	9.52	1 10 20	0 1 15 10.7	15.4	11.48
0 24 30	0 0 48 3.3	18.6	7.40	1 2 30	0 1 2 23.6	17.1	9.56	1 10 30	0 1 15 26.1	15.4	11.52
0 24 40	0 0 48 21.9	18.6	7.45	1 2 40	0 1 2 40.7	17.1	9.61	1 10 40	0 1 15 41.5	15.3	11.55
0 24 50	0 0 48 40.5	18.5	7.50	1 2 50	0 1 2 57.8	17.1	9.65	1 10 50	0 1 15 56.8	15.3	11.59
0 25 0	0 0 48 59.0	18.6	7.54	1 3 0	0 1 3 14.9	17.1	9.70	1 11 0	0 1 16 12.1	15.2	11.63
0 25 10	0 0 49 17.6	18.5	7.59	1 3 10	0 1 3 32.0	17.0	9.74	1 11 10	0 1 16 27.3	15.2	11.67
0 25 20	0 0 49 36.1	18.4	7.64	1 3 20	0 1 3 49.0	17.0	9.78	1 11 20	0 1 16 42.5	15.1	11.71
0 25 30	0 0 49 54.5	18.4	7.68	1 3 30	0 1 4 6.0	16.9	9.82	1 11 30	0 1 16 57.6	15.2	11.74
0 25 40	0 0 50 12.9	18.4	7.73	1 3 40	0 1 4 22.9	16.9	9.86	1 11 40	0 1 17 12.8	15.1	11.78
0 25 50	0 0 50 31.3	18.4	7.77	1 3 50	0 1 4 39.8	16.9	9.90	1 11 50	0 1 17 27.9	15.0	11.82
0 26 0	0 0 50 49.7	18.4	7.82	1 4 0	0 1 4 56.7	16.9	9.95	1 12 0	0 1 17 42.9	15.0	11.86
0 26 10	0 0 51 8.1	18.4	7.86	1 4 10	0 1 5 13.6	16.9	9.99	1 12 10	0 1 17 57.9	14.9	11.89
0 26 20	0 0 51 26.4	18.3	7.91	1 4 20	0 1 5 30.4	16.8	10.03	1 12 20	0 1 18 12.8	14.9	11.93
0 26 30	0 0 51 44.7	18.3	7.95	1 4 30	0 1 5 47.1	16.7	10.07	1 12 30	0 1 18 27.7	14.9	11.97
0 26 40	0 0 52 3.0	18.3	8.00	1 4 40	0 1 6 3.8	16.7	10.11	1 12 40	0 1 18 42.6	14.8	12.00
0 26 50	0 0 52 21.2	18.2	8.05	1 4 50	0 1 6 20.5	16.7	10.16	1 12 50	0 1 18 57.4	14.8	12.04
0 27 0	0 0 52 39.5	18.3	8.10	1 5 0	0 1 6 37.2	16.7	10.20	1 13 0	0 1 19 12.2	14.7	12.08
0 27 10	0 0 52 57.7	18.2	8.14	1 5 10	0 1 6 53.9	16.6	10.24	1 13 10	0 1 19 26.9	14.7	12.11
0 27 20	0 0 53 15.8	18.1	8.19	1 5 20	0 1 7 10.5	16.5	10.28	1 13 20	0 1 19 41.6	14.7	12.15
0 27 30	0 0 53 33.9	18.1	8.23	1 5 30	0 1 7 27.0	16.5	10.32	1 13 30	0 1 19 56.3	14.6	12.19
0 27 40	0 0 53 52.0	18.1	8.28	1 5 40	0 1 7 43.5	16.5	10.36	1 13 40	0 1 20 10.9	14.6	12.22
0 27 50	0 0 54 10 1	18.1	8.32	1 5 50	0 1 8 0.0	16.5	10.40	1 13 50	0 1 20 25.5	14.5	12.26
0 28 0	0 0 54 28.2	18.1	8.37	1 6 0	0 1 8 16.5	16.4	10.44	1 14 0	0 1 20 40.0	14.5	12.30
0 28 10	0 0 54 46.2	18.0	8.41	1 6 10	0 1 8 32.9	16.3	10.49	1 14 10	0 1 20 54.5	14.4	12.33
0 28 20	0 0 55 4.2	18 0	8.46	1 6 20	0 1 8 49.2	16.4	10.53	1 14 20	0 1 21 8.9	14.4	12.37
0 28 30	0 0 55 22.1	17.9	8.50	1 6 30	0 1 9 5.6	16.3	10.57	1 14 30	0 1 21 23.3	14.4	12.40
0 28 40	0 0 55 40.0	17.9	8.55	1 6 40	0 1 9 21.9	16.3	10.61	1 14 40	0 1 21 37.7	14.4	12.44
0 28 50	0 0 55 57.9	17.9	8.59	1 6 50	0 1 9 38.2	16.3	10.65	1 14 50	0 1 21 52.0	14.3	12.47
0 29 0	0 0 56 15.8	17.9	8.64	1 7 0	0 1 9 54.4	16.2	10.69	1 15 0	0 1 22 6.3	14.3	12.51
0 29 10	0 0 56 33.6	17.8	8.68	1 7 10	0 1 10 10.6	16.2	10.73	1 15 10	0 1 22 20.5	14.2	12.55
0 39 20	0 0 56 51.4	17.8	8.73	1 7 20	0 1 10 26.7	16.1	10.77	1 15 20	0 1 22 34.6	14.1	12.58
0 29 30	0 0 57 9.2	17.8	8.77	1 7 30	0 1 10 42.8	16.1	10.81	1 15 30	0 1 22 48.7	14.1	12.62
0 29 40	0 0 57 26.9	17.7	8.82	1 7 40	0 1 10 58.9	16.1	10.85	1 15 40	0 1 23 2.8	44.1	12.65
0 29 50	0 0 57 44.6	17.7	8.86	1 7 50	0 1 11 14.9	16.0	10.89	1 15 50	0 1 23 16.9	14.1	12.68
1 0 0	0 0 58 2.3	17.7	8.91	1 8 0	0 1 11 30.9	16.0	10.93	1 16 0	0 1 23 30.9	14.0	12.72
1 0 10	0 0 58 20.0	17.7	8.95	1 8 10	0 1 11 46.9	16.0	10.97	1 16 10	0 1 23 44.8	13.9	12.75
1 0 20	0 0 58 37.6	17.6	9.00	1 8 20	0 1 12 2.8	15.9	11.01	1 16 20	0 1 23 58.7	13.9	12.79
1 0 30	0 0 58 55.2	17.6	9.04	1 8 30	0 1 12 18.7	15.9	11.05	1 16 30	0 1 24 12.6	13.9	12.82
1 0 40	0 0 59 12.7	17.5	9.08	1 8 40	0 1 12 34.5	15.8	11.09	1 16 40	0 1 24 26.4	13.8	12.86
1 0 50	0 0 59 30.2	17.5	9.13	1 8 50	0 1 12 50.3	15.8	11.13	1 16 50	0 1 24 40.2	13.8	12.89
1 1 0	0 0 59 47.7	17.5	9.17	1 9 0	0 1 13 6.1	15.8	11.17	1 17 0	0 1 24 53.9	13.7	12.93
1 1 10	0 1 0 5.1	17.4	9.22	1 9 10	0 1 13 21.8	15.7	11.21	1 17 10	0 1 25 7.6	13.7	12.96
1 1 20	0 1 0 22.5	17.4	9.26	1 9 20	0 1 13 37.5	15.7	11.25	1 17 20	0 1 25 21.2	13.6	12.99
1 1 30	0 1 0 39.9	17.4	9.30	1 9 30	0 1 13 53.1	15.6	11.28	1 17 30	0 1 25 34.8	13.6	13.03
1 1 40	0 1 0 57.3	17.4	9.35	1 9 40	0 1 14 8.7	15.6	11.32	1 17 40	0 1 25 48.3	13.5	13.06
1 1 50	0 1 1 14.6	17.3	9.39	1 9 50	0 1 14 24.3	15.6	11.36	1 17 50	0 1 26 1.8	13.5	13.09
1 2 0	0 1 1 31.9	17.3	9.43	1 10 0	0 1 14 39.8	15.5	11.40	1 18 0	0 1 26 15.3	13.5	13.13

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. —	Mean Anomaly.	Equation.	Diff. +	S. V. —	Mean Anomaly.	Equation.	Diff. +	S. V. —
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
1 18 0	0 1 26 15.3	13.4	13.13	1 26 0	0 1 36 4.9	11.1	14.57	2 4 0	0 1 43 57.6	8.5	15.72
1 18 10	0 1 26 28.7	13.4	13.16	1 26 10	0 1 36 16.0	11.0	14.60	2 4 10	0 1 44 6.1	8.5	15.74
1 18 20	0 1 26 42.1	13.3	13.19	1 26 20	0 1 36 27.0	11.0	14.63	2 4 20	0 1 44 14.6	8.4	15.76
1 18 30	0 1 26 55.4	13.3	13.23	1 26 30	0 1 36 38.0	10.9	14.66	2 4 30	0 1 44 23.0	8.3	15.78
1 18 40	0 1 27 8.7	13.2	13.26	1 26 40	0 1 36 48.9	10.9	14.69	2 4 40	0 1 44 31.3	8.3	15.80
1 18 50	0 1 27 21.9	13.2	13.29	1 26 50	0 1 36 59.8	10.8	14.72	2 4 50	0 1 44 39.6	8.3	15.82
1 19 0	0 1 27 35.1	13.1	13.32	1 27 0	0 1 37 10.6	10.8	14.74	2 5 0	0 1 44 47.9	8.2	15.84
1 19 10	0 1 27 48.2	13.1	13.35	1 27 10	0 1 37 21.4	10.7	14.77	2 5 10	0 1 44 56.1	8.1	15.86
1 19 20	0 1 28 1.3	13.0	13.39	1 27 20	0 1 37 32.1	10.6	14.79	2 5 20	0 1 45 4.2	8.1	15.88
1 19 30	0 1 28 14.3	13.0	13.42	1 27 30	0 1 37 42.7	10.6	14.82	2 5 30	0 1 45 12.3	8.1	15.90
1 19 40	0 1 28 27.3	12.9	13.45	1 27 40	0 1 37 53.3	10.6	14.84	2 5 40	0 1 45 20.4	8.0	15.92
1 19 50	0 1 28 40.2	12.9	13.48	1 27 50	0 1 38 3.9	10.5	14.87	2 5 50	0 1 45 28.4	7.9	15.94
1 20 0	0 1 28 53.1	12.9	13.51	1 28 0	0 1 38 14.4	10.5	14.89	2 6 0	0 1 45 36.3	7.9	15.96
1 20 10	0 1 29 6.0	12.8	13.55	1 28 10	0 1 38 24.9	10.4	14.91	2 6 10	0 1 45 44.2	7.8	15.97
1 20 20	0 1 29 18.8	12.7	13.58	1 28 20	0 1 38 35.3	10.3	14.94	2 6 20	0 1 45 52.0	7.7	15.99
1 20 30	0 1 29 31.5	12.7	13.61	1 28 30	0 1 38 45.6	10.3	14.96	2 6 30	0 1 45 59.7	7.7	16.01
1 20 40	0 1 29 44.2	12.6	13.64	1 28 40	0 1 38 55.9	10.3	14.99	2 6 40	0 1 46 7.4	7.7	16.03
1 20 50	0 1 29 56.8	12.6	13.67	1 28 50	0 1 39 6.2	10.2	15.01	2 6 50	0 1 46 15.1	7.6	16.05
1 21 0	0 1 30 9.4	12.6	13.70	1 29 0	0 1 39 16.4	10.1	15.04	2 7 0	0 1 46 22.7	7.5	16.07
1 21 10	0 1 30 22.0	12.5	13.73	1 29 10	0 1 39 26.5	10.1	15.06	2 7 10	0 1 46 30.2	7.5	16.08
1 21 20	0 1 30 34.5	12.5	13.77	1 29 20	0 1 39 36.6	10.0	15.09	2 7 20	0 1 46 37.7	7.4	16.10
1 21 30	0 1 30 47.0	12.4	13.80	1 29 30	0 1 39 46.6	10.0	15.11	2 7 30	0 1 46 45.1	7.4	16.12
1 21 40	0 1 30 59.4	12.4	13.83	1 29 40	0 1 39 56.6	9.9	15.14	2 7 40	0 1 46 52.5	7.3	16.14
1 21 50	0 1 31 11.8	12.3	13.86	1 29 50	0 1 40 6.5	9.9	15.16	2 7 50	0 1 46 59.8	7.3	16.15
1 22 0	0 1 31 24.1	12.3	13.89	2 0 0	0 1 40 16.4	9.8	15.19	2 8 0	0 1 47 7.1	7.2	16.17
1 22 10	0 1 31 36.4	12.2	13.92	2 0 10	0 1 40 26.2	9.8	15.21	2 8 10	0 1 47 14.3	7.2	16.19
1 22 20	0 1 31 48.6	12.1	13.95	2 0 20	0 1 40 36.0	9.7	15.23	2 8 20	0 1 47 21.5	7.1	16.21
1 22 30	0 1 32 0.7	12.2	13.98	2 0 30	0 1 40 45.7	9.7	15.26	2 8 30	0 1 47 28.6	7.0	16.22
1 22 40	0 1 32 12.9	12.1	14.01	2 0 40	0 1 40 55.4	9.6	15.28	2 8 40	0 1 47 35.6	7.0	16.24
1 22 50	0 1 32 25.0	12.0	14.04	2 0 50	0 1 41 5.0	9.6	15.30	2 8 50	0 1 47 42.6	6.9	16.26
1 23 0	0 1 32 37.0	12.0	14.07	2 1 0	0 1 41 14.6	9.5	15.33	2 9 0	0 1 47 49.5	6.9	16.27
1 23 10	0 1 32 49.0	11.9	14.09	2 1 10	0 1 41 24.1	9.5	15.35	2 9 10	0 1 47 56.4	6.8	16.29
1 23 20	0 1 33 0.9	11.9	14.12	2 1 20	0 1 41 33.6	9.4	15.37	2 9 20	0 1 48 3.2	6.8	16.30
1 23 30	0 1 33 12.8	11.8	14.15	2 1 30	0 1 41 43.0	9.3	15.39	2 9 30	0 1 48 10.0	6.7	16.32
1 23 40	0 1 33 24.6	11.8	14.18	2 1 40	0 1 41 52.3	9.3	15.42	2 9 40	0 1 48 16.7	6.6	16.33
1 23 50	0 1 33 36.4	11.7	14.21	2 1 50	0 1 42 1.6	9.2	15.44	2 9 50	0 1 48 23.3	6.6	16.35
1 24 0	0 1 33 48.1	11.7	14.24	2 2 0	0 1 42 10.8	9.2	15.46	2 10 0	0 1 48 29.9	6.5	16.37
1 24 10	0 1 33 59.8	11.6	14.27	2 2 10	0 1 42 20.0	9.1	15.48	2 10 10	0 1 48 36.4	6.5	16.38
1 24 20	0 1 34 11.4	11.6	14.30	2 2 20	0 1 42 29.1	9.1	15.51	2 10 20	0 1 48 42.9	6.4	16.40
1 24 30	0 1 34 23.0	11.5	14.32	2 2 30	0 1 42 38.2	9.0	15.53	2 10 30	0 1 48 49.3	6.4	16.41
1 24 40	0 1 34 34.5	11.5	14.35	2 2 40	0 1 42 47.2	9.0	15.55	2 10 40	0 1 48 55.7	6.3	16.43
1 24 50	0 1 34 46.0	11.4	14.38	2 2 50	0 1 42 56.2	8.9	15.57	2 10 50	0 1 49 2.0	6.2	16.44
1 25 0	0 1 34 57.4	11.4	14.41	2 3 0	0 1 43 5.1	8.9	15.59	2 11 0	0 1 49 8.2	6.1	16.46
1 25 10	0 1 35 8.8	11.3	14.44	2 3 10	0 1 43 14.0	8.8	15.61	2 11 10	0 1 49 14.3	6.2	16.47
1 25 20	0 1 35 20.1	11.3	14.46	2 3 20	0 1 43 22.8	8.8	15.63	2 11 20	0 1 49 20.5	6.1	16.48
1 25 30	0 1 35 31.4	11.2	14.49	2 3 30	0 1 43 31.6	8.7	15.65	2 11 30	0 1 49 26.6	6.0	16.50
1 25 40	0 1 35 42.6	11.2	14.52	2 3 40	0 1 43 40.3	8.7	15.68	2 11 40	0 1 49 32.6	6.0	16.51
1 25 50	0 1 35 53.8	11.1	14.55	2 3 50	0 1 43 49.0	8.6	15.70	2 11 50	0 1 49 38.6	5.9	16.53
1 26 0	0 1 36 4.9		14.57	2 4 0	0 1 43 57.6		15.72	2 12 0	0 1 49 44.5		16.54

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. −	Mean Anomaly.	Equation.	Diff. +	S. V. −	Mean Anomaly.	Equation.	Diff. ±	S. V. −
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
2 12 0	0 1 49 44.5	5.8	16.54	2 20 0	0 1 53 20.1	3.1	17.03	2 28 0	0 1 54 41.2	0.3	17.18
2 12 10	0 1 49 50.3	5.8	16.55	2 20 10	0 1 53 23.2	3.0	17.04	2 28 10	0 1 54 41.5	0.2	17.18
2 12 20	0 1 49 56.1	5.7	16.57	2 20 20	0 1 53 26.2	2.9	17.04	2 28 20	0 1 54 41.7	0.1	17.18
2 12 30	0 1 50 1.8	5.7	16.58	2 20 30	0 1 53 29.1	2.9	17.05	2 28 30	0 1 54 41.8	0.0	17.18
2 12 40	0 1 50 7.5	5.6	16.59	2 20 40	0 1 53 32.0	2.8	17.06	2 28 40	0 1 54 41.8	0.0	17.18
2 12 50	0 1 50 13.1	5.6	16.61	2 20 50	0 1 53 34.8	2.8	17.06	2 28 50	0 1 54 41.8	0.1	17.18
2 13 0	0 1 50 18.7	5.5	16.62	2 21 0	0 1 53 37.6	2.7	17.07	2 29 0	0 1 54 41.7	0.1	17.18
2 13 10	0 1 50 24.2	5.4	16.63	2 21 10	0 1 53 40.3	2.7	17.07	2 29 10	0 1 54 41.6	0.1	17.18
2 13 20	0 1 50 29.6	5.4	16.65	2 21 20	0 1 53 43.0	2.6	17.08	2 29 20	0 1 54 41.5	0.2	17.18
2 13 30	0 1 50 35.0	5.3	16.66	2 21 30	0 1 53 45.6	2.5	17.08	2 29 30	0 1 54 41.3	0.3	17.17
2 13 40	0 1 50 40.3	5.3	16.67	2 21 40	0 1 53 48.1	2.5	17.09	2 29 40	0 1 54 41.0	0.3	17.17
2 13 50	0 1 50 45.6	5.2	16.68	2 21 50	0 1 53 50.6	2.4	17.09	2 29 50	0 1 54 40.7	0.4	17.17
2 14 0	0 1 50 50.8	5.2	16.70	2 22 0	0 1 53 53.0	2.4	17.10	3 0 0	0 1 54 40.3	0.5	17.17
2 14 10	0 1 50 56.0	5.1	16.71	2 22 10	0 1 53 55.4	2.3	17.10	3 0 10	0 1 54 39.8	0.5	17.17
2 14 20	0 1 51 1.1	5.1	16.72	2 22 20	0 1 53 57.7	2.2	17.11	3 0 20	0 1 54 39.3	0.6	17.16
2 14 30	0 1 51 6.2	5.0	16.73	2 22 30	0 1 53 59.9	2.2	17.11	3 0 30	0 1 54 38.7	0.6	17.16
2 14 40	0 1 51 11.2	4.9	16.74	2 22 40	0 1 54 2.1	2.1	17.12	3 0 40	0 1 54 38.1	0.7	17.16
2 14 50	0 1 51 16.1	4.9	16.75	2 22 50	0 1 54 4.2	2.1	17.12	3 0 50	0 1 54 37.4	0.7	17.15
2 15 0	0 1 51 21.0	4.8	16.76	2 23 0	0 1 54 6.3	2.0	17.13	3 1 0	0 1 54 36.7	0.8	17.15
2 15 10	0 1 51 25.8	4.8	16.78	2 23 10	0 1 54 8.3	2.0	17.13	3 1 10	0 1 54 35.9	0.9	17.15
2 15 20	0 1 51 30.6	4.7	16.79	2 23 20	0 1 54 10.3	1.9	17.13	3 1 20	0 1 54 35.0	0.9	17.14
2 15 30	0 1 51 35.3	4.6	16.80	2 23 30	0 1 54 12.2	1.8	17.14	3 1 30	0 1 54 34.1	1.0	17.14
2 15 40	0 1 51 39.9	4.6	16.81	2 23 40	0 1 54 14.0	1.7	17.14	3 1 40	0 1 54 33.1	1.0	17.14
2 15 50	0 1 51 44.5	4.5	16.82	2 23 50	0 1 54 15.7	1.7	17.15	3 1 50	0 1 54 32.1	1.1	17.13
2 16 0	0 1 51 49.0	4.5	16.83	2 24 0	0 1 54 17.4	1.7	17.15	3 2 0	0 1 54 31.0	1.2	17.13
2 16 10	0 1 51 53.5	4.4	16.84	2 24 10	0 1 54 19.1	1.6	17.16	3 2 10	0 1 54 29.8	1.2	17.13
2 16 20	0 1 51 57.9	4.3	16.85	2 24 20	0 1 54 20.7	1.6	17.16	3 2 20	0 1 54 28.6	1.3	17.12
2 16 30	0 1 52 2.2	4.2	16.86	2 24 30	0 1 54 22.3	1.5	17.16	3 2 30	0 1 54 27.3	1.3	17.12
2 16 40	0 1 52 6.4	4.3	16.87	2 24 40	0 1 54 23.8	1.4	17.16	3 2 40	0 1 54 26.0	1.4	17.11
2 16 50	0 1 52 10.7	4.2	16.88	2 24 50	0 1 54 25.2	1.4	17.16	3 2 50	0 1 54 24.6	1.4	17.11
2 17 0	0 1 52 14.9	4.1	16.89	2 25 0	0 1 54 26.6	1.3	17.17	3 3 0	0 1 54 23.2	1.5	17.11
2 17 10	0 1 52 19.0	4.0	16.90	2 25 10	0 1 54 27.9	1.3	17.17	3 3 10	0 1 54 21.7	1.6	17.10
2 17 20	0 1 52 23.0	4.0	16.90	2 25 20	0 1 54 29.2	1.2	17.17	3 3 20	0 1 54 20.1	1.6	17.09
2 17 30	0 1 52 27.0	4.0	16.91	2 25 30	0 1 54 30.4	1.1	17.17	3 3 30	0 1 54 18.5	1.7	17.09
2 17 40	0 1 52 31.0	3.9	16.92	2 25 40	0 1 54 31.5	1.0	17.17	3 3 40	0 1 54 16.8	1.7	17.09
2 17 50	0 1 52 34.9	3.8	16.93	2 25 50	0 1 54 32.5	1.0	17.17	3 3 50	0 1 54 15.1	1.8	17.08
2 18 0	0 1 52 38.7	3.8	16.94	2 26 0	0 1 54 33.5	1.0	17.18	3 4 0	0 1 54 13.3	1.9	17.08
2 18 10	0 1 52 42.5	3.7	16.95	2 26 10	0 1 54 34.5	0.9	17.18	3 4 10	0 1 54 11.4	1.9	17.07
2 18 20	0 1 52 46.2	3.6	16.96	2 26 20	0 1 54 35.4	0.8	17.18	3 4 20	0 1 54 9.5	1.9	17.06
2 18 30	0 1 52 49.8	3.6	16.96	2 26 30	0 1 54 36.2	0.8	17.18	3 4 30	0 1 54 7.6	2.0	17.06
2 18 40	0 1 52 53.4	3.6	16.97	2 26 40	0 1 54 37.0	0.7	17.18	3 4 40	0 1 54 5.6	2.1	17.05
2 18 50	0 1 52 57.0	3.5	16.98	2 26 50	0 1 54 37.7	0.7	17.18	3 4 50	0 1 54 3.5	2.1	17.05
2 19 0	0 1 53 0.5	3.4	16.99	2 27 0	0 1 54 38.4	0.6	17.18	3 5 0	0 1 54 1.4	2.2	17.04
2 19 10	0 1 53 3.9	3.3	16.99	2 27 10	0 1 54 39.0	0.6	17.18	3 5 10	0 1 53 59.2	2.3	17.03
2 19 20	0 1 53 7.2	3.3	17.00	2 27 20	0 1 54 39.6	0.5	17.18	3 5 20	0 1 53 56.9	2.3	17.03
2 19 30	0 1 53 10.5	3.3	17.01	2 27 30	0 1 54 40.1	0.4	17.18	3 5 30	0 1 53 54.6	2.4	17.02
2 19 40	0 1 53 13.8	3.2	17.02	2 27 40	0 1 54 40.5	0.4	17.18	3 5 40	0 1 53 52.2	2.5	17.01
2 19 50	0 1 53 17.0	3.1	17.02	2 27 50	0 1 54 40.9	0.3	17.18	3 5 50	0 1 53 49.7	2.5	17.00
2 20 0	0 1 53 20.1		17.03	2 28 0	0 1 54 41.2		17.18	3 6 0	0 1 53 47.2		17.00

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
3 6 0	0 1 53 47.2	2.6	17.00	3 14 0	0 1 50 40.6	5.2	16.49	3 22 0	0 1 45 26.1	7.9	15.67
3 6 10	0 1 53 44.6	2.6	16.99	3 14 10	0 1 50 35.4	5.3	16.47	3 22 10	0 1 45 18.2	8.0	15.65
3 6 20	0 1 53 42.0	2.6	16.98	3 14 20	0 1 50 30.1	5.4	16.46	3 22 20	0 1 45 10.2	8.0	15.63
3 6 30	0 1 53 39.4	2.7	16.98	3 14 30	0 1 50 24.7	5.5	16.45	3 22 30	0 1 45 2.2	8.0	15.61
3 6 40	0 1 53 36.7	2.8	16.97	3 14 40	0 1 50 19.2	5.5	16.43	3 22 40	0 1 44 54.2	8.1	15.59
3 6 50	0 1 53 33.9	2.8	16.96	3 14 50	0 1 50 13.7	5.5	16.42	3 22 50	0 1 44 46.1	8.1	15.57
3 7 0	0 1 53 31.1	2.9	16.95	3 15 0	0 1 50 8.2	5.6	16.40	3 23 0	0 1 44 38 0	8.2	15.55
3 7 10	0 1 53 28.2	2.9	16.95	3 15 10	0 1 50 2.6	5.7	16.39	3 23 10	0 1 44 29.8	8.3	15.53
3 7 20	0 1 53 25.3	3.0	16.94	3 15 20	0 1 49 56.9	5.7	16.37	3 23 20	0 1 44 21.5	8.3	15.51
3 7 30	0 1 53 22.3	3.1	16.93	3 15 30	0 1 49 51.2	5.8	16.36	3 23 30	0 1 44 13.2	8.3	15.48
3 7 40	0 1 53 19.2	3.1	16.92	3 15 40	0 1 49 45.4	5.8	16.34	3 23 40	0 1 44 4.9	8.4	15.46
3 7 50	0 1 53 16.1	3.2	16.91	3 15 50	0 1 49 39.6	5.9	16.33	3 23 50	0 1 43 56.5	8.4	15.44
3 8 0	0 1 53 12.9	3.3	16.90	3 16 0	0 1 49 33.7	5.9	16.31	3 24 0	0 1 43 48.1	8.5	15.42
3 8 10	0 1 53 9.6	3.3	16.89	3 16 10	0 1 49 27.8	6.0	16.30	3 24 10	0 1 43 39.6	8.5	15.39
3 8 20	0 1 53 6.3	3.3	16.88	3 16 20	0 1 49 21.8	6.0	16.28	3 24 20	0 1 43 31.1	8.6	15.37
3 8 30	0 1 53 3.0	3.4	16.87	3 16 30	0 1 49 15.8	6.1	16.27	3 24 30	0 1 43 22.5	8.7	15.35
3 8 40	0 1 52 59.6	3.5	16.87	3 16 40	0 1 49 9.7	6.2	16.25	3 24 40	0 1 43 13.8	8.7	15.33
3 8 50	0 1 52 56.1	3.5	16.86	3 16 50	0 1 49 3.5	6.2	16.23	3 24 50	0 1 43 5.1	8.7	15.31
3 9 0	0 1 52 52.6	3.5	16.85	3 17 0	0 1 48 57.3	6.2	16.22	3 25 0	0 1 42 56.4	8.8	15.29
3 9 10	0 1 52 49.1	3.6	16.84	3 17 10	0 1 48 51.1	6.3	16.20	3 25 10	0 1 42 47.6	8.9	15.27
3 9 20	0 1 52 45.5	3.7	16.83	3 17 20	0 1 48 44.8	6.4	16.18	3 25 20	0 1 42 38.7	8.9	15.24
3 9 30	0 1 52 41.8	3.8	16.82	3 17 30	0 1 48 38.4	6.5	16.17	3 25 30	0 1 42 29.8	9.0	15.22
3 9 40	0 1 52 38.0	3.8	16.81	3 17 40	0 1 48 31.9	6.5	16.15	3 25 40	0 1 42 20.8	9.0	15.20
3 9 50	0 1 52 34.2	3.9	16.80	3 17 50	0 1 48 25.4	6.5	16.14	3 25 50	0 1 42 11.8	9.1	15.17
3 10 0	0 1 52 30.3	3.9	16.78	3 18 0	0 1 48 18.9	6.6	16.12	3 26 0	0 1 42 2.7	9.1	15.15
3 10 10	0 1 52 26.4	4.0	16.77	3 18 10	0 1 48 12.3	6.6	16.10	3 26 10	0 1 41 53 6	9.1	15.13
3 10 20	0 1 52 22.4	4.1	16.76	3 18 20	0 1 48 5.7	6.7	16.08	3 26 20	0 1 41 44.5	9.2	15.10
3 10 30	0 1 52 18.3	4.1	16.75	3 18 30	0 1 47 59.0	6.7	16.07	3 26 30	0 1 41 35.3	9.3	15.08
3 10 40	0 1 52 14.2	4.1	16.74	3 18 40	0 1 47 52.3	6.8	16.05	3 26 40	0 1 41 26.0	9.3	15.06
3 10 50	0 1 52 10.1	4.2	16.73	3 18 50	0 1 47 45.5	6.9	16.03	3 26 50	0 1 41 16.7	9.4	15.03
3 11 0	0 1 52 5.9	4.3	16.72	3 19 0	0 1 47 38.6	6.9	16.01	3 27 0	0 1 41 7.3	9.4	15.01
3 11 10	0 1 52 1.6	4.3	16.71	3 19 10	0 1 47 31.7	7.0	16.00	3 27 10	0 1 40 57.9	9.4	14.99
3 11 20	0 1 51 57.3	4.4	16.70	3 19 20	0 1 47 24.7	7.0	15 98	3 27 20	0 1 40 48.5	9.5	14.96
3 11 30	0 1 51 52.9	4.4	16.68	3 19 30	0 1 47 17.7	7.1	15.96	3 27 30	0 1 40 39.0	9.6	14.94
3 11 40	0 1 51 48.5	4.5	16.67	3 19 40	0 1 47 10.6	7.1	15.94	3 27 40	0 1 40 29.4	9.6	14.91
3 11 50	0 1 51 44.0	4.5	16.66	3 19 50	0 1 47 3.5	7.2	15.92	3 27 50	0 1 40 19.8	9.7	14.89
3 12 0	0 1 51 39.5	4.6	16.65	3 20 0	0 1 46 56.3	7.2	15.90	3 28 0	0 1 40 10.1	9.7	14.87
3 12 10	0 1 51 34.9	4.6	16.64	3 20 10	0 1 46 49.1	7.3	15.88	3 28 10	0 1 40 0.4	9.7	14.84
3 12 20	0 1 51 30.3	4.7	16.62	3 20 20	0 1 46 41.8	7.3	15.87	3 28 20	0 1 39 50.7	9.9	14.82
3 12 30	0 1 51 25.6	4.8	16.61	3 20 30	0 1 46 34.5	7.4	15.85	3 28 30	0 1 39 40.8	9.8	14.79
3 12 40	0 1 51 20.8	4.8	16.60	3 20 40	0 1 46 27.1	7.5	15.83	3 28 40	0 1 39 31.0	9.9	14.77
3 12 50	0 1 51 16.0	4.9	16.58	3 20 50	0 1 46 19.6	7.5	15.81	3 28 50	0 1 39 21.1	10.0	14.74
3 13 0	0 1 51 11.1	4.9	16.57	3 21 0	0 1 46 12.1	7.5	15.79	3 29 0	0 1 39 11.1	10.0	14.72
3 13 10	0 1 51 6.2	5.0	16.56	3 21 10	0 1 46 4.6	7.6	15.77	3 29 10	0 1 39 1.1	10.1	14.69
3 13 20	0 1 51 1.2	5.1	16 55	3 21 20	0 1 45 57.0	7.6	15.75	3 29 20	0 1 38 51.0	10.1	14.67
3 13 30	0 1 50 56.1	5.1	16.53	3 21 30	0 1 45 49.4	7.7	15.73	3 29 30	0 1 38 40.9	10.1	14.64
3 13 40	0 1 50 51.0	5.2	16.52	3 21 40	0 1 45 41.7	7 8	15.71	3 29 40	0 1 38 30.8	10.2	14.61
3 13 50	0 1 50 45.8	5.2	16.50	3 21 50	0 1 45 33.9	7.8	15.69	3 29 50	0 1 38 20.6	10.3	14.59
3 14 0	0 1 50 40.6		16.49	3 22 0	0 1 45 26.1		15.67	4 0 0	0 1 38 10.3		14.56

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
4 0 0	0 1 38 10.3	10.4	14.56	4 8 0	0 1 29 2.8	12.5	13.19	4 16 0	0 1 18 14.5	14.5	11.58
4 0 10	0 1 37 59.9	10.3	14.53	4 8 10	0 1 28 50.3	12.6	13.16	4 16 10	0 1 18 00.0	14.5	11.54
4 0 20	0 1 37 49.6	10.4	14.51	4 8 20	0 1 28 37.7	12.6	13.12	4 16 20	0 1 17 45.5	14.6	11.50
4 0 30	0 1 37 39.2	10.4	14.48	4 8 30	0 1 28 25.1	12.7	13.09	4 16 30	0 1 17 30.9	14.6	11.47
4 0 40	0 1 37 28.8	10.5	14.46	4 8 40	0 1 28 12.4	12.7	13.06	4 16 40	0 1 17 16.3	14.7	11.43
4 0 50	0 1 37 18.3	10.5	14.43	4 8 50	0 1 27 59.7	12.7	13.03	4 16 50	0 1 17 1.6	14.7	11.40
4 1 0	0 1 37 7.8	10.6	14.40	4 9 0	0 1 27 47.0	12.8	13.00	4 17 0	0 1 16 46.9	14.7	11.36
4 1 10	0 1 36 57.2	10.6	14.38	4 9 10	0 1 27 34.2	12.8	12.97	4 17 10	0 1 16 32.2	14.8	11.32
4 1 20	0 1 36 46.6	10.7	14.35	4 9 20	0 1 27 21.4	12.8	12.93	4 17 20	0 1 16 17.4	14.8	11.28
4 1 30	0 1 36 35.9	10.7	14.32	4 9 30	0 1 27 8.6	12.9	12.90	4 17 30	0 1 16 2.6	14.8	11.25
4 1 40	0 1 36 25.2	10.8	14.30	4 9 40	0 1 26 55.7	13.0	12.87	4 17 40	0 1 15 47.8	14.8	11.21
4 1 50	0 1 36 14.4	10.9	14.27	4 9 50	0 1 26 42.7	13.0	12.84	4 17 50	0 1 15 33.0	14.9	11.18
4 2 0	0 1 36 3.5	10.9	14.24	4 10 0	0 1 26 29.7	13.0	12.81	4 18 0	0 1 15 18.1	15.0	11.14
4 2 10	0 1 35 52.6	10.9	14.22	4 10 10	0 1 26 16.7	13.1	12.77	4 18 10	0 1 15 3.1	15.0	11.10
4 2 20	0 1 35 41.7	11.0	14.19	4 10 20	0 1 26 3.6	13.1	12.74	4 18 20	0 1 14 48.1	15.0	11.07
4 2 30	0 1 35 30.7	11.0	14.16	4 10 30	0 1 25 50.5	13.2	12.71	4 18 30	0 1 14 33.1	15.1	11.03
4 2 40	0 1 35 19.7	11.1	14.14	4 10 40	0 1 25 37.3	13.2	12.68	4 18 40	0 1 14 18.0	15.0	10.99
4 2 50	0 1 35 8.6	11.1	14.11	4 10 50	0 1 25 24.1	13.3	12.64	4 18 50	0 1 14 3.0	15.1	10.96
4 3 0	0 1 34 57.5	11.1	14.09	4 11 0	0 1 25 10.8	13.3	12.61	4 19 0	0 1 13 47.9	15.2	10.92
4 3 10	0 1 34 46.4	11.2	14.06	4 11 10	0 1 24 57.5	13.3	12.58	4 19 10	0 1 13 32.7	15.2	10.88
4 3 20	0 1 34 35.2	11.3	14.03	4 11 20	0 1 24 44.2	13.4	12.54	4 19 20	0 1 13 17.5	15.2	10.84
4 3 30	0 1 34 23.9	11.3	14.00	4 11 30	0 1 24 30.8	13.4	12.51	4 19 30	0 1 13 2.3	15.3	10.81
4 3 40	0 1 34 12.6	11.3	13.97	4 11 40	0 1 24 17.4	13.4	12.48	4 19 40	0 1 12 47.0	15.3	10.77
4 3 50	0 1 34 1.3	11.4	13.94	4 11 50	0 1 24 04.0	13.5	12.44	4 19 50	0 1 12 31.7	15.3	10.73
4 4 0	0 1 33 49.9	11.4	13.91	4 12 0	0 1 23 50.5	13.6	12.41	4 20 0	0 1 12 16.4	15.4	10.69
4 4 10	0 1 33 38.5	11.5	13.88	4 12 10	0 1 23 36.9	13.6	12.38	4 20 10	0 1 12 1.0	15.4	10.66
4 4 20	0 1 33 27.0	11.5	13.85	4 12 20	0 1 23 23.3	13.6	12.34	4 20 20	0 1 11 45.6	15.5	10.62
4 4 30	0 1 33 15.5	11.6	13.82	4 12 30	0 1 23 9.7	13.7	12.31	4 20 30	0 1 11 30.1	15.5	10.58
4 4 40	0 1 33 3.9	11.6	13.79	4 12 40	0 1 22 56.0	13.7	12.28	4 20 40	0 1 11 14.6	15.5	10.54
4 4 50	0 1 32 52.3	11.7	13.76	4 12 50	0 1 22 42.3	13.7	12.24	4 20 50	0 1 10 59.1	15.6	10.50
4 5 0	0 1 32 40.6	11.7	13.73	4 13 0	0 1 22 28.6	13.8	12.21	4 21 0	0 1 10 43.5	15.5	10.46
4 5 10	0 1 32 28.9	11.8	13.70	4 13 10	0 1 22 14.8	13.8	12.17	4 21 10	0 1 10 28.0	15.6	10.43
4 5 20	0 1 32 17.1	11.8	13.67	4 13 20	0 1 22 01.0	13.8	12.14	4 21 20	0 1 10 12.4	15.6	10.39
4 5 30	0 1 32 5.3	11.9	13.64	4 13 30	0 1 21 47.2	13.9	12.10	4 21 30	0 1 9 56.8	15.7	10.35
4 5 40	0 1 31 53.4	11.9	13.61	4 13 40	0 1 21 33.3	14.0	12.07	4 21 40	0 1 9 41.1	15.7	10.31
4 5 50	0 1 31 41.5	11.9	13.58	4 13 50	0 1 21 19.3	14.0	12.04	4 21 50	0 1 9 25.4	15.8	10.27
4 6 0	0 1 31 29.6	12.0	13.55	4 14 0	0 1 21 5.3	14.0	12.00	4 22 0	0 1 9 9.6	15.8	10.23
4 6 10	0 1 31 17.6	12.0	13.52	4 14 10	0 1 20 51.3	14.1	11.97	4 22 10	0 1 8 53.8	15.9	10.19
4 6 20	0 1 31 5.6	12.1	13.49	4 14 20	0 1 20 37.2	14.1	11.93	4 22 20	0 1 8 37.9	15.8	10.15
4 6 30	0 1 30 53.5	12.1	13.46	4 14 30	0 1 20 23.1	14.1	11.90	4 22 30	0 1 8 22.1	15.9	10.11
4 6 40	0 1 30 41.4	12.2	13.43	4 14 40	0 1 20 9.0	14.2	11.86	4 22 40	0 1 8 6.2	15.9	10.07
4 6 50	0 1 30 29.2	12.2	13.40	4 14 50	0 1 19 54.8	14.2	11.83	4 22 50	0 1 7 50.3	15.9	10.04
4 7 0	0 1 30 17.0	12.3	13.37	4 15 0	0 1 19 40.6	14.3	11.79	4 23 0	0 1 7 34.4	16.0	10.00
4 7 10	0 1 30 4.7	12.3	13.34	4 15 10	0 1 19 26.3	14.3	11.76	4 23 10	0 1 7 18.4	16.1	9.95
4 7 20	0 1 29 52.4	12.3	13.31	4 15 20	0 1 19 12.0	14.3	11.72	4 23 20	0 1 7 2.3	16.0	9.91
4 7 30	0 1 29 40.1	12.4	13.28	4 15 30	0 1 18 57.7	14.4	11.69	4 23 30	0 1 6 46.3	16.1	9.88
4 7 40	0 1 29 27.7	12.4	13.25	4 15 40	0 1 18 43.3	14.4	11.65	4 23 40	0 1 6 30.2	16.1	9.84
4 7 50	0 1 29 15.3	12.5	13.22	4 15 50	0 1 18 28.9	14.4	11.61	4 23 50	0 1 6 14.1	16.1	9.80
4 8 0	0 1 29 2.8		13.19	4 16 0	0 1 18 14.5		11.58	4 24 0	0 1 5 58.0		9.76

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
4 24 0	0 1 5 58.0	16.2	9.76	5 2 0	0 0 52 27.4	17.6	7.77	5 10 0	0 0 37 58.1	18.6	5.65
4 24 10	0 1 5 41.8	16.2	9.72	5 2 10	0 0 52 9.8	17.6	7.73	5 10 10	0 0 37 39.5	18.7	5.60
4 24 20	0 1 5 25.6	16.3	9.68	5 2 20	0 0 51 52.2	17.6	7.69	5 10 20	0 0 37 20.8	18.7	5.56
4 24 30	0 1 5 9.3	16.3	9.64	5 2 30	0 0 51 34.6	17.6	7.64	5 10 30	0 0 37 2.1	18.7	5.51
4 24 40	0 1 4 53.0	16.3	9.60	5 2 40	0 0 51 17.0	17.7	7.60	5 10 40	0 0 36 43.4	18.7	5.47
4 24 50	0 1 4 36.7	16.3	9.56	5 2 50	0 0 50 59.3	17.7	7.56	5 10 50	0 0 36 24.7	18.7	5.42
4 25 0	0 1 4 20.4	16.4	9.52	5 3 0	0 0 50 41.6	17.7	7.51	5 11 0	0 0 36 6.0	18.7	5.38
4 25 10	0 1 4 4.0	16.4	9.48	5 3 10	0 0 50 23.9	17.8	7.47	5 11 10	0 0 35 47.3	18.7	5.33
4 25 20	0 1 3 47.6	16.4	9.44	5 3 20	0 0 50 6.1	17.7	7.43	5 11 20	0 0 35 28.6	18.8	5.29
4 25 30	0 1 3 31.2	16.5	9.40	5 3 30	0 0 49 48.4	17.8	7.38	5 11 30	0 0 35 9.8	18.8	5.24
4 25 40	0 1 3 14.7	16.5	9.36	5 3 40	0 0 49 30.6	17.8	7.34	5 11 40	0 0 34 51.0	18.8	5.19
4 25 50	0 1 2 58.2	16.5	9.32	5 3 50	0 0 49 12.8	17.9	7.30	5 11 50	0 0 34 32.2	18.8	5.15
4 26 0	0 1 2 41.7	16.6	9.28	5 4 0	0 0 48 54.9	17.8	7.25	5 12 0	0 0 34 13.4	18.8	5.10
4 26 10	0 1 2 25.1	16.6	9.24	5 4 10	0 0 48 37.1	17.9	7.21	5 12 10	0 0 33 54.6	18.9	5.06
4 26 20	0 1 2 8.5	16.6	9.20	5 4 20	0 0 48 19.2	17.9	7.17	5 12 20	0 0 33 35.7	18.8	5.01
4 26 30	0 1 1 51.9	16.6	9.16	5 4 30	0 0 48 1.3	18.0	7.12	5 12 30	0 0 33 16.9	18.9	4.96
4 26 40	0 1 1 35.3	16.7	9.12	5 4 40	0 0 47 43.3	17.9	7.08	5 12 40	0 0 32 58.0	18.9	4.92
4 26 50	0 1 1 18.6	16.8	9.07	5 4 50	0 0 47 25.4	18.0	7.03	5 12 50	0 0 32 39.1	18.9	4.87
4 27 0	0 1 1 1.8	16.7	9.03	5 5 0	0 0 47 7.4	18.0	6.99	5 13 0	0 0 32 20.2	19.0	4.83
4 27 10	0 1 0 45.1	16.7	8.99	5 5 10	0 0 46 49.4	18.0	6.95	5 13 10	0 0 32 1.2	18.9	4.78
4 27 20	0 1 0 28.4	16.8	8.95	5 5 20	0 0 46 31.4	18.0	6.90	5 13 20	0 0 31 42.3	19.0	4.73
4 27 30	0 1 0 11.6	16.9	8.91	5 5 30	0 0 46 13.4	18.1	6.86	5 13 30	0 0 31 23.3	18.9	4.69
4 27 40	0 0 59 54.7	16.8	8.87	5 5 40	0 0 45 55.3	18.1	6.83	5 13 40	0 0 31 4.4	19.0	4.64
4 27 50	0 0 59 37.9	16.9	8.83	5 5 50	0 0 45 37.2	18.1	6.77	5 13 50	0 0 30 45.4	19.1	4.59
4 28 0	0 0 59 21.0	16.9	8.79	5 6 0	0 0 45 19.1	18.2	6.73	5 14 0	0 0 30 26.3	19.0	4.55
4 28 10	0 0 59 4.1	16.9	8.75	5 6 10	0 0 45 0.9	18.1	6.68	5 14 10	0 0 30 7.3	19.0	4.50
4 28 20	0 0 58 47.2	17.0	8.70	5 6 20	0 0 44 42.8	18.2	6.64	5 14 20	0 0 29 48.3	19.1	4.46
4 28 30	0 0 58 30.2	17.0	8.66	5 6 30	0 0 44 24.6	18.2	6.59	5 14 30	0 0 29 29.2	19.0	4.41
4 28 40	0 0 58 13.2	17.1	8.62	5 6 40	0 0 44 6.4	18.3	6.55	5 14 40	0 0 29 10.2	19.1	4.36
4 28 50	0 0 57 56.1	17.0	8.58	5 6 50	0 0 43 48.1	18.2	6.50	5 14 50	0 0 28 51.1	19 1	4.32
4 29 0	0 0 57 39.1	17.1	8.54	5 7 0	0 0 43 29.9	18.2	6.46	5 15 0	0 0 28 32.0	19.1	4.27
4 29 10	0 0 57 22.0	17.1	8.49	5 7 10	0 0 43 11.7	18.3	6.41	5 15 10	0 0 28 12.9	19.1	4.22
4 29 20	0 0 57 4.9	17.2	8.45	5 7 20	0 0 42 53.4	18.3	6.37	5 15 20	0 0 27 53.8	19.2	4.18
4 29 30	0 0 56 47.7	17.1	8.41	5 7 30	0 0 42 35.1	18.4	6.33	5 15 30	0 0 27 34.6	19.1	4.13
4 29 40	0 0 56 30.6	17.2	8.37	5 7 40	0 0 42 16.7	18.3	6.28	5 15 40	0 0 27 15.5	19.2	4.08
4 29 50	0 0 56 13.4	17.2	8.33	5 7 50	0 0 41 58.4	18.4	6.24	5 15 50	0 0 26 56.3	19.2	4.04
5 0 0	0 0 55 56.2	17.3	8.28	5 8 0	0 0 41 40.0	18.4	6.19	5 16 0	0 0 26 37.1	19.2	3.99
5 0 10	0 0 55 38.9	17.3	8.24	5 8 10	0 0 41 21.6	18.4	6.15	5 16 10	0 0 26 17.9	19.2	3.94
5 0 20	0 0 55 21.6	17.3	8.20	5 8 20	0 0 41 3.2	18.4	6.10	5 16 20	0 0 25 58.7	19.2	3.90
5 0 30	0 0 55 4.3	17.3	8.16	5 8 30	0 0 40 44.8	18.5	6.06	5 16 30	0 0 25 39.5	19.2	3.85
5 0 40	0 0 54 47.0	17.4	8.11	5 8 40	0 0 40 26.3	18.4	6.01	5 16 40	0 0 25 20.3	19.2	3.80
5 0 50	0 0 54 29.6	17.4	8.07	5 8 50	0 0 40 7.9	18.5	5.97	5 16 50	0 0 25 1.1	19.3	3.76
5 1 0	0 0 54 12.2	17.4	8.03	5 9 0	0 0 39 49.4	18.5	5.92	5 17 0	0 0 24 41.8	19.3	3.71
5 1 10	0 0 53 54.8	17.4	7.99	5 9 10	0 0 39 30.9	18.5	5.87	5 17 10	0 0 24 22.5	19.2	3.66
5 1 20	0 0 53 37.4	17.5	7.94	5 9 20	0 0 39 12.4	18.6	5.83	5 17 20	0 0 24 3.3	19.3	3.62
5 1 30	0 0 53 19.9	17.5	7.90	5 9 30	0 0 38 53.8	18.6	5.78	5 17 30	0 0 23 44.0	19.3	3.57
5 1 40	0 0 53 2.4	17.5	7.86	5 9 40	0 0 38 35.2	18.5	5.72	5 17 40	0 0 23 24.7	19.3	3.52
5 1 50	0 0 52 44.9	17.5	7.82	5 9 50	0 0 38 16.7	18.6	5.69	5 17 50	0 0 23 5.4	19.4	3.48
5 2 0	0 0 52 27.4	17.5	7.77	5 10 0	0 0 37 58.1		5.65	5 18 0	0 0 22 46.0		3.43

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. —	S. V. —	Mean Anomaly.	Equation.	Diff. —	S. V. ∓	Mean Anomaly.	Equation.	Diff. —	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
5 18 0	0 0 22 46.0		3.43	5 26 0	0 0 7 8.3		1.16	6 4 0	11 29 51 21.7		1.16
5 18 10	0 0 22 26.7	19.3	3.38	5 26 10	0 0 6 48.6	19.7	1.11	6 4 10	11 29 51 2.1	19.6	1.21
5 18 20	0 0 22 7.3	19.4	3.34	5 26 20	0 0 6 28.9	19.7	1.06	6 4 20	11 29 50 42.4	19.7	1.25
5 18 30	0 0 21 48.0	19.3	3.29	5 26 30	0 0 6 9.2	19.7	1.01	6 4 30	11 29 50 22.7	19.7	1.30
5 18 40	0 0 21 28.6	19.4	3.24	5 26 40	0 0 5 49.5	19.7	0.96	6 4 40	11 29 50 3.0	19.7	1.35
5 18 50	0 0 21 9.3	19.3	3.19	5 26 50	0 0 5 29.8	19.7	0.91	6 4 50	11 29 49 43.3	19.7	1.39
5 19 0	0 0 20 49.9	19.4	3.15	5 27 0	0 0 5 10.1	19.7	0.86	6 5 0	11 29 49 23.7	19.6	1.44
5 19 10	0 0 20 30.5	19.4	3.10	5 27 10	0 0 4 50.4	19.7	0.81	6 5 10	11 29 49 4.1	19.6	1.48
5 19 20	0 0 20 11.0	19.5	3.05	5 27 20	0 0 4 30.6	19.8	0.77	6 5 20	11 29 48 44.4	19.7	1.53
5 19 30	0 0 19 51.6	19.4	3.00	5 27 30	0 0 4 10.9	19.7	0.72	6 5 30	11 29 48 24.8	19.6	1.58
5 19 40	0 0 19 32.2	19.4	2.95	5 27 40	0 0 3 51.2	19.7	0.67	6 5 40	11 29 48 5.2	19.6	1.63
5 19 50	0 0 19 12.8	19.4	2.91	5 27 50	0 0 3 31.5	19.7	0.62	6 5 50	11 29 47 45.5	19.7	1.67
5 20 0	0 0 18 53.4	19.4	2.86	5 28 0	0 0 3 11.8	19.7	0.57	6 6 0	11 29 47 25.8	19.7	1.72
5 20 10	0 0 18 33.9	19.5	2.82	5 28 10	0 0 2 52.0	19.8	0.53	6 6 10	11 29 47 6.2	19.6	1.77
5 20 20	0 0 18 14.4	19.5	2.77	5 28 20	0 0 2 32.3	19.7	0.48	6 6 20	11 29 46 46.6	19.6	1.82
5 20 30	0 0 17 55.0	19.4	2.72	5 28 30	0 0 2 12.6	19.7	0.43	6 6 30	11 29 46 27.0	19.6	1.87
5 20 40	0 0 17 35.5	19.5	2.67	5 28 40	0 0 1 52.9	19.7	0.38	6 6 40	11 29 46 7.4	19.6	1.91
5 20 50	0 0 17 16.0	19.5	2.63	5 28 50	0 0 1 33.1	19.8	0.33	6 6 50	11 29 45 47.8	19.6	1.96
5 21 0	0 0 16 56.5	19.5	2.58	5 29 0	0 0 1 13.4	19.7	0.29	6 7 0	11 29 45 28.2	19.6	2.01
5 21 10	0 0 16 37.0	19.5	2.53	5 29 10	0 0 0 53.7	19.7	0.24	6 7 10	11 29 45 8.6	19.6	2.06
5 21 20	0 0 16 17.4	19.6	2.49	5 29 20	0 0 0 33.9	19.8	0.19	6 7 20	11 29 44 49.0	19.6	2.10
5 21 30	0 0 15 57.9	19.5	2.44	5 29 30	0 0 0 14.2	19.7	0.15	6 7 30	11 29 44 29.4	19.6	2.15
5 21 40	0 0 15 38.4	19.5	2.39	5 29 40	11 29 59 54.5	19.7	0.10	6 7 40	11 29 44 9.9	19.5	2.20
5 21 50	0 0 15 18.8	19.6	2.34	5 29 50	11 29 59 34.8	19.7	0.05	6 7 50	11 29 43 50.3	19.6	2.25
5 22 0	0 0 14 59.3	19.5	2.29	6 0 0	11 29 59 15.0	19.8	0.00	6 8 0	11 29 43 30.7	19.6	2.29
5 22 10	0 0 14 39.7	19.6	2.25	6 0 10	11 29 58 55.3	19.7	0.05	6 8 10	11 29 43 11.2	19.5	2.35
5 22 20	0 0 14 20.1	19.6	2.20	6 0 20	11 29 58 35.5	19.8	0.10	6 8 20	11 29 42 51.6	19.6	2.39
5 22 30	0 0 14 0.6	19.5	2.15	6 0 30	11 29 58 15.8	19.7	0.15	6 8 30	11 29 42 32.1	19.5	2.44
5 22 40	0 0 13 41.0	19.6	2.10	6 0 40	11 29 57 56.1	19.7	0.19	6 8 40	11 29 42 12.6	19.5	2.49
5 22 50	0 0 13 21.4	19.6	2.06	6 0 50	11 29 57 36.3	19.8	0.24	6 8 50	11 29 41 53.1	19.5	2.53
5 23 0	0 0 13 1.8	19.6	2.01	6 1 0	11 29 57 16.6	19.7	0.29	6 9 0	11 29 41 33.5	19.6	2.58
5 23 10	0 0 12 42.2	19.6	1.96	6 1 10	11 29 56 56.9	19.7	0.33	6 9 10	11 29 41 14.0	19.5	2.63
5 23 20	0 0 12 22.6	19.6	1.91	6 1 20	11 29 56 37.1	19.8	0.38	6 9 20	11 29 40 54.5	19.5	2.67
5 23 30	0 0 12 3.0	19.6	1.87	6 1 30	11 29 56 17.4	19.7	0.43	6 9 30	11 29 40 35.0	19.5	2.72
5 23 40	0 0 11 43.4	19.6	1.82	6 1 40	11 29 55 57.7	19.7	0.48	6 9 40	11 29 40 15.6	19.4	2.77
5 23 50	0 0 11 23.8	19.6	1.77	6 1 50	11 29 55 38.0	19.7	0.53	6 9 50	11 29 39 56.1	19.5	2.82
5 24 0	0 0 11 4.2	19.6	1.72	6 2 0	11 29 55 18.2	19.8	0.57	6 10 0	11 29 39 36.6	19.5	2.86
5 24 10	0 0 10 44.5	19.7	1.67	6 2 10	11 29 54 58.5	19.7	0.62	6 10 10	11 29 39 17.2	19.4	2.91
5 24 20	0 0 10 24.8	19.7	1.63	6 2 20	11 29 54 38.8	19.7	0.67	6 10 20	11 29 38 57.8	19.4	2.96
5 24 30	0 0 10 5.2	19.6	1.58	6 2 30	11 29 54 19.1	19.7	0.72	6 10 30	11 29 38 38.4	19.4	3.00
5 24 40	0 0 9 45.6	19.6	1.53	6 2 40	11 29 53 59.4	19.7	0.77	6 10 40	11 29 38 19.0	19.4	3.05
5 24 50	0 0 9 25.9	19.7	1.48	6 2 50	11 29 53 39.6	19.8	0.81	6 10 50	11 29 37 59.5	19.5	3.10
5 25 0	0 0 9 6.3	19.6	1.44	6 3 0	11 29 53 19.9	19.7	0.86	6 11 0	11 29 37 40.1	19.4	3.15
5 25 10	0 0 8 46.7	19.6	1.39	6 3 10	11 29 53 00.2	19.7	0.91	6 11 10	11 29 37 20.7	19.4	3.19
5 25 20	0 0 8 27.0	19.7	1.35	6 3 20	11 29 52 40.5	19.7	0.96	6 11 20	11 29 37 1.4	19.3	3.24
5 25 30	0 0 8 7.3	19.7	1.30	6 3 30	11 29 52 20.8	19.7	1.01	6 11 30	11 29 36 42.0	19.4	3.29
5 25 40	0 0 7 47.6	19.7	1.25	6 3 40	11 29 52 1.1	19.7	1.06	6 11 40	11 25 36 22.7	19.3	3.34
5 25 50	0 0 7 27.9	19.7	1.21	6 3 50	11 29 51 41.4	19.7	1.11	6 11 50	11 29 36 3.3	19.4	3.38
5 26 0	0 0 7 8.3	19.6	1.16	6 4 0	11 29 51 21.7	19.7	1.16	6 12 0	11 29 35 44.0	19.3	3.43

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.	Mean Anomaly.	Equation.	Diff.	S. V.
		—	+			—	+			—	+
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
6 12 0	11 29 35 44.0	19.4	3.43	6 20 0	11 29 20 31.9	18.6	5.65	6 28 0	11 29 6 2.6	17.5	7.77
6 12 10	11 29 35 24.6	19.3	3.48	6 20 10	11 29 20 13.3	18.5	5.69	6 28 10	11 29 5 45.1	17.5	7.81
6 12 20	11 29 35 5.3	19.3	3.52	6 20 20	11 29 19 54.8	18.6	5.73	6 28 20	11 29 5 27.6	17.5	7.85
6 12 30	11 29 34 46.0	19.3	3.57	6 20 30	11 29 19 36.2	18.6	5.78	6 28 30	11 29 5 10.1	17.5	7.90
6 12 40	11 29 34 26.7	19.2	3.62	6 20 40	11 29 19 17.6	18.5	5.83	6 28 40	11 29 4 52.6	17.4	7.94
6 12 50	11 29 34 7.5	19.3	3.66	6 20 50	11 29 18 59.1	18.5	5.87	6 28 50	11 29 4 35.2	17.4	7.99
6 13 0	11 29 33 48.2	19.3	3.71	6 21 0	11 29 18 40.6	18.5	5.92	6 29 0	11 29 4 17.8	17.4	8.03
6 13 10	11 29 33 28.9	19.2	3.76	6 21 10	11 29 18 22.1	18.4	5.97	6 29 10	11 29 4 0.4	17.4	8.07
6 13 20	11 29 33 9.7	19.2	3.80	6 21 20	11 29 18 3.7	18.5	6.01	6 29 20	11 29 3 43.0	17.3	8.11
6 13 30	11 29 32 50.5	19.2	3.85	6 21 30	11 29 17 45.2	18.4	6.06	6 29 30	11 29 3 25.7	17.3	8.16
6 13 40	11 29 32 31.3	19.2	3.90	6 21 40	11 29 17 26.8	18.4	6.10	6 29 40	11 29 3 8.4	17.3	8.20
6 13 50	11 29 32 12.1	19.2	3.94	6 21 50	11 29 17 8.4	18.4	6.15	6 29 50	11 29 2 51.1	17.3	8.24
6 14 0	11 29 31 52.9	19.2	3.99	6 22 0	11 29 16 50.0	18.4	6.19	7 0 0	11 29 2 33.8	17.2	8.28
6 14 10	11 29 31 33.7	19.2	4.04	6 22 10	11 29 16 31.6	18.3	6.24	7 0 10	11 29 2 16.6	17.2	8.33
6 14 20	11 29 31 14.5	19.1	4.08	6 22 20	11 29 16 13.3	18.4	6.28	7 0 20	11 29 1 59.4	17.1	8.37
6 14 30	11 29 30 55.4	19.2	4.13	6 22 30	11 29 15 54.9	18.3	6.33	7 0 30	11 29 1 42.3	17.2	8.41
6 14 40	11 29 30 36.2	19.1	4.18	6 22 40	11 29 15 36.6	18.3	6.37	7 0 40	11 29 1 25.1	17.1	8.45
6 14 50	11 29 30 17.1	19.1	4.22	6 22 50	11 29 15 18.3	18.2	6.41	7 0 50	11 29 1 8.0	17.1	8.49
6 15 0	11 29 29 58.0	19.1	4.27	6 23 0	11 29 15 .1	18.2	6.46	7 1 0	11 29 0 50.9	17.0	8.54
6 15 10	11 29 29 38.9	19.1	4.32	6 23 10	11 29 14 41.9	18.3	6.50	7 1 10	11 29 0 33.9	17.1	8.58
6 15 20	11 29 29 19.8	19.0	4.36	6 23 20	11 29 14 23.6	18.2	6.55	7 1 20	11 29 0 16.8	17.0	8.62
6 15 30	11 29 29 0.8	19.1	4.41	6 23 30	11 29 14 5.4	18.2	6.59	7 1 30	11 28 59 59.8	17.0	8.66
6 15 40	11 29 28 41.7	19.0	4.46	6 23 40	11 29 13 47.2	18.1	6.64	7 1 40	11 28 59 42.8	16.9	8.70
6 15 50	11 29 28 22.7	19.0	4.50	6 23 50	11 29 13 29.1	18.2	6.68	7 1 50	11 28 59 25.9	16.9	8.75
6 16 0	11 29 28 3.7	19.1	4.55	6 24 0	11 29 13 10.9	18.1	6.73	7 2 0	11 28 59 9.0	16.9	8.79
6 16 10	11 29 27 44.6	19.0	4.59	6 24 10	11 29 12 52.8	18.1	6.77	7 2 10	11 28 58 52.1	16.8	8.83
6 16 20	11 29 27 25.6	18.9	4.64	6 24 20	11 29 12 34.7	18.1	6.81	7 2 20	11 28 58 35.3	16.9	8.87
6 16 30	11 29 27 6.7	19.0	4.69	6 24 30	11 29 12 16.6	18.0	6.86	7 2 30	11 28 58 18.4	16.8	8.91
6 16 40	11 29 26 47.7	18.9	4.73	6 24 40	11 29 11 58.6	18.0	6.90	7 2 40	11 28 58 1.6	16.7	8.95
6 16 50	11 29 26 28.8	19.0	4.78	6 24 50	11 29 11 40.6	18.0	6.95	7 2 50	11 28 57 44.9	16.7	8.99
6 17 0	11 29 26 9.8	18.9	4.83	6 25 0	11 29 11 22.6	18.0	6.99	7 3 0	11 28 57 28.2	16.8	9.03
6 17 10	11 29 25 50.9	18.9	4.87	6 25 10	11 29 11 4.6	17.9	7.03	7 3 10	11 28 57 11.4	16.7	9.07
6 17 20	11 29 25 32.0	18.9	4.92	6 25 20	11 29 10 46.7	18.0	7.08	7 3 20	11 28 56 54.7	16.6	9.12
6 17 30	11 29 25 13.1	18.8	4.96	6 25 30	11 29 10 28.7	17.9	7.12	7 3 30	11 28 56 38.1	16.6	9.16
6 17 40	11 29 24 54.3	18.9	5.01	6 25 40	11 29 10 10.8	17.9	7.17	7 3 40	11 28 56 21.5	16.6	9.20
6 17 50	11 29 24 35.4	18.8	5.06	6 25 50	11 29 9 52.9	17.8	7.21	7 3 50	11 28 56 4.9	16.6	9.24
6 18 0	11 29 24 16.6	18.8	5.10	6 26 0	11 29 9 35.1	17.9	7.25	7 4 0	11 28 55 48.3	16.5	9.28
6 18 10	11 29 23 57.8	18.8	5.15	6 26 10	11 29 9 17.2	17.8	7.30	7 4 10	11 28 55 31.8	16.5	9.32
6 18 20	11 29 23 39.0	18.8	5.19	6 26 20	11 29 8 59.4	17.8	7.34	7 4 20	11 28 55 15.3	16.5	9.36
6 18 30	11 29 23 20.2	18.8	5.24	6 26 30	11 29 8 41.6	17.7	7.38	7 4 30	11 28 54 58.8	16.4	9.40
6 18 40	11 29 23 1.4	18.7	5.29	6 26 40	11 29 8 23.9	17.8	7.43	7 4 40	11 28 54 42.4	16.4	9.44
6 18 50	11 29 22 42.7	18.7	5.33	6 26 50	11 29 8 6.1	17.7	7.47	7 4 50	11 28 54 26.0	16.4	9.48
6 19 0	11 29 22 24.0	18.7	5.38	6 27 0	11 29 7 48.4	17.7	7.51	7 5 0	11 28 54 9.6	16.3	9.52
6 19 10	11 29 22 5.3	18.7	5.42	6 27 10	11 29 7 30.7	17.7	7.56	7 5 10	11 28 53 53.3	16.3	9.56
6 19 20	11 29 21 46.6	18.7	5.47	6 27 20	11 29 7 13.0	17.6	7.60	7 5 20	11 28 53 37.0	16.3	9.60
6 19 30	11 29 21 27.9	18.7	5.51	6 27 30	11 29 6 55.4	17.6	7.64	7 5 30	11 28 53 20.7	16.3	9.64
6 19 40	11 29 21 9.2	18.7	5.56	6 27 40	11 29 6 37.8	17.7	7.69	7 5 40	11 28 53 4.4	16.2	9.68
6 19 50	11 29 20 50.5	18.6	5.60	6 27 50	11 29 6 20.1	17.5	7.73	7 5 50	11 28 52 48.2	16.2	9.72
6 20 0	11 29 20 31.9		5.65	6 28 0	11 29 6 2.6		7.77	7 6 0	11 28 52 32.0		9.76

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. —	S. V. +	Mean Anomaly.	Equation.	Diff. —	S. V. +	Mean Anomaly.	Equation.	Diff. —	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
7 6 0	11 28 52 32.0	16.1	9.76	7 14 0	11 28 40 15.5	14.4	11.58	7 22 0	11 28 29 27.2	12.5	13.19
7 6 10	11 28 52 15.9	16.1	9.79	7 14 10	11 28 40 1.1	14.4	11.61	7 22 10	11 28 29 14.7	12.4	13.22
7 6 20	11 28 51 59.8	16.1	9.84	7 14 20	11 28 39 46.7	14.4	11.65	7 22 20	11 28 29 2.3	12.4	13.25
7 6 30	11 28 51 43.7	16.0	9.88	7 14 30	11 28 39 32.3	14.3	11.69	7 22 30	11 28 28 49.9	12.3	13.28
7 6 40	11 28 51 27.7	16.1	9.91	7 14 40	11 28 39 18.0	14.3	11.72	7 22 40	11 28 28 37.6	12.3	13.31
7 6 50	11 28 51 11.6	16.0	9.95	7 14 50	11 28 39 3.7	14.3	11.76	7 22 50	11 28 28 25.3	12.3	13.34
7 7 0	11 28 50 55.6	15.9	10.00	7 15 0	11 28 38 49.4	14.2	11.79	7 23 0	11 28 28 13.0	12.2	13.37
7 7 10	11 28 50 39.7	15.9	10.04	7 15 10	11 28 38 35.2	14.2	11.83	7 23 10	11 28 28 0.8	12.2	13.40
7 7 20	11 28 50 23.8	15.9	10.07	7 15 20	11 28 38 21.0	14.1	11.86	7 23 20	11 28 27 48.6	12.1	13.43
7 7 30	11 28 50 7.9	15.8	10.11	7 15 30	11 28 38 6.9	14.1	11.90	7 23 30	11 28 27 36.5	12.1	13.46
7 7 40	11 28 49 52.1	15.9	10.15	7 15 40	11 28 37 52.8	14.1	11.93	7 23 40	11 28 27 24.4	12.0	13.49
7 7 50	11 28 49 36.2	15.8	10.19	7 15 50	11 28 37 38.7	14.0	11.97	7 23 50	11 28 27 12.4	12.0	13.52
7 8 0	11 28 49 20.4	15.8	10.23	7 16 0	11 28 37 24.7	14.0	12.00	7 24 0	11 28 27 0.4	11.9	13.55
7 8 10	11 28 49 4.6	15.7	10.27	7 16 10	11 28 37 10.7	14.0	12.04	7 24 10	11 28 26 48.5	11.9	13.58
7 8 20	11 28 48 48.9	15.7	10.31	7 16 20	11 28 36 56.7	13.9	12.07	7 24 20	11 28 26 36.6	11.9	13.61
7 8 30	11 28 48 33.2	15.6	10.35	7 16 30	11 28 36 42.8	13.8	12.10	7 24 30	11 28 26 24.7	11.8	13.64
7 8 40	11 28 48 17.6	15.6	10.39	7 16 40	11 28 36 29.0	13.8	12.14	7 24 40	11 28 26 12.9	11.8	13.67
7 8 50	11 28 48 2.0	15.5	10.43	7 16 50	11 28 36 15.2	13.8	12.17	7 24 50	11 28 26 1.1	11.7	13.70
7 9 0	11 28 47 46.5	15.6	10.46	7 17 0	11 28 36 1.4	13.7	12.21	7 25 0	11 28 25 49.4	11.7	13.73
7 9 10	11 28 47 30.9	15.5	10.50	7 17 10	11 28 35 47.7	13.7	12.24	7 25 10	11 28 25 37.7	11.6	13.76
7 9 20	11 28 47 15.4	15.5	10.54	7 17 20	11 28 35 34.0	13.7	12.28	7 25 20	11 28 25 26.1	11.6	13.79
7 9 30	11 28 46 59.9	15.5	10.58	7 17 30	11 28 35 20.3	13.6	12.31	7 25 30	11 28 25 14.5	11.5	13.82
7 9 40	11 28 46 44.4	15.4	10.62	7 17 40	11 28 35 6.7	13.6	12.34	7 25 40	11 28 25 3.0	11.5	13.85
7 9 50	11 28 46 29.0	15.4	10.65	7 17 50	11 28 34 53.1	13.6	12.38	7 25 50	11 28 24 51.5	11.4	13.88
7 10 0	11 28 46 13.6	15.3	10.69	7 18 0	11 28 34 39.5	13.5	12.41	7 26 0	11 28 24 40.1	11.4	13.91
7 10 10	11 28 45 58.3	15.3	10.73	7 18 10	11 28 34 26.0	13.4	12.44	7 26 10	11 28 24 28.7	11.3	13.94
7 10 20	11 28 45 43.0	15.2	10.77	7 18 20	11 28 34 12.6	13.4	12.48	7 26 20	11 28 24 17.4	11.3	13.97
7 10 30	11 28 45 27.8	15.3	10.81	7 18 30	11 28 33 59.2	13.4	12.51	7 26 30	11 28 24 6.1	11.3	14.00
7 10 40	11 28 45 12.5	15.2	10.84	7 18 40	11 28 33 45.8	13.3	12.54	7 26 40	11 28 23 54.8	11.2	14.03
7 10 50	11 28 44 57.3	15.2	10.88	7 18 50	11 28 33 32.5	13.3	12.58	7 26 50	11 28 23 43.6	11.1	14.06
7 11 0	11 28 44 42.1	15.1	10.92	7 19 0	11 28 33 19.2	13.2	12.61	7 27 0	11 28 23 32.5	11.1	14.09
7 11 10	11 28 44 27.0	15.1	10.96	7 19 10	11 28 33 5.9	13.2	12.64	7 27 10	11 28 23 21.4	11.1	14.11
7 11 20	11 28 44 11.9	15.0	10.99	7 19 20	11 28 32 52.7	13.2	12.68	7 27 20	11 28 23 10.3	11.0	14.14
7 11 30	11 28 43 56.9	15.0	11.03	7 19 30	11 28 32 39.5	13.1	12.71	7 27 30	11 28 22 59.3	11.0	14.16
7 11 40	11 28 43 41.9	15.0	11.07	7 19 40	11 28 32 26.4	13.1	12.74	7 27 40	11 28 22 48.3	10.9	14.19
7 11 50	11 28 43 26.9	15.0	11.10	7 19 50	11 28 32 13.3	13.0	12.77	7 27 50	11 28 22 37.4	10.9	14.22
7 12 0	11 28 43 11.9	14.9	11.14	7 20 0	11 28 32 0.3	13.0	12.81	7 28 0	11 28 22 26.5	10.9	14.24
7 12 10	11 28 42 57.0	14.8	11.18	7 20 10	11 28 31 47.3	13.0	12.84	7 28 10	11 28 22 15.6	10.8	14.27
7 12 20	11 28 42 42.2	14.8	11.21	7 20 20	11 28 31 34.3	12.9	12.87	7 28 20	11 28 22 4.8	10.7	14.30
7 12 30	11 28 42 27.4	14.8	11.25	7 20 30	11 28 31 21.4	12.8	12.90	7 28 30	11 28 21 54.1	10.7	14.32
7 12 40	11 28 42 12.6	14.8	11.29	7 20 40	11 28 31 8.6	12.8	12.93	7 28 40	11 28 21 43.4	10.6	14.35
7 12 50	11 28 41 57.8	14.7	11.32	7 20 50	11 28 30 55.8	12.8	12.97	7 28 50	11 28 21 32.8	10.6	14.38
7 13 0	11 28 41 43.1	14.7	11.36	7 21 0	11 28 30 43.0	12.7	13.00	7 29 0	11 28 21 22.2	10.5	14.40
7 13 10	11 28 41 28.4	14.7	11.40	7 21 10	11 28 30 30.3	12.7	13.03	7 29 10	11 28 21 11.7	10.5	14.43
7 13 20	11 28 41 13.7	14.6	11.43	7 21 20	11 28 30 17.6	12.7	13.06	7 29 20	11 28 21 1.2	10.4	14.46
7 13 30	11 28 40 59.1	14.6	11.47	7 21 30	11 28 30 4.9	12.6	13.09	7 29 30	11 28 20 50.8	10.4	14.48
7 13 40	11 28 40 44.5	14.5	11.50	7 21 40	11 28 29 52.3	12.6	13.12	7 29 40	11 28 20 40.4	10.3	14.51
7 13 50	11 28 40 30.0	14.5	11.54	7 21 50	11 28 29 39.7	12.5	13.16	7 29 50	11 28 20 30.1	10.4	14.53
7 14 0	11 28 40 15.5		11.58	7 22 0	11 28 29 27.2		13.19	8 0 0	11 28 20 19.7		14.56

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. —	S. V. +	Mean Anomaly.	Equation.	Diff. —	S. V. +	Mean Anomaly.	Equation.	Diff. —	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
8 0 0	11 28 20 19.7	10.3	14.56	8 8 0	11 28 13 3.9	7.8	15.67	8 16 0	11 28 7 49.4	5.2	16.49
8 0 10	11 28 20 9.4	10.2	14.59	8 8 10	11 28 12 56.1	7.8	15.69	8 16 10	11 28 7 44.2	5.2	16.50
8 0 20	11 28 19 59.2	10.1	14.61	8 8 20	11 28 12 48.3	7.7	15.71	8 16 20	11 28 7 39.0	5.1	16.52
8 0 30	11 28 19 49.1	10.1	14.64	8 8 30	11 28 12 40.6	7.6	15.73	8 16 30	11 28 7 33.9	5.1	16.53
8 0 40	11 28 19 39.0	10.1	14.67	8 8 40	11 28 12 33.0	7.6	15.75	8 16 40	11 28 7 28.8	5.0	16.55
8 0 50	11 28 19 28.9	10.0	14.69	8 8 50	11 28 12 25.4	7.5	15.77	8 16 50	11 28 7 23.8	4.9	16.56
8 1 0	11 28 19 18.9	10.0	14.72	8 9 0	11 28 12 17.9	7.5	15.79	8 17 0	11 28 7 18.9	4.9	16.57
8 1 10	11 28 19 8.9	9.9	14.74	8 9 10	11 28 12 10.4	7.5	15.81	8 17 10	11 28 7 14.0	4.8	16.58
8 1 20	11 28 18 59.0	9.9	14.77	8 9 20	11 28 12 2.9	7.4	15.83	8 17 20	11 28 7 9.2	4.8	16.60
8 1 30	11 28 18 49.1	9.8	14.79	8 9 30	11 28 11 55.5	7.3	15.85	8 17 30	11 28 7 4.4	4.7	16.61
8 1 40	11 28 18 39.3	9.7	14.82	8 9 40	11 28 11 48.2	7.3	15.87	8 17 40	11 28 6 59.7	4.6	16.62
8 1 50	11 28 18 29.6	9.7	14.84	8 9 50	11 28 11 40.9	7.2	15.88	8 17 50	11 28 6 55.1	4.6	16.64
8 2 0	11 28 18 19.9	9.7	14.87	8 10 0	11 28 11 33.7	7.2	15.90	8 18 0	11 28 6 50.5	4.5	16.65
8 2 10	11 28 18 10.2	9.6	14.89	8 10 10	11 28 11 26.5	7.1	15.92	8 18 10	11 28 6 46.0	4.5	16.66
8 2 20	11 28 18 0.6	9.6	14.91	8 10 20	11 28 11 19.4	7.1	15.94	8 18 20	11 28 6 41.5	4.4	16.67
8 2 30	11 28 17 51.0	9.5	14.94	8 10 30	11 28 11 12.3	7.0	15.96	8 18 30	11 28 6 37.1	4.4	16.68
8 2 40	11 28 17 41.5	9.4	14.96	8 10 40	11 28 11 5.3	7.0	15.99	8 18 40	11 28 6 32.7	4.3	16.70
8 2 50	11 28 17 32.1	9.4	14.99	8 10 50	11 28 10 58.3	6.9	16.00	8 18 50	11 28 6 28.4	4.3	16.71
8 3 0	11 28 17 22.7	9.4	15.01	8 11 0	11 28 10 51.4	6.9	16.01	8 19 0	11 28 6 24.1	4.2	16.72
8 3 10	11 28 17 13.3	9 3	15.03	8 11 10	11 28 10 44.5	6.8	16.03	8 19 10	11 28 6 19.9	4.1	16.73
8 3 20	11 28 17 4.0	9.3	15.06	8 11 20	11 28 10 37.7	6.7	16.05	8 19 20	11 28 6 15.8	4.1	16.74
8 3 30	11 28 16 54.7	9.2	15.08	8 11 30	11 28 10 31.0	6.7	16.07	8 19 30	11 28 6 11.7	4.1	16.75
8 3 40	11 28 16 45.5	9.1	15.10	8 11 40	11 28 10 24.3	6.6	16.08	8 19 40	11 28 6 7.6	4.0	16.76
8 3 50	11 28 16 36.4	9.1	15.13	8 11 50	11 28 10 17.7	6.6	16.10	8 19 50	11 28 6 3.6	3.9	16.77
8 4 0	11 28 16 27.3	9.1	15.15	8 12 0	11 28 10 11.1	6.5	16.12	8 20 0	11 28 5 59.7	3.9	16.78
8 4 10	11 28 16 18.2	9.0	15.17	8 12 10	11 28 10 4.6	6.5	16.13	8 20 10	11 28 5 55.8	3.8	16.79
8 4 20	11 28 16 9.2	9.0	15.20	8 12 20	11 28 9 58.1	6.5	16.15	8 20 20	11 28 5 52.0	3.8	16.81
8 4 30	11 28 16 0.2	8.9	15.22	8 12 30	11 28 9 51.6	6.4	16.17	8 20 30	11 28 5 48.2	3.7	16.82
8 4 40	11 28 15 51.3	8.9	15.24	8 12 40	11 28 9 45.2	6.3	16.18	8 20 40	11 28 5 44.5	3.6	16.83
8 4 50	11 28 15 42.4	8.8	15.27	8 12 50	11 28 9 38.9	6.2	16.20	8 20 50	11 28 5 40.9	3.5	16.84
8 5 0	11 28 15 33.6	8.7	15.29	8 13 0	11 28 9 32.7	6.2	16.22	8 21 0	11 28 5 37.4	3.5	16.85
8 5 10	11 28 15 24.9	8.7	15.31	8 13 10	11 28 9 26.5	6.2	16.23	8 21 10	11 28 5 33.9	3.5	16.86
8 5 20	11 28 15 16.2	8.7	15.33	8 13 20	11 28 9 20.3	6.2	16.25	8 21 20	11 28 5 30.4	3.5	16.87
8 5 30	11 28 15 7.5	8.7	15.35	8 13 30	11 28 9 14.2	6.1	16.27	8 21 30	11 28 5 27.0	3.4	16.87
8 5 40	11 28 14 58.9	8.6	15.37	8 13 40	11 28 9 8.2	6.0	16.28	8 21 40	11 28 5 23.7	3.3	16.88
8 5 50	11 28 14 50.4	8.5	15.39	8 13 50	11 28 9 2.2	6.0	16.30	8 21 50	11 28 5 20.4	3.3	16.89
8 6 0	11 28 14 41.9	8.5	15.42	8 14 0	11 28 8 56.2	6.0	16.31	8 22 0	11 28 5 17.1	3.3	16.90
8 6 10	11 28 14 33.5	8.4	15.44	8 14 10	11 28 8 50.4	5.8	16.33	8 22 10	11 28 5 13.9	3.2	16.91
8 6 20	11 28 14 25.1	8.4	15.46	8 14 20	11 28 8 44.6	5.8	16.34	8 22 20	11 28 5 10.8	3 1	16.92
8 6 30	11 28 14 16.8	8.3	15.48	8 14 30	11 28 8 38.8	5.8	16.36	8 22 30	11 28 5 7.7	3.1	16.93
8 6 40	11 28 14 8.5	8.3	15.51	8 14 40	11 28 8 33.1	5.7	16.37	8 22 40	11 28 5 4.7	3.0	16.94
8 6 50	11 28 14 0.2	8.3	15.53	8 14 50	11 28 8 27.4	5.7	16.39	8 22 50	11 28 5 1.8	2.9	16.94
8 7 0	11 28 13 52.0	8.2	15.55	8 15 0	11 28 8 21.8	5.6	16.40	8 23 0	11 28 4 58.9	2.9	16.95
8 7 10	11 28 13 43.9	8.1	15.57	8 15 10	11 28 8 16.3	5.5	16.42	8 23 10	11 28 4 56.1	2.8	16.96
8 7 20	11 28 13 35.8	8.1	15.59	8 15 20	11 28 8 10.8	5.5	16.43	8 23 20	11 28 4 53.3	2.8	16.97
8 7 30	11 28 13 27.8	8.0	15.61	8 15 30	11 28 8 5.3	5.5	16.45	8 23 30	11 28 4 50.6	2.7	16.98
8 7 40	11 28 13 19.8	8.0	15.63	8 15 40	11 28 7 59.9	5.4	16.46	8 23 40	11 28 4 48.0	2.6	16.98
8 7 50	11 28 13 11.8	8.0	15.65	8 15 50	11 28 7 54.6	5.3	16.47	8 23 50	11 28 4 45.4	2.6	16.99
8 8 0	11 28 13 3.9	7.9	15.67	8 16 0	11 28 7 49.4	5.2	16.49	8 24 0	11 28 4 42.8	2.6	17.00

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. ∓	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
8 24 0	11 28 4 42.8	2.5	17.00	9 2 0	11 28 3 48.8	0.3	17.18	9 10 0	11 28 5 9.9	3.1	17.03
8 24 10	11 28 4 40.3	2.5	17.00	9 2 10	11 28 3 49.1	0.4	17.18	9 10 10	11 28 5 13.0	3.2	17.02
8 24 20	11 28 4 37.8	2.4	17.01	9 2 20	11 28 3 49.5	0.4	17.18	9 10 20	11 28 5 16.2	3.3	17.02
8 24 30	11 28 4 35.4	2.3	17.02	9 2 30	11 28 3 49.9	0.5	17.18	9 10 30	11 28 5 19.5	3.3	17.01
8 24 40	11 28 4 33.1	2.3	17.03	9 2 40	11 28 3 50.4	0.6	17.18	9 10 40	11 28 5 22.8	3.3	17.00
8 24 50	11 28 4 30.8	2.2	17.03	9 2 50	11 28 3 51.0	0.6	17.18	9 10 50	11 28 5 26.1	3.4	16.99
8 25 0	11 28 4 28.6	2.1	17.04	9 3 0	11 28 3 51.6	0.7	17.18	9 11 0	11 28 5 29.5	3.5	16.99
8 25 10	11 28 4 26.5	2.1	17.05	9 3 10	11 28 3 52.3	0.7	17.18	9 11 10	11 28 5 33.0	3.6	16.98
8 25 20	11 28 4 24.4	2.0	17.05	9 3 20	11 28 3 53.0	0.8	17.18	9 11 20	11 28 5 36.6	3.6	16.97
8 25 30	11 28 4 22.4	1.9	17.06	9 3 30	11 28 3 53.8	0.8	17.18	9 11 30	11 28 5 40.2	3.6	16.96
8 25 40	11 28 4 20.5	1.9	17.07	9 3 40	11 28 3 54.6	0.9	17.18	9 11 40	11 28 5 43.8	3.7	16.96
8 25 50	11 28 4 18.6	1.9	17.07	9 3 50	11 28 3 55.5	1.0	17.18	9 11 50	11 28 5 47.5	3.8	16.95
8 26 0	11 28 4 16.7	1.8	17.08	9 4 0	11 28 3 56.5	1.0	17.18	9 12 0	11 28 5 51.3	3.8	16.94
8 26 10	11 28 4 14.9	1.7	17.08	9 4 10	11 28 3 57.5	1.0	17.17	9 12 10	11 28 5 55.1	3.9	16.93
8 26 20	11 28 4 13.2	1.7	17.09	9 4 20	11 28 3 58.5	1.1	17.17	9 12 20	11 28 5 59.0	4.0	16.92
8 26 30	11 28 4 11.5	1.6	17.09	9 4 30	11 28 3 59.6	1.2	17.17	9 12 30	11 28 6 3.0	4.0	16.91
8 26 40	11 28 4 9.9	1.6	17.10	9 4 40	11 28 4 0.8	1.3	17.17	9 12 40	11 28 6 7.0	4.0	16.90
8 26 50	11 28 4 8.3	1.5	17.10	9 4 50	11 28 4 2.1	1.3	17.17	9 12 50	11 28 6 11.0	4.1	16.90
8 27 0	11 28 4 6.8	1.4	17.11	9 5 0	11 28 4 3.4	1.4	17.16	9 13 0	11 28 6 15.1	4.2	16.89
8 27 10	11 28 4 5.4	1.4	17.11	9 5 10	11 28 4 4.8	1.4	17.16	9 13 10	11 28 6 19.3	4.3	16.88
8 27 20	11 28 4 4.0	1.3	17.11	9 5 20	11 28 4 6.2	1.5	17.16	9 13 20	11 28 6 23.6	4.2	16.87
8 27 30	11 28 4 2.7	1.3	17.12	9 5 30	11 28 4 7.7	1.6	17.16	9 13 30	11 28 6 27.8	4.3	16.86
8 27 40	11 28 4 1.4	1.2	17.12	9 5 40	11 28 4 9.3	1.6	17.16	9 13 40	11 28 6 32.1	4.4	16.85
8 27 50	11 28 4 0.2	1.2	17.13	9 5 50	11 28 4 10.9	1.7	17.15	9 13 50	11 28 6 36.5	4.5	16.84
8 28 0	11 28 3 59.0	1.1	17.13	9 6 0	11 28 4 12.6	1.7	17.15	9 14 0	11 28 6 41.0	4.5	16.83
8 28 10	11 28 3 57.9	1.0	17.13	9 6 10	11 28 4 14.3	1.7	17.15	9 14 10	11 28 6 45.5	4.6	16.82
8 28 20	11 28 3 56.9	1.0	17.14	9 6 20	11 28 4 16.0	1.8	17.14	9 14 20	11 28 6 50.1	4.6	16.81
8 28 30	11 28 3 55.9	0.9	17.14	9 6 30	11 28 4 17.8	1.9	17.14	9 14 30	11 28 6 54.7	4.7	16.80
8 28 40	11 28 3 55.0	0.9	17.14	9 6 40	11 28 4 19.7	2.0	17.13	9 14 40	11 28 6 59.4	4.8	16.79
8 28 50	11 28 3 54.1	0.8	17.15	9 6 50	11 28 4 21.7	2.0	17.13	9 14 50	11 28 7 4.2	4.8	16.78
8 29 0	11 28 3 53.3	0.7	17.15	9 7 0	11 28 4 23.7	2.1	17.13	9 15 0	11 28 7 9.0	4.9	16.76
8 29 10	11 28 3 52.6	0.7	17.15	9 7 10	11 28 4 25.8	2.1	17.12	9 15 10	11 28 7 13.9	4.9	16.75
8 29 20	11 28 3 51.9	0.6	17.16	9 7 20	11 28 4 27.9	2.2	17.12	9 15 20	11 28 7 18.8	5.0	16.74
8 29 30	11 28 3 51.3	0.6	17.16	9 7 30	11 28 4 30.1	2.2	17.11	9 15 30	11 28 7 23.8	5.1	16.73
8 29 40	11 28 3 50.7	0.5	17.16	9 7 40	11 28 4 32.3	2.3	17.11	9 15 40	11 28 7 28.9	5.1	16.72
8 29 50	11 28 3 50.2	0.5	17.17	9 7 50	11 28 4 34.6	2.4	17.10	9 15 50	11 28 7 34.0	5.2	16.71
9 0 0	11 28 3 49.7	0.4	17.17	9 8 0	11 28 4 37.0	2.4	17.10	9 16 0	11 28 7 39.2	5.2	16.70
9 0 10	11 28 3 49.3	0.3	17.17	9 8 10	11 28 4 39.4	2.5	17.09	9 16 10	11 28 7 44.4	5.3	16.68
9 0 20	11 28 3 49.0	0.3	17.17	9 8 20	11 28 4 41.9	2.5	17.09	9 16 20	11 28 7 49.7	5.3	16.67
9 0 30	11 28 3 48.7	0.2	17.17	9 8 30	11 28 4 44.4	2.6	17.08	9 16 30	11 28 7 55.0	5.4	16.66
9 0 40	11 28 3 48.5	0.1	17.18	9 8 40	11 28 4 47.0	2.7	17.08	9 16 40	11 28 8 0.4	5.4	16.65
9 0 50	11 28 3 48.4	0.1	17.18	9 8 50	11 28 4 49.7	2.7	17.07	9 16 50	11 28 8 5.8	5.5	16.63
9 1 0	11 28 3 48.3	0.1	17.18	9 9 0	11 28 4 52.4	2.8	17.07	9 17 0	11 28 8 11.3	5.6	16.62
9 1 10	11 28 3 48.2	0.0	17.18	9 9 10	11 28 4 55.2	2.8	17.06	9 17 10	11 28 8 16.9	5.6	16.61
9 1 20	11 28 3 48.2	0.0	17.18	9 9 20	11 28 4 58.0	2.9	17.06	9 17 20	11 28 8 22.5	5.7	16.59
9 1 30	11 28 3 48.2	0.1	17.18	9 9 30	11 28 5 0.9	2.9	17.05	9 17 30	11 28 8 28.2	5.7	16.58
9 1 40	11 28 3 48.3	0.2	17.18	9 9 40	11 28 5 3.8	3.0	17.04	9 17 40	11 28 8 33.9	5.8	16.57
9 1 50	11 28 3 48.5	0.3	17.18	9 9 50	11 28 5 6.8	3.1	17.04	9 17 50	11 28 8 39.7	5.8	16.55
9 2 0	11 28 3 48.8			9 10 0	11 28 5 9.9		17.03	9 18 0	11 28 8 45.5		16.54

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
9 18 0	11 28 8 45.5	5.9	16.54	9 26 0	11 28 14 32.4	8.6	15.72	10 4 0	11 28 22 25.1	11.1	14.57
9 18 10	11 28 8 51.4	6.0	16.53	9 26 10	11 28 14 41.0	8.7	15.70	10 4 10	11 28 22 36.2	11.2	14.55
9 18 20	11 28 8 57.4	6.0	16.51	9 26 20	11 28 14 49.7	8.7	15.68	10 4 20	11 28 22 47.4	11.2	14.52
9 18 30	11 28 9 3.4	6.1	16.50	9 26 30	11 28 14 58.4	8.8	15.65	10 4 30	11 28 22 58.6	11.3	14.49
9 18 40	11 28 9 9.5	6.2	16.48	9 26 40	11 28 15 7.2	8.8	15.63	10 4 40	11 28 23 9.9	11.3	14.46
9 18 50	11 28 9 15.7	6.1	16.47	9 26 50	11 28 15 16.0	8.9	15.61	10 4 50	11 28 23 21.2	11.4	14.44
9 19 0	11 28 9 21.8	6.2	16.46	9 27 0	11 28 15 24.9	8.9	15.59	10 5 0	11 28 23 32.6	11.4	14.41
9 19 10	11 28 9 28.0	6.3	16.44	9 27 10	11 28 15 33.8	9.0	15.57	10 5 10	11 28 23 44.0	11.5	14.38
9 19 20	11 28 9 34.3	6.4	16.43	9 27 20	11 28 15 42.8	9.0	15.55	10 5 20	11 28 23 55.5	11.5	14.35
9 19 30	11 28 9 40.7	6.4	16.41	9 27 30	11 28 15 51.8	9.1	15.53	10 5 30	11 28 24 7.0	11.6	14.32
9 19 40	11 28 9 47.1	6.5	16.40	9 27 40	11 28 16 0.9	9.1	15.51	10 5 40	11 28 24 18.6	11.6	14.30
9 19 50	11 28 9 53.6	6.5	16.38	9 27 50	11 28 16 10.0	9.2	15.48	10 5 50	11 28 24 30.2	11.7	14.27
9 20 0	11 28 10 0.1	6.6	16.37	9 28 0	11 28 16 19.2	9.2	15.46	10 6 0	11 28 24 41.9	11.7	14.24
9 20 10	11 28 10 6.7	6.7	16.35	9 28 10	11 28 16 28.4	9.3	15.44	10 6 10	11 28 24 53.6	11.8	14.21
9 20 20	11 28 10 13.4	6.6	16.33	9 28 20	11 28 16 37.7	9.3	15.42	10 6 20	11 28 25 5.4	11.8	14.18
9 20 30	11 28 10 20.0	6.8	16.32	9 28 30	11 28 16 47.0	9.4	15.39	10 6 30	11 28 25 17.2	11.9	14.15
9 20 40	11 28 10 26.8	6.8	16.30	9 28 40	11 28 16 56.4	9.5	15.37	10 6 40	11 28 25 29.1	11.9	14.12
9 20 50	11 28 10 33.6	6.9	16.29	9 28 50	11 28 17 5.9	9.5	15.35	10 6 50	11 28 25 41.0	12.0	14.09
9 21 0	11 28 10 40.5	6.9	16.27	9 29 0	11 28 17 15.4	9.6	15.33	10 7 0	11 28 25 53.0	12.0	14.07
9 21 10	11 28 10 47.4	7.0	16.26	9 29 10	11 28 17 25.0	9.6	15.30	10 7 10	11 28 26 5.0	12.1	14.04
9 21 20	11 28 10 54.4	7.0	16.24	9 29 20	11 28 17 34.6	9.7	15.28	10 7 20	11 28 26 17.1	12.2	14.01
9 21 30	11 28 11 1.4	7.1	16.23	9 29 30	11 28 17 44.3	9.7	15.26	10 7 30	11 28 26 29.3	12.1	13.98
9 21 40	11 28 11 8.5	7.2	16.21	9 29 40	11 28 17 54.0	9.8	15.23	10 7 40	11 28 26 41.4	12.2	13.95
9 21 50	11 28 11 15.7	7.2	16.19	9 29 50	11 28 18 3.8	9.8	15.21	10 7 50	11 28 26 53.6	12.3	13.92
9 22 0	11 28 11 22.9	7.3	16.17	10 0 0	11 28 18 13.6	9.9	15.19	10 8 0	11 28 27 5.9	12.3	13.89
9 22 10	11 28 11 30.2	7.3	16.15	10 0 10	11 28 18 23.5	9.9	15.16	10 8 10	11 28 27 18.2	12.4	13.86
9 22 20	11 28 11 37.5	7.4	16.14	10 0 20	11 28 18 33.4	10.0	15.14	10 8 20	11 28 27 30.6	12.4	13.83
9 22 30	11 28 11 44.9	7.4	16.12	10 0 30	11 28 18 43.4	10.0	15.11	10 8 30	11 28 27 43.0	12.5	13.80
9 22 40	11 28 11 52.3	7.5	16.10	10 0 40	11 28 18 53.4	10.1	15.09	10 8 40	11 28 27 55.5	12.5	13.77
9 22 50	11 28 11 59.8	7.5	16.08	10 0 50	11 28 19 3.5	10.1	15.06	10 8 50	11 28 28 8.0	12.6	13.73
9 23 0	11 28 12 7.3	7.6	16.07	10 1 0	11 28 19 13.6	10.2	15.04	10 9 0	11 28 28 20.6	12.6	13.70
9 23 10	11 28 12 14.9	7.7	16.05	10 1 10	11 28 19 23.8	10.3	15.01	10 9 10	11 28 28 33.2	12.6	13.67
9 23 20	11 28 12 22.6	7.7	16.03	10 1 20	11 28 19 34.1	10.3	14.99	10 9 20	11 28 28 45.8	12.7	13.64
9 23 30	11 28 12 30.3	7.7	16.01	10 1 30	11 28 19 44.4	10.3	14.96	10 9 30	11 28 28 58.5	12.7	13.61
9 23 40	11 28 12 38.0	7.8	15.99	10 1 40	11 28 19 54.7	10.4	14.94	10 9 40	11 28 29 11.2	12.8	13.58
9 23 50	11 28 12 45.8	7.9	15.97	10 1 50	11 28 20 5.1	10.5	14.91	10 9 50	11 28 29 24.0	12.9	13.55
9 24 0	11 28 12 53.7	7.9	15.96	10 2 0	11 28 20 15.6	10.5	14.89	10 10 0	11 28 29 36.9	12.9	13.51
9 24 10	11 28 13 1.6	8.0	15.94	10 2 10	11 28 20 26.1	10.6	14.87	10 10 10	11 28 29 49.8	12.9	13.48
9 24 20	11 28 13 9.6	8.1	15.92	10 2 20	11 28 20 36.7	10.6	14.84	10 10 20	11 28 30 2.7	13.0	13.45
9 24 30	11 28 13 17.7	8.1	15.90	10 2 30	11 28 20 47.3	10.6	14.82	10 10 30	11 28 30 15.7	13.0	13.42
9 24 40	11 28 13 25.8	8.1	15.88	10 2 40	11 28 20 57.9	10.7	14.79	10 10 40	11 28 30 28.7	13.1	13.39
9 24 50	11 28 13 33.9	8.2	15.86	10 2 50	11 28 21 8.6	10.8	14.77	10 10 50	11 28 30 41.8	13.1	13.35
9 25 0	11 28 13 42.1	8.3	15.84	10 3 0	11 28 21 19.4	10.8	14.74	10 11 0	11 28 30 54.9	13.2	13.32
9 25 10	11 28 13 50.4	8.3	15.82	10 3 10	11 28 21 30.2	10.9	14.72	10 11 10	11 28 31 8.1	13.2	13.29
9 25 20	11 28 13 58.7	8.3	15.80	10 3 20	11 28 21 41.1	10.9	14.69	10 11 20	11 28 31 21.3	13.3	13.26
9 25 30	11 28 14 7.0	8.4	15.78	10 3 30	11 28 21 52.0	11.0	14.66	10 11 30	11 28 31 34.6	13.3	13.23
9 25 40	11 28 14 15.4	8.5	15.76	10 3 40	11 28 22 3.0	11.0	14.63	10 11 40	11 28 31 47.9	13.4	13.19
9 25 50	11 28 14 23.9	8.5	15.74	10 3 50	11 28 22 14.0	11.1	14.60	10 11 50	11 28 32 1.3	13.4	13.16
9 26 0	11 28 14 32.4		15.72	10 4 0	11 28 22 25.1		14.57	10 12 0	11 28 32 14.7		13.13

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +	Mean Anomaly.	Equation.	Diff. +	S. V. +
S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.	S. D. M.	S. D. M. S.	S.	S.
10 12 0	11 28 32 14.7	13.5	13.13	10 20 0	11 28 43 50.2	15.5	11.40	10 28 0	11 28 56 58.1	17.3	9.43
10 12 10	11 28 32 28.2	13.5	13.09	10 20 10	11 28 44 5.7	15.6	11.36	10 28 10	11 28 57 15.4	17.3	9.39
10 12 20	11 28 32 41.7	13.5	13.06	10 20 20	11 28 44 21.3	15.6	11.32	10 28 20	11 28 57 32.7	17.4	9.35
10 12 30	11 28 32 55.2	13.6	13.03	10 20 30	11 28 44 36.9	15.6	11.28	10 28 30	11 28 57 50.1	17.4	9.30
10 12 40	11 28 33 8.8	13.6	12.99	10 20 40	11 28 44 52.5	15.7	11.25	10 28 40	11 28 58 7.5	17.4	9.26
10 12 50	11 28 33 22.4	13.7	12.96	10 20 50	11 28 45 8.2	15.7	11.21	10 28 50	11 28 58 24.9	17.4	9.21
10 13 0	11 28 33 36.1	13.7	12.93	10 21 0	11 28 45 23.9	15.8	11.17	10 29 0	11 28 58 42.3	17.5	9.17
10 13 10	11 28 33 49.8	13.8	12.89	10 21 10	11 28 45 39.7	15.8	11.13	10 29 10	11 28 58 59.8	17.5	9.13
10 13 20	11 28 34 3.6	13.8	12.86	10 21 20	11 28 45 55.5	15.8	11.09	10 29 20	11 28 59 17.3	17.5	9.08
10 13 30	11 28 34 17.4	13.9	12.82	10 21 30	11 28 46 11.3	15.9	11.05	10 29 30	11 28 59 34.8	17.6	9.04
10 13 40	11 28 34 31.3	13.9	12.79	10 21 40	11 28 46 27.2	15.9	11.01	10 29 40	11 28 59 52.4	17.6	9.00
10 13 50	11 28 34 45.2	13.9	12.75	10 21 50	11 28 46 43.1	16.0	10.97	10 29 50	11 29 0 10.0	17.7	8.95
10 14 0	11 28 34 59.1	14.0	12.72	10 22 0	11 28 46 59.1	16.0	10.93	11 0 0	11 29 0 27.7	17.7	8.91
10 14 10	11 28 35 13.1	14.1	12.68	10 22 10	11 28 47 15.1	16.0	10.89	11 0 10	11 29 0 45.4	17.7	8.86
10 14 20	11 28 35 27.2	14.1	12.65	10 22 20	11 28 47 31.1	16.1	10.85	11 0 20	11 29 1 3.1	17.7	8.82
10 14 30	11 28 35 41.3	14.1	12.62	10 22 30	11 28 47 47.2	16.1	10.81	11 0 30	11 29 1 20.8	17.8	8.77
10 14 40	11 28 35 55.4	14.1	12.58	10 22 40	11 28 48 3.3	16.1	10.77	11 0 40	11 29 1 38.6	17.8	8.73
10 14 50	11 28 36 9.5	14.2	12.55	10 22 50	11 28 48 19.4	16.2	10.73	11 0 50	11 29 1 56.4	17.8	8.68
10 15 0	11 28 36 23.7	14.3	12.51	10 23 0	11 28 48 35.6	16.2	10.69	11 1 0	11 29 2 14.2	17.9	8.64
10 15 10	11 28 36 38.0	14.3	12.47	10 23 10	11 28 48 51.8	16.3	10.65	11 1 10	11 29 2 32.1	17.9	8.59
10 15 20	11 28 36 52.3	14.4	12.44	10 23 20	11 28 49 8.1	16.3	10.61	11 1 20	11 29 2 50.0	17.9	8.55
10 15 30	11 28 37 6.7	14.4	12.40	10 23 30	11 28 49 24.4	16.4	10.57	11 1 30	11 29 3 7.9	17.9	8.50
10 15 40	11 28 37 21.1	14.4	12.37	10 23 40	11 28 49 40.8	16.3	10.52	11 1 40	11 29 3 25.8	18.0	8.46
10 15 50	11 28 37 35.5	14.5	12.33	10 23 50	11 28 49 57.1	16.4	10.48	11 1 50	11 29 3 43.8	18.0	8.41
10 16 0	11 28 37 50.0	14.5	12.30	10 24 0	11 28 50 13.5	16.5	10.44	11 2 0	11 29 4 1.8	18.1	8.37
10 16 10	11 28 38 4.5	14.6	12.26	10 24 10	11 28 50 30.0	16.5	10.40	11 2 10	11 29 4 19.9	18.1	8.32
10 16 20	11 28 38 19.1	14.6	12.22	10 24 20	11 28 50 46.5	16.5	10.36	11 2 20	11 29 4 38.0	18.1	8.28
10 16 30	11 28 38 33.7	14.7	12.19	10 24 30	11 28 51 3.0	16.5	10.32	11 2 30	11 29 4 56.1	18.1	8.23
10 16 40	11 28 38 48.4	14.7	12.15	10 24 40	11 28 51 19.5	16.6	10.28	11 2 40	11 29 5 14.2	18.1	8.19
10 16 50	11 28 39 3.1	14.7	12.11	10 24 50	11 28 51 36.1	16.7	10.24	11 2 50	11 29 5 32.3	18.1	8.14
10 17 0	11 28 39 17.8	14.8	12.08	10 25 0	11 28 51 52.8	16.7	10.20	11 3 0	11 29 5 50.5	18.2	8.10
10 17 10	11 28 39 32.6	14.8	12.04	10 25 10	11 28 52 9.5	16.7	10.16	11 3 10	11 29 6 8.8	18.3	8.05
10 17 20	11 28 39 47.4	14.9	12.00	10 25 20	11 28 52 26.2	16.7	10.11	11 3 20	11 29 6 27.0	18.2	8.00
10 17 30	11 28 40 2.3	14.9	11.97	10 25 30	11 28 52 42.9	16.7	10.07	11 3 30	11 29 6 45.3	18.3	7.96
10 17 40	11 28 40 17.2	14.9	11.93	10 25 40	11 28 52 59.6	16.8	10.03	11 3 40	11 29 7 3.6	18.3	7.92
10 17 50	11 28 40 32.1	15 0	11.89	10 25 50	11 28 53 16.4	16.9	9.99	11 3 50	11 29 7 21.9	18.3	7.87
10 18 0	11 28 40 47.1	15.0	11.86	10 26 0	11 28 53 33.3	16.9	9.95	11 4 0	11 29 7 40.3	18.4	7.82
10 18 10	11 28 41 2.1	15.1	11.82	10 26 10	11 28 53 50.2	16.9	9.90	11 4 10	11 29 7 58.7	18.4	7.77
10 18 20	11 28 41 17.2	15.2	11.78	10 26 20	11 28 54 7.1	16.9	9.86	11 4 20	11 29 8 17.1	18.4	7.73
10 18 30	11 28 41 32.4	15.1	11.74	10 26 30	11 28 54 24.0	17.0	9.82	11 4 30	11 29 8 35.5	18.4	7.68
10 18 40	11 28 41 47.5	15.2	11.71	10 26 40	11 28 54 41.0	17.0	9.78	11 4 40	11 29 8 53.9	18.4	7.64
10 18 50	11 28 42 2.7	15.2	11.67	10 26 50	11 28 54 58.0	17.1	9.73	11 4 50	11 29 9 12.4	18.5	7.59
10 19 0	11 28 42 17.9	15.3	11.63	10 27 0	11 28 55 15.1	17.1	9.69	11 5 0	11 29 9 31.0	18.6	7.54
10 19 10	11 28 42 33.2	15.3	11.59	10 27 10	11 28 55 32.2	17.1	9.65	11 5 10	11 29 9 49.5	18.5	7.50
10 19 20	11 28 42 48.5	15.4	11.55	10 27 20	11 28 55 49.3	17.1	9.61	11 5 20	11 29 10 8.1	18.6	7.45
10 19 30	11 28 43 3.9	15.4	11.52	10 27 30	11 28 56 6.4	17.2	9.56	11 5 30	11 29 10 26.7	18.6	7.40
10 19 40	11 28 43 19.3	15.4	11.48	10 27 40	11 28 56 23.6	17.2	9.52	11 5 40	11 29 10 45.3	18.6	7.35
10 19 50	11 28 43 34.7	15.5	11.44	10 27 50	11 28 56 40.8	17.3	9.48	11 5 50	11 29 11 4.0	18.7	7.31
10 20 0	11 28 43 50.2		11.40	10 28 0	11 28 56 58.1		9.43	11 6 0	11 29 11 22.7	18.7	7.26

TABLE VII.

Equation of the Sun's Center for 1810, with the Secular Variation.

Mean Anomaly.	Equation.	Diff. +	S. V. +
S. D. M.	S. D. M. S.	S.	S.
11 6 0	11 29 11 22.7	18.7	7.26
11 6 10	11 29 11 41.4	18.7	7.21
11 6 20	11 29 12 0.1	18.8	7.17
11 6 30	11 29 12 18.9	18.7	7.12
11 6 40	11 29 12 37.6	18.8	7.07
11 6 50	11 29 12 56.4	18.9	7.02
11 7 0	11 29 13 15.3	18.9	6.98
11 7 10	11 29 13 34.2	18.9	6.93
11 7 20	11 29 13 53.1	18.9	6.88
11 7 30	11 29 14 12.0	18.9	6.83
11 7 40	11 29 14 30.9	18.9	6.79
11 7 50	11 29 14 49.8	19.0	6.74
11 8 0	11 29 15 8.8	19.0	6.69
11 8 10	11 29 15 27.8	19.0	6.64
11 8 20	11 29 15 46.8	19.0	6.59
11 8 30	11 29 16 5.9	19.1	6.55
11 8 40	11 29 16 24.9	19.0	6.50
11 8 50	11 29 16 44.0	19.1	6.45
11 9 0	11 29 17 3.1	19.1	6.40
11 9 10	11 29 17 22.3	19.2	6.35
11 9 20	11 29 17 41.4	19.1	6.31
11 9 30	11 29 18 0.6	19.2	6.26
11 9 40	11 29 18 19.8	19.2	6.21
11 9 50	11 29 18 39.0	19.2	6.16
11 10 0	11 29 18 58.3	19.3	6.11
11 10 10	11 29 19 17.6	19.3	6.06
11 10 20	11 29 19 36.9	19.3	6.02
11 10 30	11 29 19 56.2	19.3	5.97
11 10 40	11 29 20 15.5	19.3	5.92
11 10 50	11 29 20 34.9	19.4	5.87
11 11 0	11 29 20 54.3	19.4	5.82
11 11 10	11 29 21 13.7	19.4	5.77
11 11 20	11 29 21 33.1	19.4	5.72
11 11 30	11 29 21 52.6	19.5	5.67
11 11 40	11 29 22 12.0	19.4	5.62
11 11 50	11 29 22 31.5	19.5	5.57
11 12 0	11 29 22 51.0	19.5	5.52
11 12 10	11 29 23 10.5	19.5	5.48
11 12 20	11 29 23 30.0	19.5	5.43
11 12 30	11 29 23 49.6	19.6	5.38
11 12 40	11 29 24 9.2	19.6	5.33
11 12 50	11 29 24 28.8	19.6	5.28
11 13 0	11 29 24 48.4	19.6	5.23
11 13 10	11 29 25 8.0	19.6	5.18
11 13 20	11 29 25 27.6	19.6	5.13
11 13 30	11 29 25 47.3	19.7	5.08
11 13 40	11 29 26 7.0	19.7	5.03
11 13 50	11 29 26 26.7	19.7	4.98
11 14 0	11 29 26 46.5	19.8	4.93
11 14 10	11 29 27 6.2	19.7	4.88
11 14 20	11 29 27 26.0	19.8	4.83
11 14 30	11 29 27 45.7	19.7	4.78
11 14 40	11 29 28 5.5	19.8	4.73
11 14 50	11 29 28 25.3	19.8	4.68
11 15 0	11 29 28 45.2	19.9	4.63
11 15 10	11 29 29 5.0	19.8	4.58
11 15 20	11 29 29 24.9	19.9	4.53
11 15 30	11 29 29 44.7	19.8	4.48
11 15 40	11 29 30 4.6	19.9	4.43
11 15 50	11 29 30 24.5	19.9	4.38
11 16 0	11 29 30 44.4	19.9	4.33
11 16 10	11 29 31 4.4	20.0	4.28
11 16 20	11 29 31 24.3	19.9	4.23
11 16 30	11 29 31 44.3	20.0	4.18
11 16 40	11 29 32 4.3	20.0	4.13
11 16 50	11 29 32 24.3	20.0	4.08
11 17 0	11 29 32 44.3	20.0	4.03
11 17 10	11 29 33 4.4	20.1	3.98
11 17 20	11 29 33 24.4	20.0	3.93
11 17 30	11 29 33 44.5	20.1	3.87
11 17 40	11 29 34 4.5	20.0	3.82
11 17 50	11 29 34 24.6	20.1	3.77
11 18 0	11 29 34 44.7	20.1	3.72
11 18 10	11 29 35 4.8	20.1	3.67
11 18 20	11 29 35 24.9	20.1	3.62
11 18 30	11 29 35 45.1	20.2	3.56
11 18 40	11 29 36 5.2	20.1	3.52
11 18 50	11 29 36 25.3	20.1	3.47
11 19 0	11 29 36 45.5	20.2	3.42
11 19 10	11 29 37 5.7	20.2	3.36
11 19 20	11 29 37 25.9	20.2	3.31
11 19 30	11 29 37 46.1	20.2	3.26
11 19 40	11 29 38 6.3	20.2	3.21
11 19 50	11 29 38 26.6	20.3	3.16
11 20 0	11 29 38 46.8	20.2	3.11
11 20 10	11 29 39 7.1	20.3	3.06
11 20 20	11 29 39 27.3	20.2	3.01
11 20 30	11 29 39 47.6	20.3	2.96
11 20 40	11 29 40 7.9	20.3	2.90
11 20 50	11 29 40 28.2	20.3	2.85
11 21 0	11 29 40 48.5	20.3	2.80
11 21 10	11 29 41 8.8	20.3	2.75
11 21 20	11 29 41 29.1	20.3	2.70
11 21 30	11 29 41 49.5	20.4	2.65
11 21 40	11 29 42 9 9	20.4	2.60
11 21 50	11 29 42 30.2	20.3	2.54
11 22 0	11 29 42 50.5	20.3	2.49
11 22 10	11 29 43 10.9	20.4	2.44
11 22 20	11 29 43 31.3	20.4	2.39
11 22 30	11 29 43 51.7	20.4	2.34
11 22 40	11 29 44 12.1	20.4	2.29
11 22 50	11 29 44 32.5	20.4	2.23
11 23 0	11 29 44 52.9	20.4	2.18
11 23 10	11 29 45 13.3	20.4	2.13
11 23 20	11 29 45 33.7	20.5	2.08
11 23 30	11 29 45 54.2	20.5	2.03
11 23 40	11 29 46 14.7	20.4	1.98
11 23 50	11 29 46 35.1	20.4	1.92
11 24 0	11 29 46 55.5	20.5	1.87
11 24 10	11 29 47 16.0	20.5	1.82
11 24 20	11 29 47 36.5	20.5	1.77
11 24 30	11 29 47 57.0	20.5	1.72
11 24 40	11 29 48 17.5	20.4	1.67
11 24 50	11 29 48 37.9	20.5	1.61
11 25 0	11 29 48 58.4	20.6	1.56
11 25 10	11 29 49 19.0	20.5	1.51
11 25 20	11 29 49 39.5	20.5	1.46
11 25 30	11 29 50 0.0	20.5	1.40
11 25 40	11 29 50 20.5	20.5	1.35
11 25 50	11 29 50 41.0	20.5	1.29
11 26 0	11 29 51 1.5	20.5	1.24
11 26 10	11 29 51 22.0	20.6	1.19
11 26 20	11 29 51 42.6	20.5	1.14
11 26 30	11 29 52 3.1	20.5	1.09
11 26 40	11 29 52 23.6	20.6	1.04
11 26 50	11 29 52 44.2	20.5	0.99
11 27 0	11 29 53 4.7	20.6	0.94
11 27 10	11 29 53 25.3	20.6	0.89
11 27 20	11 29 53 45.9	20.6	0.83
11 27 30	11 29 54 6.5	20.5	0.78
11 27 40	11 29 54 27.0	20.5	0.73
11 27 50	11 29 54 47.5	20.6	0.68
11 28 0	11 29 55 8.1	20.5	0.62
11 28 10	11 29 55 28.6	20.6	0.57
11 28 20	11 29 55 49.2	20.5	0.52
11 28 30	11 29 56 9.7	20.6	0.47
11 28 40	11 29 56 30.3	20.6	0.42
11 28 50	11 29 56 50.9	20.6	0.36
11 29 0	11 29 57 11.5	20.6	0.30
11 29 10	11 29 57 32.1	20.6	0.26
11 29 20	11 29 57 52.7	20.6	0.21
11 29 30	11 29 58 13.3	20.6	0.16
11 29 40	11 29 58 33.8	20.5	0.10
11 29 50	11 29 58 54.4	20.6	0.05
12 0 0	11 29 59 15.0	20.6	0.00

TABLE VIII.

NUTATION.

ARGUMENT. Supplement of the Node, or N.

N	Longitude.	Right Ascension.	Obliquity.	Equation of Time.	N	Longitude.	Right Ascension.	Obliquity.	Equation of Time.
	s.	s.	s.	s.		s.	s.	s.	s.
00	+ 0.0	+ 0.0	+ 9.6	+ 0.00	500	— 0.0	— 0.0	— 9.6	— 0.00
10	1.1	1.1	9.6	0.01	510	1.0	1.1	9.6	0.01
20	2.2	2.1	9.6	0.01	520	2.2	2.1	9.5	0.01
30	3.4	3.1	9.5	0.02	530	3.4	3.1	9.5	0.02
40	4.5	4.1	9.3	0.02	540	4.5	4.1	9.3	0.02
50	+ 5.6	+ 5.1	9.2	+ 0.03	550	5.6	5.1	9.2	0.03
60	6.6	6.1	+ 9.0	0.04	560	— 6.6	— 6.1	— 9.0	— 0.04
70	7.7	7.0	8.7	0.04	570	7.7	7.0	8.7	0.04
80	8.7	8.0	8.5	0.05	580	8.7	8.0	8.5	0.05
90	9.7	8.9	8.1	0.05	590	9.7	8.9	8.1	0.05
100	+ 10.6	+ 9.7	7.8	+ 0.06	600	10.6	9.7	7.8	— 0.06
110	11.5	10.5	+ 7.4	0.06	610	— 11.5	— 10.5	— 7.4	— 0.06
120	12.3	11.3	7.0	0.07	620	12.3	11.3	7.0	0.07
130	13.1	12.1	6.6	0.07	630	13.1	12.1	6.6	0.07
140	13.9	12.7	6.1	0.08	640	13.9	12.7	6.1	0.08
150	+ 14.6	+ 13.4	5.7	+ 0.08	650	14.6	13.4	5.7	0.08
160	15.2	14.0	+ 5.2	0.08	660	— 15.2	— 14.0	— 5.2	— 0.08
170	15.8	14.5	4.6	0.09	670	15.8	14.5	4.6	0.09
180	16.3	15.0	4.1	0.09	680	16.3	15.0	4.1	0.09
190	16.8	15.4	3.5	0.09	690	16.8	15.4	3.5	0.09
200	+ 17.2	+ 15.7	3.0	+ 0.09	700	17.2	15.7	3.0	0.09
210	17.5	16.0	+ 2.4	0.10	710	— 17.5	— 16.0	— 2.4	— 0.10
220	17.7	16.3	1.8	0.10	720	17.7	16.3	1.8	0.10
230	17.9	16.4	1.2	0.10	730	17.9	16.4	1.2	0.10
240	18.0	16.5	0.6	0.10	740	18.0	16.5	0.6	0.10
250	+ 18.0	+ 16.5	+ 0.0	+ 0.10	750	— 18.0	16.5	— 0.0	0.10
260	18.0	16.5	— 0.6	0.10	760	18.0	— 16.5	+ 0.6	— 0.10
270	17.9	16.4	1.2	0.10	770	17.9	16.4	1.2	0.10
280	17.7	16.3	1.8	0.10	780	17.7	16.3	1.8	0.10
290	17.5	16.0	2.4	0.10	790	17.5	16.0	2.4	0.10
300	+ 17.2	+ 15.7	— 3.0	+ 0.09	800	— 17.2	15.7	3.0	0.09
310	16.8	15.4	3.5	0.09	810	16.8	— 15.4	+ 3.5	— 0.09
320	16.3	15.0	4.1	0.09	820	16.3	15.0	4.1	0.09
330	15.8	14.5	4.6	0.09	830	15.8	14.5	4.6	0.09
340	15.2	14.0	5.2	0.08	840	15.2	14.0	5.2	0.08
350	+ 14.6	+ 13.4	— 5.7	+ 0.08	850	— 14.6	13.4	5.7	0.08
360	13.9	12.7	6.1	0.08	860	13.9	— 12.7	+ 6.1	— 0.08
370	13.1	12.1	6.6	0.07	870	13.1	12.1	6.6	0.07
380	12.3	11.3	7.0	0.07	880	12.3	11.3	7.0	0.07
390	11.5	10.5	7.4	0.06	890	11.5	10.5	7.4	0.06
400	+ 10.6	+ 9.7	— 7.8	+ 0.06	900	— 10.6	9.7	7.8	0.06
410	9.7	8.9	8.1	0.05	910	9.7	— 8.9	+ 8.1	— 0.05
420	8.7	8.0	8.5	0.05	920	8.7	8.0	8.5	0.05
430	7.7	7.0	8.7	0.04	930	7.7	7.0	8.7	0.04
440	6.6	6.1	9.0	0.04	940	6.6	6.1	9.0	0.04
450	+ 5.6	+ 5.1	— 9.2	+ 0.03	950	— 5.6	5.1	9.2	0.03
460	4.5	4.1	9.3	0.02	960	4.5	— 4.1	+ 9.3	— 0.02
470	3.4	3.1	9.5	0.02	970	3.4	3.1	9.5	0.02
480	2.2	2.1	9.6	0.01	980	2.2	2.1	9.6	0.01
490	1.0	1.0	9.6	0.01	990	1.0	1.0	9.6	0.01
500	+ 0.0	+ 0.0	— 9.6	+ 0.00	1000	— 0.0	— 0.0	+ 9.6	— 0.00

Continuation of Table VIII.

Solar Nutation.

Months and Days.		Long.	Obliq.
21 March.	23 Sept.	— 0.″0	+ 0.″4
26	28	0.2	0.4
31	3 Oct.	0.3	0.4
5 April.	8	0.5	0.4
10	14	0.7	0.3
15	19	0.8	0.3
20	24	0.9	0.2
25	29	0.9	0.1
1 May.	3 Nov.	1.0	+ 0.1
6	8	1.0	0.0
11	13	1.0	— 0.1
16	18	0.9	0.1
21	22	0.9	0.2
27	27	0.8	0.3
1 June.	2 Dec.	0.7	0.3
6	7	0.5	0.4
11	12	0.3	0.4
16	17	— 0.2	0.4
22	22	0.0	0.4
27	27	+ 0.2	0.4
2 July.	1 Jan.	0.3	0.4
7	6	0.5	0.4
13	11	0.7	0.3
18	16	0.8	0.3
23	20	0.9	0.2
28	25	0.9	0.1
3 August.	30	1.0	— 0.1
8	4 Feb.	1.0	0.0
13	9	1.0	+ 0.1
18	14	0.9	0.1
23	19	0.9	0.2
29	24	0.8	0.3
3 Sept.	1 March.	0.7	0.3
8	6	0.5	0.4
13	11	0.3	0.4
18	16	0.2	0.4
23	21	+ 0.0	+ 0.4

TABLE IX.

Mean Motion of the Sun in Right Ascension for Sidereal Hours.

Hours.	Motion.	Minutes.	Motion.	Minutes	Motion.	Seconds.	Motion.
	′ ″		″		″		″
1	0 9.83	1	0.16	31	5.08	3	0.01
2	0 19.66	2	0.33	32	5.24	6	0.02
3	0 29.49	3	0.49	33	5.41	9	0.02
4	0 39.32	4	0.65	34	5.57	12	0.03
5	0 49.15	5	0.82	35	5.73	15	0.04
6	0 58.98	6	0.98	36	5.90	18	0.05
7	1 8.81	7	1.15	37	6.06	21	0.06
8	1 18.64	8	1.31	38	6.22	24	0.06
9	1 28.46	9	1.47	39	6.39	27	0.07
10	1 38.29	10	1.64	40	6.55	30	0.08
11	1 48.12	11	1.80	41	6.72	33	0.09
12	1 57.95	12	1.97	42	6.88	36	0.10
13	2 7.78	13	2.13	43	7.04	39	0.11
14	2 17.61	14	2.29	44	7.21	42	0.11
15	2 27.44	15	2.46	45	7.37	45	0.12
16	2 37.27	16	2.62	46	7.54	48	0.13
17	2 47.10	17	2.78	47	7.70	51	0.14
18	2 56.93	18	2.95	48	7.86	54	0.15
19	3 6.76	19	3.11	49	8.03	57	0.16
20	3 16.59	20	3.28	50	8.19	60	0.16
21	3 26.42	21	3.44	51	8.35		
22	3 36.25	22	3.60	52	8.51		
23	3 46.08	23	3.77	53	8.68		
24	3 55.91	24	3.93	54	8.85		
25	4 5.74	25	4.10	55	9.01		
26	4 15.57	26	4.26	56	9.17		
27	4 25.40	27	4.42	57	9.34		
28	4 35.22	28	4.59	58	9.50		
29	4 45.05	29	4.75	59	9.67		
30	4 54.88	30	4.91	60	9.83		

Sidereal Days.	Motion. M. S.	Sideral Days.	Motion. H. M. S.	Sidereal Days.	Motion. H. M. S.
1	3 55.9	11	0 43 15.0	21	1 22 34.1
2	7 51.8	12	0 47 10.9	22	1 26 30.0
3	11 47.7	13	0 51 6.8	23	1 30 25.9
4	15 43.6	14	0 55 2.7	24	1 34 21.8
5	19 39.5	15	0 58 58.6	25	1 38 17.7
6	23 35.4	16	1 2 54.5	26	1 42 13.6
7	27 31.4	17	1 6 50.4	27	1 46 9.5
8	31 27.3	18	1 10 46.3	28	1 50 5.4
9	35 23.2	19	1 14 42.3	29	1 54 1.3
10	39 19.1	20	1 18 38.2	30	1 57 57.2

TABLE X.

Lunar Equation, 1st Part.

ARGUMENT A.

A.	Equation A.	A.	Equation A.
0	7.5	500	7.5
10	8.0	510	7.0
20	8.4	520	6.6
30	8.9	530	6.1
40	9.4	540	5.6
50	9.8	550	5.2
60	10.3	560	4.7
70	10.7	570	4.3
80	11.1	580	3.9
90	11.5	590	3.5
100	11.9	600	3.1
110	12.3	610	2.7
120	12.6	620	2.4
130	13.0	630	2.0
140	13.3	640	1.7
150	13.6	650	1.4
160	13.8	660	1.2
170	14.1	670	0.9
180	14.3	680	0.7
190	14.5	690	0.5
200	14.6	700	0.4
210	14.8	710	0.2
220	14.9	720	0.1
230	14.9	730	0.1
240	15.0	740	0.0
250	15.0	750	0.0
260	15.0	760	0.0
270	14.9	770	0.1
280	14.9	780	0.1
290	14.8	790	0.2
300	14.6	800	0.4
310	14.5	810	0.5
320	14.2	820	0.7
330	14.1	830	0.9
340	13.8	840	1.2
350	13.6	850	1.4
360	13.3	860	1.7
370	13.0	870	2.0
380	12.6	880	2.4
390	12.3	890	2.7
400	11.9	900	3.1
410	11.5	910	3.5
420	11.1	920	3.9
430	10.7	930	4.3
440	10.3	940	4.7
450	9.8	950	5.2
460	9.4	960	5.6
470	8.9	970	6.1
480	8.4	980	6.6
490	8.0	990	7.0
500	7.5	1000	7.5

Continuation of TABLE X.

Lunar Equation, Second Part.

ARGUMENTS. First Part, and Mean Anomaly.

Mean Anomaly.	0″	1″	2″	3″	4″	11″	12″	13″	14″	15″
S. D.										
0 0	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
0 20	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
I 10	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
20	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2
II. 0	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
III. 0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
IV. 0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0
10	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0
IV. 20	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	0.0
V. 0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	0.0
VI. 0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	0.0
VII. 0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	0.0
20	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0
VIII. 0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0
IX. 0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
X. 10	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
20	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
XI. 0	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
XI. 20	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
XII. 0	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2

From the sum of these three Equations subtract the constant 0″.5.

Continuation of TABLE X.

Lunar Equation, Third Part.

ARGUMENTS. M and A.

A.

M	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	0.4	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.4	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.4
50	0.4	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.7	0.6	0.7	0.7	0.6	0.5	0.4
100	0.4	0.3	0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.6	0.6	0.6	0.6	0.6	0.5	0.4
150	0.4	0.3	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.5	0.6	0.6	0.6	0.5	0.5	0.6	0.5	0.5	0.4
200	0.4	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.4
250	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.3	0.3	0.3	0.4	0.4	0.4
300	0.4	0.4	0.4	0.5	0.5	0.6	0.6	0.6	0.5	0.4	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.3	0.4	0.4
350	0.4	0.5	0.5	0.6	0.6	0.6	0.7	0.6	0.6	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.3	0.4
400	0.4	0.5	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.5	0.4	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.3	0.4
450	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.1	0.0	0.0	0.1	0.2	0.3	0.4
500	0.4	0.5	0.6	0.7	0.8	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	0.0	0.0	0.1	0.2	0.3	0.4
550	0.4	0.5	0.6	0.7	0.8	0.8	0.7	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	0.0	0.1	0.1	0.2	0.3	0.4
600	0.4	0.5	0.6	0.7	0.7	0.7	0.7	0.6	0.6	0.5	0.4	0.3	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.4
650	0.4	0.5	0.6	0.6	0.7	0.6	0.6	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.1	0.2	0.2	0.3	0.3	0.3	0.4
700	0.4	0.4	0.5	0.6	0.6	0.6	0.5	0.5	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.4
750	0.4	0.4	0.4	0.5	0.5	0.5	0.4	0.4	0.3	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.4	0.4	0.5	0.4	0.4
800	0.4	0.4	0.3	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.4
850	0.4	0.3	0.3	0.3	0.2	0.3	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.5	0.4
900	0.4	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.6	0.6	0.6	0.7	0.6	0.5	0.4
950	0.4	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.4	0.5	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.4
1000	0.4	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.4	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.4

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

C

B	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
0	21.6	20.8	19.8	19.0	17.9	16.8	15.9	14.7	14.0	13.2	12.8	12.5	12.2	12.2	12.3	12.4	12.8	13.3	13.9
20	23.1	22.7	21.6	21.0	20.1	19.3	18.4	17.4	16.4	15.5	14.5	13.8	13.4	12.9	12.6	12.3	12.2	12.4	12.9
40	23.5	23.2	22.9	22.7	22.0	21.1	20.4	19.5	18.7	17.9	16.9	16.1	15.3	14.4	14.0	13.5	13.0	12.9	12.6
60	22.2	22.5	23.1	22.7	22.8	22.5	21.9	21.3	20.5	19.9	19.1	18.2	17.4	16.7	16.0	15.2	14.5	14.0	13.6
80	20.0	20.7	21.4	21.7	22.1	22.3	22.2	22.2	21.7	21.3	20.7	19.9	19.3	18.7	17.7	17.1	16.4	15.9	15.4
100	17.6	18.6	19.2	19.9	20.5	21.0	21.6	21.7	21.6	21.6	21.5	21.1	20.5	20.2	19.5	18.9	18.2	17.5	17.1
120	15.3	16.0	16.9	17.7	18.4	19.2	19.8	20.2	20.7	20.8	21.1	21.1	20.8	20.7	20.4	20.0	19.7	19.2	18.5
140	13.6	14.2	14.8	15.5	16.2	17.0	17.6	18.3	19.0	19.4	20.0	20.0	20.4	20.4	20.2	20.0	20.1	19.7	19.5
160	12.7	13.2	13.6	14.1	14.6	15.0	15.7	16.4	17.0	17.3	18.1	18.7	19.2	19.1	19.4	19.7	19.5	19.6	19.3
180	12.7	12.9	13.1	13.5	13.9	14.0	14.5	14.8	15.0	15.8	16.4	16.8	17.2	17.7	18.5	18.5	18.5	18.8	18.4
200	13.2	13.2	13.2	13.4	13.7	13.8	14.1	14.2	14.5	14.5	14.8	15.2	16.0	16.2	16.6	16.8	17.5	17.6	17.7
220	13.5	13.6	13.9	14.1	14.1	14.1	14.2	14.3	14.5	14.6	14.6	14.7	14.8	15.0	15.3	15.7	16.1	16.2	16.6
240	13.6	13.8	14.1	14.4	14.6	14.8	14.8	14.9	15.1	15.1	15.1	14.9	14.8	14.7	14.8	15.0	15.1	15.4	15.7
260	12.8	13.3	13.8	14.2	14.6	15.0	15.3	15.6	15.5	15.5	15.6	15.6	15.6	15.7	15.3	14.8	15.0	15.0	15.1
280	11.5	12.3	13.0	13.4	14.0	14.6	15.1	15.4	16.0	16.2	16.2	16.3	16.2	16.2	16.2	15.9	15.8	15.8	15.5
300	10.1	10.9	11.3	12.1	12.9	13.7	14.2	14.9	15.4	16.0	16.4	16.5	16.7	17.0	17.1	16.9	16.9	16.6	16.5
320	8.2	8.8	9.6	10.6	11.3	12.0	12.9	13.7	14.3	15.0	15.8	16.3	16.8	17.3	17.5	17.6	17.7	17.6	17.5
340	6.9	7.5	8.1	8.4	9.4	10.1	11.1	11.9	12.7	13.6	14.4	15.2	16.0	16.4	17.2	17.8	17.9	18.1	18.3
360	6.5	6.5	6.8	7.4	8.0	8.4	9.1	9.9	10.8	11.5	12.6	13.4	14.4	15.2	16.0	16.7	17.4	18.1	18.4
380	6.8	6.5	6.3	6.4	6.7	7.0	7.6	8.2	8.9	9.6	10.6	11.4	12.4	13.4	14.3	15.3	16.1	16.9	17.5
400	7.5	7.1	6.7	6.4	6.2	6.4	6.5	6.9	7.5	7.9	8.7	9.4	10.3	11.2	12.3	13.2	14.2	15.1	16.0
420	9.1	8.4	7.6	7.1	6.7	6.5	6.3	6.2	6.7	6.8	7.2	7.8	8.4	9.2	10.0	11.0	12.2	13.0	14.1
440	10.6	9.8	9.0	8.6	7.9	7.2	6.7	6.4	6.4	6.4	6.6	6.8	7.1	7.6	8.4	9.0	9.9	10.9	11.8
460	12.1	11.5	10.5	9.6	9.0	8.5	8.0	7.3	6.8	6.6	6.5	6.4	6.5	6.8	7.2	7.4	8.1	9.0	9.7
480	13.6	12.8	11.9	11.0	10.4	9.6	8.8	8.2	7.7	7.2	6.8	6.4	6.5	6.5	6.4	6.6	7.0	7.5	8.2
500	15.1	14.4	13.4	12.4	11.6	10.8	10.1	9.3	8.6	8.1	7.5	7.1	6.8	6.7	6.5	6.3	6.5	6.6	7.0
520	16.5	15.6	14.8	13.9	13.1	12.3	11.3	10.5	9.7	9.1	8.6	7.9	7.4	7.0	6.8	6.5	6.3	6.1	6.3
540	18.1	17.5	16.4	15.5	14.5	13.7	12.8	11.8	11.1	10.4	9.7	8.9	8.2	7.6	7.2	6.8	6.5	6.3	6.2
560	20.4	19.3	18.2	17.6	16.5	15.4	14.4	13.4	12.7	11.6	10.8	10.2	9.2	8.6	7.9	7.5	6.8	6.6	6.3
580	22.8	21.7	20.7	19.7	18.4	17.6	16.6	15.5	14.3	13.4	12.5	11.6	10.6	9.8	9.1	8.4	7.7	7.3	6.6
600	25.2	24.1	23.1	22.2	21.2	19.9	18.6	17.8	16.6	15.6	14.5	13.4	12.6	11.4	10.5	9.5	8.7	8.1	7.4
620	27.3	26.5	25.6	24.7	23.5	22.5	21.6	20.4	19.0	18.1	16.8	15.7	14.7	13.5	12.4	11.4	10.4	9.5	8.7
640	29.0	28.5	27.7	26.9	26.2	25.1	24.1	22.9	21.8	20.8	19.6	18.4	17.2	16.2	14.9	13.7	12.5	11.4	10.4
660	29.8	29.6	29.2	28.5	28.1	27.4	26.5	25.6	24.5	23.4	22.5	21.2	19.8	19.0	17.6	16.5	15.1	13.9	12.8
680	29.7	29.6	29.5	29.5	29.1	28.8	28.2	27.6	27.0	26.0	25.0	25.8	22.8	21.7	20.4	19.3	18.1	16.8	15.7
700	28.8	29.2	29.3	29.5	29.5	29.5	29.2	28.8	28.4	27.8	27.2	26.4	25.2	24.3	23.3	22.1	20.7	19.7	18.5
720	26.9	27.6	28.3	29.0	29.2	29.4	29.4	29.3	29.1	28.9	28.4	27.9	27.3	26.4	25.7	24.5	23.7	22.5	21.1
740	24.7	25.7	26.6	27.3	27.9	28.5	29.1	29.0	29.2	29.3	29.1	28.8	28.4	27.7	27.4	26.6	25.9	24.9	24.0
760	22.2	23.5	24.3	25.3	26.2	27.0	27.6	28.3	28.6	28.7	28.9	29.1	29.0	28.7	28.3	27.8	27.3	26.8	25.9
780	19.6	21.0	22.0	23.2	24.2	25.1	25.9	26.7	27.3	27.8	28.4	28.5	28.7	28.7	28.8	28.7	28.3	28.0	27.2
800	17.2	18.5	19.3	20.9	21.8	22.9	23.9	25.0	25.8	26.4	26.9	27.6	28.1	28.3	28.4	28.5	28.5	28.4	28.2
820	15.2	15.9	17.0	18.4	18.9	20.7	21.7	22.8	23.8	24.8	25.6	26.2	26.6	27.3	27.8	28.1	28.3	28.1	28.1
840	13.2	14.0	15.0	16.0	17.0	18.2	18.8	20.3	21.7	22.7	23.6	24.5	25.3	26.2	26.7	27.2	27.5	27.9	28.1
860	11.5	12.2	13.0	13.9	14.9	15.9	17.1	18.0	18.9	20.3	21.4	22.6	23.5	24.5	25.1	25.9	26.6	27.1	27.4
880	11.0	11.2	11.5	12.2	13.0	13.7	14.8	15.7	16.8	18.1	19.1	20.2	21.1	22.4	23.3	24.2	25.1	25.8	26.5
900	11.2	11.2	10.9	11.5	12.5	12.1	12.8	13.7	14.5	15.5	14.6	17.9	18.5	20.1	21.3	22.1	23.1	24.7	25.0
920	12.1	11.6	11.5	11.1	11.2	11.3	11.7	12.1	12.7	13.4	16.4	15.2	16.4	17.7	18.4	20.0	21.0	22.2	23.0
940	14.0	13.3	12.6	12.3	11.6	11.5	11.3	11.4	11.6	12.0	12.8	13.3	14.2	14.9	16.1	17.5	18.2	19.6	20.8
960	16.7	15.6	14.6	13.7	13.1	12.5	11.9	11.7	11.6	11.4	11.7	12.1	12.6	13.3	14.1	14.4	15.9	17.2	17.9
980	19.5	18.3	17.3	16.4	15.2	14.2	13.4	12.7	12.2	12.0	11.9	11.8	11.8	12.1	12.7	13.3	14.1	14.8	15.6
1000	21.6	20.8	19.8	19.0	17.9	16.8	15.9	14.7	14.0	13.2	12.8	12.5	12.2	12.2	12.3	12.4	12.8	13.3	13.9
	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

C

B	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360
0	13.9	14.7	15.6	16.5	17.6	18.8	20.1	21.1	22.2	23.4	24.3	25.2	25.8	26.6	27.2	27.6	27.7	27.6	27.6
20	12.9	13.3	14.0	14.6	15.5	16.4	17.3	18.6	19.7	20.9	21.9	23.0	24.2	24.9	25.8	26.6	27.0	27.4	27.7
40	12.6	12.6	13.1	13.5	14.0	14.4	15.4	16.5	17.3	18.3	19.4	20.5	21.6	22.7	23.7	24.9	25.5	26.3	26.9
60	13.6	13.3	13.2	13.2	13.4	13.5	14.1	14.6	15.2	16.3	17.2	18.1	18.9	20.3	21.2	22.3	23.4	24.5	25.3
80	15.4	14.6	14.3	13.9	13.8	13.7	13.6	14.0	14.5	14.9	15.5	16.3	17.3	18.2	19.0	20.0	21.1	22.0	23.1
100	17.1	16.3	15.9	15.4	14.8	14.6	14.3	14.3	14.3	14.4	14.6	15.0	15.5	16.2	16.9	17.7	18.9	19.8	20.8
120	18.5	18.0	17.3	16.9	16.5	16.2	15.6	15.2	14.8	14.8	15.0	14.9	15.0	15.2	15.9	16.3	17.0	17.7	18.5
140	19.5	19.3	18.8	18.2	17.7	17.4	17.0	16.6	16.4	15.9	15.5	15.4	15.6	15.6	15.5	15.6	16.1	16.7	17.1
160	19.5	19.6	19.2	19.0	18.7	18.4	18.1	17.7	17.5	17.3	16.9	16.6	16.3	15.9	16.1	16.3	16.3	16.2	16.5
180	18.3	18.8	19.0	19.0	18.9	18.6	18.5	18.5	18.3	18.1	17.9	17.6	17.5	17.3	17.0	16.9	16.7	16.8	16.9
200	17.7	17.9	18.1	18.2	18.3	18.3	18.3	18.4	18.2	18.2	18.2	18.2	18.1	18.1	17.8	17.7	17.6	17.5	17.7
220	16 6	16.8	17.1	17.5	17.1	17.4	17.5	17.6	17.8	17.8	18.0	18.0	18.2	18.1	18.1	18.3	18.4	18.3	18.3
240	15.7	15.8	16.0	16.1	16.1	16.3	16.4	16.5	16.7	16.9	17.1	17.3	17.3	17.7	17.5	18.0	18.3	18.4	18.6
260	15.1	15.0	15.1	15.2	15.2	15.1	15.2	15.5	15.5	15.6	15.8	16.1	16.4	16.6	16.8	16.9	17.4	17.7	18.2
280	15 5	15.4	15.1	14.9	14.8	14.7	15.0	14.9	14.9	14.9	14.9	14.7	15.0	15.3	15.5	15.9	16.1	16.4	16.8
300	16.5	16.3	15.9	15.7	15.2	14.9	14.8	14.6	14.6	14.2	14.0	14.0	13.9	13.9	14.2	14.5	14.8	15.0	15.5
320	17.5	17.2	17.0	16.8	16.5	16.1	15.6	15.3	14.7	14.5	14.4	13.1	13.6	13.4	13.3	13.1	13.4	13.6	13.8
340	18.3	18.2	18.2	17.9	17.5	17.3	16.8	16.6	16.0	15.5	15.2	14.5	14.3	13.7	13.1	13.0	12.7	12.6	12.6
360	18.4	18.6	18.8	18.8	18.8	18.7	18.4	17.9	17.5	17.0	16.5	15.9	15.4	14.9	14.3	13.7	13.0	12.6	12.3
380	17.5	18.1	18.6	19.1	19.3	19.5	19.5	19.2	18.9	18.5	17.9	17.7	16.9	16.4	15.8	15.0	14.5	13.6	13.1
400	16.0	16.8	17.8	18.4	18.8	19.3	19.8	19.8	20.1	19.7	19.4	19.1	18.6	18.1	17.5	17.0	16.1	15.2	14.8
420	14.1	15.0	15.9	16.9	17.7	18.5	19.0	19.5	20.0	20.3	20.3	20.3	20.1	19.4	19.0	18.9	18.1	17.3	16.5
440	11.8	12.9	13.8	14.9	16.0	16.7	17.8	18.7	19.2	19.7	20.1	20.4	20.7	20.7	20.5	20.2	19.8	19.5	18.6
460	9.7	10.6	11.7	12.6	13.8	14.6	15.9	16.8	17.6	18.6	19.2	19.9	20.3	20.6	21.0	20.9	20.9	20.8	20.3
480	8.2	8.8	9.6	10.4	11.5	12.5	13.5	14.6	15.5	16.6	17.7	18.5	19.3	19.9	20.5	20.8	21.1	21.2	21.2
500	7.0	7.4	8.2	8.6	9.4	10.4	11.3	12.4	13.4	14.4	15.5	16.5	17.7	18.6	19.1	19.9	20.7	21.0	21.4
520	6.3	6.6	7.0	7.5	8.0	8.8	9.3	10.2	11.2	12.2	13.3	14.2	15.4	16.4	17.6	18.4	19.2	19.8	20.6
540	6.2	6.0	6.2	6.5	6.9	7.4	7.9	8.6	9.4	10.1	11.1	12.1	13.1	14.2	15.3	16.3	17.4	18.3	19.2
560	6.3	6.1	6.0	6.1	6.2	6.5	6.9	7.2	7.8	8.4	9.2	10.1	11.0	11.9	13.1	14.1	15.2	16.2	17.2
580	6.6	6.3	6.1	5.9	5.7	5.9	6.0	6.3	6.6	7.0	7.6	8.4	9.1	9.9	10.9	11.9	12.9	14.1	15.0
600	7.4	7.0	6.4	6.1	5.8	5.5	5.6	5.6	5.8	6.1	6.5	6.8	7.4	8.1	8.8	9.9	10.7	11.8	12.8
620	8.7	7.9	7.3	6.7	6.2	5.6	5.2	5.4	5.3	5.3	5.5	5.9	6.3	6.6	7.2	8.0	8.7	9.5	10.6
640	10.4	9.5	8.7	7.8	7.0	6.5	5.9	5.6	5.2	4.9	5.0	5.0	5.2	5.5	5.8	6.4	7.0	7.6	8.5
660	12.8	11.5	10.5	9.6	8.6	7.7	6.9	6.3	5.7	5.4	5.0	4.8	4.5	4.7	4.9	5.1	5.5	6.0	6.8
680	15.7	14.2	13.0	11.9	10.7	9.6	8.6	7.6	6.9	6.2	5.6	5.1	4.8	4.6	4.2	4.2	4.5	4.6	5.1
700	18.5	17.3	16.0	14.3	13.4	12.1	11.0	10.0	8.7	7.8	6.8	6.3	5.6	5.0	4.6	4.2	4.2	4.0	4.2
720	21.1	20.2	18.8	17.7	16.4	15.3	13.9	12.5	11.2	10.3	9.1	7.9	7.1	6.2	5.6	4.8	4.5	4.2	3.8
740	24.0	22.8	21.5	20.6	19.2	18.1	16.8	15.5	14.4	13.0	11.7	10.5	9.4	8.4	7.2	6.5	5.6	5.0	4.3
760	25.9	25.2	24.3	23.0	21.7	20.7	19.7	18.5	17.2	15.9	14.7	13.5	12.2	10.8	9 8	8.9	7.6	6.7	5.9
780	27.2	26.1	26.1	25.2	24.3	23.3	22.2	21.2	20.1	19.0	17.6	16.3	15.1	14.0	12.6	11.6	10.2	9.2	8.1
800	28.2	27.3	27.3	26.7	25.9	25.1	24.4	23.4	22.2	21.3	20.3	19.2	18.0	16.7	15.4	14.3	13.2	11.9	10.8
820	28.1	28.0	27.9	27.7	27.2	26.5	25.9	25.1	24.4	23.3	22.3	21.6	20.4	19.4	18.2	17.2	15.9	14.6	13.6
840	28.1	28.1	27.9	27.9	27.6	27.3	27.2	26.6	25.8	25.0	24.3	23.5	22.4	21.6	20.5	19.4	18.4	17.3	16.4
860	27.4	27.7	27.9	28.0	27.9	27.7	27.5	27.1	26.8	26.4	25.5	24.8	24.3	23.3	22.2	21.5	20.5	19.6	18.4
880	26.5	27.0	27.3	27.5	27.8	28.0	27.7	27.5	27.2	27.0	26.5	26.0	25.5	24.7	24.1	23.2	22.0	21.4	20.4
900	25.0	25.7	26.3	26.9	27.3	27.5	27.6	27.8	27.9	27.6	27.1	26.7	26.5	25.7	25.3	24.6	23.9	23.0	22.0
920	23.0	23.9	24.9	25.7	26.2	26.9	27.3	27.5	27.5	27.6	27.7	27.5	27.2	26.7	26.3	25.7	25.1	24.3	23.6
940	20.8	21.9	23.0	23.9	24.7	25.7	26.1	26.7	27.2	27.4	27.7	27.7	27.6	27.5	27.1	26.6	26.2	25.6	25.5
960	17.9	19.5	20.5	21.7	22.7	23.9	24.7	25.4	26.2	26.6	27.2	27.5	27.7	27.7	27.6	27.4	27.1	27.0	26.2
980	15.6	16.8	17.6	19.3	20.2	21.4	22.6	23.7	24.6	25.3	25.9	26.8	27.2	27.5	27.7	27.8	27.6	27.5	27.1
1000	13.9	14.7	15.6	16.5	17.6	18.8	20.1	21.1	22.2	23.4	24.3	25.2	25.8	26.6	27.2	27.6	27.7	27.6	27.6
	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

C.

B	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540
0	27.6	27.7	27.3	26.7	26.2	25.5	24.7	23.8	23.1	22.3	21.3	20.2	19.3	18.3	17.4	16.6	15.7	15.0	14.2
20	27.7	27.8	27.8	27.6	27.4	26.8	26.2	25.6	24.8	24.0	23.1	22.0	20.9	20.2	19.1	18.2	17.1	16.2	15.5
40	26.9	27.3	27.6	27.9	27.9	27.7	27.5	27.1	26.3	25.6	24.9	24.0	23.2	22.0	20.8	20.1	18.9	17.9	17.1
60	25.3	26.0	26.8	27.1	27.5	27.9	27.8	27.7	27.3	27.1	26.7	25.9	25.0	24.0	23.2	22.0	20.7	19.9	18.9
80	23.1	24.0	25.1	25.9	26.5	27.3	27.5	27.9	28.2	28.0	27.6	27.5	27.2	26.4	25.6	24.1	23.2	22.1	20.8
100	20.8	21.8	22.6	23.6	24.6	25.5	26.2	26.7	27.2	27.5	27.6	27.8	27.4	27.2	26.8	26.3	25.4	24.5	23.5
120	18.5	19.6	20.6	21.5	22.4	23.2	24.1	25.1	25.8	26.4	26.9	27.3	27.5	27.5	27.6	27.1	26.8	26.3	25.4
140	17.1	17.9	18.6	19.3	20.3	21.3	22.0	22.9	23.7	24.7	25.5	26.0	26.7	27.0	27.2	27.4	27.3	27.4	26.9
160	16.5	17.1	17.4	18.1	18.8	19.3	20.1	21.0	21.9	22.6	23.5	24.2	25.1	25.6	26.1	26.7	26.9	27.3	27.1
180	16.9	17.0	17.1	17.4	18.0	18.4	18.9	19.4	20.1	20.7	21.2	22.2	23.0	23.8	24.5	25.0	25.7	26.3	26.7
200	17.7	17.5	17.7	17.7	17.6	18.1	18.3	18.7	19.2	19.7	20.1	20.8	21.5	22.2	22.8	23.5	24.1	24.7	25.5
220	18.3	18.2	18.3	18.3	18.3	18.3	18.6	18.7	18.9	19.3	19.5	20.0	20.4	21.0	21.5	22.0	22.6	23.2	23.8
240	18.6	18.8	18.9	18.9	18.9	19.0	19.2	19.1	19.2	19.5	19.6	19.7	19.9	20.4	20.8	21.2	21.6	21.8	22.2
260	18.2	18.5	18.7	18.8	19.0	19.3	19.5	19.6	19.9	19.9	20.0	20.1	20.2	20.3	20.6	21.2	21.4	21.7	21.9
280	16.8	17.4	17.9	18.3	18.7	19.1	19.3	19.8	20.0	20.2	20.4	20.6	20.8	20.8	21.0	21.1	21.3	21.4	21.5
300	15.5	15.8	16.2	16.6	17.6	18.1	18.5	19.2	19.4	19.9	20.6	20.8	20.9	21.0	21.5	21.7	21.7	22.0	22.0
320	13.8	14.2	14.6	15.1	15.6	16.2	16.8	17.7	18.3	18.9	19.5	20.1	20.8	21.2	21.5	21.6	22.0	22.3	22.5
340	12.6	12.9	13.0	13.3	13.7	14.4	14.9	15.5	16.2	17.1	18.0	18.6	19.4	20.2	20.8	21.5	21.9	22.1	22.6
360	12.3	12.1	11.9	12.0	12.3	12.5	13.0	13.4	14.2	14.9	15.7	16.5	17.3	18.4	19.5	20.0	20.6	21.5	22.2
380	13.1	12.5	11.9	11.6	11.5	11.4	11.6	11.7	12.3	12.7	13.3	14.0	15.0	15.9	16.9	17.8	18.6	19.6	20.6
400	14.8	13.9	13.1	12.5	11.7	11.2	11.1	10.9	11.0	11.1	11.4	12.0	12.6	13.2	14.2	15.4	16.2	17.3	18.1
420	16.5	15.7	15.1	14.3	13.4	12.5	11.7	11.1	10.8	10.8	10.5	10.6	10.7	11.2	12.0	12.5	13.5	14.5	15.6
440	18.6	17.9	17.1	16.1	15.6	14.4	13.5	12.8	11.9	11.1	10.6	10.3	10.3	10.2	10.3	10.5	11.3	12.0	12.9
460	20.3	19.8	19.3	18.5	17.6	16.8	15.9	14.7	13.7	12.9	12.0	11.1	10.9	10.1	9.9	9.9	9.9	10.1	10.7
480	21.2	21.1	20.8	20.3	19.7	19.1	18.3	17.4	16.4	15.0	14.1	13.2	12.2	11.4	10.7	10.1	9.7	9.5	9.7
500	21.4	21.4	21.4	21.3	21.1	20.8	20.0	19.5	18.8	17.8	17.0	15.7	14.4	13.6	12.5	11.6	10.9	10.2	9.8
520	20.6	21.2	21.7	21.7	21.5	21.5	21.4	21.1	20.5	19.8	19.1	18.2	17.6	16.2	15.1	13.9	12.9	11.9	10.9
540	19.2	20.0	20.7	21.1	21.8	22.0	21.8	21.7	21.5	21.2	20.9	20.3	19.6	18.6	18.0	16.7	15.4	14.5	13.2
560	17.2	18.4	19.0	20.0	20.8	21.1	21.7	21.9	22.2	22.1	21.9	21.7	21.1	20.4	19.8	19.0	18.2	17.2	16.0
580	15.0	16.0	17.3	18.2	19.1	19.9	20.8	21.1	21.7	22.0	22.2	22.3	22.1	21.8	21.5	20.9	20.3	19.3	18.6
600	12.8	13.9	15.1	15.9	17.2	18.0	19.0	19.9	20.6	21.3	21.8	22.0	22.4	22.4	22.2	22.2	21.5	21.2	20.6
620	10.6	11.5	12.7	13.7	14.9	16.0	17.1	18.3	19.1	19.9	20.8	21.3	22.0	22.3	22.4	22.4	22.3	22.3	21.9
640	8.5	9.5	10.4	11.3	12.3	13.7	14.9	16.0	17.1	18.1	19.0	19.9	20.7	21.7	22.0	22.3	22.6	22.5	22.6
660	6.8	7.4	8.2	9.1	10.1	11.1	12.2	13.6	14.6	15.8	17.1	18.1	19.0	20.0	20.8	21.3	22.1	22.3	22.6
680	5.1	5.7	6.4	7.1	7.9	8.7	9.7	11.0	12.1	13.1	14.1	15.7	16.8	18.0	19.0	19.9	20.8	21.5	22.1
700	4.2	4.4	4.7	5.1	5.8	6.7	7.4	8.4	9.4	10.6	11.5	13.0	14.1	15.2	16.8	17.9	18.8	20.0	21.1
720	3.8	3.8	3.8	4.0	4.4	4.8	5.4	5.9	6.9	8.0	9.1	10.1	11.5	12.7	13.9	15.0	16.4	17.9	18.6
740	4.3	3.9	3.8	3.7	3.6	3.8	3.9	4.4	4.9	5.7	6.4	7.4	8.9	9.8	10.9	12.2	13.6	14.8	16.2
760	5.9	5.1	4.4	4.0	3.6	3.4	3.4	3.5	3.9	4.3	4.7	5.2	5.9	6.8	8.0	9.3	10.3	11.8	13.2
780	8.1	7.1	6.1	5.3	4.6	4.1	3.7	3.3	3.3	3.1	3.4	3.6	4.1	4.9	5.6	6.4	7.5	8.6	9.9
800	10.8	9.7	8.5	7.5	6.5	5.6	4.9	4.2	3.8	3.4	3.2	3.1	3.1	3.3	4.4	4.8	5.5	6.1	6.9
820	13.6	12.5	11.2	10.1	9.0	8.0	6.9	6.1	5.3	4.7	3.9	3.7	3.1	3.1	3.2	3.1	3.6	3.9	4.8
840	16.4	15.1	13.7	12.9	11.7	10.6	9.5	8.6	7.5	6.6	5.7	4.9	4.4	3.7	3.5	3.2	3.2	3.1	3.4
860	18.4	17.5	16.6	15.4	14.3	13.1	12.1	11.1	10.0	9.1	7.9	7.0	6.3	5.5	4.6	4.1	3.6	3.4	3.3
880	20.4	19.6	18.7	17.5	16.6	15.6	14.5	13.6	12.5	11.5	10.4	9.5	8.6	7.6	6.7	5.9	5.2	4.5	4.1
900	22.0	21.1	20.2	19.4	18.7	17.7	16.5	15.7	14.7	13.8	12.5	11.9	10.9	10.0	9.1	8.3	7.2	6.5	5.8
920	23.6	22.7	21.7	21.1	20.1	19.4	18.4	17.5	16.7	15.6	14.8	13.9	13.1	12.1	11.2	10.3	9.6	8.7	7.7
940	25.5	24.1	23.4	22.4	21.4	20.6	19.9	19.0	18.2	17.3	16.6	15.7	14.8	14.1	13.1	12.4	11.5	10.8	9.8
960	26.2	25.6	24.7	24.1	23.3	22.3	21.3	20.6	19.0	18.9	17.9	17.1	16.3	15.4	14.6	14.0	13.2	12.6	11.7
980	27.1	26.7	26.3	25.5	24.9	23.8	23.4	22.2	21.0	20.4	19.4	18.6	17.7	16.8	16.2	15.2	14.5	13.9	13.1
1000	27.6	27.7	27.3	26.7	26.2	25.5	24.7	23.8	23.1	22.3	21.3	20.2	19.3	18.3	17.4	16.6	15.7	15.0	14.2
	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

C

B	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700	710	720
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	14.2	13.6	13.1	12.3	11.7	11.3	10.8	10.2	9.5	9.1	8.4	7.9	7.4	7.0	6.6	6.3	5.9	5.5	5.4
20	15.5	14.7	14.1	13.3	12.7	12.2	11.5	11.3	10.7	10.4	9.8	9.4	8.9	8.5	7.9	7.7	7.3	6.7	6.6
40	17.1	15.9	15.1	14.4	13.7	13.0	12.3	12.0	11.5	11.0	10.7	10.3	10.0	9.6	9.3	8.9	8.5	8.1	7.8
60	18.9	17.7	16.8	15.8	14.9	14.0	13.3	12.7	12.1	11.6	11.2	10.9	10.5	10.2	10.0	9.8	9.5	9.2	8.9
80	20.8	20.0	18.7	17.9	16.6	15.6	14.8	13.6	12 9	12.4	11.8	11.3	10.9	10.7	10.3	9.9	9.8	9.8	9.6
100	23.5	22.2	20.9	20.0	18.6	17.6	16.6	15.4	14.4	13.4	12.6	12.1	11.5	11.0	10.6	10.2	10.0	9.9	9.6
120	25.4	24.6	23.7	22.4	21.0	20.1	18.8	17.7	16.4	15.3	14.3	13.2	12.4	11.6	11.2	10.6	10.1	10.1	9.6
140	26.9	26.2	25.4	24.6	23.9	22.6	21.1	20.1	18.9	17.7	16.5	15.2	14.2	13.0	12.3	11.6	11.1	10.3	9.9
160	27.1	27.0	26.9	26.4	25.5	24.7	23.9	22.9	21.5	20 4	19.2	17.9	16.6	15.3	14.1	13.1	12.0	11.2	10.5
180	26.7	26.8	27.0	26.8	26.6	26.2	25.6	24.8	23.9	22.9	21.6	20.6	19.1	18.0	16.7	15.5	14.3	12.9	12.0
200	25.5	25.8	26.3	26.6	26.6	26.6	26.4	26.0	25.6	24.9	24.0	22.9	21.7	20.8	19.3	18.1	16.9	15.5	14.4
220	23.8	24.5	25.0	25.4	25.8	26.0	26.2	26.3	26.1	25.8	25.3	24.9	24.1	23.1	21.2	20.9	19.7	18.3	17.1
240	22.2	22.6	23.1	23.3	23.9	24.2	24.6	25.1	25.1	25.3	25.2	25.1	24.7	24.3	24.0	23.0	21.9	21.3	20.2
260	21.9	22.2	22.3	22.7	23.1	23.3	23.6	23.9	24.2	24.5	24.7	24.8	24.9	24.6	24.3	23.8	23.4	22.9	21.6
280	21.5	21.8	22.0	22.2	22.7	23.0	23.3	23.6	23.9	24.2	24.7	24.8	25.0	24.9	24.9	24.8	24.4	24.0	23.5
300	22.0	22.1	22.1	22.2	22.4	22.6	22.8	23.0	23.3	23.4	23.8	24.0	24.1	24.5	24.5	24.6	24.5	24.4	24.0
320	22.5	22.5	22.6	22.7	22.8	22.8	22.9	23.0	23.1	23.2	23.4	23.3	23.6	23.8	24.0	23.9	24.2	24.2	24.2
340	22.6	23.0	23.2	23.4	23.3	23.4	23.5	23.5	23.5	23.4	23.5	23.6	23.6	23.5	23.5	23.6	23.9	23.8	23.8
360	22.2	22.7	23.0	23.7	23.7	24.0	24.2	24.2	24.3	24.2	24.2	24.0	23.7	23.9	24.0	23.7	23.7	23.6	23.6
380	20.6	21.5	22.3	22.9	23.5	23.9	24.5	24.6	24.8	25.1	24.8	24.9	25.0	24.9	24.6	24.5	24.5	24.3	24.0
400	18.1	19.2	20.3	21.4	22.4	23.0	23.7	24.3	24.7	25.0	25.4	25.7	25.7	25.5	25.5	25.4	25.2	24.8	24.6
420	15.6	16.7	17.7	18.7	20.1	21.0	22.0	23.0	23.7	24.6	25.0	25.7	26.1	26.2	26.3	26.5	26.2	26.0	25.9
440	12.9	13.6	14.7	16.0	17.0	18.3	19.5	20.8	21.7	22.7	23.7	24.6	25.4	26.0	26.5	26.7	26.9	27.0	26.9
460	10.7	11.3	12.2	13.0	14.0	15.1	16.5	17.8	19.0	20.1	21.4	22.3	23.5	24.8	25.4	26.1	26.7	27.1	27.3
480	9.7	9.9	10.2	10.7	11.7	12.5	13.4	14.5	15.6	17.0	18.5	19.7	20.9	22.1	23.2	24.4	25.4	26.2	26.8
500	9.8	9.4	9.3	9.6	9.8	10.2	11.1	12.0	13.0	13.8	14.9	16.3	17.9	19.1	20.5	21.6	22.9	24.2	25.1
520	10.9	10.3	9.8	9.5	9.2	9.2	9.6	9.8	10.5	11.5	12.4	13.4	14.4	15.5	17.1	18.4	19.9	21.2	22.3
540	12.2	12.3	11.3	10.5	10.1	9.5	9.3	9.0	9.2	9.6	10.3	11.0	11.9	12.8	13.9	15.1	16.5	17.9	19.4
560	16.0	14.8	13.7	12.7	11.7	10.9	10.2	9.7	9.3	9.1	9.1	9.4	10.0	10.6	11.5	12.4	13.3	14.5	16.0
580	18.6	17.3	16.5	15.4	14.0	12.9	12.2	11.3	10.4	9.9	9.4	9.0	9.2	9.3	9.7	10.4	11.0	12.0	12.7
600	20.6	19.5	19.1	17.7	16.8	15.8	14.4	13.3	12.5	11.6	10.8	10.1	9.6	9.4	9.1	9.3	9.9	10.0	10.8
620	21.9	21.5	20.9	20.0	19.3	18.0	16.9	16.1	14.9	13.7	12.7	12.0	11.1	10.4	9.8	9.5	9.5	9.3	9.7
640	22.6	22.4	22.0	21.6	21.1	20.3	19.6	18.4	17.4	16 3	15.2	14.2	13.1	12.1	11.3	10.6	10.1	9.6	9.5
660	22.6	22.8	22.7	22.6	22.2	21.8	21.3	20.6	19.9	18.7	17.8	16.7	15.6	14.4	13.4	12.4	11.7	11.0	10.2
680	22.1	22.6	22.7	23.0	23.0	22.8	22.4	22.0	21.5	20.8	20.2	19.0	18.1	17.0	15.8	14.7	13.7	12.8	12.0
700	22.1	21.5	22.2	22.6	22.9	23.0	23.2	23.2	22.6	22.2	21.7	21.0	20.5	19.3	18.3	17.3	16.0	15.0	14.1
720	18.6	19.7	20.8	21.6	22.3	22.7	23.0	23.3	23.2	23.4	23.1	22.4	21.9	21.3	20.8	19.5	18.5	17.6	16.4
740	16.2	17.5	18.7	19.5	20.6	21.6	22.3	22.8	23.2	23.4	23.6	23.6	23.3	22.8	22.2	21.6	21.1	19.9	18.8
760	13.2	14.5	15.9	17.4	18.2	19.5	20.5	21.4	22.5	22.8	23.3	23.7	23.6	23.8	23.5	23.3	22.7	21.8	21.3
780	9.9	11.1	12.6	14.0	15.6	16.8	18.1	19.2	20.4	21.3	22.3	23.0	23.3	23.7	23.8	24.0	23.8	23.5	23.0
800	6.9	7.9	9.4	10.7	12.1	13.4	14.9	16.4	17.7	19.1	20.1	21.2	21.1	22.9	23.4	23.8	24.1	24.2	23.9
820	4.8	5.7	6.5	7.5	8.7	10.0	11.5	12.9	14.3	15.8	17.8	18.7	20.0	20.9	22.0	22.7	23.5	23.9	24.0
840	3.4	3.7	4.1	5.0	6.2	7.0	8.2	9.5	10.8	12.2	13.8	15.2	16.6	18.1	19.5	20.6	21.7	22.6	23.3
860	3.3	3.2	3.4	3.4	4.0	4.5	5.6	6.8	7.7	8.8	10.2	11.5	13.2	14.7	16.0	17.4	19.0	20.2	21.3
880	4.1	3.8	3.5	3.4	3.4	3.6	3.9	4.4	5.2	6.1	7.2	8.2	9.7	10.9	12.5	14.1	15.4	16.8	18.2
900	5.8	5.1	4.4	4.2	3.8	3.6	3.6	3.6	3.9	4.2	5.0	5.7	6.6	7.8	9.1	10.3	11.8	13.4	14.8
920	7.7	6.9	6.3	5.8	5.1	4.6	4.2	3.8	3.9	3.9	4.0	4.3	4.7	5.4	6.4	7.3	8.6	9.8	11.2
940	9.8	9.1	8.3	7.6	6.8	6.3	5.9	5.1	4.6	4.4	4.2	4.3	4.3	4.3	4.9	5.3	6.3	7.0	8.0
960	11.7	11.0	10.1	9.6	8.8	8.1	7.5	6.9	6.3	5.8	5.3	4.7	4.7	4.6	4.6	4.6	4.9	5.4	6.0
980	13.1	12.5	11.8	11.2	10.5	9.7	9.3	8.7	7.9	7.4	6.8	6.4	6.0	5.6	5.2	5.0	4.9	5.1	5.1
1000	14.2	13.6	13.1	12.3	11.7	11.3	10.8	10.2	9.5	9.1	8.4	7.9	7.4	7.0	6.6	6.3	5.9	5.5	5.4
	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700	710	720

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

C

B	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860	870	880	890	900
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	5.4	5.5	5.8	6.0	6.3	6.8	7.6	8.4	9.3	10.4	11.7	12.9	14.3	15.5	16.9	18.2	19.2	20.2	21.4
20	6.6	6.3	6.0	6.1	6.1	6.2	6.5	6.9	7.7	8.3	9.4	10.2	11.2	12.4	13.6	14.9	16.2	17.3	18.6
40	7.8	7.4	7.1	7.0	6.7	6.6	6.8	6.8	6.9	7.2	7.7	8.5	9.3	10.2	10.9	11.8	13.3	14.2	15.5
60	8.9	8.8	8.3	8.1	7.8	7.6	7.4	7.4	7.3	7.4	7.4	7.7	8.3	8.7	9.5	10.1	10.8	11.6	12.7
80	9.6	9.5	9.1	9.1	9.0	8.8	8.4	8.2	8.1	8.1	8.0	8.1	8.2	8.3	8.6	8.9	9.6	10.3	10.7
100	9.6	9.5	9.6	9.5	9.5	9.3	9.3	9.2	9.2	9.0	8.7	8.7	8.7	8.7	8.9	9.0	9.1	9.4	9.9
120	9.6	9.6	9.5	9.3	9.4	9.6	9.6	9.5	9.5	9.6	9.6	9.6	9.6	9.5	9.3	9.6	9.6	9.7	9.9
140	9.9	9.5	9.6	9.4	9.3	9.3	9.0	9.3	9.5	9.8	9.7	9.8	10.0	10.2	10.1	10.2	10.1	10.3	10.4
160	10.5	9.9	9.5	9.1	8.9	9.0	8.9	9.0	9.0	9.0	9.5	9.6	9.9	10.0	10.2	10.4	10.6	11.0	11.0
180	12.0	11.0	10.1	9.7	9.1	8.8	8.7	8.3	8.5	8.7	8.8	9.0	9.1	9.6	9.9	10.1	10.4	10.7	11.0
200	14.4	13.3	12.0	11.0	10.1	9.4	8.9	8.5	8.2	8.0	8.0	8.3	8.5	8.8	9.1	9.5	9.7	10.0	10.5
220	17.1	15.7	14.6	13.2	12.0	10.9	10.2	9.2	8.7	8.3	7.9	7.7	7.7	7.7	8.1	8.4	8.8	9.2	9.7
240	20.2	19.1	17.8	16.5	14.5	13.4	12.2	11.1	10.0	9.4	8.4	8.0	7.7	7.3	7.4	7.4	7.7	8.0	8.4
260	21.6	21.1	20.1	19.2	17.3	15.9	14.6	13.4	12.4	11.3	10.1	9.1	8.6	7.9	7 4	7.2	7.1	7.1	7.3
280	23.5	22.7	21.6	21.0	19.8	18.8	17.3	16.1	15.0	13.5	12.5	11.5	10.2	9.2	8.3	7.9	7.4	7.1	7.0
300	24.0	23.4	23.2	22.4	21.4	20.5	19.8	18.7	17.5	16.1	15.0	13.7	12.4	11.4	10.4	9.3	8.5	7.8	7.4
320	24.2	23.9	23.5	23.1	22.7	22.2	21.2	20.6	19.6	18.6	17.5	16.3	15.1	13.9	12.5	11.4	10.5	9.7	8.6
340	23.8	23.9	23.7	23.5	23.2	22.8	22.3	21.4	20.9	20.5	19.2	18.6	17.4	16.4	15.2	13.9	12.7	11.6	10.6
360	23.6	23.6	23.6	23.3	23.3	23.1	22.9	22.4	22.0	21.4	20.4	19.9	18.9	18.1	17.4	16.3	15.1	13.8	12.8
380	24.0	24.0	23.7	23.5	23.3	23.1	23.1	22.7	22.4	22.2	21.6	20.8	20.0	19.6	18.8	17.7	16.9	16.0	15.1
400	24.6	24.4	24.4	24.0	23.8	23.4	23.2	23.0	22.8	22.4	22.1	21.6	21.3	20.6	19.6	19.4	18.4	17.6	16.5
420	25.9	25.6	25.2	24.8	24.7	24.3	23.9	23.6	23.3	22.9	22.7	22.3	21.7	21.1	20.8	20.3	19.3	18.9	18.2
440	26.9	26.6	26.4	26.2	25.9	25.5	25.2	24.9	24.5	23.8	23.4	23.0	22.8	22.1	21.6	20.8	20.6	19.7	19.0
460	27.3	27.6	27.6	27.4	27.0	26.9	26.5	26.1	25.6	25.0	24.6	24.2	23.7	23.3	22.7	22.0	21.6	20.9	20.2
480	26.8	27.4	27.6	28.0	28.1	28.2	27.7	27.4	27.3	26.6	26.2	25.7	25.1	24.4	23.9	23.3	22.8	22.0	21.4
500	25.1	26.1	26.8	27.5	28.1	28.2	28.6	28.5	28.4	28.3	27.6	27.2	26.7	26.3	25.7	24.9	24.3	23.6	23.0
520	22.3	23.9	24.8	25.9	26.8	27.5	28.1	28.5	28.7	29.0	28.8	28.6	28.4	27.8	27.3	26.8	26.3	25.6	24.7
540	19.4	20.7	22.1	23.4	24.6	25.6	26.5	27.4	28.0	28.7	28.9	29.1	29.2	29.2	28.9	28.5	27.8	27.4	26.8
560	16.0	17.3	18.6	19.9	21.4	22.9	24.1	25.5	26.4	27.3	28.2	28.6	29.2	29.3	29.5	29.6	29.3	29.1	28.8
580	12.7	14.1	15.5	16.8	18.0	19.3	20.9	22.2	23.5	24.9	26.1	27.0	27.8	28.6	29.0	29.4	29.6	29.8	29.8
600	10.8	11.6	12.7	13.6	14.9	16.2	17.5	18.7	20.2	21.8	23.0	24.4	25.5	26.7	27.6	28.4	28.9	29.2	29.6
620	9.7	10.0	10.5	10.7	12.2	13.2	14.4	15.6	17.0	18.3	19.6	21.2	22.6	23.8	25.0	26.2	27.1	27.9	28.8
640	9.5	9.4	9.6	10.1	10.4	11.1	12.0	13.0	14.0	15.2	16.5	17.9	19.2	20.6	21.6	23.3	24.6	25.2	26.6
660	10.2	10.0	9.7	9.5	9.5	9.9	10.4	11.0	11.7	12.7	13.8	14.9	16.2	17.5	18.8	20.2	21.1	22.9	24.0
680	12.0	11.2	10.5	10.0	9.7	9.5	9.6	10.0	10.4	11.0	11.6	12.5	13.8	14.7	15.8	16.9	18.4	19.9	20 6
700	14.1	13.1	12.3	11.3	10.7	10.1	9.7	9.7	9.9	9.9	10.4	10.9	11.5	12.3	13.4	14.6	15.6	16.7	18.0
720	16.4	15.3	14.4	13.3	12.2	11.6	10.9	10.2	10.1	9.9	10.0	10.1	10.4	11.0	11.4	12.3	13.3	14.3	15.6
740	18.8	17.7	16.7	15.6	14.4	13.5	12.4	11.5	11.1	10.7	10.1	10.0	10.3	10.4	10.5	11.0	11.4	12.2	13.3
760	21.3	20.1	19.2	18.1	16.6	15.6	14.7	13.6	12.8	11.9	11.3	10.7	10.3	10.0	10.2	10.3	10.7	11.0	11.5
780	23.0	22.3	21.5	20.5	19.4	18.4	17.2	15.8	14.9	14.0	13.0	12.2	11.3	10.8	10.6	10.2	10.2	10.5	10.7
800	23.9	23.9	23.4	22.6	21.9	20.7	19.8	18.8	17.5	16.2	15.1	14.2	13.4	12.5	11.7	11.0	10.6	10.3	10.3
820	24.0	24.5	24.2	23.9	23.3	22.6	22.3	21.3	20.3	19.4	18.3	17.3	16.2	15.2	14.4	13.5	12.5	11.9	11.4
840	23.3	24.0	24.3	24.5	24.4	24.3	23.8	23.4	22.7	21.7	20.8	19.6	18.3	17.1	16.2	14.9	14.1	13.0	12.4
860	21.3	22.3	23.3	23.9	24.2	24.7	24.5	24.5	24.3	23.6	23.1	21.9	21.0	20.2	18.7	17.7	16.6	15.4	14.3
880	18.2	19.7	20.9	22.0	22.8	23.8	24.1	24.6	24.8	24.7	24.5	24.0	23.5	22.4	21.3	20.4	19.3	18.0	17.0
900	14.8	16.1	17.6	19.0	20.6	21.5	22.5	23.2	24.1	24.5	24.2	24.8	24.5	24.2	23.8	22.7	21.9	20.9	19.7
920	11.2	12.6	14.0	15.5	17.0	18.4	19.9	21.0	22.0	22.9	23.5	24.5	24.5	24.8	24.7	24.3	24.1	23.2	22.3
940	8.0	9.3	10.7	12.0	13.3	14.8	16.4	17.6	19.1	20.4	21.4	22.4	23.2	24.0	24.5	24.6	24.5	24.5	24.2
960	6.0	6.9	7.8	8.6	10.2	11.5	12.7	14.1	15.6	16.9	18.5	19.5	20.7	21.9	22.8	23.6	24.0	24.5	24.5
980	5.1	5.5	6.0	6.7	7.7	8.5	9.7	10.9	12.2	13.6	14.8	16.1	17.6	18.7	20.1	21.2	22.2	23.1	23.6
1000	5.4	5.5	5.8	5.8	6.3	6.8	7.6	8.4	9.3	10.5	11.7	12.9	14.3	15.5	16.9	18.2	19.2	20.2	21.4
	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860	870	880	890	900

TABLE XI.

Perturbations produced by *Venus*. ARGUMENTS. B and C.

B \ C	900	910	920	930	940	950	960	970	980	990	1000
0	21.4	22.5	23.0	23.5	24.0	24.2	24.2	23.7	23.1	22.5	21.6
20	18.6	19.6	20.5	21.5	22.4	23.1	23.6	23.7	24.0	23.4	23.1
40	15.5	16.6	17.8	18.8	19.7	20.7	21.6	22.4	22.9	23.5	23.5
60	12.7	13.8	14.9	15.9	17.0	18.1	19.1	20.1	20.7	21.5	22.2
80	10.7	11.6	12.5	13.3	14.5	15.2	16.2	17.3	18.4	19.0	20.0
100	9.9	10.4	11.0	11.7	12.4	12.9	14.0	14.8	15.6	16.5	17.6
120	9.9	9.8	10.4	10.9	11.3	11.8	12.3	12.9	13.7	14.3	15.3
140	10.4	10.5	10.5	10.6	10.9	11.4	11.5	12.0	12.6	12.8	13.6
160	11.0	10.9	11.0	11.3	11.3	11.3	11.6	11.8	12.1	12.3	12.7
180	11.0	11.3	11.5	11.7	11.7	11.9	12.2	12.2	12.3	12.5	12.7
200	10.5	11.0	11.2	11.6	12.0	12.2	12.4	12.7	12.8	13.1	13.2
220	9.7	10.1	10.6	11.0	11.4	11.8	12.3	12.7	13.0	13.3	13.5
240	8.4	9.0	9.6	10.0	10.5	11.0	11.5	12.1	12.4	13.1	13.6
260	7.3	7.6	8.1	8.5	9.3	10.0	10.4	11.0	11.5	12.2	12.8
280	7.0	6.9	7.0	7.3	7.7	8.5	8.8	9.6	10.4	10.7	11.5
300	7.4	6.9	6.7	6.8	6.8	7.0	7.5	7.9	8.6	9.0	10.1
320	8.6	7.8	7.4	7.0	6.6	6.5	6.7	6.8	7.3	7.8	8.3
340	10.6	9.7	8.7	8.0	7.3	6.8	6.6	6.4	6.6	6.7	6.9
360	12.8	11.7	10.6	9.8	8.8	8.0	7.4	6.9	6.5	6.5	6.5
380	15.1	13.9	12.7	11.8	10.8	9.8	8.9	8.2	7.5	6.9	6.8
400	16.5	15.7	14.8	13.7	12.8	11.8	10.9	10.0	9.0	8.3	7.5
420	18.2	17.2	16.3	15.3	14.5	13.7	12.6	11.6	10.7	9.9	9.1
440	19.0	18.6	17.7	16.6	15.9	15.1	14.2	13.3	12.5	11.6	10.6
460	20.2	19.5	18.5	18.1	17.3	16.7	15.7	14.8	13.9	13.0	12.1
480	21.4	20.9	20.2	19.3	18.3	17.7	16.9	16.3	15.5	14.5	13.6
500	23.0	22.3	21.4	20.7	20.3	19.1	18.1	17.6	16.6	15.8	15.1
520	24.7	23.9	23.3	22.6	21.8	20.8	20.1	19.2	18.1	17.4	16.5
540	26.8	26.1	25.3	24.4	23.7	23.0	22.0	21.0	20.2	19.2	18.1
560	28.8	28.0	27.4	26.9	26.1	25.1	24.3	23.5	22.6	21.5	20.6
580	29.8	29.3	29.0	28.7	27.9	27.3	26.6	25.7	24.9	23.8	23.0
600	29.6	29.9	29.9	29.8	29.3	29.0	28.5	27.8	27.0	26.3	25.4
620	28.8	29.3	29.6	29.8	30.1	29.8	29.6	29.2	28.8	28.2	27.4
640	26.6	27.8	28.3	28.9	29.4	29.7	29.9	30.0	29.9	29.5	29.5
660	24.0	25.1	26.2	27.1	28.2	28.8	29.2	29.5	29.7	29.8	29.9
680	20.6	22.3	23.6	24.9	25.8	26.7	27.5	28.6	28.9	29.2	29.7
700	18.0	19.5	20.7	22.0	23.1	24.2	25.1	26.4	27.3	27.8	28.7
720	15.6	16.4	17.7	19.3	20.3	21.6	22.6	23.9	25.0	26.1	26.8
740	13.3	14.2	15.3	16.5	17.4	18.8	19.5	21.3	22.5	23.6	24.6
760	11.5	12.2	13.1	14.2	15.1	16.0	17.3	18.6	19.4	21.0	22.1
780	10.7	11.1	11.5	12.3	13.2	14.0	15.0	15.8	17.1	18.5	19.3
800	10.3	10.4	10.7	11.0	11.6	12.3	13.2	14.1	14.8	15.9	17.0
820	11.4	11.0	10.9	10.8	10.8	11.2	11.4	12.0	12.5	13.4	15.4
840	12.4	11.7	11.2	10.7	10.6	11.1	11.2	11.3	11.7	12.2	13.2
860	14.3	13.3	12.5	11.9	11.4	11.0	10.9	10.8	10.9	11.2	11.5
880	17.0	15.9	14.8	13.7	12.8	12.0	11.6	11.3	11.1	10.8	11.0
900	19.7	18.6	17.2	16.4	15.3	14.1	13.3	12.3	11.9	11.3	11.2
920	22.3	21.3	20.0	19.3	18.0	16.7	15.7	14.6	13.7	12.8	12.1
940	24.2	23.5	22.7	21.8	20.6	19.5	18.4	17.3	16.2	15.1	14.0
960	24.5	24.2	24.3	23.7	22.9	22.1	21.0	20.0	18.9	17.9	16.7
980	23.6	24.0	24.3	24.3	24.3	23.7	23.0	22.4	21.4	20.3	19.5
1000	21.4	22.5	23.0	23.5	24.0	24.2	24.2	23.7	23.1	22.5	21.6
	900	910	920	930	940	950	960	970	980	990	1000

TABLE XII.

Perturs. prod. by *Mars*. ARGU. B & D.

D	0	10	20	30	40	50	60	70
	9.5	10.2	10.8	11.2	11.5	11.7	11.8	11.5
	8.3	9.1	9.8	10.5	10.9	11.2	11.5	11.6
	7.1	7.9	8.8	9.4	10.0	10.6	10.8	11.2
	5.8	6.7	7.6	8.4	9.1	9.8	10.3	10.5
	4.3	5.3	6.4	7.2	8.0	8.9	9.3	9.9
	3.3	4.2	5.0	5.9	6.8	7.6	8.4	9.1
	2.4	3.1	3.9	4.8	5.6	6.4	7.3	8.0
	2.1	2.4	2.9	3.8	4.6	5.5	6.3	7.0
	2.0	2.2	2.4	2.7	3.5	4.4	5.1	5.9
	1.9	2.0	2.3	2.6	2.9	3.4	3.9	4.9
	2.3	2.2	2.2	2.4	2.7	3.0	3.4	3.8
	3.0	2.6	2.5	2.4	2.5	2.7	3.1	3.5
	3.7	3.3	3.0	2.9	2.7	2.8	2.9	3.2
	4.8	4.1	3.7	3.5	3.1	3.1	3.0	3.1
	5.5	5.1	4.6	4.1	3.8	3.5	3.5	3.4
	6.2	5.8	5.6	5.0	4.8	4.2	3.9	3.8
	6.9	6.6	6.1	5.9	5.4	5.1	4.7	4.3
	7.2	7.1	6.9	6.5	6.2	5.8	5.5	5.1
	7.5	7.4	7.1	7.0	6.8	6.4	6.2	5.8
	7.5	7.6	7.3	7.3	7.2	7.1	6.7	6.5
	7.3	7.3	7.5	7.4	7.4	7.4	7.1	7.0
	6.9	7.0	7.3	7.4	7.4	7.4	7.3	7.5
	6.5	6.8	6.8	7.1	7.2	7.3	7.3	7.4
	6.2	6.2	6.5	6.7	6.8	7.1	7.1	7.3
	5.8	5.9	6.0	6.2	6.4	6.5	7.0	6.9
	5.3	5.4	5.7	5.8	6.0	6.0	6.3	6.6
	5.1	5.1	5.1	5.3	5.4	5.6	5.8	6.0
	4.7	4.8	4.8	4.8	5.0	5.1	5.4	5.5
	4.4	4.5	4.6	4.6	4.7	4.8	4.8	5.0
	4.2	4.3	4.4	4.3	4.5	4.4	4.4	4.5
	4.0	4.2	4.3	4.2	4.2	4.2	4.2	4.3
	4.2	4.0	4.1	4.0	4.0	4.0	4.0	3.9
	4.3	4.2	4.1	4.0	4.1	4.0	3.9	3.9
	4.6	4.4	4.3	4.1	4.1	4.1	4.0	3.8
	4.8	4.6	4.5	4.3	4.2	4.1	4.0	3.9
	5.3	5.0	4.8	4.5	4.6	4.0	4.1	4.1
	5.8	5.5	5.1	5.0	4.7	4.5	4.1	4.1
	6.5	6.1	5.7	5.4	5.2	4.9	4.6	4.3
	7.4	6.7	6.4	6.0	5.6	5.3	5.1	5.0
	8.2	7.6	6.9	6.5	6.4	5.8	5.6	5.3
	9.2	8.5	8.0	7.3	6.8	6.5	6.1	5.8
	10.1	9.6	8.8	8.2	7.6	7.1	6.7	6.5
	10.9	10.4	9.8	9.1	8.4	7.9	7.5	6.9
	11.7	11.0	10.4	10.0	9.4	8.7	8.2	7.7
	12.3	11.9	11.3	10.6	10.2	9.7	8.9	8.4
	12.4	12.2	11.8	11.6	10.8	10.3	9.7	9.3
	12.3	12.3	12.2	11.9	11.6	11.0	10.5	9.9
	12.1	12.1	12.2	12.2	11.8	11.4	11.0	10.6
	11.4	11.9	11.9	12.0	12.0	11.7	11.4	11.0
	10.6	11.1	11.6	11.8	11.9	11.9	11.7	11.4
	9.5	10.2	10.8	11.2	11.5	11.7	11.8	11.5
	0	10	20	30	40	50	60	70

TABLE XII.

Perturbations produced by *Mars*. ARGUMENTS. B and D.

D.

B	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	11.5	11.2	11.0	10.6	10.1	9.9	9.5	9.0	8.6	8.2	8.1	7.8	7.6	7.4	7.2	7.0	6.6	6.4	6.2
20	11.6	11.4	11.0	10.9	10.6	10.2	9.7	9.1	9.1	8.8	8.4	8.1	7.9	7.8	7.2	7.3	7.2	7.0	6.6
40	11.2	11.3	11.2	11.0	10.8	10.5	10.3	9.8	9.4	9.3	9.1	8.7	8.4	8.2	8.1	7.6	7.5	7.3	7.2
60	10.5	10.9	11.1	10.9	11.0	10.9	10.4	10.0	9.7	9.5	9.2	8.8	8.7	8.4	8.0	7.9	7.8	7.6	7.5
80	9.9	10.0	10.5	10.9	10.8	10.7	10.4	10.3	10.0	9.7	9.3	9.0	8.8	8.6	8.5	8.2	8 0	7.6	7.7
100	9.1	9.5	9.8	10.1	10.6	10.5	10.4	10.3	10.1	9.9	9.6	9.3	9.0	8.8	8.5	8.6	8.4	8.2	7.6
120	8.0	8.8	9.3	9.5	9.9	10.2	10.2	10.1	10.0	9.8	9.6	9.4	9.1	8.9	8.7	8.4	8.4	8.3	8.3
140	7.0	7.9	8.4	9.0	9.3	9.6	9.9	9.9	9.9	9.7	9.7	9.4	9.3	8.9	8.7	8.4	8.3	8.2	8.1
160	5.9	6.5	7.2	8.0	8.5	8.9	9.2	9.6	9.5	9.6	9.5	9.5	9.3	9.1	8.9	8.7	8.4	8.3	8.3
180	4.9	5.6	6.4	6.9	7.7	8.3	8.6	8.9	9.4	9.3	9.3	9.3	9.2	9.1	8.8	8.7	8.5	8.4	8.2
200	3.8	4.6	5.3	6.0	6.7	7.4	7.9	8.3	8.6	8.9	9.1	9.0	9.0	8.9	8.8	8.6	8.4	8.4	8.3
220	3.5	3.9	4.4	5.1	5.8	6.4	7.1	7.6	7.9	8.4	8.6	8.8	8.8	8.7	8.7	8.6	8.4	8.2	8.1
240	3.2	3.6	4.0	4.4	5.0	5.5	6.2	6.8	7.4	7.6	8.1	8.4	8.4	8.5	8.4	8.5	8.3	8.1	8.0
260	3.1	3.2	3.8	4.1	4.5	4.9	5.4	5.9	6.6	7.1	7.5	7.7	8.0	8.2	8.2	8.1	8.1	8.1	7.8
280	3.4	3.4	3.5	3.8	4.2	4.5	4.9	5.5	5.6	6.2	6.8	7.1	7.5	7.8	7.8	8.0	7.8	7.9	7.9
300	3.8	3.7	3.7	3.7	3.9	4.4	4.7	4.9	5.4	5.7	6.0	6.6	6.9	7.3	7.6	7.5	7.6	7.7	7.6
320	4.3	4.2	4.1	4.0	4.1	4.2	4.4	4.7	5.0	5.4	5.8	6.0	6.4	6.6	7.1	7.3	7.4	7.4	7.3
340	5.1	4.9	4.6	4.4	4.4	4.3	4.5	4.5	5.0	5.2	5.5	5.8	6.0	6.3	6.4	6.7	7.2	7.1	7.2
360	5.8	5.6	5.3	5.0	4.8	4.8	4.7	4.8	4.9	5.1	5.4	5.5	5.9	6.1	6.2	6.4	6.5	6.9	6.9
380	6.5	6.4	5.9	5.7	5.5	5.4	5.1	5.1	5.1	5.1	5.4	5.5	5.7	5.8	6.1	6.3	6.4	6.6	6.7
400	7.0	6.7	6.7	6.3	6.1	5.9	5.7	5.6	5.5	5.5	5.5	5.6	5.7	5.9	6.0	6.2	6.3	6.4	6.5
420	7.4	7.2	6.9	7.1	6.7	6.4	6.3	6.1	6.0	5.9	5.9	5.8	5.8	6.1	6.3	6.2	6.4	6.3	6.4
440	7.5	7.4	7.4	7.0	7.1	7.4	6.8	6.7	6.5	6.3	6.3	6.4	6.2	6.3	6.4	6.4	6.6	6.5	6.6
460	7.3	7.4	7.4	7.5	7.4	7.3	7.3	7.2	7.1	7.1	6.7	6.7	6.7	6.7	6.5	6.5	6.6	6.7	6.9
480	6.9	7.1	7.3	7.4	7.5	7.3	7.6	7.5	7.4	7.5	7.4	7.2	7.1	7.1	7.1	7.0	6.9	6.9	6.9
500	6.6	6.8	6.9	7.2	7.3	7.5	7.5	7.6	7.8	7.7	7.8	7.7	7.6	7.4	7.5	7.4	7.4	7.3	7.2
520	6.0	6.3	6.5	6.7	7.1	7.2	7.5	7.5	7.7	7.8	7.9	7.6	7.9	7.9	7.8	7.8	7.8	7.8	7.6
540	5.5	5.7	6.0	6.3	6.6	6.9	7.1	7.3	7.4	7.7	7.9	8.0	8.2	8.3	8.3	8.3	8.2	8.2	8.1
560	5.0	5.2	5.4	5.8	5.9	6.2	6.6	6.9	7.1	7.4	7.7	7.8	8.1	8.2	8.6	8.4	8.6	8.7	8.5
580	4.5	4.7	4.9	5.0	5.3	5.7	6.0	6.6	6.8	7.1	7.2	7.5	7.9	8.2	8.3	8.6	8.8	8.8	9.0
600	4.3	4.3	4.4	4.6	4.6	5.0	5.3	5.6	5.9	6.5	6.9	7.0	7.4	7.7	8.1	8.5	8.6	8.9	9.1
620	3.9	4.0	4.0	4.1	4.3	4.4	4.6	4.9	5.3	5.4	6.1	6.6	6.9	7.4	7.6	8.0	8.5	8.7	9.0
640	3.9	3.8	3.8	3.8	3.9	3.9	4.1	4.3	4.5	5.0	5.2	5.8	6.3	6.7	7.2	7.5	7.9	8.3	8.7
660	3.8	3.7	3.7	3.6	3.6	3.7	3.8	3.9	4.1	4.2	4.5	5.0	5.3	6.0	6.3	7.0	7.3	7.7	8.2
680	3.9	3.8	3.6	3.4	3.5	3.4	3.5	3.5	3.6	3.7	3.8	4.2	4.6	4.9	5.6	6.0	6.6	7.1	7.7
700	4.1	3.9	3.8	3.6	3.5	3.3	3.3	3.2	3.2	3.2	3.5	3.6	3.8	4.2	4.5	5.2	5.8	6.4	6.8
720	4.1	4.1	4.0	3.8	3.6	3.5	3.3	3.2	3.3	3.2	3.0	3.2	3.4	3.6	3.9	4.3	4.7	5.3	5.9
740	4.3	4.3	4.2	4.0	3.8	3.7	3.5	3.2	3.0	3.0	2.9	2.8	2.9	3.1	3.3	3.6	3.9	4.4	4.8
760	5.0	4.7	4.4	4.3	4.1	3.8	3.7	3.4	3.1	3.0	2.9	2.7	2.7	2.8	2.8	3.0	3.3	3.6	4.0
780	5.3	5.1	4·7	4.6	4.4	4.4	4.0	3.8	3.4	3.2	2.9	2.8	2.7	2.5	2.6	2.5	2.7	3.1	3.3
800	5.8	5.5	5.4	4.8	4.7	4.7	4.5	4.2	3.9	3.5	3.3	2.9	2.8	2.7	2.5	2.5	2.5	2.5	2.7
820	6.5	6.1	5.8	5.6	5.0	5.0	4.9	4.6	4.3	4.1	3.6	3.3	3.0	2.9	2.6	2.4	2.3	2.2	2.3
840	6.9	6.7	6.3	6.1	5.8	5.3	5.2	4.9	4.9	4.5	4.2	3.9	3.5	3.1	2.8	2.6	2.4	2.3	2.2
860	7.7	7.4	6.9	6.6	6.2	6.2	5.5	5.4	5.2	5.0	4.8	4.4	4.1	3.6	3.3	3.0	2.7	2.4	2.3
880	8.4	7.9	7.6	7.1	6.9	6.4	6.4	5.8	5.7	5.4	5.2	5.0	4.6	4.3	3.8	3.6	3.2	2.8	2.5
900	9.3	8.7	8.3	7.7	7.4	7.1	6.7	6.6	6.1	6.0	5.6	5.4	5.2	4.9	4.6	4.2	3.6	3.4	2.9
920	9.9	9.3	8.8	8.4	7.9	7.7	7.3	6.9	6.6	6.3	6.2	6.1	5.6	5.4	5.1	4.6	4.5	3.9	3.5
940	10.6	10.1	9.5	8.9	8.7	8.2	7.8	7.6	7.2	7.1	6.5	6.5	6.3	5.9	5.7	5.3	4.9	4.7	4.3
960	11.0	10.7	10.3	9.7	9.1	8.7	8.4	8.0	7.8	7.4	7.2	6.9	6.7	6.5	6.2	5.9	5.5	5.1	4.9
980	11.4	11.0	10.6	10.2	9.8	9.2	8.9	8.4	8.1	8.0	7.6	7.3	7.2	6.9	6.8	6.4	6.1	5.8	5.4
1000	11.5	11.2	11.0	10.6	10.0	9.9	9.5	9.0	8.6	8.2	8.1	7.4	7.6	7.4	7.2	7.0	6.6	6.4	6.2
	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250

TABLE XII.

Perturbations produced by *Mars*. ARGUMENTS. B and D.

D

B	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390	400	410	420	430
0	″ 6.2	″ 5.7	″ 5.3	″ 4.9	″ 4.7	″ 4.1	″ 3.8	″ 3.4	″ 2.8	″ 2.6	″ 2.4	″ 2.2	″ 2.3	″ 2.3	″ 2.5	″ 2.7	″ 2.9	″ 3.4	″ 4.0
20	6.6	6.3	6.0	5.7	5.3	5.0	4.4	3.9	3.5	3.1	2.7	2.6	2.4	2.4	2.3	2.5	2.7	3.0	3.3
40	7.2	6.8	6.6	6.2	5.9	5.6	5.2	4.7	4.2	3.9	3.5	3.0	2.8	2.7	2.6	2.5	2.6	2.8	2.9
60	7.5	7.3	7.1	6.8	6.4	6.1	5.8	5.4	5.0	4.6	4.2	3.8	3.4	3.1	2.8	2.8	2.7	2.7	2.7
80	7.7	7.6	7.4	7.1	7.0	6.7	6.3	6.0	5.7	5.4	4.8	4.4	4.0	3.6	3.4	3.1	2.9	2.9	2.9
100	7.6	7.7	7.8	7.6	7.3	7.2	6.9	6.6	6.3	5.9	5.6	5.2	4.8	4.3	4.0	3.7	3.5	3.2	3.0
120	8.3	8.0	7.9	7.7	7.6	7.5	7.3	7.0	6.9	6.4	6.1	5.8	5.3	5.2	4.6	4.3	4.0	3.8	3.6
140	8.1	8.3	8.0	7.9	7.8	7.7	7.5	7.4	7.2	6.9	6.6	6.5	6.1	5.6	5.4	5.0	4.6	4.3	4.0
160	8.3	8.2	8.1	8.0	7.9	7.9	7.7	7.6	7.5	7.3	7.0	6.8	6.6	6.2	5.9	5.5	5.3	4.9	4.6
180	8.2	8.0	8.0	8.1	7.9	7.8	8.0	7.8	7.7	7.5	7.4	7.3	6.9	6.7	6.5	6.2	5.8	5.6	5.3
200	8.3	8.1	8.0	7.9	7.8	7.8	7.9	7.9	7.8	7.7	7.6	7.5	7.3	7.1	6.9	6.6	6.4	6.1	5.6
220	8.1	8.0	7.9	7.8	7.7	7.7	7.6	7.7	7.7	7.7	7.8	7.7	7.5	7.3	7.2	7.0	6.7	6.5	6.2
240	8.0	7.8	7.8	7.8	7.8	7.8	7.8	7.6	7.6	7.6	7.6	7.7	7.6	7.5	7.3	7.2	7.1	6.9	6.6
260	7.8	7.8	7.7	7.6	7.6	7.6	7.5	7.4	7.3	7.5	7.5	7.5	7.6	7.6	7.5	7.5	7.3	7.1	7.0
280	7.9	7.7	7.5	7.5	7.3	7.3	7.4	7.3	7.4	7.3	7.3	7.4	7.4	7.3	7.4	7.3	7.5	7.2	7.1
300	7.6	7.6	7.6	7.4	7.3	7.1	7.0	7.1	7.1	7.1	7.0	7.2	7.3	7.3	7.3	7.2	7.2	7.3	7.2
320	7.3	7.4	7.4	7.3	7.1	7.0	7.0	6.8	6.8	6.9	6.9	6.8	7.0	7.1	7.1	7.1	7.1	7.1	7.0
340	7.2	7.2	7.1	7.1	7.0	6.9	6.8	6.8	6.7	6.6	6.6	6.6	6.8	6.9	6.9	7.0	7.0	6.9	6.9
360	6.9	7.0	7.0	6.9	6.8	6.7	6.6	6.5	6.5	6.4	6.3	6.4	6.5	6.6	6.7	6.8	6.8	6.8	6.8
380	6.7	6.6	6.6	6.7	6.8	6.7	6.6	6.5	6.3	6.3	6.2	6.2	6.2	6.3	6.3	6.4	6.5	6.6	6.7
400	6.5	6.6	6.6	6.5	6.6	6.6	6.5	6.4	6.2	6.2	6.0	6.1	6.0	6.0	6.0	6.0	6.1	6.2	6.3
420	6.4	6.5	6.6	6.5	6.5	6.5	6.5	6.4	6.2	6.1	6.0	5.9	5.8	5.9	5.9	5.9	5.9	5.9	6.0
440	6.6	6.5	6.5	6.5	6.5	6.3	6.3	6.2	6.1	6.0	5.8	5.8	5.7	5.6	5.6	5.6	5.7	5.7	5.8
460	6.9	6.7	6.6	6.6	6.6	6.5	6.3	6.2	6.0	5.9	5.8	5.7	5.5	5.5	5.4	5.5	5.4	5.5	5.3
480	6.9	7.0	7.0	6.8	6.7	6.6	6.5	6.3	6.2	6.0	5.7	5.6	5.5	5.4	5.3	5.2	5.2	5.2	5.3
500	7.2	7.3	7.2	7.1	6.9	6.8	6.8	6.6	6.4	6.2	6.0	5.7	5.4	5.3	5.2	5.1	5.1	5.1	5.0
520	7.6	7.6	7.5	7.5	7.4	7.1	7.0	6.9	6.7	6.4	6.1	6.1	5.7	5.5	5.1	5.1	5.0	4.9	5.0
540	8.1	8.0	7.9	7.9	7.8	7.6	7.5	7.2	7.1	6.7	6.5	6.2	6.1	5.8	5.5	5.2	5.0	4.9	4.8
560	8.5	8.5	8.4	8.2	8.3	8.2	8.0	7.6	7.4	7.3	7.0	6.6	6.3	6.0	5.8	5.4	5.3	5.0	4.7
580	9.0	8.9	8.9	8.7	8.7	8.6	8.4	8.4	8.0	7.8	7.5	7.0	6.8	6.3	6.2	5.9	5.5	5.3	5.0
600	9.1	9.1	9.2	9.2	9.1	9.0	8.8	8.7	8.6	8.3	8.0	7.8	7.4	7.0	6.6	6.3	6.0	5.6	5.3
620	9.0	9.2	9.5	9.5	9.5	9.4	9.3	9.2	9.1	8.9	8.6	8.4	8.1	7.6	7.2	6.8	6.5	6.1	5.7
640	8.7	9.0	9.3	9.5	9.8	9.8	9.7	9.7	9.6	9.4	9.3	9.0	8.7	8.2	7.8	7.4	7.0	6.6	6.3
660	8.2	8.7	9.0	9.4	9.7	9.8	10.1	10.0	10.0	9.9	9.8	9.6	9.3	8.9	8.5	8.2	7.7	7.2	6.8
680	7.7	8.1	8.5	9.0	9.3	9.8	10.0	10.2	10.4	10.3	10.2	10.1	9.9	9.6	9.3	9.0	8.5	8.1	7.5
700	6.8	7.4	8.0	8.5	8.9	9.2	9.8	10.1	10.3	10.5	10.6	10.4	10.3	10.1	9.8	9.6	9.3	8.9	8.3
720	5.9	6.6	7.0	7.8	8.3	8.8	9.1	9.7	10.1	10.3	10.6	10.7	10.6	10.5	10.5	10.2	10.0	9.6	9.2
740	4.8	5.6	6.2	6.9	7.5	8.0	8.7	9.2	9.6	10.0	10.3	10.6	10.7	10.8	10.9	10.6	10.5	10.2	9.9
760	4.0	4.4	5.1	5.8	6.5	7.2	7.8	8.4	9.0	9.5	9.8	10.2	10.6	10.9	11.0	11.0	11.0	10.7	10.5
780	3.3	3.7	4.1	4.8	5.4	6.1	6.9	7.6	8.2	8.9	9.4	9.9	10.3	10.6	10.9	11.1	11.2	11.0	10.8
800	2.7	3.0	3.4	3.8	4.4	5.0	5.6	6.6	7.3	7.9	8.5	9.2	9.8	10.1	10.6	10.8	11.1	11.3	11.1
820	2.3	2.6	2.8	3.1	3.4	4.1	4.7	5.4	6.0	7.0	7.6	8.2	8.9	9.6	10.0	10.5	10.8	11.0	11.3
840	2.2	2.3	2.4	2.6	2.8	3.2	3.8	4.3	5.0	5.6	6.5	7.2	7.9	9.8	9.2	9.9	10.3	10.7	10.9
860	2.3	2.1	2.2	2.3	2.5	2.7	3.0	3.4	4.0	4.6	5.3	6.1	6.9	7.5	8.4	9.1	9.6	10.1	10.7
880	2.5	2.3	2.1	2.0	2.2	2.3	2.5	2.6	3.1	3.7	4.3	5.0	5.7	6.6	7.1	8.1	8.7	9.4	9.8
900	2.9	2.6	2.3	2.2	2.2	2.1	2.2	2.4	2.7	3.0	3.4	4.0	4.6	5.4	6.1	6.9	7.6	8.4	9.1
920	3.5	3.2	2.9	2.6	2.2	2.0	2.1	2.2	2.3	2.3	2.8	3.3	3.7	4.3	5.0	5.8	6.5	7.2	8.0
940	4.3	3.8	3.4	3.0	2.7	2.4	2.1	2.0	2.1	2.3	2.3	2.7	2.9	3.4	4.1	4.7	5.5	6.1	7.0
960	4.9	4.5	4.0	3.4	3.1	2.8	2.4	2.3	2.2	2.2	2.3	2.3	2.5	2.8	3.2	3.9	4.5	5.1	5.7
980	5.4	5.1	4.8	4.3	3.9	3.5	3.0	2.7	2.4	2.2	2.3	2.3	2.4	2.5	2.8	3.0	3.6	4.1	4.7
1000	6.2	5.7	5.3	4.9	4.7	4.1	3.8	3.4	2.8	2.6	2.4	2.2	2.3	2.3	2.5	2.7	2.9	3.4	4.0
	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390	400	410	420	430

TABLE XII.

Perturbations produced by *Mars*. ARGUMENTS. B and D.

D

B	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600	610
0	″4.0	″4.5	″5.2	″5.9	″6.6	″7.3	″8.0	″8.5	″9.0	″9.5	″10.0	″10.4	″10.7	″10.9	″10.8	″10.6	″10.4	″10.3	″10.0
20	3.3	3.8	4.3	4.9	5.6	6.2	6.9	7.6	8.2	8.8	9.3	9.7	10.0	10.4	10.6	10.7	10.6	10.4	10.2
40	2.9	3.2	3.7	4.2	4.8	5.4	5.9	6.6	7.3	7.9	8.4	8.9	9.4	9.8	10.1	10.4	10.4	10.5	10.3
60	2.7	3.0	3.2	3.6	4.0	4.5	5.1	5.7	6.3	6.9	7.5	8.0	8.6	9.1	9.4	9.8	10.2	10.2	10.3
80	2.9	2.9	3.1	3.3	3.5	3.9	4.4	4.9	5.4	5.9	6.5	7.1	7.7	8.2	8.7	9.0	9.3	9.6	9.8
100	3.0	3.0	3.1	3.2	3.5	3.6	3.8	4.2	4.8	5.3	5.9	6.4	6.9	7.4	7.9	8.4	8.7	9.0	9.4
120	3.6	3.4	3.3	3.3	3.4	3.5	3.6	3.9	4.2	4.7	5.1	5.6	6.0	6.6	6.9	7.6	8.1	8.3	8.6
140	4.0	3.9	3.8	3.6	3.6	3.6	3.7	4.0	4.0	4.2	4.6	5.0	5.4	5.9	6.3	6.8	7.2	7.7	8.0
160	4.6	4.4	4.2	3.9	4.1	3.8	3.7	4.0	4.1	4.2	4.5	4.6	4.9	5.3	5.8	6.0	6.5	6.9	7.4
180	5.3	5.0	4.8	4.4	4.2	4.2	4.2	4.0	4.1	4.3	4.4	4.4	4.7	5.0	5.2	5.6	6.0	6.3	6.7
200	5.6	5.5	5.2	5.1	4.8	4.6	4.5	4.5	4.4	4.5	4.5	4.7	4.6	4.8	5.0	5.3	5.4	5.8	6.1
220	6.2	5.9	5.7	5.5	5.3	5.1	4.9	4.9	4.8	4.7	4.8	4.8	4.9	5.0	5.0	5.1	5.3	5.5	5.7
240	6.6	6.4	6.2	5.9	5.8	5.6	5.4	5.3	5.2	5.1	5.1	5.1	5.2	5.2	5.2	5.3	5.3	5.4	5.5
260	7.0	6.7	6.6	6.4	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.4	5.4	5.5	5.5	5.5	5.5	5.5
280	7.1	6.9	6.8	6.7	6.5	6.3	6.2	6.1	6.0	5.9	5.9	5.9	5.8	5.8	5.8	5.8	5.9	5.8	5.8
300	7.2	7.1	7.0	6.8	6.8	6.6	6.5	6.4	6.3	6.2	6.2	6.2	6.2	6.2	6.1	6.2	6.1	6.1	6.1
320	7.0	7.2	7.1	6.9	6.8	6.8	6.7	6.6	6.5	6.5	6.5	6.5	6.6	6.6	6.5	6.6	6.6	6.5	6.5
340	6.9	6.9	6.9	7.0	6.9	6.9	6.8	6.7	6.8	6.7	6.6	6.7	6.8	6.9	6.9	6.9	7.0	7.0	6.9
360	6.8	6.9	6.8	6.8	6.8	6.8	6.7	6.7	6.6	6.6	6.8	6.8	6.8	6.9	7.0	7.2	7.3	7.3	7.3
380	6.7	6.7	6.5	6.5	6.6	6.7	6.6	6.6	6.7	6.7	6.7	6.8	6.9	6.9	7.0	7.2	7.4	7.5	7.6
400	6.3	6.4	6.4	6.3	6.3	6.4	6.5	6.5	6.5	6.6	6.7	6.7	6.8	6.8	7.0	7.1	7.3	7.6	7.9
420	6.0	6.0	6.2	6.3	6.3	6.2	6.2	6.3	6.3	6.3	6.3	6.5	6.6	6.7	6.9	7.0	7.2	7.6	7.8
440	5.8	5.9	5.9	6.0	6.0	6.0	6.0	6.0	6.1	6.0	6.1	6.2	6.2	6.4	6.6	6.9	7.0	7.3	7.5
460	5.3	5.4	5.5	5.7	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.9	6.0	6.1	6.2	6.5	6.9	7.1	7.2
480	5.3	5.3	5.3	5.5	5.5	5.5	5.6	5.5	5.6	5.4	5.6	5.7	5.5	5.8	5.9	6.0	6.2	6.7	7.1
500	5.0	5.0	5.0	5.1	5.2	5.3	5.3	5.3	5.2	5.2	5.2	5.3	5.4	5.4	5.5	5.6	5.9	6.1	6.4
520	5.0	4.9	4.9	4.9	4.8	5.0	5.1	5.1	5.1	5.1	5.1	5.0	5.0	5.1	5.2	5.2	5.3	5.6	5.9
540	4.8	4.8	4.8	4.7	4.8	4.8	4.9	4.9	5.0	4.9	4.8	4.8	4.9	4.8	4.8	4.8	5.0	5.1	5.4
560	4.7	4.7	4.6	4.6	4.7	4.7	4.6	4.7	4.7	4.7	4.7	4.6	4.6	4.6	4.5	4.5	4.5	4.7	4.8
580	5.0	4.9	4.6	4.5	4.5	4.6	4.5	4.4	4.4	4.5	4.5	4.5	4.4	4.4	4.3	4.3	4.3	4.3	4.3
600	5.3	5.1	4.9	4.6	4.5	4.4	4.4	4.4	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.2	4.1	4.0	4.0
620	5.7	5.3	5.1	4.9	4.7	4.6	4.4	4.3	4.1	4.2	4.2	4.2	4.2	4.1	4.0	4.0	3.9	3.9	3.8
640	6.3	5.8	5.4	5.2	5.0	4.7	4.6	4.4	4.1	4.1	4.1	4.2	4.2	4.0	3.9	4.0	3.8	3.8	3.8
660	6.8	6.4	6.0	5.7	5.4	5.0	4.8	4.7	4.5	4.3	4.2	4.2	4.1	4.0	4.0	3.9	3.8	3.7	3.5
680	7.5	7.1	6.6	6.2	5.7	5.4	5.1	4.9	4.7	4.5	4.4	4.3	4.0	3.9	4.0	3.9	3.8	3.6	3.5
700	8.3	7.8	7.2	6.8	6.4	6.0	5.6	5.3	5.0	4.7	4.6	4.6	4.3	4.1	3.9	3.9	3.9	3.7	3.5
720	9.2	8.6	8.0	7.6	7.1	6.6	6.2	5.7	5.5	5.2	4.9	4.6	4.6	4.3	4.1	4.0	3.9	3.8	3.8
740	9.9	9.4	9.0	8.4	8.0	7.4	6.9	6.3	6.0	5.6	5.3	5.0	4.7	4.5	4.2	4.2	4.2	4.0	3.7
760	10.5	10.3	9.7	9.3	8.6	8.1	7.6	7.2	6.5	6.2	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.2	4.1
780	10.8	10.7	10.5	9.9	9.6	9.0	8.5	7.8	7.4	7.0	6.4	6.1	5.7	5.5	5.1	4.9	4.5	4.4	4.3
800	11.1	11.0	11.0	10.6	10.2	9.9	9.3	8.8	8.1	7.7	7.3	6.7	6.3	5.8	5.6	5.2	5.0	4.6	4.5
820	11.3	11.3	11.1	10.9	10.6	10.3	10.0	9.6	9.1	8.5	7.9	7.4	7.0	6.6	6.1	5.8	5.5	5.3	5.0
840	10.9	11.2	11.3	11.2	11.1	11.0	10.7	10.2	9.9	9.4	8.8	8.2	7.7	7.3	6.8	6.5	6.1	5.7	5.5
860	10.7	10.9	11.1	11.4	11.3	11.3	11.2	10.7	10.4	9.9	9.6	9.2	8.5	7.9	7.5	7.0	6.7	6.4	5.9
880	9.8	10.4	10.8	11.0	11.3	11.2	11.2	11.2	10.9	10.5	10.3	9.8	9.3	8.7	8.2	7.8	7.3	6.9	6.6
900	9.1	9.7	10.1	10.6	11.0	11.2	11.2	11.2	11.0	10.9	10.7	10.2	10.0	9.4	9.0	8.5	8.0	7.6	7.2
920	8.0	8.7	9.3	9.9	10.3	10.8	11.0	11.1	11.2	11.2	11.0	10.7	10.4	10.1	9.8	9.2	8.7	8.3	7.8
940	7.0	7.7	8.2	8.8	9.5	10.1	10.4	10.9	11.0	11.2	11.2	11.0	10.7	10.5	10.2	9.8	9.4	8.8	8.5
960	5.7	6.5	7.3	8.1	8.6	9.3	9.8	10.2	10.6	10.8	11.1	11.2	10.9	10.8	10.5	10.2	10.0	9.5	9.1
980	4.7	5.5	6.2	7.0	7.7	8.3	8.9	9.5	10.0	10.4	10.6	10.8	11.0	10.9	10.7	10.3	10.2	9.9	9.6
1000	4.0	4.5	5.2	5.9	6.6	7.3	8.0	8.5	9.0	9.5	10.0	10.4	10.7	10.9	10.8	10.6	10.4	10.3	10.0
	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600	610

TABLE XII.

Perturbations produced by *Mars*. ARGUMENTS. B and D.

D

B	610	620	630	640	650	660	670	680	690	700	710	720	730	740	750	760	770	780	790
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	10.0	9.7	9.2	8.9	8.5	8.1	7.9	7.7	7.4	6.9	6.8	6.7	6.4	6.1	5.8	5.5	5.2	4.8	4.4
20	10.2	9.9	9.7	9.3	9.0	8.8	8.5	8.1	7.8	7.4	7.0	7.1	6.9	6.7	6.4	6.1	5.8	5.5	5.1
40	10.3	10.2	9.9	9.6	9.4	9.1	8.9	8.5	8.3	7.8	7.5	7.2	7.1	7.0	6.9	6.6	6.4	6.1	5.8
60	10.3	10.2	10.1	9.9	9.6	9.3	9.0	8.8	8.6	8.3	8.1	7.8	7.6	7.5	7.4	7.1	6.9	6.7	6.3
80	9.8	10.0	9.9	9.8	9.7	9.5	9.3	9.1	8.9	8.7	8.4	8.1	8.0	7·8	7.6	7.4	7.3	7.1	6.9
100	9.4	9.6	9.7	9.8	9.7	9.7	9.5	9.2	8.9	8.8	8.7	8.6	8.3	8.0	7.7	7.6	7.6	7.6	7.3
120	8.6	9.0	9.2	9.4	9.5	9.5	9.4	9.3	9.2	9.0	8.7	8.6	8.4	8.2	8.1	7.9	7.8	7.7	7.6
140	8.0	8.3	8.7	8.9	9.1	9.2	9.3	9.3	9.2	9.0	9.0	8.7	8.5	8.4	8.3	8.0	7.8	7.7	7.7
160	7.4	7.7	8.0	8.4	8.5	8.8	8.9	9.0	9.0	8.9	8.8	8.7	8.6	8.5	8.4	8.2	8.0	7.9	7.8
180	6.7	7.1	7.2	7.7	8.1	8.1	8.4	8.6	8.6	8.7	8.7	8.7	8.6	8.5	8.3	8.3	8.0	8.2	7.8
200	6.1	6.5	6.7	7.1	7.3	7.7	7.8	8.0	8.2	8.3	8.3	8.5	8.4	8.4	8.4	8.2	8.1	8.1	8.1
220	5.7	6.0	6.3	6.6	6.8	7.0	7.3	7.5	7.7	7.9	8.1	8.2	8.2	8.1	8.2	8.2	8.0	8.1	8.0
240	5.5	5.7	5.9	6.1	6.4	6.6	6.8	7.1	7.2	7.4	7.5	7.6	7.7	7.8	7.8	7.9	8.0	8.0	7.8
260	5.5	5.5	5.7	5.8	6.0	6.3	6.4	6.5	6.7	6.9	7.1	7.2	7.3	7.4	7.5	7.6	7.6	7.7	7.7
280	5.8	5.8	5.9	5.9	5.9	6.0	6.1	6.2	6.3	6.5	6.7	6.7	6.9	7.1	7.2	7.3	7.3	7.3	7.3
300	6.1	6.2	6.1	6.0	5.9	5.9	6.0	6.1	6.0	6.2	6.4	6.4	6.5	6.6	6.7	6.9	6.9	6.9	7.1
320	6.5	6.6	6.5	6.5	6.3	6.1	6.0	6.0	6.0	6.0	6.0	6.2	6.1	6.2	6.3	6.5	6.5	6.6	6.6
340	6.9	6.8	6.9	6.9	6.8	6.6	6.5	6.3	6.2	6.0	6.0	6.0	6.0	6.1	6.1	6.2	6.2	6.3	6.3
360	7.3	7.4	7.3	7.3	7.1	7.1	7.0	6.7	6.6	6.4	6.1	6.0	5.9	6.0	5.9	5.9	5.9	6.0	6.1
380	7.6	7.7	7.7	7.7	7.6	7.5	7.4	7.2	7.1	6.8	6.6	6.4	6.2	6.1	5.9	5.8	5.7	5.6	5.8
400	7.9	8.0	8.0	8.1	8.1	8.1	7.9	7.8	7.7	7.4	7.1	6.8	6.6	6.4	6.1	6.0	5.8	5.6	5.5
420	7.8	8.0	8.2	8.3	8.4	8.4	8.5	8.4	8.2	8.0	7.8	7.5	7.2	6.8	6.5	6.2	6.0	5.7	5.5
440	7.5	7.9	8.2	8.4	8.6	8.8	8.8	8.9	8.8	8.7	8.4	8.2	7.8	7.5	7.1	6.6	6.2	6.0	5.7
460	7.2	7.6	8.0	8.4	8.7	9.0	9.1	9.2	9.2	9.2	9.0	8.8	8.5	8.2	7.9	7.5	6.9	6.5	6.3
480	7.1	7.2	7.6	7.9	8.5	8.9	9.2	9.3	9.5	9.6	9.6	9.4	9.2	9.1	8.6	8.3	7.8	7.2	6.9
500	6.4	6.9	7.2	7.7	7.9	8.4	9.0	9.4	9.6	9.8	10.0	9.9	9.8	9.6	9.4	9.1	8.7	8.2	7.6
520	5.9	6.3	6.7	7.0	7.6	8.0	8.4	9.0	9.5	9.8	10.1	10.2	10.3	10.3	10.0	9.8	9.5	9.1	8.5
540	5.4	5.6	6.0	6.4	6.7	7.5	8.1	8.5	9.1	9.5	10.0	10.3	10.5	10.6	10.6	10.4	10.1	9.8	9.5
560	4.8	5.0	5.3	5.8	6.2	6.6	7.1	7.8	8.5	9.0	9.5	9.9	10.4	10.8	10.8	10.9	10.8	10.6	10.2
580	4.3	4.5	4.7	5.2	5.5	5.9	6.4	6.9	7.6	8.3	9.0	9.7	10.0	10.4	10.7	11.1	11.2	11.0	11.0
600	4.0	4.1	4.2	4.5	4.8	5.1	5.7	6.2	6.8	7.4	8.0	8.9	9.6	10.1	10.4	10.9	11.3	11.4	11.3
620	3.8	3.8	3.8	3.8	4.0	4.4	4.9	5.4	5.9	6.5	7.1	7.8	8.6	9.4	10.3	10.6	11.0	11.5	11.7
640	3.8	3.7	3.5	3.5	3.6	3.8	4.0	4.5	5.0	5.5	6.2	6.8	7.6	8.4	9.2	10.0	10.7	11.1	11.6
660	3.5	3.5	3.4	3.3	3.3	3.4	3.5	3.7	4.1	4.7	5.2	5.9	6.5	7.3	8.3	9.1	9.8	10.5	11.2
680	3.5	3.4	3.3	3.2	3.1	3.1	3.1	3.1	3.4	3.8	4.3	4.8	5.5	6.2	7.0	7.8	8.7	9.6	10.2
700	3.5	3.4	3.3	3.2	3.0	3.0	3.0	2.9	2.8	3.0	3.4	3.9	4.5	5.2	6.0	6.7	7.5	8.5	9.4
720	3.8	3.5	3.4	3.1	2.9	2.9	2.7	2.7	2.6	2.5	2.7	3.1	3.5	4.0	4.8	5.6	6.4	7.3	8.2
740	3.7	3.6	3.4	3.3	3.0	2.8	2.6	2.5	2.4	2.4	2.4	2.5	2.7	3.1	3.6	4.5	5.2	6.1	6.9
760	4.1	3.8	3.6	3.3	3.1	2.9	2.8	2.5	2.3	2.2	2.1	2.1	2.3	2.4	2.8	3.2	4.1	4.7	5.7
780	4.3	4.1	3.9	3.8	3.4	3.2	3.0	2.7	2.5	2.3	2.1	2.0	1.9	2.1	2.2	2.5	2.9	3.6	4.4
800	4.5	4.4	4.3	4.1	3.8	3.5	3.1	2.8	2.7	2.4	2.2	2.0	1.8	1.8	1.8	2.0	2.3	2.5	3.2
820	5.0	4.8	4.6	4.4	4.2	4.0	3.6	3.3	3.0	2.7	2.3	2.1	1.9	1.8	1.5	1.7	1.7	2.0	2.2
840	5.5	5.2	5.0	4.7	4.6	4.3	4.1	3.8	3.5	3.0	2.6	2.3	2.1	1.9	1.6	1.5	1.5	1.6	1.7
860	5.9	5.8	5.4	5.1	5.0	4.8	4.6	4.4	4.0	3.5	3.2	2.8	2.3	1.9	1.7	1.4	1.3	1.2	1.4
880	6.6	6.3	6.0	5.7	5.4	5.2	5.0	4.7	4.4	4.1	3.7	3.3	3.0	2.5	2.1	1.7	1.4	1.3	1.2
900	7.2	6.8	6.6	6.3	5.9	5.6	5.4	5.2	5.0	4.6	4.3	4.0	3.6	3.2	2.7	2.2	1.6	1.3	1.2
920	7.8	7.4	7.0	6.7	6.4	6.0	5.8	5.7	5.3	5.1	5.0	4.6	4.2	3.8	3.4	2.9	2.3	1.9	1.3
940	8.5	8.0	7.6	7.3	6.9	6.6	6.2	6.1	5.9	5.6	5.4	5.2	4.8	4.5	3.9	3.5	3.1	2.6	2.1
960	9.1	8.6	8.2	7.8	7.5	7.1	6.8	6.6	6.4	6.2	5.9	5.6	5.4	5.1	4.7	4.3	3.7	3.2	2.8
980	9.6	9.2	9.0	8.5	8.0	7.7	7.4	7.2	6.9	6.6	6.4	6.2	5.9	5.6	5.3	5.0	4.6	4.0	3.5
1000	10.0	9.7	9.2	8.9	8.5	8.1	7.9	7.7	7.4	6.9	6.8	6.7	6.4	6.1	5.8	5.5	5.2	4.8	4.4
	610	620	630	640	650	660	670	680	690	700	710	720	730	740	750	760	770	780	790

TABLE XII.

Perturbations produced by *Mars*. ARGUMENTS. B and D.

D.

B	790	800	810	820	830	840	850	860	870	880	890	900	910	920	930	940	950	960	970
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
0	4.4	3.7	3.2	2.6	2.1	1.7	1.3	0.9	0.7	0.7	1.0	1.2	1.6	2.2	3.0	3.8	4.8	5.8	6.9
20	5.1	4.7	4.2	3.6	3.1	2.4	1.9	1.5	1.2	0.8	0.6	0.9	1.2	1.5	2.1	2.6	3.4	4.4	5.5
40	5.8	5.3	4.9	4.5	3.8	3.3	2.7	2.0	1.7	1.4	1.0	0.8	0.9	1.0	1.4	1.8	2.5	3.2	4.0
60	6.3	6.0	5.7	5.2	4.7	4.1	3.6	3.1	2.6	2.0	1.5	1.2	0.9	1.0	1.1	1.3	1.8	2.3	3.0
80	6.9	6.5	6.3	6.0	5.5	5.0	4.6	4.0	3.4	2.7	2.2	1.8	1.5	1.3	1.1	1.2	1.4	1.6	2.1
100	7.3	7.0	6.7	6.5	6.3	5.9	5.3	4.9	4.4	3.7	3.1	2.5	2.1	1.7	1.3	1.2	1.2	1.3	1.6
120	7.6	7.5	7.3	7.0	6.8	6.5	6.2	5.7	5.1	4.7	4.1	3.5	2.9	2.4	2.0	1.5	1.4	1.4	1.4
140	7.7	7.7	7.7	7.5	7.3	7.0	6.7	6.4	6.0	5.6	5.1	4.5	3.8	3.3	2.8	2.3	2.0	1.7	1.5
160	7.8	7.8	7.9	7.7	7.6	7.4	7.2	7.0	6.8	6.3	5.8	5.4	4.8	4.2	3.6	3.1	2.6	2.1	2.0
180	7.8	7.9	7.8	7.9	7.9	7.7	7.6	7.5	7.1	7.0	6.6	6.1	5.7	5.2	4.6	4.0	3.5	3.1	2.5
200	8.1	7.9	7.9	7.8	7.9	7.8	7.7	7.6	7.5	7.5	7.1	6.8	6.3	6.1	5.5	5.0	4.4	3.9	3.5
220	8.0	8.0	7.9	7.8	7.8	7.8	7.8	7.8	7.8	7.6	7.5	7.4	7.1	6.7	6.3	5.8	5.4	4.9	4.4
240	7.8	7.8	7.7	7.7	7.7	7.7	7.7	7.8	7.8	7.7	7.6	7.6	7.5	7.2	6.9	6.6	6.1	5.6	5.3
260	7.7	7.8	7.7	7.7	7.6	7.7	7.7	7.7	7.7	7.7	7.7	7.8	7.8	7.6	7.5	7.1	6.8	6.5	6.0
280	7.3	7.4	7.4	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.6	7.6	7.8	7.7	7.7	7.5	7.3	7.1	6.7
300	7.1	7.1	7.2	7.3	7.3	7.3	7.3	7.3	7.4	7.5	7.4	7.5	7.5	7.7	7.7	7.7	7.7	7.4	7.2
320	6.6	6.8	6.9	6.8	7.0	7.1	7.1	7.1	7.1	7.3	7.3	7.3	7.4	7.4	7.4	7.6	7.7	7.6	7.6
340	6.3	6.4	6.5	6.6	6.6	6.7	6.7	6.8	6.9	7.0	7.1	7.2	7.2	7.2	7.2	7.3	7.5	7.7	7.6
360	6.1	6.2	6.2	6.2	6.3	6.4	6.4	6.5	6.6	6.7	6.7	6.9	6.9	7.1	7.1	7.1	7.2	7.2	7.6
380	5.8	5.9	5.8	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.4	6.4	6.6	6.8	6.9	7.0	7.0	7.0	7.1
400	5.5	5.6	5.6	5.6	5.7	5.7	5.7	5.8	5.9	5.9	6.0	6.1	6.2	6.4	6.6	6.6	6.7	6.7	6.9
420	5.5	5.4	5.4	5.5	5.5	5.5	5.5	5.5	5.5	5.6	5.6	5.6	5.7	5.8	5.9	6.2	6.3	6.6	6.5
440	5.7	5.6	5.3	5.3	5.3	5.3	5.2	5.2	5.2	5.2	5.1	5.0	5.3	5.5	5.6	5.7	5.8	6.0	6.1
460	6.3	6.0	5.6	5.4	5.3	5.2	5.2	5.1	5.0	5.1	5.2	5.2	5.2	5.3	5.3	5.4	5.7	5.7	5.7
480	6.9	6.5	6.0	5.7	5.4	5.2	5.2	5.1	4.9	4.9	4.9	4.9	5.0	5.0	5.0	5.0	5.1	5.3	5.4
500	7.6	7.2	6.8	6.3	5.9	5.6	5.3	5.0	4.8	4.9	4.8	4.8	4.8	4.9	4.9	5.0	5.0	5.0	5.1
520	8.5	8.0	7.4	7.0	6.5	6.1	5.5	5.4	5.1	4.9	4.7	4.7	4.7	4.8	4.8	4.8	4.8	4.8	4.7
540	9.5	9.0	8.4	7.8	7.3	6.7	6.3	5.8	5.4	5.2	4.9	4.7	4.7	4.7	4.7	4.6	4.6	4.6	4.5
560	10.2	9.9	9.5	8.8	8.2	7.7	7.1	6.5	6.0	5.7	5.3	5.0	4.8	4.6	4.5	4.5	4.4	4.5	4.5
580	11.0	10.6	10.2	9.8	9.3	8.8	8.1	7.2	6.8	6.4	6.0	5.6	5.1	4.9	4.7	4.6	4.5	4.4	4.4
600	11.3	11.2	11.0	10.7	10.3	9.6	9.1	8.5	7.7	7.1	6.4	6.1	5.6	5.3	4.9	4.8	4.7	4.5	4.4
620	11.7	11.7	11.5	11.4	11.0	10.6	9.9	9.5	8.9	8.1	7.4	6.8	6.3	5.9	5.5	5.1	4.8	4.6	4.5
640	11.6	11.8	11.9	11.8	11.7	11.3	11.0	10.4	9.8	9.3	8.5	7.8	7.1	6.6	6.1	5.6	5.4	5.0	4.7
660	11.2	11.5	11.8	12.0	12.1	11.9	11.6	11.2	10.8	10.2	9.6	8.9	8.2	7.5	6.8	6.3	5.9	5.5	5.3
680	10.2	11.0	11.6	12.1	12.2	12.1	12.2	12.1	11.5	11.1	10.6	10.1	9.2	8.5	7.8	7.3	6.5	6.1	5.6
700	9.4	10.1	10.9	11.6	12.1	12.4	12.3	12.3	12.3	11.9	11.4	10.8	10.4	9.7	8.9	8.1	7.6	7.0	6.3
720	8.2	9.1	10.0	10.6	11.4	11.9	12.4	12.6	12.5	12.4	12.0	11.6	11.2	10.8	10.0	9.3	8.5	7.9	7.2
740	6.9	7.8	8.8	9.7	10.5	11.3	11.8	12.3	12.8	12.6	12.6	12.3	11.9	11.5	11.0	10.2	9.7	8.9	8.2
760	5.7	6.6	7.6	8.5	9.4	10.3	11.0	11.7	12.1	12.6	12.8	12.7	12.5	12.1	11.8	11.3	10.5	10.0	9.3
780	4.4	5.2	6.3	7.1	8.1	9.2	10.1	10.7	11.6	12.0	12.4	12.8	12.9	12.8	12.3	11.9	11.4	10.9	10 2
800	3.2	4.0	4.8	5.7	6.7	7.7	8.7	9.7	10.5	11.3	11.9	12.3	12.5	12.9	12.9	12.5	12.1	11.7	11.2
820	2.2	2.9	3.6	4.4	5.4	6.4	7.3	8.4	9.5	10.3	11.0	11.7	12.1	12.5	12.7	12.8	12.7	12.2	11.9
840	1.7	2.2	2.7	3.3	4.0	4.9	6.0	7.0	8.0	9.1	10.0	10.8	11.4	12.0	12.4	12.6	12.8	12.6	12.4
860	1.4	1.6	1.6	2.2	2.9	3.6	4.6	5.6	6.6	7.6	8.6	9.6	10.5	11.2	11.8	12.3	12.5	12.7	12.5
880	1.2	1.2	1.3	1.5	1.9	2.6	3.3	4.1	5.2	6.1	7.1	8.2	9.2	10.1	11.0	11.5	12.1	12 3	12.6
900	1.2	1.1	1.1	1.2	1.3	1.7	2.2	2.9	3.8	4.8	5.7	6.8	7.9	8.8	9.8	10.6	11.3	11.8	12.2
920	1.3	1.1	1.0	1.0	1.1	1.1	1.4	1.9	2.6	3.4	4.4	5.3	6.3	7.4	8.4	9.3	10.2	11 0	11.5
940	2.1	1.5	1.1	0.8	0.9	1.0	1.1	1.3	1.6	2.3	3.1	3.9	5.0	5.9	7.1	8.1	8.9	9.9	10.7
960	2.8	2.3	1.7	1.3	0.9	0.7	0.8	0.9	1.2	1.4	2.0	2.8	3.5	4.6	5.6	6.7	7.7	8.7	9.4
980	3.5	3.0	2.5	1.9	1.4	1.2	1.0	0.8	0.9	1.2	1.4	1.7	2.4	3.3	4.2	5.2	6.2	7.3	8 2
1000	4.4	3.7	3.2	2.6	2.1	1.7	1.3	0.9	0.7	0.7	1.0	1.2	1.6	2.2	3.0	3.8	4.8	5.8	6.9
	790	800	810	820	830	840	850	860	870	880	890	900	910	920	930	940	950	960	970

TABLE XII.

Perturb. produ. by *Mars*.
ARGUMENTS. B & D. D

B	970	980	990	1000
	″	″	″	″
0	6.9	7.8	8.4	9.5
20	5.5	6.5	7.6	8.7
40	4.0	5.2	6.0	7.1
60	3.0	3.7	4.8	5.8
80	2.1	2.7	3.6	4.5
100	1.6	2.0	2.6	3.3
120	1.4	1.7	1.9	2.4
140	1.5	1.5	1.8	2.1
160	2.0	1.7	1.7	1.9
180	2.5	2.0	2.0	1.9
200	3.5	2.8	2.7	2.9
220	4.4	3.9	3.2	3.0
240	5.3	4.8	4.2	3.7
260	6.0	5.6	5.2	4.8
280	6.7	6.3	5.9	5.5
300	7.2	7.0	6.6	6.1
320	7.6	7.3	7.1	6.9
340	7.6	7.6	7.6	7.7
360	7.6	7.6	7.6	7.5
380	7.1	7.3	7.5	7.5
400	6.9	7.0	7.1	7.3
420	6.5	6.7	6.7	6.9
440	6.1	6.3	6.5	6.5
460	5.7	5.9	6.1	6.2
480	5.4	5.5	5.6	5.8
500	5.1	5.2	5.3	5.3
520	4.7	4.9	5.0	5.1
540	4.5	4.6	4.6	4.7
560	4.5	4.5	4.5	4.4
580	4.4	4.4	4.4	4.2
600	4.4	4.4	4.3	4.1
620	4.5	4.4	4.3	4.2
640	4.7	4.6	4.5	4.3
660	5.3	4.9	4.8	4.6
680	5.6	5.4	5.1	4.8
700	6.3	5.9	5.6	5.3
720	7.2	6.6	6.1	5.8
740	8.2	7.6	6.9	6.5
760	9.3	8.5	7.9	7.3
780	10.2	9.6	9.0	8.2
800	11.2	10.5	9.8	9.2
820	11.9	11.2	10.7	10.1
840	12.4	12.2	11.5	10.9
860	12.5	12.5	12.3	11.7
880	12.6	12.6	12.4	12.3
900	12.2	12.4	12.5	12.4
920	11.5	12.1	12.2	12.3
940	10.7	11.2	11.7	12.1
960	9.4	10.2	10.9	11.4
980	8.2	8.9	9.9	10.6
1000	6.9	7.8	8.7	9.5
	970	980	990	1000

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E

0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
15.3	15.1	15.0	15.0	14.8	14.7	14.7	14.6	14.5	14.5	14.4	14.5	14.5	14.6	14.7
14.9	14.9	14.7	14.8	14.7	14.6	14.4	14.4	14.2	14.2	14.1	14.1	14.1	14.1	14.1
14.7	14.6	14.6	14.5	14.4	14.4	14.3	14.2	14.1	13.9	13.8	13.8	13.8	13.8	13.8
14.4	14.4	14.4	14.4	14.3	14.3	14.2	14.1	13.9	13.8	13.6	13.5	13.5	13.5	13.4
13.4	13.9	14.0	14.2	14.2	14.1	14.5	14.0	13.8	13.7	13.5	13.4	13.2	13.1	13.0
13.2	13.4	13.6	13.7	13.7	13.9	13.9	13.8	13.7	13.6	13.5	13.4	13.2	13.0	12.8
12.3	12.7	13.0	13.3	13.4	13.4	13.5	13.6	13.5	13.5	13.3	13.3	13.2	13.0	12.8
11.3	11.8	12.1	12.5	12.8	13.0	13.1	13.2	13.2	13.3	13.2	13.1	13.0	12.9	12.8
10.2	10.7	11.2	11.7	12.0	12.4	12.6	12.7	12.8	12.9	12.9	13.0	12.9	12.8	12.7
9.1	9.6	10.1	10.7	11.1	11.6	11.9	12.2	12.3	12.5	12.5	12.6	12.7	12.8	12.6
7.8	8.3	8.9	9.5	10.0	10.6	11.0	11.5	11.7	11.9	12.2	12.2	12.3	12.4	12.3
6.8	7.2	7.7	8.3	8.8	9.5	9.9	10.4	10.8	11.3	11.5	11.8	11.9	12.0	12.0
5.7	6.2	6.6	7.2	7.7	8.2	8.9	9.4	9.8	10.3	10.6	11.0	11.3	11.5	11.7
4.8	5.2	5.6	6.1	6.5	7.1	7.6	8.3	8.8	9.3	9.7	10.1	10.5	10.9	11.0
3.9	4.1	4.7	5.2	5.5	6.0	6.5	7.1	7.6	8.2	8.7	9.2	9.6	10.0	10.4
3.4	3.5	3.9	4.3	4.7	5.1	5.5	6.1	6.6	7.1	7.6	8.1	8.7	9.1	9.4
3.2	3.1	3.4	3.6	3.9	4.3	4.6	5.1	5.4	6.0	6.6	7.2	7.7	8.1	8.5
3.2	3.0	3.0	3.1	3.3	3.5	3.8	4.1	4.5	5.0	5.4	6.1	6.6	7.2	7.6
3.5	3.2	2.9	2.9	3.0	3.1	3.3	3.6	3.8	4.1	4.5	5.0	5.5	6.1	6.6
4.5	4.0	3.4	3.1	2.8	2.8	2.7	2.8	2.9	3.0	3.2	3.5	4.1	4.6	5.0
5.0	4.3	3.8	3.5	3.1	2.9	2.9	2.8	2.8	3.0	3.1	3.4	3.8	4.2	4.7
6.1	5.2	4.6	4.1	3.6	3.3	3.1	2.8	2.7	2.8	2.9	3.1	3.2	3.5	3.8
7.5	6.6	5.8	4.9	4.4	3.9	3.4	3.1	2.7	2.8	2.7	2.8	3.1	3.1	3.2
9.0	7.9	7.0	6.3	5.4	4.8	4.3	3.7	3.2	2.9	2.8	2.8	2.7	2.7	2.8
10.5	9.5	8.5	7.6	6.7	5.9	5.2	4.6	4.1	3.6	3.1	3.0	2.8	2.8	2.6
12.3	11.3	10.0	9.1	8.1	7.2	6.4	5.7	5.0	4.4	3.9	3.4	3.2	3.1	2.9
14.0	12.7	11.7	10.7	9.5	8.7	7.7	6.9	6.1	5.5	4.8	4.2	3.8	3.5	3.2
15.6	14.5	13.3	12.3	11.1	10.2	9.1	8.4	7.4	6.6	5.9	5.3	4.7	4.1	3.8
17.1	16.1	15.1	14.0	13.0	11.9	10.8	9.9	8.7	7.9	7.1	6.4	5.8	5.2	4.5
18.6	17.4	16.5	15.7	14.5	13.6	12.5	11.4	10.4	9.3	8.3	7.7	6.9	6.2	5.5
19.8	19.0	17.9	17.0	16.0	15.0	14.0	13.1	12.0	11.0	10.1	9.2	8.2	7.5	6.7
20.8	20.1	19.2	18.4	17.4	16.5	15.5	14.7	13.6	12.6	11.6	10.7	9.8	9.0	8.0
21.6	20.9	20.2	19.5	18.5	17.9	17.0	16.0	15.1	14.2	13.1	12.2	11.3	10.4	9.4
22.1	21.6	21.0	20.4	19.7	18.9	18.1	17.4	16.3	15.6	14.6	13.7	12.8	11.9	11.0
22.3	22.0	21.6	21.2	20.5	19.9	19.1	18.5	17.6	16.8	16.0	15.1	14.2	13.5	12.5
22.2	22.0	21.7	21.5	21.0	20.6	20.0	19.3	18.7	18.0	17.1	16.5	15.6	14.7	13.8
22.0	21.9	21.7	21.6	21.2	21.0	20.5	20.0	19.3	18.9	18.3	17.5	16.8	16.1	15.1
21.6	21.6	21.5	21.5	21.2	21.1	20.8	20.5	20.0	19.4	18.9	18.4	17.7	17.2	16.8
21.2	21.1	21.1	21.2	21.0	21.0	20.8	20.7	20.3	20.0	19.4	19.0	18.6	17.9	17.4
20.4	20.5	20.6	20.7	20.7	20.7	20.6	20.6	20.3	20.2	19.8	19.4	19.1	18.7	18.1
19.6	19.8	19.9	20.1	20.2	20.3	20.3	20.4	20.3	20.1	19.9	19.7	19.3	19.1	18.7
18.8	19.0	19.2	19.4	19.5	19.7	19.8	19.9	19.9	19.9	19.8	19.8	19.6	19.2	18.9
18.1	18.2	18.4	18.6	18.8	18.9	19.0	19.2	19.3	19.4	19.4	19.4	19.4	19.4	19.0
17.4	17.5	17.6	17.9	18.0	18.3	18.4	18.6	18.7	18.8	18.9	19.0	19.1	19.1	19.0
16.9	16.9	16.9	17.1	17.2	17.5	17.6	17.9	18.0	18.2	18.3	18.5	18.6	18.6	18.6
16.3	16.4	16.4	16.5	16.6	16.8	16.9	17.1	17.1	17.4	17.5	17.7	17.9	18.1	18.2
16.0	15.9	15.9	16.0	16.0	16.1	16.2	16.4	16.5	16.7	16.8	17.0	17.2	17.4	17.5
15.8	15.7	15.7	15.6	15.5	15.6	15.6	15.7	15.8	16.0	16.1	16.3	16.5	16.8	16.8
15.5	15.4	15.3	15.4	15.3	15.3	15.2	15.2	15.2	15.3	15.4	15.6	15.7	15.9	16.0
15.3	15.2	15.2	15.1	15.0	15.0	14.9	14.9	14.8	14.9	14.9	14.9	15.0	15.2	15.3
15.3	15.1	15.0	15.0	14.8	14.7	14.7	14.6	14.5	14.5	14.4	14.5	14.5	14.6	14.7
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E.

B	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320
0	14.7	14.8	15.0	15.3	15.5	15.8	15.9	16.2	16.3	16.7	17.0	17.1	17.3	17.5	17.5	17.7	17.8	17.9	17.9
20	14.1	14.2	14.3	14.6	14.8	14.9	15.2	15.5	15.7	15.9	16.2	16.6	16.8	17.1	17.3	17.5	17.6	17.8	17.8
40	13.8	13.7	13.7	13.9	14.1	14.3	14.5	14.8	15.0	15.3	15.5	15.8	16.2	16.4	16.8	16.9	17.2	17.6	17.7
60	13.4	13.3	13.2	13.4	13.5	13.6	13.8	14.1	14.3	14.6	14.8	15.1	15.5	15.8	16.0	16.4	16.7	16.9	17.3
80	13.0	13.1	13.0	13.0	13.0	13.1	13.1	13.3	13.5	13.8	14.1	14.4	14.5	15.1	15.4	15.7	16.1	16.4	16.7
100	12.8	12.7	12.7	12.7	12.6	12.7	12.6	12.8	12.9	13.1	13.4	13.7	14.0	14.2	14.6	15.1	15.0	15.8	16.1
120	12.8	12.6	12.5	12.5	12.4	12.3	12.2	12.3	12.3	12.6	12.8	13.0	13.3	13.6	13.7	14.2	14.5	15.0	15.4
140	12.8	12.6	12.4	12.4	12.3	12.1	12.0	12.0	12.0	12.1	12.1	12.3	12.5	12.8	13.1	13.3	13.7	14.2	14.4
160	12.7	12.5	12.3	12.2	12.1	12.1	11.9	11.8	11.8	11.8	11.8	11.9	12.0	12.2	12.4	12.6	12.9	13.4	13.8
180	12.6	12.5	12.3	12.2	12.1	11.9	11.8	11.7	11.5	11.5	11.5	11.6	11.7	11.8	11.9	12.1	12.3	12.5	12.8
200	12.3	12.3	12.2	12.2	12.0	11.9	11.7	11.7	11.5	11.4	11.3	11.2	11.3	11.5	11.5	11.6	11.7	12.0	12.1
220	12.0	12.0	12.0	12.1	12.0	11.8	11.6	11.6	11.5	11.4	11.3	11.2	11.1	11.1	11.1	11.2	11.3	11.6	11.7
240	11.7	11.8	11.8	11.9	11.9	11.8	11.6	11.5	11.4	11.3	11.2	11.1	11.1	11.0	10.9	10.9	11.0	11.2	11.3
260	11.0	11.2	11.5	11.6	11.6	11.6	11.5	11.3	11.3	11.3	11.2	11.1	11.0	10.9	10.8	10.8	10.8	10.9	10.9
280	10.4	10.6	10.8	11.1	11.2	11.2	11.2	11.3	11.3	11.2	11.2	11.1	11.0	10.9	10.8	10.7	10.6	10.7	10.6
300	9.4	9.9	10.1	10.5	10.8	10.9	11.0	11.1	11.0	11.0	11.0	11.0	11.1	10.9	10.8	10.7	10.6	10.6	10.5
320	8.5	8.9	9.4	9.7	10.1	10.4	10.5	10.7	10.8	10.8	10.8	10.8	10.8	10.9	10.7	10.7	10.6	10.6	10.5
340	7.6	8.0	8.5	9.1	9.3	9.6	9.9	10.2	10.3	10.5	10.6	10.6	10.7	10.7	10.7	10.6	10.5	10.5	10.4
360	6.6	7.1	7.5	8.0	8.4	8.9	9.2	9.5	9.8	10.1	10.3	10.4	10.5	10.5	10.5	10.5	10.5	10.5	10.4
380	5.0	5.6	6.2	6.8	7.3	7.8	8.3	8.9	9.3	9.7	10.0	10.0	10.1	10.2	10.3	10.3	10.3	10.4	10.3
400	4.7	5.2	5.6	6.2	6.6	7.0	7.5	7.9	8.4	8.8	9.1	9.4	9.7	9.9	10.0	10.0	10.2	10.3	10.2
420	3.8	4.3	4.8	5.3	5.8	6.2	6.6	7.1	7.4	7.9	8.4	8.7	9.1	9.4	9.6	9.8	9.9	10.1	10.2
440	3.2	3.5	3.9	4.4	4.9	5.4	5.7	6.2	6.7	7.1	7.6	7.9	8.4	8.7	9.0	9.2	9.4	9.7	9.8
460	2.8	3.2	3.3	3.8	4.1	4.5	4.9	5.4	5.7	6.3	6.7	7.2	7.7	8.0	8.4	8.6	8.8	9.1	9.3
480	2.6	2.7	2.9	3.2	3.6	3.9	4.3	4.7	5.0	5.4	5.9	6.3	6.8	7.3	7.6	7.9	8.4	8.7	8.9
500	2.9	2.7	2.7	2.9	3.1	3.4	3.6	4.0	4.4	4.8	5.2	5.7	5.9	6.4	6.9	7.2	7.6	8.0	8.3
520	3.2	3.1	2.8	2.9	3.0	3.1	3.2	3.5	3.8	4.2	4.7	4.9	5.4	5.7	6.1	6.6	6.9	7.3	7.6
540	3.8	3.5	3.2	3.1	3.0	3.0	3.0	3.3	3.5	3.7	4.1	4.3	4.7	5.1	5.4	5.8	6.2	6.7	7.0
560	4.5	4.1	3.8	3.6	3.3	3.2	3.2	3.2	3.3	3.5	3.7	4.0	4.3	4.5	4.9	5.1	5.5	6.0	6.3
580	5.5	5.0	4.6	4.2	4.0	3.6	3.5	3.3	3.2	3.4	3.5	3.7	4.0	4.2	4.4	4.8	5.0	5.3	5.7
600	6.7	6.0	5.4	5.1	4.6	4.3	3.9	3.7	3.5	3.5	3.6	3.7	3.8	4.0	4.2	4.3	4.7	4.9	5.2
620	8.0	7.3	6.6	6.0	5.6	5.1	4.6	4.2	4.0	3.9	3.8	3.9	3.9	4.0	4.0	4.1	4.3	4.7	4.8
640	9.4	8.7	7.8	7.3	6.6	6.1	5.5	5.2	4.7	4.4	4.2	4.0	4.0	4.1	4.1	4.2	4.2	4.4	4.6
660	11.0	10.1	9.3	8.6	7.7	7.2	6.5	6.2	5.9	5.3	4.9	4.6	4.5	4.4	4.3	4.3	4.3	4.5	4.5
680	12.5	11.6	10.8	10.0	9.3	8.5	7.5	7.3	6.7	6.3	5.8	5.5	5.2	4.9	4.9	4.7	4.6	4.7	4.5
700	13.8	13.0	12.1	11.5	10.7	9.9	9.0	8.5	7.8	7.4	6.9	6.3	6.0	5.8	5.4	5.2	5.1	5.0	4.9
720	15.1	14.3	13.5	12.8	12.1	11.3	10.6	9.8	9.1	8.7	8.0	7.6	7.0	6.6	6.2	5.9	5.7	5.6	5.5
740	16.8	15.7	14.9	14.2	13.4	12.7	12.0	11.2	10.5	9.7	9.3	8.9	8.2	7.7	7.2	6.8	6.5	6.4	6.1
760	17.4	16.7	15.9	15.5	14.7	13.9	13.3	12.6	11.8	11.2	10.5	10.0	9.5	9.0	8.2	7.9	7.5	7.2	6.9
780	18.1	17.6	17.0	16.4	15.7	15.1	14.6	13.8	13.2	12.6	11.9	11.2	10.8	10.2	9.7	9.1	8.4	8.2	7.7
800	18.7	18.2	17.8	17.3	16.8	16.2	15.6	15.0	14.3	13.7	13.1	12.6	12.0	11.5	11.0	10.4	9.8	9.4	8.7
820	18.9	18.7	18.3	18.0	17.6	17.0	16.6	16.0	15.3	14.9	14.3	13.7	13.1	12.6	12.1	11.7	11.2	10.6	10.1
840	19.0	18.9	18.7	18.4	18.2	17.7	17.2	16.8	16.3	15.8	15.3	14.9	14.4	13.8	13.2	12.8	12.3	11.9	11.3
860	19.0	18.8	18.7	18.6	18.4	18.3	17.9	17.4	17.1	16.7	16.3	15.9	15.4	15.0	14.4	13.8	13.5	13.1	12.6
880	18.6	18.7	18.5	18.6	18.5	18.3	18.2	18.0	17.7	17.4	17.1	16.6	16.3	15.9	15.4	15.0	14.4	14.2	13.7
900	18.2	18.2	18.2	18.3	18.3	18.3	18.1	18.1	18.0	17.8	17.6	17.3	17.0	16.7	16.4	15.9	15.5	15.2	14.8
920	17.5	17.7	17.9	18.0	18.0	18.1	18.1	18.0	18.0	18.0	17.8	17.7	17.6	17.3	17.1	16.8	16.5	16.2	15.7
940	16.8	17.1	17.1	17.4	17.6	17.6	17.7	17.8	17.8	17.9	18.0	17.8	17.8	17.7	17.5	17.3	17.1	16.9	16.6
960	16.0	16.2	16.5	16.8	16.9	17.1	17.2	17.4	17.5	17.6	17.8	17.9	18.0	17.9	17.8	17.6	17.5	17.4	17.2
980	15.3	15.5	15.7	16.1	16.3	16.5	16.7	16.8	17.0	17.2	17.3	17.6	17.7	17.9	17.8	17.8	17.8	17.8	17.8
1000	14.7	14.8	15.0	15.3	15.5	15.8	15.9	16.2	16.3	16.7	17.0	17.1	17.3	17.5	17.7	17.7	17.8	17.9	17.9
	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E

B	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500
0	17.9	18.0	18.0	17.9	17.7	17.6	17.5	17.5	17.1	17.0	16.7	16.5	16.3	16.1	15.8	15.6	15.1	14.6	14.3
20	17.8	18.0	18.1	18.1	18.1	18.0	18.0	18.0	18.1	17.7	17.5	17.5	17.2	17.1	16.8	16.7	16.3	16.0	15.6
40	17.7	17.9	18.1	18.3	18.3	18.4	18.4	18.6	18.6	18.5	18.4	18.3	18.1	18.0	17.8	17.6	17.3	17.2	16.8
60	17.3	17.6	17.9	18.2	18.3	18.5	18.5	18.7	18.9	18.9	18.9	18.9	18.7	18.8	18.6	18.7	18.4	18.1	17.9
80	16.7	17.0	17.5	17.8	18.0	18.3	18.5	18.8	18.9	19.2	19.3	19.4	19.3	19.3	19.3	19.3	19.2	19.2	18.9
100	16.1	16.5	17.0	17.2	17.5	17.9	18.3	18.7	18.9	19.1	19.4	19.7	19.8	19.8	19.8	19.8	19.8	19.9	19.7
120	15.4	15.8	16.2	16.7	17.1	17.3	17.9	18.3	18.6	18.9	19.2	19.5	19.8	20.0	20.1	20.3	20.3	20.4	20.4
140	14.4	15.1	15.5	15.9	16.3	16.8	17.3	17.7	18.2	18.6	18.9	19.2	19.6	20.0	20.3	20.5	20.6	20.7	20.8
160	13.8	14.1	14.6	15.2	15.5	16.0	16.5	17.1	17.6	17.9	18.5	19.0	19.3	19.8	20.2	20.5	20.6	20.9	21.1
180	12.8	13.3	13.7	14.4	14.7	15.2	15.7	16.3	16.8	17.3	17.9	18.3	18.8	19.3	19.8	20.3	20.6	20.9	21.1
200	12.1	12.5	13.0	13.4	13.8	14.3	14.7	15.5	16.0	16.5	17.1	17.7	18.2	18.6	19.1	19.8	20.2	20.7	21.0
220	11.7	11.9	12.3	12.7	13.0	13.5	14.0	14.5	15.0	15.6	16.1	16.9	17.4	18.0	18.6	19.0	19.7	20.3	20.7
240	11.3	11.5	11.8	12.1	12.3	12.8	13.2	13.8	14.2	14.7	15.2	15.9	16.5	17.1	17.7	18.4	18.9	19.5	20.1
260	10.9	11.1	11.3	11.4	11.6	12.0	12.3	13.0	13.4	13.9	14.4	15.0	15.5	16.3	16.9	17.5	18.0	18.6	19.3
280	10.6	10.8	11.0	11.2	11.3	11.5	11.8	12.2	12.7	13.0	13.5	14.2	14.7	15.3	15.9	16.7	17.2	17.8	18.4
300	10.5	10.6	10.7	10.8	10.9	11.1	11.4	11.8	11.9	12.4	12.8	13.3	13.8	14.4	14.9	15.7	16.3	17.0	17.6
320	10.5	10.5	10.6	10.7	10.6	10.7	11.0	11.2	11.5	11.8	12.2	12.7	13.0	13.6	14.1	14.7	15.3	16.0	16.6
340	10.4	10.5	10.5	10.6	10.5	10.6	10.7	10.8	11.2	11.4	11.6	12.1	12.4	12.9	13.4	13.9	14.4	15.1	15.7
360	10.4	10.4	10.4	10.4	10.3	10.5	10.6	10.8	10.8	11.0	11.2	11.6	11.9	12.3	12.6	13.2	13.6	14.2	14.8
380	10.3	10.4	10.4	10.4	10.3	10.3	10.4	10.6	10.6	10.7	10.9	11.2	11.4	11.9	12.2	12.6	12.9	13.5	13.9
400	10.2	10.2	10.3	10.4	10.3	10.3	10.3	10.5	10.5	10.6	10.6	10.9	11.1	11.4	11.8	12.2	12.5	12.9	13.3
420	10.2	10.1	10.2	10.2	10.2	10.3	10.3	10.4	10.4	10.5	10.6	10.7	10.9	11.2	11.3	11.7	11.9	12.4	12.8
440	9.8	10.0	10.1	10.2	10.1	10.1	10.2	10.4	10.4	10.4	10.5	10.7	10.8	10.9	11.1	11.3	11.6	11.9	12.2
460	9.3	9.6	9.9	10.1	10.0	10.0	10.2	10.3	10.4	10.4	10.4	10.6	10.6	10.7	10.9	11.2	11.3	11.7	11.9
480	8.9	9.1	9.4	9.6	9.7	9.8	10.0	10.1	10.2	10.3	10.4	10.6	10.6	10.7	10.8	11.0	11.2	11.4	11.7
500	8.3	8.6	8.9	9.2	9.4	9.5	9.7	9.9	10.0	10.1	10.2	10.4	10.5	10.7	10.8	10.9	11.0	11.2	11.3
520	7.6	7.9	8.3	8.6	8.9	9.1	9.4	9.7	9.8	9.8	10.0	10.2	10.3	10.5	10.6	10.9	10.8	11.1	11.3
540	7.0	7.4	7.7	8.0	8.3	8.6	8.9	9.2	9.4	9.6	9.8	10.0	10.2	10.3	10.4	10.6	10.7	10.9	11.1
560	6.3	6.7	7.2	7.5	7.7	8.0	8.3	8.7	8.9	9.1	9.3	9.7	9.8	10.1	10.3	10.5	10.6	10.7	10.8
580	5.7	6.1	6.6	6.9	7.1	7.4	7.7	8.1	8.5	8.7	8.7	9.2	9.4	9.7	9.9	10.2	10.4	10.6	10.7
600	5.2	5.6	6.0	6.3	6.5	6.8	7.2	7.6	7.9	8.2	8.5	8.8	9.0	9.3	9.5	9.8	10.0	10.3	10.5
620	4.8	5.1	5.5	5.8	6.1	6.4	6.7	7.0	7.3	7.6	7.9	8.2	8.5	8.8	9.0	9.4	9.6	10.0	10.1
640	4.6	4.8	5.1	5.4	5.6	5.9	6.3	6.6	6.8	7.1	7.4	7.7	7.9	8.2	8.6	8.9	9.1	9.4	9.7
660	4.5	4.7	4.9	5.1	5.3	5.5	5.8	6.2	6.4	6.6	6.9	7.3	7.6	7.9	8.1	8.3	8.6	8.9	9.2
680	4.5	4.6	4.8	5.0	5.1	5.3	5.5	5.8	6.1	6.2	6.5	6.8	7.0	7.4	7.6	7.9	8.1	8.4	8.7
700	4.9	4.9	4.9	5.1	5.2	5.3	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.9	7.1	7.4	7.6	7.9	8.2
720	5.5	5.4	5.3	5.3	5.3	5.3	5.4	5.5	5.6	5.7	5.9	6.2	6.3	6.5	6.8	7.1	7.2	7.5	7.7
740	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.7	5.7	5.7	5.8	6.0	6.1	6.2	6.4	6.7	6.9	7.1	7.2
760	6.9	6.7	6.5	6.3	6.1	5.9	5.9	6.0	6.0	6.0	6.0	6.0	6.1	6.2	6.3	6.4	6.5	6.7	6.8
780	7.7	7.6	7.4	7.2	6.9	6.6	6.5	6.5	6.3	6.2	6.2	6.3	6.3	6.3	6.3	6.4	6.4	6.5	6.7
800	8.7	8.5	8.3	8.0	7.7	7.6	7.3	7.1	7.0	6.7	6.6	6.7	6.5	6.5	6.4	6.5	6.5	6.5	6.6
820	10.1	9.7	9.2	9.1	8.6	8.3	8.1	7.9	7.6	7.5	7.3	7.2	7.0	7.0	6.8	6.8	6.7	6.6	6.6
840	11.3	10.9	10.5	10.2	9.6	9.4	9.1	8.9	8.6	8.3	8.1	7.8	7.7	7.6	7.4	7.3	7.1	7.0	6.8
860	12.6	12.1	11.7	11.2	10.7	10.4	10.1	10.0	9.7	9.3	9.0	8.7	8.4	8.2	8.1	7.9	7.7	7.6	7.3
880	13.7	13.4	12.9	12.5	12.0	11.5	11.3	11.1	10.5	10.4	10.0	9.7	9.5	9.2	8.9	8.7	8.4	8.2	7.9
900	14.8	14.4	14.1	13.7	13.2	12.8	12.4	12.2	11.8	11.5	11.0	10.8	10.5	10.3	9.9	9.7	9.4	9.0	8.8
920	15.7	15.5	15.2	14.8	14.3	14.0	13.6	13.3	13.0	12.6	12.3	12.1	11.5	11.3	11.0	10.6	10.2	10.1	9.7
940	16.6	16.3	16.1	16.0	15.5	15.0	14.7	14.5	14.1	13.8	13.5	13.2	12.8	12.5	11.9	11.8	11.3	11.0	10.7
960	17.2	17.0	16.9	16.8	16.4	16.2	15.8	15.6	15.3	14.9	14.6	14.4	14.0	13.7	13.3	13.0	12.5	12.1	11.8
980	17.8	17.6	17.5	17.3	17.2	17.0	16.8	16.6	16.3	16.0	15.7	15.6	15.2	14.9	14.6	14.2	13.8	13.6	12.9
1000	17.9	18.0	18.0	17.9	17.7	17.6	17.5	17.5	17.1	17.0	16.7	16.5	16.3	16.1	15.8	15.6	15.1	14.6	14.3
	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E

B	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680
0	14.3	13.9	13.4	13.1	12.7	12.1	11.8	11.3	10.8	10.2	9.9	9.4	8.9	8.4	8.0	7.7	7.3	6.9	6.7
20	15.6	15.3	14.9	14.4	13.9	13.5	13.1	12.5	12.1	11.5	11.0	10.4	10.0	9.4	9.0	8.4	8.0	7.5	7.1
40	16.8	16.5	16.3	15.7	15.4	15.0	14.3	13.8	13.4	12.8	12.3	11.7	11.1	10.5	10.1	9.4	8.9	8.3	7.8
60	17.9	17.7	17.3	17.0	16.6	16.1	15.8	15.3	14.7	14.3	13.7	13.0	12.4	11.8	11.3	10.6	10.1	9.3	8.7
80	18.9	18.8	18.5	18.1	17.9	17.4	17.1	16.6	16.2	15.7	15.1	14.5	13.9	13.2	12.7	12.0	11.3	10.5	9.9
100	19.7	19.7	19.5	19.2	19.0	18.8	18.4	17.9	17.6	17.0	16.5	16.0	15.2	14.7	14.1	13.4	12.8	12.0	11.3
120	20.4	20.4	20.3	20.2	20.0	19.7	19.5	19.1	18.8	18.4	18.0	17.3	16.8	16.2	15.4	14.9	14.2	13.4	12.7
140	20.8	21.0	21.1	21.0	20.8	20.7	20.4	20.2	19.9	19.6	19.3	18.8	18.3	17.7	17.2	16.4	15.6	14.9	14.2
160	21.1	21.2	21.5	21.5	21.6	21.5	21.3	21.2	21.0	20.6	20.4	20.1	19.6	19.1	18.6	17.9	17.3	16.6	15.7
180	21.1	21.4	21.6	21.8	22.0	22.0	22.1	21.9	21.8	21.6	21.4	21.1	20.7	20.3	19.9	19.4	18.8	18.0	17.3
200	21.0	21.4	21.7	21.9	22.1	22.3	22.5	22.5	22.5	22.4	22.3	22.1	21.8	21.5	21.2	20.8	20.2	19.3	18.9
220	20.7	21.1	21.5	21.8	22.2	22.5	22.8	23.1	23.1	22.9	22.8	22.9	22.6	22.5	22.3	21.9	21.5	21.0	20.3
240	20.1	20.7	21.1	21.5	21.9	22.3	22.7	23.0	23.3	23.4	23.5	23.4	23.3	23.2	23.0	22.9	22.5	22.0	21.6
260	19.3	20.0	20.6	21.0	21.6	22.0	22.4	22.8	23.2	23.5	23.8	23.8	23.8	23.9	23.8	23.7	23.5	23.1	22.7
280	18.4	19.1	19.9	20.4	20.9	21.5	22.0	22.4	23.0	23.3	23.7	24.0	24.1	24.1	24.3	24.2	24.2	24.0	23.7
300	17.6	18.2	19.0	19.6	20.3	20.7	21.3	21.8	22.3	23.0	23.4	23.8	24.1	24.3	24.5	24.6	24.6	24.5	24.4
320	16.6	17.4	18.9	18.7	19.4	20.0	20.6	21.1	21.8	22.3	22.9	23.3	23.7	24.2	24.5	24.7	24.9	24.8	24.8
340	15.7	16.4	17.0	17.6	18.5	19.2	19.9	20.4	21.1	21.6	22.2	22.8	23.3	23.7	24.2	24.5	24.7	25.0	25.2
360	14.8	15.5	16.2	16.7	17.4	18.2	18.9	19.5	20.1	20.8	21.5	22.0	22.6	23.2	23.7	24.2	24.5	24.7	25.0
380	13.9	14.5	15.2	15.9	16.6	17.1	17.9	18.6	19.3	19.8	20.5	21.1	21.8	22.5	23.1	23.6	24.1	24.4	24.7
400	13.3	13.8	14.4	14.9	15.6	16.2	16.8	17.6	18.4	19.1	19.7	20.3	20.9	21.5	22.3	22.8	23.4	23.9	24.3
420	12.8	13.3	13.7	14.2	14.8	15.3	16.0	16.5	17.4	18.0	18.7	19.4	20.0	20.6	21.3	22.0	22.6	23.1	23.6
440	12.2	12.7	13.1	13.6	14.1	14.6	15.2	15.7	16.4	17.1	17.8	18.4	18.9	19.6	20.3	21.0	21.8	22.3	22.9
460	11.9	12.2	12.7	13.0	13.5	13.9	14.4	15.0	15.6	16.1	16.9	17.5	18.2	18.7	19.4	20.1	20.7	21.3	21.9
480	11.7	12.0	12.2	12.5	13.0	13.4	13.9	14.3	14.8	15.3	15.9	16.6	17.3	17.9	18.5	19.1	19.7	20.3	21.0
500	11.3	11.7	12.0	12.2	12.6	12.9	13.3	13.8	14.3	14.7	15.2	15.7	16.4	16.9	17.6	18.2	18.8	19.3	19.9
520	11.3	11.5	11.9	12.0	12.3	12.6	13.0	13.2	13.8	14.2	14.7	15.1	15.5	16.2	16.8	17.3	17.9	18.4	19.0
540	11.1	11.4	11.6	11.9	12.2	12.4	12.7	12.9	13.3	13.7	14.2	14.6	15.0	15.4	16.1	16.6	17.2	17.5	18.1
560	10.8	11.2	11.4	11.5	11.9	12.1	12.4	12.7	13.1	13.4	13.8	14.1	14.5	14.9	15.4	16.0	16.5	16.9	17.3
580	10.7	10.9	11.2	11.4	11.6	11.9	12.2	12.4	12.8	13.1	13.5	13.8	14.2	14.5	15.0	15.3	15.9	16.3	16.7
600	10.5	10.7	10.8	11.1	11.5	11.7	12.0	12.2	12.5	12.8	13.1	13.4	13.8	14.2	14.6	14.9	15.3	15.8	16.3
620	10.1	10.4	10.7	10.7	11.1	11.4	11.6	12.0	12.3	12.5	12.9	13.1	13.4	13.8	14.2	14.6	14.9	15.1	15.7
640	9.7	10.1	10.4	10.6	10.7	11.0	11.3	11.6	12.0	12.3	12.6	12.9	13.2	13.5	14.0	14.2	14.6	14.8	15.1
660	9.2	9.5	9.9	10.2	10.5	10.6	11.0	11.3	11.6	11.9	12.3	12.6	12.9	13.2	13.5	13.9	14.3	14.6	14.9
680	8.7	9.0	9.3	9.6	10.0	10.3	10.5	10.8	11.3	11.5	11.9	12.2	12.4	12.8	13.2	13.5	13.9	14.2	14.5
700	8.2	8.5	8.9	9.1	9.5	9.8	10.1	10.3	10.7	11.1	11.4	11.8	12.1	12.4	12.9	13.3	13.5	13.8	14.2
720	7.7	8.0	8.3	8.5	9.0	9.2	9.6	9.9	10.2	10.5	10.9	11.3	11.7	12.0	12.4	12.8	13.2	13.5	13.8
740	7.2	7.5	7.8	8.0	8.3	8.6	9.0	9.3	9.7	9.9	10.4	10.8	11.1	11.5	11.9	12.2	12.6	12.9	13.3
760	6.8	7.1	7.3	7.5	7.9	8.1	8.4	8.6	9.1	9.4	9.7	10.1	10.5	10.9	11.4	11.8	12.2	12.4	12.8
780	6.7	6.8	7.0	7.1	7.3	7.6	7.9	8.1	8.5	8.8	9.2	9.4	9.8	10.2	10.6	11.2	11.6	11.9	12.4
800	6.6	6.7	6.8	6.8	7.0	7.1	7.3	7.5	7.8	8.2	8.5	8.8	9.1	9.5	10.0	10.3	10.9	11.3	11.6
820	6.6	6.7	6.8	6.6	6.8	6.9	7.0	7.1	7.4	7.6	7.9	8.1	8.4	8.7	9.3	9.7	10.0	10.5	10.9
840	6.8	6.8	6.8	6.8	6.8	6.8	6.9	6.9	7.1	7.2	7.4	7.6	7.9	8.1	8.4	8.8	9.3	9.6	10.1
860	7.3	7.2	7.1	7.1	7.0	6.9	6.9	6.8	6.8	6.9	7.1	7.2	7.3	7.6	7.9	8.1	8.5	8.8	9.2
880	7.9	7.7	7.5	7.4	7.3	7.1	7.0	6.8	6.8	6.7	6.8	6.8	7.0	7.2	7.4	7.6	7.8	8.1	8.5
900	8.8	8.5	8.2	7.9	7.7	7.5	7.3	7.2	7.1	6.9	6.9	6.8	6.8	6.8	7.0	7.1	7.3	7.4	7.8
920	9.7	9.4	9.2	8.7	8.4	8.1	7.9	7.6	7.4	7.1	7.0	6.9	6.8	6.7	6.8	6.8	6.9	7.0	7.0
940	10.7	10.4	10.0	9.7	9.4	8.9	8.6	8.3	8.1	7.7	7.4	7.1	6.9	6.7	6.7	6.7	6.8	6.7	6.8
960	11.8	11.5	11.2	10.7	10.4	9.8	9.5	9.1	8.8	8.5	8.1	7.7	7.4	7.1	7.0	6.8	6.7	6.5	6.5
980	12.9	12.7	12.3	11.8	11.5	11.1	10.6	10.0	9.7	9.2	8.9	8.5	8.1	7.7	7.4	7.1	6.9	6.6	6.5
1000	14.3	13.9	13.4	13.1	12.7	12.1	11.8	11.3	10.8	10.2	9.9	9.4	8.9	8.4	8.0	7.7	7.3	6.9	6.7
	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E

B	680	690	700	710	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	6.7	6.5	6.5	6.3	6.2	6.2	6.4	6.5	6.8	7.2	7.5	8.0	8.4	8.8	9.5	10.1	10.5	11.0	11.6
20	7.1	6.9	6.7	6.4	6.3	6.0	6.1	6.1	6.2	6.5	6.7	7.0	7.4	7.9	8.4	9.0	9.5	10.0	10.6
40	7.8	7.4	7.0	6.6	6.4	6.2	5.9	5.8	5.9	5.9	6.2	6.4	6.6	6.9	7.4	7.8	8.2	8.8	9.5
60	8.7	8.2	7.7	7.2	6.8	6.4	6.2	5.8	5.7	5.7	5.7	5.9	6.1	6.2	6.5	6.9	7.2	7.7	8.3
80	9.9	9.2	8.7	8.1	7.6	7.1	6.6	6.2	5.8	5.7	5.6	5.4	5.6	5.7	5.9	6.1	6.3	6.7	7.3
100	11.3	10.6	9.9	9.1	8.5	7.9	7.3	6.8	6.3	5.9	5.6	5.5	5.3	5.3	5.4	5.4	5.6	5.9	6.3
120	12.7	12.0	11.3	10.4	9.8	8.9	8.2	7.6	7.4	6.5	6.0	5.7	5.5	5.1	5.2	5.1	5.1	5.2	5.5
140	14.2	13.4	12.7	11.9	11.1	10.2	9.6	8.8	8.1	7.4	6.8	6.2	5.8	5.4	5.2	5.0	4.9	4.8	5.0
160	15.7	15.0	14.2	13.3	12.6	11.7	10.9	10.0	9.3	8.5	7.8	7.2	6.5	5.9	5.5	5.1	4.9	4.7	4.7
180	17.3	16.7	15.8	15.0	14.1	13.2	12.4	11.5	10.6	9.7	9.0	8.2	7.5	6.9	6.3	5.8	5.2	4.8	4.7
200	18.9	18.1	17.5	16.6	15.7	14.9	14.0	13.1	12.2	11.2	10.4	9.5	8.8	7.9	7.1	6.5	5.9	5.3	5.0
220	20.3	19.7	19.0	18.2	17.5	16.6	15.5	14.7	13.8	12.9	12.0	11.1	10.2	9.3	8.4	7.5	6.7	6.1	5.5
240	21.6	21.1	20.5	19.8	19.1	18.2	17.3	16.4	15.3	14.5	13.6	12.6	11.7	10.7	9.8	8.8	7.9	7.0	6.5
260	22.7	22.3	21.8	21.2	20.6	19.8	19.1	18.1	17.2	16.3	15.3	14.3	13.3	12.2	11.4	10.4	9.4	8.3	7.7
280	23.7	23.5	23.1	22.4	21.8	21.2	20.5	19.8	18.9	17.9	17.0	16.1	15.0	14.0	13.0	11.9	10.9	9.9	8.9
300	24.4	24.2	23.9	23.6	23.1	22.5	21.9	21.2	20.4	19.6	18.7	17.7	16.8	15.8	14.7	13.7	12.6	11.5	10.5
320	24.8	24.8	24.7	24.4	24.1	23.7	23.1	22.5	21.9	21.2	20.4	19.4	18.5	17.4	16.5	15.5	14.2	13.2	12.3
340	25.2	25.1	25.0	25.0	24.9	24.6	24.1	23.7	23.0	22.4	21.8	21.1	20.2	19.2	18.3	17.1	16.1	15.0	13.9
360	25.0	25.1	25.3	25.4	25.3	25.1	24.9	24.5	24.0	23.6	23.0	22.4	21.6	20.8	19.9	18.9	17.9	16.8	15.9
380	24.7	25.1	25.2	25.4	25.5	25.4	25.3	25.2	24.9	24.5	24.0	23.5	22.8	22.1	21.4	20.5	19.5	18.5	17.6
400	24.3	24.7	25.1	25.2	25.4	25.6	25.6	25.5	25.4	25.1	24.8	24.5	23.9	23.4	22.7	21.9	21.0	20.1	19.2
420	23.6	24.1	24.5	25.0	25.2	25.4	25.6	25.7	25.6	25.5	25.3	25.0	24.5	24.2	23.7	23.2	22.3	21.5	20.7
440	22.9	23.4	23.9	24.3	24.8	25.0	25.2	25.6	25.6	25.7	25.7	25.5	25.3	24.9	24.6	24.1	23.4	22.7	22.0
460	21.9	22.6	23.3	23.6	24.1	24.6	24.8	25.1	25.3	25.5	25.6	25.8	25.7	25.4	25.2	24.8	24.3	23.7	23.1
480	21.0	21.6	22.2	22.8	23.3	23.8	24.3	24.6	24.9	25.2	25.4	25.6	25.6	25.5	25.4	25.2	24.9	24.5	24.1
500	19.9	20.7	21.4	21.9	22.5	22.9	23.4	23.9	24.2	24.7	25.0	25.3	25.4	25.5	25.5	25.4	25.2	24.9	24.7
520	19.0	19.7	20.4	21.0	21.6	22.1	22.6	23.0	23.6	23.9	24.3	24.7	24.9	25.2	25.4	25.4	25.3	25.2	25.1
540	18.1	18.7	19.3	19.9	20.5	21.2	21.7	22.2	22.6	23.2	23.6	24.0	24.4	24.6	24.9	25.1	25.0	25.1	25.1
560	17.3	17.9	18.4	18.9	19.6	20.1	20.7	21.3	21.7	22.2	22.8	23.2	23.7	24.0	24.3	24.6	24.7	24.8	24.9
580	16.7	17.1	17.6	18.1	18.7	19.3	19.8	20.3	20.8	21.3	21.8	22.3	22.7	23.2	23.7	23.9	24.1	24.4	24.6
600	16.3	16.6	17.0	17.4	17.9	18.3	18.9	19.4	19.9	20.4	20.8	21.4	21.9	22.2	22.7	23.1	23.4	23.7	24.1
620	15.7	16.2	16.6	16.9	17.3	17.6	18.0	18.5	19.0	19.5	20.1	20.5	20.9	21.4	21.8	22.2	22.6	22.9	23.3
640	15.1	15.6	16.1	16.5	16.8	17.1	17.5	17.9	18.3	18.7	19.2	19.7	20.1	20.5	21.0	21.3	21.7	22.1	22.5
660	14.9	15.2	15.6	15.9	16.4	16.6	17.0	17.3	17.6	18.1	18.5	18.9	19.4	19.6	20.1	20.5	20.7	21.2	21.7
680	14.5	14.9	15.2	15.6	16.0	16.2	16.5	16.8	17.1	17.4	17.8	18.2	18.6	18.9	19.4	19.7	20.1	20.4	20.7
700	14.2	14.5	14.9	15.1	15.6	15.9	16.2	16.4	16.7	16.9	17.3	17.7	18.0	18.3	18.7	18.9	19.2	19.6	20.0
720	13.8	14.2	14.5	14.8	15.1	15.5	15.8	16.1	16.3	16.5	16.9	17.2	17.6	17.8	18.0	18.3	18.5	18.7	19.2
740	13.3	13.8	14.2	14.5	14.8	15.1	15.4	15.7	16.0	16.2	16.5	16.7	17.0	17.3	17.6	17.8	17.9	18.1	18.5
760	12.8	13.2	13.7	14.1	14.5	14.7	15.0	15.4	15.7	16.0	16.1	16.4	16.6	16.7	17.2	17.4	17.4	17.8	18.0
780	12.4	12.8	13.2	13.5	13.9	14.3	14.6	14.9	15.3	15.6	15.9	16.1	16.3	16.5	16.7	16.9	17.1	17.3	17.6
800	11.6	12.1	12.6	12.9	13.4	13.8	14.2	14.5	14.7	15.2	15.5	15.8	15.9	16.2	16.5	16.6	16.8	16.9	17.1
820	10.9	11.4	11.9	12.3	12.8	13.2	13.6	14.0	14.4	14.7	15.1	15.4	15.7	15.8	16.1	16.3	16.4	16.6	16.9
840	10.1	10.6	11.1	11.6	12.1	12.5	13.0	13.4	13.7	14.1	14.5	15.1	15.4	15.4	15.8	15.9	16.0	16.2	16.6
860	9.2	9.7	10.2	10.7	11.2	11.7	12.1	12.6	13.1	13.5	13.9	14.3	14.8	15.2	15.5	15.6	15.8	16.0	16.3
880	8.5	8.8	9.4	9.8	10.2	10.7	11.2	11.8	12.3	12.8	13.3	13.7	14.1	14.5	15.0	15.3	15.4	15.6	15.9
900	7.8	8.2	8.5	8.9	9.4	9.8	10.3	10.8	11.3	11.9	12.4	13.0	13.4	13.7	14.2	14.7	15.0	15.2	15.5
920	7.0	7.4	7.8	8.1	8.6	8.9	9.4	9.9	10.3	10.8	11.4	12.0	12.5	12.9	13.4	14.0	14.3	14.7	15.0
940	6.8	6.8	7.1	7.4	7.7	8.1	8.4	8.9	9.4	9.9	10.4	11.0	11.6	12.1	12.5	13.0	13.6	13.9	14.4
960	6.5	6.6	6.7	6.8	7.1	7.3	7.7	8.0	8.3	8.8	9.4	10.0	10.6	11.1	11.7	12.2	12.5	13.1	13.7
980	6.5	6.4	6.4	6.3	6.5	6.8	6.9	7.3	7.6	7.9	8.4	8.9	9.5	9.9	10.5	11.1	11.6	12.1	12.8
1000	6.7	6.5	6.5	6.3	6.2	6.2	6.4	6.5	6.8	7.2	7.5	8.0	8.4	9.5	8.9	10.0	10.5	11.0	11.6
	680	690	700	710	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860

TABLE XIII.

Perturbations produced by *Jupiter*. ARGUMENTS. B and E.

E

B	860	870	880	890	900	910	920	930	940	950	960	970	980	990	1000
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	11.6	12.4	12.9	13.2	13.6	13.9	14.2	14.4	14.8	15.0	15.1	15.1	15.2	15.2	15.3
20	10.6	11.1	11.7	12.2	12.7	13.2	13.6	13.8	14.1	14.4	14.7	14.8	15.0	14.9	14.9
40	9.5	10.0	10.5	11.1	11.7	12.3	12.6	13.0	13.4	13.7	14.1	14.3	14.6	14.7	14.7
60	8.3	8.8	9.4	9.9	10.6	11.2	11.8	12.1	12.6	12.9	13.3	13.6	13.9	14.2	14.4
80	7.3	7.8	8.3	8.7	9.3	10.0	10.5	11.1	11.6	12.1	12.5	12.8	13.2	13.5	13.8
100	6.3	6.8	7.2	7.6	8.1	8.6	9.4	9.9	10.5	10.9	11.4	12.0	12.4	12.8	13.2
120	5.5	5.8	6.1	6.6	7.1	7.6	8.1	8.7	9.4	9.9	10.4	10.8	11.4	11.8	12.3
140	5.0	5.1	5.3	5.6	6.0	6.5	7.0	7.5	8.2	8.7	9.3	9.7	10.3	10.8	11.3
160	4.7	4.7	4.8	4.8	5.2	5.6	5.9	6.3	6.8	7.4	8.0	8.6	9.2	9.7	10.2
180	4.7	4.5	4.5	4.4	4.5	4.8	5.1	5.4	5.8	6.2	6.9	7.4	8.0	8.4	9.1
200	5.0	4.7	4.5	4.2	4.2	4.2	4.4	4.6	5.0	5.3	5.7	6.3	6.9	7.4	7.8
220	5.5	5.2	4.7	4.3	4.2	4.1	4.1	4.0	4.3	4.5	4.8	5.1	5.7	6.2	6.8
240	6.5	5.9	5.3	4.7	4.3	4.1	4.0	3.8	3.9	4.0	4.2	4.3	4.7	5.2	5.7
260	7.7	6.9	6.1	5.4	4.9	4.4	4.1	3.8	3.7	3.6	3.7	3.8	4.1	4.3	4.9
280	8.9	8.0	7.2	6.3	5.7	5.2	4.6	4.1	3.8	3.5	3.5	3.5	3.6	3.7	3.9
300	10.5	9.4	8.5	7.5	6.8	6.1	5.4	4.7	4.3	3.9	3.6	3.3	3.3	3.3	3.4
320	12.3	11.2	10.1	9.1	8.1	7.3	6.5	5.7	5.0	4.4	4.0	3.6	3.4	3.2	3.2
340	13.9	12.9	11.8	10.7	9.6	8.7	7.7	6.8	6.0	5.2	4.6	4.1	3.7	3.4	3.2
360	15.9	14.7	13.4	12.3	11.1	10.1	9.2	8.3	7.4	6.4	5.7	4.9	4.3	3.8	3.5
380	17.6	16.5	15.4	14.2	13.0	11.8	10.8	9.7	8.7	7.8	6.9	6.1	5.4	4.6	4.1
400	19.2	18.2	17.2	16.0	14.9	13.8	12.4	11.4	10.4	9.3	8.3	7.3	6.4	5.6	5.0
420	20.7	19.8	18.8	17.7	16.7	15.5	14.4	13.1	11.9	10.9	9.8	8.8	8.0	6.9	6.1
440	22.0	21.2	20.3	19.3	18.3	17.3	16.2	14.9	13.8	12.7	11.5	10.5	9.5	8.4	7.5
460	23.1	22.5	21.6	20.6	19.7	18.9	17.9	16.7	15.6	14.3	13.3	12.2	10.9	10.0	9.0
480	24.1	23.5	22.7	22.0	21.1	20.2	19.3	18.2	17.3	16.2	15.0	13.8	12.8	11.6	10.5
500	24.7	24.3	23.8	23.0	22.3	21.6	20.7	19.7	18.8	17.8	16.7	15.4	14.5	13.4	12.3
520	25.1	24.8	24.3	23.7	23.2	22.7	21.9	21.1	20.2	19.2	18.3	17.2	16.1	15.0	14.0
540	25.1	25.0	24.8	24.3	23.9	23.4	22.8	22.1	21.3	20.6	19.7	18.7	17.6	16.6	15.6
560	24.9	24.9	24.8	24.7	24.4	24.0	23.6	22.9	22.4	21.6	20.8	20.0	19.1	18.2	17.1
580	24.6	24.7	24.7	24.6	24.5	24.3	23.9	23.5	23.1	22.5	21.9	21.1	20.3	19.5	18.6
600	24.1	24.3	24.3	24.3	24.3	24.3	24.1	23.8	23.5	23.0	22.5	22.0	21.4	20.6	19.8
620	23.3	23.6	23.7	23.9	24.0	24.1	24.1	23.9	23.7	23.4	23.1	22.6	22.1	21.4	20.8
640	22.5	22.8	23.1	23.2	23.4	23.6	23.7	23.8	23.7	23.5	23.2	22.9	22.6	22.1	21.6
660	21.7	22.0	22.3	22.5	22.8	23.0	23.2	23.2	23.3	23.2	23.1	23.0	22.8	22.5	22.1
680	20.7	21.2	21.5	21.7	22.0	22.3	22.5	22.6	22.8	22.9	22.9	22.8	22.7	22.7	22.3
700	20.0	20.3	20.7	20.9	21.2	21.5	21.7	21.9	22.2	22.3	22.5	22.5	22.5	22.4	22.2
720	19.2	19.5	19.8	20.1	20.4	20.8	21.1	21.2	21.4	21.6	21.8	21.9	22.0	22.0	22.0
740	18.5	18.8	19.0	19.2	19.6	19.9	20.2	20.5	20.7	20.9	21.1	21.2	21.5	21.5	21.6
760	18.0	18.2	18.5	18.4	18.8	19.1	19.4	19.6	19.9	20.1	20.3	20.5	20.8	21.0	21.2
780	17.6	17.7	17.8	18.0	18.1	18.4	18.7	18.8	19.1	19.3	19.5	19.7	20.0	20.2	20.4
800	17.1	17.3	17.4	17.4	17.7	17.9	18.0	18.1	18.4	18.6	18.9	18.9	19.1	19.4	19.6
820	16.9	17.0	17.2	17.2	17.2	17.4	17.4	17.6	17.8	17.8	18.1	18.3	18.5	18.6	18.8
840	16.6	16.7	16.8	16.8	16.9	17.2	17.2	17.1	17.1	17.3	17.4	17.5	17.8	17.9	18.1
860	16.3	16.4	16.5	16.5	16.6	16.6	16.7	16.8	16.9	16.9	17.0	17.0	17.1	17.2	17.4
880	15.9	16.1	16.3	16.3	16.5	16.5	16.5	16.6	16.6	16.6	16.6	16.6	16.7	16.7	16.9
900	15.5	15.7	15.9	16.1	16.2	16.3	16.4	16.3	16.3	16.2	16.2	16.2	16.3	16.3	16.3
920	15.0	15.3	15.5	15.6	15.9	16.0	16.1	16.1	16.1	16.0	16.1	16.1	16.1	16.0	16.0
940	14.4	14.7	14.9	15.2	15.4	15.7	15.8	15.8	16.0	15.9	15.9	15.9	15.8	15.7	15.8
960	13.7	14.1	14.3	14.5	14.8	15.2	15.5	15.5	15.7	15.7	15.7	15.6	15.6	15.5	15.5
980	12.8	13.3	13.7	13.9	14.2	14.5	14.8	15.1	15.3	15.4	15.5	15.4	15.4	15.4	15.3
1000	11.6	12.4	12.9	13.2	13.6	13.9	14.2	14.4	14.8	15.0	15.1	15.1	15.2	15.2	15.3
	860	870	880	890	900	910	920	930	940	950	960	970	980	990	1000

TABLE XIV.

Perturbations produced by *Saturn.*

ARGUMENTS. B and F.

F.

B	000	100	200	300	400	500	600	700	800	900	1000
	″	″	″	″	″	″	″	″	″	″	″
0	1.2	1.5	1.4	1.0	0.7	0.6	0.5	0.5	0.4	0.8	1.2
100	0.9	1.2	1.3	1.1	0.9	0.8	0.7	0.7	0.6	0.7	0.9
200	0.7	0.9	1.0	1.1	1.0	0.9	0.8	0.8	0.9	0.8	0.7
300	0.9	0.8	0.7	0.8	0.9	1.0	1.0	1.0	1.0	1.0	0.9
400	1.0	0.9	0.6	0.4	0.6	0.9	1.0	1.1	1.1	1.1	1.0
500	1.1	1.0	0.8	0.4	0.2	0.5	1.0	1.3	1.3	1.2	1.1
600	1.2	1.1	0.9	0.6	0.2	0.2	0.5	1.1	1.5	1.5	1.2
700	1.4	1.1	1.0	0.8	0.4	0.1	0.3	0.8	1.4	1.7	1.4
800	1.6	1.3	1.0	0.8	0.6	0.4	0.1	0.3	1.0	1.6	1.6
900	1.5	1.4	1.1	0.9	0.7	0.6	0.3	0.2	0.6	1.2	1.5
1000	1.2	1.5	1.4	1.0	0.7	0.6	0.5	0.5	0.4	0.8	1.2

TABLE. XV.

Variable Part of the Aberration of the SUN.

ARGUMENT. Mean Anomaly of the Sun.

D.	O— VI+	I— VII+	II— VIII+	
	″	″	″	
0	0.34	0.29	0.17	30
3	0.33	0.28	0.15	27
6	0.33	0.27	0.13	24
9	0.33	0.26	0.12	21
12	0.33	0.25	0.10	18
15	0.32	0.23	0.08	15
18	0.32	0.22	0.07	12
21	0.31	0.21	0.05	9
24	0.30	0.20	0.03	6
27	0.30	0.18	0.02	3
30	0.29	0.17	0.00	0
	XI— V+	X— IV+	IX— III+	D.

The constant quantity —20″ is contained in the Mean Longitude: Thus, to find the heliocentric Longitude of the Earth, we omit the variable part of the Aberration, and add to the true place of the Sun 180° 0′ 20″.

TABLE XVI.

Semi-diameter, Horary Motion, and Horizontal Parallax.

ARGUMENT. Sun's Mean Anomaly.

	Sig. O.			Sig. I.			Sig. II.			
Deg.	Semi-diam.	Hor. Mot.	Parall.	Semi-diam.	Hor. Mot.	Parall.	Semi-diam.	Hor. Mot.	Parall.	
	′ ″	′ ″	″	′ ″	′ ″	″	′ ″	′ ″	″	
0	16 17.79	2 32.92	8.95	16 15.49	2 32.20	8.93	16 9.30	2 30.28	8.87	30
1	16 17.79	2 32.92	8.95	16 15.34	2 32.15	8.93	16 9.05	2 30.20	8.87	29
2	16 17.78	2 32.92	8.95	16 15.18	2 32.10	8.93	16 8.79	2 30.12	8.87	28
3	16 17.77	2 32.91	8.95	16 15.02	2 32.05	8.92	16 8.54	2 30.04	8.87	27
4	16 17.75	2 32.91	8.95	16 14.85	2 32.00	8.92	16 8.28	2 29.96	8.86	26
5	16 17.73	2 32.90	8.95	16 14.69	2 31.95	8.92	16 8.02	2 29.88	8.86	25
6	16 17.70	2 32.89	8.95	16 14.51	2 31.90	8.92	16 7.75	2 29.80	8.86	24
7	16 17.66	2 32.88	8.95	16 14.34	2 31.84	8.92	16 7.49	2 29.71	8.86	23
8	16 17.62	2 32.87	8.95	16 14.16	2 31.78	8.92	16 7.22	2 29.63	8.85	22
9	16 17.58	2 32.86	8.95	16 13.97	2 31.73	8.91	16 6.95	2 29.54	8.85	21
10	16 17.53	2 32.84	8.95	16 13.78	2 31.67	8.91	16 6.68	2 29.46	8.85	20
11	16 17.47	2 32.82	8.95	16 13.59	2 31.61	8.91	16 6.41	2 29.38	8.85	19
12	16 17.41	2 32.80	8.95	16 13.39	2 31.55	8.91	16 6.14	2 29.30	8.84	18
13	16 17.35	2 32.78	8.95	16 13.20	2 31.48	8.91	16 5.86	2 29.21	8.84	17
14	16 17.28	2 32.76	8.95	16 12.99	2 31.42	8.91	16 5.59	2 29.13	8.84	16
15	16 17.20	2 32.73	8.95	16 12.78	2 31.36	8.90	16 5.31	2 29.04	8.84	15
16	16 17.12	2 32.71	8.94	16 12.57	2 31.29	8.90	16 5.04	2 28.96	8.83	14
17	16 17.04	2 32.68	8.94	16 12.36	2 31.22	8.90	16 4.76	2 28.87	8.83	13
18	16 16.95	2 32.65	8.94	16 12.14	2 31.16	8.90	16 4.48	2 28.78	8.83	12
19	16 16.85	2 32.62	8.94	16 11.92	2 31.09	8.90	16 4.20	2 28.70	8.83	11
20	16 16.75	2 32.59	8.94	16 11.69	2 31.02	8.89	16 3.92	2 28.61	8.82	10
21	16 16.65	2 32.56	8.94	16 11.46	2 30.95	8.89	16 3.64	2 28.52	8.82	9
22	16 16.54	2 32.53	8.94	16 11.24	2 30.88	8.89	16 3.36	2 28.44	8.82	8
23	16 16.42	2 32.49	8.94	16 11.01	2 30.80	8.89	16 3.07	2 28.35	8.82	7
24	16 16.30	2 32.45	8.94	16 10.77	2 30.73	8.89	16 2.79	2 28.26	8.81	6
25	16 16.18	2 32.41	8.94	16 10.54	2 30.66	8.88	16 2.51	2 28.18	8.81	5
26	16 16.05	2 32.37	8.93	16 10.29	2 30.58	8.88	16 2.23	2 28.09	8.81	4
27	16 15.92	2 32.33	8.93	16 10.05	2 30.51	8.88	16 1.95	2 28.00	8.80	3
28	16 15.78	2 32.29	8.93	16 9.80	2 30.43	8.88	16 1.67	2 27.92	8.80	2
29	16 15.63	2 32.24	8.93	16 9.55	2 30.35	8.87	16 1.38	2 27.83	8.80	1
30	16 15.49	2 32.20	8.93	16 9.30	2 30.28	8.87	16 1.10	2 27.74	8.80	0
	Sig. XI.			Sig. X.			Sig. IX.			D.

If we suppose the semi-diameter at the mean distance to vary by 1″, there results the variation of 1″+0″.017 cos. ($\odot$ + 2*s*. 20½°) for any other semi-diameter.

Continuation of TABLE XVI.

ARGUMENT. Sun's Mean Anomaly.

	Sig. III.			Sig. IV.			Sig. V.			
Deg.	Semi-diam.	Hor. Mot.	Parall.	Semi-diam.	Hor. Mot.	Parall.	Semi-diam.	Hor. Mot.	Parall.	
0	16 1.10	2 27.74	8.80	15 53.17	2 25.32	8.72	15 47.53	2 23.60	8.67	30
1	16 0.82	2 27.66	8.79	15 52.94	2 25.24	8.72	15 47.40	2 23.56	8.67	29
2	16 0.54	2 27.57	8.79	15 52.70	2 25.17	8.72	15 47.27	2 23.52	8.67	28
3	16 0.26	2 27.48	8.79	15 52.48	2 25.10	8.72	15 47.15	2 23.48	8.67	27
4	15 59.98	2 27.40	8.78	15 52.25	2 25.03	8.72	15 47.03	2 23.45	8.67	26
5	15 59.70	2 27.31	8.78	15 52.03	2 24.96	8.71	15 46.92	2 23.41	8.67	25
6	15 59.42	2 27.23	8.78	15 51.81	2 24.90	8.71	15 46.81	2 23.38	8.67	24
7	15 59.14	2 27.14	8.78	15 51.59	2 24.84	8.71	15 46.70	2 23.35	8.67	23
8	15 58.87	2 27.06	8.78	15 51.37	2 24.77	8.71	15 46.60	2 23.32	8.66	22
9	15 58.59	2 26.97	8.77	15 51.16	2 24.70	8.71	15 46.50	2 23.29	8.66	21
10	15 58.32	2 26.89	8.77	15 50.96	2 24.64	8.70	15 46.41	2 23.26	8.66	20
11	15 58.05	2 26.80	8.77	15 50.75	2 24.58	8.70	15 46.32	2 23.23	8.66	19
12	15 57.77	2 26.72	8.77	15 50.55	2 24.52	8.70	15 46.24	2 23.20	8.66	18
13	15 57.50	2 26.64	8.77	15 50.35	2 24.46	8.70	15 46.16	2 23.18	8.66	17
14	15 57.23	2 26.56	8.77	15 50.15	2 24.40	8.70	15 46.08	2 23.16	8.66	16
15	15 56.96	2 26.47	8.77	15 49.96	2 24.34	8.69	15 46.01	2 23.14	8.66	15
16	15 56.70	2 26.39	8.76	15 49.77	2 24.28	8.69	15 45.95	2 23.12	8.66	14
17	15 56.44	2 26.31	8.75	15 49.59	2 24.23	8.69	15 45.88	2 23.10	8.66	13
18	15 56.17	2 26.23	8.75	15 49.41	2 24.17	8.69	15 45.83	2 23.09	8.66	12
19	15 55.91	2 26.15	8.75	15 49.23	2 24.12	8.69	15 45.78	2 23.07	8.66	11
20	15 55.65	2 26.07	8.75	15 49.06	2 24.06	8.69	15 45.73	2 23.05	8.66	10
21	15 55.39	2 25.99	8.74	15 48.89	2 24.01	8.69	15 45.68	2 23.04	8.66	9
22	15 55.13	2 25.91	8.74	15 48.72	2 23.96	8.68	15 45.64	2 23.03	8.66	8
23	15 54.88	2 25.84	8.74	15 48.56	2 23.91	8.68	15 45.61	2 23.02	8.66	7
24	15 54.63	2 25.76	8.74	15 48.40	2 23.86	8.68	15 45.58	2 23.01	8.65	6
25	15 54.38	2 25.68	8.74	15 48.24	2 23.82	8.68	15 45.55	2 23.00	8.65	5
26	15 54.14	2 25.61	8.73	15 48.09	2 23.77	8.68	15 45.53	2 23.00	8.65	4
27	15 53.89	2 25.53	8.73	15 47.94	2 23.73	8.68	15 45.52	2 22.99	8.65	3
28	15 53.65	2 25.46	8.73	15 47.80	2 23.68	8.68	15 45.51	2 22.99	8.65	2
29	15 53.41	2 25.39	8.73	15 47.66	2 23.64	8.67	15 45.50	2 22.99	8.65	1
30	15 53.17	2 25.32	8.72	15 47.53	2 23.60	8.67	15 45.50	2 22.99	8.65	0
	Sig. VIII.			Sig. VII.			Sig. VI.			D

TABLE XVII.

2d. PART.—Mean Refraction, or Correction of a Fixed Star in Altitude.

First Part. Correction of the Sun in Altitude. Subtract.

Subtract.

Ap. Alt.	Cor.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt	Refr.
° ′	′ ″	′ ″	′ ″	° ′	′ ″	° ′	′ ″	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	32 51	0 0	33 0	5 0	9 54	10 0	5 15	20 0	2 35	32 20	1 30	67 0	0 24
0 5	32 1	0 5	32 10	5 5	9 40	10 10	5 10	20 10	2 34	32 40	1 29	68 0	0 23
0 10	31 13	0 10	31 22	5 10	9 38	10 20	5 5	20 20	2 32	33 0	1 28	69 0	0 22
0 15	31 26	0 15	30 35	5 15	9 30	10 30	5 0	20 30	2 31	33 20	1 26	70 0	0 21
0 20	29 41	0 20	29 50	5 20	9 23	10 40	4 56	20 40	2 29	33 40	1 25	71 0	0 19
0 25	28 57	0 25	29 6	5 25	9 15	10 50	4 51	20 50	2 28	34 0	1 24	72 0	0 18
0 30	28 14	0 30	28 23	5 30	9 8	11 0	4 47	21 0	2 27	34 20	1 23	73 0	0 17
0 35	27 32	0 35	27 41	5 35	9 1	11 10	4 43	21 10	2 26	34 40	1 22	74 0	0 16
0 40	26 51	0 40	27 0	5 40	8 54	11 20	4 39	21 20	2 25	35 0	1 21	75 0	0 15
0 45	26 11	0 45	26 20	5 45	8 47	11 30	4 34	21 30	2 24	35 20	1 20	76 0	0 14
0 50	25 33	0 50	25 42	5 50	8 41	11 40	4 31	21 40	2 23	35 40	1 19	77 0	0 13
0 55	24 56	0 55	25 5	5 55	8 34	11 50	4 27	21 50	2 21	36 0	1 18	78 0	0 12
1 0	24 20	1 0	24 29	6 0	8 28	12 0	4 23	22 0	2 20	36 30	1 17	79 0	0 11
1 10	23 11	1 5	23 54	6 5	8 21	12 10	4 20	22 10	2 19	37 0	1 16	80 0	0 10
1 20	22 6	1 10	23 20	6 10	8 15	12 20	4 16	22 20	2 18	37 30	1 14	81 0	0 9
1 30	21 6	1 15	22 47	6 15	8 9	12 30	4 13	22 30	2 17	38 0	1 13	82 0	0 8
1 40	20 9	1 20	22 15	6 20	8 3	12 40	4 9	22 40	2 16	38 30	1 11	83 0	0 7
1 50	19 16	1 25	21 44	6 25	7 57	12 50	4 6	22 50	2 15	39 0	1 10	84 0	0 6
2 0	18 26	1 30	21 15	6 30	7 51	13 0	4 3	23 0	2 14	39 30	1 9	85 0	0 5
2 20	16 55	1 35	20 46	6 35	7 45	13 10	4 0	23 10	2 13	40 0	1 8	86 0	0 4
2 40	15 36	1 40	20 18	6 40	7 40	13 20	3 57	23 20	2 12	40 30	1 7	87 0	0 3
3 0	14 27	1 45	19 51	6 45	7 35	13 30	3 54	23 30	2 11	41 0	1 5	88 0	0 2
3 20	13 25	1 50	19 25	6 50	7 30	13 40	3 51	23 40	2 10	41 30	1 4	89 0	0 1
3 40	12 31	1 55	19 0	6 55	7 25	13 50	3 48	23 50	2 9	42 0	1 3	90 0	0 0
4 0	11 42	2 0	18 35	7 0	7 20	14 0	3 45	24 0	2 8	42 30	1 2		
4 30	10 39	2 5	18 11	7 5	7 15	14 10	3 43	24 10	2 7	43 0	1 1		
5 0	9 45	2 10	17 48	7 10	7 11	14 20	3 40	24 20	2 6	43 30	1 0		
5 30	8 59	2 15	17 26	7 15	7 6	14 30	3 38	24 30	2 5	44 0	0 59		
6 0	8 19	2 20	17 4	7 20	7 2	14 40	3 35	24 40	2 4	44 30	0 58		
6 30	7 42	2 25	16 44	7 25	6 57	14 50	3 33	24 50	2 3	45 0	0 57		
7 0	7 11	2 30	16 24	7 30	6 53	15 0	3 30	25 0	2 2	45 30	0 56		
8 0	6 20	2 35	16 4	7 35	6 49	15 10	3 28	25 10	2 1	46 0	0 55		
9 0	5 39	2 40	15 45	7 40	6 45	15 20	3 26	25 20	2 0	46 30	0 54		
10 0	5 6	2 45	15 27	7 45	6 41	15 30	3 24	25 30	1 59	47 0	0 53		
11 0	4 38	2 50	15 9	7 50	6 37	15 40	3 21	25 40	1 58	47 30	0 52		
12 0	4 15	2 55	14 52	7 55	6 33	15 50	3 19	25 50	1 57	48 0	0 51		
13 0	3 55	3 0	14 36	8 0	6 29	16 0	3 17	26 0	1 56	48 30	0 50		
14 0	3 37	3 5	14 20	8 5	6 25	16 10	3 15	26 10	1 55	49 0	0 49		
15 0	3 22	3 10	14 4	8 10	6 22	16 20	3 12	26 20	1 55	49 30	0 49		
16 0	3 9	3 15	13 49	8 15	6 18	16 30	3 10	26 30	1 54	50 0	0 48		
18 0	2 46	3 20	13 34	8 20	6 15	16 40	3 8	26 40	1 53	50 30	0 47		
20 0	2 17	3 25	13 20	8 25	6 11	16 50	3 6	26 50	1 52	51 0	0 46		
22 0	2 12	3 30	13 6	8 30	6 8	17 0	3 4	27 0	1 51	51 30	0 45		
24 0	2 0	3 35	12 53	8 35	6 5	17 10	3 3	27 15	1 50	52 0	0 44		
26 0	1 48	3 40	12 40	8 40	6 1	17 20	3 1	27 30	1 49	52 30	0 44		
28 0	1 39	3 45	12 27	8 45	5 58	17 30	2 59	27 45	1 48	53 0	0 43		
30 0	1 30	3 50	12 15	8 50	5 55	17 40	2 57	28 0	1 47	53 30	0 42		
32 0	1 24	3 55	12 3	8 55	5 52	17 50	2 55	28 15	1 46	54 0	0 41		
34 0	1 17	4 0	11 51	9 0	5 48	18 0	2 54	28 30	1 45	55 0	0 40		
36 0	1 11	4 5	11 40	9 5	5 45	18 10	2 52	28 45	1 44	56 0	0 38		
38 0	1 6	4 10	11 29	9 10	5 42	18 20	2 51	29 0	1 42	57 0	0 37		
40 0	1 1	4 15	11 18	9 15	5 39	18 30	2 49	29 20	1 41	58 0	0 35		
45 0	0 51	4 20	11 8	9 20	5 36	18 40	2 47	29 40	1 40	59 0	0 34		
50 0	0 42	4 25	10 58	9 25	5 34	18 50	2 46	30 0	1 38	60 0	0 33		
55 0	0 35	4 30	10 48	9 30	5 31	19 0	2 44	30 20	1 37	61 0	0 32		
60 0	0 29	4 35	10 39	9 35	5 28	19 10	2 43	30 40	1 36	62 0	0 30		
65 0	0 22	4 40	10 29	9 40	5 25	19 20	2 41	31 0	1 35	63 0	0 29		
70 0	0 18	4 45	10 20	9 45	5 23	19 30	2 40	31 20	1 33	64 0	0 28		
80 0	0 9	4 50	10 11	9 50	5 20	19 40	2 38	31 40	1 32	65 0	0 26		
90 0	0 0	4 55	10 2	9 55	5 18	19 50	2 37	32 0	1 31	66 0	0 25		

Depression of the Horizon.

height of the Eye in Feet	depr. of the visi. horizon.
	′ ″
1	0 57
2	1 21
3	1 39
4	1 55
5	2 8
6	2 20
7	2 31
8	2 42
9	2 52
10	3 1
11	3 10
12	3 18
13	3 26
14	3 34
15	3 42
16	3 49
17	3 56
18	4 3
19	4 10
20	4 16
21	4 22
22	4 28
23	4 34
24	4 40
26	4 52
28	5 3
30	5 14
35	5 39
40	6 2
45	6 24
50	6 44
60	7 23
70	7 59
80	8 32
90	9 3
100	9 33

3d. PART.—Correction of the Mean Refraction.—ARGUMENT.—Height of Farenheit's Thermometer.

ap. alt.	20°	24°	28°	32°	36°	40°	44°	48°	52°	56°	60°	64°	68°	72°	76°	80°
° ′	+ ″	+ ″	+ ″	+ ″	+ ″	+ ″	+ ″	+ ″	− ″	− ″	− ″	− ″	− ″	− ″	− ″	− ″
0 0	2 40	2 18	1 55	1 33	1 11	0 51	0 31	0 10	0 10	0 29	0 48	1 7	1 25	1 43	2 1	2 19
0 10	2 32	2 12	1 49	1 28	1 8	0 48	0 29	0 10	0 9	0 27	0 45	1 4	1 21	1 38	1 54	2 12
0 20	2 25	2 5	1 44	1 24	1 4	0 46	0 28	0 9	0 9	0 26	0 44	1 1	1 17	1 33	1 49	2 5
0 30	2 18	1 59	1 39	1 20	1 1	0 44	0 26	0 9	0 8	0 25	0 41	0 58	1 13	1 28	1 43	1 59
0 40	2 11	1 53	1 34	1 16	0 58	0 42	0 25	0 8	0 8	0 24	0 39	0 55	1 10	1 24	1 38	1 53
0 50	2 5	1 48	1 29	1 12	0 55	0 40	0 24	0 8	0 8	0 23	0 37	0 52	1 6	1 20	1 34	1 48
1 0	1 59	1 43	1 25	1 9	0 53	0 38	0 23	0 8	0 7	0 21	0 36	0 50	1 3	1 17	1 30	1 43
1 10	1 53	1 38	1 21	1 6	0 50	0 36	0 22	0 7	0 7	0 20	0 34	0 48	1 0	1 13	1 26	1 38
1 20	1 48	1 33	1 17	1 3	0 48	0 34	0 21	0 7	0 6	0 19	0 32	0 45	0 57	1 9	1 21	1 33
1 30	1 43	1 29	1 14	1 0	0 46	0 32	0 20	0 7	0 6	0 18	0 31	0 43	0 54	1 6	1 18	1 29
1 40	1 39	1 25	1 11	0 57	0 44	0 31	0 18	0 6	0 6	0 18	0 30	0 41	0 52	1 4	1 15	1 25
1 50	1 35	1 21	1 8	0 55	0 42	0 30	0 17	0 6	0 6	0 17	0 28	0 39	0 50	1 1	1 11	1 21
2 0	1 31	1 18	1 5	0 53	0 41	0 29	0 17	0 6	0 5	0 16	0 27	0 37	0 48	0 58	1 8	1 18
2 20	1 23	1 11	1 0	0 48	0 37	0 26	0 16	0 5	0 5	0 15	0 25	0 35	0 44	0 54	1 3	1 11
2 40	1 17	1 6	0 55	0 44	0 34	0 24	0 14	0 5	0 5	0 14	0 23	0 32	0 41	0 50	0 58	1 6
3 0	1 11	1 1	0 51	0 41	0 32	0 22	0 13	0 4	0 4	0 13	0 21	0 30	0 38	0 46	0 54	1 1
3 20	1 6	0 57	0 47	0 38	0 29	0 21	0 13	0 4	0 4	0 12	0 20	0 28	0 35	0 43	0 50	0 57
3 40	1 2	0 53	0 44	0 36	0 28	0 20	0 12	0 4	0 4	0 11	0 18	0 26	0 33	0 40	0 47	0 53
4 0	0 58	0 49	0 41	0 33	0 26	0 18	0 11	0 4	0 4	0 10	0 17	0 24	0 31	0 37	0 44	0 50
4 30	0 53	0 45	0 38	0 31	0 24	0 17	0 10	0 3	0 3	0 9	0 16	0 22	0 28	0 34	0 40	0 45
5 0	0 48	0 41	0 35	0 28	0 22	0 16	0 9	0 3	0 3	0 9	0 14	0 20	0 26	0 31	0 36	0 40
5 30	0 45	0 38	0 32	0 26	0 20	0 14	0 9	0 3	0 3	0 8	0 13	0 19	0 24	0 29	0 34	0 38
6 0	0 41	0 35	0 30	0 24	0 19	0 13	0 8	0 3	0 2	0 7	0 12	0 17	0 22	0 26	0 31	0 35
6 30	0 38	0 33	0 28	0 22	0 17	0 12	0 7	0 2	0 2	0 7	0 11	0 15	0 20	0 24	0 29	0 33
7 0	0 36	0 31	0 26	0 21	0 17	0 11	0 7	0 2	0 2	0 6	0 10	0 14	0 19	0 23	0 27	0 31
8 0	0 32	0 27	0 23	0 19	0 15	0 10	0 6	0 2	0 2	0 5	0 9	0 13	0 16	0 20	0 24	0 27
9 0	0 28	0 24	0 20	0 16	0 13	0 9	0 5	0 2	0 2	0 5	0 8	0 11	0 14	0 18	0 21	0 24
10 0	0 26	0 22	0 18	0 15	0 12	0 8	0 5	0 2	0 1	0 4	0 7	0 10	0 13	0 16	0 19	0 22
11 0	0 23	0 20	0 17	0 14	0 11	0 8	0 5	0 2	0 1	0 4	0 7	0 9	0 12	0 15	0 18	0 20
12 0	0 21	0 18	0 15	0 13	0 10	0 7	0 4	0 1	0 1	0 4	0 6	0 9	0 11	0 13	0 16	0 18
13 0	0 20	0 17	0 15	0 12	0 9	0 7	0 4	0 1	0 1	0 3	0 6	0 8	0 10	0 12	0 15	0 17
14 0	0 18	0 16	0 13	0 11	0 8	0 6	0 4	0 1	0 1	0 3	0 5	0 7	0 9	0 11	0 14	0 16
15 0	0 17	0 15	0 12	0 10	0 8	0 6	0 3	0 1	0 1	0 3	0 5	0 7	0 9	0 11	0 13	0 15
16 0	0 16	0 14	0 12	0 9	0 7	0 5	0 3	0 1	0 1	0 3	0 5	0 6	0 8	0 10	0 12	0 14
17 0	0 15	0 13	0 11	0 9	0 7	0 5	0 3	0 1	0 1	0 3	0 4	0 6	0 8	0 9	0 11	0 13
18 0	0 14	0 12	0 10	0 8	0 6	0 5	0 3	0 1	0 1	0 2	0 4	0 6	0 7	0 9	0 10	0 12
19 0	0 13	0 11	0 10	0 8	0 6	0 4	0 3	0 1	0 1	0 2	0 4	0 5	0 7	0 8	0 10	0 11
20 0	0 13	0 11	0 9	0 7	0 6	0 4	0 2	0 1	0 1	0 2	0 4	0 5	0 6	0 8	0 9	0 11
21 0	0 12	0 10	0 9	0 7	0 5	0 4	0 2	0 1	0 1	0 2	0 3	0 5	0 6	0 7	0 9	0 10
22 0	0 11	0 10	0 8	0 7	0 5	0 4	0 2	0 1	0 1	0 2	0 3	0 5	0 6	0 7	0 8	0 10
23 0	0 11	0 9	0 8	0 6	0 5	0 4	0 2	0 1	0 1	0 2	0 3	0 4	0 6	0 7	0 8	0 9
24 0	0 10	0 9	0 7	0 6	0 5	0 3	0 2	0 1	0 1	0 2	0 3	0 4	0 5	0 6	0 8	0 9
25 0	0 10	0 8	0 7	0 6	0 5	0 3	0 2	0 1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8
26 0	0 9	0 8	0 7	0 6	0 4	0 3	0 2	0 1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8
27 0	0 9	0 8	0 6	0 5	0 4	0 3	0 2	0 1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8
28 0	0 9	0 7	0 6	0 5	0 4	0 3	0 2	0 1	0 0	0 1	0 2	0 3	0 5	0 5	0 6	0 7
30 0	0 8	0 7	0 6	0 5	0 4	0 3	0 2	0 1	0 0	0 1	0 2	0 3	0 4	0 5	0 6	0 7
32 0	0 7	0 6	0 5	0 4	0 3	0 2	0 1	0 0	0 0	0 1	0 2	0 3	0 4	0 5	0 5	0 6
34 0	0 7	0 6	0 5	0 4	0 3	0 2	0 1	0 0	0 0	0 1	0 2	0 3	0 3	0 4	0 5	0 6
36 0	0 6	0 5	0 5	0 4	0 3	0 2	0 1	0 0	0 0	0 1	0 2	0 3	0 3	0 4	0 5	0 5
38 0	0 6	0 5	0 4	0 4	0 3	0 2	0 1	0 0	0 0	0 1	0 2	0 2	0 3	0 4	0 4	0 5
40 0	0 6	0 5	0 4	0 3	0 3	0 2	0 1	0 0	0 0	0 1	0 2	0 2	0 3	0 3	0 4	0 5
45 0	0 5	0 4	0 3	0 3	0 2	0 2	0 1	0 0	0 0	0 1	0 1	0 2	0 2	0 3	0 3	0 4
50 0	0 4	0 3	0 3	0 2	0 2	0 1	0 1	0 0	0 0	0 1	0 1	0 2	0 2	0 2	0 3	0 3
55 0	0 3	0 3	0 2	0 2	0 2	0 1	0 1	0 0	0 0	0 1	0 1	0 1	0 2	0 2	0 2	0 3
60 0	0 3	0 2	0 2	0 2	0 1	0 1	0 1	0 0	0 0	0 0	0 1	0 1	0 1	0 2	0 2	0 2
65 0	0 2	0 2	0 2	0 1	0 1	0 1	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 2	0 2
70 0	0 2	0 1	0 1	0 1	0 1	0 1	0 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 1
80 0	0 1	0 1	0 1	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1
90 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
				−	−	−	−	−	+	+	+	+				
				28.26	28.56	28.85	29.15	29.45	29.75	30.05	30.35	30.64	30.93			

ARGUMENT.—Height of the Barometer.

TABLE XVIII.

Radius Vector in Natural Numbers for 1810.

Note.—All the Radii have been diminished by 0.000100, that the Perturbations may be additive.

ARGUMENT. Sun's Mean Anomaly.

	Sig. O.			Sig. I			Sig. II.			
Deg.	Rad. Vector.	Diff.	Sec. Var +	Rad. Vector.	Diff.	Sec. Var +	Rad. Vector.	Diff.	Sec. Var ±	Deg.
0	0.983109		41.7	0.985431		35.7	0.991719		19.8	30
1	0.983112	003	41.6	0.985584	153	35.3	0.991978	259	19.1	29
2	0.983120	008	41.6	0.985742	158	34.9	0.992240	262	18.4	28
3	0.983133	013	41.6	0.985904	162	34.5	0.992504	264	17.8	27
4	0.983152	019	41.5	0.986070	166	34.1	0.992770	266	17.1	26
5	0.983176	024	41.5	0.986241	171	33.6	0.993038	268	16.4	25
6	0.983205	029	41.4	0.986416	175	33.2	0.993308	270	15.8	24
7	0.983239	034	41.3	0.986595	179	32.7	0.993581	273	15.1	23
8	0.983279	040	41.2	0.986778	183	32.3	0.993855	274	14.4	22
9	0.983323	044	41.1	0.986965	187	31.8	0.994131	276	13.7	21
10	0.983373	050	41.0	0.987157	192	31.3	0.994408	278	13.0	20
11	0.983429	056	40.8	0.987352	195	30.8	0.994688	280	12.3	19
12	0.983489	060	40.7	0.987551	199	30.3	0.994968	281	11.6	18
13	0.983555	066	40.5	0.987754	203	29.8	0.995251	282	10.9	17
14	0.983625	070	40.3	0.987961	207	29.3	0.995534	283	10.2	16
15	0.983701	076	40.1	0.988171	210	28.7	0.995819	285	09.5	15
16	0.983782	081	39.9	0.988385	214	28.2	0.996105	286	08.7	14
17	0.983868	086	39.7	0.988602	217	27.6	0.996392	287	08.0	13
18	0.983959	091	39.5	0.988823	221	27.1	0.996680	288	07.3	12
19	0.984055	096	39.2	0.989048	225	26.5	0.996969	289	06.6	11
20	0.984156	101	39.0	0.989275	227	25.9	0.997259	290	05.9	10
21	0.984262	106	38.7	0.989506	231	25.3	0.997550	291	05.1	9
22	0.984373	111	38.4	0.989741	235	24.7	0.997841	291	04.4	8
23	0.984488	115	38.1	0.989978	237	24.1	0.998132	292	03.7	7
24	0.984609	121	37.8	0.990218	240	23.5	0.998425	293	03.0	6
25	0.984734	125	37.5	0.990461	243	22.9	0.998717	293	02.2	5
26	0.984864	130	37.2	0.990707	246	22.3	0.999010	293	01.5	4
27	0.984999	135	36.8	0.990956	249	21.7	0.999303	293	00.8	3
28	0.985138	139	36.5	0.991208	252	21.0	0.999596	293	00.1	2
29	0.985283	145	36.1	0.991462	254	20.4	0.999889	293	00.7	1
30	0.985431	148	35.7	0.991719	257	19.8	1.000182	293	01.4	0
	Sig. XI.			Sig. X.			Sig. IX.			d

Note.—The Secular Variation, as inserted in this Table, is for 100 years, and if the Decimal Point be removed two places to the left hand, we have the Secular Variation for 1 year; but if we reject the Decimal Point altogether, then the three figures give the Secular Variation for 1000 years. For time before 1810, the Sign of Variation must be changed.

Continuation of TABLE XVIII.

ARGUMENT. Sun's Mean Anomaly.

	Sig. III.			Sig. IV.			Sig. V.			
Deg.	Rad. Vector.	Diff.	Sec. Var —	Rad. Vector.	Diff.	Sec. Var —	Rad. Vector.	Diff.	Sec. Var —	Deg.
0	1.000182		01.4	1.008504		21.8	1.014510		36.4	30
		293			248			140		
1	1.000475		02.1	1.008752		22.5	1.014650		36.7	29
		292			245			136		
2	1.000767		02.9	1.008998		23.0	1.014786		37.1	28
		293			242			131		
3	1.001060		03.6	1.009240		23.6	1.014917		37.4	27
		291			240			127		
4	1.001351		04.3	1.009480		24.2	1.015044		37.7	26
		292			237			123		
5	1.001643		05.0	1.009717		24.8	1.015167		38.0	25
		290			234			118		
6	1.001933		05.7	1.009951		25.4	1.015285		38.3	24
		290			231			113		
7	1.002223		06.4	1.010182		25.9	1.015398		38.5	23
		289			228			109		
8	1.002512		07.2	1.010410		26.5	1.015507		38.8	22
		289			224			104		
9	1.002801		07.9	1.010634		27.0	1.015611		39.1	21
		287			222			099		
10	1.003088		08.6	1.010856		27.6	1.015710		39.3	20
		286			218			095		
11	1.003374		09.3	1.011074		28.1	1.015805		39.5	19
		285			214			090		
12	1.003659		10.0	1.011288		28.6	1.015895		39.7	18
		284			211			085		
13	1.003943		10.7	1.011499		29.1	1.015980		39.9	17
		283			208			081		
14	1.004226		11.4	1.011707		29.6	1.016061		40.1	16
		281			204			076		
15	1.004507		12.1	1.011911		30.1	1.016137		40.3	15
		280			201			071		
16	1.004787		12.8	1.012112		30.6	1.016208		40.5	14
		278			197			066		
17	1.005065		13.4	1.012309		31.1	1.016274		40.6	13
		277			193			062		
18	1.005342		14.1	1.012502		31.6	1.016336		40.8	12
		275			189			056		
19	1.005617		14.8	1.012691		32.0	1.016392		40.9	11
		273			186			052		
20	1.005890		15.4	1.012877		32.5	1.016444		41.1	10
		271			181			047		
21	1.006161		16.1	1.013058		32.9	1.016491		41.2	9
		269			178			042		
22	1.006430		16.8	1.013236		33.3	1.016533		41.3	8
		267			174			037		
23	1.006697		17.4	1.013410		33.7	1.016570		41.4	7
		265			169			032		
24	1.006962		18.1	1.013579		34.2	1.016602		41.4	6
		263			166			027		
25	1.007225		18.7	1.013745		34.6	1.016629		41.5	5
		261			162			022		
26	1.007486		19.4	1.013906		34.9	1.016651		41.6	4
		258			158			017		
27	1.007744		20.0	1.014064		35.3	1.016668		41.6	3
		256			153			013		
28	1.008000		20.6	1.014217		35.7	1.016681		41.6	2
		253			149			007		
29	1.008253		21.2	1.014366		36.1	1.016688		41.6	1
		251			144			003		
30	1.008504		21.8	1.014510		36.4	1.016691		41.6	0
	Sig. VIII.			Sig. VII.			Sig. VI.			

TABLE XIX.

Perturbations of the Radius Vector produced by the Moon and Planets.

By the *Moon*.

1*st*. Part.

ARGUMENT. A.

0	1000	74
50	950	73
100	900	67
150	850	59
200	800	49
250	750	38
300	700	27
350	650	17
400	600	9
450	550	3
500	500	2

2d Part.

ARGUMENTS. M. and A.

M.

A	0 / 1000	250 / 750	500
0 / 1000	0	3	6
250 / 750	3	3	3
500	6	3	1

By *Saturn*.

ARGUMENTS. B and F.

F.

B	0	200	400
0	2	2	1
200	3	2	2
400	1	2	2
600	1	1	2
800	2	1	1
1000	2	2	1

B	600	800	1000
0	1	2	2
200	1	1	3
400	2	1	1
600	2	2	1
800	2	3	2
1000	1	2	2

By *Venus*. ARGUMENTS. B and C.

C.

B	0	100	200	300	400	500	600	700	800	900	1000
0	43	30	7	11	39	49	36	20	9	18	43
100	20	39	30	11	11	37	50	35	18	11	20
200	13	23	38	26	13	13	35	51	35	16	13
300	13	12	27	38	23	13	16	35	51	36	13
400	37	11	10	29	40	20	11	19	35	50	37
500	49	39	11	7	30	43	15	8	20	36	49
600	38	48	40	13	4	29	46	19	5	20	38
700	18	40	47	40	16	3	26	48	22	3	18
800	3	15	40	47	39	17	3	22	48	26	3
900	29	4	13	39	48	38	20	5	19	46	29
1000	43	30	7	11	39	49	36	20	8	18	43

By *Mars*. ARGUMENTS. B and D.

D.

B	0	100	200	300	400	500	600	700	800	900	1000
0	20	17	12	7	15	23	14	9	12	16	20
100	15	21	17	10	8	18	21	13	12	10	15
200	7	17	22	16	10	9	18	18	16	12	7
300	9	7	20	20	15	11	10	16	18	19	9
400	19	6	10	21	17	17	12	8	15	20	19
500	22	16	6	13	17	16	20	10	7	16	22
600	18	20	14	9	13	14	19	21	7	9	18
700	12	17	19	16	11	11	14	23	18	7	12
800	9	12	14	20	17	9	9	16	23	15	9
900	15	12	10	13	22	16	8	12	18	20	15
1000	20	17	12	7	15	23	14	9	12	16	20

By *Jupiter*. ARGUMENTS. B and E.

E.

B	0	100	200	300	400	500	600	700	800	900	1000
0	39	36	29	18	13	12	9	16	33	42	39
100	41	38	36	29	19	14	10	8	18	36	41
200	37	39	36	36	30	21	14	7	7	22	37
300	24	36	36	36	37	31	22	13	5	9	24
400	11	25	32	34	37	39	32	22	11	4	11
500	5	14	24	29	33	39	40	33	22	10	5
600	9	8	16	22	26	34	41	40	33	21	9
700	20	9	10	16	19	25	37	42	39	32	20
800	31	19	11	12	15	16	27	39	42	38	31
900	37	30	18	12	13	12	15	30	42	41	37
1000	39	36	29	18	13	12	9	16	33	42	39

Note.—The perturbations produced by the Moon and planets, taken from this Table, are all to be added to the Radius Vector in Natural Numbers, as in Table XVIII, and then we have the Radius Vector corrected; of which we are to take its Logarithm from any Table of Common Logarithms; making the Index 9, if less than 1,000000; but 10 if more than 1,000000.

TABLE XX.

Equation of Time to convert *Apparent* into *Mean* Time, for 1810, with the Secular Variation.

ARGUMENT. Sun's Mean Longitude.

Deg.	Sig. O.	Diff.	Secular Variation. ∓		Sig. I.	Diff.	Secular Variation. +		Sig. II.	Diff.	Secular Variation. +	
	′ ″	″	″		′ ″	″	″		′ ″	″	″	
0	+ 6 59.2	18.8	2.97	25	− 1 31.2	12.5	4.31	24	− 3 43.4	4.5	10.83	18
1	6 40.4	18.9	2.72	24	1 43.7	12.0	4.55	24	3 38.9	5.0	11.01	17
2	6 21.5	18.9	2.48	24	1 55.7	11.4	4.79	23	3 33.9	5.5	11.18	18
3	6 2.6	18.8	2.24	24	2 7.1	10.8	5.02	24	3 28.4	6.1	11.36	17
4	5 43.8	18.9	2.00	25	2 17.9	10.4	5.26	23	3 22.3	6.6	11.53	16
5	5 24.9	19.0	1.75	24	2 28.3	9.8	5.49	23	3 15.7	6.9	11.69	16
6	5 5.9	18.8	1.51	24	2 38.1	9.3	5.73	23	3 8.8	7.7	11.85	16
7	4 47.1	18.8	1.27	25	2 47.4	8.8	5.96	23	3 1.1	8.0	12.01	15
8	4 28.3	18.7	1.02	24	2 56.2	8.3	6.19	23	2 53.1	8.4	12.16	15
9	4 9.6	18.6	0.78	24	3 4.5	7.6	6.42	23	2 44.7	8.8	12.31	14
10	3 51.0	18.4	0.54	25	3 12.1	7.1	6.65	23	2 35.9	9.2	12.45	14
11	3 32.6	18.5	0.29	24	3 19.2	6.6	6.88	23	2 26.7	9.8	12.59	14
12	3 14.1	18.3	0.05	25	3 25.8	5.9	7.11	22	2 16.9	10.0	12.73	13
13	2 55.8	18.0	0.20	24	3 31.7	5.4	7.33	23	2 6.9	10.3	12.86	13
14	2 37.8	17.9	0.44	24	3 37.1	4.9	7.56	22	1 59.6	10.8	12.99	12
15	2 19.9	17.6	0.68	25	3 42.0	4.2	7.78	22	1 45.8	11.2	13.11	12
16	2 2.3	17.3	0.93	24	3 46.2	3.4	8.00	22	1 34.6	11.4	13.23	11
17	1 45.0	17.1	1.17	24	3 49.6	3.0	8.22	21	1 23.2	11.6	13.34	11
18	1 27.9	16.9	1.41	25	3 52.6	2.5	8.43	22	1 11.6	11.9	13.45	10
19	1 11.0	16.7	1.66	24	3 55.1	1.9	8.65	20	0 59.7	12.1	13.55	9
20	0 54.3	16.4	1.90	24	3 57.0	1.3	8.85	21	0 47.6	12.3	13.64	9
21	0 37.9	16.0	2.14	25	3 58.3	0.6	9.06	21	0 35.3	12.5	13.73	9
22	0 21.9	15.5	2.39	24	3 58.9	0.1	9.27	21	0 22.8	12.7	13.82	8
23	+ 0 6.4	15.1	2.63	24	3 59.0	0.5	9.48	20	− 0 10.1	12.8	13.90	8
24	− 0 8.7	14.7	2.87	24	3 58.5	1.1	9.68	20	+ 0 2.7	12.9	13.98	7
25	0 23.4	14.3	3.11	24	3 57.4	1.7	9.88	19	0 15.6	13.1	14.05	6
26	0 37.7	13.9	3.35	24	3 55.7	2.2	10.07	20	0 28.7	13.1	14.11	6
27	0 51.6	13.5	3.59	24	3 53.5	2.8	10.27	20	0 41.8	13.2	14.17	5
28	1 5.1	13.2	3.83	24	3 50.7	3.4	10.47	18	0 55.0	13.1	14.22	5
29	1 18.3	12.9	4.07	24	3 47.3	3.9	10.65	18	1 8.1	13.0	14.27	4
30	− 1 31.2		4.31		− 3 43.4		10.83		+ 1 21.1		14.31	

Continuation of TABLE XX.

ARGUMENT. Sun's Mean Longitude.

Deg.	Sig. III.	Diff.	Secular Variation. +		Sig. IV.	Diff.	Secular Variation. +		Sig. V.	Diff.	Secular Variation. +	
	′ ″	″	″		′ ″	″	″		′ ″	″	″	
0	+ 1 21.1	13.0	14.31	4	+ 5 59.3	2.1	13.14	11	+ 2 46.2	15.0	8.78	17
1	1 34.1	13.1	14.35	3	6 1.4	1.6	13.03	12	2 31.2	15.5	8.61	17
2	1 47.2	13.1	14.38	2	6 3.0	1.2	12.91	12	2 15.7	15.9	8.44	17
3	2 0.3	13.0	14.40	2	6 4.2	0.6	12.79	12	1 59.8	16.4	8.27	17
4	2 13.3	12.8	14.42	2	6 4.8	0.1	12.67	12	1 43.4	16.8	8.10	17
5	2 26.1	12.6	14.44	1	6 4.7	0.7	12.55	13	1 26.6	17.2	7.93	17
6	2 38.7	12.4	14.45	0	6 4.0	1.4	12.42	13	1 9.4	17.6	7.76	17
7	2 51.1	12.2	14.45	0	6 2.6	2.0	12.29	13	0 51.8	18.0	7.59	17
8	3 3.3	12.0	14.45	1	6 0.6	2.5	12.16	13	0 33.8	18.4	7.42	17
9	3 15.3	11.7	14.44	1	5 58.1	3.3	12.03	14	+ 0 15.4	18.6	7.25	17
10	3 27.0	11.4	14.43	2	5 54.8	3.8	11.89	14	— 0 3.2	18.9	7.08	17
11	3 38.4	11.1	14.41	2	5 51.0	4.4	11.75	14	0 22.1	19.2	6.91	17
12	3 49.5	10.8	14.39	3	5 46.6	5.0	11.61	14	0 41.3	19.5	6.74	18
13	4 0.3	10.5	14.36	4	5 41.6	5.7	11.47	15	1 0.8	19.8	6.56	17
14	4 10.8	10.1	14.32	4	5 35.9	6.4	11.32	15	1 20.6	20.0	6.39	18
15	4 20.9	9.7	14.28	4	5 29.5	6.9	11.17	15	1 40.6	20.2	6.21	17
16	4 30.6	9.4	14.24	5	5 22.6	7.5	11.02	15	2 0.8	20.5	6.04	17
17	4 40 0	9.0	14.19	6	5 15.1	8.1	10.87	15	2 21.3	20.6	5.87	18
18	4 49.0	8.5	14.13	6	5 7.0	8.7	10.72	15	2 41.9	20.8	5.69	17
19	4 57.5	8.1	14.07	6	4 58.3	9.3	10.57	16	3 2.7	20.9	5.52	18
20	5 5.6	7.6	14.01	7	4 49.0	9.8	60.41	16	3 23.6	21.1	5.34	18
21	5 13.2	7.4	13.94	7	4 39.2	10.4	10.25	16	3 44.7	21.1	5.16	18
22	5 20.6	6.9	13.87	8	4 28.8	10.9	10.09	16	4 5.8	21.2	4.98	17
23	5 27.5	6.2	13.79	8	4 17.9	11.5	9.93	16	4 27.0	21.3	4.81	18
24	5 33.7	5.7	13.71	9	4 6.4	12.1	9.77	17	4 48.3	21.4	4.63	18
25	5 39.4	5.1	13.62	9	3 54.3	12.7	9.60	16	5 9.7	21.3	4.45	18
26	5 44.5	4.5	13.53	9	3 41.6	13.0	9.44	16	5 31.0	21.3	4.27	18
27	5 49.0	3.9	13.44	10	3 28.6	13.6	9.28	16	5 52.3	21.4	4.09	18
28	5 52.9	3.4	13.34	10	3 15.0	14.2	9.12	17	6 13.7	21.3	3.91	18
29	5 56.3	3.0	13.24	10	3 0.8	14.6	8.95	17	6 35.0	21.3	3.73	19
30	+ 5 59.3		13.14		+ 2 46.2		8.78		— 6 36.3		3.54	

Continuation of TABLE XX.

ARGUMENT. Sun's Mean Longitude.

Deg.	Sig. VI.	Diff.	Secular Variation. ±		Sig. VII.	Diff.	Secular Variation. —		Sig. VIII.	Diff.	Secular Variation. —	
0	— 6′ 56.3″	21.2	3.54″	18	— 15′ 17.8″	8.8	2.56″	23	— 13′ 54.9″	16.0	9.64″	24
1	7 17.5	21.0	3.36	19	15 26.6	8.1	2.79	23	13 38.9	16.8	9.88	23
2	7 38.5	20.8	3.17	18	15 34.7	7.4	3.02	23	13 22.1	17.5	10.11	22
3	7 59.3	20.7	2.99	19	15 42.1	6.7	3.25	23	13 4.6	18.2	10.33	22
4	8 20.0	20.6	2.80	19	15 48.8	6.0	3.48	23	12 46.4	19.1	10.55	22
5	8 40.6	20.4	2.61	19	15 54.8	5.2	3.71	23	12 27.3	19.9	10.77	21
6	9 1.0	20.1	2.42	19	16 0.0	4.4	3.94	24	12 7.4	20.6	10.98	21
7	9 21.1	19.9	2.23	19	16 4.4	3.7	4.18	23	11 46.8	21.3	11.19	21
8	9 41.0	19.7	2.04	19	16 8.1	2.9	4.41	24	11 25.5	22.0	11.40	21
9	10 0.7	19.4	1.85	19	16 11.0	2.0	4.65	24	11 3.5	22.7	11.61	20
10	10 20.1	19.2	1.66	20	16 13.0	1.2	4.89	24	10 40.8	23.4	11.81	20
11	10 39.3	18.8	1.46	20	16 14.2	0.5	5.13	24	10 17.4	24.0	12.01	19
12	10 58.1	18.4	1.26	20	16 14.7	0.5	5.37	24	9 53.4	24.5	12.20	19
13	11 16.5	18.0	1.06	20	16 14.2	1.2	5.61	24	9 28.9	25.2	12.39	19
14	11 34.5	17.7	0.86	20	16 12.9	2.1	5.85	24	9 3.7	25.8	12.58	19
15	11 52.2	17.3	0.66	20	16 10.8	3.0	6.09	24	8 37.9	26.4	12.77	18
16	12 9.5	16.8	0.46	21	16 7.8	3.8	6.33	24	8 11.5	26.9	12.95	17
17	12 26.3	16.4	0.25	21	16 4.0	4.7	6.57	24	7 44.6	27.3	13.12	17
18	12 42.7	16.0	0.04	20	15 59.3	5.6	6.81	24	7 17.3	27.6	13.29	16
19	12 58.7	15.5	0.16	21	15 53.7	6.4	7.05	24	6 49.7	28.1	13.45	16
20	13 14.2	14.9	0.37	21	15 47.3	7.3	7.29	24	6 21.6	28.5	13.61	15
21	13 29.1	14.4	0.58	22	15 40.0	8.3	7.53	24	5 53.1	28.8	13.76	15
22	13 43.5	13.9	0.80	21	15 31.7	9.1	7.77	24	5 24.3	29.2	13.91	14
23	13 57.4	13.4	1.01	22	15 22.6	10.0	8.01	24	4 55.1	29.5	14.05	14
24	14 10.8	12.8	1.23	22	15 12.6	10.9	8.25	24	4 25.6	29.7	14.19	13
25	14 23.6	12.1	1.45	22	15 1.7	11.7	8.49	23	3 55.9	30.0	14.32	12
26	14 35.7	11.5	1.67	22	14 50.0	12.5	8.72	24	3 25.9	30.1	14.44	12
27	14 47.2	10.8	1.89	22	14 37.5	13.4	8.96	23	2 55.8	30.3	14.56	11
28	14 58.0	10.2	2.11	22	14 24.1	14.2	9.19	23	2 25.5	30.5	14.67	11
29	15 8.2	9.6	2.33	23	14 9.9	15.0	9.42	22	1 55.0	30.3	14.78	10
30	— 15 17.8		2.56		— 13 54.9		9.64		— 1 24.7		14.88	

Continuation of TABLE XX.

ARGUMENT. Sun's Mean Longitude.

Deg.	Sig. IX.	Diff.	Secular Variation. —		Sig. X.	Diff.	Secular Variation. —		Sig. XI.	Diff.	Secular Variation. —	
0	— 1′ 24.7″	30.5″	14.88″	10	+ 11′ 36.3″	17.0″	14.89″	10	+ 14′ 6.9″	7.0″	9.96″	22
1	0 54.2	30.5	14.98	9	11 53.3	16.2	14.79	11	13 59.9	7.8	9.74	22
2	— 0 23.7	30.5	15.07	8	12 9.5	15.3	14.68	11	13 52.1	8.5	9.52	22
3	+ 0 6.8	30.4	15.15	8	12 24.8	14.5	14.57	11	13 43.6	9.1	9.30	22
4	0 37.2	30.3	15.23	7	12 39.3	13.8	14.46	12	13 34.5	9.6	9.08	22
5	1 7.5	30.1	15.30	6	12 53.1	13.0	14.34	13	13 24.9	10.2	8.86	23
6	1 37.6	29.9	15.36	5	13 6.1	12.1	14.21	13	13 14.7	10.7	8.63	22
7	2 7.5	29.7	15.41	5	13 18.2	11.2	14.08	14	13 4.0	11.6	8.41	23
8	2 37.2	29.5	15.46	5	13 29.4	10.4	13.94	14	12 52.4	12.0	8.18	22
9	3 6.7	29.3	15.51	4	13 39.8	9.5	13.80	15	12 40.4	12.5	7.96	23
10	3 36.0	28.9	15.55	3	13 49.3	8.7	13.65	15	12 27.9	13.0	7.73	24
11	4 4.9	28.6	15.58	2	13 58.0	7.9	13.50	15	12 14.9	13.4	7.49	23
12	4 33.5	28.3	15.60	1	14 5.9	7.0	13.35	16	12 1.5	14.0	7.26	23
13	5 1.8	27.8	15.61	1	14 12.9	6.1	13.19	16	11 47.5	14.4	7.03	24
14	5 29.6	27.3	15.62	0	14 19.0	5.2	13.03	17	11 33.1	14.8	6.79	24
15	5 56.9	26.8	15.62	0	14 24.2	4.4	12.86	17	11 18.3	15.1	6.55	23
16	6 23.7	26.3	15.62	1	14 28.6	3.7	12.69	17	11 3.2	15.6	6.32	23
17	6 50.0	25.9	15.61	2	14 32.3	2.8	12.52	18	10 47.6	16.0	6.09	24
18	7 15.9	25.3	15.59	2	14 35.1	2.0	12.34	18	10 31.6	16.3	5.85	24
19	7 41.2	24.6	15.57	3	14 37.1	1.1	12.16	19	10 15.3	16.7	5.61	24
20	8 5.8	24.1	15.54	4	14 38.2	0.3	11.97	19	9 58.6	17.0	5.37	24
21	8 29.9	23.4	15.50	5	14 38.5	0.6	11.78	19	9 41.6	17.2	5.13	23
22	8 53.3	22.8	15.45	4	14 37.9	1.3	11.59	19	9 24.4	17.4	4.90	24
23	9 16.1	22.1	15.41	6	14 36.6	2.0	11.40	20	9 7.0	17.7	4.66	24
24	9 38.2	21.5	15.35	6	14 34.6	2.7	11.20	20	8 49.3	17.9	4.42	24
25	9 59.7	20.9	15.29	7	14 31.9	3.6	11.00	21	8 31.4	18.1	4.18	24
26	10 20.6	20.2	15.22	8	14 28.3	4.4	10.79	20	8 13.3	18.3	3.94	24
27	10 40.8	19.3	15.14	8	14 23.9	5.0	10.59	21	7 55.0	18.5	3.70	24
28	11 0.1	18.5	15.06	8	14 18.9	5.6	10.38	21	7 36.5	18.6	3.45	25
29	11 18.6	17.7	14.98	9	14 13.3	6.4	10.17	21	7 17.9	18.7	3.21	24
30	+ 11 36.3		14.89		+ 14 6.9		9.96		+ 6 59.2		2.97	24

TABLE XXI.

Equation of Time. Perturbations.

C

B	0	100	200	300	400	500	600	700	800	900	1000
	″	″	″	″	″	″	″	″	″	″	″
0	1.4	0.8	1.0	1.7	1.7	1.2	0.7	0.4	0.6	1.4	1.4
100	1.2	1.4	1.1	1.0	1.6	1.8	1.1	0.7	0.6	0.7	1.2
200	0.9	1.0	1.2	1.2	1.2	1.5	1.7	1.1	0.5	0.7	0.9
300	0.7	1.1	1.1	0.9	1.2	1.4	1.5	1.6	1.2	0.5	0.7
400	0.5	0.6	1.2	1.2	0.8	1.0	1.6	1.7	1.5	1.2	0.5
500	1.0	0.5	0.6	1.2	1.4	0.8	0.8	1.5	1.9	1.5	1.0
600	1.7	1.0	0.4	0.5	1.2	1.4	0.9	0.6	1.3	2.0	1.7
700	1.9	1.8	1.1	0.4	0.4	1.1	1.6	1.1	0.7	1.2	1.9
800	1.2	1.8	1.8	1.2	0.4	0.3	1.0	1.6	1.2	0.7	1.2
900	0.7	1.1	1.7	1.8	1.2	0.6	0.2	0.8	1.6	1.3	0.7
1000	1.4	0.8	1.0	1.7	1.7	1.2	0.7	0.4	0.6	1.4	1.4

D

B											
	″	″	″	″	″	″	″	″	″	″	″
0	0.6	0.7	0.5	0.3	0.2	0.6	0.7	0.5	0.2	0.1	0.6
100	0.2	0.7	0.6	0.5	0.2	0.3	0.6	0.9	0.5	0.2	0.2
200	0.2	0.4	0.6	0.5	0.4	0.3	0.4	0.6	0.5	0.5	0.2
300	0.4	0.2	0.5	0.5	0.5	0.4	0.4	0.4	0.5	0.5	0.4
400	0.5	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.4	0.4	0.5
500	0.4	0.5	0.5	0.5	0.4	0.4	0.3	0.4	0.5	0.3	0.4
600	0.3	0.3	0.5	0.6	0.4	0.4	0.3	0.5	0.7	0.4	0.3
700	0.4	0.2	0.3	0.6	0.6	0.4	0.2	0.2	0.7	0.7	0.4
800	0.6	0.3	0.2	0.3	0.7	0.6	0.3	0.2	0.3	0.8	0.6
900	0.8	0.5	0.3	0.1	0.4	0.7	0.5	0.3	0.1	0.5	0.8
1000	0.6	0.7	0.5	0.3	0.2	0.6	0.7	0.5	0.2	0.1	0.6

E

B											
	″	″	″	″	″	″	″	″	″	″	″
0	1.0	1.0	1.1	1.2	1.1	1.0	0.7	0.4	0.6	0.9	1.0
100	0.9	0.9	0.8	1.0	1.3	1.3	1.0	0.7	0.4	0.5	0.9
200	0.5	0.7	0.7	0.8	1.0	1.0	1.1	1.2	0.9	0.3	0.5
300	0.2	0.5	0.7	0.7	0.8	1.2	1.5	1.5	1.1	0.5	0.2
400	0.3	0.2	0.5	0.7	0.7	0.9	1.3	1.4	1.4	1.0	0.3
500	0.8	0.3	0.2	0.5	0.7	0.7	1.0	1.4	1.4	1.4	0.8
600	1.3	0.7	0.3	0.3	0.5	0.7	0.9	1.1	1.4	1.6	1.3
700	1.5	1.1	0.7	0.3	0.4	0.5	0.8	1.0	1.2	1.4	1.5
800	1.3	1.3	1.0	0.7	0.4	0.4	0.6	0.8	1.0	1.2	1.3
900	1.1	1.2	1.2	1.0	0.8	0.6	0.5	0.6	0.9	1.1	1.1
1000	1.0	1.0	1.1	1.2	1.1	1.0	0.7	0.4	0.6	0.9	1.0

F

B											
	″	″	″	″	″	″	″	″	″	″	″
0	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1
200	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.1
400	0.1	0.1	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1
600	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
800	0.1	0.1	0.1	0.1	0.0	0.1	0.0	0.0	0.1	0.1	0.1
1000	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1

Moon and Nutation.

A	″ 0.5	′ 0.8	″ 1.0	″ 1.0	″ 0.8	″ 0.5	″ 0.2	″ 0.0	″ 0.0	″ 0.2	″ 0.5
N	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1

Constant Quantity — 3″.1.

TABLES OF THE MOON.

TABLE I.

Motion for Past and Future Centuries.

Quantities to be added to the Epochs in Table II: to obtain the Epochs for other Centuries.

Years.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
— 2100	08869	8936	8201	0558	7491	4716	3280	9264	3466	329	403	071	240	127	510	581	166	967
— 900	05678	4000	9219	9350	4083	3143	3230	5218	7366	193	672	514	049	449	142	786	788	878
— 800	05413	1088	1803	0916	9632	2179	0726	0714	4358	599	527	217	200	309	945	637	089	038
— 700	05147	8177	4388	2482	5181	1214	8222	6210	1350	004	383	921	351	169	748	487	391	197
— 600	04881	5266	6973	4048	0731	0250	5718	1707	8341	410	239	624	503	029	550	338	693	356
— 500	04615	2354	9558	5614	6280	9286	3213	7203	5333	815	095	328	654	889	353	188	995	515
— 400	04349	9443	2143	7180	1829	8321	0709	2699	2325	220	950	031	805	749	156	038	297	674
— 300 I	04083	6532	4728	8746	7379	7357	8205	8195	9316	626	806	735	956	609	958	889	599	834
— 300 G	01345	0033	4326	5876	4023	3637	7628	4292	9074	924	495	030	615	618	653	522	178	574
— 200	01079	7122	6911	7442	9572	2673	5124	9788	6065	330	351	734	766	478	456	372	480	733
— 100	00540	3561	8455	8721	4786	1336	2562	4894	3033	665	175	367	883	239	228	186	740	867
+ 100	99460	6439	1545	1279	5214	8664	7438	5106	6967	335	825	633	117	761	772	814	260	133

TABLE I. Continued

Years.	19	20	21	22	23	24	25	26	27	28	29	30	31	Evection.	Anomaly.
— 2100	604	817	78	41	74	26	53	33	59	02	66	45	11	1s 21° 58′ 48″	10 1 35 56.3
— 900	416	925	53	38	87	58	94	11	87	58	35	06	93	11 27 1 8	5 17 24 59.9
— 800	817	600	93	13	13	44	89	25	98	13	91	61	50	1 22 26 20	0 6 14 5.2
— 700	218	276	32	87	39	30	84	40	09	67	47	16	06	3 17 51 31	6 25 3 10.5
— 600	619	952	72	62	65	16	79	55	19	22	03	71	63	5 13 16 43	1 13 52 15.8
— 500	020	627	11	37	91	02	74	70	30	77	58	27	20	7 8 41 55	8 2 41 21.1
— 400	421	303	51	12	17	88	69	85	41	31	14	82	76	9 4 7 06	2 21 30 26.4
— 300 I	822	979	90	86	43	74	64	00	52	86	70	37	33	10 29 32 18	9 10 19 31.7
— 300 G	801	974	83	78	42	47	23	63	82	44	39	35	30	7 6 22 24	4 29 40 32.0
— 200	202	650	22	52	68	33	18	78	93	99	95	90	87	9 1 47 35	1 1 18 29 37.3
— 100	601	325	61	26	84	17	09	89	96	49	47	45	43	10 15 53 48	5 24 14 48.7
+ 100	399	675	39	74	16	83	91	11	04	51	53	55	57	1 14 6 12	6 5 45 11.3

TABLE I. Continued.

Years.	Variation.	Longitude ☽	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
— 2100	5 26 47 22	5s 22° 36′ 16″8	2s 3° 3′ 55″6	3s 27° 56′	566	743	238	416	022	348	582	511
— 900	8 22 13 05	8 27 10 58.8	7 23 06 19.6	0 24 9	333	446	010	124	540	863	766	398
— 800	6 29 20 13	7 5 3 52.3	0 7 16 31.6	6 16 20	563	672	491	599	417	489	281	472
— 700	5 6 27 27	5 12 56 45.8	4 21 26 43.6	0 8 31	794	897	972	075	293	115	796	546
— 600	3 13 34 30	3 20 49 39.3	9 5 36 55.6	6 0 43	025	122	453	551	170	742	312	620
— 500	1 20 41 44	1 28 42 32.8	1 19 47 7.6	11 22 54	255	347	934	026	047	368	827	694
— 400	11 27 48 47	0 6 35 26.3	6 3 57 19 6	5 15 05	486	573	415	502	923	994	342	768
— 300 I	10 4 55 56	10 14 28 19.8	10 18 7 31.6	11 7 16	716	798	896	978	800	620	858	842
— 300 G	6 3 1 28	6 2 42 29.6	10 17 35 45.3	7 15 45	376	403	613	640	442	204	524	789
— 200	4 10 8 36	4 10 35 23.1	3 1 45 57.3	1 7 56	607	628	094	116	318	831	037	863
— 100	2 5 4 18	2 5 17 41.5	7 15 52 58.6	6 18 58	803	814	547	558	159	415	518	931
+ 100	9 24 55 42	9 24 42 18.5	4 14 7 1.4	5 11 2	197	186	453	442	841	585	482	069

Note.—I means Julian or Old Style, and G. Gregorian or New Style. The sign — denotes past centuries, + future ones.

TABLE II.

EPOCHS.

Years.	1	2	3	4	5	6	7	8	9	10	11	12	13
1801	00320	0948	6871	5055	5860	9907	4820	5924	0402	138	546	100	512
1802	00249	8157	6536	9799	8332	5705	5881	8382	9266	744	907	820	985
1803	00178	5365	6202	4543	0803	1502	6941	0839	8130	351	267	539	458
1804 B	00107	2574	5868	9287	3275	7299	8002	3296	6993	957	628	259	931
1805	00309	0432	6574	4318	6082	3469	9120	6144	5882	634	020	049	438
1806	00238	7640	6240	9062	8554	9266	0180	8602	4745	240	381	768	911
1807	00167	4849	5906	3806	1025	5063	1241	1059	3609	847	742	487	384
1808 B	00096	2057	5571	8550	3497	0861	2301	3516	2473	453	102	207	857
1809	00299	9915	6277	3580	6304	7030	3420	6364	1361	130	494	997	364
1810	00228	7124	5943	8324	8776	2827	4480	8821	0225	737	855	716	837
1811	00157	4332	5609	3068	1247	8625	5541	1279	9089	343	216	436	310
1812 B	00086	1541	5275	7812	3719	4422	6601	3736	7953	950	577	155	783
1813	00288	9399	5981	2843	6526	0592	7719	6584	6841	626	969	945	290
1814	00217	6607	5646	7587	8998	6389	8780	9041	5705	233	329	664	763
1815	00146	3816	5312	2331	1469	2186	9840	1499	4569	839	690	384	236
1816 B	00075	1024	4978	7075	3941	7984	0901	3956	3432	446	051	103	709
1817	00277	8883	5684	2106	6748	4153	2019	6804	2321	122	443	893	216
1818	00206	6091	5350	6850	9219	9950	3080	9261	1184	729	803	612	689
1819	00135	3299	5016	1594	1691	5748	4140	1719	0048	335	164	332	162
1820 B	00064	0508	4682	6337	4163	1545	5201	4176	8912	942	525	051	635
1821	00267	8366	5388	1368	6970	7714	6319	7024	7800	619	917	841	142
1822	00196	5574	5053	6112	9441	3512	7380	9481	6664	225	278	561	615
1823	00125	2783	4719	0856	1913	9309	8440	1938	5528	832	639	280	088
1824 B	00054	9991	4385	5600	4385	5106	9501	4396	4392	438	999	999	561
1825	00256	7850	5091	0631	7192	1276	0619	7243	3280	115	391	789	068
1826	00185	5058	4757	5375	9663	7073	1680	9701	2144	721	752	509	541
1827	00114	2266	4423	0119	2135	2871	2740	2158	1008	328	113	228	014
1828 B	00043	9475	4088	4863	4607	8668	3801	4616	9871	934	474	948	487
1829	00245	7333	4794	9894	7414	4837	4919	7463	8760	611	866	737	994
1830	00174	4542	4460	4638	9885	0635	5979	9921	7623	218	226	457	467
1831	00103	1750	4126	9381	2357	6432	7040	2378	6487	824	587	176	940
1832 B	00032	8958	3792	4125	4829	2229	8100	4835	5351	431	948	896	413
1833	00235	6817	4498	9156	7636	8399	9219	7683	4239	107	340	686	920
1834	00164	4025	4163	3900	0107	4196	0279	0140	3103	714	701	405	393
1835	00093	1233	3829	8644	2579	9993	1340	2598	1967	320	061	124	866
1836 B	00022	8442	3495	3388	5051	5791	2400	5055	0831	927	422	844	339
1837	00224	6300	4201	8419	7858	1960	3518	7903	9719	604	814	634	846
1838	00153	3509	3867	3163	0329	7757	4579	0360	8583	210	175	353	319
1839	00082	0717	3533	7907	2801	3555	5639	2818	7447	817	536	073	792
1840 B	00011	7926	3198	2651	5273	9352	6700	5275	6310	423	896	792	265
1841	00213	5784	3905	7682	8080	5522	7818	8123	5199	100	288	582	772
1842	00142	2992	3570	2425	0551	1319	8879	0580	4062	706	649	301	245
1843	00071	0201	3236	7169	3023	7116	9939	3038	2926	313	010	021	718
1844 B	00000	7409	2902	1913	5495	2914	1000	5495	1790	919	371	740	191
1845	00203	5267	3608	6944	8302	9083	2118	8343	0678	596	763	530	698
1846	00132	2476	3274	1688	0773	4880	3179	0800	9542	202	123	249	171
1847	00061	9684	2940	6432	3245	0678	4239	3257	8406	809	484	969	644
1848 B	99990	6893	2605	1176	5717	6475	5300	5715	7270	415	845	688	117
1849	00192	4751	3311	6207	8524	2644	6418	8563	6158	092	237	478	624
1850	00121	1959	2977	0951	0995	8442	7479	1020	5022	699	597	198	097

TABLE II. continued.

EPOCHS

Years.	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1801	606	107	552	071	502	077	701	08	48	68	42	67	52	17	59	59	75	97
1802	800	247	911	423	977	851	867	85	59	65	13	69	99	66	84	83	38	88
1803	994	387	270	776	452	624	034	63	69	62	84	71	46	16	08	08	00	80
1804 B	188	528	630	128	928	398	201	40	80	58	55	73	93	65	33	33	63	71
1805	481	699	026	523	429	173	368	18	91	65	26	79	44	21	62	61	25	63
1806	675	839	385	876	904	947	534	95	02	62	96	81	91	71	86	86	88	54
1807	869	980	744	228	379	720	701	72	12	59	67	83	39	19	11	10	50	46
1808 B	062	120	104	581	854	494	868	50	23	55	38	85	86	69	35	35	13	37
1809	355	291	500	975	355	269	035	27	34	62	09	91	37	26	64	63	75	29
1810	549	431	859	328	830	043	201	04	45	59	79	92	84	75	88	88	38	21
1811	743	572	218	680	305	816	368	82	55	56	50	94	31	24	13	13	00	12
1812 B	937	712	578	033	780	590	535	59	66	52	21	96	79	74	37	38	63	04
1813	230	883	974	427	281	365	702	37	77	59	92	02	29	30	66	65	26	95
1814	424	023	333	780	756	139	868	14	88	56	62	04	77	80	91	90	88	87
1815	617	164	692	132	232	912	035	91	98	53	33	06	24	29	15	15	51	78
1816 B	811	304	052	485	707	686	202	69	09	49	04	08	71	78	40	40	13	70
1817	104	475	448	879	208	461	369	46	20	56	75	14	22	35	68	68	76	62
1818	298	616	807	232	683	235	535	24	31	53	45	16	69	84	93	92	38	53
1819	492	756	166	584	158	008	702	01	41	50	16	18	17	33	17	17	01	45
1820 B	686	896	526	937	633	782	869	78	52	46	87	20	64	83	42	42	63	36
1821	979	067	922	331	134	557	036	56	63	54	58	26	15	39	70	70	26	28
1822	172	208	281	684	609	331	202	33	74	50	28	28	62	88	95	95	89	20
1823	366	348	640	036	084	104	369	11	84	47	99	30	09	38	19	19	51	11
1824 B	560	488	000	389	559	878	536	88	95	44	70	32	56	87	44	44	14	03
1825	853	659	396	783	060	653	703	65	06	51	41	38	07	44	73	72	76	94
1826	047	800	755	136	536	427	869	43	17	47	11	40	54	93	97	97	39	86
1827	241	940	114	488	011	200	036	20	27	44	82	42	02	42	22	22	01	77
1828 B	434	080	474	841	486	974	203	97	38	41	53	44	49	91	46	46	64	69
1829	727	251	870	235	987	749	370	75	49	48	24	50	00	48	75	74	26	61
1830	921	392	229	588	462	523	536	52	60	44	94	51	47	97	99	99	89	52
1831	115	532	588	940	937	296	703	30	70	41	65	53	94	47	24	24	51	44
1832 B	309	673	948	293	412	070	870	07	81	38	36	55	42	96	48	49	14	35
1833	602	844	344	688	913	845	037	85	92	45	07	61	92	52	77	77	77	27
1834	796	984	703	040	388	619	203	62	03	42	77	63	40	02	01	01	39	18
1835	989	124	062	393	863	392	370	39	13	38	48	65	87	50	26	26	02	10
1836 B	183	265	422	745	338	166	537	17	24	35	19	67	34	00	50	51	64	01
1837	476	436	818	140	840	942	704	94	35	42	90	73	85	57	79	79	27	93
1838	670	576	177	492	315	715	870	72	46	38	60	75	32	06	04	04	89	84
1839	864	716	536	845	790	489	037	49	56	35	31	77	80	55	28	28	52	76
1840 B	058	857	896	197	265	262	204	26	67	32	02	79	27	05	53	53	14	67
1841	351	028	292	592	766	038	371	04	78	39	73	85	77	61	81	81	77	59
1842	544	168	651	944	241	811	537	81	89	35	43	87	25	11	06	06	40	51
1843	738	308	011	297	716	584	704	58	99	32	14	89	72	60	30	31	02	42
1844 B	932	449	370	649	191	358	871	36	10	29	85	91	19	09	55	55	65	34
1845	225	620	766	044	692	134	038	13	21	36	56	97	70	66	84	83	27	26
1846	419	760	125	396	167	907	204	91	32	32	26	99	17	15	08	08	90	17
1847	613	901	485	749	643	681	371	68	42	29	97	01	65	64	33	33	52	09
1848 B	806	041	844	101	118	454	538	45	53	26	68	03	12	14	57	58	15	00
1849	099	212	240	496	619	230	705	23	64	33	39	09	63	70	86	86	77	92
1850	293	352	599	848	094	003	871	00	75	29	09	10	10	19	10	10	40	83

TABLE II. continued.

EPOCHS.

Years.	Evection.	Sec. —	Anomaly.	Eq. Sec. +	Variation.	Sec. +	Mean Long. ☾	Sec. +
	s ° ′ ″	″	s ° ′ ″	″	s ° ′ ″	″	s ° ′ ″	″
1801	6 2 38 27	26	7 0 6 39.8	36.6	6 8 37 33	5	3 18 32 3.0	5.3
1802	11 23 9 57	27	9 28 49 58.8	37.1	10 18 14 57	5	7 27 55 7.9	5.1
1803	5 13 41 27	27	0 27 33 17.7	37.5	2 27 52 22	5	0 7 18 12.9	5.0
1804 B	11 4 12 57	28	3 26 16 36.7	37.9	7 7 29 46	5	4 16 41 17.8	4.8
1805	5 6 3 27	29	7 8 3 49.6	38.5	11 29 18 38	5	9 9 14 57.9	4.6
1806	10 26 34 57	30	10 6 47 8.6	39.0	4 8 56 2	4	1 18 38 2.9	4.5
1807	4 17 6 27	31	1 5 30 27.5	39.5	8 18 33 27	4	5 28 1 7.8	4.3
1808 B	10 7 37 58	32	4 4 13 46.5	40.0	0 28 10 51	4	10 7 24 12.7	4.2
1809	4 9 28 28	32	7 16 0 59.4	40.6	5 19 59 43	4	2 29 57 52.9	4.1
1810	9 29 59 58	33	10 14 44 18.4	41.1	9 29 37 7	4	7 9 20 57.8	4.0
1811	3 20 31 28	34	1 13 27 37.3	41.7	2 9 14 32	4	11 18 44 2.7	3.9
1812 B	9 11 2 58	35	4 12 10 56.3	42.3	6 18 51 57	4	3 28 7 7.7	3.8
1813	3 12 53 28	35	7 23 58 9.2	42.9	11 10 40 48	4	8 20 40 47.8	3.7
1814	9 3 24 58	36	10 22 41 27.2	43.6	3 20 18 12	4	1 0 3 52.8	3.7
1815	2 23 56 28	37	1 21 24 47.1	44.2	7 29 55 37	4	5 9 26 57.7	3.6
1816 B	8 14 27 59	38	4 20 8 6.1	44.9	0 9 33 2	4	9 18 50 2.6	3.6
1817	2 16 18 28	38	8 1 55 19.0	45.6	5 1 21 53	4	2 11 23 42.8	3.6
1818	8 6 49 59	39	11 0 38 38.0	46.4	9 10 59 18	4	6 20 46 47.7	3.6
1819	1 27 21 29	40	1 29 21 57.0	47.1	1 20 36 42	4	11 0 9 52.6	3.6
1820 B	7 17 52 59	41	4 28 5 15.9	48.0	6 0 14 7	4	3 9 32 57.6	3.7
1821	1 19 43 29	41	8 9 52 28.9	48.7	10 22 2 58	4	8 2 6 37.7	3.7
1822	7 10 14 59	42	11 8 35 47.8	49.5	3 1 40 23	4	0 11 29 42.6	3.8
1823	1 0 46 29	43	2 7 19 6.8	50.3	7 11 17 47	4	4 20 52 47.6	3.8
1824 B	6 21 17 59	43	5 6 2 25.7	51.1	11 20 55 12	4	9 0 15 52.5	3.9
1825	0 23 8 29	44	8 17 49 38.7	52.0	4 12 44 3	4	1 22 49 32.6	4.0
1826	6 13 40 0	45	11 16 32 57.6	53.0	8 22 21 28	4	6 2 12 37.6	4.2
1827	0 4 11 30	45	2 15 16 16.6	53.9	1 1 58 52	4	10 11 35 42.5	4.3
1828 B	5 24 43 0	46	5 13 59 35.5	54.8	5 11 36 17	5	2 20 58 47.4	4.5
1829	11 26 33 30	46	8 25 46 48.5	55.8	10 3 25 8	5	7 13 32 27.6	4.7
1830	5 17 5 0	47	11 24 30 7.4	56.8	2 13 2 33	5	11 22 55 32.5	4.9
1831	11 7 36 30	48	2 23 13 26.4	57.9	6 22 39 58	5	4 2 18 37.4	5.1
1832 B	4 28 8 0	48	5 21 56 45.3	58.8	11 2 17 22	5	8 11 41 42.4	5.3
1833	10 29 58 30	49	9 3 43 58.3	59.9	3 24 6 14	6	1 4 15 22.5	5.6
1834	4 20 30 1	49	0 2 27 17.2	1 1.0	8 3 43 38	6	5 13 38 27.5	5.8
1835	10 11 1 31	50	3 1 10 36.2	1 2.1	0 13 21 3	6	9 23 1 32.4	6.1
1836 B	4 1 33 1	51	5 29 53 55.1	1 3.3	4 22 58 27	6	2 2 24 37.3	6.4
1837	10 3 23 31	51	9 11 41 8.1	1 4.4	9 14 47 19	7	6 24 58 17.5	6.7
1838	3 23 55 1	51	0 10 24 27.0	1 5.6	1 24 24 43	7	11 4 21 22.4	7.1
1839	9 14 26 31	52	3 9 7 46.0	1 6.8	6 4 2 8	7	3 13 44 27.3	7.4
1840 B	3 4 58 1	52	6 7 51 4.9	1 8.0	10 13 39 33	8	7 23 7 32.3	7.8
1841	9 6 48 31	53	9 19 38 17.9	1 9.3	3 5 28 24	8	0 15 41 12.4	8.2
1842	2 27 20 1	53	0 18 21 36.8	1 10.5	7 15 5 48	9	4 25 4 17.3	8.6
1843	8 17 51 32	54	3 17 4 55.8	1 11.8	11 24 43 13	9	9 4 27 22.3	9.0
1844 B	2 8 23 2	54	6 15 48 14.7	1 13.1	4 4 20 38	9	1 13 50 27.2	9.5
1845	8 10 13 32	55	9 27 35 27.7	1 14.5	8 26 9 29	10	6 6 24 7.3	9.9
1846	2 0 45 2	55	0 26 18 46.6	1 15.9	1 5 46 54	10	10 15 47 12.3	10.4
1847	7 21 16 32	56	3 25 2 5.6	1 17.3	5 15 24 18	11	2 25 10 17.2	10.8
1848 B	1 11 48 2	56	6 23 45 24.5	1 18.7	9 25 1 43	11	7 4 33 22.1	11.3
1849	7 13 38 32	56	10 5 32 37.5	1 20.1	2 16 50 34	12	11 27 7 2.3	11.9
1850	1 4 10 2	57	1 4 15 56.5	1 21.5	6 26 27 59	12	4 6 30 7.2	12.4

We correct all the arguments of these two pages, except the last of all (the XII) by adding the sum of the preceding equations.

TABLE II. continued.

EPOCHS.

Years.	Suppl. Node.	Sec. —	II	V	VI	VII	VIII	IX	X	XI	XII
	s ° ′ ″	″	s ° ′								
1801	11 16 13 38.0	7.7	9 11 15	260	266	778	784	861	344	172	782
1802	0 5 33 20.5	7.9	1 1 47	673	679	085	091	941	529	449	721
1803	0 24 53 2.9	8.1	4 22 19	087	091	393	397	021	715	725	660
1804 B	1 14 12 45.4	8.2	8 12 51	501	504	701	702	101	901	002	600
1805	2 3 35 38.5	8.3	0 14 32	949	955	037	043	216	129	312	544
1806	2 22 55 20.9	8.5	4 5 4	363	368	345	350	296	315	589	484
1807	3 12 15 3.4	8.7	7 25 36	777	780	652	656	376	500	866	423
1808 B	4 1 34 45.9	8.8	11 16 8	190	192	960	961	456	686	143	363
1809	4 20 57 39.0	9.0	3 17 49	637	644	296	302	572	914	453	307
1810	5 10 17 21.4	9.1	7 8 21	051	057	604	609	651	100	730	247
1811	5 29 37 3.9	9.3	10 28 53	465	469	912	915	731	285	006	186
1812 B	6 18 56 46.3	9.5	2 19 25	879	882	219	220	811	471	283	126
1813	7 8 19 39.4	9.7	6 21 6	328	333	555	561	927	699	594	070
1814	7 27 39 21.9	9.8	10 11 38	741	746	863	868	006	885	870	010
1815	8 16 59 4.4	10.0	2 2 10	155	158	171	174	086	070	147	949
1816 B	9 6 18 46.8	10.2	5 22 42	569	571	479	479	166	256	424	889
1817	9 25 41 39.9	10.4	9 24 23	017	022	815	820	282	484	734	833
1818	10 15 1 22.4	10.5	1 14 55	431	435	122	127	361	670	011	773
1819	11 4 21 4.8	10.7	5 5 27	844	847	430	433	441	855	288	712
1820 B	11 23 40 47.3	10.9	8 26 0	258	260	738	739	521	041	564	652
1821	0 13 3 40.4	11.1	0 27 41	706	711	074	079	637	269	875	596
1822	1 2 23 22.9	11.3	4 18 13	120	124	382	386	717	455	151	536
1823	1 21 43 5.3	11.5	8 8 45	533	536	689	692	796	641	428	475
1824 B	2 11 2 47.8	11.6	11 29 17	947	949	997	998	876	826	705	415
1825	3 0 25 40.9	11.8	4 0 58	395	401	333	338	992	054	016	359
1826	3 19 45 23.3	12.0	7 21 30	809	813	641	645	072	240	292	299
1827	4 9 5 5.8	12.2	11 12 2	223	225	949	951	151	426	569	238
1828 B	4 28 24 48.3	12.4	3 2 34	636	638	256	257	231	611	846	177
1829	5 17 47 41.4	12.6	7 4 15	084	090	592	597	347	839	156	122
1830	6 7 7 23.8	12.8	10 24 47	498	502	900	904	427	025	433	062
1831	6 26 27 6.3	13.0	2 15 19	912	914	208	210	506	211	710	001
1832 B	7 15 46 48.7	13.2	6 5 51	326	327	516	516	586	397	986	940
1833	8 5 9 41.8	13.4	10 7 32	774	779	852	856	702	624	297	885
1834	8 24 29 24.3	13.6	1 28 4	187	191	159	163	782	810	573	825
1835	9 13 49 6.8	13.8	5 18 36	601	603	467	469	861	996	850	764
1836 B	10 3 8 49.2	14.0	9 9 9	015	016	775	775	941	182	127	703
1837	10 22 31 42.3	14.2	1 10 50	463	468	111	116	057	409	437	648
1838	11 11 51 24.8	14.4	5 1 22	876	880	419	423	137	595	714	588
1839	0 1 11 7.2	14.6	8 21 54	290	292	726	729	217	781	991	527
1840 B	0 20 30 49.7	14.8	0 12 26	704	705	034	035	296	967	268	466
1841	1 9 53 42.8	15.1	4 14 7	152	157	370	375	412	194	578	411
1842	1 29 13 25.3	15.3	8 4 39	566	569	678	682	492	380	855	350
1843	2 18 33 7.7	15.5	11 25 11	980	980	986	988	572	566	131	290
1844 B	3 7 52 50.2	15.7	3 15 43	393	394	293	294	651	752	408	229
1845	3 27 15 43.3	15.9	7 17 24	840	846	629	634	767	979	718	174
1846	4 16 35 25.7	16.1	11 7 56	254	258	937	941	847	165	995	113
1847	5 5 55 8.2	16.4	2 28 38	668	670	245	247	927	351	272	053
1848 B	5 25 14 50.7	16.6	6 19 0	082	083	553	553	006	537	549	992
1849	6 14 37 43.8	16.8	10 20 41	531	535	889	893	122	764	859	937
1850	7 3 57 26.2	17.0	2 11 13	944	947	196	200	202	950	136	876

TABLE II. continued.

EPOCHS.

Years.	1	2	3	4	5	6	7	8	9	10	11	12	13
1851	00050	9168	2643	5695	3467	4239	8539	3477	3885	305	958	917	570
1852 B	99979	6376	2309	0439	5939	0036	9600	5935	2749	912	319	636	043
1853	00181	4234	3015	5469	8746	6206	0718	8782	1637	588	711	426	550
1854	00110	1443	2680	0213	1217	2003	1778	1240	0501	195	072	146	023
1855	00039	8651	2346	4957	3689	7801	2839	3697	9365	801	432	865	496
1856 B	99968	5860	2012	9701	6161	3598	3899	6155	8229	408	793	585	969
1857	00171	3718	2718	4732	8968	9767	5018	9002	7117	085	185	374	476
1858	00100	0926	2384	9476	1439	5565	6078	1460	5981	691	546	094	949
1859	00029	8135	2050	4220	3911	1362	7139	3917	4845	298	907	813	422
1860 B	99958	5343	1715	8964	6383	7159	8199	6374	3709	904	267	533	895
1861	00160	3201	2422	3995	9190	3329	9317	9222	2597	581	659	323	402
1862	00089	0410	2087	8739	1661	9126	0378	1679	1461	187	020	042	875
1863	00018	7618	1753	3483	4133	4923	1438	4137	0324	794	381	761	348
1864 B	99947	4827	1419	8227	6605	0721	2499	6594	9188	400	742	481	821
1865	00149	2685	2125	3257	9412	6890	3617	9442	8076	077	134	271	328
1866	00078	9894	1791	8001	1883	2687	4678	1899	6940	684	494	990	801
1867	00007	7102	1456	2745	4355	8485	5738	4357	5804	290	855	710	274
1868 B	99936	4310	1122	7489	6827	4282	6799	6814	4668	897	216	430	747
1869	00138	2169	1828	2520	9634	0452	7917	9662	3556	573	608	219	254
1870	00067	9377	1494	7264	2105	6249	8978	2119	2420	180	968	939	727
1871	99996	6586	1160	2008	4577	2046	0038	4577	1284	786	329	658	200
1872 B	99925	3794	0826	6752	7049	7844	1099	7034	0148	393	690	378	673
1873	00128	1652	1532	1783	9856	4013	2217	9882	9036	069	082	167	181
1874	00057	8861	1197	6527	2327	9810	3278	2339	7900	676	443	887	654
1875	99986	6069	0863	1271	4799	5608	4338	4796	6763	282	804	606	127
1876 B	99915	3278	0529	6014	7271	1405	5399	7254	5627	889	164	326	600
1877	00117	1136	1235	1045	0078	7574	6517	0101	4515	566	556	115	107
1878	00046	8344	0901	5789	2549	3372	7577	2559	3379	172	917	835	580
1879	99975	5553	0567	0533	5021	9169	8638	5016	2243	779	278	554	053
1880 B	99904	2761	0232	5277	7493	4966	9698	7474	1107	385	639	273	526
1881	00107	0619	0938	0308	0299	1136	0817	0321	9995	062	031	063	033
1882	00036	7828	0604	5052	2771	6933	1877	2779	8859	668	391	783	506
1883	99965	5036	0270	9796	5243	2731	2938	5236	7723	275	752	502	979
1884 B	99894	2245	9936	4540	7714	8528	3998	7694	6587	881	113	222	452
1885	00097	0103	0642	9571	0521	4697	5116	0541	5475	558	505	011	959
1886	00026	7311	0308	4315	2993	0495	6177	2999	4339	165	866	731	432
1887	99955	4520	9973	9058	5465	6292	7237	5456	3202	771	226	450	905
1888 B	99884	1728	9639	3802	7936	2089	8298	7913	2066	378	587	170	378
1889	00086	9586	0345	8833	0743	8259	9416	0761	0954	054	979	960	885
1890	00015	6795	0011	3577	3215	4056	0477	3218	9818	661	340	679	358
1891	99944	4003	9677	8321	5687	9853	1537	5676	8682	267	701	398	831
1892 B	99873	1212	9343	3065	8158	5651	2598	8133	7546	874	061	118	304
1893	00075	9070	0049	8096	0965	1820	3716	0981	6434	551	453	908	811
1894	00004	6278	9714	2840	3437	7617	4777	3438	5298	157	814	627	284
1895	99933	3487	9380	7584	5909	3415	5837	5896	4162	764	175	347	757
1896 B	99862	0695	9046	2328	8380	9212	6898	8353	3026	370	536	067	230
1897	00065	8553	9752	7359	1187	5382	8016	1201	1914	047	928	856	737
1898	99994	5762	9418	2102	3659	1179	9077	3658	0778	653	288	576	210
1899	99923	2970	9084	6846	6131	6976	0137	6116	9641	260	649	295	683
1900	99851	0179	8750	1590	8602	2774	1198	8573	8505	866	010	015	156

TABLE II. continued.

EPOCHS.

Years.	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1851	487	493	959	201	569	777	038	78	85	26	80	12	57	69	35	35	02	75
1852 B	681	633	318	553	044	550	205	55	96	23	51	14	04	18	59	60	65	66
1853	974	804	714	948	545	326	372	33	07	30	22	20	55	75	88	88	28	58
1854	168	944	073	300	020	099	539	10	18	26	93	22	03	24	12	12	90	50
1855	361	085	433	653	495	873	705	87	28	23	63	24	50	73	37	37	53	41
1856 B	555	225	792	005	970	646	872	65	39	20	34	26	97	22	61	62	15	33
1857	848	396	188	400	471	422	039	42	50	27	05	32	48	79	90	90	78	24
1858	042	537	547	752	947	195	206	20	61	24	76	34	95	28	15	15	40	16
1859	236	677	907	105	422	969	372	97	71	20	46	36	42	78	39	40	03	07
1860 B	430	817	266	457	897	742	539	74	82	17	17	38	89	27	64	64	65	99
1861	723	988	662	852	398	518	706	52	93	24	88	44	41	83	92	92	28	91
1862	916	129	021	204	873	291	873	29	04	20	60	46	88	33	17	17	91	82
1863	110	269	381	557	348	065	039	06	14	17	29	48	35	81	41	42	53	74
1864 B	304	409	740	909	823	838	206	84	25	14	00	50	82	31	66	66	16	65
1865	597	580	136	304	324	614	373	61	36	21	71	56	33	88	95	94	78	57
1866	791	721	495	657	799	387	540	39	47	17	42	58	80	37	19	19	41	49
1867	985	861	855	009	274	161	707	16	57	14	12	60	28	86	44	44	03	40
1868 B	178	001	214	362	749	934	873	93	68	11	83	62	75	36	68	69	66	32
1869	471	172	610	756	251	710	040	71	79	18	54	68	26	92	97	97	28	23
1870	665	313	969	109	726	483	207	48	90	15	26	69	73	42	21	21	91	15
1871	859	453	329	461	201	257	373	25	00	11	95	71	20	91	46	46	53	06
1872 B	053	594	688	814	676	030	540	03	11	08	66	73	67	40	70	71	16	98
1873	346	765	084	208	177	806	707	81	22	15	37	79	18	97	99	99	79	90
1874	540	905	443	561	652	579	874	58	33	12	08	81	66	46	23	24	41	81
1875	733	045	803	913	127	353	040	35	43	08	78	83	13	95	48	48	04	73
1876 B	927	186	162	266	602	126	207	13	54	05	49	85	60	45	72	73	66	64
1877	220	357	558	660	103	902	374	90	65	12	29	91	11	01	01	01	29	56
1878	414	497	917	013	578	675	541	67	76	09	91	93	58	50	25	26	91	47
1879	608	637	277	365	053	449	707	45	86	05	61	95	05	00	50	51	54	39
1880 B	802	778	636	718	528	222	874	22	97	02	32	97	53	49	74	75	16	30
1881	095	949	032	112	030	998	041	00	08	09	03	03	04	06	03	03	79	22
1882	288	089	391	465	505	771	208	77	19	06	74	05	51	55	28	28	42	13
1883	482	229	751	817	980	545	374	54	29	02	44	07	98	04	52	53	04	05
1884 B	676	370	110	170	455	318	541	32	40	99	15	09	45	53	77	78	67	96
1885	969	541	506	565	956	094	708	09	51	06	86	15	96	10	05	05	29	88
1886	163	681	865	917	431	868	875	87	62	03	57	17	43	59	30	30	92	80
1887	357	822	225	270	906	641	041	64	72	99	27	19	91	09	54	55	54	71
1888 B	550	962	584	622	381	415	208	41	83	96	98	21	38	58	79	80	17	63
1889	843	133	979	017	882	190	374	19	94	03	69	27	89	14	08	08	79	55
1890	037	273	338	369	357	964	542	96	05	00	40	28	36	64	32	33	42	46
1891	231	414	698	722	832	737	708	74	15	96	10	30	83	12	57	57	04	38
1892 B	425	554	057	074	308	511	875	51	26	93	81	32	30	62	81	82	67	29
1893	718	725	453	469	809	286	041	28	37	00	52	38	81	19	10	10	30	21
1894	912	865	812	821	284	060	208	06	48	97	23	40	29	68	34	35	92	12
1895	105	006	172	174	759	833	374	83	58	94	93	42	76	17	59	60	55	04
1896 B	299	146	531	526	234	607	541	60	69	90	64	44	23	67	83	84	17	95
1897	592	317	927	921	735	382	709	38	80	97	35	50	74	23	12	12	80	87
1898	786	458	286	273	210	156	876	15	91	94	06	52	21	73	36	37	42	79
1899	980	598	646	626	685	929	043	93	01	91	76	54	68	22	61	62	05	70
1900	174	738	005	978	160	703	209	70	12	87	47	57	16	71	85	86	67	62

TABLE II. continued.

EPOCHS.

Years.	Evection.	Sec. −	Anomaly.	Eq. Sec. +	Variation.	Sec. +	Long. ☾	Sec. +
	s ° ′ ″	′ ″	s. ° ′ ″	′ ″	s. ° ′ ″		s. ° ′ ″	″
1851	6 24 41 33	0 57	4 2 59 15.4	1 22.9	11 6 5 23	13	8 15 53 12.1	13.0
1852 B	0 15 13 3	0 57	7 1 42 34.4	1 24.5	3 15 42 48	14	0 25 16 17.1	13.6
1853	6 17 3 33	0 58	10 13 29 47.4	1 26.0	8 7 31 39	14	5 17 49 57.2	14.1
1854	0 7 35 3	0 58	1 12 13 6.3	1 27.5	0 17 9 4	15	9 27 13 2.2	14.7
1855	5 28 6 33	0 59	4 10 56 25.3	1 29.1	4 26 46 28	15	2 6 36 7.1	15.3
1856 B	11 18 38 3	0 59	7 9 39 44.2	1 30.7	9 6 23 53	16	6 15 59 12.0	15.9
1857	5 20 28 33	0 59	10 21 26 57.2	1 32.3	1 28 12 44	17	11 8 32 52.2	16.6
1858	11 11 0 3	1 0	1 20 10 16.1	1 33.9	6 7 50 9	17	3 17 55 57.1	17.2
1859	5 1 31 34	1 0	4 18 53 35.1	1 35.5	10 17 27 34	18	7 27 19 2.0	17.8
1860 B	10 22 3 4	1 0	7 17 36 54.0	1 37.1	2 27 4 58	19	0 6 42 7.0	18.5
1861	4 23 53 34	1 1	10 29 24 7.0	1 38.8	7 18 53 50	19	4 29 15 47.1	19.2
1862	10 14 25 4	1 1	1 28 7 25.9	1 40.4	11 28 31 14	20	9 8 38 52.0	19.9
1863	4 4 56 34	1 1	4 26 50 44.9	1 42.1	4 8 8 39	21	1 18 1 57.0	20.6
1864 B	9 25 28 4	1 1	7 25 34 3.8	1 43.9	8 17 46 3	21	5 27 25 1.9	21.3
1865	3 27 18 34	1 2	11 7 21 16.8	1 45.6	1 9 34 55	22	10 19 58 42.0	22.0
1866	9 17 50 4	1 2	2 6 4 35.7	1 47.3	5 19 12 19	23	2 29 21 47.0	22.8
1867	3 8 21 34	1 2	5 4 47 54.7	1 49.0	9 28 49 44	24	7 8 44 51.9	23.5
1868 B	8 28 53 5	1 2	8 3 31 13.6	1 50.8	2 8 27 9	24	11 18 7 56.8	24.3
1869	3 0 43 35	1 2	11 15 18 26.6	1 52.7	7 0 16 0	25	4 10 41 37.0	25.0
1870	8 21 15 5	1 3	2 14 1 45.5	1 54.5	11 9 53 24	26	8 20 4 41.9	25.8
1871	2 11 46 35	1 3	5 12 45 4.5	1 56.3	3 19 30 49	26	0 29 27 46.8	26.5
1872 B	8 2 18 5	1 3	8 11 28 23.4	1 58.1	7 29 8 14	27	5 8 50 51.8	27.3
1873	2 4 8 35	1 4	11 23 15 36.4	2 0.0	0 20 57 5	28	10 1 24 31.9	28.1
1874	7 24 40 5	1 4	2 21 58 55.3	2 1.8	5 0 34 30	29	2 10 47 36.9	28.9
1875	1 15 11 35	1 4	5 20 42 14.3	2 3.7	9 10 11 54	30	6 20 10 41.8	29.7
1876 B	7 5 43 6	1 5	8 19 25 33.2	2 5.6	1 19 49 19	31	10 29 33 46.7	30.5
1877	1 7 33 36	1 5	0 1 12 46.2	2 7.6	6 11 38 10	31	3 22 7 26.9	31.3
1878	6 28 5 6	1 5	2 29 56 5.1	2 9.5	10 21 15 35	32	8 1 30 31.8	32.1
1879	0 18 36 36	1 6	5 28 39 24.1	2 11.5	3 0 52 59	33	0 10 53 36.7	32.9
1880 B	6 9 8 6	1 6	8 27 22 43.0	2 13.4	7 10 30 24	34	4 20 16 41.7	33.7
1881	0 10 58 36	1 6	0 9 9 56.0	2 15.2	0 2 19 15	34	9 12 50 21.8	34.5
1882	6 1 30 6	1 7	3 7 53 14.9	2 17.2	4 11 56 40	35	1 22 13 26.7	35.3
1883	11 22 1 36	1 7	6 6 36 33.9	2 19.1	8 21 34 4	36	6 1 36 31.7	36.1
1884 B	5 12 33 6	1 7	9 5 19 52.8	2 21.0	1 1 11 29	37	10 10 59 36.6	36.9
1885	11 14 23 36	1 8	0 17 7 5.8	2 22.9	5 23 0 20	38	3 3 33 16.7	37.7
1886	5 4 55 7	1 8	3 15 50 24.8	2 24.9	10 2 37 45	39	7 12 56 21.7	38.5
1887	10 25 26 37	1 8	6 14 33 43.7	2 26.8	2 12 15 10	39	11 22 19 26.6	39.3
1888 B	4 15 58 7	1 9	9 13 17 2.7	2 28.8	6 21 52 34	40	4 1 42 31.5	40.1
1889	10 17 48 37	1 9	0 25 4 15.7	2 30.7	11 13 41 26	41	8 24 16 11.7	40.9
1890	4 8 20 7	1 9	3 23 47 34.6	2 32.6	3 23 18 50	42	1 3 39 16.6	41.7
1891	9 28 51 37	1 10	6 22 30 53.6	2 34.6	8 2 56 15	42	5 13 2 21.6	42.5
1892 B	3 19 23 7	1 10	9 21 14 12.5	2 36.6	0 12 33 39	43	9 22 25 26.6	43.3
1893	9 21 13 37	1 10	1 3 1 25.5	2 38.6	5 4 22 31	44	2 14 59 6.7	44.1
1894	3 11 45 8	1 11	4 1 44 44.4	2 40.6	9 13 59 55	45	6 24 22 11.7	44.8
1895	9 2 16 38	1 11	7 0 28 3.4	2 42.5	1 23 37 20	46	11 3 45 16.6	45.6
1896 B	2 22 48 8	1 12	9 29 11 22.3	2 44.5	6 3 14 45	46	3 13 8 21.7	46.4
1897	8 24 38 8	1 12	1 10 58 35.3	2 46.5	10 25 3 36	47	8 5 42 1.7	47.1
1898	2 15 10 8	1 13	4 9 41 54.2	2 48.5	3 4 41 0	48	0 15 5 6.6	47.9
1899	8 5 41 8	1 13	7 8 25 13.2	2 50.4	7 14 18 25	49	4 24 28 11.6	48.6
1900	1 26 13 8	1 14	10 7 8 32.1	2 52.4	11 23 55 50	49	9 3 51 16.5	49.4

We correct all the arguments of these two pages, except the last of all (the XII) by adding the sum of the preceding eqnations.

End of TABLE II.

EPOCHS.

Years.	Suppl. Node.	Sec.	II	V	VI	VII	VIII	IX	X	XI	XII
	s ° ′ ″	″	s ° ′								
1851	7 23 17 8.7	17.2	5 31 45	358	359	504	506	282	136	413	816
1852 B	8 12 36 51.1	17.5	9 22 18	772	772	812	812	362	322	689	755
1853	9 1 59 44.2	17.7	1 23 59	220	223	148	152	477	549	000	700
1854	9 21 19 26.7	17.9	5 14 31	634	636	456	459	557	735	276	639
1855	10 10 39 9.2	18.1	9 5 3	047	048	763	765	637	921	553	579
1856 B	10 29 58 51.6	18.3	0 25 35	461	461	071	071	717	107	830	518
1857	11 19 21 44.7	18.5	4 27 16	909	912	407	411	832	334	140	463
1858	0 8 41 27.1	18.7	8 17 48	323	325	715	718	912	520	417	402
1859	0 28 1 9.6	18.9	0 8 20	736	737	023	024	992	706	694	342
1860 B	1 17 20 52.1	19.2	3 28 52	150	150	330	330	072	892	971	281
1861	2 6 43 45.2	19.4	8 0 33	598	601	666	670	187	119	281	226
1862	2 26 3 27.7	19.7	11 21 5	012	014	974	977	267	305	558	165
1863	3 15 23 10.1	20.0	3 11 37	426	426	282	283	347	491	834	105
1864 B	4 4 42 52.6	20.3	7 2 9	839	839	590	589	427	677	111	044
1865	4 24 5 45.7	20.5	11 3 50	287	291	926	929	542	904	422	989
1866	5 13 25 28.1	20.8	2 24 22	701	703	233	236	622	090	698	928
1867	6 2 45 10.6	21.1	6 14 54	115	115	541	542	702	276	975	868
1868 B	6 22 4 53.1	21.4	10 5 27	529	528	849	848	782	462	252	807
1869	7 11 27 46.2	21.6	2 7 8	977	980	185	188	897	689	562	752
1870	8 0 47 28.6	21.9	5 27 40	390	392	493	495	977	875	839	691
1871	8 20 7 11.1	22.1	9 18 12	804	804	800	801	057	061	116	631
1872 B	9 9 26 53.5	22.4	1 8 44	218	217	108	107	137	247	392	570
1873	9 28 49 46.6	22.6	5 10 25	666	669	444	447	253	474	703	515
1874	10 18 9 29.1	22.9	9 0 57	079	081	752	754	332	660	979	454
1875	11 7 29 11.6	23.1	0 21 29	493	493	060	060	412	846	256	394
1876 B	11 26 48 54.0	23.4	4 12 1	907	906	367	366	492	032	533	333
1877	0 16 11 47.1	23.7	8 13 42	355	358	703	706	608	260	843	278
1878	1 5 31 29.6	23.9	0 4 14	769	770	011	013	687	445	120	217
1879	1 24 51 12.0	24.3	3 24 46	183	182	319	319	767	631	397	157
1880 B	2 14 10 54.5	24.6	7 15 18	596	595	627	625	847	817	674	096
1881	3 3 33 47.6	24.8	11 16 59	043	047	963	965	963	045	984	041
1882	3 22 53 30.1	25.1	3 7 31	457	459	270	272	042	230	261	980
1883	4 12 13 12.5	25.3	6 28 3	871	870	578	578	122	416	537	920
1884 B	5 1 32 55.0	25.6	10 18 36	285	284	886	884	202	602	814	859
1885	5 20 55 48.1	25.9	2 20 17	734	736	222	224	318	830	125	804
1886	6 10 15 30.5	26.1	6 10 49	147	148	530	531	398	016	401	743
1887	6 29 35 13.0	26.4	10 1 21	561	560	837	837	477	201	678	682
1888 B	7 18 54 55.5	26.6	1 21 53	975	973	145	143	557	387	955	622
1889	8 8 17 48.6	26.9	5 23 34	423	425	481	483	673	615	265	567
1890	8 27 37 31.0	27.1	9 14 6	837	837	789	790	753	801	542	506
1891	9 16 57 13.5	27.4	1 4 38	250	249	097	096	832	986	819	445
1892 B	10 6 16 55.9	27.7	4 25 10	664	662	404	402	911	172	095	385
1893	10 25 39 49.0	27.9	8 26 51	112	113	740	742	028	400	406	330
1894	11 14 59 31.5	28.2	0 17 23	526	526	048	049	108	586	682	269
1895	0 4 19 14.0	28.5	4 7 55	939	938	356	355	187	771	959	208
1896 B	0 23 38 56.4	28.8	7 28 27	353	350	664	661	267	957	236	148
1897	1 13 1 49.5	29.1	0 0 8	801	802	000	001	383	185	546	093
1898	2 2 21 32.0	29.5	3 20 40	215	215	307	308	463	371	823	032
1899	2 21 41 14.4	29.9	7 11 12	629	627	615	614	542	556	100	971
1900	3 11 0 56.9	30.3	11 1 45	042	040	923	920	622	742	377	911

TABLE III.

Motion for Days. JANUARY.

Comm.	Bissex.	1	2	3	4	5	6	7	8	9	10	11	12
1	1	00000	0000	0000	0000	0000	0000	0000	0000	0000	000	000	000
2	2	00274	0650	1040	0287	0336	0372	0058	0390	0024	070	031	070
3	3	00548	1300	2080	0574	0671	0744	0115	0781	0049	140	062	141
4	4	00821	1950	3121	0861	1007	1116	0173	1171	0073	210	093	211
5	5	01095	2600	4161	1148	1342	1488	0231	1561	0097	281	125	282
6	6	01369	3249	5201	1435	1678	1860	0289	1952	0121	351	156	352
7	7	01643	3899	6241	1722	2013	2232	0346	2342	0146	421	187	423
8	8	01916	4549	7281	2009	2349	2604	0404	2732	0170	491	218	493
9	9	02190	5199	8321	2296	2684	2976	0462	3122	0194	561	249	564
10	10	02464	5849	9362	2583	3020	3348	0519	3513	0219	631	280	634
11	11	02738	6499	0402	2870	3355	3720	0577	3903	0243	702	311	705
12	12	03012	7149	1442	3157	3691	4093	0635	4293	0267	772	342	775
13	13	03285	7799	2482	3444	4026	4465	0692	4684	0291	842	374	845
14	14	03559	8449	3522	3731	4362	4837	0750	5074	0316	912	405	916
15	15	03833	9098	4563	4018	4698	5209	0808	5464	0340	982	436	986
16	16	04107	9748	5603	4305	5033	5581	0866	5854	0364	052	467	057
17	17	04380	0398	6643	4592	5369	5953	0923	6245	0389	122	498	127
18	18	04654	1048	7683	4878	5704	6325	0981	6635	0413	193	529	198
19	19	04928	1698	8723	5165	6040	6697	1039	7025	0437	263	560	268
20	20	05202	2348	9763	5452	6375	7069	1096	7416	0461	333	591	339
21	21	05476	2998	0804	5739	6711	7441	1154	7806	0486	403	623	409
22	22	05749	3648	1844	6026	7046	7813	1212	8196	0510	473	654	480
23	23	06023	4298	2884	6313	7382	8185	1269	8586	0534	543	685	550
24	24	06297	4947	3924	6600	7717	8557	1327	8977	0559	614	716	621
25	25	06571	5597	4964	6887	8053	8929	1385	9367	0583	684	747	691
26	26	06844	6247	6005	7174	8389	9301	1443	9757	0607	754	778	762
27	27	07118	6897	7045	7461	8724	9673	1500	0148	0631	824	809	832
28	28	07392	7547	8085	7748	9060	0045	1558	0538	0656	894	840	903
29	29	07666	8197	9125	8035	9395	0417	1616	0928	0680	964	872	973
30	30	07940	8847	0165	8322	9731	0789	1673	1319	0704	034	903	043
31	31	08213	9497	1205	8609	0066	1161	1731	1709	0729	105	934	114

FEBRUARY.

1	1	08487	0146	2246	8896	0402	1533	1789	2099	0753	175	965	184
2	2	08761	0796	3286	9183	0737	1906	1846	2489	0777	245	996	255
3	3	09035	1446	4326	9470	1073	2278	1904	2880	0801	315	027	325
4	4	09308	2096	5366	9757	1408	2650	1962	3270	0826	385	058	396
5	5	09582	2746	6406	0044	1744	3022	2020	3660	0850	455	089	466
6	6	09856	3396	7446	0331	2079	3394	2077	4051	0874	526	121	537
7	7	10130	4046	8487	0618	2415	3766	2135	4441	0899	596	152	607
8	8	10404	4696	9527	0905	2751	4138	2193	4831	0923	666	183	678
9	9	10677	5346	0567	1192	3086	4510	2250	5222	0947	736	214	748
10	10	10951	5995	1607	1479	3422	4882	2308	5612	0971	806	245	818
11	11	11224	6645	2647	1766	3757	5254	2366	6002	0996	876	276	889
12	12	11499	7295	3688	2053	4093	5626	2423	6392	1020	946	307	959
13	13	11772	7945	4728	2340	4428	5998	2481	6783	1044	017	338	030
14	14	12046	8595	5768	2627	4764	6370	2539	7173	1069	087	370	100
15	15	12320	9245	6808	2914	5099	6742	2597	7563	1093	157	401	171

TABLE III. continued.

JANUARY.

Comm.	Bissex.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1	000	000	000	000	000	000	000	000	00	00	00	00	00	00	00
2	2	034	099	031	037	042	026	002	000	01	01	10	03	04	04	07
3	3	068	198	061	073	084	052	004	001	02	02	20	05	08	07	14
4	4	103	297	092	110	126	078	006	001	02	03	30	08	12	11	21
5	5	137	397	122	146	168	104	008	002	03	03	41	11	16	15	28
6	6	171	496	153	183	210	130	011	002	04	04	51	13	21	18	35
7	7	205	595	183	220	252	156	013	003	05	05	61	16	25	22	42
8	8	239	694	214	256	294	182	015	003	05	06	71	19	29	26	49
9	9	273	793	244	293	336	208	017	004	06	07	81	21	33	30	56
10	10	308	892	275	329	379	234	019	004	07	08	91	24	37	33	63
11	11	342	992	305	366	421	260	021	005	08	09	01	27	41	37	70
12	12	376	091	336	403	463	286	023	005	08	09	11	29	45	41	77
13	13	410	190	366	439	505	312	025	005	09	10	22	32	49	44	84
14	14	444	289	397	476	547	337	028	006	10	11	32	34	53	48	91
15	15	478	388	427	512	589	363	030	006	11	12	42	37	58	52	98
16	16	513	487	458	549	631	389	032	007	11	13	52	40	62	55	05
17	17	547	587	488	586	673	415	034	007	12	14	62	42	66	59	12
18	18	581	686	519	622	715	441	036	008	13	14	72	45	70	63	19
19	19	615	785	549	659	757	467	038	008	14	15	82	48	74	66	26
20	20	649	884	580	695	799	493	040	009	14	16	92	50	78	70	33
21	21	683	983	611	732	841	519	042	009	15	17	03	53	82	74	40
22	22	718	082	641	769	883	545	044	010	16	18	13	56	86	77	47
23	23	752	182	672	805	925	571	047	010	17	19	23	58	90	81	54
24	24	786	281	702	842	967	597	049	011	17	20	33	61	95	85	61
25	25	820	380	733	878	009	623	051	011	18	20	43	64	99	89	68
26	26	854	479	763	915	052	649	053	011	19	21	53	66	03	92	75
27	27	888	578	794	952	094	675	055	012	20	22	63	69	07	96	82
28	28	923	677	824	988	136	701	057	012	20	23	73	72	11	00	89
29	29	957	777	855	025	178	727	059	013	21	24	84	74	15	03	96
30	30	991	876	885	061	220	753	061	013	22	25	94	77	19	07	03
31	31	025	975	916	098	262	779	064	014	23	26	04	80	23	11	10

FEBRUARY.

1	1	059	074	946	135	304	805	066	014	24	26	14	82	28	14	17
2	2	094	173	977	171	346	831	068	015	24	27	24	85	32	18	24
3	3	128	272	007	208	388	857	070	015	25	28	34	88	36	22	30
4	4	162	371	038	244	430	883	072	016	26	29	44	90	40	25	37
5	5	196	471	068	281	472	909	074	016	27	30	54	93	44	29	44
6	6	230	570	099	318	514	935	076	016	27	31	65	96	48	33	51
7	7	264	669	129	354	556	961	078	017	28	31	75	98	52	37	58
8	8	299	768	160	391	598	986	081	017	29	32	85	01	56	40	65
9	9	333	867	190	427	640	012	083	018	30	33	95	04	60	44	72
10	10	367	966	221	464	682	038	085	018	30	34	05	06	65	48	79
11	11	401	066	251	501	725	064	087	019	31	35	15	09	69	51	86
12	12	435	165	282	537	767	090	089	019	32	36	25	12	73	55	93
13	13	469	264	313	574	809	116	091	020	33	37	36	14	77	59	00
14	14	504	363	343	610	851	142	093	020	33	37	46	17	81	62	07
15	15	538	462	374	647	893	168	095	021	34	38	56	20	85	66	14

TABLE III. continued.

Motion for Days. JANUARY.

Comm.	Bissex.	28	29	30	31	Evection.	Anomally.	Variation.	Longitude ☽
						☌ ° ′ ″	☌ ° ′ ″	☌ ° ′ ″	☌ ° ′ ″
1	1	00	00	00	00	0 0 0 0	0 0 0 00	0 0 0 0	0 0 0 0
2	2	04	03	00	00	0 11 18 59	0 13 3 54.0	0 12 11 27	0 13 10 35.0
3	3	08	06	00	00	0 22 37 59	0 26 7 47.9	0 24 22 53	0 26 21 10.1
4	4	13	09	01	01	1 3 56 58	1 9 11 41.9	1 6 34 20	1 9 31 45.1
5	5	17	12	01	01	1 15 15 58	1 22 15 35.9	1 18 45 47	1 22 42 20.1
6	6	21	15	01	01	1 26 34 57	2 5 19 29.8	2 0 57 13	2 5 52 55.1
7	7	25	18	01	01	2 7 53 57	2 18 23 23.8	2 13 8 40	2 19 3 30.2
8	8	29	22	01	02	2 19 12 56	3 1 27 17.8	2 25 20 7	3 2 14 5.2
9	9	33	25	01	02	3 0 31 55	3 14 31 11.7	3 7 31 34	3 15 24 40.2
10	10	38	28	02	02	3 11 50 55	3 27 35 5.7	3 19 43 0	3 28 35 15.2
11	11	42	31	02	02	3 23 9 54	4 10 38 59.7	4 1 54 27	4 11 45 50.3
12	12	46	34	02	03	4 4 28 54	4 23 42 53.7	4 14 5 54	4 24 56 25.3
13	13	50	37	02	03	4 15 47 53	5 6 46 47.6	4 26 17 20	5 8 7 0.3
14	14	54	40	02	03	4 27 6 53	5 19 50 41.6	5 8 28 47	5 21 17 35.4
15	15	58	43	02	03	5 8 25 52	6 2 54 35.6	5 20 40 14	6 4 28 10.4
16	16	63	46	03	04	5 19 44 51	6 15 58 29.5	6 2 51 40	6 17 38 45.4
17	17	67	49	03	04	6 1 3 51	6 29 2 23.5	6 15 3 7	7 0 49 20.4
18	18	71	52	03	04	6 12 22 50	7 12 6 17.5	6 27 14 34	7 13 59 55.5
19	19	75	55	03	04	6 23 41 50	7 25 10 11.4	7 9 26 1	7 27 10 30.5
20	20	79	59	03	05	7 5 0 49	8 8 14 5.4	7 21 37 27	8 10 21 5.5
21	21	84	62	03	05	7 16 19 49	8 21 17 59.4	8 3 48 54	8 23 31 40.5
22	22	88	65	04	05	7 27 38 48	9 4 21 53.4	8 16 0 21	9 6 42 15.6
23	23	92	68	04	05	8 8 57 47	9 17 25 47.3	8 28 11 47	9 19 52 50.6
24	24	96	71	04	06	8 20 16 47	10 0 29 41.3	9 10 23 14	10 3 3 25.6
25	25	00	74	04	06	9 1 35 46	10 13 33 35.3	9 22 34 41	10 16 14 0.7
26	26	04	77	04	06	9 12 54 46	10 26 37 29.2	10 4 46 7	10 29 24 35.7
27	27	09	80	04	06	9 24 13 45	11 9 41 23.2	10 16 57 34	11 12 35 10.7
28	28	13	83	05	06	10 5 32 45	11 22 45 17.2	10 29 9 1	11 25 45 45.7
29	29	17	86	05	07	10 16 51 44	0 5 49 11.1	11 11 20 28	0 8 56 20.8
30	30	21	89	05	07	10 28 10 43	0 18 53 5.1	11 23 31 54	0 22 6 55.8
31	31	25	92	05	07	11 9 29 43	1 1 56 59.1	0 5 43 21	1 5 17 30.8
						FEBRUARY.			
1	1	29	96	05	07	11 20 48 42	1 15 0 53.1	0 17 54 48	1 18 28 5.8
2	2	34	99	06	08	0 2 7 42	1 28 4 47.0	1 0 6 14	2 1 38 40.9
3	3	38	02	06	08	0 13 26 41	2 11 8 41.0	1 12 17 41	2 14 49 15.9
4	4	42	05	06	08	0 24 45 41	2 24 12 35.0	1 24 29 8	2 27 59 50.9
5	5	46	08	06	08	1 6 4 40	3 7 16 28.9	2 6 40 34	3 11 10 26.0
6	6	50	11	06	09	1 17 23 39	3 20 20 22.9	2 18 52 1	3 24 21 1.0
7	7	55	14	06	09	1 28 42 39	4 3 24 16.9	3 1 3 28	4 7 31 36.0
8	8	59	17	07	09	2 10 1 38	4 16 28 10.8	3 13 14 54	4 20 42 11.0
9	9	63	20	07	09	2 21 20 38	4 29 32 4.8	3 25 26 21	5 3 52 46.1
10	10	67	23	07	10	3 2 39 37	5 12 35 58.8	4 7 37 48	5 17 3 21.1
11	11	71	26	07	10	3 13 58 36	5 25 39 52.7	4 19 49 15	6 0 13 56.1
12	12	75	29	07	10	3 25 17 36	6 8 43 46.7	5 2 0 41	6 13 24 31.1
13	13	80	33	07	10	4 6 36 35	6 21 47 40.7	5 14 12 8	6 26 35 6.2
14	14	84	36	07	11	4 17 55 35	7 4 51 34.7	5 26 23 35	7 9 45 41.2
15	15	88	39	08	11	4 29 14 34	7 17 55 28.6	6 8 35 1	7 22 56 16.2

TABLE III. continued.

Motion for Days. JANUARY.

Comm.	Bissex.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
		s ° ′ ″	s ° ′								
1	1	0 0 0 0.0	0 0 0	000	000	000	000	000	000	000	000
2	2	0 0 3 10.6	0 11 9	034	039	028	034	036	042	034	005
3	3	0 0 6 21.3	0 22 18	068	079	056	067	072	083	067	011
4	4	0 0 9 31.9	1 3 27	102	118	085	101	108	125	101	016
5	5	0 0 12 42.5	1 14 37	136	158	113	135	143	166	135	021
6	6	0 0 15 53.2	1 25 46	170	197	141	169	179	208	168	027
7	7	0 0 19 3.8	2 6 55	204	237	169	202	215	250	202	032
8	8	0 0 22 14.5	2 18 4	238	276	198	236	251	291	235	037
9	9	0 0 25 25.1	2 29 13	272	316	226	270	287	333	269	043
10	10	0 0 28 35.7	3 10 22	306	355	254	303	323	374	303	048
11	11	0 0 31 46.4	3 21 31	340	395	282	337	358	416	336	053
12	12	0 0 34 57.0	4 2 40	374	434	311	371	394	458	370	058
13	13	0 0 38 7.6	4 13 50	408	474	339	405	430	499	404	064
14	14	0 0 41 18.3	4 24 59	442	513	367	438	466	541	437	069
15	15	0 0 44 28.9	5 6 8	476	553	395	472	502	583	471	074
16	16	0 0 47 39.5	5 17 17	510	592	424	506	538	624	505	080
17	17	0 0 50 50.2	5 28 26	544	632	452	539	573	666	538	085
18	18	0 0 54 0.8	6 9 35	578	671	480	573	609	707	572	090
19	19	0 0 57 11.5	6 20 44	612	711	508	607	645	749	605	096
20	20	0 1 0 22.1	7 1 53	646	750	537	641	681	791	639	101
21	21	0 1 3 32.7	7 13 3	680	790	565	674	717	832	673	106
22	22	0 1 6 43.4	7 24 12	714	829	593	708	753	874	706	112
23	23	0 1 9 54.0	8 5 21	748	869	621	742	788	915	740	117
24	24	0 1 13 4.6	8 16 30	782	908	650	775	824	957	774	122
25	25	0 1 16 15.3	8 27 39	816	948	678	809	860	999	807	128
26	26	0 1 19 25.9	9 8 48	850	987	706	843	896	040	841	133
27	27	0 1 22 36.5	9 19 57	884	027	734	877	932	082	875	138
28	28	0 1 25 47.2	10 1 6	918	066	762	910	968	123	908	143
29	29	0 1 28 57.8	10 12 16	952	106	791	944	003	165	942	149
30	30	0 1 32 8.5	10 23 25	986	145	819	978	039	207	975	154
31	31	0 1 35 19.1	11 4 34	020	185	847	011	075	248	009	159

FEBRUARY.

1	1	0 1 38 29.7	11 15 43	054	224	875	045	111	290	043	165
2	2	0 1 41 40.4	11 26 52	088	264	904	079	147	331	076	170
3	3	0 1 44 51.0	0 8 1	122	303	932	113	183	373	110	175
4	4	0 1 48 1.6	0 19 10	156	343	960	146	218	415	144	181
5	5	0 1 51 12.3	1 0 20	190	382	988	180	254	456	177	186
6	6	0 1 54 22.9	1 11 29	224	421	017	214	290	498	211	191
7	7	0 1 57 33.5	1 22 38	258	461	045	247	326	539	244	197
8	8	0 2 0 44.2	2 3 47	292	500	073	281	362	581	278	202
9	9	0 2 3 54.8	2 14 56	326	540	101	315	398	623	312	207
10	10	0 2 7 5.5	2 26 5	360	579	130	349	433	664	345	213
11	11	0 2 10 16.1	3 7 14	394	619	158	382	469	706	379	218
12	12	0 2 13 26.7	3 18 23	428	658	186	416	505	747	413	223
13	13	0 2 16 37.4	3 29 33	462	698	214	449	541	789	446	228
14	14	0 2 19 48.0	4 10 42	496	737	243	484	577	831	480	234
15	15	0 2 22 58.6	4 21 51	530	777	271	517	613	872	514	239

TABLE III. continued.

Motion for Days. FEBRUARY.

Comm.	Bissex.	1	2	3	4	5	6	7	8	9	10	11	12
16	16	12594	9895	7848	3201	5435	7114	2654	7954	1117	227	432	241
17	17	12868	0545	8888	3488	5770	7486	2712	8344	1141	297	463	312
18	18	13141	1195	9929	3775	6106	7858	2770	8734	1166	367	494	382
19	19	13415	1844	0969	4062	6441	8230	2827	9124	1190	438	525	453
20	20	13689	2494	2009	4348	6777	8602	2885	9515	1214	508	556	523
21	21	13963	3144	3049	4635	7112	8974	2943	9905	1239	578	587	594
22	22	14236	3794	4089	4922	7448	9346	3001	0295	1263	648	619	664
23	23	14510	4444	5130	5209	7784	9719	3058	0686	1287	718	650	735
24	24	14784	5094	6170	5496	8119	0091	3116	1076	1311	788	681	805
25	25	15058	5744	7210	5783	8455	0463	3174	1466	1336	858	712	876
26	26	15332	6394	8250	6070	8790	0835	3231	1856	1360	929	743	946
27	27	15605	7044	9290	6357	9126	1207	3289	2247	1384	999	774	016
28	28	15879	7693	0330	6644	9461	1579	3347	2637	1409	069	805	087
	29	16153	8343	1371	6931	9797	1951	3404	3027	1433	139	836	157

MARCH.

1	0	16153	8343	1371	6931	9797	1951	3404	3027	1433	139	836	157
2	1	16427	8993	2411	7218	0132	2323	3462	3418	1457	209	868	228
3	2	16700	9643	3451	7505	0468	2695	3520	3808	1481	279	899	298
4	3	16974	0293	4491	7792	0803	3067	3578	4198	1506	350	930	369
5	4	17248	0943	5531	8079	1139	3439	3635	4589	1530	420	961	439
6	5	17522	1593	6571	8366	1474	3811	3693	4979	1554	490	992	510
7	6	17796	2243	7612	8653	1810	4183	3751	5369	1579	560	023	580
8	7	18069	2893	8652	8940	2146	4555	3808	5759	1603	630	054	651
9	8	18343	3542	9692	9227	2481	4927	3866	6150	1627	700	085	721
10	9	18617	4192	0732	9514	2817	5299	3924	6540	1651	770	117	791
11	10	18891	4842	1772	9801	3152	5671	3981	6930	1676	841	148	862
12	11	19164	5492	2813	0088	3488	6043	4039	7321	1700	911	179	932
13	12	19438	6142	3853	0375	3823	6415	4097	7711	1724	981	210	003
14	13	19712	6792	4893	0662	4159	6787	4155	8101	1749	051	241	073
15	14	19986	7442	5933	0949	4494	7159	4212	8491	1773	121	272	144
16	15	20260	8092	6973	1236	4830	7532	4270	8882	1797	191	303	214
17	16	20533	8741	8013	1523	5165	7904	4328	9272	1821	262	334	285
18	17	20807	9391	9054	1810	5501	8276	4385	9662	1846	332	366	355
19	18	21081	0041	0094	2097	5837	8648	4443	0053	1870	402	397	426
20	19	21355	0691	1134	2384	6172	9020	4501	0443	1894	472	428	496
21	20	21628	1341	2174	2671	6508	9392	4558	0833	1918	542	459	567
22	21	21902	1991	3214	2958	6843	9764	4616	1224	1943	612	490	637
23	22	22176	2641	4255	3245	7179	0136	4674	1614	1967	682	521	708
24	23	22450	3291	5295	3532	7514	0508	4732	2004	1991	753	552	778
25	24	22724	3941	6335	3818	7850	0880	4789	2394	2016	823	583	849
26	25	22997	4590	7375	4105	8185	1252	4847	2785	2040	893	615	919
27	26	23271	5240	8415	4392	8521	1624	4905	3175	2064	963	646	989
28	27	23544	5890	9455	4679	8856	1996	4962	3565	2089	033	677	060
29	28	23819	6540	0496	4966	9192	2368	5020	3956	2113	103	708	130
30	29	24092	7190	1536	5253	9527	2740	5078	4346	2137	174	739	201
31	30	24366	7840	2576	5540	9863	3112	5135	4736	2161	244	770	271
	31	24640	8490	3616	5827	0199	3484	5193	5126	2186	314	801	342

TABLE III. continued.

Motion for Days. FEBRUARY.

Comm.	Bissex.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
16	16	572	561	404	684	935	194	097	021	35	39	66	22	89	70	21
17	17	606	661	435	720	977	220	100	021	36	40	76	25	93	73	28
18	18	640	760	465	757	019	246	102	022	36	41	86	28	97	77	35
19	19	674	859	496	793	061	272	104	022	37	42	96	30	02	81	42
20	20	709	958	526	830	103	298	106	023	38	43	06	33	06	85	49
21	21	743	057	557	867	145	324	108	023	39	43	17	35	10	88	56
22	22	777	156	587	903	187	350	110	024	39	44	27	38	14	92	63
23	23	811	256	618	940	229	376	112	024	40	45	37	41	18	96	70
24	24	845	355	648	976	271	402	114	025	41	46	47	44	22	99	77
25	25	879	454	679	013	313	428	117	025	42	47	57	46	26	03	84
26	26	914	553	709	050	355	454	119	026	43	48	67	49	30	07	91
27	27	948	652	740	086	398	480	121	026	43	48	77	51	35	10	98
28	28	982	751	770	123	440	506	123	026	44	49	87	54	39	14	05
	29	016	851	801	159	482	532	125	027	45	50	98	57	43	18	12

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1	0	016	851	801	159	482	532	125	027	45	50	98	57	43	18	12
2	1	050	950	831	196	524	558	127	027	46	51	08	59	47	21	19
3	2	085	049	862	233	566	584	129	028	46	52	18	62	51	25	26
4	3	119	148	892	269	608	609	131	028	47	53	28	65	55	29	33
5	4	153	247	923	306	650	635	134	029	48	54	38	67	59	33	40
6	5	187	346	953	342	692	661	136	029	49	54	48	70	63	36	47
7	6	221	445	984	379	734	687	138	030	49	55	58	73	67	40	54
8	7	255	545	015	416	776	713	140	030	50	56	68	75	72	44	61
9	8	290	644	045	452	818	739	142	031	51	57	79	78	76	47	68
10	9	324	743	076	489	860	765	144	031	52	58	89	81	80	51	75
11	10	358	842	106	525	902	791	146	032	52	59	99	83	84	55	82
12	11	392	941	137	562	944	817	148	032	53	60	09	86	88	58	89
13	12	426	040	167	599	986	843	150	032	54	60	19	89	92	62	96
14	13	460	140	198	635	028	869	153	033	55	61	29	91	96	66	03
15	14	495	239	228	672	070	895	155	033	55	62	39	94	00	69	10
16	15	529	338	259	708	113	921	157	034	56	63	49	97	05	73	17
17	16	563	437	289	745	155	947	159	034	57	64	60	99	09	77	24
18	17	597	536	320	782	197	973	161	035	58	65	70	02	13	81	31
19	18	631	635	350	818	239	999	163	035	58	65	80	05	17	84	38
20	19	665	735	381	855	281	025	165	036	59	66	90	07	21	88	45
21	20	700	834	411	891	323	051	167	036	60	67	00	10	25	92	52
22	21	734	933	442	928	365	077	170	037	61	68	10	13	29	95	59
23	22	768	032	472	965	407	103	172	037	61	69	20	15	33	99	66
24	23	802	131	503	001	449	129	174	037	62	70	31	18	37	03	73
25	24	836	230	533	038	491	155	176	038	63	71	41	21	42	06	80
26	25	870	330	564	074	533	181	178	038	64	71	51	23	46	10	87
27	26	905	429	594	111	575	207	180	039	64	72	61	26	50	14	94
28	27	939	528	625	148	617	233	182	039	65	73	71	29	54	17	01
29	28	973	627	655	184	659	258	184	040	66	74	81	31	58	21	08
30	29	007	726	686	221	701	284	187	040	67	75	91	34	62	25	15
31	30	041	825	717	257	743	310	189	041	68	76	01	37	66	29	22
	31	076	925	747	294	786	336	191	041	68	77	12	39	70	32	29

TABLE III. continued.

Motion for Days. FEBRUARY.

Comm.	Bissex.	28	29	30	31	Evection.	Anomally.	Variation.	Longitude ☽
16	16	92	42	08	11	5s 10° 33′ 34″	8s 0° 59′ 22″6	6s 20° 46′ 28″	8s 6° 6′ 51″3
17	17	96	45	08	11	5 21 52 33	8 14 3 16.6	7 2 57 55	8 19 17 26.3
18	18	00	48	08	12	6 3 11 32	8 27 7 10.5	7 15 9 21	9 2 28 1.3
19	19	05	51	08	12	6 14 30 32	9 10 11 4.5	7 27 20 48	9 15 38 36.3
20	20	09	54	09	12	6 25 49 31	9 23 14 58.5	8 9 32 15	9 28 49 11.4
21	21	13	57	09	13	7 7 8 31	10 6 18 52.4	8 21 43 42	10 11 59 46.4
22	22	17	60	09	13	7 18 27 30	10 19 22 46.4	9 3 55 8	10 25 10 21.4
23	23	21	63	09	13	7 29 46 30	11 2 26 40.4	9 16 6 35	11 8 20 56.4
24	24	26	66	09	13	8 11 5 29	11 15 30 34.4	9 28 18 2	11 21 31 31.5
25	25	30	69	09	14	8 22 24 28	11 28 34 28.3	10 10 29 28	0 4 42 6.5
26	26	34	73	10	14	9 3 43 28	0 11 38 22.3	10 22 40 55	0 17 52 41.5
27	27	38	76	10	14	9 15 2 27	0 24 42 16.3	11 4 52 22	1 1 3 16.6
28	28	42	79	10	14	9 26 21 27	1 7 46 10.2	11 17 3 48	1 14 13 51.6
	29	46	82	10	15	10 7 40 26	1 20 50 4.2	11 29 15 15	1 27 24 26.6

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1	0	46	82	10	15	10 7 40 26	1 20 50 4.2	11 29 15 15	1 27 24 26.6
2	1	51	85	10	15	10 18 59 26	2 3 53 58.2	0 11 26 42	2 10 35 1.6
3	2	55	88	10	15	11 0 18 25	2 16 57 52.1	0 23 38 9	2 23 45 36.7
4	3	59	91	11	15	11 11 37 24	3 0 1 46.1	1 5 49 35	3 6 56 11.7
5	4	63	94	11	16	11 22 56 24	3 13 5 40.1	1 18 1 2	3 20 6 46.7
6	5	67	97	11	16	0 4 15 23	3 26 9 34.1	2 0 12 29	4 3 17 21.7
7	6	71	00	11	16	0 15 34 23	4 9 13 28.0	2 12 23 55	4 16 27 56.8
8	7	76	03	11	16	0 26 53 22	4 22 17 22.0	2 24 35 22	4 29 38 31.8
9	8	80	06	11	17	1 8 12 22	5 5 21 16.0	3 6 46 49	5 12 49 6.8
10	9	84	10	12	17	1 19 31 21	5 18 25 9.9	3 18 58 15	5 25 59 41.9
11	10	88	13	12	17	2 0 50 20	6 1 29 3.9	4 1 9 42	6 9 10 16.9
12	11	92	16	12	17	2 12 9 20	6 14 32 57.9	4 13 21 9	6 22 20 51.9
13	12	97	19	12	18	2 23 28 19	6 27 36 51.8	4 25 32 36	7 5 31 26.9
14	13	01	22	12	18	3 4 47 19	7 10 40 45.8	5 7 44 2	7 18 42 2.0
15	14	05	25	12	18	3 16 6 18	7 23 44 39.8	5 19 55 29	8 1 52 37.0
16	15	09	28	13	19	3 27 25 18	8 6 48 33.7	6 2 6 56	8 15 3 12.0
17	16	13	31	13	19	4 8 44 17	8 19 52 27.7	6 14 18 22	8 28 13 47.0
18	17	17	34	13	19	4 20 3 16	9 2 56 21.7	6 26 29 49	9 11 24 22.1
19	18	22	37	13	19	5 1 22 16	9 16 0 15.7	7 8 41 16	9 24 34 57.1
20	19	26	40	13	20	5 12 41 15	9 29 4 9.6	7 20 52 42	10 7 45 32.1
21	20	30	43	14	20	5 24 0 15	10 12 8 3.6	8 3 4 9	10 20 56 7.2
22	21	34	47	14	20	6 5 19 14	10 25 11 57.6	8 15 15 36	11 4 6 42.2
23	22	38	50	14	20	6 16 38 13	11 8 15 51.5	8 27 27 2	11 17 17 17.2
24	23	42	53	14	21	6 27 57 13	11 21 19 45.5	9 9 38 29	0 0 27 52.2
25	24	47	56	14	21	7 9 16 12	0 4 23 39.5	9 21 49 56	0 13 38 27.3
26	25	51	59	14	21	7 20 35 12	0 17 27 33.4	10 4 1 23	0 26 49 2.3
27	26	55	62	15	21	8 1 54 11	1 0 31 27.4	10 16 12 49	1 9 59 37.3
28	27	59	65	15	22	8 13 13 11	1 13 35 21.4	10 28 24 16	1 23 10 12.3
29	28	63	68	15	22	8 24 32 10	1 26 39 15.4	11 10 35 43	2 6 20 47.4
30	29	68	71	15	22	9 5 51 9	2 9 43 9.3	11 22 47 9	2 19 31 22.4
31	30	72	74	15	22	9 17 10 9	2 22 47 3.3	0 4 58 36	3 2 41 57.4
	31	76	77	15	23	9 28 29 8	3 5 50 57.3	0 17 10 3	3 15 52 32.5

TABLE III. continued.

Motion for Days. FEBRUARY.

Comm.	Bissex.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
16	16	0s 2° 26′ 9″3	5s 3° 0′	564	816	299	551	648	914	547	244
17	17	0 2 29 19.9	5 14 9	598	856	327	585	684	955	581	250
18	18	0 2 32 30.5	5 25 18	632	895	356	618	720	997	614	255
19	19	0 2 35 41.2	6 6 27	667	935	384	652	756	039	648	260
20	20	0 2 38 51.8	6 17 36	701	974	412	685	792	080	682	266
21	21	0 2 42 2.5	6 28 46	735	014	440	720	828	122	715	271
22	22	0 2 45 13.1	7 9 55	769	053	468	753	863	164	749	276
23	23	0 2 48 23.7	7 21 4	803	093	497	787	899	205	783	282
24	24	0 2 51 34.4	8 2 13	837	132	525	821	935	247	816	287
25	25	0 2 54 45.0	8 13 22	871	172	553	854	971	288	850	292
26	26	0 2 57 55.6	8 24 31	905	211	581	888	007	330	884	298
27	27	0 3 1 6.3	9 5 40	939	251	610	921	043	372	917	303
28	28	0 3 4 16.9	9 16 49	973	290	638	956	078	413	951	308
	29	0 3 7 27.5	9 27 59	007	330	666	989	114	455	984	313

MARCH.

1	0	0 3 7 27.5	9 27 59	007	330	666	989	114	455	984	313
2	1	0 3 10 38.2	10 9 8	041	369	694	023	150	496	018	319
3	2	0 3 13 48.8	10 20 17	075	409	723	057	186	538	052	324
4	3	0 3 16 59.5	11 1 26	109	448	751	090	222	580	085	329
5	4	0 3 20 10.1	11 12 35	143	488	779	124	258	621	119	335
6	5	0 3 23 20.7	11 23 44	177	527	807	157	294	663	153	340
7	6	0 3 26 31.4	0 4 53	211	567	836	192	329	704	186	345
8	7	0 3 29 42.0	0 16 3	245	606	864	225	365	746	220	351
9	8	0 3 32 52.6	0 27 12	279	646	892	259	401	788	254	356
10	9	0 3 36 3.3	1 8 21	313	685	920	293	437	829	287	361
11	10	0 3 39 13.9	1 19 30	347	725	949	326	473	871	321	367
12	11	0 3 42 24.5	2 0 39	381	764	977	360	509	912	354	372
13	12	0 3 45 35.2	2 11 48	415	804	005	393	544	954	388	377
14	13	0 3 48 45.8	2 22 57	449	843	033	428	580	996	422	383
15	14	0 3 51 56.5	3 4 6	483	882	062	461	616	037	455	388
16	15	0 3 55 7.1	3 15 16	517	922	090	495	652	079	489	393
17	16	0 3 58 17.7	3 26 25	551	961	118	529	688	120	523	399
18	17	0 4 1 28.4	4 7 34	585	001	146	562	724	162	556	404
19	18	0 4 4 39.0	4 18 43	619	040	175	596	759	204	590	409
20	19	0 4 7 49.6	4 29 52	653	080	203	629	795	245	624	414
21	20	0 4 11 0.3	5 11 1	687	119	231	664	831	287	657	420
22	21	0 4 14 10.9	5 22 10	721	159	259	697	867	328	691	425
23	22	0 4 17 21.5	6 3 19	755	198	287	731	903	370	724	430
24	23	0 4 20 32.2	6 14 29	789	238	316	765	939	412	758	436
25	24	0 4 23 42.8	6 25 38	823	277	344	798	974	453	792	441
26	25	0 4 26 53.5	7 6 47	857	317	372	832	010	495	825	446
27	26	0 4 30 4.1	7 17 57	891	356	400	866	046	536	859	452
28	27	0 4 33 14.7	7 29 5	925	396	429	900	082	578	893	457
29	28	0 4 36 25.4	8 10 14	959	435	457	933	118	620	926	462
30	29	0 4 39 36.0	8 21 23	993	475	485	967	154	661	960	468
31	30	0 4 42 46.6	9 2 32	027	514	513	001	189	703	994	473
	31	0 4 45 57.3	9 13 42	061	554	542	034	225	745	027	478

TABLE III. continued.

Motion for Days. APRIL.

Comm.	Bissex.	1	2	3	4	5	6	7	8	9	10	11	12
1	0	24640	8490	3616	5827	0199	3484	5193	5126	2186	314	801	342
2	1	24914	9140	4657	6114	0534	3856	5251	5517	2210	384	832	412
3	2	25188	9790	5697	6401	0870	4228	5309	5907	2234	454	864	483
4	3	25461	0439	6737	6688	1205	4600	5366	6297	2258	524	895	553
5	4	25735	1089	7777	6975	1541	4972	5424	6688	2283	595	926	624
6	5	26009	1739	8817	7262	1876	5345	5482	7078	2307	665	957	694
7	6	26283	2389	9857	7549	2212	5717	5539	7468	2331	735	988	764
8	7	26556	3039	0897	7836	2547	6089	5597	7859	2356	805	019	835
9	8	26830	3689	1938	8123	2883	6461	5655	8249	2380	875	050	905
10	9	27104	4339	2978	8410	3218	6833	5712	8639	2404	945	081	976
11	10	27378	4989	4018	8697	3554	7205	5770	9029	2428	015	113	046
12	11	27652	5639	5058	8984	3889	7577	5828	9420	2453	086	144	117
13	12	27925	6288	6098	9271	4225	7949	5886	9810	2477	156	175	187
14	13	28199	6938	7139	9558	4561	8321	5943	0200	2501	226	206	258
15	14	28473	7588	8179	9845	4896	8693	6001	0591	2526	296	237	328
16	15	28747	8238	9219	0132	5232	9065	6059	0981	2550	366	268	399
17	16	29020	8888	0259	0419	5567	9437	6116	1371	2574	436	299	469
18	17	29294	9538	1299	0706	5903	9809	6174	1761	2598	507	330	540
19	18	29568	0188	2339	0993	6238	0181	6232	2152	2623	577	362	610
20	19	29842	0838	3380	1280	6574	0553	6289	2542	2647	647	393	681
21	20	30116	1487	4420	1567	6909	0925	6347	2932	2671	717	424	751
22	21	30389	2137	5460	1854	7245	1297	6405	3323	2696	787	455	822
23	22	30663	2787	6500	2141	7580	1669	6463	3713	2720	857	486	892
24	23	30937	3437	7540	2428	7916	2041	6520	4103	2744	927	517	962
25	24	31211	4087	8581	2715	8251	2413	6578	4494	2768	998	548	033
26	25	31484	4737	9621	3002	8587	2785	6636	4884	2793	068	579	103
27	26	31758	5387	0661	3288	8923	3158	6693	5274	2817	138	611	174
28	27	32032	6037	1701	3575	9258	3530	6751	5665	2841	208	642	244
29	28	32306	6687	2741	3862	9594	3902	6809	6055	2866	278	673	315
30	29	32580	7336	3781	4149	9929	4274	6866	6445	2890	348	704	385
	30	32853	7986	4822	4436	0265	4646	6924	6835	2914	419	735	456

MAY.

1	0	32853	7986	4822	4436	0265	4646	6924	6835	2914	419	735	456
2	1	33127	8636	5862	4723	0600	5018	6982	7226	2938	489	766	526
3	2	33401	9286	6902	5010	0936	5390	7040	7616	2963	559	797	597
4	3	33675	9936	7942	5297	1271	5762	7097	8006	2987	629	828	667
5	4	33948	0586	8982	5584	1607	6134	7155	8397	3011	699	860	737
6	5	34222	1236	0023	5871	1942	6506	7213	8787	3036	769	891	808
7	6	34496	1886	1063	6158	2278	6878	7270	9177	3060	839	922	878
8	7	34770	2536	2103	6445	2613	7250	7328	9567	3084	910	953	949
9	8	35044	3185	3143	6732	2949	7622	7386	9958	3108	980	984	019
10	9	35317	3835	4183	7019	3284	7994	7443	0348	3133	050	015	090
11	10	35591	4485	5223	7306	3620	8366	7501	0738	3157	120	046	160
12	11	35864	5135	6264	7593	3956	8738	7559	1129	3181	190	077	231
13	12	36139	5785	7304	7880	4291	9110	7617	1519	3206	260	109	301
14	13	36412	6435	8344	8167	4627	9482	7674	1909	3230	331	140	372
15	14	36686	7085	9384	8454	4962	9854	7732	2299	3254	401	171	442

TABLE III. continued.

Motion for Days. APRIL.

Comm.	Bissex.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0	076	925	747	294	786	336	191	041	68	77	12	39	70	32	29
2	1	110	024	778	331	828	362	193	042	69	77	22	42	74	36	36
3	2	144	123	808	367	870	388	195	042	70	78	32	45	79	40	43
4	3	178	222	839	404	912	414	197	042	71	79	42	47	83	43	50
5	4	212	321	869	440	954	440	199	043	71	80	52	50	87	47	57
6	5	246	420	900	477	996	466	201	043	72	81	62	53	91	51	64
7	6	281	519	930	514	038	492	203	044	73	82	72	55	95	54	71
8	7	315	619	961	550	080	518	206	044	74	82	82	58	99	58	78
9	8	349	718	991	587	122	544	208	045	74	83	93	60	03	62	84
10	9	383	817	022	623	164	570	210	045	75	84	03	63	07	65	91
11	10	417	916	052	660	206	596	212	046	76	85	13	66	12	68	98
12	11	451	015	083	697	248	622	214	046	77	86	23	68	16	73	05
13	12	486	114	113	733	290	648	216	047	77	87	33	71	20	77	12
14	13	520	214	144	770	332	674	218	047	78	88	43	74	24	80	19
15	14	554	313	174	807	374	700	220	047	79	88	53	76	28	84	26
16	15	588	412	205	843	416	726	223	048	80	89	63	79	32	88	33
17	16	622	511	235	880	459	752	225	048	81	90	74	82	36	91	40
18	17	656	610	266	916	501	778	227	049	81	91	84	84	40	95	47
19	18	691	709	296	953	543	804	229	049	82	92	94	87	44	99	54
20	19	725	809	327	990	585	830	231	050	83	93	04	90	49	02	61
21	20	759	908	357	026	627	856	233	050	84	94	14	92	53	06	68
22	21	793	007	388	063	669	881	235	051	84	94	24	95	57	10	75
23	22	827	106	419	099	711	907	237	051	85	95	34	98	61	13	82
24	23	861	205	449	136	753	933	240	052	86	96	44	00	65	17	89
25	24	896	304	480	173	795	959	242	052	87	97	55	03	69	21	96
26	25	930	404	510	209	837	985	244	053	87	98	65	06	73	25	03
27	26	964	503	541	246	879	011	246	053	88	99	75	08	77	28	10
28	27	998	602	571	282	921	037	248	053	89	99	85	11	81	32	17
29	28	032	701	602	319	963	063	250	054	90	00	95	14	86	36	24
30	29	067	800	632	356	005	089	252	054	90	01	05	16	90	39	31
	30	101	899	663	392	047	115	254	055	91	02	15	19	94	43	38

MAY.

1	0	101	899	663	392	047	115	254	055	91	02	15	19	94	43	38
2	1	135	999	693	429	089	141	256	055	92	03	26	22	98	47	45
3	2	169	098	724	465	132	167	259	056	93	04	36	24	02	50	52
4	3	203	197	754	502	174	193	261	056	93	05	46	27	06	54	59
5	4	237	296	785	539	216	219	263	057	94	06	56	30	10	58	66
6	5	272	395	815	575	258	245	265	057	95	06	66	32	14	61	73
7	6	306	494	846	612	300	271	267	058	96	07	76	35	18	65	80
8	7	340	593	876	648	342	297	269	058	96	08	86	38	23	69	87
9	8	374	693	907	685	384	323	271	058	97	09	96	40	27	72	94
10	9	408	792	937	722	426	349	273	059	98	10	07	43	31	76	01
11	10	442	891	968	758	468	375	275	059	99	11	17	45	35	80	08
12	11	477	990	998	795	510	401	278	060	99	11	27	48	39	84	15
13	12	511	089	029	831	552	427	280	060	00	12	37	51	43	87	22
14	13	545	188	059	868	594	453	282	061	01	13	47	53	47	91	29
15	14	579	288	090	905	636	479	284	061	02	14	57	56	51	95	36

TABLE III. continued.

Motion for Days. APRIL.

Comm.	Bissex.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
1	0	76	77	15	23	9s 28° 29′ 8″	3s 5° 50′ 57″3	0s 17° 10′ 3″	3s 15° 52′ 32″5
2	1	80	80	16	23	10 9 48 8	3 18 54 51.2	0 29 21 29	3 29 3 7.5
3	2	84	84	16	23	10 21 7 7	4 1 58 45.2	1 11 32 56	4 12 13 42.5
4	3	88	87	16	23	11 2 26 7	4 15 2 39.2	1 23 44 23	4 25 24 17.5
5	4	93	90	16	24	11 13 45 6	4 28 6 33.1	2 5 55 50	5 8 34 52.6
6	5	97	93	16	24	11 25 4 5	5 11 10 27.1	2 18 7 16	5 21 45 27.6
7	6	01	96	17	24	0 6 23 5	5 24 14 21.1	3 0 18 43	6 4 56 2.6
8	7	05	99	17	24	0 17 42 4	6 7 18 15.1	3 12 30 10	6 18 6 37.6
9	8	09	02	17	25	0 29 1 4	6 20 22 9.0	3 24 41 36	7 1 17 12.7
10	9	13	05	17	25	1 10 20 3	7 3 26 3.0	4 6 53 3	7 14 27 47.7
11	10	18	08	17	25	1 21 39 3	7 16 29 57.0	4 19 4 30	7 27 38 22.7
12	11	22	11	17	25	2 2 58 2	7 29 33 50.9	5 1 15 56	8 10 48 57.8
13	12	26	14	18	26	2 14 17 1	8 12 37 44.9	5 13 27 23	8 23 59 32.8
14	13	30	17	18	26	2 25 36 1	8 25 41 38.9	5 25 38 50	9 7 10 7.8
15	14	34	20	18	26	3 6 55 0	9 8 45 32.8	6 7 50 17	9 20 20 42.8
16	15	39	24	18	26	3 18 14 0	9 21 49 26.8	6 20 1 43	10 3 31 17.9
17	16	43	27	18	27	3 29 32 59	10 4 53 20.8	7 2 13 10	10 16 41 52.9
18	17	47	30	18	27	4 10 51 59	10 17 57 14.7	7 14 24 37	10 29 52 27.9
19	18	51	33	19	27	4 22 10 58	11 1 1 8.7	7 26 36 3	11 13 3 2.9
20	19	55	36	19	27	5 3 29 57	11 14 5 2.7	8 8 47 30	11 26 13 38.0
21	20	59	39	19	28	5 14 48 57	11 27 8 56.7	8 20 58 57	0 9 24 13.0
22	21	64	42	19	28	5 26 7 56	0 10 12 50.6	9 3 10 23	0 22 34 48.0
23	22	68	45	19	28	6 7 26 56	0 23 16 44.6	9 15 21 50	1 5 45 23.1
24	23	72	48	19	28	6 18 45 55	1 6 20 38.6	9 27 33 17	1 18 55 58.1
25	24	76	51	20	29	7 0 4 55	1 19 24 32.5	10 9 44 43	2 2 6 33.1
26	25	80	54	20	29	7 11 23 54	2 2 28 26.5	10 21 56 10	2 15 17 8.1
27	26	84	57	20	29	7 22 42 53	2 15 32 20.5	11 4 7 37	2 28 27 43.2
28	27	89	61	20	29	8 4 1 53	2 28 36 14.4	11 16 19 4	3 11 38 18.2
29	28	93	64	20	30	8 15 20 52	3 11 40 8.4	11 28 30 30	3 24 48 53.2
30	29	97	67	20	30	8 26 39 52	3 24 44 2.4	0 10 41 57	4 7 59 28.2
	30	01	70	21	30	9 7 58 51	4 7 47 56.4	0 22 53 24	4 21 10 3.3

MAY.

1	0	01	70	21	30	9 7 58 51	4 7 47 56.4	0 22 53 24	4 21 10 3.3
2	1	05	73	21	30	9 19 17 50	4 20 51 50.3	1 5 4 50	5 4 20 38.3
3	2	10	76	21	31	10 0 36 50	5 3 55 44.3	1 17 16 17	5 17 31 13.3
4	3	14	79	21	31	10 11 55 49	5 16 59 38.3	1 29 27 44	6 0 41 48.4
5	4	18	82	21	31	10 23 14 49	6 0 3 32.2	2 11 39 10	6 13 52 23.4
6	5	22	85	21	31	11 4 33 48	6 13 7 26.2	2 23 50 37	6 27 2 58.4
7	6	26	88	22	32	11 15 52 48	6 26 11 20.2	3 6 2 4	7 10 13 33.4
8	7	30	91	22	32	11 27 11 47	7 9 15 14.1	3 18 13 31	7 23 24 8.5
9	8	35	94	22	32	0 8 30 46	7 22 19 8.1	4 0 24 57	8 6 34 43.5
10	9	39	98	22	32	0 19 49 46	8 5 23 2.1	4 12 36 24	8 19 45 18.5
11	10	43	01	22	33	1 1 8 45	8 18 26 56.1	4 24 47 51	9 2 55 53.5
12	11	47	04	22	33	1 12 27 45	9 1 30 50.0	5 6 59 17	9 16 6 28.6
13	12	51	07	23	33	1 23 46 44	9 14 34 44.0	5 19 10 44	9 29 17 3.6
14	13	55	10	23	33	2 5 5 44	9 27 38 38.0	6 1 22 11	10 12 27 38.6
15	14	60	13	23	34	2 16 24 43	10 10 42 31.9	6 13 33 37	10 25 38 13.7

TABLE III. continued.

Motion for Days. APRIL.

Comm.	Bissex.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
1	0	0s 4° 45′ 57″3	9s 13° 42′	061	554	542	034	225	745	027	478
2	1	0 4 49 7.9	9 24 51	095	593	570	068	261	787	061	484
3	2	0 4 52 18.5	10 6 0	129	633	598	102	297	828	094	489
4	3	0 4 55 29.2	10 17 9	163	672	626	136	333	869	128	494
5	4	0 4 58 39.8	10 28 18	197	712	655	169	369	911	162	499
6	5	0 5 1 50.5	11 9 27	231	751	683	203	404	953	195	505
7	6	0 5 5 1.1	11 20 36	265	791	711	237	440	994	229	510
8	7	0 5 8 11.7	0 1 46	299	830	739	270	476	036	263	515
9	8	0 5 11 22.4	0 12 55	333	870	768	304	512	077	296	521
10	9	0 5 14 33.0	0 24 4	367	909	796	338	548	119	330	526
11	10	0 5 17 43.6	1 5 13	401	949	824	372	584	161	363	531
12	11	0 5 20 54.3	1 16 22	435	988	852	405	619	202	397	537
13	12	0 5 24 4.9	1 27 31	469	028	881	439	655	244	431	542
14	13	0 5 27 15.5	2 8 40	503	067	909	473	691	285	464	547
15	14	0 5 30 26.2	2 19 49	537	107	937	506	727	327	498	553
16	15	0 5 33 36.8	3 0 59	571	146	965	540	763	369	532	558
17	16	0 5 36 47.5	3 12 8	605	186	994	574	799	410	565	563
18	17	0 5 39 58.1	3 23 17	639	225	022	608	834	452	599	569
19	18	0 5 43 8.7	4 4 26	673	264	050	641	870	493	633	574
20	19	0 5 46 19.4	4 15 35	707	304	078	675	906	535	666	579
21	20	0 5 49 30.0	4 26 44	741	343	106	709	942	577	700	584
22	21	0 5 52 40.6	5 7 53	775	383	135	742	978	618	733	590
23	22	0 5 55 51.3	5 19 2	809	422	163	776	014	660	767	595
24	23	0 5 59 1.9	6 0 12	843	462	191	810	049	701	801	600
25	24	0 6 2 12.5	6 11 21	877	501	219	844	085	743	834	606
26	25	0 6 5 23.2	6 22 30	911	541	248	877	121	785	868	611
27	26	0 6 8 33.8	7 3 39	945	580	276	911	157	826	902	616
28	27	0 6 11 44.5	7 14 48	979	620	304	945	193	868	935	621
29	28	0 6 14 55.1	7 25 57	013	659	332	978	229	909	969	627
30	29	0 6 18 5.7	8 7 6	047	699	361	012	264	951	003	632
	30	0 6 21 16.4	8 18 15	081	738	389	046	300	993	036	638

MAY.

1	0	0 6 21 16.4	8 18 15	081	738	389	046	300	993	036	638
2	1	0 6 24 27.0	8 29 25	115	778	417	080	336	034	070	643
3	2	0 6 27 37.6	9 10 34	149	817	445	113	372	076	103	648
4	3	0 6 30 48.3	9 21 43	183	857	474	147	408	117	137	654
5	4	0 6 33 58.9	10 2 52	217	896	502	180	444	159	171	659
6	5	0 6 37 9.5	10 14 1	251	936	530	215	480	201	204	664
7	6	0 6 40 20.2	10 25 10	285	975	558	248	515	242	238	669
8	7	0 6 43 30.8	11 6 19	319	015	587	282	551	284	272	675
9	8	0 6 46 41.5	11 17 29	353	054	615	316	587	326	305	680
10	9	0 6 49 52.1	11 28 38	387	094	643	349	623	367	339	685
11	10	0 6 53 2.7	0 9 47	421	133	671	383	659	409	373	691
12	11	0 6 56 13.4	0 20 56	455	173	700	416	695	450	406	696
13	12	0 6 59 24.0	1 2 5	489	212	728	451	730	492	440	701
14	13	0 7 2 34.6	1 13 14	523	252	756	484	766	534	473	707
15	14	0 7 5 45.3	1 24 23	557	291	784	518	802	575	507	712

TABLE III. continued.

Motion for Days. MAY.

Comm.	Bissex.	1	2	3	4	5	6	7	8	9	10	11	12
16	15	36960	7735	0424	8741	5298	0226	7790	2690	3278	471	202	513
17	16	37234	8385	1465	9028	5633	0598	7847	3080	3303	541	233	583
18	17	37508	9034	2505	9315	5969	0971	7905	3470	3327	611	264	654
19	18	37781	9684	3545	9602	6304	1343	7963	3861	3351	681	295	724
20	19	38055	0334	4585	9889	6640	1715	8020	4251	3376	751	326	795
21	20	38329	0984	5625	0176	6975	2087	8078	4641	3400	822	358	865
22	21	38603	1634	6665	0463	7311	2459	8136	5031	3424	892	389	935
23	22	38876	2284	7706	0750	7647	2831	8194	5422	3448	962	420	006
24	23	39150	2934	8746	1037	7982	3203	8251	5812	3473	032	451	076
25	24	39424	3584	9786	1324	8318	3575	8309	6202	3497	102	482	147
26	25	39698	4234	0826	1611	8653	3947	8367	6593	3521	172	513	217
27	26	39972	4883	1866	1898	8989	4319	8424	6983	3546	243	544	288
28	27	40245	5533	2906	2185	9324	4691	8482	7373	3570	313	575	358
29	28	40519	6183	3947	2472	9660	5063	8540	7764	3594	383	607	429
30	29	40793	6833	4987	2758	9995	5435	8597	8154	3618	453	638	499
31	30	41067	7483	6027	3045	0331	5807	8655	8544	3643	523	669	570
	31	41340	8133	7067	3332	0666	6179	8713	8934	3667	593	700	640

JUNE.

1	0	41340	8133	7067	3332	0666	6179	8713	8934	3667	593	700	640
2	1	41614	8783	8107	3619	1002	6551	8771	9325	3691	663	731	710
3	2	41888	9433	9148	3906	1337	6923	8828	9715	3716	734	762	781
4	3	42162	0082	0188	4193	1673	7295	8886	0105	3740	804	793	851
5	4	42436	0732	1228	4480	2009	7667	8944	0496	3764	874	824	922
6	5	42709	1382	2268	4767	2344	8039	9001	0886	3788	944	856	992
7	6	42983	2032	3308	5054	2680	8411	9059	1276	3813	014	887	063
8	7	43257	2682	4348	5341	3015	8784	9117	1667	3837	084	918	133
9	8	43531	3332	5389	5628	3351	9156	9174	2057	3861	155	949	204
10	9	43804	3982	6429	5915	3686	9528	9232	2447	3886	225	980	274
11	10	44078	4632	7469	6202	4022	9900	9290	2837	3910	295	011	345
12	11	44352	5282	8509	6489	4357	0272	9348	3228	3934	365	042	415
13	12	44626	5931	9549	6776	4693	0644	9405	3618	3958	435	073	486
14	13	44900	6581	0590	7063	5028	1016	9463	4008	3983	505	105	556
15	14	45173	7231	1630	7350	5364	1388	9521	4399	4007	575	136	627
16	15	45447	7881	2670	7637	5699	1760	9578	4789	4031	646	167	697
17	16	45721	8531	3710	7924	6035	2132	9636	5179	4056	716	198	768
18	17	45995	9181	4750	8211	6370	2504	9694	5569	4080	786	229	838
19	18	46268	9831	5790	8498	6706	2876	9752	5960	4104	856	260	908
20	19	46542	0481	6831	8785	7042	3248	9809	6350	4128	926	291	979
21	20	46816	1131	7871	9072	7377	3620	9867	6740	4153	996	322	049
22	21	47090	1780	8911	9359	7713	3992	9925	7131	4177	067	354	120
23	22	47364	2430	9951	9646	8048	4364	9982	7521	4201	137	385	190
24	23	47637	3080	0991	9933	8384	4736	0040	7911	4226	207	416	261
25	24	47911	3730	2031	0220	8719	5108	0098	8301	4250	277	447	331
26	25	48184	4380	3072	0507	9055	5480	0155	8692	4274	347	478	402
27	26	48459	5030	4112	0794	9390	5852	0213	9082	4298	417	509	472
28	27	48732	5680	5152	1081	9726	6224	0271	9472	4323	487	540	543
29	28	49006	6330	6192	1368	0061	6597	0329	9863	4347	558	571	613
30	29	49280	6980	7232	1655	0397	6969	0386	0253	4371	628	603	683
	30	49554	7629	8273	1942	0732	7341	0444	0643	4396	698	634	754

TABLE III. continued.

Motion for Days. MAY.

Comm.	Bissex.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
16	15	613	387	121	941	678	505	286	062	03	15	67	59	55	98	43
17	16	647	486	151	978	720	530	288	062	03	16	77	61	60	02	50
18	17	682	585	182	014	762	556	290	063	04	17	88	64	64	06	57
19	18	716	684	212	051	805	582	292	063	05	17	98	67	68	09	64
20	19	750	783	243	088	847	608	295	063	06	18	08	69	72	13	71
21	20	784	883	273	124	889	634	297	064	06	19	18	72	76	17	78
22	21	818	982	304	161	931	660	299	064	07	20	28	75	80	20	85
23	22	852	081	334	197	973	686	301	065	08	21	38	77	84	24	92
24	23	887	180	365	234	015	712	303	065	09	22	48	80	88	28	99
25	24	921	279	395	271	057	738	305	066	09	23	59	83	93	32	06
26	25	955	378	426	307	099	764	307	066	10	23	69	85	97	35	13
27	26	989	478	456	344	141	790	309	067	11	24	79	88	01	39	20
28	27	023	577	487	380	183	816	311	067	12	25	89	91	05	43	27
29	28	058	676	517	417	225	842	314	068	12	26	99	93	09	46	34
30	29	092	775	548	454	267	868	316	068	13	27	09	96	13	50	41
31	30	126	874	578	490	309	894	318	068	14	28	19	99	17	54	48
	31	160	973	609	527	351	920	320	069	15	28	29	01	21	57	55

JUNE.

1	0	160	973	609	527	351	920	320	069	15	28	29	01	21	57	55
2	1	194	073	639	563	393	946	322	069	15	29	40	04	25	61	62
3	2	228	172	670	600	435	972	324	070	16	30	50	07	30	65	69
4	3	263	271	700	637	477	998	326	070	17	31	60	09	34	68	76
5	4	297	370	731	673	520	024	328	071	18	32	70	12	38	72	83
6	5	331	469	761	710	562	050	331	071	18	33	80	15	42	76	90
7	6	365	568	792	746	604	076	333	072	19	34	90	17	46	80	97
8	7	399	667	823	783	646	102	335	072	20	34	00	20	50	83	04
9	8	433	767	853	820	688	128	337	073	21	35	11	23	54	87	11
10	9	468	866	884	856	730	153	339	073	22	36	21	25	58	91	18
11	10	502	965	914	893	772	179	341	074	22	37	31	28	63	94	25
12	11	536	064	945	929	814	205	343	074	23	38	41	31	67	98	32
13	12	570	163	975	966	856	231	345	074	24	39	51	33	71	02	38
14	13	604	262	006	003	898	257	348	075	25	40	61	36	75	05	45
15	14	638	362	036	039	940	283	350	075	25	40	71	39	79	09	52
16	15	673	461	067	076	982	309	352	076	26	41	81	41	83	13	59
17	16	707	560	097	112	024	335	354	076	27	42	92	44	87	16	66
18	17	741	659	128	149	066	361	356	077	28	43	02	46	91	20	73
19	18	775	758	158	186	108	387	358	077	28	44	12	49	95	24	80
20	19	809	857	189	222	150	413	360	078	29	45	22	52	00	28	87
21	20	843	957	219	259	193	439	362	078	30	45	32	55	04	31	94
22	21	878	056	250	295	235	465	364	079	31	46	42	57	08	35	01
23	22	912	155	280	332	277	491	367	079	31	47	52	60	12	39	08
24	23	946	254	311	369	319	517	369	079	32	48	62	62	16	42	15
25	24	980	353	341	405	361	543	371	080	33	49	73	65	20	46	22
26	25	014	452	372	442	403	569	373	080	34	50	83	68	24	50	29
27	26	049	552	402	478	445	595	375	081	34	51	93	70	28	53	36
28	27	083	651	433	515	487	621	377	081	35	51	03	73	32	57	43
29	28	117	750	463	552	529	647	379	082	36	52	13	76	37	61	50
30	29	151	849	494	588	571	673	381	082	37	53	23	78	41	64	57
	30	185	948	525	625	613	699	384	083	37	54	33	81	45	68	64

TABLE III. continued.

Motion for Days. MAY.

Comm.	Bissex.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
16	15	64	16	23	34	2s 27° 43′ 42	10s 23° 46′ 25″9	6s 25° 45′ 4″	11s 8° 48′ 48″7
17	16	68	19	23	34	3 9 2 42	11 6 50 19.9	7 7 56 31	11 21 59 23.7
18	17	72	22	23	34	3 20 21 41	11 19 54 13.8	7 20 7 58	0 5 9 58.7
19	18	76	25	24	35	4 1 40 41	0 2 58 7.8	8 2 19 24	0 18 20 33.8
20	19	81	28	24	35	4 12 59 40	0 16 2 1.8	8 14 30 51	1 1 31 8.8
21	20	85	31	24	35	4 24 18 40	0 29 5 55.7	8 26 42 18	1 14 41 43.8
22	21	89	35	24	35	5 5 37 39	1 12 9 49.7	9 8 53 44	1 27 52 18.8
23	22	93	38	24	36	5 16 56 38	1 25 13 43.7	9 21 5 11	2 11 2 53.9
24	23	97	41	24	36	5 28 15 38	2 8 17 37.7	10 3 16 38	2 24 13 28.9
25	24	01	44	25	36	6 9 34 37	2 21 21 31.6	10 15 28 4	3 7 24 3.9
26	25	06	47	25	36	6 20 53 37	3 4 25 25.6	10 27 39 31	3 20 34 39.0
27	26	10	50	25	37	7 2 12 36	3 17 29 19.6	11 9 50 58	4 3 45 14.0
28	27	14	53	25	37	7 13 31 36	4 0 33 13.5	11 22 2 24	4 16 55 49.0
29	28	18	56	25	37	7 24 50 35	4 13 37 7.5	0 4 13 51	5 0 6 24.0
30	29	22	59	25	37	8 6 9 34	4 26 41 1.5	0 16 25 18	5 13 16 59.1
31	30	26	62	26	38	8 17 28 34	5 9 44 55.4	0 28 36 45	5 26 27 34.1
	31	31	65	26	38	8 28 47 33	5 22 48 49.4	1 10 48 11	6 9 38 9.1

JUNE.

1	0	31	65	26	38	8 28 47 33	5 22 48 49.4	1 10 48 11	6 9 38 9.1
2	1	35	68	26	38	9 10 6 33	6 5 52 43.4	1 22 59 38	6 22 48 44.1
3	2	39	71	26	38	9 21 25 32	6 18 56 37.4	2 5 11 05	7 5 59 19.2
4	3	43	75	26	39	10 2 44 32	7 2 0 31.3	2 17 22 31	7 19 9 54.2
5	4	47	78	26	39	10 14 3 31	7 15 4 25.3	2 29 33 58	8 2 20 29.2
6	5	52	81	27	39	10 25 22 30	7 28 8 19.3	3 11 45 25	8 15 31 4.3
7	6	56	84	27	39	11 6 41 30	8 11 12 13.2	3 23 56 51	8 28 41 39.3
8	7	60	87	27	40	11 18 0 29	8 24 16 7.2	4 6 8 18	9 11 52 14.3
9	8	64	90	27	40	11 29 19 29	9 7 20 1.2	4 18 19 45	9 25 2 49.3
10	9	68	93	27	40	0 10 38 28	9 20 23 55.1	5 0 31 12	10 8 13 24.4
11	10	72	96	28	40	0 21 57 27	10 3 27 49.1	5 12 42 38	10 21 23 59.4
12	11	77	99	28	41	1 3 16 27	10 16 31 43.1	5 24 54 5	11 4 34 34.4
13	12	81	02	28	41	1 14 35 26	10 29 35 37.1	6 7 5 32	11 17 45 9.4
14	13	85	05	28	41	1 25 54 26	11 12 39 31.0	6 19 16 58	0 0 55 44.5
15	14	89	08	28	41	2 7 13 25	11 25 43 25.0	7 1 28 25	0 14 6 19.5
16	15	93	12	28	42	2 18 32 25	0 8 47 19.0	7 13 39 52	0 27 16 54.5
17	16	97	15	29	42	2 29 51 24	0 21 51 12.9	7 25 51 18	1 10 27 29.6
18	17	02	18	29	42	3 11 10 23	1 4 55 6.9	8 8 2 45	1 23 38 4.6
19	18	06	21	29	42	3 22 29 23	1 17 59 0.9	8 20 14 12	2 6 48 39.6
20	19	10	24	29	43	4 3 48 22	2 1 2 54.8	9 2 25 39	2 19 59 14.6
21	20	14	27	29	43	4 15 7 22	2 14 6 48.8	9 14 37 5	3 3 9 49.7
22	21	18	30	29	43	4 26 26 21	2 27 10 42.8	9 26 48 32	3 16 20 24.7
23	22	22	33	30	43	5 7 45 21	3 10 14 36.7	10 8 59 59	3 29 30 59.7
24	23	27	36	30	44	5 19 4 20	3 23 18 30.7	10 21 11 25	4 12 41 34.7
25	24	31	39	30	44	6 0 23 19	4 6 22 24.7	11 3 22 52	4 25 52 9.8
26	25	35	42	30	44	6 11 42 19	4 19 26 18.7	11 15 34 19	5 9 2 44.8
27	26	39	45	30	44	6 23 1 18	5 2 30 12.6	11 27 45 45	5 22 13 19.8
28	27	43	49	30	45	7 4 20 18	5 15 34 6.6	0 9 57 12	6 5 23 54.9
29	28	48	52	31	45	7 15 39 17	5 28 38 0.6	0 22 8 39	6 18 34 29.9
30	29	52	55	31	45	7 26 58 17	6 11 41 54.5	1 4 20 6	7 1 45 4.9
	30	56	58	31	45	8 8 17 16	6 24 45 48.5	1 16 31 32	7 14 55 39.9

TABLE III. continued.

Motion for Days. MAY.

Comm.	Bissex.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
16	15	0s 7° 8′ 55″9	2s 5° 32′	591	331	812	552	838	617	541	717
17	16	0 7 12 6.6	2 16 42	625	370	841	585	874	658	574	723
18	17	0 7 15 17.2	2 27 51	659	410	869	619	910	700	608	728
19	18	0 7 18 27.8	3 9 0	693	449	897	652	945	742	642	733
20	19	0 7 21 38.5	3 20 9	727	489	925	687	981	783	675	739
21	20	0 7 24 49.1	4 1 18	761	528	954	720	017	825	709	744
22	21	0 7 27 59.7	4 12 27	795	568	982	754	053	866	743	749
23	22	0 7 31 10.4	4 23 36	829	607	010	788	089	908	776	755
24	23	0 7 34 21.0	5 4 45	863	647	038	821	125	950	810	760
25	24	0 7 37 31.6	5 15 55	897	686	067	855	160	991	843	765
26	25	0 7 40 42.3	5 27 4	932	725	095	888	196	033	877	770
27	26	0 7 43 52.9	6 8 13	966	765	123	923	232	074	911	776
28	27	0 7 47 3.6	6 19 22	000	804	151	956	268	116	944	781
29	28	0 7 50 14.2	7 0 31	034	844	180	990	304	158	978	786
30	29	0 7 53 24.8	7 11 40	068	883	208	024	340	199	012	792
31	30	0 7 56 35.5	7 22 49	102	923	236	057	375	241	045	797
	31	0 7 59 46.1	8 3 58	136	962	264	091	411	282	079	802

JUNE.

1	0	0 7 59 46.1	8 3 58	136	962	264	091	411	282	079	802
2	1	0 8 2 56.7	8 15 8	170	002	293	124	447	324	113	808
3	2	0 8 6 7.4	8 26 17	204	041	321	159	483	366	146	813
4	3	0 8 9 18.0	9 7 26	238	081	349	192	519	407	180	818
5	4	0 8 12 28.6	9 18 35	272	120	377	226	555	449	213	824
6	5	0 8 15 39.3	9 29 44	306	160	406	260	590	490	247	829
7	6	0 8 18 49.9	10 10 53	340	199	434	293	626	532	281	834
8	7	0 8 22 0.6	10 22 2	374	239	462	327	662	574	314	840
9	8	0 8 25 11.2	11 3 12	408	278	490	360	698	615	348	845
10	9	0 8 28 21.8	11 14 21	442	318	518	395	734	657	382	850
11	10	0 8 31 32.5	11 25 30	476	357	547	428	770	698	415	855
12	11	0 8 34 43.1	0 6 39	510	397	575	462	805	740	449	861
13	12	0 8 37 53.7	0 17 48	544	436	603	496	841	782	482	866
14	13	0 8 41 4.4	0 28 57	578	476	631	529	877	823	516	871
15	14	0 8 44 15.0	1 10 6	612	515	660	563	913	865	550	877
16	15	0 8 47 25.6	1 21 15	646	555	688	597	949	907	583	882
17	16	0 8 50 36.3	2 2 25	680	594	716	631	985	948	617	887
18	17	0 8 53 46.9	2 13 34	714	634	744	664	020	990	651	893
19	18	0 8 56 57.6	2 24 43	748	673	773	698	056	031	684	898
20	19	0 9 0 8.2	3 5 52	782	713	801	732	092	073	718	903
21	20	0 9 3 18.8	3 17 1	816	752	829	765	128	115	752	909
22	21	0 9 6 29.5	3 28 10	850	792	857	799	164	156	785	914
23	22	0 9 9 40.1	4 9 19	884	831	886	833	200	198	819	919
24	23	0 9 12 50.7	4 20 28	918	871	914	867	235	239	852	925
25	24	0 9 16 1.4	5 1 38	952	910	942	900	271	281	886	930
26	25	0 9 19 12.0	5 12 47	986	950	970	934	307	323	920	935
27	26	0 9 22 22.6	5 23 56	020	989	999	968	343	364	953	940
28	27	0 9 25 33.3	6 5 5	054	029	027	001	379	406	987	946
29	28	0 9 28 43.9	6 16 14	088	068	055	035	415	447	021	951
30	29	0 9 31 54.6	6 27 23	122	107	083	069	450	489	054	956
	30	0 9 35 5.2	7 8 32	156	147	112	103	486	531	088	962

TABLE III. continued.

Motion for Days. JULY.

Comm.	Bissex.	1	2	3	4	5	6	7	8	9	10	11	12
1	0	49554	7629	8273	1942	0732	7341	0444	0643	4396	698	634	754
2	1	49828	8279	9313	2228	1068	7713	0502	1034	4420	768	665	824
3	2	50101	8929	0353	2515	1404	8085	0559	1424	4444	838	696	895
4	3	50375	9579	1393	2802	1739	8457	0617	1814	4468	908	727	965
5	4	50649	0229	2433	3089	2075	8829	0675	2204	4493	979	758	036
6	5	50923	0879	3473	3376	2410	9201	0732	2595	4517	049	789	106
7	6	51196	1529	4514	3663	2746	9573	0790	2985	4541	119	820	177
8	7	51470	2179	5554	3950	3081	9945	0848	3375	4566	189	852	247
9	8	51744	2828	6594	4237	3417	0317	0906	3766	4590	259	883	318
10	9	52018	3478	7634	4524	3752	0689	0963	4156	4614	329	914	388
11	10	52292	4128	8674	4811	4088	1061	1021	4546	4638	399	945	459
12	11	52565	4778	9715	5098	4423	1433	1079	4936	4663	470	976	529
13	12	52839	5428	0755	5385	4759	1805	1136	5327	4687	540	007	600
14	13	53113	6078	1795	5672	5095	2177	1194	5717	4711	610	038	670
15	14	53387	6728	2835	5959	5430	2549	1252	6107	4735	680	069	741
16	15	53660	7378	3875	6246	5766	2921	1309	6498	4760	750	101	811
17	16	53934	8028	4915	6533	6101	3293	1367	6888	4784	821	132	881
18	17	54208	8677	5956	6820	6437	3665	1425	7278	4808	891	163	952
19	18	54482	9327	6996	7107	6772	4037	1483	7669	4833	961	194	022
20	19	54756	9977	8036	7394	7108	4410	1540	8059	4857	031	225	093
21	20	55029	0627	9076	7681	7443	4782	1598	8449	4881	101	256	163
22	21	55303	1277	0116	7968	7779	5154	1656	8839	4905	171	287	234
23	22	55577	1927	1157	8255	8114	5526	1713	9230	4930	241	318	304
24	23	55851	2577	2197	8542	8450	5898	1771	9620	4954	312	350	375
25	24	56124	3227	3237	8829	8785	6270	1829	0010	4978	382	381	445
26	25	56398	3877	4277	9116	9121	6642	1886	0401	5003	452	412	516
27	26	56672	4526	5317	9403	9457	7014	1944	0791	5027	522	443	586
28	27	56946	5176	6357	9690	9792	7386	2002	1181	5051	592	474	656
29	28	57220	5826	7398	9977	0128	7758	2060	1571	5075	662	505	727
30	29	57493	6476	8438	0264	0463	8130	2117	1962	5100	733	536	797
31	30	57767	7126	9478	0551	0799	8502	2175	2352	5124	803	567	868
	31	58041	7776	0518	0838	1134	8874	2233	2742	5148	873	599	938

AUGUST.

1	0	58041	7776	0518	0838	1134	8874	2233	2742	5148	873	599	938
2	1	58315	8426	1558	1125	1470	9246	2290	3133	5173	943	630	009
3	2	58588	9076	2599	1412	1805	9618	2348	3523	5197	013	661	079
4	3	58862	9726	3639	1698	2141	9990	2406	3913	5221	083	692	150
5	4	59136	0375	4679	1985	2476	0362	2463	4304	5245	153	723	220
6	5	59410	1025	5719	2272	2812	0734	2521	4694	5270	224	754	291
7	6	59684	1675	6759	2559	3147	1106	2579	5084	5294	294	785	361
8	7	59957	2325	7799	2846	3483	1478	2637	5474	5318	364	816	432
9	8	60231	2975	8840	3133	3819	1850	2694	5865	5343	434	848	502
10	9	60504	3625	9880	3420	4154	2223	2752	6255	5367	504	879	573
11	10	60779	4275	0920	3707	4490	2595	2810	6645	5391	574	910	643
12	11	61052	4925	1960	3994	4825	2967	2867	7036	5415	645	941	714
13	12	61326	5575	3000	4281	5161	3339	2925	7426	5440	715	972	784
14	13	61600	6224	4041	4568	5496	3711	2983	7816	5464	785	003	854
15	14	61874	6874	5081	4855	5832	4083	3040	8206	5488	855	034	925

TABLE III. continued.

Motion for Days. JULY.

Comm.	Bissex.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0	185	948	525	625	613	699	384	083	37	54	33	81	45	68	64
2	1	219	047	555	661	655	725	386	083	38	55	43	84	49	72	71
3	2	254	147	586	698	697	751	388	084	39	56	54	86	53	75	78
4	3	288	246	616	735	739	777	390	084	40	57	64	89	57	79	85
5	4	322	345	647	771	781	802	392	084	40	57	74	92	61	83	92
6	5	356	444	677	808	823	828	394	085	41	58	84	94	65	87	99
7	6	390	543	708	844	866	854	396	085	42	59	94	97	69	90	06
8	7	424	642	738	881	908	880	398	086	43	60	04	00	74	94	13
9	8	459	741	769	918	950	906	401	086	44	61	14	02	78	98	20
10	9	493	841	799	954	992	932	403	087	44	62	24	05	82	01	27
11	10	527	940	830	991	034	958	405	087	45	62	35	08	86	05	34
12	11	561	039	860	027	076	984	407	088	46	63	45	10	90	09	41
13	12	595	138	891	064	118	010	409	088	47	64	55	13	94	12	48
14	13	629	237	921	101	160	036	411	089	47	65	65	16	98	16	55
15	14	664	336	952	137	202	062	413	089	48	66	75	18	02	20	62
16	15	698	436	982	174	244	088	415	089	49	67	85	21	07	23	69
17	16	732	535	013	210	286	114	417	090	50	68	95	24	11	27	76
18	17	766	634	043	247	328	140	420	090	50	68	06	26	15	31	83
19	18	800	733	074	284	370	166	422	091	51	69	16	29	19	35	90
20	19	834	832	104	320	412	192	424	091	52	70	26	32	23	38	97
21	20	869	931	135	357	454	218	426	092	53	71	36	34	27	42	04
22	21	903	031	165	393	496	244	428	092	53	72	46	37	31	46	11
23	22	937	130	196	430	539	270	430	093	54	73	56	40	35	49	18
24	23	971	229	227	467	581	296	432	093	55	74	66	42	39	53	25
25	24	005	328	257	503	623	322	434	094	56	74	76	45	44	57	32
26	25	040	427	288	540	665	348	437	094	56	75	87	48	48	60	39
27	26	074	526	318	576	707	374	439	095	57	76	97	50	52	64	46
28	27	108	626	349	613	749	400	441	095	58	77	07	53	56	68	53
29	28	142	725	379	650	791	425	443	095	59	78	17	56	60	71	60
30	29	176	824	410	686	833	451	445	096	60	79	27	58	64	75	67
31	30	210	923	440	723	875	477	447	096	60	79	37	61	68	79	74
	31	245	022	471	759	917	503	449	097	61	80	47	64	72	82	81

AUGUST.

1	0	245	022	471	759	917	503	449	097	61	80	47	64	72	82	81
2	1	279	121	501	796	959	529	451	097	62	81	57	66	77	86	88
3	2	313	221	532	833	001	555	454	098	63	82	68	69	81	90	95
4	3	347	320	562	869	043	581	456	098	63	83	78	71	85	94	02
5	4	381	419	593	906	085	607	458	099	64	84	88	74	89	97	09
6	5	415	518	623	942	127	633	460	099	65	85	98	77	93	01	16
7	6	450	617	654	979	169	659	462	100	66	85	08	79	97	05	23
8	7	484	716	684	016	212	685	464	100	66	86	18	82	01	08	30
9	8	518	815	715	052	254	711	466	100	67	87	28	85	05	12	37
10	9	552	915	745	089	296	737	468	101	68	88	38	87	09	16	44
11	10	586	014	776	125	338	763	470	101	69	89	49	90	13	19	51
12	11	620	113	806	162	380	789	473	102	69	90	59	93	18	23	58
13	12	655	212	837	199	422	815	475	102	70	91	69	95	22	27	65
14	13	689	311	867	235	464	841	477	103	71	91	79	98	26	30	72
15	14	723	410	898	272	506	867	479	103	72	92	89	01	30	34	79

TABLE III. continued.

Motion for Days. JULY.

Com.	Biss.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
1	0	56	58	31	45	8s 8°17′ 16″	6s 24° 45′ 48″5	1s 16°31′ 32″	7s 14°55′ 39″9
2	1	60	61	31	46	8 19 36 15	7 7 49 42.5	1 28 42 59	7 28 6 15.0
3	2	64	64	31	46	9 0 55 15	7 20 53 36.4	2 10 54 26	8 11 16 50.0
4	3	68	67	31	46	9 12 14 14	8 3 57 30.4	2 23 5 52	8 24 27 25.0
5	4	73	70	32	46	9 23 33 14	8 17 1 24.4	3 5 17 19	9 7 38 0.0
6	5	77	73	32	47	10 4 52 13	9 0 5 18.4	3 17 28 46	9 20 48 35.1
7	6	81	76	32	47	10 16 11 13	9 13 9 12.3	3 29 40 12	10 3 59 10.1
8	7	85	79	32	47	10 27 30 12	9 26 13 6.3	4 11 51 39	10 17 9 45.1
9	8	89	82	32	47	11 8 49 11	10 9 17 0.3	4 24 3 6	11 0 20 20.2
10	9	94	86	32	48	11 20 8 11	10 22 20 54.2	5 6 14 32	11 13 30 55.2
11	10	98	89	33	48	0 1 27 10	11 5 24 48.2	5 18 25 59	11 26 41 30.2
12	11	02	92	33	48	0 12 46 10	11 18 28 42.2	6 0 37 26	0 9 52 5.2
13	12	06	95	33	48	0 24 5 9	0 1 32 36.1	6 12 48 53	0 23 2 40.3
14	13	10	98	33	49	1 5 24 9	0 14 36 30.1	6 25 0 19	1 6 13 15.3
15	14	14	01	33	49	1 16 43 8	0 27 40 24.1	7 7 11 46	1 19 23 50.3
16	15	19	04	34	49	1 28 2 7	1 10 44 18.1	7 19 23 13	2 2 34 25.3
17	16	23	07	34	49	2 9 21 7	1 23 48 12.0	8 1 34 39	2 15 45 0.4
18	17	27	10	34	50	2 20 40 6	2 6 52 6.0	8 13 46 6	2 28 55 35.4
19	18	31	13	34	50	3 1 59 6	2 19 56 0.0	8 25 57 33	3 12 6 10.4
20	19	35	16	34	50	3 13 18 5	3 2 59 53.9	9 8 8 59	3 25 16 45.5
21	20	39	19	34	50	3 24 37 4	3 16 3 47.9	9 20 20 26	4 8 27 20.5
22	21	44	22	35	51	4 5 56 4	3 29 7 41.9	10 2 31 53	4 21 37 55.5
23	22	48	26	35	51	4 17 15 3	4 12 11 35.8	10 14 43 20	5 4 48 30.5
24	23	52	29	35	51	4 28 34 3	4 25 15 29.8	10 26 54 46	5 17 59 5.6
25	24	56	32	35	51	5 9 53 2	5 8 19 23.8	11 9 6 13	6 1 9 40.6
26	25	60	35	35	52	5 21 12 2	5 21 23 17.7	11 21 17 40	6 14 20 15.6
27	26	65	38	35	52	6 2 31 1	6 4 27 11.7	0 3 29 6	6 27 30 50.6
28	27	69	41	36	52	6 13 50 1	6 17 31 5.7	0 15 40 33	7 10 41 25.7
29	28	73	44	36	52	6 25 9 0	7 0 34 59.7	0 27 52 0	7 23 52 0.7
30	29	77	47	36	53	7 6 28 0	7 13 38 53.6	1 10 3 26	8 7 2 35.7
31	30	81	50	36	53	7 17 46 59	7 26 42 47.6	1 22 14 53	8 20 13 10.8
	31	85	53	36	53	7 29 5 59	8 9 46 41.6	2 4 26 20	9 3 23 45.8

AUGUST.

1	0	85	53	36	53	7 29 5 59	8 9 46 41.6	2 4 26 20	9 3 23 45.8
2	1	90	56	36	53	8 10 24 58	8 22 50 35.5	2 16 37 47	9 16 34 20.8
3	2	94	59	37	54	8 21 43 58	9 5 54 29.5	2 28 49 13	9 29 44 55.8
4	3	98	63	37	54	9 3 2 57	9 18 58 23.5	3 11 0 40	10 12 55 30.9
5	4	02	66	37	54	9 14 21 56	10 2 2 17.4	3 23 12 7	10 26 6 5.9
6	5	06	69	37	54	9 25 40 56	10 15 6 11.4	4 5 23 33	11 9 16 40.9
7	6	10	72	37	55	10 6 59 55	10 28 10 5.4	4 17 35 0	11 22 27 15.9
8	7	15	75	37	55	10 18 18 55	11 11 13 59.4	4 29 46 27	0 5 37 51.0
9	8	19	78	38	55	10 29 37 54	11 24 17 53.3	5 11 57 53	0 18 48 26.0
10	9	23	81	38	55	11 10 56 54	0 7 21 47.3	5 24 9 20	1 1 59 1.0
11	10	27	84	38	56	11 22 15 53	0 20 25 41.3	6 6 20 47	1 15 9 36.1
12	11	31	87	38	56	0 3 34 52	1 3 29 35.2	6 18 32 13	1 28 20 11.1
13	12	36	90	38	56	0 14 53 52	1 16 33 29.2	7 0 43 40	2 11 30 46.1
14	13	40	93	38	56	0 26 12 51	1 29 37 23.2	7 12 55 7	2 24 41 21.1
15	14	44	96	39	57	1 7 31 51	2 12 41 17.1	7 25 6 34	3 7 51 56.2

TABLE III. continued.

Motion for Days. JULY.

Com.	Biss.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
1	0	0s 9°35′ 5″2	7s 8°32′	156	147	112	103	486	531	088	962
2	1	0 9 38 15.9	7 19 41	190	186	140	136	522	572	122	967
3	2	0 9 41 26.5	8 0 51	224	226	168	170	558	614	155	972
4	3	0 9 44 37.2	8 12 0	258	265	196	204	594	655	189	978
5	4	0 9 47 47.8	8 23 9	292	305	225	237	630	697	222	983
6	5	0 9 50 58.4	9 4 18	326	344	253	271	666	739	256	988
7	6	0 9 54 9.1	9 15 27	360	384	281	305	701	780	290	994
8	7	0 9 57 19.7	9 26 36	394	423	309	339	737	822	323	999
9	8	0 10 0 30.4	10 7 45	428	463	337	372	773	863	357	004
10	9	0 10 3 41.0	10 18 55	462	502	366	406	809	905	391	010
11	10	0 10 6 51.6	11 0 4	496	542	394	440	845	947	424	015
12	11	0 10 10 2.3	11 11 13	530	581	422	473	881	988	458	020
13	12	0 10 13 12.9	11 22 22	564	621	450	507	916	030	492	025
14	13	0 10 16 23.5	0 3 31	598	660	479	541	952	071	525	031
15	14	0 10 19 34.2	0 14 40	632	700	507	575	988	113	559	036
16	15	0 10 22 44.8	0 25 49	666	739	535	608	024	155	592	041
17	16	0 10 25 55.4	1 6 58	700	779	563	642	060	196	626	047
18	17	0 10 29 6.1	1 18 8	734	818	592	676	096	238	660	052
19	18	0 10 32 16.7	1 29 17	768	858	620	709	131	279	693	057
20	19	0 10 35 27.4	2 10 26	802	897	648	743	167	321	727	063
21	20	0 10 38 38.0	2 21 35	836	937	676	777	203	363	761	068
22	21	0 10 41 48.6	3 2 44	870	976	705	811	239	404	794	073
23	22	0 10 44 59.3	3 13 53	904	016	733	844	275	446	828	079
24	23	0 10 48 9.9	3 25 2	938	055	761	878	311	488	862	084
25	24	0 10 51 20.5	4 6 11	972	095	789	911	346	529	895	089
26	25	0 10 54 31.2	4 17 21	006	134	818	945	382	571	929	095
27	26	0 10 57 41.8	4 28 30	040	174	846	979	418	612	962	100
28	27	0 11 0 52.4	5 9 39	074	213	874	013	454	654	996	105
29	28	0 11 4 3.1	5 20 48	108	253	902	047	490	696	030	111
30	29	0 11 7 13.7	6 1 57	142	292	931	080	526	737	063	116
31	30	0 11 10 24.4	6 13 6	176	332	959	114	561	779	097	121
	31	0 11 13 35.0	6 24 15	210	371	987	147	597	820	131	126

AUGUST.

1	0	0 11 13 35.0	6 24 15	210	371	987	147	597	820	131	126
2	1	0 11 16 45.6	7 5 24	244	411	015	182	633	862	164	132
3	2	0 11 19 56.3	7 16 34	278	450	043	215	669	904	198	137
4	3	0 11 23 6.9	7 27 43	312	490	072	249	705	945	232	142
5	4	0 11 26 17.5	8 8 52	346	529	100	283	741	987	265	148
6	5	0 11 29 28.2	8 20 1	380	568	128	316	776	028	299	153
7	6	0 11 32 38.8	9 1 10	414	608	156	350	812	070	332	158
8	7	0 11 35 49.4	9 12 19	448	647	185	383	848	112	366	164
9	8	0 11 39 0.1	9 23 28	482	687	213	418	884	153	400	169
10	9	0 11 42 10.7	10 4 38	516	726	241	451	920	195	433	174
11	10	0 11 45 21.4	10 15 47	550	766	269	485	956	236	467	180
12	11	0 11 48 32.0	10 26 56	584	805	298	519	991	278	501	185
13	12	0 11 51 42.6	11 8 5	618	845	326	552	027	320	534	190
14	13	0 11 54 53.3	11 19 14	652	884	354	586	063	361	568	196
15	14	0 11 58 3.9	0 0 23	686	924	382	619	099	403	601	201

TABLE III. continued.

Motion for Days. AUGUST.

Com.	Biss.	1	2	3	4	5	6	7	8	9	10	11	12
16	15	62148	7524	6121	5142	6167	4455	3098	8597	5513	925	065	995
17	16	62421	8174	7161	5429	6503	4827	3156	8987	5537	995	097	066
18	17	62695	8824	8201	5716	6838	5199	3214	9377	5561	065	128	136
19	18	62969	9474	9241	6003	7174	5571	3271	9768	5585	136	159	207
20	19	63243	0124	0282	6290	7509	5943	3329	0158	5610	206	190	277
21	20	63516	0774	1322	6577	7845	6315	3387	0548	5634	276	221	348
22	21	63790	1423	2362	6864	8181	6687	3444	0939	5658	346	252	418
23	22	64064	2073	3402	7151	8516	7059	3502	1329	5683	416	283	489
24	23	64338	2723	4442	7438	8852	7431	3560	1719	5707	486	314	559
25	24	64612	3373	5483	7725	9187	7803	3617	2110	5731	557	346	629
26	25	64885	4023	6523	8012	9523	8175	3675	2500	5755	627	377	700
27	26	65159	4673	7563	8299	9858	8547	3733	2890	5780	697	408	770
28	27	65433	5323	8603	8586	0194	8919	3791	3280	5804	767	439	841
29	28	65707	5973	9643	8873	0529	9291	3848	3671	5828	837	470	911
30	29	65980	6623	0683	9160	0865	9663	3906	4061	5853	907	501	982
31	30	66254	7272	1724	9447	1200	0036	3964	4451	5877	977	532	052
	31	66528	7922	2764	9734	1536	0408	4021	4842	5901	048	563	123

SEPTEMBER.

1	0	66528	7922	2764	9734	1536	0408	4021	4842	5901	048	563	123
2	1	66802	8572	3804	0021	1871	0780	4079	5232	5925	118	595	193
3	2	67076	9222	4844	0308	2207	1152	4137	5622	5950	188	626	264
4	3	67349	9872	5884	0595	2542	1524	4194	6012	5974	258	657	334
5	4	67623	0522	6925	0882	2878	1896	4252	6403	5998	328	688	405
6	5	67897	1172	7965	1168	3214	2268	4310	6793	6023	398	719	475
7	6	68171	1822	9005	1455	3549	2640	4368	7183	6047	469	750	546
8	7	68444	2472	0045	1742	3885	3012	4425	7574	6071	539	781	616
9	8	68718	3121	1085	2029	4220	3384	4483	7964	6095	609	812	687
10	9	68992	3771	2125	2316	4556	3756	4541	8354	6120	679	844	757
11	10	69266	4421	3166	2603	4891	4128	4598	8744	6144	749	875	827
12	11	69540	5071	4206	2890	5227	4500	4656	9135	6168	819	906	898
13	12	69813	5721	5246	3177	5562	4872	4714	9525	6193	889	937	968
14	13	70087	6371	6286	3464	5898	5244	4772	9915	6217	960	968	039
15	14	70361	7021	7326	3751	6233	5616	4829	0306	6241	030	999	109
16	15	70635	7671	8367	4038	6569	5988	4887	0696	6265	100	030	180
17	16	70908	8321	9407	4325	6905	6360	4945	1086	6290	170	061	250
18	17	71182	8970	0447	4612	7240	6732	5002	1476	6314	240	093	321
19	18	71456	9620	1487	4899	7576	7104	5060	1867	6338	310	124	391
20	19	71730	0270	2527	5186	7911	7476	5118	2257	6362	381	155	462
21	20	72004	0920	3567	5473	8247	7849	5175	2647	6387	451	186	532
22	21	72277	1570	4608	5760	8582	8221	5233	3038	6411	521	217	602
23	22	72551	2220	5648	6047	8918	8593	5291	3428	6435	591	248	673
24	23	72824	2870	6688	6334	9253	8965	5349	3818	6460	661	279	743
25	24	73099	3520	7728	6621	9589	9337	5406	4209	6484	731	310	814
26	25	73372	4170	8768	6908	9924	9709	5464	4599	6508	801	342	884
27	26	73646	4819	9809	7195	0260	0081	5522	4989	6533	872	373	955
28	27	73920	5469	0849	7482	0595	0453	5579	5379	6557	942	404	025
29	28	74194	6119	1889	7769	0931	0825	5637	5770	6581	012	435	096
30	29	74468	6769	2929	8056	1267	1197	5695	6160	6605	082	466	166
	30	74741	7419	3969	8343	1602	1569	5752	6550	6630	152	497	237

TABLE III. continued.

Motion for Days. AUGUST.

Com.	Biss.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
16	15	757	510	929	308	548	893	481	104	72	93	99	03	34	38	86
17	16	791	609	959	345	590	919	483	104	73	94	09	06	38	41	92
18	17	825	708	990	382	632	945	485	105	74	95	19	09	42	45	99
19	18	860	807	020	418	674	971	487	105	75	96	30	11	46	49	06
20	19	894	906	051	455	716	997	490	105	75	96	40	14	51	53	13
21	20	928	005	081	491	758	023	492	106	76	97	50	17	55	56	20
22	21	962	105	112	528	800	049	494	106	77	98	60	19	59	60	27
23	22	996	204	142	565	842	074	496	107	78	99	70	22	63	64	34
24	23	031	303	173	601	884	100	498	107	78	00	80	25	67	67	41
25	24	065	402	203	638	927	126	500	108	79	01	90	27	71	71	48
26	25	099	501	234	674	969	152	502	108	80	02	01	30	75	75	55
27	26	133	600	264	711	011	178	504	109	81	02	11	33	79	78	62
28	27	167	700	295	748	053	204	507	109	82	03	21	35	84	82	69
29	28	201	799	325	784	095	230	509	110	82	04	31	38	88	86	76
30	29	236	898	356	821	137	256	511	110	83	05	41	41	92	89	83
31	30	270	997	386	857	179	282	513	111	84	06	51	43	96	93	90
	31	304	096	417	894	221	308	515	111	85	07	61	46	00	97	97

SEPTEMBER.

1	0	304	096	417	894	221	308	515	111	85	07	61	46	00	97	97
2	1	338	195	447	931	263	334	517	111	85	08	71	49	04	01	04
3	2	372	295	478	967	305	360	519	112	86	08	82	51	08	04	11
4	3	406	394	508	004	347	386	521	112	87	09	92	54	12	08	18
5	4	441	493	539	040	389	412	523	113	88	10	02	56	16	12	25
6	5	475	592	570	077	431	438	526	113	88	11	12	59	21	15	32
7	6	509	691	600	114	473	464	528	114	89	12	22	62	25	19	39
8	7	543	790	631	150	515	490	530	114	90	13	32	64	29	23	46
9	8	577	889	662	187	557	516	532	115	91	13	42	67	33	26	53
10	9	611	989	692	223	600	542	534	115	91	14	52	70	37	30	60
11	10	646	088	723	260	642	568	536	116	92	15	63	72	41	34	67
12	11	680	187	753	297	684	594	538	116	93	16	73	75	45	37	74
13	12	714	286	784	333	726	620	540	117	94	17	83	78	49	41	81
14	13	748	385	814	370	768	646	542	117	94	18	93	80	53	45	88
15	14	782	484	845	406	810	672	545	118	95	19	03	83	58	49	95
16	15	816	584	875	443	852	697	547	118	96	19	13	86	62	52	02
17	16	851	683	906	480	894	723	549	119	97	20	23	88	66	56	09
18	17	885	782	936	516	936	749	551	119	97	21	33	91	70	60	16
19	18	919	881	967	553	978	775	553	120	98	22	44	94	74	63	23
20	19	953	980	997	589	020	801	555	120	99	23	54	96	78	67	30
21	20	987	079	028	626	062	827	557	121	00	24	64	99	82	71	37
22	21	022	179	058	663	104	853	559	121	01	25	74	02	86	74	44
23	22	056	278	089	699	146	879	562	121	01	25	84	04	90	78	51
24	23	090	377	119	736	188	905	564	122	02	26	94	07	95	82	58
25	24	124	476	150	772	230	931	566	122	03	27	04	10	99	85	65
26	25	158	575	180	809	273	957	568	123	04	28	14	12	03	89	72
27	26	192	674	211	846	315	983	570	123	04	29	25	15	07	93	79
28	27	227	774	241	882	357	009	572	124	05	30	35	18	11	97	86
29	28	261	873	272	919	399	035	574	124	06	31	45	20	15	00	93
30	29	295	972	302	955	441	061	576	125	07	31	55	23	19	04	00
	30	329	071	333	992	483	087	578	125	07	32	65	26	23	08	07

TABLE III. continued.

Motion for Days. AUGUST.

Com.	Biss.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
16	15	48	00	39	57	1s 18° 50′ 50″	2s 25° 45′ 11″1	8s 7° 18′ 0″	3s 21° 2′ 31″2
17	16	52	03	39	57	2 0 9 50	3 8 49 5.1	8 19 29 27	4 4 13 6.2
18	17	56	06	39	57	2 11 28 49	3 21 52 59.1	9 1 40 54	4 17 23 41.2
19	18	61	09	39	58	2 22 47 48	4 4 56 53.0	9 13 52 20	5 0 34 16.3
20	19	65	12	40	58	3 4 6 48	4 18 0 47.0	9 26 3 47	5 13 44 51.3
21	20	69	15	40	58	3 15 25 47	5 1 4 41.0	10 8 15 14	5 26 55 26.3
22	21	73	18	40	58	3 26 44 47	5 14 8 34.9	10 20 26 40	6 10 6 1.4
23	22	77	21	40	59	4 8 3 46	5 27 12 28.9	11 2 38 7	6 23 16 36.4
24	23	81	24	40	59	4 19 22 46	6 10 16 22.9	11 14 49 34	7 6 27 11.4
25	24	86	27	40	59	5 0 41 45	6 23 20 16.8	11 27 1 1	7 19 37 46.4
26	25	90	30	41	59	5 12 0 44	7 6 24 10.8	0 9 12 27	8 2 48 21.5
27	26	94	33	41	60	5 23 19 44	7 19 28 4.8	0 21 23 54	8 15 58 56.5
28	27	98	37	41	60	6 4 38 43	8 2 31 58.7	1 3 35 21	8 29 9 31.5
29	28	02	40	41	60	6 15 57 43	8 15 35 52.7	1 15 46 47	9 12 20 6.5
30	29	07	43	41	60	6 27 16 42	8 28 39 46.7	1 27 58 14	9 25 30 41.6
31	30	11	46	41	61	7 8 35 41	9 11 43 40.7	2 10 9 41	10 8 41 16.6
	31	15	49	42	61	7 19 54 41	9 24 47 34.6	2 22 21 7	10 21 51 51.6

SEPTEMBER.

1	0	15	49	42	61	7 19 54 41	9 24 47 34.6	2 22 21 7	10 21 51 51.6
2	1	19	52	42	61	8 1 13 40	10 7 51 28.6	3 4 32 34	11 5 2 26.7
3	2	23	55	42	61	8 12 32 40	10 20 55 22.6	3 16 44 1	11 18 13 1.7
4	3	27	58	42	62	8 23 51 39	11 3 59 16.5	3 28 55 28	0 1 23 36.7
5	4	32	61	42	62	9 5 10 39	11 17 3 10.5	4 11 6 54	0 14 34 11.7
6	5	36	64	42	62	9 16 29 38	0 0 7 4.5	4 23 18 21	0 27 44 46.8
7	6	40	67	43	62	9 27 48 37	0 13 10 58.4	5 5 29 48	1 10 55 21.8
8	7	44	70	43	63	10 9 7 37	0 26 14 52.4	5 17 41 14	1 24 5 56.8
9	8	48	73	43	63	10 20 26 36	1 9 18 46.4	5 29 52 41	2 7 16 31.8
10	9	52	77	43	63	11 1 45 36	1 22 22 40.4	6 12 4 8	2 20 27 6.9
11	10	57	80	43	63	11 13 4 35	2 5 26 34.3	6 24 15 34	3 3 37 41.9
12	11	61	83	43	64	11 24 23 35	2 18 30 28.3	7 6 27 1	3 16 48 16.9
13	12	65	86	44	64	0 5 42 34	3 1 34 22.3	7 18 38 28	3 29 58 52.0
14	13	69	89	44	64	0 17 1 33	3 14 38 16.2	8 0 49 54	4 13 9 27.0
15	14	73	92	44	64	0 28 20 33	3 27 42 10.2	8 13 1 21	4 26 20 2.0
16	15	78	95	44	65	1 9 39 32	4 10 46 4.2	8 25 12 48	5 9 30 37.0
17	16	82	98	44	65	1 20 58 32	4 23 49 58.1	9 7 24 15	5 22 41 12.1
18	17	86	01	44	65	2 2 17 31	5 6 53 52.1	9 19 35 41	6 5 51 47.1
19	18	90	04	45	65	2 13 36 31	5 19 57 46.1	10 1 47 8	6 19 2 22.1
20	19	94	07	45	66	2 24 55 30	6 3 1 40.1	10 13 58 35	7 2 12 57.1
21	20	98	10	45	66	3 6 14 29	6 16 5 34.0	10 26 10 1	7 15 23 32.2
22	21	03	14	45	66	3 17 33 29	6 29 9 28.0	11 8 21 28	7 28 34 7.2
23	22	07	17	45	66	3 28 52 28	7 12 13 22.0	11 20 32 55	8 11 44 42.2
24	23	11	20	46	67	4 10 11 28	7 25 17 15.9	0 2 44 21	8 24 55 17.3
25	24	15	23	46	67	4 21 30 27	8 8 21 9.9	0 14 55 48	9 8 5 52.3
26	25	19	26	46	67	5 2 49 27	8 21 25 3.9	0 27 7 15	9 21 16 27.3
27	26	23	29	46	67	5 14 8 26	9 4 28 57.8	1 9 18 42	10 4 27 2.3
28	27	28	32	46	68	5 25 27 25	9 17 32 51.8	1 21 30 8	10 17 37 37.4
29	28	32	35	46	68	6 6 46 25	10 0 36 45.8	2 3 41 35	11 0 48 12.4
30	29	36	38	47	68	6 18 5 24	10 13 40 39.7	2 15 53 2	11 13 58 47.4
	30	40	41	47	68	6 29 24 24	10 26 44 33.7	2 28 4 28	11 27 9 22.4

TABLE III. continued.

Motion for Days. AUGUST.

Com.	Biss.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
16	15	0s12° 1 14′5	0s11°32′	720	963	411	654	155	444	635	206
17	16	0 12 4 25.2	0 22 41	754	003	439	687	171	486	669	211
18	17	0 12 7 35.8	1 3 51	788	042	467	721	205	528	702	217
19	18	0 12 10 46.4	1 15 0	822	082	495	755	242	569	736	222
20	19	0 12 13 57.1	1 26 9	856	121	524	788	278	611	770	227
21	20	0 12 17 7.7	2 7 18	890	161	552	822	314	652	803	233
22	21	0 12 20 18.4	2 18 27	924	200	580	855	350	694	837	238
23	22	0 12 23 29.0	2 29 36	958	240	608	890	386	736	871	243
24	23	0 12 26 39.6	3 10 45	992	279	637	923	421	777	904	249
25	24	0 12 29 50.3	3 21 54	026	319	665	957	457	819	938	254
26	25	0 12 33 0.9	4 3 4	060	358	693	991	493	860	971	259
27	26	0 12 36 11.5	4 14 13	094	398	721	024	529	902	005	265
28	27	0 12 39 22.2	4 25 22	128	437	749	058	565	944	039	270
29	28	0 12 42 32.8	5 6 31	162	477	778	091	601	985	072	275
30	29	0 12 45 43.4	5 17 40	197	516	806	126	636	027	106	281
31	30	0 12 48 54.1	5 28 49	231	556	834	159	672	069	140	286
	31	0 12 52 4.7	6 9 58	265	595	862	193	708	110	173	291

SEPTEMBER.

1	0	0 12 52 4.7	6 9 58	265	595	862	193	708	110	173	291
2	1	0 12 55 15.4	6 21 7	299	635	891	227	744	152	207	296
3	2	0 12 58 26.0	7 2 17	333	674	919	260	780	193	241	302
4	3	0 13 1 36.6	7 13 26	367	714	947	294	816	235	274	307
5	4	0 13 4 47.3	7 24 35	401	753	975	328	852	277	308	312
6	5	0 13 7 57.9	8 5 44	435	793	004	362	887	318	341	318
7	6	0 13 11 8.5	8 16 53	469	832	032	395	923	360	375	323
8	7	0 13 14 19.2	8 28 2	503	872	060	429	959	401	409	328
9	8	0 13 17 29.8	9 9 11	537	911	088	463	995	443	442	334
10	9	0 13 20 40.4	9 20 21	571	950	117	496	031	485	476	339
11	10	0 13 23 51.1	10 1 30	605	990	145	530	067	526	510	344
12	11	0 13 27 1.7	10 12 39	639	029	173	564	102	568	543	350
13	12	0 13 30 12.4	10 23 48	673	069	201	598	138	609	577	355
14	13	0 13 33 23.0	11 4 57	707	108	230	631	174	651	611	360
15	14	0 13 36 33.6	11 16 6	741	148	258	665	210	693	644	366
16	15	0 13 39 44.3	11 27 15	775	187	286	699	246	734	678	371
17	16	0 13 42 54.9	0 8 24	809	227	314	732	282	776	711	376
18	17	0 13 46 5.5	0 19 34	843	266	343	766	317	817	745	381
19	18	0 13 49 16.2	1 0 43	877	306	371	800	353	859	779	387
20	19	0 13 52 26.8	1 11 52	911	345	399	834	389	901	812	392
21	20	0 13 55 37.4	1 23 1	945	385	427	867	425	942	846	397
22	21	0 13 58 48.1	2 4 10	979	424	455	901	461	984	880	403
23	22	0 14 1 58.7	2 15 19	013	464	484	935	497	025	913	408
24	23	0 14 5 9.4	2 26 28	047	503	512	968	532	067	947	413
25	24	0 14 8 20.0	3 7 37	081	543	540	002	568	109	981	419
26	25	0 14 11 30.6	3 18 47	115	582	568	036	604	150	014	424
27	26	0 14 14 41.3	3 29 56	149	622	597	070	640	192	048	429
28	27	0 14 17 51.9	4 11 5	183	661	625	103	676	233	081	435
29	28	0 14 21 2.5	4 22 14	217	701	653	137	712	275	115	440
30	29	0 14 24 13.2	5 3 23	251	740	681	171	747	317	149	445
	30	0 14 27 23.8	5 14 32	285	780	710	204	783	358	182	451

TABLE III. continued.

Motion for Days. OCTOBER.

Com.	Biss.	1	2	3	4	5	6	7	8	9	10	11	12
1	0	74741	7419	3969	8343	1602	1569	5752	6550	6630	152	497	237
2	1	75015	8069	5009	8630	1938	1941	5810	6941	6654	222	528	307
3	2	75289	8719	6050	8917	2273	2313	5868	7331	6678	293	559	378
4	3	75563	9369	7090	9204	2609	2685	5926	7721	6703	363	590	448
5	4	75836	0018	8130	9491	2944	3057	5983	8112	6727	433	622	519
6	5	76110	0668	9170	9778	3280	3429	6041	8502	6751	503	653	589
7	6	76384	1318	0210	0065	3615	3801	6099	8892	6775	573	684	660
8	7	76658	1968	1250	0352	3951	4173	6156	9282	6800	643	715	730
9	8	76932	2618	2291	0638	4286	4545	6214	9673	6824	713	746	800
10	9	77205	3268	3331	0925	4622	4917	6272	0063	6848	784	777	871
11	10	77479	3918	4371	1212	4957	5289	6329	0453	6873	854	808	941
12	11	77753	4568	5411	1499	5293	5662	6387	0844	6897	924	840	012
13	12	78027	5218	6451	1786	5628	6034	6445	1234	6921	994	871	082
14	13	78300	5867	7492	2073	5964	6406	6502	1624	6945	064	902	153
15	14	78574	6517	8532	2360	6300	6778	6560	2014	6970	134	933	223
16	15	78848	7167	9572	2647	6635	7150	6618	2405	6994	205	964	294
17	16	79122	7817	0612	2934	6971	7522	6676	2795	7018	275	995	364
18	17	79396	8467	1652	3221	7306	7894	6733	3185	7043	345	026	435
19	18	79669	9117	2692	3508	7642	8266	6791	3576	7067	415	057	505
20	19	79943	9767	3733	3795	7977	8638	6849	3966	7091	485	089	575
21	20	80217	0417	4773	4082	8313	9010	6906	4356	7115	555	120	646
22	21	80491	1067	5813	4369	8648	9382	6964	4746	7140	625	151	716
23	22	80764	1716	6853	4656	8984	9754	7022	5137	7164	696	182	787
24	23	81038	2366	7893	4943	9319	0126	7080	5527	7188	766	213	857
25	24	81312	3016	8934	5230	9655	0498	7137	5917	7212	836	244	928
26	25	81586	3666	9974	5517	9990	0870	7195	6308	7237	906	275	998
27	26	81860	4316	1014	5804	0326	1242	7253	6698	7261	976	306	069
28	27	82133	4966	2054	6091	0662	1614	7310	7088	7285	047	338	139
29	28	82407	5616	3094	6378	0997	1986	7368	7479	7310	117	369	210
30	29	82681	6266	4134	6665	1333	2358	7426	7869	7334	187	400	280
31	30	82955	6916	5175	6952	1668	2730	7483	8259	7358	257	431	351
	31	83228	7565	6215	7239	2004	3102	7541	8649	7382	327	462	421

NOVEMBER.

1	0	83228	7565	6215	7239	2004	3102	7541	8649	7382	327	462	421
2	1	83502	8215	7255	7526	2339	3475	7599	9040	7407	397	493	492
3	2	83776	8865	8295	7813	2675	3847	7657	9430	7431	467	524	562
4	3	84050	9515	9335	8100	3010	4219	7714	9820	7455	538	555	633
5	4	84324	0165	0376	8387	3346	4591	7772	0211	7480	608	587	703
6	5	84597	0815	1416	8674	3681	4963	7830	0601	7504	678	618	773
7	6	84871	1465	2456	8961	4017	5335	7887	0991	7528	748	649	844
8	7	85144	2115	3496	9248	4353	5707	7945	1381	7552	818	680	914
9	8	85419	2764	4536	9535	4688	6079	8003	1772	7577	888	711	985
10	9	85692	3414	5576	9822	5024	6451	8060	2162	7601	959	742	055
11	10	85966	4064	6617	0108	5359	6823	8118	2552	7625	029	773	126
12	11	86240	4714	7657	0395	5695	7195	8176	2943	7650	099	804	196
13	12	86514	5364	8697	0682	6030	7567	8234	3333	7674	169	836	267
14	13	86788	6014	9737	0969	6366	7939	8291	3723	7698	239	867	337
15	14	87061	6664	0777	1256	6701	8311	8349	4114	7722	309	898	408

TABLE III. continued.

Motion for Days. OCTOBER.

Com.	Biss.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0	329	071	333	992	483	087	578	125	07	32	65	26	23	08	07
2	1	363	170	363	029	525	113	581	126	08	33	75	28	28	11	14
3	2	397	269	394	065	567	139	583	126	09	34	85	31	32	15	21
4	3	432	369	424	102	609	165	585	126	10	35	96	34	36	19	28
5	4	466	468	455	138	651	191	587	127	10	36	06	36	40	22	35
6	5	500	567	485	175	693	217	589	127	11	37	16	39	44	26	42
7	6	534	666	516	212	735	243	591	128	12	37	26	42	48	30	49
8	7	568	765	546	248	777	269	593	128	13	38	36	44	52	33	56
9	8	602	864	577	285	819	295	595	129	13	39	46	47	56	37	63
10	9	637	963	607	321	861	321	598	129	14	40	56	50	60	41	70
11	10	671	063	638	358	903	346	600	130	15	41	66	52	65	45	77
12	11	705	162	668	395	946	372	602	130	16	42	77	55	69	48	84
13	12	739	261	699	431	988	398	604	131	16	43	87	57	73	52	91
14	13	773	360	729	468	030	424	606	131	17	44	97	60	77	56	98
15	14	807	459	760	504	072	450	608	131	18	45	07	63	81	59	05
16	15	842	558	790	541	114	476	610	132	19	46	17	66	85	63	12
17	16	876	658	821	578	156	502	612	132	19	47	27	68	89	67	19
18	17	910	757	851	614	198	528	615	133	20	48	37	71	93	70	26
19	18	944	856	882	651	240	554	617	133	21	48	47	73	97	74	33
20	19	978	955	912	687	282	580	619	134	22	49	58	76	02	78	40
21	20	013	054	943	724	324	606	621	134	23	50	68	79	06	81	46
22	21	047	153	973	761	366	632	623	135	23	51	78	81	10	85	53
23	22	081	253	004	797	408	658	625	135	24	52	88	84	14	89	60
24	23	115	352	035	834	450	684	627	136	25	23	98	87	18	93	67
25	24	149	451	065	870	492	710	629	136	26	54	08	89	22	96	74
26	25	183	550	096	907	534	736	631	137	26	54	18	92	26	00	81
27	26	218	649	126	944	576	762	634	137	27	55	28	95	30	04	88
28	27	252	748	157	980	619	788	636	137	28	56	39	97	35	07	95
29	28	286	848	187	017	661	814	638	138	29	57	49	00	39	11	02
30	29	320	947	218	053	703	840	640	138	29	58	59	03	43	15	09
31	30	354	046	248	090	745	866	641	139	30	59	69	05	47	18	16
	31	388	145	279	127	787	892	644	139	31	59	79	08	51	22	23

NOVEMBER.

1	0	388	145	279	127	787	892	644	139	31	59	79	08	51	22	20
2	1	423	244	309	163	829	918	646	140	32	60	89	11	55	26	30
3	2	457	343	340	200	871	944	648	140	32	61	99	13	59	29	37
4	3	491	443	370	236	913	969	651	141	33	62	09	16	63	33	44
5	4	525	542	401	273	955	995	653	141	34	63	20	19	67	37	51
6	5	559	641	431	310	997	021	655	142	35	64	30	21	72	41	58
7	6	593	740	462	346	039	047	657	142	35	65	40	24	76	44	65
8	7	628	839	492	383	081	073	659	142	36	65	50	27	80	48	72
9	8	662	938	523	419	123	099	661	143	37	66	60	29	84	52	79
10	9	696	037	553	456	165	125	663	143	38	67	70	32	88	55	86
11	10	730	137	584	493	207	151	665	144	39	68	80	35	92	59	93
12	11	764	236	614	529	249	177	668	144	39	69	91	37	96	63	00
13	12	798	335	645	566	291	203	670	145	40	70	01	40	00	66	07
14	13	833	434	675	602	334	229	672	145	41	71	11	43	05	70	14
15	14	867	533	706	639	376	255	674	146	42	71	21	45	09	74	21

TABLE III. continued.

Motion for Days. OCTOBER.

Com.	Biss.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
1	0	40	41	47	68	6s 29° 24′ 24″	10s 26° 44′ 33″7	2s 28° 4′ 28″	11s 27° 9′ 22″4
2	1	44	44	47	69	7 10 43 23	11 9 48 27.7	3 10 15 55	0 10 19 57.5
3	2	49	47	47	69	7 22 2 23	11 22 52 21.7	3 22 27 22	0 23 30 32.5
4	3	53	51	47	69	8 3 21 22	0 5 56 15.6	4 4 38 48	1 6 41 7.5
5	4	57	54	47	69	8 14 40 21	0 19 0 9.6	4 16 50 15	1 19 51 42.6
6	5	61	57	48	70	8 25 59 21	1 2 4 3.6	4 29 1 42	2 3 2 17.6
7	6	65	60	48	70	9 7 18 20	1 15 7 57.5	5 11 13 9	2 16 12 52.6
8	7	69	63	48	70	9 18 37 20	1 28 11 51.5	5 23 24 35	2 29 23 27.6
9	8	74	66	48	70	9 29 56 19	2 11 15 45.5	6 5 36 2	3 12 34 2.7
10	9	78	69	48	71	10 11 15 18	2 24 19 39.4	6 17 47 29	3 25 44 37.7
11	10	82	72	48	71	10 22 34 18	3 7 23 33.4	6 29 58 55	4 8 55 12.7
12	11	86	75	49	71	11 3 53 17	3 20 27 27.4	7 12 10 22	4 22 5 47.7
13	12	90	78	49	71	11 15 12 17	4 3 31 21.4	7 24 21 49	5 5 16 22.8
14	13	94	81	49	72	11 26 31 16	4 16 35 15.3	8 6 33 15	5 18 26 57.8
15	14	99	84	49	72	0 7 50 16	4 29 39 9.3	8 18 44 42	6 1 37 32.8
16	15	03	88	49	72	0 19 9 15	5 12 43 3.3	9 0 56 9	6 14 48 7.9
17	16	07	91	49	72	1 0 28 14	5 25 46 57.2	9 13 7 36	6 27 58 42.9
18	17	11	94	50	73	1 11 47 14	6 8 50 51.2	9 25 19 2	7 11 9 17.9
19	18	15	97	50	73	1 23 6 13	6 21 54 45.2	10 7 30 29	7 24 19 52.9
20	19	20	00	50	73	2 4 25 13	7 4 58 39.1	10 19 41 56	8 7 30 28.0
21	20	24	03	50	73	2 15 44 12	7 18 2 33.1	11 1 53 22	8 20 41 3.0
22	21	28	06	50	74	2 27 3 12	8 1 6 27.1	11 14 4 49	9 3 51 38.0
23	22	32	09	50	74	3 8 22 11	8 14 10 21.1	11 26 16 16	9 17 2 13.0
24	23	36	12	51	74	3 19 41 10	8 27 14 15.0	0 8 27 42	10 0 12 48.1
25	24	40	15	51	74	4 1 0 10	9 10 18 9.0	0 20 39 9	10 13 23 23.1
26	25	45	18	51	75	4 12 19 9	9 23 22 3.0	1 2 50 36	10 26 33 58.1
27	26	49	21	51	75	4 23 38 9	10 6 25 56.9	1 15 2 2	11 9 44 33.2
28	27	53	24	51	75	5 4 57 8	10 19 29 50.9	1 27 13 29	11 22 55 8.2
29	28	57	28	51	75	5 16 16 8	11 2 33 44.9	2 9 24 56	0 6 5 43.2
30	29	61	31	52	76	5 27 35 7	11 15 37 38.8	2 21 36 23	0 19 16 18.2
31	30	65	34	52	76	6 8 54 6	11 28 41 32.8	3 3 47 49	1 2 26 53.3
	31	70	37	52	76	6 20 13 6	0 11 45 26.8	3 15 59 16	1 15 37 28.3

NOVEMBER.

1	0	70	37	52	76	6 20 13 6	0 11 45 26.8	3 15 59 16	1 15 37 28.3
2	1	74	40	52	76	7 1 32 5	0 24 49 20.7	3 28 10 43	1 28 48 3.3
3	2	78	43	52	77	7 12 51 5	1 7 53 14.7	4 10 22 9	2 11 58 38.3
4	3	82	46	53	77	7 24 10 4	1 20 57 8.7	4 22 33 36	2 25 9 13.4
5	4	86	49	53	77	8 5 29 4	2 4 1 2.7	5 4 45 3	3 8 19 48.4
6	5	91	52	53	77	8 16 48 3	2 17 4 56.6	5 16 56 29	3 21 30 23.4
7	6	95	55	53	78	8 28 7 2	3 0 8 50.6	5 29 7 56	4 4 40 58.5
8	7	99	58	53	78	9 9 26 2	3 13 12 44.6	6 11 19 23	4 17 51 33.5
9	8	03	61	53	78	9 20 45 1	3 26 16 38.5	6 23 30 50	5 1 2 8.5
10	9	07	65	54	78	10 2 4 1	4 9 20 32.5	7 5 42 16	5 14 12 43.5
11	10	11	68	54	79	10 13 23 0	4 22 24 26.5	7 17 53 43	5 27 23 18.6
12	11	16	71	54	79	10 24 42 0	5 5 28 20.4	8 0 5 10	6 10 33 53.6
13	12	20	74	54	79	11 6 0 59	5 18 32 14.4	8 12 16 36	6 23 44 28.6
14	13	24	77	54	79	11 17 19 58	6 1 36 8.4	8 24 28 3	7 6 55 3.6
15	14	28	80	54	80	11 28 38 58	6 14 40 2.4	9 6 39 30	7 20 5 38.7

TABLE III. continued.

Motion for Days. OCTOBER.

Com.	Biss.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI	XII
1	0	0s 14°27′ 23″8	5s 14°32′	285	780	710	204	783	358	182	451
2	1	0 14 30 34.4	5 25 41	319	819	738	238	819	400	216	456
3	2	0 14 33 45.1	6 6 50	353	859	766	272	855	441	250	461
4	3	0 14 36 55.7	6 18 0	387	898	794	306	891	483	283	467
5	4	0 14 40 6.4	6 29 9	421	938	823	339	927	525	317	472
6	5	0 14 43 17.0	7 10 18	455	977	851	373	962	566	351	477
7	6	0 14 46 27.6	7 21 27	489	017	879	407	998	608	384	482
8	7	0 14 49 38.3	8 2 36	523	056	907	440	034	650	418	488
9	8	0 14 52 48.9	8 13 45	557	096	936	474	070	691	451	493
10	9	0 14 55 59.5	8 24 54	591	135	964	508	106	733	485	498
11	10	0 14 59 10.2	9 6 4	625	175	992	542	142	774	519	504
12	11	0 15 2 20.8	9 17 13	659	214	020	575	177	816	552	509
13	12	0 15 5 31.4	9 28 22	693	254	049	609	213	858	586	514
14	13	0 15 8 42.1	10 9 31	727	293	077	642	249	899	620	520
15	14	0 15 11 52.7	10 20 40	761	333	105	677	285	941	653	525
16	15	0 15 15 3.4	11 1 49	795	372	133	710	321	982	687	530
17	16	0 15 18 14.0	11 12 58	829	411	162	744	357	024	720	536
18	17	0 15 21 24.6	11 24 7	863	451	190	778	392	066	754	541
19	18	0 15 24 35.3	0 5 17	897	490	218	811	428	107	788	546
20	19	0 15 27 45.9	0 16 26	931	530	246	845	464	149	821	552
21	20	0 15 30 56.5	0 27 35	965	569	274	878	500	190	855	557
22	21	0 15 34 7.2	1 8 44	999	609	303	913	536	232	889	562
23	22	0 15 37 17.8	1 19 53	033	648	331	946	572	274	922	567
24	23	0 15 40 28.5	2 1 2	067	688	359	980	607	315	956	573
25	24	0 15 43 39.1	2 12 11	101	727	387	014	643	357	990	578
26	25	0 15 46 49.7	2 23 20	135	767	416	047	679	398	023	583
27	26	0 15 50 00.4	3 4 30	169	806	444	081	715	440	057	589
28	27	0 15 53 11.0	3 15 39	203	846	472	114	751	482	090	594
29	28	0 15 56 21.6	3 26 48	237	885	500	149	787	523	124	599
30	29	0 15 59 32.3	4 7 57	271	925	529	182	822	565	158	605
31	30	0 16 2 42.9	4 19 6	305	964	557	216	858	606	191	610
	31	0 16 5 53.5	5 0 15	339	004	585	250	894	648	225	615

NOVEMBER.

1	0	0 16 5 53.5	5 0 15	339	004	585	250	894	648	225	615
2	1	0 16 9 4.2	5 11 24	373	043	613	283	930	690	259	621
3	2	0 16 12 14.8	5 22 33	407	083	642	317	966	731	292	626
4	3	0 16 15 25.5	6 3 43	441	122	670	350	002	773	326	631
5	4	0 16 18 36.1	6 14 52	475	162	698	385	038	814	360	637
6	5	0 16 21 46.7	6 26 1	509	201	726	418	073	856	393	642
7	6	0 16 24 57.4	7 7 10	543	241	755	452	109	898	427	647
8	7	0 16 28 8.0	7 18 19	577	280	783	486	145	939	460	652
9	8	0 16 31 18.6	7 29 28	611	320	811	519	181	981	494	658
10	9	0 16 34 29.3	8 10 37	645	359	839	553	217	022	528	663
11	10	0 16 37 39.9	8 21 47	679	399	868	587	253	064	561	668
12	11	0 16 40 50.5	9 2 56	713	438	896	621	288	106	595	674
13	12	0 16 44 1.2	9 14 5	747	478	924	654	324	147	629	679
14	13	0 16 47 11.8	9 25 14	781	517	952	688	360	189	662	684
15	14	0 16 50 22.5	10 6 23	815	557	980	722	396	231	696	690

TABLE III. continued.

Motion for Days. NOVEMBER.

Com.	Biss.	1	2	3	4	5	6	7	8	9	10	11	12
16	15	87335	7314	1818	1543	7037	8683	8407	4504	7747	379	929	478
17	16	87609	7964	2858	1830	7372	9055	8464	4894	7771	450	960	548
18	17	87883	8613	3898	2117	7708	9427	8522	5284	7795	520	991	619
19	18	88156	9263	4938	2404	8043	9799	8580	5675	7820	590	022	689
20	19	88430	9913	5978	2691	8379	0171	8637	6065	7844	660	054	760
21	20	88704	0563	7018	2978	8715	0543	8695	6455	7868	730	085	830
22	21	88978	1213	8059	3265	9050	0915	8753	6846	7892	800	116	901
23	22	89252	1863	9099	3552	9386	1288	8811	7236	7917	871	147	971
24	23	89525	2513	0139	3839	9721	1660	8868	7626	7941	941	178	042
25	24	89799	3163	1179	4126	0057	2032	8926	8016	7965	011	209	112
26	25	90073	3813	2219	4413	0392	2404	8984	8407	7990	081	240	183
27	26	90347	4462	3260	4700	0728	2776	9041	8797	8014	151	271	253
28	27	90620	5112	4300	4987	1063	3148	9099	9187	8038	221	303	324
29	28	90894	5762	5340	5274	1399	3520	9157	9578	8062	291	334	394
30	29	91168	6412	6380	5561	1734	3892	9214	9968	8087	362	365	465
	30	91442	7062	7420	5848	2070	4264	9272	0358	8111	432	396	535

DECEMBER.

1	0	91442	7062	7420	5848	2070	4264	9272	0358	8111	432	396	535
2	1	91716	7712	8460	6135	2405	4636	9330	0749	8135	502	427	606
3	2	91989	8362	9501	6422	2741	5008	9388	1139	8160	572	458	676
4	3	92263	9012	0541	6709	3077	5380	9445	1529	8184	642	489	746
5	4	92537	9662	1581	6996	3412	5752	9503	1919	8208	712	520	817
6	5	92811	0311	2621	7283	3748	6124	9561	2310	8232	783	552	887
7	6	93084	0961	3661	7570	4083	6496	9618	2700	8257	853	583	958
8	7	93358	1611	4701	7857	4419	6868	9676	3090	8281	923	614	028
9	8	93632	2261	5742	8144	4754	7240	9734	3481	8305	993	645	099
10	9	93906	2911	6782	8431	5090	7612	9791	3871	8330	063	676	169
11	10	94180	3561	7822	8718	5425	7984	9849	4261	8354	133	707	240
12	11	94453	4211	8862	9005	5761	8352	9907	4651	8378	203	738	310
13	12	94727	4861	9902	9292	6096	8728	9965	5042	8402	274	769	381
14	13	95001	5510	0943	9578	6432	9101	0022	5432	8427	344	801	451
15	14	95275	6160	1983	9865	6767	9473	0080	5822	8451	414	832	521
16	15	95548	6810	3023	0152	7103	9845	0138	6213	8475	484	863	592
17	16	95822	7460	4063	0439	7439	0217	0195	6603	8500	554	894	662
18	17	96096	8110	5103	0726	7774	0589	0253	6993	8524	624	925	733
19	18	96370	8760	6143	1013	8110	0961	0311	7384	8548	695	956	803
20	19	96644	9410	7184	1300	8445	1333	0369	7774	8572	765	987	874
21	20	96917	0060	8224	1587	8781	1705	0426	8164	8597	835	018	944
22	21	97191	0710	9264	1874	9116	2077	0484	8555	8621	905	050	015
23	22	97464	1359	0304	2161	9452	2449	0542	8945	8645	975	081	085
24	23	97739	2009	1344	2448	9787	2821	0599	9335	8670	045	112	156
25	24	98012	2659	2385	2735	0123	3193	0657	9725	8694	115	143	226
26	25	98286	3309	3425	3022	0458	3565	0715	0116	8718	186	174	297
27	26	98560	3959	4465	3309	0794	3937	0772	0506	8742	256	205	367
28	27	98834	4609	5505	3596	1129	4309	0830	0896	8767	326	236	438
29	28	99108	5259	6545	3883	1465	4681	0888	1287	8791	396	267	508
30	29	99381	5909	7585	4170	1800	5053	0946	1677	8815	466	298	579
31	30	99655	6559	8626	4457	2136	5425	1003	2067	8840	536	330	649
	31	99929	7208	9666	4744	2472	5797	1061	2457	8864	607	361	719

TABLE III. continued.

Motion for Days. NOVEMBER.

Com.	Biss.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
16	15	901	632	737	676	418	281	676	146	42	72	31	48	13	77	28
17	16	935	732	767	712	460	307	678	147	43	73	41	51	17	81	35
18	17	969	831	798	749	502	333	680	147	44	74	51	53	21	85	42
19	18	004	930	828	785	544	359	682	147	45	75	61	56	25	89	49
20	19	038	029	859	822	586	385	684	148	45	76	72	59	29	92	56
21	20	072	128	889	859	628	411	687	148	46	76	82	61	33	96	63
22	21	106	227	920	895	670	437	689	149	47	77	92	64	37	00	70
23	22	140	327	950	932	712	463	691	149	48	78	02	67	42	03	77
24	23	174	426	981	968	754	489	693	150	48	79	12	69	46	07	84
25	24	209	525	011	005	796	515	695	150	49	80	22	72	50	11	91
26	25	243	624	042	042	838	541	697	151	50	81	32	75	54	14	98
27	26	277	723	072	078	880	567	699	151	51	82	42	77	58	18	05
28	27	311	822	103	115	922	593	701	151	51	82	53	80	62	22	12
29	28	345	922	133	151	964	618	704	152	52	83	63	82	66	25	19
30	29	379	021	164	188	007	644	706	152	53	84	73	85	70	29	26
	30	414	120	194	225	049	670	708	153	54	85	83	88	74	33	33

DECEMBER.

1	0	414	120	194	225	049	670	708	153	54	85	83	88	74	33	33
2	1	448	219	225	261	091	696	710	153	54	86	93	90	79	37	40
3	2	482	318	255	298	133	722	712	154	55	87	03	93	83	40	47
4	3	516	417	286	334	175	748	714	154	56	88	13	96	87	44	54
5	4	550	517	316	371	217	774	716	155	57	88	23	98	91	48	61
6	5	584	616	347	408	259	800	718	155	57	89	34	01	95	51	68
7	6	619	715	377	444	301	826	721	155	58	90	44	04	99	55	75
8	7	653	814	408	481	343	852	723	156	59	91	54	06	03	59	82
9	8	687	913	439	517	385	878	725	156	60	92	64	09	07	62	89
10	9	721	012	469	554	427	904	727	157	61	93	74	12	12	66	96
11	10	755	111	500	591	469	930	729	157	61	93	84	14	16	70	03
12	11	789	211	530	627	511	956	731	158	62	94	94	17	20	73	10
13	12	824	310	561	664	553	982	733	158	63	95	04	20	24	77	17
14	13	858	409	591	700	595	008	735	159	64	96	15	22	28	81	24
15	14	892	508	622	737	637	034	737	159	64	97	25	25	32	84	31
16	15	926	607	652	774	680	060	740	159	65	98	35	28	36	88	38
17	16	960	706	683	810	722	086	742	160	66	99	45	30	40	92	45
18	17	995	806	713	847	764	112	744	160	67	99	55	33	44	96	52
19	18	029	905	744	883	806	138	746	161	67	00	65	36	49	99	59
20	19	063	004	774	920	848	164	748	161	68	01	75	38	53	03	66
21	20	097	103	805	957	890	190	750	162	69	02	86	41	57	07	73
22	21	131	202	835	993	932	216	752	162	70	03	96	44	61	10	80
23	22	165	301	866	030	974	241	754	163	70	04	06	46	65	14	87
24	23	200	401	896	066	016	267	757	163	71	05	16	49	69	18	94
25	24	234	500	927	103	058	293	759	164	72	05	26	52	73	21	00
26	25	268	599	957	140	100	319	761	164	73	06	36	54	77	25	07
27	26	302	698	988	176	142	345	763	165	73	07	46	57	81	29	14
28	27	336	797	018	213	184	371	765	165	74	08	56	60	86	32	21
29	28	370	896	049	249	226	397	767	166	75	09	67	62	90	36	28
30	29	405	996	079	286	268	423	769	166	76	10	77	65	94	40	35
31	30	439	095	110	323	310	449	771	166	76	10	87	67	98	44	42
	31	473	194	140	359	353	475	773	167	77	11	97	70	02	47	49

TABLE III. continued.

Motion for Days. NOVEMBER.

Com.	Biss.	28	29	30	31	Evection.	Anomaly.	Variation.	Longitude ☽
16	15	32	83	55	80	0s 9° 57′ 57″	6s 27° 43′ 56″3	9s 18° 50′ 56″	8s 3°16′ 13″7
17	16	36	86	55	80	0 21 16 57	7 10 47 50.3	10 1 2 23	8 16 26 48.7
18	17	41	89	55	80	1 2 35 56	7 23 51 44.3	10 13 13 50	8 29 37 23.8
19	18	45	92	55	81	1 13 54 55	8 6 55 38.2	10 25 25 17	9 12 47 58.8
20	19	49	95	55	81	1 25 13 55	8 19 59 32.2	11 7 36 43	9 25 58 33.8
21	20	53	98	55	81	2 6 32 54	9 3 3 26.2	11 19 48 10	10 9 9 8.8
22	21	57	02	56	81	2 17 51 54	9 16 7 20.1	0 1 59 37	10 22 19 43.9
23	22	62	05	56	82	2 29 10 53	9 29 11 14.1	0 14 11 3	11 5 30 18.9
24	23	66	08	56	82	3 10 29 53	10 12 15 8.1	0 26 22 30	11 18 40 53.9
25	24	70	11	56	82	3 21 48 52	10 25 19 2.1	1 8 33 57	0 1 51 28.9
26	25	74	14	56	82	4 3 7 51	11 8 22 56.0	1 20 45 23	0 15 2 4.0
27	26	78	17	56	83	4 14 26 51	11 21 26 50.0	2 2 56 50	0 28 12 39.0
28	27	82	20	57	83	4 25 45 50	0 4 30 44.0	2 15 8 17	1 11 23 14.0
29	28	87	23	57	83	5 7 4 50	0 17 34 37.9	2 27 19 43	1 24 33 49.1
30	29	91	26	57	83	5 18 23 49	1 0 38 31.9	3 9 31 10	2 7 44 24.1
	30	95	29	57	84	5 29 42 49	1 13 42 25.9	3 21 42 37	2 20 54 59.1

DECEMBER.

1	0	95	29	57	84	5 29 42 49	1 13 42 25.9	3 21 42 37	2 20 54 59.1
2	1	99	32	57	84	6 11 1 48	1 26 46 19.8	4 3 54 4	3 4 5 34.1
3	2	03	35	57	84	6 22 20 47	2 9 50 13.8	4 16 5 30	3 17 16 9.2
4	3	07	39	58	84	7 3 39 47	2 22 54 7.8	4 28 16 57	4 0 26 44.2
5	4	12	42	58	85	7 14 58 47	3 5 58 1.7	5 10 28 24	4 13 37 19.2
6	5	16	45	58	85	7 26 17 46	3 19 1 55.7	5 22 39 50	4 26 47 54.2
7	6	20	48	58	85	8 7 36 46	4 2 5 49.7	6 4 51 17	5 9 58 29.3
8	7	24	51	58	85	8 18 55 45	4 15 9 43.7	6 17 2 44	5 23 9 4.3
9	8	28	54	58	86	9 0 14 45	4 28 13 37.6	6 29 14 10	6 6 19 39.3
10	9	33	57	59	86	9 11 33 44	5 11 17 31.6	7 11 25 37	6 19 30 14.4
11	10	37	60	59	86	9 22 52 43	5 24 21 25.6	7 23 37 4	7 2 40 49.4
12	11	41	63	59	86	10 4 11 43	6 7 25 19.5	8 5 48 31	7 15 51 24.4
13	12	45	66	59	87	10 15 30 42	6 20 29 13.5	8 17 59 57	7 29 1 59.4
14	13	49	69	59	87	10 26 49 42	7 3 33 7.5	9 0 11 24	8 12 12 34.5
15	14	53	72	59	87	11 8 8 41	7 16 37 1.4	9 12 22 51	8 25 23 9.5
16	15	58	75	60	87	11 19 27 41	7 29 40 55.4	9 24 34 17	9 8 33 44.5
17	16	62	79	60	88	0 0 46 40	8 12 44 49.4	10 6 45 44	9 21 44 19.5
18	17	66	82	60	88	0 12 5 39	8 25 48 43.4	10 18 57 11	10 4 54 54.6
19	18	70	85	60	88	0 23 24 39	9 8 52 37.3	11 1 8 37	10 18 5 29.6
20	19	74	88	60	88	1 4 43 38	9 21 56 31.3	11 13 20 4	11 1 16 4.6
21	20	78	91	61	89	1 16 2 38	10 5 0 25.3	11 25 31 31	11 14 26 39.6
22	21	83	94	61	89	1 27 21 37	10 18 4 19.2	0 7 42 58	11 27 37 14.7
23	22	87	97	61	89	2 8 40 37	11 1 8 13.2	0 19 54 24	0 10 47 49.7
24	23	91	00	61	89	2 19 59 36	11 14 12 7.2	1 2 5 51	0 23 58 24.7
25	24	95	03	61	90	3 1 18 35	11 27 16 1.1	1 14 17 18	1 7 8 59.7
26	25	99	06	61	90	3 12 37 35	0 10 19 55.1	1 26 28 44	1 20 19 34.8
27	26	04	09	62	90	3 23 56 34	0 23 23 49.1	2 8 40 11	2 3 30 9.8
28	27	08	12	62	90	4 5 15 33	1 6 27 43.1	2 20 51 38	2 16 40 44.8
29	28	12	16	62	91	4 16 34 32	1 19 31 37.0	3 3 3 4	2 29 51 19.8
30	29	16	19	62	91	4 27 53 32	2 2 35 31.0	3 15 14 31	3 13 1 54.9
31	30	20	22	62	91	5 9 12 31	2 15 39 25.0	3 27 25 58	3 26 12 29.9
	31	25	25	62	91	5 20 31 30	2 28 43 19.0	4 9 37 25	4 9 23 4.9

End of TABLE III.

Motion for Days. NOVEMBER.

Com.	Biss.	Suppl. Node	II	V	VI	VII	VIII	IX	X	XI	XII
16	15	0s 16°53′ 33″1	10s 17°32′	849	596	009	755	432	272	730	695
17	16	0 16 56 43.7	10 28 41	883	636	037	789	468	314	763	700
18	17	0 16 59 54.4	11 9 50	917	675	065	822	503	355	797	706
19	18	0 17 3 5.0	11 21 0	951	715	093	857	539	397	830	711
20	19	0 17 6 15.6	0 2 9	985	754	122	890	575	439	864	716
21	20	0 17 9 26.3	0 13 18	019	793	150	924	611	480	898	722
22	21	0 17 12 36.9	0 24 27	053	833	178	958	647	522	931	727
23	22	0 17 15 47.5	1 5 36	087	872	206	991	683	563	965	732
24	23	0 17 18 58.2	1 16 45	121	912	235	025	718	605	999	737
25	24	0 17 22 8.8	1 27 54	155	951	263	059	754	647	032	743
26	25	0 17 25 19.5	2 9 3	189	991	291	093	790	688	066	748
27	26	0 17 28 30.1	2 20 13	223	030	319	126	826	730	100	753
28	27	0 17 31 40.7	3 1 22	257	070	348	160	862	771	133	759
29	28	0 17 34 51.4	3 12 31	291	109	376	194	898	813	167	764
30	29	0 17 38 2.0	3 23 40	325	149	404	227	933	855	200	769
	30	0 17 41 12.6	4 4 49	359	188	432	261	969	896	234	775

DECEMBER.

1	0	0 17 41 12.6	4 4 49	359	188	432	261	969	896	234	775
2	1	0 17 44 23.3	4 15 58	393	228	461	295	005	938	268	780
3	2	0 17 47 33.9	4 27 7	427	267	489	329	041	979	301	785
4	3	0 17 50 44.5	5 8 16	462	307	517	362	077	021	335	791
5	4	0 17 53 55.2	5 19 26	496	346	545	396	113	063	369	796
6	5	0 17 57 5.8	6 0 35	530	386	574	430	148	104	402	801
7	6	0 18 0 16.5	6 11 44	564	425	602	463	184	146	436	807
8	7	0 18 3 27.1	6 22 53	598	465	630	497	220	187	470	812
9	8	0 18 6 37.7	7 4 2	632	504	658	531	256	229	503	817
10	9	0 18 9 48.4	7 15 11	666	544	686	565	292	271	537	823
11	10	0 18 12 59.0	7 26 20	700	583	715	598	328	312	570	828
12	11	0 18 16 9.6	8 7 30	734	623	743	632	363	354	604	833
13	12	0 18 19 20.3	8 18 39	768	662	771	666	399	395	638	838
14	13	0 18 22 30.9	8 29 48	802	702	799	699	435	437	671	844
15	14	0 18 25 41.5	9 10 57	836	741	828	733	471	479	705	849
16	15	0 18 28 52.2	9 22 6	870	781	856	767	507	520	739	854
17	16	0 18 32 2.8	10 3 15	904	820	884	801	543	562	772	860
18	17	0 18 35 13.5	10 14 24	938	860	912	834	578	603	806	865
19	18	0 18 38 24.1	10 25 33	972	899	941	868	614	645	839	870
20	19	0 18 41 34.7	11 6 43	006	939	969	901	650	687	873	876
21	20	0 18 44 45.4	11 17 52	040	978	997	935	686	728	907	881
22	21	0 18 47 56.0	11 29 1	074	018	025	969	722	770	940	886
23	22	0 18 51 6.6	0 10 10	108	057	054	003	758	812	974	892
24	23	0 18 54 17.3	0 21 19	142	097	082	037	793	853	008	897
25	24	0 18 57 27.9	1 2 28	176	136	110	070	829	895	041	902
26	25	0 19 0 38.5	1 13 37	210	176	138	104	865	936	075	908
27	26	0 19 3 49.2	1 24 46	244	215	167	138	901	978	109	913
28	27	0 19 6 59.8	2 5 56	278	254	195	171	937	020	142	918
29	28	0 19 10 10.5	2 17 5	312	294	223	205	973	061	176	923
30	29	0 19 13 21.1	2 28 14	346	333	251	239	008	103	209	929
31	30	0 19 16 31.7	3 9 23	380	373	280	273	044	144	243	934
	31	0 19 19 42.5	3 20 32	414	412	308	306	080	186	277	939

TABLE IV.

Motion for Hours.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	11	27	43	12	14	16	2	16	1	3	1	3	1	4	1	2	2	1	0	0	0	0	0	0	0	0	0	0	0
2	23	54	87	24	28	31	5	33	2	6	3	6	3	8	3	3	4	2	0	0	0	0	1	0	0	0	1	0	0
3	34	81	130	36	42	47	7	49	3	9	4	9	4	12	4	5	5	3	0	0	0	0	1	0	1	0	1	1	0
4	46	108	173	48	56	62	10	65	4	12	5	12	6	16	5	6	7	4	0	0	0	0	2	0	1	1	1	1	1
5	57	135	217	60	70	78	12	81	5	15	6	15	7	21	6	8	9	5	0	0	0	0	2	1	1	1	1	1	1
6	68	162	260	72	84	93	14	98	6	18	8	18	9	25	8	9	11	6	0	0	0	0	3	1	1	1	2	1	1
7	80	190	303	84	98	109	17	114	7	20	9	20	10	29	9	11	12	8	1	0	0	0	3	1	1	1	2	1	1
8	91	217	347	96	112	124	19	130	8	23	10	23	11	33	10	12	14	9	1	0	0	0	3	1	1	1	2	1	1
9	103	244	390	108	126	140	22	146	9	26	12	26	13	37	11	14	16	10	1	0	0	0	4	1	2	1	3	1	1
10	114	271	433	120	140	155	24	163	10	29	13	29	14	41	13	15	18	11	1	0	0	0	4	1	2	2	3	2	1
11	125	298	477	131	154	171	26	179	11	32	14	32	16	45	14	17	19	12	1	0	0	0	5	1	2	2	3	2	1
12	137	325	520	143	168	186	29	195	12	35	16	35	17	49	15	18	21	13	1	0	0	0	5	1	2	2	3	2	2
13	148	352	563	155	182	202	31	211	13	38	17	38	18	54	16	20	23	14	1	0	0	0	5	1	2	2	4	2	2
14	160	379	607	167	196	217	34	228	14	41	18	41	20	58	18	21	25	15	1	0	0	0	6	2	2	2	4	2	2
15	171	406	650	179	210	233	36	244	15	44	19	44	21	62	19	23	26	16	1	0	0	0	6	2	3	2	4	3	2
16	182	433	693	191	224	248	38	260	16	47	21	47	23	66	20	25	28	17	1	0	1	1	7	2	3	2	5	3	2
17	194	460	737	203	238	264	41	276	17	50	22	50	24	70	21	26	30	18	1	0	1	1	7	2	3	3	5	3	2
18	205	487	780	215	252	279	43	293	18	53	23	53	25	74	23	28	32	19	2	0	1	1	8	2	3	3	5	3	2
19	217	515	823	227	266	295	46	309	19	56	25	56	27	78	24	29	33	21	2	0	1	1	8	2	3	3	6	3	3
20	228	542	867	239	280	310	48	325	20	58	26	58	28	83	25	31	35	22	2	0	1	1	8	2	3	3	6	3	3
21	239	569	910	251	294	326	50	341	21	61	27	61	30	87	26	32	37	23	2	0	1	1	9	2	4	3	6	4	3
22	251	596	953	263	308	341	53	358	22	64	28	64	31	91	28	34	39	24	2	0	1	1	9	2	4	3	6	4	3
23	262	623	997	275	322	357	55	374	23	67	30	67	33	95	29	35	40	25	2	0	1	1	10	3	4	4	7	4	3
24	274	650	1040	287	336	372	58	390	24	70	31	70	34	99	31	37	42	26	2	0	1	1	10	3	4	4	7	4	3

Continuation of TABLE IV.

Motion for Hours.

Hour.	Evection.	Anomaly.	Variation.	Longit. ☽	Suppl. node.	II	V	VI	VII	VIII	IX	X	XI	XII
1	0°28′ 17″	0°32′ 39″7	0°30′ 29″	0°32′ 56″5	0′ 7″9	0°28′	1	2	1	1	1	2	1	0
2	0 56 35	1 5 19.5	1 0 57	1 5 52.9	0 15.9	0 56	3	3	2	3	3	3	3	0
3	1 24 52	1 37 59.2	1 31 26	1 38 49.4	0 23.8	1 24	4	5	4	4	4	5	4	1
4	1 53 10	2 10 39.0	2 1 54	2 11 45.8	0 31.8	1 52	6	7	5	6	6	7	6	1
5	2 21 27	2 43 18.7	2 32 23	2 44 42.3	0 39.7	2 19	7	8	6	7	7	9	7	1
6	2 49 45	3 15 58.5	3 2 52	3 17 38.8	0 47.7	2 47	9	10	7	9	9	10	9	1
7	3 18 2	3 48 38.2	3 33 20	3 50 35.2	0 55.6	3 15	10	12	8	10	10	12	10	2
8	3 46 20	4 21 18.0	4 3 49	4 23 31.7	1 3.6	3 43	11	13	9	11	12	14	11	2
9	4 14 37	4 53 57.7	4 34 17	4 56 28.1	1 11.5	4 11	13	15	11	13	13	15	13	2
10	4 42 55	5 26 37.5	5 4 46	5 29 24.6	1 19.4	4 39	14	16	12	14	15	17	14	2
11	5 11 12	5 59 17.2	5 35 15	6 2 21.0	1 27.4	5 7	16	18	13	15	16	19	15	2
12	5 39 30	6 31 57.0	6 5 43	6 35 17.5	1 35.3	5 35	17	20	14	17	18	21	17	3
13	6 7 47	7 4 36.7	6 36 12	7 8 14.0	1 43.3	6 2	18	21	15	18	19	23	18	3
14	6 36 5	7 37 16.5	7 6 40	7 41 10.4	1 51.2	6 30	20	23	16	19	21	24	19	3
15	7 4 22	8 9 56.2	7 37 9	8 14 6.9	1 59.2	6 58	21	25	18	21	22	26	21	3
16	7 32 40	8 42 36.0	8 7 38	8 47 3.4	2 7.1	7 26	23	26	19	22	24	28	22	4
17	8 0 57	9 15 15.7	8 38 6	9 19 59.8	2 15.0	7 54	24	28	20	24	25	29	24	4
18	8 29 15	9 47 55.5	9 8 35	9 52 56.3	2 23.0	8 22	26	29	21	25	27	31	25	4
19	8 57 32	10 20 35.2	9 39 3	10 25 52.7	2 30.9	8 50	27	31	22	27	28	33	27	4
20	9 25 50	10 53 15.0	10 9 32	10 58 49.2	2 38.9	9 18	28	32	24	28	30	35	28	4
21	9 54 7	11 25 54.7	10 40 1	11 31 45.6	2 46.8	9 45	30	34	25	29	31	37	29	5
22	10 22 24	11 58 34.5	11 10 29	12 4 42.1	2 54.8	10 13	31	36	26	31	33	38	31	5
23	10 50 42	12 31 14.2	11 40 58	12 37 38.6	3 2.7	10 41	33	38	27	32	34	40	32	5
24	11 18 59	13 3 54.0	12 11 27	13 10 35.0	3 10.6	11 9	34	39	28	34	36	42	34	5

TABLE V.

Motion for Minutes.

Minut.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
3	1	1	2	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
4	1	2	3	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
5	1	2	4	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
6	1	3	4	1	1	2	0	2	0	0	0	0	0	0	0	0	0	0
7	1	3	5	1	2	2	0	2	0	0	0	0	0	0	0	0	0	0
8	2	4	6	2	2	2	0	2	0	0	0	0	0	1	0	0	0	0
9	2	4	6	2	2	2	0	2	0	0	0	0	0	1	0	0	0	0
10	2	5	7	2	2	3	0	3	0	0	0	0	0	1	0	0	0	0
11	2	5	8	2	3	3	0	3	0	1	0	1	0	1	0	0	0	0
12	2	5	9	2	3	3	0	3	0	1	0	1	0	1	0	0	0	0
13	2	6	9	3	3	3	1	4	0	1	0	1	0	1	0	0	0	0
14	3	6	10	3	3	4	1	4	0	1	0	1	0	1	0	0	0	0
15	3	7	11	3	3	4	1	4	0	1	0	1	0	1	0	0	0	0
16	3	7	12	3	4	4	1	4	0	1	0	1	0	1	0	0	0	0
17	3	8	12	3	4	4	1	5	0	1	0	1	0	1	0	0	0	0
18	3	8	13	4	4	5	1	5	0	1	0	1	0	1	0	0	1	0
19	4	9	14	4	4	5	1	5	0	1	0	1	0	1	0	0	1	0
20	4	9	14	4	5	5	1	5	0	1	0	1	0	1	0	1	1	0
21	4	10	15	4	5	5	1	6	0	1	0	1	0	1	0	1	1	0
22	4	10	16	4	5	6	1	6	0	1	0	1	1	2	0	1	1	0
23	4	10	17	5	5	6	1	6	0	1	0	1	1	2	0	1	1	0
24	5	11	17	5	6	6	1	7	0	1	1	1	1	2	1	1	1	0
25	5	11	18	5	6	6	1	7	0	1	1	1	1	2	1	1	1	0
26	5	12	19	5	6	7	1	7	0	1	1	1	1	2	1	1	1	0
27	5	12	19	5	6	7	1	7	0	1	1	1	1	2	1	1	1	0
28	5	13	20	6	7	7	1	8	0	1	1	1	1	2	1	1	1	0
29	6	13	21	6	7	7	1	8	0	1	1	1	1	2	1	1	1	0
30	6	14	22	6	7	8	1	8	0	1	1	1	1	2	1	1	1	0
31	6	14	22	6	7	8	1	8	0	1	1	1	1	2	1	1	1	1

Continuation of TABLE V.

Motion for Minutes.

Minut.	Evection.	Anomaly.	Variat.	Long. ☽	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI
1	0′ 28″	0′ 32″7	0′ 30″	0′ 32″9	0″1	0′	0	0	0	0	0	0	0
2	0 57	1 5.3	1 1	1 5.9	0.3	1	0	0	0	0	0	0	0
3	1 25	1 38.0	1 31	1 38.8	0.4	1	0	0	0	0	0	0	0
4	1 53	2 10.6	2 2	2 11.8	0.5	2	0	0	0	0	0	0	0
5	2 21	2 43.3	2 32	2 44.7	0.7	2	0	0	0	0	0	0	0
6	2 50	3 16.0	3 3	3 17.6	0.8	3	0	0	0	0	0	0	0
7	3 18	3 48.6	3 33	3 50.6	0.9	3	0	0	0	0	0	0	0
8	3 46	4 21.3	4 4	4 23.5	1.1	4	0	0	0	0	0	0	0
9	4 15	4 54.0	4 34	4 56.5	1.2	4	0	0	0	0	0	0	0
10	4 43	5 26.6	5 5	5 29.4	1.3	5	0	0	0	0	0	0	0
11	5 11	5 59.3	5 35	6 2.4	1.5	5	0	0	0	0	0	0	0
12	5 40	6 31.9	6 6	6 35.3	1.6	6	0	0	0	0	0	0	0
13	6 8	7 4.6	6 36	7 8.2	1.7	6	0	0	0	0	0	0	0
14	6 36	7 37.3	7 7	7 41.2	1.9	7	0	0	0	0	0	0	0
15	7 4	8 9.9	7 37	8 14.1	2.0	7	0	0	0	0	0	0	0
16	7 33	8 42.6	8 8	8 47.1	2.1	7	0	0	0	0	0	0	0
17	8 1	9 15.3	8 38	9 20.0	2.3	8	0	0	0	0	0	0	0
18	8 29	9 47.9	9 9	9 52.9	2.4	8	0	0	0	0	0	1	0
19	8 58	10 20.6	9 39	10 25.9	2.5	9	0	0	0	0	0	1	0
20	9 26	10 53.2	10 10	10 58.8	2.6	9	0	1	0	0	0	1	0
21	9 54	11 25.9	10 40	11 31.8	2.8	10	0	1	0	0	0	1	0
22	10 22	11 58.6	11 11	12 4.7	2.9	10	1	1	0	0	1	1	0
23	10 51	12 31.2	11 41	12 37.6	3.0	11	1	1	0	0	1	1	0
24	11 19	13 3.9	12 12	13 10.6	3.2	11	1	1	0	1	1	1	1
25	11 47	13 36.6	12 42	13 43.5	3.3	12	1	1	0	1	1	1	1
26	12 16	14 9.2	13 13	14 16.5	3.4	12	1	1	1	1	1	1	1
27	12 44	14 41.9	13 43	14 49.4	3.6	13	1	1	1	1	1	1	1
28	13 12	15 14.6	14 13	15 22.3	3.7	13	1	1	1	1	1	1	1
29	13 40	15 47.2	14 44	15 55.3	3.8	13	1	1	1	1	1	1	1
30	14 9	16 19.9	15 14	16 28.2	4.0	14	1	1	1	1	1	1	1
31	14 37	16 52.5	15 45	17 1.2	4.1	14	1	1	1	1	1	1	1

TABLE V.

Motion for Minutes.

Minut.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
31	6	14	22	6	7	8	1	8	0	1	1	1	1	2	1	1	1	1
32	6	14	23	6	7	8	1	9	1	2	1	2	1	2	1	1	1	1
33	6	15	24	7	8	9	1	9	1	2	1	2	1	2	1	1	1	1
34	6	15	25	7	8	9	1	9	1	2	1	2	1	2	1	1	1	1
35	7	16	25	7	8	9	1	10	1	2	1	2	1	2	1	1	1	1
36	7	16	26	7	8	9	1	10	1	2	1	2	1	3	1	1	1	1
37	7	17	27	7	9	10	1	10	1	2	1	2	1	3	1	1	1	1
38	7	17	27	8	9	10	2	10	1	2	1	2	1	3	1	1	1	1
39	7	18	28	8	9	10	2	11	1	2	1	2	1	3	1	1	1	1
40	8	18	29	8	9	10	2	11	1	2	1	2	1	3	1	1	1	1
41	8	19	30	8	10	11	2	11	1	2	1	2	1	3	1	1	1	1
42	8	19	30	8	10	11	2	11	1	2	1	2	1	3	1	1	1	1
43	8	19	31	9	10	11	2	12	1	2	1	2	1	3	1	1	1	1
44	8	20	32	9	10	11	2	12	1	2	1	2	1	3	1	1	1	1
45	9	20	32	9	10	12	2	12	1	2	1	2	1	3	1	1	1	1
46	9	21	33	9	11	12	2	12	1	2	1	2	1	3	1	1	1	1
47	9	21	34	9	11	12	2	13	1	2	1	2	1	3	1	1	1	1
48	9	22	35	10	11	12	2	13	1	2	1	2	1	3	1	1	1	1
49	9	22	35	10	11	13	2	13	1	2	1	2	1	3	1	1	1	1
50	9	23	36	10	11	13	2	13	1	2	1	2	1	3	1	1	1	1
51	10	23	37	10	12	13	2	14	1	2	1	2	1	4	1	1	1	1
52	10	24	38	10	12	13	2	14	1	3	1	3	1	4	1	1	1	1
53	10	24	38	11	12	14	2	14	1	3	1	3	1	4	1	1	1	1
54	10	24	39	11	12	14	2	14	1	3	1	3	1	4	1	1	2	1
55	10	25	40	11	13	14	2	15	1	3	1	3	1	4	1	1	2	1
56	11	25	40	11	13	14	2	15	1	3	1	3	1	4	1	1	2	1
57	11	26	41	11	13	15	2	15	1	3	1	3	1	4	1	1	2	1
58	11	26	42	12	13	15	2	16	1	3	1	3	1	4	1	2	2	1
59	11	27	43	12	14	15	2	16	1	3	1	3	1	4	1	2	2	1
60	11	27	43	12	14	15	2	16	1	3	1	3	1	4	1	2	2	1

End of TABLE V.

Motion for Minutes.

Minut.	Evection.	Anomaly.	Variation.	Longitude.	Suppl. Node.	II	V	VI	VII	VIII	IX	X	XI
31	14′ 37″	16′ 52″5	15′ 45″	17′ 1″2	4″1	14	1	1	1	1	1	1	1
32	15 5	17 25.2	16 15	17 34.1	4.2	15	1	1	1	1	1	1	1
33	15 34	17 57.9	16 46	18 7.1	4.4	15	1	1	1	1	1	1	1
34	16 2	18 30.5	17 16	18 40.0	4.5	16	1	1	1	1	1	1	1
35	16 30	19 3.2	17 47	19 12.9	4.7	16	1	1	1	1	1	1	1
36	16 58	19 35.8	18 17	19 45.9	4.8	17	1	1	1	1	1	1	1
37	17 27	20 8.5	18 48	20 18.8	4.9	17	1	1	1	1	1	1	1
38	17 55	20 41.2	19 18	20 51.8	5.0	18	1	1	1	1	1	1	1
39	18 23	21 13.8	19 49	21 24.7	5.2	18	1	1	1	1	1	1	1
40	18 52	21 46.5	20 19	21 57.6	5.3	19	1	1	1	1	1	1	1
41	19 20	22 19.2	20 50	22 30.6	5.4	19	1	1	1	1	1	1	1
42	19 48	22 51.8	21 20	23 3.5	5.6	20	1	1	1	1	1	1	1
43	20 16	23 24.5	21 51	23 36.5	5.7	20	1	1	1	1	1	1	1
44	20 45	23 57.1	22 21	24 9.4	5.8	21	1	1	1	1	1	1	1
45	21 13	24 29.8	22 52	24 42.3	6.0	21	1	1	1	1	1	1	1
46	21 41	25 2.5	23 22	25 15.3	6.1	21	1	1	1	1	1	1	1
47	22 10	25 35.1	23 53	25 48.2	6.2	22	1	1	1	1	1	1	1
48	22 38	26 7.8	24 23	26 21.2	6.4	22	1	1	1	1	1	1	1
49	23 6	26 40.5	24 54	26 54.1	6.5	23	1	1	1	1	1	1	1
50	23 34	27 13.1	25 24	27 27.0	6.6	23	1	1	1	1	1	1	1
51	24 3	27 45.8	25 55	28 0.0	6.8	24	1	1	1	1	1	1	1
52	24 31	28 18.5	26 25	28 32.9	6.9	24	1	1	1	1	1	1	1
53	24 59	28 51.1	26 56	29 5.9	7.0	25	1	1	1	1	1	1	1
54	25 28	29 23.8	27 26	29 38.8	7.1	25	1	1	1	1	1	2	1
55	25 56	29 56.4	27 56	30 11.8	7.3	26	1	1	1	1	1	2	1
56	26 24	30 29.1	28 27	30 44.7	7.4	26	1	1	1	1	1	2	1
57	26 52	31 1.8	28 57	31 17.6	7.5	27	1	2	1	1	1	2	1
58	27 21	31 34.4	29 28	31 50.6	7.7	27	1	2	1	1	1	2	1
59	27 49	32 7.1	29 58	32 23.5	7.8	28	1	2	1	1	1	2	1
60	28 17	32 39.8	30 29	32 56.5	7.9	28	1	2	1	1	1	2	1

TABLE VI.

Motion for Seconds.

Sec.	Evec.	Anom.	Var.	Long.	Sec.	Evec.	Anom.	Var.	Long.
1	0″	0″5	1″	0″5	31	15″	16″9	16″	17″0
2	1	1.1	1	1.1	32	15	17.4	16	17.6
3	1	1.6	2	1.6	33	16	18.0	17	18.1
4	2	2.2	2	2.2	34	16	18.5	17	18.7
5	2	2.7	3	2.7	35	17	19.1	18	19.2
6	3	3.3	3	3.3	36	17	19.6	18	19.8
7	3	3.8	4	3.8	37	18	20.1	19	20.3
8	4	4.3	4	4.4	38	18	20.7	19	20.9
9	4	4.9	5	4.9	39	18	21.2	20	21.4
10	5	5.4	5	5.5	40	19	21.8	20	22.0
11	5	6.0	6	6.0	41	19	22.3	21	22.5
12	6	6.5	6	6.6	42	20	22.9	21	23.1
13	6	7.1	7	7.1	43	20	23.4	22	23.6
14	7	7.6	7	7.7	44	21	24.0	22	24.2
15	7	8.2	8	8.2	45	21	24.5	23	24.7
16	8	8.7	8	8.8	46	22	25.0	23	25.3
17	8	9.2	9	9.3	47	22	25.6	24	25.8
18	9	9.8	9	9.9	48	23	26.1	24	26.4
19	9	10.3	10	10.4	49	23	26.7	25	26.9
20	9	10.9	10	11.0	50	24	27.2	25	27.4
21	10	11.4	11	11.5	51	24	27.8	26	28.0
22	10	12.0	11	12.1	52	25	28.3	26	28.5
23	11	12.5	12	12.6	53	25	28.9	27	29.1
24	11	13.1	12	13.2	54	26	29.4	27	29.6
25	12	13.6	13	13.7	55	26	29.9	28	30.2
26	12	14.1	13	14.3	56	26	30.5	28	30.7
27	13	14.7	14	14.8	57	27	31.0	29	31.3
28	13	15.2	14	15.4	58	27	31.6	29	31.8
29	14	15.8	15	15.9	59	28	32.1	30	32.4
30	14	16.3	15	16.5	60	28	32.7	30	32.9

TABLE VII.

LUNAR NUTATION.

Argum. Supplement Node ☽.

Deg.	Os VIs +	Os VIs −	Is VIIs +	Is VIIs −	IIs VIIIs +	IIs VIIIs −	
0		0″0		9″0		15″6	30
1		0.3		9.3		15.7	29
2		0.6		9.5		15.9	28
3		0.9		9.8		16.0	27
4		1.3		10.1		16.2	26
5		1.6		10.3		16.3	25
6		1.9		10.6		16.4	24
7		2.2		10.8		16.6	23
8		2.5		11.1		16.7	22
9		2.8		11.3		16.8	21
10		3.1		11.6		16.9	20
11		3.4		11.8		17.0	19
12		3.7		12.0		17.1	18
13		4.0		12.3		17.2	17
14		4.3		12.5		17.3	16
15		4.7		12.7		17.4	15
16		5.0		12.9		17.5	14
17		5.3		13.1		17.5	13
18		5.6		13.4		17.6	12
19		5.9		13.6		17.7	11
20		6.2		13.8		17.7	10
21		6.5		14.0		17.8	9
22		6.7		14.2		17.8	8
23		7.0		14.4		17.9	7
24		7.3		14.6		17.9	6
25		7.6		14.7		17.9	5
26		7.9		14.9		18.0	4
27		8.2		15.1		18.0	3
28		8.5		15.3		18.0	2
29		8.7		15.4		18.0	1
30		9.0		15.6		18.0	0
	+ XIs	− Vs	+ Xs	− IVs	+ IXs	− IIIs	Deg.

SOLAR NUTATION.

+	+	+	+		−	−	−	−
23 sept.	21 march.	22 decem.	22 june.	0″0	23 sept.	22 decem.	21 march.	22 june.
20 sept.	18 march.	25 decem.	25 june.	0.1	26 sept.	19 decem.	24 march.	19 june.
17 sept.	15 march.	28 decem.	28 june.	0.2	29 sept.	16 decem.	27 march.	16 june.
14 sept.	12 march.	31 decem.	1 july.	0.3	2 octob.	13 decem.	30 march.	13 june.
11 sept.	9 march.	2 jan.	4 july.	0.4	5 octob.	10 decem.	2 april.	9 june.
8 sept.	6 march.	5 jan.	7 july.	0.5	8 octob.	7 decem.	5 april.	6 june.
4 sept.	2 march.	9 jan.	11 july.	0.6	12 octob.	4 decem.	8 april.	2 june.
31 aug.	27 feb.	12 jan.	15 july.	0.7	16 octob.	30 nov.	12 april.	29 may.
27 aug.	22 feb.	17 jan.	20 july.	0.8	20 octob.	26 nov.	17 april.	25 may.
21 aug.	17 feb.	22 jan.	25 july.	0.9	26 octob.	20 nov.	23 april.	19 may.
8 aug.	4 feb.	4 feb.	8 aug.	1.0	8 nov.	8 nov.	6 may.	6 may.

TABLE VIII.

1st. Equation of the Moon's Longitude. ARGUMENT 1

Argum.		Diff. pr. 100	Argum.		Diff. pr. 100.	Argum.		Diff. pr. 100.	Argum.		Diff. pr. 100.
00000	12′ 40″0	4″24	25000	1′ 40″7	0″16	50000	12′ 40″0	4″06	75000	23′ 39″3	0″02
00500	12 18.8	4.22	25500	1 41.5	0.28	50500	13 0.3	4.04	75500	23 39.4	0.10
01000	11 57.7	4.22	26000	1 42.9	0.42	51000	13 20.5	4.04	76000	23 38.9	0.24
01500	11 36.6	4.20	26500	1 45.0	0.54	51500	13 40.7	4.04	76500	23 37.7	0.38
02000	11 15.6	4.18	27000	1 47.7	0.66	52000	14 0.9	4.00	77000	23 35.8	0 50
02500	10 54.7	4.16	27500	1 51.0	0.80	52500	14 20.9	4.00	77500	23 33.3	0.62
03000	10 33.9	4.14	28000	1 55.0	0.92	53000	14 40.9	3.98	78000	23 30.2	0.76
03500	10 13.2	4.12	28500	1 59.6	1.04	53500	15 0.8	3.94	78500	23 26.4	0.88
04000	9 52.6	4.06	29000	2 4.8	1.18	54000	15 20.5	3.92	79000	23 22.0	1.02
04500	9 32.3	4.04	29500	2 10.7	1.28	54500	15 40.1	3.90	79500	23 16.9	1.14
05000	9 12.1	4.00	30000	2 17.1	1.42	55000	15 59.6	3.84	80000	23 11.2	1.26
05500	8 52.1	3.94	30500	2 24.2	1.54	55500	16 18.8	3.80	80500	23 4.9	1.40
06000	8 32.4	3.88	31000	2 31.9	1.64	56000	16 37.8	3.78	81000	22 57.9	1.52
06500	8 13.0	3.84	31500	2 40.1	1.76	56500	16 56.7	3.72	81500	22 50.3	1.66
07000	7 53.8	3.78	32000	2 48.9	1.88	57000	17 15.3	3.66	82000	22 42.0	1.76
07500	7 34.9	3.70	32500	2 58.3	1.98	57500	17 33.6	3.60	82500	22 33.2	1.90
08000	7 16.4	3.64	33000	3 8.2	2.10	58000	17 51.6	3.56	83000	22 23.7	2.00
08500	6 58.2	3.58	33500	3 18.7	2.20	58500	18 9.4	3.50	83500	22 13.7	2.12
09000	6 40 3	3.50	34000	3 29.7	2.32	59000	18 26.9	3.42	84000	22 3 1	2.24
09500	6 22.8	3.42	34500	3 41.3	2.42	59500	18 44.0	3.36	84500	21 51.9	2.36
10000	6 5.7	3.34	35000	3 53.4	2.50	60000	19 0.8	3.28	85000	21 40.1	2.46
10500	5 49.0	3.24	35500	4 5.9	2.62	60500	19 17.2	3.22	85500	21 27.8	2.56
11000	5 32.8	3.16	36000	4 19.0	2.70	61000	19 33.3	3.14	86000	21 15.0	2.68
11500	5 17.0	3.08	36500	4 32.5	2.80	61500	19 49.0	3.04	86500	21 1.6	2.78
12000	5 1.6	2.98	37000	4 46.5	2.88	62000	20 4.2	2.98	87000	20 47.7	2.88
12500	4 46.7	2.88	37500	5 0.9	2.98	62500	20 19.1	2.88	87500	20 33.3	2.98
13000	4 32.3	2.78	38000	5 15.8	3.04	63000	20 33.5	2.80	88000	20 18.4	3.08
13500	4 18.4	2.68	38500	5 31.0	3.14	63500	20 47.5	2.70	88500	20 3.0	3.16
14000	4 5.0	2.56	39000	5 46.7	3.22	64000	21 1.0	2.62	89000	19 47.2	3.24
14500	3 52.2	2.46	39500	6 2.8	3.28	64500	21 14.1	2.50	89500	19 31.0	3.34
15000	3 39.9	2.36	40000	6 19.2	3.36	65000	21 26.6	2.42	90000	19 14.3	3.42
15500	3 28.1	2.24	40500	6 36.0	3.42	65500	21 38.7	2.32	90500	18 57.2	3.50
16000	3 16.9	2.12	41000	6 53.1	3.50	66000	21 50.3	2.20	91000	18 39.7	3.58
16500	3 6.3	2.00	41500	7 10.6	3.56	66500	22 1.3	2.10	91500	18 21.8	3.64
17000	2 56.3	1.90	42000	7 28.4	3.60	67000	22 11.8	1.98	92000	18 3.6	3.70
17500	2 46.8	1.76	42500	7 46.4	3.66	67500	22 21.7	1.88	92500	17 45.1	3.78
18000	2 38.0	1.66	43000	8 4.7	3.72	68000	22 31.1	1.76	93000	17 26.2	3.84
18500	2 29.7	1.52	43500	8 23.3	3.78	68500	22 39.9	1.64	93500	17 7.0	3.88
19000	2 22.1	1.40	44000	8 42.2	3.80	69000	22 48.1	1.54	94000	16 47.6	3.94
19500	2 15.1	1.26	44500	9 1.2	3.84	69500	22 55.8	1.42	94500	16 27.9	4.00
20000	2 8.8	1.14	45000	9 20.4	3.90	70000	23 2.9	1.28	95000	16 7.9	4.04
20500	2 3.1	1.02	45500	9 39.9	3.92	70500	23 9.3	1.18	95500	15 47.7	4.06
21000	1 58.0	0.88	46000	9 59.5	3.94	71000	23 15.2	1.04	96000	15 27.4	4.12
21500	1 53.6	0.76	46500	10 19.2	3.98	71500	23 20.4	0.92	96500	15 6.8	4.14
22000	1 49.8	0.62	47000	10 39.1	4.00	72000	23 25.0	0.80	97000	14 46.1	4.16
22500	1 46.7	0.50	47500	10 59.1	4.00	72500	23 29.0	0.66	97500	14 25.3	4.18
23000	1 44.2	0.38	48000	11 19.1	4.04	73000	23 32.3	0.54	98000	14 4.4	4.20
23500	1 42.3	0.24	48500	11 39.3	4.04	73500	23 35.0	0.42	98500	13 43.4	4.22
24000	1 41.1	0.10	49000	11 59.5	4.04	74000	23 37.1	0.28	99000	13 22.3	4.22
24500	1 40.6	0.02	49500	12 19.7	4.04	74500	23 38.5	0.16	99500	13 1.2	4.24
25000	1 40.7		50000	12 40.0	4.06	75000	23 39.3		100000	12 40.0	

TABLE IX.

Equations of the Moon's Longitude, from 2 to 7, which depend on their respective Arguments.

Arg.	2	Diff.	3	Diff.	4	Diff.	5	Diff.	6	Diff.	7	Diff.	Arg.
2500	4′ 57″3		0′ 2″3		6′ 30″3		3′ 39″4		0′ 6″2		0 0″8		2500
2600	4 57.0	0″3	0 2.4	0″1	6 29.9	0″4	3 39.2	0″2	0 6.4	0′2	0 0.9	0″1	2400
2700	4 56.1	0.9	0 2.8	0.4	6 28.8	1.1	3 38.5	0.7	0 6.9	0.5	0 1.3	0.4	2300
2800	4 54.7	1.4	0 3.3	0.5	6 26.9	1.9	3 37.5	1.0	0 7.7	0.8	0 1.8	0.5	2200
2900	4 52.7	2.0	0 4.1	0.8	6 24.3	2.6	3 36.0	1.5	0 8.8	1.1	0 2.7	0.9	2100
3000	4 50.1	2.6	0 5.1	1.0	6 21.0	3.3	3 34.1	1.9	0 10.3	1.5	0 3.7	1.0	2000
3100	4 47.0	3.1	0 6.4	1.3	6 16.9	4.1	3 31.7	2.4	0 12.1	1.8	0 5.0	1.3	1900
3200	4 43.3	3.7	0 7.8	1.4	6 12.2	4.7	3 29.0	2.7	0 14.2	2.1	0 6.4	1.4	1800
3300	4 39.1	4.2	0 9.4	1.6	6 6.8	5.4	3 25.9	3.1	0 16.6	2.4	0 8.1	1.7	1700
3400	4 34.4	4.7	0 11.3	1.9	6 0.7	6.1	3 22.4	3.5	0 19.2	2.6	0 10.0	1.9	1600
3500	4 29.2	5.2	0 13.3	2.0	5 54.0	6.7	3 18.5	3.9	0 22.2	3.0	0 12.1	2.1	1500
3600	4 23.5	5.7	0 15.5	2.2	5 46.6	7.4	3 14.3	4.2	0 25.4	3.2	0 14.4	2.3	1400
3700	4 17.4	6.1	0 17.9	2.4	5 38.7	7.9	3 9.7	4.6	0 28.9	3.5	0 16.8	2.4	1300
3800	4 10.8	6.6	0 20.5	2.6	5 30.3	8.4	3 4.9	4.8	0 32.7	3.8	0 19.5	2.7	1200
3900	4 3.9	6.9	0 23.2	2.7	5 21.3	9.0	2 59.7	5.2	0 36.6	3.9	0 22.3	2.8	1100
4000	3 56.6	7.3	0 26.1	2.9	5 11.9	9.4	2 54.3	5.4	0 40.7	4.1	0 25.2	2.9	1000
4100	3 48.9	7.7	0 29.1	3.0	5 2.0	9.9	2 48.6	5.7	0 45.1	4.4	0 28.3	3.1	0900
4200	3 41.0	7.9	0 32.2	3.1	4 51.7	10.3	2 42.7	5.9	0 49.6	4.5	0 31.5	3.2	0800
4300	3 32.7	8.3	0 35.4	3.2	4 41.0	10.7	2 36.6	6.1	0 54.3	4.7	0 34.8	3.3	0700
4400	3 24.2	8.5	0 38.8	3.4	4 30.1	10.9	2 30.3	6.3	0 59.2	4.9	0 38.2	3.4	0600
4500	3 15.5	8.7	0 42.2	3.4	4 18.8	11.3	2 23.8	6.5	1 4.1	4.9	0 41.7	3.5	0500
4600	3 6.6	8.9	0 45.7	3.5	4 7.3	11.5	2 17.2	6.6	1 9.2	5.1	0 45.3	3.6	0400
4700	2 57.6	9.0	0 49.2	3.5	3 55.7	11.6	2 10.5	6.7	1 14.3	5.1	0 48.9	3.6	0300
4800	2 48.5	9.1	0 52.8	3.6	3 43.9	11.8	2 3.7	6.8	1 19.5	5.2	0 52.6	3.7	0200
4900	2 39.2	9.3	0 56.4	3.6	3 31.9	12.0	1 56.9	6.8	1 24.7	5.2	0 56.3	3.7	0100
5000	2 30.0	9.2	1 0.0	3.6	3 20.0	11.9	1 50.0	6.9	1 30.0	5.3	1 0.0	3.7	0000
5100	2 20.8	9.2	1 3.6	3.6	3 8.1	11.9	1 43.1	6.9	1 35.3	5.3	1 3.7	3.7	9900
5200	2 11.5	9.3	1 7.2	3.6	2 56.1	12.0	1 36.3	6.8	1 40.5	5.2	1 7.4	3.7	9800
5300	2 2.4	9.1	1 10.8	3.6	2 44.3	11.8	1 29.5	6.8	1 45.7	5.2	1 11.1	3.7	9700
5400	1 53.4	9.0	1 14.3	3.5	2 32.7	11.6	1 22.8	6.7	1 50.8	5.1	1 14.7	3.6	9600
5500	1 44.5	8.9	1 17.8	3.5	2 21.2	11.5	1 16.2	6.6	1 55.9	5.1	1 18.3	3.6	9500
5600	1 35.8	8.7	1 21.2	3.4	2 9.9	11.3	1 9.7	6.5	2 0.8	4.9	1 21.8	3.5	9400
5700	1 27.3	8.5	1 24.6	3.4	1 59.0	10.9	1 3.4	6.3	2 5.7	4.9	1 25.2	3.4	9300
5800	1 19.0	8.3	1 27.8	3.2	1 48.3	10.7	0 57.3	6.1	2 10.4	4.7	1 28.5	3.3	9200
5900	1 11.1	7.9	1 30.9	3.1	1 38.0	10.3	0 51.4	5.9	2 14.9	4.5	1 31.7	3.2	9100
6000	1 3.4	7.7	1 33.9	3.0	1 28.1	9.9	0 45.7	5.7	2 19.3	4.4	1 34.8	3.1	9000
6100	0 56.1	7.3	1 36.8	2.9	1 18.7	9.4	0 40.3	5.4	2 23.4	4.1	1 37.7	2.9	8900
6200	0 49.2	6.9	1 39.5	2.7	1 9.7	9.0	0 35.1	5.2	2 27.3	3.9	1 40.5	2.8	8800
6300	0 42.6	6.6	1 42.1	2.6	1 1.3	8.4	0 30.3	4.8	2 31.1	3.8	1 43.2	2.7	8700
6400	0 36.5	6.1	1 44.5	2.4	0 53.4	7.9	0 25.7	4.6	2 34.6	3.5	1 45.6	2.4	8600
6500	0 30.8	5.7	1 46.7	2.2	0 46.0	7.4	0 21.5	4.2	2 37.8	3.2	1 47.9	2.3	8500
6600	0 25.6	5.2	1 48.7	2.0	0 39.3	6.7	0 17.6	3.9	2 40.8	3.0	1 50.0	2.1	8400
6700	0 20.9	4.7	1 50.6	1.9	0 33.2	6.1	0 14.1	3.5	2 43.4	2.6	1 51.9	1.9	8300
6800	0 16.7	4.2	1 52.2	1.6	0 27.8	5.4	0 11.0	3.1	2 45.8	2.4	1 53.6	1.7	8200
6900	0 13.0	3.7	1 53.6	1.4	0 23.1	4.7	0 8.3	2.7	2 47.9	2.1	1 55.0	1.4	8100
7000	0 9.9	3.1	1 54.9	1.3	0 19.0	4.1	0 5.9	2.4	2 49.7	1.8	1 56.3	1.3	8000
7100	0 7.3	2.6	1 55.9	1.0	0 15.7	3.3	0 4.0	1.9	2 51.2	1.5	1 57.3	1.0	7900
7200	0 5.3	2.0	1 56.7	0.8	0 13.1	2.6	0 2.5	1.5	2 52.3	1.1	1 58.2	0.9	7800
7300	0 3.9	1.4	1 57.2	0.5	0 11.2	1.9	0 1.5	1.0	2 53.1	0.8	1 58.7	0.5	7700
7400	0 3.0	0.9	1 57.6	0.4	0 10.1	1.1	0 0.8	0.7	2 53.6	0.5	1 59.1	0.4	7600
7500	0 2.7	0.3	1 57.7	0.1	0 9.7	0.4	0 0.6	0.2	2 53.8	0.2	1 59.2	0.1	7500

TABLE X.

Equations 8 and 9 of the Moon's Longitude.

Arg.	8	Dif	9	Dif	Arg.	8	Dif	9	Dif
0000	1′ 20″0	4″5	1′ 20″0	8″7	5000	1′ 20″0	4″4	1′ 20″0	5″8
0100	1 15.5	4.4	1 28.7	8.6	5100	1 24.4	4.4	1 25.8	5.6
0200	1 11.1	4.4	1 37.3	8.4	5200	1 28.8	4.3	1 31.4	5.5
0300	1 6.7	4.4	1 45.7	8.0	5300	1 33.1	4.3	1 36.9	5.1
0400	1 2.3	4.3	1 53.7	7.6	5400	1 37.4	4.2	1 42.0	4.8
0500	0 58.0	4.2	2 1.3	7.0	5500	1 41.6	4.2	1 46.8	4.2
0600	0 53.8	4.1	2 8.3	6.4	5600	1 45.8	4.0	1 51.0	3.6
0700	0 49.7	4.0	2 14.7	5.5	5700	1 49.8	4.0	1 54.6	3.0
0800	0 45.7	3.8	2 20.2	4.8	5800	1 53.8	3.8	1 57.6	2.2
0900	0 41.9	3.7	2 25.0	3.9	5900	1 57.6	3.6	1 59.8	1.5
1000	0 38.2	3.5	2 28.9	3.0	6000	2 1.2	3.5	2 1.3	0.6
1100	0 34.7	3.3	2 31.9	2.0	6100	2 4.7	3.3	2 1.9	0.2
1200	0 31.4	3.2	2 33.9	1.0	6200	2 8.0	3.2	2 1.7	1.0
1300	0 28.2	2.9	2 34.9	0.1	6300	2 11.2	2.9	2 0.7	1.9
1400	0 25.3	2.7	2 35.0	0.9	6400	2 14.1	2.7	1 58.8	2.7
1500	0 22.6	2.5	2 34.1	1.9	6500	2 16.8	2.5	1 56.1	3.6
1600	0 20.1	2.2	2 32.2	2.7	6600	2 19.3	2.3	1 52.5	4.2
1700	0 17.9	2.0	2 29.5	3.6	6700	2 21.6	2.1	1 48.3	4.9
1800	0 15.9	1.7	2 25.9	4.4	6800	2 23.7	1.7	1 43.4	5.6
1900	0 14.2	1.5	2 21.5	5.1	6900	2 25.4	1.6	1 37.8	6.1
2000	0 12.7	1.2	2 16.4	5.7	7000	2 27.0	1.2	1 31.7	6.6
2100	0 11.5	1.0	2 10.7	6.3	7100	2 28.2	1.0	1 25.1	6.9
2200	0 10.5	0.6	2 4.4	6.7	7200	2 29.2	0.8	1 18.2	7.1
2300	0 9.9	0.4	1 57.7	7.0	7300	2 30.0	0.4	1 11.1	7.3
2400	0 9.5	0.1	1 50.7	7.2	7400	2 30.4	0.2	1 3.8	7.3
2500	0 9.4	0.2	1 43.5	7.3	7500	2 30.6	0.1	0 56.5	7.2
2600	0 9.6	0.5	1 36.2	7.3	7600	2 30.5	0.4	0 49.3	7.0
2700	0 10.1	0.7	1 28 9	7.1	7700	2 30.1	0.6	0 42.3	6.7
2800	0 10.8	1.0	1 21.8	6.9	7800	2 29.5	1.0	0 35.6	6.3
2900	0 11.8	1.2	1 14.9	6.6	7900	2 28.5	1.2	0 29.3	5.7
3000	0 13.0	1.6	1 8.3	6.1	8000	2 27.3	1.5	0 23.6	5.1
3100	0 14.6	1.7	1 2.2	5.6	8100	2 25.8	1.7	0 18.5	4.4
3200	0 16.3	2.1	0 56.6	4.9	8200	2 24.1	2.0	0 14.1	3.6
3300	0 18.4	2.3	0 51.7	4.2	8300	2 22.1	2.2	0 10.5	2.7
3400	0 20.7	2.5	0 47.5	3.6	8400	2 19.9	2.5	0 7.8	1.9
3500	0 23.2	2.7	0 43.9	2.7	8500	2 17.4	2.7	0 5.9	0.9
3600	0 25.9	2.9	0 41.2	1.9	8600	2 14.7	2.9	0 5.0	0.1
3700	0 28.8	3.2	0 39.3	1.0	8700	2 11.8	3.2	0 5.1	1.0
3800	0 32.0	3.3	0 38.3	0.2	8800	2 8.6	3.3	0 6.1	2.0
3900	0 35.3	3.5	0 38.1	0.6	8900	2 5.3	3.5	0 8.1	3.0
4000	0 38.8	3.6	0 38.7	1.5	9000	2 1.8	3.7	0 11.1	3.9
4100	0 42.4	3.8	0 40.2	2.2	9100	1 58.1	3.8	0 15.0	4.8
4200	0 46.2	4.0	0 42.4	3.0	9200	1 54.3	4.0	0 19.8	5.5
4300	0 50.2	4.0	0 45.4	3.6	9300	1 50.3	4.1	0 25.3	6.4
4400	0 54.2	4.2	0 49.0	4.2	9400	1 46.2	4.2	0 31.7	7.0
4500	0 58.4	4.2	0 53.2	4.8	9500	1 42.0	4.3	0 38.7	7.6
4600	1 2.6	4.3	0 58.0	5.1	9600	1 37.7	4.4	0 46.3	8.0
4700	1 6.9	4.3	1 3.1	5.5	9700	1 33.3	4.4	0 54.3	8.4
4800	1 11.2	4.4	1 8.6	5.6	9800	1 28.9	4.4	1 2.7	8.6
4900	1 15.6	4.4	1 14.2	5.8	9900	1 24.5	4.5	1 11.3	8.7
5000	1 20.0		1 20.0		10000	1 20.0		1 20.0	

TABLE XI.

Equation 10 & 11 of the Moon's Longitude.

Arg.	10	Dif	11	Dif	Arg.	10	Dif	11	Dif
000	10″0	0″7	10″0	1″1	500	10″0	0″4	10″0	0″8
010	9.3	0.7	11.1	1.0	510	9.6	0.4	10.8	0.7
020	8.6	0.6	12.1	1.0	520	9.2	0.3	11.5	0.8
030	8.0	0.6	13.1	1.0	530	8.9	0.4	12.3	0.6
040	7.4	0.6	14.1	0.9	540	8.5	0.3	12.9	0.7
050	6.8	0.6	15.0	0.8	550	8.2	0.3	13.6	0.6
060	6.2	0.5	15.8	0.8	560	7.9	0.2	14.2	0.4
070	5.7	0.4	16.6	0.7	570	7.7	0.2	14.6	0.4
080	5.3	0.4	17.3	0.6	580	7.5	0.1	15.0	0.4
090	4.9	0.3	17.9	0.4	590	7.4	0.1	15.4	0.2
100	4.6	0.3	18.3	0.3	600	7.3	0.1	15.6	0.1
110	4.3	0.2	18.6	0.3	610	7.2	0.1	15.7	0.0
120	4.1	0.1	18.9	0.1	620	7.3	0.1	15.7	0.1
130	4.0	0.0	19.0	0.1	630	7.4	0.1	15.6	0.2
140	4.0	0.0	18.9	0.1	640	7.5	0.3	15.4	0.3
150	4.0	0.2	18.8	0.2	650	7.8	0.3	15.1	0.4
160	4.2	0.2	18.6	0.4	660	8.1	0.3	14.7	0.5
170	4.4	0.2	18.2	0.5	670	8.4	0.3	14.2	0.7
180	4.6	0.3	17.7	0.6	680	8.7	0.5	13.5	0.7
190	4.9	0.4	17.1	0.6	690	9.2	0.5	12.8	0.7
200	5.3	0.4	16.5	0.8	700	9.7	0.5	12.1	0.8
210	5.7	0.5	15.7	0.8	710	10.2	0.5	11.3	0.9
220	6.2	0.5	14.9	0.8	720	10.7	0.5	10.4	0.9
230	6.7	0.5	14.1	0.9	730	11.2	0.5	9.5	0.9
240	7.2	0.5	13.2	0.9	740	11.7	0.6	8.6	0.9
250	7.7	0.6	12.3	0.9	750	12.3	0.5	7.7	0.9
260	8.3	0.5	11.4	0.9	760	12.8	0.5	6.8	0.9
270	8.8	0.5	10.5	0.9	770	13 3	0.5	5.9	0.8
280	9.3	0.5	9.6	0.9	780	13.8	0.5	5.1	0.8
290	9.8	0.5	8.7	0.8	790	14.3	0.4	4.3	0.8
300	10.3	0.5	7.9	0.7	800	14.7	0.4	3.5	0.6
310	10.8	0.5	7.2	0.7	810	15.1	0.3	2.9	0.6
320	11.3	0.3	6.5	0.7	820	15.4	0.2	2.3	0.5
330	11.6	0.3	5.8	0.5	830	15.6	0.2	1.8	0.4
340	11.9	0.3	5.3	0.4	840	15.8	0.2	1.4	0.2
350	12.2	0.3	4.9	0.3	850	16.0	0.0	1.2	0.1
360	12.5	0.1	4.6	0.2	860	16.0	0.0	1.1	0.1
370	12.6	0.1	4.4	0.1	870	16.0	0.1	1.0	0.1
380	12.7	0.1	4.3	0.0	880	15.9	0.2	1.1	0.3
390	12.8	0.1	4.3	0.1	890	15.7	0.3	1.4	0.3
400	12.7	0.1	4.4	0.2	900	15.4	0.3	1.7	0.4
410	12.6	0.1	4.6	0.4	910	15.1	0.4	2.1	0.6
420	12.5	0.2	5.0	0.4	920	14.7	0.4	2.7	0.7
430	12.3	0.2	5.4	0.4	930	14.3	0.5	3.4	0.8
440	12.1	0.3	5.8	0.6	940	13.8	0.6	4.2	0.8
450	11.8	0.3	6.4	0.7	950	13.2	0.6	5.0	0.9
460	11.5	0.4	7.1	0.6	960	12.6	0.6	5.9	1.0
470	11.1	0.3	7.7	0.8	970	12.0	0.6	6.9	1.0
480	10.8	0.4	8.5	0.7	980	11.4	0.7	7.9	1.0
490	10.4	0.4	9.2	0.8	990	10.7	0.7	8.9	1.1
500	10.0		10.0		1000	10.0		10.0	

TABLE XII.

Equation from 12 to 19 of the Moon's Longitude.

Arg.	12	Diff.	13	Diff.	14	Diff.	15	Diff.	16	Diff.	17	Diff.	18	Dif.	19	Dif.	Arg.
250	2″3		1″6		7″8		0″0		33″7		3″4		16″7		0″4		250
260	2.3	0″0	1.6	0″0	7.8	0″0	0.0	0″0	33.7	0″0	3.4	0″0	16.7	0″0	0.4	0″0	240
270	2.4	0.1	1.7	0.1	7.9	0.1	0.1	0.1	33.6	0.1	3.5	0.1	16.6	0.1	0.4	0.0	230
280	2.6	0.2	1.9	0.2	8.0	0.1	0.2	0.1	33.5	0.1	3.5	0.0	16.6	0.0	0.5	0.1	220
290	2.9	0.3	2.2	0.3	8.2	0.2	0.3	0.1	33.2	0.3	3.6	0.1	16.5	0.1	0.5	0.0	210
300	3.2	0.3	2.5	0.4	8.4	0.2	0.5	0.2	33.0	0.2	3.7	0.2	16.4	0.2	0.6	0.1	200
310	3.5	0.3	2.9	0.5	8.7	0.3	0.7	0.2	32.7	0.3	3.9	0.1	16.2	0.1	0.7	0.1	190
320	4.0	0.5	3.4	0.5	9.0	0.3	1.0	0.3	32.4	0.4	4.0	0.2	16.1	0.2	0.8	0.2	180
330	4.5	0.5	3.9	0.5	9.3	0.3	1.2	0.2	32.0	0.4	4.2	0.2	15.9	0.2	1.0	0.1	170
340	5.1	0.6	4.4	0.7	9.7	0.4	1.6	0.4	31.6	0.5	4.4	0.3	15.7	0.3	1.1	0.2	160
350	5.7	0.6	5.1	0.7	10.1	0.4	1.9	0.3	31.1	0.5	4.7	0.2	15.4	0.2	1.3	0.2	150
360	6.4	0.7	5.8	0.8	10.6	0.5	2.3	0.4	30.6	0.5	4.9	0.3	15.2	0.3	1.5	0.2	140
370	7.1	0.7	6.6	0.8	11.1	0.5	2.7	0.4	30.1	0.7	5.2	0.3	14.9	0.3	1.7	0.2	130
380	7.9	0.8	7.4	0.8	11.7	0.6	3.2	0.5	29.4	0.7	5.5	0.3	14.6	0.3	1.9	0.2	120
390	8.7	0.8	8.3	0.9	12.2	0.5	3.6	0.4	28.7	0.7	5.8	0.3	14.3	0.4	2.1	0.2	110
400	9.6	0.9	9.2	0.9	12.8	0.6	4.1	0.5	28.0	0.7	6.1	0.4	13.9	0.3	2.3	0.2	100
410	10.5	0.9	10.1	1.0	13.5	0.7	4.6	0.5	27.3	0.7	6.5	0.3	13.6	0.4	2.5	0.3	090
420	11.5	1.0	11.1	1.1	14.1	0.6	5.2	0.6	26.6	0.8	6.8	0.4	13.2	0.3	2.8	0.3	080
430	12.5	1.0	12.2	1.0	14.8	0.7	5.7	0.5	25.8	0.8	7.2	0.4	12.9	0.4	3.1	0.2	070
440	13.5	1.0	13.2	1.1	15.5	0.7	6.3	0.6	25.0	0.8	7.6	0.4	12.5	0.4	3.3	0.3	060
450	14.5	1.1	14.3	1.1	16.2	0.8	6.9	0.6	24.2	0.8	8.0	0.4	12.1	0.4	3.6	0.3	050
460	15.6	1.1	15.4	1.1	17.0	0.7	7.5	0.6	23.4	0.8	8.4	0.4	11.7	0.4	3.9	0.2	040
470	16.7	1.1	16.5	1.2	17.7	0.8	8.1	0.6	22.6	0.9	8.8	0.4	11.3	0.5	4.1	0.3	030
480	17.8	1.1	17.7	1.1	18.5	0.7	8.7	0.7	21.7	0.8	9.2	0.4	10.8	0.4	4.4	0.3	020
490	18.9	1.1	18.8	1.2	19.2	0.8	9.4	0.6	20.9	0.9	9.6	0.4	10.4	0.4	4.7	0.3	010
500	20.0	1.1	20.0	1.2	20.0	0.8	10.0	0.6	20.0	0.9	10.0	0.4	10.0	0.4	5.0	0.3	000
510	21.1	1.1	21.2	1.1	20.8	0.7	10.6	0.7	19.1	0.8	10.4	0.4	9.6	0.4	5.3	0.3	990
520	22.2	1.1	22.3	1.2	21.5	0.8	11.3	0.6	18.3	0.9	10.8	0.4	9.2	0.5	5.6	0.3	980
530	23.3	1.1	23.5	1.1	22.3	0.7	11.9	0.6	17.4	0.8	11.2	0.4	8.7	0.4	5.9	0.2	970
540	24.4	1.1	24.6	1.1	23.0	0.8	12.5	0.6	16.6	0.8	11.6	0.4	8.3	0.4	6.1	0.3	960
550	25.5	1.0	25.7	1.1	23.8	0.7	13.1	0.6	15.8	0.8	12.0	0.4	7.9	0.4	6.4	0.3	950
560	26.5	1.0	26.8	1.0	24.5	0.7	13.7	0.6	15.0	0.8	12.4	0.4	7.5	0.4	6.7	0.2	940
570	27.5	1.0	27.8	1.1	25.2	0.7	14.3	0.5	14.2	0.8	12.8	0.4	7.1	0.3	6.9	0.3	930
580	28.5	1.0	28.9	1.0	25.9	0.6	14.8	0.6	13.4	0.7	13.2	0.3	6.8	0.4	7.2	0.3	920
590	29.5	0.9	29.9	0.9	26.5	0.7	15.4	0.5	12.7	0.7	13.5	0.4	6.4	0.3	7.5	0.2	910
600	30.4	0.9	30.8	0.9	27.2	0.6	15.9	0.5	12.0	0.7	13.9	0.3	6.1	0.4	7.7	0.2	900
610	31.3	0.9	31.7	0.9	27.8	0.5	16.4	0.4	11.3	0.7	14.2	0.3	5.7	0.3	7.9	0.2	890
620	32.1	0.8	32.6	0.8	28.3	0.6	16.8	0.5	10.6	0.7	14.5	0.3	5.4	0.3	8.1	0.2	880
630	32.9	0.8	33.4	0.8	28.9	0.5	17.3	0.4	9.9	0.5	14.8	0.3	5.1	0.3	8.3	0.2	870
640	33.6	0.7	34.2	0.7	29.4	0.5	17.7	0.4	9.4	0.5	15.1	0.2	4.8	0.2	8.5	0.2	860
650	34.3	0.6	34.9	0.7	29.9	0.4	18.1	0.3	8.9	0.5	15.3	0.3	4.6	0.3	8.7	0.2	850
660	34.9	0.6	35.6	0.5	30.3	0.4	18.4	0.4	8.4	0.4	15.6	0.2	4.3	0.2	8.9	0.1	840
670	35.5	0.5	36.1	0.5	30.7	0.3	18.8	0.2	8.0	0.4	15.8	0.2	4.1	0.2	9.0	0.2	830
680	36.0	0.5	36.6	0.5	31.0	0.3	19.0	0.3	7.6	0.3	16.0	0.1	3.9	0.1	9.2	0.1	820
690	36.5	0.3	37.1	0.4	31.3	0.3	19.3	0.2	7.3	0.3	16.1	0.2	3.8	0.2	9.3	0.1	810
700	36.8	0.3	37.5	0.3	31.6	0.2	19.5	0.2	7.0	0.2	16.3	0.1	3.6	0.1	9.4	0.1	800
710	37.1	0.3	37.8	0.3	31.8	0.2	19.7	0.1	6.8	0.3	16.4	0.1	3.5	0.1	9.5	0.0	790
720	37.4	0.2	38.1	0.2	32.0	0.1	19.8	0.1	6.5	0.1	16.5	0.0	3.4	0.0	9.5	0.1	780
730	37.6	0.1	38.3	0.1	32.1	0.1	19.9	0.1	6.4	0.1	16.5	0.1	3.4	0.1	9.6	0.0	770
740	37.7	0.1	38.4	0.0	32.2	0.0	20.0	0.0	6.3	0.0	16.6	0.0	3.3	0.0	9.6	0.0	760
750	37.7	0.0	38.4		32.2		20.0		6.3		16.6		3.3		9.6		750

TABLE XIII.

Equation 20.

Arg.	Arg.	20	Diff.	Arg.	Arg.	20	Diff.
000	500	10″0	0″9	250	750	10″0	0″9
010	510	10.9	0.9	260	760	9.1	0.9
020	520	11.8	0.9	270	770	8.2	0.9
030	530	12.7	0.8	280	780	7.3	0.8
040	540	13.5	0.8	290	790	6.5	0.8
050	550	14.3	0.7	300	800	5.7	0.7
060	560	15.0	0.7	310	810	5.0	0.7
070	570	15.7	0.5	320	820	4.3	0.5
080	580	16.2	0.5	330	830	3.8	0.5
090	590	16.7	0.3	340	840	3.3	0.3
100	600	17.0	0.2	350	850	3.0	0.2
110	610	17.2	0.2	360	860	2.8	0.2
120	620	17.4	0.0	370	870	2.6	0.0
130	630	17.4	0.2	380	880	2.6	0.2
140	640	17.2	0.2	390	890	2.8	0.2
150	650	17.0	0.3	400	900	3.0	0.3
160	660	16.7	0.5	410	910	3.3	0.5
170	670	16.2	0.5	420	920	3.8	0.5
180	680	15.7	0.7	430	930	4.3	0.7
190	690	15.0	0.7	440	940	5.0	0.7
200	700	14.3	0.8	450	950	5.7	0.8
210	710	13.5	0.8	460	960	6.5	0.8
220	720	12.7	0.9	470	970	7.3	0.9
230	730	11.8	0.9	480	980	8.2	0.9
240	740	10.9	0.9	490	990	9.1	0.9
250	750	10.0		500	1000	10.0	

TABLE XIV

Equations from 21 to 29.

Arg.	21	22	23	24	25	26	27	28	29	Arg.
25	7″8	3″2	7″1	6″1	5″9	4.1	5″8	4″3	5″7	25
27	7.8	3.2	7.1	6.1	5.9	4.1	5.8	4.3	5.7	23
29	7.7	3.3	7.0	6.1	5.9	4.1	5.8	4.3	5.7	21
31	7.6	3.3	7.0	6.0	5.8	4.2	5.7	4.3	5.7	19
33	7.5	3.4	6.8	6.0	5.8	4.2	5.7	4.4	5.6	17
35	7.3	3.5	6.7	5.9	5.7	4.3	5.6	4.4	5.6	15
37	7.0	3.7	6.5	5.8	5.7	4.3	5.6	4.5	5.5	13
39	6.8	3.9	6.3	5.7	5.6	4.4	5.5	4.6	5.4	11
41	6.5	4.0	6.1	5.6	5.5	4.5	5.4	4.6	5.4	09
43	6.2	4.2	5.9	5.5	5.4	4.6	5.3	4.7	5.3	07
45	5.9	4.4	5.6	5.3	5.3	4.7	5.2	4.8	5.2	05
47	5.5	4.7	5.4	5.2	5.2	4.8	5.1	4.9	5.1	03
49	5.2	4.9	5.1	5.1	5.1	4.9	5.0	5.0	5.0	01
51	4.8	5.1	4.9	4.9	4.9	5.1	5.0	5.0	5.0	99
53	4.5	5.3	4.6	4.8	4.8	5.2	4.9	5.1	4.9	97
55	4.1	5.6	4.4	4.7	4.7	5.3	4.8	5.2	4.8	95
57	3.8	5.8	4.1	4.5	4.6	5.4	4.7	5.3	4.7	93
59	3.5	6.0	3.9	4.4	4.5	5.5	4.6	5.4	4.6	91
61	3.2	6.1	3.7	4.3	4.4	5.6	4.5	5.4	4.6	89
63	3.0	6.3	3.5	4.2	4.3	5.7	4.4	5.5	4.5	87
65	2.7	6.5	3.3	4.1	4.3	5.7	4.4	5.6	4.4	85
67	2.5	6.6	3.2	4.0	4.2	5.8	4.3	5.6	4.4	83
69	2.4	6.7	3.0	4.0	4.2	5.8	4.3	5.7	4.3	81
71	2.3	6.7	3.0	3.9	4.1	5.9	4.2	5.7	4.3	79
73	2.2	6.8	2.9	3.9	4.1	5.9	4.2	5.7	4.3	77
75	2.2	6.8	2.9	3.9	4.1	5.9	4.2	5.7	4.3	75

TABLE XV.

Equations 30 and 31.

Arg.	30	31	Arg.	30	31	Arg.	30	31
00	5″0	5″0	34	3″7	5″9	68	6″3	4″1
02	5.0	5.0	36	3.7	5.9	70	6.3	4.1
04	4.9	5.1	38	3.8	5.8	72	6.2	4.1
06	4.9	5.1	40	3.9	5.7	74	6.2	4.2
08	4.8	5.2	42	4.1	5.6	76	6.0	4.2
10	4.8	5.2	44	4.3	5.5	78	5.9	4.3
12	4.7	5.3	46	4.5	5.3	80	5.8	4.4
14	4.6	5.4	48	4.8	5.2	82	5.7	4.5
16	4.5	5.5	50	5.0	5.0	84	5.5	4.6
18	4.4	5.5	52	5.2	4.8	86	5.4	4.6
20	4.2	5.6	54	5.5	4.7	88	5.3	4.7
22	4.1	5.7	56	5.7	4.5	90	5.2	4.8
24	4.0	5.8	58	5.9	4.4	92	5.1	4.8
26	3.9	5.8	60	6.1	4.3	94	5.1	4.9
28	3.8	5.9	62	6.2	4.2	96	5.0	4.9
30	3.7	5.9	64	6.3	4.1	98	5.0	5.0
32	3.7	5.9	66	6.3	4.1	100	5.0	5.0
34	3.7	5.9	68	6.3	4.1			

TABLE XVI.

Equation 32. *Argum.* supplem. of ☽ Node.

Deg.	III*s*	IV*s*	V*s*	VI*s*	VII*s*	VIII*s*	Deg.
0	3″1	4″0	6″5	10″0	13″5	16.0	30
2	3.1	4.2	6.8	10.2	13.7	16.1	28
4	3.1	4.3	7.0	10.5	13.8	16.2	26
6	3.1	4.4	7.2	10.7	14.0	16.3	24
8	3.2	4.6	7.4	11.0	14.2	16.4	22
10	3.2	4.7	7.6	11.2	14.4	16.5	20
12	3.3	4.9	7.9	11.4	14.6	16.6	18
14	3.3	5.0	8.1	11.7	14.8	16.6	16
16	3.4	5.2	8.3	11.9	15.0	16.7	14
18	3.4	5.4	8.6	12.1	15.1	16.7	12
20	3.5	5.6	8.8	12.4	15.3	16.8	10
22	3.6	5.8	9.0	12.6	15.4	16.8	8
24	3.7	6.0	9.3	12.8	15.6	16.9	6
26	3.8	6.2	9.5	13.0	15.7	16.9	4
28	3.9	6.3	9.8	13.2	15.8	16.9	2
30	4.0	6.5	10.0	13.5	16.0	16.9	0
	II	I	0	XI	X	IX	

TABLE XVII.

EVECTION.

Deg.	Os	Diff	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs	Diff.	Vs	Diff.
	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″
0	1 30 00.0	85.5	2 10 43.5	73.2	2 40 9.7	40.9	2 50 25.5	2.0	2 39 8.3	43.4	2 9 42.0	72.7
1	1 31 25.5	85.4	2 11 56.7	72.3	2 40 50.6	39.5	2 50 23.5	3.4	2 38 24.9	44.5	2 8 29.3	73.3
2	1 32 50.9	85.4	2 13 9.0	71.6	2 41 30.1	38.2	2 50 20.1	4.9	2 37 40.4	45.8	2 7 16.0	74.0
3	1 34 16.3	85.3	2 14 20.6	70.7	2 42 8.3	36.8	2 50 15.2	6.4	2 36 54.6	47.0	2 6 2.0	74.6
4	1 35 41.6	85.1	2 15 31.3	69.8	2 42 45.1	35.5	2 50 8.8	7.8	2 36 7.6	48.1	2 4 47.4	75.2
5	1 37 6.7	85.1	2 16 41.1	69.0	2 43 20.6	34.1	2 50 1.0	9.3	2 35 19.5	49.3	2 3 32.2	75.9
6	1 38 31.8	84.9	2 17 50.1	68.1	2 43 54.7	32.7	2 49 51.7	10.7	2 34 30.2	50.5	2 2 16.3	76.4
7	1 39 56.7	84.7	2 18 58.2	67.1	2 44 27.4	31.4	2 49 41.0	12.2	2 33 39.7	51.6	2 0 59.9	76.9
8	1 41 21.4	84.4	2 20 5.3	66.2	2 44 58.8	29.9	2 49 28.8	13.7	2 32 48.1	52.7	1 59 43.0	77.4
9	1 42 45.8	84.3	2 21 11.5	65.2	2 45 28.7	28.6	2 49 15.1	14.9	2 31 55.4	53.8	1 58 25.6	78.0
10	1 44 10.1	83.9	2 22 16.7	64.3	2 45 57.3	27.2	2 49 0.2	16.7	2 31 1.6	54.9	1 57 7.6	78.4
11	1 45 34.0	83.7	2 23 21.0	63.2	2 46 24.5	25.7	2 48 43.5	17.9	2 30 6.7	56.0	1 55 49.2	78.9
12	1 46 57.7	83.4	2 24 24.2	62.2	2 46 50.2	24.3	2 48 25.6	19.3	2 29 10.7	57.0	1 54 30.3	79.3
13	1 48 21.1	83.0	2 25 26.4	61.2	2 47 14.5	22.9	2 48 6.3	20.8	2 28 13.7	58.0	1 53 11.0	79.7
14	1 49 44.1	82.6	2 26 27.6	60.0	2 47 37.4	21.4	2 47 45.5	22.2	2 27 15.7	59.1	1 51 51.3	80.1
15	1 51 6.7	82.2	2 27 27.6	59.0	2 47 58.8	20.0	2 47 23.3	23.5	2 26 16.6	60.0	1 50 31.2	80.5
16	1 52 28.9	81.8	2 28 26.6	58.0	2 48 18.8	18.6	2 46 59.8	25.0	2 25 16.6	61.0	1 49 10.7	80.8
17	1 53 50.7	81.3	2 29 24.6	56.8	2 48 37.4	17.1	2 46 34.8	26.3	2 24 15.6	62.0	1 47 49.9	81.1
18	1 55 12.0	80.9	2 30 21.4	55.6	2 48 54.5	15.6	2 46 8.5	27.8	2 23 13.6	62.9	1 46 28.8	81.3
19	1 56 32.9	80.3	2 31 17.0	54.5	2 49 10.1	14.3	2 45 40.7	29.1	2 22 10.7	63.9	1 45 7.5	81.7
20	1 57 53.2	79.8	2 32 11.5	53.3	2 49 24.4	12.7	2 45 11.6	30.4	2 21 6.8	64.7	1 43 45.8	81.9
21	1 59 13.0	79.3	2 33 4.8	52.2	2 49 37.1	11.2	2 44 41.2	31.7	2 20 2.1	65.7	1 42 23.9	82.1
22	2 0 32.3	78.7	2 33 57.0	50.9	2 49 48.3	9.8	2 44 9.5	33.1	2 18 56.4	66.5	1 41 1.8	82.3
23	2 1 51.0	78.1	2 34 47.9	49.8	2 49 58.1	8.3	2 43 36.4	34.5	2 17 49.9	67.3	1 39 39.5	82.5
24	2 3 9.1	77.4	2 35 37.7	48.5	2 50 6.4	6.9	2 43 1.9	35.7	2 16 42.6	68.2	1 38 17.0	82.6
25	2 4 26.5	76.8	2 36 26.2	47.2	2 50 13.3	5.4	2 42 26.2	37.0	2 15 34.4	68.9	1 36 54.4	82.7
26	2 5 43.3	76.1	2 37 13.4	46.0	2 50 18.7	3.9	2 41 49.2	38.4	2 14 25.5	69.8	1 35 31.7	82.9
27	2 6 59.4	75.5	2 37 59.4	44.8	2 50 22.6	2.4	2 41 10.8	39.6	2 13 15.7	70.5	1 34 8.8	82.9
28	2 8 14.9	74.7	2 38 44.2	43.4	2 50 25.0	1.0	2 40 31.2	40.8	2 12 5.2	71.2	1 32 45.9	82.9
29	2 9 29.6	73.9	2 39 27.6	42.1	2 50 26.0	0.5	2 39 50.4	42.1	2 10 54.0	72.0	1 31 23.0	83.0
30	2 10 43.5		2 40 9.7		2 50 25.5		2 39 8.3		2 9 42.0		1 30 0.0	

TABLE XVII. Continued.

EVECTION.

Deg.	VIs	Diff.	VIIs	Diff.	VIIIs	Diff.	IXs	Diff.	Xs	Diff.	XIs	Diff.
	° ′ ″	″	′ ″	″	′ ″	″	′ ″	″	′ ″	″	° ′ ″	″
0	1 30 0.0	83.0	50 18.0	72.0	20 51.7	42.1	9 34.5	0.5	19 50.3	42.1	0 49 16.5	73.9
1	1 28 37.0	82.9	49 6.0	71.2	20 9.6	40.8	9 34.0	1.0	20 32.4	43.4	0 50 30.4	74.7
2	1 27 14.1	82.9	47 54.8	70.5	19 28.8	39.6	9 35.0	2.4	21 15.8	44.8	0 51 45.1	75.5
3	1 25 51.2	82.9	46 44.3	69.8	18 49.2	38.4	9 37.4	3.9	22 0.6	46.0	0 53 0.6	76.1
4	1 24 28.3	82.7	45 34.5	68.9	18 10.8	37.0	9 41.3	5.4	22 46.6	47.2	0 54 16.7	76.8
5	1 23 5.6	82.6	44 25.6	68.2	17 33.8	35.7	9 46.7	6.9	23 33.8	48.5	0 55 33.5	77.4
6	1 21 43.0	82.5	43 17.4	67.3	16 58.1	34.5	9 53.6	8.3	24 22.3	49.8	0 56 50.9	78.1
7	1 20 20.5	82.3	42 10.1	66.5	16 23.6	33.1	10 1.9	9.8	25 12.1	50.9	0 58 9.0	78.7
8	1 18 58.2	82.1	41 3.6	65.7	15 50.5	31.7	10 11.7	11.2	26 3.0	52.2	0 59 27.7	79.3
9	1 17 36.1	81.9	39 57.9	64.7	15 18.8	30.4	10 22.9	12.7	26 55.2	53.3	1 0 47.0	79.8
10	1 16 14.2	81.7	38 53.2	63.9	14 48.4	29.1	10 35.6	14.3	27 48.5	54.5	1 2 6.8	80.3
11	1 14 52.5	81.3	37 49.3	62.9	14 19.3	27.8	10 49.9	15.6	28 43.0	55.6	1 3 27.1	80.9
12	1 13 31.2	81.1	36 46.4	62.0	13 51.5	26.3	11 5.5	17.1	29 38.6	56.8	1 4 48.0	81.3
13	1 12 10.1	80.8	35 44.4	61.0	13 25.2	25.0	11 22.6	18.6	30 35.4	58.0	1 6 9.3	81.8
14	1 10 49.3	80.5	34 43.4	60.0	13 0.2	23.5	11 41.2	20.0	31 33.4	59.0	1 7 31.1	82.2
15	1 9 28.8	80.1	33 43.4	59.1	12 36.7	22.2	12 1.2	21.4	32 32.4	60.0	1 8 53.3	82.6
16	1 8 8.7	79.7	32 44.3	58.0	12 14.5	20.8	12 22.6	22.9	33 32.4	61.2	1 10 15.9	83.0
17	1 6 49.0	79.3	31 46.3	57.0	11 53.7	19.3	12 45.5	24.3	34 33.6	62.2	1 11 38.9	83.4
18	1 5 29.7	78.9	30 49.3	56.0	11 34.4	17.9	13 9.8	25.7	35 35.8	63.2	1 13 2.3	83.7
19	1 4 10.8	78.4	29 53.3	54.9	11 16.5	16.7	13 35.5	27.2	36 39.0	64.3	1 14 26.0	83.9
20	1 2 52.4	78.0	28 58.4	53.8	10 59.8	14.9	14 2.7	28.6	37 43.3	65.2	1 15 49.9	84.3
21	1 1 34.4	77.4	28 4.6	52.7	10 44.9	13.7	14 31.3	29.9	38 48.5	66.2	1 17 14.2	84.4
22	1 0 17.0	76.9	27 11.9	51.6	10 31.2	12.2	15 1.2	31.4	39 54.7	67.1	1 18 38.6	84.7
23	0 59 0.1	76.4	26 20.3	50.5	10 19.0	10.7	15 32.6	32.7	41 1.8	68.1	1 20 3.3	84.9
24	0 57 43.7	75.9	25 29.8	49.3	10 8.3	9.3	16 5.3	34.1	42 9.9	69.0	1 21 28.2	85.1
25	0 56 27.8	75.2	24 40.5	48.1	9 59.0	7.8	16 39.4	35.5	43 18.9	69.8	1 22 53.3	85.1
26	0 55 12.6	74.6	23 52.4	47.0	9 51.2	6.4	17 14.9	36.8	44 28.7	70.7	1 24 18.4	85.3
27	0 53 58.0	74.0	23 5.4	45.8	9 44.8	4.9	17 51.7	38.2	45 39.4	71.6	1 25 43.7	85.4
28	0 52 44.0	73.3	22 19.6	44.5	9 39.9	3.4	18 29.9	39.5	46 51.0	72.3	1 27 9.1	85.4
29	0 51 30.7	72.7	21 35.1	43.4	9 36.5	2.0	19 9.4	40.9	48 3.3	73.2	1 28 34.5	85.5
30	0 50 18.0		20 51.7		9 34.5		19 50.3		49 16.5		1 30 0.0	

TABLE XVIII.

EQUATION OF THE CENTRE.

Argum.—Anomaly.

	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs	Diff.	Vs	Diff.
0.00	7 0 0.0	70.9	10 20 57.9	59.4	12 38 43.6	30.3	13 17 35.2	4.6	12 16 20.8	35.1	9 58 28.9	55.0
0.10	7 1 10.9	70.9	10 21 57.3	59.2	12 39 13.9	30.1	13 17 30.6	4.8	12 15 45.7	35.1	9 57 33.9	55.0
0.20	7 2 21.8	70.8	10 22 56.5	59.1	12 39 44.0	30.0	13 17 25.8	4.9	12 15 10.6	35.3	9 56 38.9	55.1
0.30	7 3 32.6	70.9	10 23 55.6	59.0	12 40 14.0	29.7	13 17 20.9	5.2	12 14 35.3	35.5	9 55 43.8	55.2
0.40	7 4 43.5	70.9	10 24 54.6	58.9	12 40 43.7	29.6	13 17 15.7	5.4	12 13 59.8	35.6	9 54 48.6	55.3
0.50	7 5 54.4	70.8	10 25 53.5	58.7	12 41 13.3	29.4	13 17 10.3	5.5	12 13 24.2	35.7	9 53 53.3	55.3
1. 0	7 7 5.2	70.9	10 26 52.2	58.6	12 41 42.7	29.1	13 17 4.8	5.7	12 12 48.5	35.9	9 52 58.0	55.4
1.10	7 8 16.1	70.8	10 27 50.8	58.5	12 42 11.8	29.0	13 16 59.1	5.9	12 12 12.6	36.0	9 52 2.6	55.5
1.20	7 9 26.9	70.9	10 28 49.3	58.4	12 42 40.8	28.8	13 16 53.2	6.1	12 11 36.6	36.2	9 51 7.1	55.5
1.30	7 10 37.8	70.8	10 29 47.7	58.2	12 43 9.6	28 6	13 16 47.1	6.3	12 11 0.4	36.3	9 50 11.6	55.6
1.40	7 11 48.6	70.9	10 30 45.9	58.1	12 43 38.2	28.5	13 16 40.8	6.5	12 10 24.1	36.4	9 49 16.0	55.7
1.50	7 12 59.5	70.8	10 31 44.0	58.0	12 44 6.7	28.2	13 16 34.3	6.7	12 9 47.7	36.6	9 48 20.3	55.8
2. 0	7 14 10.3	70.8	10 32 42.0	57.9	12 44 34.9	28.0	13 16 27.6	6.8	12 9 11.1	36.7	9 47 24.5	55.8
2.10	7 15 21.1	70.8	10 33 39.9	57.7	12 45 2.9	27.9	13 16 20.8	7.1	12 8 34.4	36.9	9 46 28.7	55.9
2.20	7 16 31.9	70.8	10 34 37.6	57.6	12 45 30.8	27.6	13 16 13.7	7.2	12 7 57.5	37.0	9 45 32.8	56.0
2.30	7 17 42.7	70.8	10 35 35.2	57.4	12 45 58.4	27.5	13 16 6.5	7.4	12 7 20.5	37.1	9 44 36.8	56.0
2.40	7 18 53.5	70.8	10 36 32.6	57.3	12 46 25.9	27.2	13 15 59.1	7.6	12 6 43.4	37.3	9 43 40.8	56.1
2.50	7 20 4.3	70.7	10 37 29.9	57.2	12 46 53.1	27.1	13 15 51.5	7.8	12 6 6.1	37.4	9 42 44.7	56.2
3. 0	7 21 15.0	70.8	10 38 27.1	57.1	12 47 20.2	26.9	13 15 43.7	8.0	12 5 28.7	37.6	9 41 48.5	56.3
3.10	7 22 25.8	70.8	10 39 24.2	57.0	12 47 47.1	26.7	13 15 35.7	8.1	12 4 51.1	37.7	9 40 52.2	56.3
3.20	7 23 36.6	70.7	10 40 21.2	56.8	12 48 13.8	26.5	13 15 27.6	8.4	12 4 13.4	37.8	9 39 55.9	56.4
3.30	7 24 47.3	70.7	10 41 18.0	56.6	12 48 40.3	26.3	13 15 19.2	8.5	12 3 35.6	37.9	9 38 59.5	56.4
3.40	7 25 58.0	70.7	10 42 14.6	56.6	12 49 6.6	26.1	13 15 10.7	8.7	12 2 57.7	38.1	9 38 3.1	56.5
3.50	7 27 8.7	70.7	10 43 11.2	56.4	12 49 32.7	26.0	13 15 2.0	8.9	12 2 19.6	38.3	9 37 6.6	56.6
4. 0	7 28 19.4	70.6	10 44 7.6	56.3	12 49 58.7	25.7	13 14 53.1	9.1	12 1 41.3	38.3	9 36 10.0	56.7
4.10	7 29 30.0	70.6	10 45 3.9	56.1	12 50 24.4	25.5	13 14 44.0	9.3	12 1 3.0	38.5	9 35 13.3	56.7
4.20	7 30 40.6	70.6	10 46 0.0	56.0	12 50 49.9	25.4	13 14 34.7	9.5	12 0 24.5	38.7	9 34 16.6	56.8
4.30	7 31 51.2	70.6	10 46 56.0	55.8	12 51 15.3	25.2	13 14 25.2	9.6	11 59 45.8	38.8	9 33 19.8	56.8
4.40	7 33 1.8	70.6	10 47 51.8	55.8	12 51 40.5	24.9	13 14 15.6	9.8	11 59 7.0	38.9	9 32 23.0	56.9
4.50	7 34 12.4	70.6	10 48 47.6	55.6	12 52 5.4	24.8	13 14 5.8	10.0	11 58 28.1	39.0	9 31 26.1	57.0
5. 0	7 35 23.0	70.5	10 49 43.2	55.4	12 52 30.2	24.6	13 13 55.8	10.2	11 57 49.1	39.2	9 30 29.1	57.1
5.10	7 36 33.5	70.5	10 50 38.6	55.3	12 52 54.8	24.3	13 13 45.6	10.3	11 57 9.9	39.3	9 29 32.0	57.1
5.20	7 37 44.0	70.5	10 51 33.9	55.2	12 53 19.1	24.2	13 13 35.3	10.6	11 56 30.6	39.5	9 28 34.9	57.1
5.30	7 38 54.5	70.4	10 52 29.1	55.0	12 53 43.3	24.0	13 13 24.7	10.8	11 55 51.1	39.5	9 27 37.8	57.2
5.40	7 40 4.9	70.5	10 53 24.1	54.9	12 54 7.3	23.8	13 13 13.9	10.9	11 55 11.6	39.7	9 26 40.6	57.3
5.50	7 41 15.4	70.4	10 54 19.0	54.8	12 54 31.1	23.6	13 13 3.0	11.1	11 54 31.9	39.9	9 25 43.3	57.4
6. 0	7 42 25.8	70.4	10 55 13.8	54.6	12 54 54.7	23.5	13 12 51.9	11.3	11 53 52.0	39.9	9 24 45.9	57.4
6.10	7 43 36.2	70.3	10 56 8.4	54.5	12 55 18.2	23.2	13 12 40.6	11.5	11 53 12.1	40.1	9 23 48.5	57.5
6.20	7 44 46.5	70.4	10 57 2.9	54.3	12 55 41.4	23.0	13 12 29.1	11.7	11 52 32.0	40.3	9 22 51.0	57.5
6.30	7 45 56.9	70.3	10 57 57.2	54.2	12 56 4.4	22.8	13 12 17.4	11.8	11 51 51.7	40.3	9 21 53.5	57.6
6.40	7 47 7.2	70.3	10 58 51.4	54.0	12 56 27.2	22.7	13 12 5.6	12.0	11 51 11.4	40.5	9 20 55.9	57.6
6.50	7 48 17.5	70.2	10 59 45.4	53.9	12 56 49.9	22.4	13 11 53.6	12.2	11 50 30.9	40.6	9 19 58.3	57.7
7.00	7 49 27.7	70.2	11 0 39.3	53.8	12 57 12.3	22.3	13 11 41.4	12.4	11 49 50.3	40.8	9 19 0.6	57.8
7.10	7 50 37.9	70.2	11 1 33.1	53.6	12 57 34.6	22.0	13 11 29.0	12.6	11 49 9.5	40.9	9 18 2.8	57.8
7.20	7 51 48.1	70.1	11 2 26.7	53.4	12 57 56.6	21.9	13 11 16.4	12.7	11 48 28.6	41.0	9 17 5.0	57.9
7.30	7 52 58.2		11 3 20.1		12 58 18.5		13 11 3.7		11 47 47.6		9 16 7.1	

TABLE XVIII. continued.

	VI*s*.	Diff.	VII*s*.	Diff.	VIII*s*.	Diff.	IX*s*.	Diff.	X*s*.	Diff.	XI*s*	Diff.
° ′	° ′ ″	″	° ′ ″	″	° ′ ″	″	′ ″	″	° ′ ″	″	° ′ ″	″
0 0	7 0 0.0	61.8	4 1 31.1	54.8	1 43 39.2	34.9	42 24.8	4.4	1 21 16.4	30.5	3 39 2.1	59.5
0 10	6 58 58.2	61.8	4 0 36.3	54.8	1 43 4.3	34.7	42 20.4	4.2	1 21 46.9	30.7	3 40 1.6	59.5
0 20	6 57 56.4	61.8	3 59 41.5	54.8	1 42 29.6	34.6	42 16.2	4.1	1 22 17.6	30.9	3 41 1.1	59.7
0 30	6 56 54.6	61.8	3 58 46.7	54.6	1 41 55.0	34.4	42 12.1	3.8	1 22 48.5	31.0	3 42 0.8	59.9
0 40	6 55 52.8	61.8	3 57 52.1	54.6	1 41 20.6	34.3	42 8.3	3.6	1 23 19.5	31.3	3 43 0.7	59.9
0 50	6 54 51.0	61.8	3 56 57.5	54.5	1 40 46.3	34.3	42 4.7	3.5	1 23 50.8	31.4	3 44 0.6	60.1
1 0	6 53 49.2	61.7	3 56 3.0	54.4	1 40 12.0	34.0	42 1.2	3.3	1 24 22.2	31.6	3 45 0.7	60.2
1 10	6 52 47.5	61.8	3 55 8.6	54.3	1 39 38.0	33.8	41 57.9	3.0	1 24 53.8	31.9	3 46 0.9	60.3
1 20	6 51 45.7	61.8	3 54 14.3	54.3	1 39 4.2	33.7	41 54.9	2.9	1 25 25.7	32.0	3 47 1.2	60.5
1 30	6 50 43.9	61.8	3 53 20.0	54.2	1 38 30.5	33.5	41 52.0	2.7	1 25 57.7	32.2	3 48 1.7	60.5
1 40	6 49 42.1	61.7	3 52 25.8	54.1	1 37 57.0	33.4	41 49.3	2.6	1 26 29.9	32.4	3 49 2.2	60.7
1 50	6 48 40.4	61.8	3 51 31.7	54.0	1 37 23.6	33.3	41 46.7	2.3	1 27 2.3	32.5	3 50 2.9	60.8
2 0	6 47 38.6	61.7	3 50 37.7	53.9	1 36 50.3	33.1	41 44.4	2.1	1 27 34.8	32.8	3 51 3.7	60.9
2 10	6 46 36.9	61.8	3 49 43.8	53.8	1 36 17.2	33.0	41 42.3	1.9	1 28 7.6	32.9	3 52 4.6	61.0
2 20	6 45 35.1	61.7	3 48 50.0	53.8	1 35 44.2	32.9	41 40.4	1.7	1 28 40.5	33.2	3 53 5.6	61.1
2 30	6 44 33.4	61.8	3 47 56.2	53.7	1 35 11.3	32.7	41 38.7	1.6	1 29 13.7	33.3	3 54 6.7	61.2
2 40	6 43 31.6	61.7	3 47 2.5	53.6	1 34 38.6	32.5	41 37.1	1.3	1 29 47.0	33.5	3 55 7.9	61.4
2 50	6 42 29.9	61.8	3 46 8.9	53.5	1 34 6.1	32.4	41 35.8	1.2	1 30 20.5	33.7	3 56 9.3	61.4
3 0	6 41 28.1	61.7	3 45 15.4	53.5	1 33 33.7	32.2	41 34.6	1.0	1 30 54.2	33.8	3 57 10.7	61.6
3 10	6 40 26.4	61.7	3 44 21.9	53.4	1 33 1.5	32.1	41 33.6	0.8	1 31 28.0	34.1	3 58 12.3	61.7
3 20	6 39 24.7	61.7	3 43 28.5	53.2	1 32 29.4	31.9	41 32.8	0.6	1 32 2.1	34.2	3 59 14.0	61.8
3 30	6 38 23.0	61.7	3 42 35.3	53.2	1 31 57.5	31.8	41 32.2	0.4	1 32 36.3	34.5	4 0 15.8	61.9
3 40	6 37 21.3	61.6	3 41 42.1	53.1	1 31 25.7	31.6	41 31.8	0.2	1 33 10.8	34.6	4 1 17.7	62.0
3 50	6 36 19.7	61.7	3 40 49.0	53.0	1 30 54.1	31.5	41 31.6	0.0	1 33 45.4	34.8	4 2 19.7	62.1
4 0	6 35 18.0	61.7	3 39 56.0	53.0	1 30 22.6	31.4	41 31.6	0.2	1 34 20.2	34.9	4 3 21.8	62.2
4 10	6 34 16.3	61.6	3 39 3.0	52.8	1 29 51.2	31.2	41 31.8	0.4	1 34 55.1	35.2	4 4 24.0	62.4
4 20	6 33 14.7	61.7	3 38 10.2	52.8	1 29 20.0	31.0	41 32.2	0.5	1 35 30.3	35.3	4 5 26.4	62.4
4 30	6 32 13.0	61.6	3 37 17.4	52.7	1 28 49.0	30.9	41 32.7	0.8	1 36 5.6	35.6	4 6 28.8	62.6
4 40	6 31 11.4	61.7	3 36 24.7	52.6	1 28 18.1	30.7	41 33.5	0.9	1 36 41.2	35.7	4 7 31.4	62.6
4 50	6 30 9.7	61.6	3 35 32.1	52.5	1 27 47.4	30.6	41 34.4	1.2	1 37 16.9	35.9	4 8 34.0	62.8
5 0	6 29 8.1	61.6	3 34 39.6	52.4	1 27 16.8	30.4	41 35.6	1.3	1 37 52.8	36.0	4 9 36.8	62.9
5 10	6 28 6.5	61.5	3 33 47.2	52.3	1 26 46.4	30.2	41 36.9	1.5	1 38 28.8	36.3	4 10 39.7	62.9
5 20	6 27 5.0	61.6	3 32 54.9	52.2	1 26 16.2	30.1	41 38.4	1.7	1 39 5.1	36.4	4 11 42.6	63.1
5 30	6 26 3.4	61.5	3 32 2.7	52.2	1 25 46.1	30.0	41 40.1	1.9	1 39 41.5	36.7	4 12 45.7	63.2
5 40	6 25 1.9	61.6	3 31 10.5	52.0	1 25 16.1	29.8	41 42.0	2.2	1 40 18.2	36.8	4 13 48.9	63.2
5 50	6 24 0.3	61.5	3 30 18.5	52.0	1 24 46.3	29.6	41 44.2	2.2	1 40 55.0	37.0	4 14 52.1	63.4
6 0	6 22 58.8	61.5	3 29 26.5	51.9	1 24 16.7	29.5	41 46.4	2.5	1 41 32.0	37.1	4 15 55.5	63.5
6 10	6 21 57.3	61.5	3 28 34.6	51.8	1 23 47.2	29.3	41 48.9	2.7	1 42 9.1	37.4	4 16 59.0	63.5
6 20	6 20 55.8	61.5	3 27 42.8	51.7	1 23 17.9	29.2	41 51.6	2.9	1 42 46.5	37.5	4 18 2.5	63.7
6 30	6 19 54.3	61.4	3 26 51.1	51.6	1 22 48.7	29.1	41 54.5	3.1	1 43 24.0	37.7	4 19 6.2	63.8
6 40	6 18 52.9	61.5	3 25 59.5	51.5	1 22 19.6	28.8	41 57.6	3.3	1 44 1.7	37.9	4 20 10.0	63.9
6 50	6 17 51.4	61.4	3 25 8.0	51.4	1 21 50.8	28.7	42 0.9	3.4	1 44 39.6	38.1	4 21 13.9	63.9
7 0	6 16 50.0	61.4	3 24 16.6	51.3	1 21 22.1	28.6	42 4.3	3.7	1 45 17.7	38.2	4 22 17.8	64.1
7 10	6 15 48.6	61.4	3 23 25.3	51.3	1 20 53.5	28.4	42 8.0	3.8	1 45 55.9	38.4	4 23 21.9	64.2
7 20	6 14 47.2	61.4	3 22 34.0	51.1	1 20 25.1	28.2	42 11.8	4.1	1 46 34.3	38.6	4 24 26.1	64.2
7 30	6 13 45.8		3 21 42.9		1 19 56.9		42 15.9		1 47 12.9		4 25 30.3	

EQUATION OF THE CENTRE.

	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs	Diff.	Vs	Diff.
	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″
7 30	7 52 58.2	70.1	11 3 20.1	53.4	12 58 18.5	21.7	13 11 3.7	13.0	11 47 47.6	41.1	9 16 7.1	57.9
7 40	7 54 8.3	70.1	11 4 13.5	53.1	12 58 40.2	21.4	13 10 50.7	13.1	11 47 6.5	41.3	9 15 9.2	58.0
7 50	7 55 18.4	70.1	11 5 6.6	53.1	12 59 1.6	21.3	13 10 37.6	13.3	11 46 25.2	41.4	9 14 11.2	58.1
8 0	7 56 28.5	70.0	11 5 59.7	52.9	12 59 22.9	21.1	13 10 24.3	13.5	11 45 43.8	41.5	9 13 13.1	58.1
8 10	7 57 38.5	69.9	11 6 52.6	52.7	12 59 44.0	20.9	13 10 10.8	13.6	11 45 2.3	41.7	9 12 15.0	58.2
8 20	7 58 48.4	70.0	11 7 45.3	52.6	13 0 4.9	20.7	13 9 57.2	13.8	11 44 20.6	41.7	9 11 16.8	58.2
8 30	7 59 58.4	69.9	11 8 37.9	52.4	13 0 25.6	20.5	13 9 43.4	14.0	11 43 38.9	41.9	9 10 18.6	58.3
8 40	8 1 8.3	69.8	11 9 30.3	52.3	13 0 46.1	20.3	13 9 29.4	14.2	11 42 57.0	42.0	9 9 20.3	58.3
8 50	8 2 18.1	69.9	11 10 22.6	52.2	13 1 6.4	20.1	13 9 15.2	14.4	11 42 15.0	42.2	9 8 22.0	58.4
9 0	8 3 28.0	69.7	11 11 14.8	52.0	13 1 26.5	19.9	13 9 0.8	14.5	11 41 32.8	42.3	9 7 23.6	58.4
9 10	8 4 37.7	69.8	11 12 6.8	51.8	13 1 46.4	19.7	13 8 46.3	14.8	11 40 50.5	42.4	9 6 25.2	58.5
9 20	8 5 47.5	69.7	11 12 58.6	51.7	13 2 6.1	19.6	13 8 31.5	14.9	11 40 8.1	42.5	9 5 26.7	58.6
9 30	8 6 57.2	69.7	11 13 50.3	51.6	13 2 25.7	19.3	13 8 16.6	15.1	11 39 25.6	42.6	9 4 28.1	58.6
9 40	8 8 6.9	69.6	11 14 41.9	51.4	13 2 45.0	19.1	13 8 1.5	15.2	11 38 43.0	42.8	9 3 29.5	58.6
9 50	8 9 16.5	69.5	11 15 33.3	51.2	13 3 4.1	18.9	13 7 46.3	15.5	11 38 0.2	42.9	9 2 30.9	58.7
10 0	8 10 26.0	69.6	11 16 24.5	51.1	13 3 23.0	18.8	13 7 30.8	15.6	11 37 17.3	43.0	9 1 32.2	58.8
10 10	8 11 35.6	69.4	11 17 15.6	50.9	13 3 41.8	18.5	13 7 15.2	15.8	11 36 34.3	43.1	9 0 33.4	58.8
10 20	8 12 45.0	69.5	11 18 6.5	50.8	13 4 0.3	18.4	13 6 59.4	16.0	11 35 51.2	43.3	8 59 34.6	58.8
10 30	8 13 54.5	69.4	11 18 57.3	50.7	13 4 18.7	18.1	13 6 43.4	16.1	11 35 7.9	43.4	8 58 35.8	58.9
10 40	8 15 3.9	69.3	11 19 48.0	50.5	13 4 36.8	18.0	13 6 27.3	16.4	11 34 24.5	43.5	8 57 36 9	59.0
10 50	8 16 13.2	69.3	11 20 38.5	50.3	13 4 54.8	17.7	13 6 10.9	16.5	11 33 41.0	43.6	8 56 37.9	59.0
11 0	8 17 22.5	69.3	11 21 28.8	50.2	13 5 12.5	17.6	13 5 54.4	16.6	11 32 57.4	43.8	8 55 38.9	59.0
11 10	8 18 31.8	69.2	11 22 19.0	50.0	13 5 30.1	17.3	13 5 37.8	16.9	11 32 13.6	43.8	8 54 39.9	59.1
11 20	8 19 41.0	69.1	11 23 9.0	49.8	13 5 47.4	17.2	13 5 20.9	17.0	11 31 29.8	44.0	8 53 40.8	59.1
11 30	8 20 50.1	69.1	11 23 58.8	49.7	13 6 4.6	17.0	13 5 3.9	17.3	11 30 45.8	44.1	8 52 41.7	59.2
11 40	8 21 59.2	69.1	11 24 48.5	49.6	13 6 21.6	16.8	13 4 46.6	17.4	11 30 1.7	44.2	8 51 42.5	59.3
11 50	8 23 8.3	69.0	11 25 38.1	49.4	13 6 38.4	16.5	13 4 29.2	17.5	11 29 17.5	44.4	8 50 43.2	59.3
12 0	8 24 17.3	68.9	11 26 27.5	49.2	13 6 54.9	16.4	13 4 11.7	17.7	11 28 33.1	44.4	8 49 43.9	59.3
12 10	8 25 26.2	68.9	11 27 16.7	49.1	13 7 11.3	16.2	13 3 54.0	17.9	11 27 48.7	44.6	8 48 44.6	59.4
12 20	8 26 35.1	68.9	11 28 5.8	48.9	13 7 27.5	16.0	13 3 36.1	18.1	11 27 4.1	44.7	8 47 45.2	59.4
12 30	8 27 44.0	68.8	11 28 54.7	48.8	13 7 43.5	15.8	13 3 18.0	18.3	11 26 19.4	44.8	8 46 45.8	59.4
12 40	8 28 52.8	68.7	11 29 43.5	48.6	13 7 59.3	15.6	13 2 59.7	18.4	11 25 34.6	44.9	8 45 46.4	59.5
12 50	8 30 1.5	68.7	11 30 32.1	48.4	13 8 14.9	15.4	13 2 41.3	18.6	11 24 49.7	45.1	8 44 46.9	59.6
13 0	8 31 10.2	68.6	11 31 20.5	48.3	13 8 30.3	15.2	13 2 22.7	18.8	11 24 4.6	45.1	8 43 47.3	59.6
13 10	8 32 18.8	68.5	11 32 8.8	48.2	13 8 45.5	15.0	13 2 3.9	19.0	11 23 19.5	45.3	8 42 47.7	59.6
13 20	8 33 27.3	68.5	11 32 57.0	47.9	13 9 0.5	14.9	13 1 44.9	19.1	11 22 34.2	45.4	8 41 48.1	59.7
13 30	8 34 35.8	68.5	11 33 44.9	47.8	13 9 15.4	14.6	13 1 25.8	19.3	11 21 48.8	45.5	8 40 48.4	59.7
13 40	8 35 44.3	68.4	11 34 32.7	47.7	13 9 30.0	14.4	13 1 6.5	19.5	11 21 3.3	45.6	8 39 48.7	59.8
13 50	8 36 52.7	68.3	11 35 20.4	47.5	13 9 44.4	14.2	13 0 47.0	19.6	11 20 17.7	45.8	8 38 48.9	59.8
14 0	8 38 1.0	68.2	11 36 7.9	47.3	13 9 58.6	14.1	13 0 27.4	19.9	11 19 31.9	45.8	8 37 49.1	59.8
14 10	8 39 9.2	68.2	11 36 55.2	47.2	13 10 12.7	13.8	13 0 7.5	20.0	11 18 46.1	46.0	8 36 49.3	59.9
14 20	8 40 17.4	68.2	11 37 42.4	47.0	13 10 26.5	13.6	12 59 47.5	20.1	11 18 0.1	46.0	8 35 49.4	59.9
14 30	8 41 25.6	68.1	11 38 29.4	46.8	13 10 40.1	13.5	12 59 27.4	20.3	11 17 14.1	46.2	8 34 49.5	60.0
14 40	8 42 33.7	68.0	11 39 16.2	46.6	13 10 53.6	13.2	12 59 7.1	20.5	11 16 27.9	46.3	8 33 49.5	60.0
14 50	8 43 41.7	67.9	11 40 2.8	46.5	13 11 6.8	13.1	12 58 46.6	20.7	11 15 41.6	46.4	8 32 49.5	60.1
15 0	8 44 49.6		11 40 49.3		13 11 19.9		12 58 25.9		11 14 55.2		8 31 49.4	

TABLE XVIII. continued.

	VI*s*	Diff.	VII*s*	Diff.	VIII*s*	Diff.	XI*s*	Diff.	X*s*	Diff.	XI*s*	Diff.
° ′	° ′ ″	″	° ′ ″	″	° ′ ″	″	′ ″	″	° ′ ″	″	° ′ ″	″
7 30	6 13 45.8	61.3	3 21 42.9	51.0	1 19 56.9	28.1	42 15.9	4.2	1 47 12.9	38.8	4 25 30.3	64.4
7 40	6 12 44.5	61.3	3 20 51.9	51.0	1 19 28.8	27.9	42 20.1	4.4	1 47 51.7	39.0	4 26 34.7	64.4
7 50	6 11 43.2	61.3	3 20 0.9	50.9	1 19 0.9	27.8	42 24.5	4.7	1 48 30.7	39.1	4 27 39.1	64.6
8 0	6 10 41.9	61.3	3 19 10.0	50.7	1 18 33.1	27.6	42 29.2	4.8	1 49 9.8	39.4	4 28 43.7	64.6
8 10	6 9 40.6	61.3	3 18 19.3	50.7	1 18 5.5	27.4	42 34.0	5.0	1 49 49.2	39.5	4 29 48.3	64.7
8 20	6 8 39.3	61.3	3 17 28.6	50.6	1 17 38.1	27.3	42 39.0	5.2	1 50 28.7	39.6	4 30 53.0	64.8
8 30	6 7 38.0	61.2	3 16 38.0	50.4	1 17 10.8	27.2	42 44.2	5.5	1 51 8.3	39.9	4 31 57.8	65.0
8 40	6 6 36.8	61.2	3 15 47.6	50.4	1 16 43.6	27.0	42 49.7	5.6	1 51 48.2	40.0	4 33 2.8	65.0
8 50	6 5 35.6	61.2	3 14 57.2	50.3	1 16 16.6	26.8	42 55.3	5.8	1 52 28.2	40.2	4 34 7.8	65.1
9 0	6 4 34.4	61.1	3 14 6.9	50.2	1 15 49.8	26.6	43 1.1	5.9	1 53 8.4	40.4	4 35 12.9	65.1
9 10	6 3 33.3	61.2	3 13 16.7	50.1	1 15 23.2	26.5	43 7.0	6.2	1 53 48.8	40.6	4 36 18.0	65.3
9 20	6 2 32.1	61.1	3 12 26.6	50.0	1 14 56.7	26.3	43 13.2	6.4	1 54 29.4	40.7	4 37 23.3	65.4
9 30	6 1 31.0	61.1	3 11 36.6	49.8	1 14 30.4	26.1	43 19.6	6.6	1 55 10.1	40.9	4 38 28.7	65.4
9 40	6 0 29.9	61.0	3 10 46.8	49.8	1 14 4.3	26.0	43 26.2	6.8	1 55 51.0	41.1	4 39 34.1	65.5
9 50	5 59 28.9	61.1	3 9 57.0	49.7	1 13 38.3	25.8	43 33.0	6.9	1 56 32.1	41.2	4 40 39.6	65.6
10 0	5 58 27.8	61.0	3 9 7.3	49.6	1 13 12.5	25.7	43 39.9	7.2	1 57 13.3	41.5	4 41 45.2	65.7
10 10	5 57 26.8	60.9	3 8 17.7	49.5	1 12 46.8	25.5	43 47.1	7.4	1 57 54.8	41.6	4 42 50.9	65.8
10 20	5 56 25.9	61.0	3 7 28.2	49.3	1 12 21.3	25.4	43 54.5	7.5	1 58 36.4	41.8	4 43 56.7	65.9
10 30	5 55 24.9	60.9	3 6 38.9	49.3	1 11 55.9	25.2	44 2.0	7.8	1 59 18.2	41.9	4 45 2.6	66.0
10 40	5 54 24.0	60.9	3 5 49.6	49.2	1 11 30.7	25.0	44 9.8	7.9	2 0 0.1	42.1	4 46 8.6	66.0
10 50	5 53 23.1	60.9	3 5 0.4	49.1	1 11 5.7	24.8	44 17.7	8.2	2 0 42.2	42.3	4 47 14.6	66.1
11 0	5 52 22.2	60.9	3 4 11.3	49.0	1 10 40.9	24.7	44 25.9	8.3	2 1 24.5	42.5	4 48 20.7	66.2
11 10	5 51 21.3	60.8	3 3 22.3	48.9	1 10 16.2	24.5	44 34.2	8.5	2 2 7.0	42.6	4 49 26.9	66.3
11 20	5 50 20.5	60.8	3 2 33.4	48.7	1 9 51.7	24.4	44 42.7	8.8	2 2 49.6	42.8	4 50 33.2	66.4
11 30	5 49 19.7	60.7	3 1 44.7	48.7	1 9 27.3	24.2	44 51.5	8.9	2 3 32.4	43.0	4 51 39.6	66.4
11 40	5 48 19.0	60.8	3 0 56.0	48.6	1 9 3.1	24.0	45 0.4	9.1	2 4 15.4	43.2	4 52 46.0	66.5
11 50	5 47 18.2	60.7	3 0 7.4	48.5	1 8 39.1	23.9	45 9.5	9.3	2 4 58.6	43.3	4 53 52.5	66.6
12 0	5 46 17.5	60.6	2 59 18.9	48.3	1 8 15.2	23.7	45 18.8	9.5	2 5 41.9	43.5	4 54 59.1	66.7
12 10	5 45 16.9	60.7	2 58 30.6	48.3	1 7 51.5	23.6	45 28.3	9.8	2 6 25.4	43.7	4 56 5.8	66.7
12 20	5 44 16.2	60.6	2 57 42.3	48.1	1 7 27.9	23.3	45 38.1	9.9	2 7 9.1	43.8	4 57 12.5	66.8
12 30	5 43 15.6	60.6	2 56 54.2	48.1	1 7 4.6	23.2	45 48.0	10.1	2 7 52.9	44.0	4 58 19.3	66.9
12 40	5 42 15.0	60.5	2 56 6.1	47.9	1 6 41.4	23.1	45 58.1	10.3	2 8 36.9	44.2	4 59 26.2	67.0
12 50	5 41 14.5	60.5	2 55 18.2	47.8	1 6 18.3	22.9	46 8.4	10.5	2 9 21.1	44.4	5 0 33.2	67.1
13 0	5 40 14.0	60.5	2 54 30.4	47.7	1 5 55.4	22.7	46 18.9	10.7	2 10 5.5	44.5	5 1 40.3	67.1
13 10	5 39 13.5	60.4	2 53 42.7	47.7	1 5 32.7	22.5	46 29.6	10.9	2 10 50.0	44.7	5 2 47.4	67.2
13 20	5 38 13.1	60.5	2 52 55.0	47.5	1 5 10.2	22.4	46 40.5	11.0	2 11 34.7	44.8	5 3 54.6	67.3
13 30	5 37 12.6	60.3	2 52 7.5	47.4	1 4 47.8	22.2	46 51.5	11.3	2 12 19.5	45.0	5 5 1.9	67.3
13 40	5 36 12.3	60.4	2 51 20.1	47.3	1 4 25.6	22.0	47 2.8	11.5	2 13 4.5	45.2	5 6 9.2	67.4
13 50	5 35 11.9	60.3	2 50 32.8	47.2	1 4 3.6	21.9	47 14.3	11.7	2 13 49.7	45.4	5 7 16.6	67.5
14 0	5 34 11.6	60.2	2 49 45.6	47.1	1 3 41.7	21.7	47 26.0	11.8	2 14 35.1	45.5	5 8 24.1	67.5
14 10	5 33 11.4	60.3	2 48 58.5	46.9	1 3 20.0	21.5	47 37.8	12.1	2 15 20.6	45.7	5 9 31.6	67.6
14 20	5 32 11.1	60.2	2 48 11.6	46.9	1 2 58.5	21.4	47 49.9	12.3	2 16 6.3	45.8	5 10 39.2	67.7
14 30	5 31 10.9	60.1	2 47 24.7	46.7	1 2 37.1	21.1	48 2.2	12.4	2 16 52.1	46.0	5 11 46.9	67.8
14 40	5 30 10.8	60.1	2 46 38.0	46.6	1 2 16.0	21.0	48 14.6	12.7	2 17 38.1	46.2	5 12 54.7	67.8
14 50	5 29 10.7	60.1	2 45 51.4	46.6	1 1 55.0	20.9	48 27.3	12.8	2 18 24.3	46.4	5 14 2.5	67.9
15 0	5 28 10.6		2 45 4.8		1 1 34.1		48 40.1		2 19 10.7		5 15 10.4	

EQUATION OF THE CENTRE.

	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs	Diff.	Vs	Diff.
° ′	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″
15 0	8 44 49.6	67.9	11 40 49.3	46.4	13 11 19.9	12.8	12 58 25.9	20.9	11 14 55.2	46.6	8 31 49.4	60.1
15 10	8 45 57.5	67.8	11 41 35.7	46.2	13 11 32.7	12.7	12 58 5.0	21.0	11 14 8.6	46.6	8 30 49.3	60.1
15 20	8 47 5.3	67.8	11 42 21.9	46.0	13 11 45.4	12.4	12 57 44.0	21.1	11 13 22.0	46.7	8 29 49.2	60.1
15 30	8 48 13.1	67.7	11 43 7.9	45.8	13 11 57.1	12.3	12 57 22.9	21.4	11 12 35.3	46.9	8 28 49.1	60.2
15 40	8 49 20.8	67.6	11 43 53.7	45.7	13 12 10.1	12.1	12 57 1.5	21.5	11 11 48.4	46.9	8 27 48.9	60.3
15 50	8 50 28.4	67.5	11 44 39.4	45.5	13 12 22.2	11.8	12 56 40.0	21.7	11 11 1.5	47.1	8 26 48.6	60.2
16 0	8 51 35.9	67.5	11 45 24.9	45.4	13 12 34.0	11.7	12 56 18.3	21.9	11 10 14.4	47.2	8 25 48.4	60.3
16 10	8 52 43.4	67.4	11 46 10.3	45.2	13 12 45.7	11.5	12 55 56.4	22.0	11 9 27.2	47.3	8 24 48.1	60.4
16 20	8 53 50.8	67.3	11 46 55.5	45.0	13 12 57.2	11.3	12 55 34.4	22.2	11 8 39.9	47.4	8 23 47.7	60.3
16 30	8 54 58.1	67.3	11 47 40.5	44.8	13 13 8.5	11.0	12 55 12.2	22.4	11 7 52.5	47.5	8 22 47.4	60.5
16 40	8 56 5.4	67.2	11 48 25.3	44.7	13 13 19.5	10.9	12 54 49.8	22.5	11 7 5.0	47.7	8 21 46.9	60.4
16 50	8 57 12.6	67.1	11 49 10.0	44.5	13 13 30.4	10.7	12 54 27.3	22.7	11 6 17.3	47.7	8 20 46.5	60.5
17 0	8 58 19.7	67.1	11 49 54.5	44.4	13 13 41.1	10.5	12 54 4.6	22.9	11 5 29.6	47.8	8 19 46.0	60.5
17 10	8 59 26.8	67.0	11 50 38.9	44.2	13 13 51.6	10.3	12 53 41.7	23.1	11 4 41.8	47.9	8 18 45.5	60.5
17 20	9 0 33.8	66.9	11 51 23.1	44.0	13 14 1.9	10.1	12 53 18.6	23.2	11 3 53.9	48.1	8 17 45.0	60.6
17 30	9 1 40.7	66.8	11 52 7.1	43.8	13 14 12.0	9.9	12 52 55.4	23.3	11 3 5.8	48.1	8 16 44.4	60.6
17 40	9 2 47.5	66.7	11 52 50.9	43.7	13 14 21.9	9.8	12 52 32.1	23.6	11 2 17.7	48.3	8 15 43.8	60.7
17 50	9 3 54.2	66.7	11 53 34.6	43.5	13 14 31.7	9.5	12 52 8.5	23.7	11 1 29.4	48.3	8 14 43.1	60.6
18 0	9 5 0.9	66.6	11 54 18.1	43.3	13 14 41.2	9.3	12 51 44.8	23.9	11 0 41.1	48.5	8 13 42.5	60.7
18 10	9 6 7.5	66.5	11 55 1.4	43.2	13 14 50.5	9.1	12 51 20.9	24.0	10 59 52.6	48.6	8 12 41.8	60.8
18 20	9 7 14.0	66.4	11 55 44.6	43.0	13 14 59.6	8.9	12 50 56.9	24.2	10 59 4.0	48.7	8 11 41.0	60.7
18 30	9 8 20.4	66.4	11 56 27.6	42.8	13 15 8.5	8.8	12 50 32.7	24.4	10 58 15.3	48.7	8 10 40.3	60.8
18 40	9 9 26.8	66.3	11 57 10.4	42.6	13 15 17.3	8.5	12 50 8.3	24.5	10 57 26.6	48.9	8 9 39.5	60.8
18 50	9 10 33.1	66.2	11 57 53.0	42.5	13 15 25.8	8.3	12 49 43.8	24.7	10 56 37.7	49.0	8 8 38.7	60.9
19 0	9 11 39.3	66.1	11 58 35.5	42.3	13 15 34.1	8.2	12 49 19.1	24.8	10 55 48.7	49.1	8 7 37.8	60.9
19 10	9 12 45.4	66.0	11 59 17.8	42.1	13 15 42.3	7.9	12 48 54.3	25.0	10 54 59.6	49.2	8 6 36.9	60.9
19 20	9 13 51.4	66.0	11 59 59.9	41.9	13 15 50.2	7.8	12 48 29.3	25.2	10 54 10.4	49.3	8 5 36.0	60.9
19 30	9 14 57.4	65.9	12 0 41.8	41.8	13 15 58.0	7.5	12 48 4.1	25.4	10 53 21.1	49.3	8 4 35.1	61.0
19 40	9 16 3.3	65.8	12 1 23.6	41.6	13 16 5.5	7.4	12 47 38.7	25.5	10 52 31.8	49.5	8 3 34.1	60.9
19 50	9 17 9.1	65.7	12 2 5.2	41.5	13 16 12.9	7.2	12 47 13.2	25.7	10 51 42.3	49.6	8 2 33.2	61.0
20 0	9 18 14.8	65.6	12 2 46.7	41.2	13 16 20.1	6.9	12 46 47.5	25.8	10 50 52.7	49.7	8 1 32.2	61.1
20 10	9 19 20.4	65.5	12 3 27.9	41.1	13 16 27.0	6.8	12 46 21.7	26.0	10 50 3.0	49.8	8 0 31.1	61.0
20 20	9 20 25.9	65.4	12 4 9.0	40.9	13 16 33.8	6.6	12 45 55.7	26.1	10 49 13.2	49.8	7 59 30.1	61.1
20 30	9 21 31.3	65.4	12 4 49.9	40.7	13 16 40.4	6.4	12 45 29.6	26.3	10 48 23.4	50.0	7 58 29.0	61.1
20 40	9 22 36.7	65.3	12 5 30.6	40.6	13 16 46.8	6.2	12 45 3.3	26.5	10 47 33.4	50.1	7 57 27.9	61.2
20 50	9 23 42.0	65.1	12 6 11.2	40.4	13 16 53.0	5.9	12 44 36.8	26.6	10 46 43.3	50.2	7 56 26.7	61.1
21 0	9 24 47.1	65.1	12 6 51.6	40.2	13 16 58.9	5.8	12 44 10.2	26.8	10 45 53.1	50.3	7 55 25.6	61.2
21 10	9 25 52.2	65.0	12 7 31.8	40.0	13 17 4.7	5.6	12 43 43.4	27.0	10 45 2.8	50.4	7 54 24.4	61.2
21 20	9 26 57.2	65.0	12 8 11.8	39.9	13 17 10.3	5.5	12 43 16.4	27.2	10 44 12.4	50.4	7 53 23.2	61.2
21 30	9 28 2.2	64.8	12 8 51.7	39.6	13 17 15.8	5.2	12 42 49.2	27.3	10 43 22.0	50.6	7 52 22.0	61.3
21 40	9 29 7.0	64.7	12 9 31.3	39.5	13 17 21.0	5.0	12 42 21.9	27.4	10 42 31.4	50.7	7 51 20.7	61.3
21 50	9 30 11.7	64.6	12 10 10.8	39.4	13 17 26.0	4.8	12 41 54.5	27.6	10 41 40.7	50.7	7 50 19.4	61.3
22 0	9 31 16.3	64.6	12 10 50.2	39.1	13 17 30.8	4.7	12 41 26.9	27.8	10 40 50.0	50.9	7 49 18.1	61.3
22 10	9 32 20.9	64.4	12 11 29.3	39.0	13 17 35.5	4.4	12 40 59.1	27.9	10 39 59.1	51.0	7 48 16.8	61.3
22 20	9 33 25.3	64.4	12 12 8.3	38.8	13 17 39.9	4.2	12 40 31.2	28.1	10 39 8.1	51.0	7 47 15.5	61.3
22 30	9 34 29.7		12 12 47.1		13 17 44.1		12 40 3.1		10 38 17.1		7 46 14.2	

TABLE XVIII. continued.

	VI*s*.	Diff.	VII*s*.	Diff.	VIII*s*.	Diff.	IX*s*.	Diff.	X*s*.	Diff.	XI*s*	Diff.
° ′	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″
15 0	5 28 10.6	60.1	2 45 4.8	46.4	1 1 34.1	20.7	0 48 40.1	13.1	2 19 10.7	46.5	5 15 10.4	67.9
15 10	5 27 10.5	60.0	2 44 18.4	46.3	1 1 13.4	20.5	0 48 53.2	13.2	2 19 57.2	46.6	5 16 18.3	68.0
15 20	5 26 10.5	60.0	2 43 32.1	46.2	1 0 52.9	20.3	0 49 6.4	13.5	2 20 43.8	46.8	5 17 26.3	68.1
15 30	5 25 10.5	59.9	2 42 45.9	46.0	1 0 32.6	20.1	0 49 19.9	13.6	2 21 30.6	47.0	5 8 34.4	68.2
15 40	5 24 10.6	59.9	2 41 59.9	46.0	1 0 12.5	20.0	0 49 33.5	13.8	2 22 17.6	47.2	5 19 42.6	68.2
15 50	5 23 10.7	59.8	2 41 13.9	45.8	0 59 52.5	19.9	0 49 47.3	14.1	2 23 4.8	47.3	5 20 50.8	68.2
16 0	5 22 10.9	59.8	2 40 28.1	45.8	0 59 32.6	19.6	0 50 1.4	14.2	2 23 52.1	47.5	5 21 59.0	68.3
16 10	5 21 11.1	59.8	2 39 42.3	45.6	0 59 13.0	19.5	0 50 15.6	14.4	2 24 39.6	47.7	5 23 7.3	68.4
16 20	5 20 11.3	59.7	2 38 56.7	45.5	0 58 53.5	19.3	0 50 30.0	14.6	2 25 27.3	47.8	5 24 15.7	68.5
16 30	5 19 11.6	59.7	2 38 11.2	45.4	0 58 34.2	19.1	0 50 44.6	14.9	2 26 15.1	47.9	5 25 24.2	68.5
16 40	5 18 11.9	59.6	2 37 25.8	45.3	0 58 15.1	19.0	0 50 59.5	15.0	2 27 3.0	48.2	5 26 32.7	68.5
16 50	5 17 12.3	59.6	2 36 40.5	45.1	0 57 56.1	18.8	0 51 14.5	15.2	2 27 51.2	48.3	5 27 41.2	68.6
17 0	5 16 12.7	59.6	2 35 55.4	45.1	0 57 37.3	18.6	0 51 29.7	15.4	2 28 39.5	48.4	5 28 49.8	68.7
17 10	5 15 13.1	59 5	2 35 10.3	44.9	0 57 18.7	18.4	0 51 45.1	15.6	2 29 27.9	48.6	5 29 58.5	68.7
17 20	5 14 13.6	59.4	2 34 25.4	44.8	0 57 0.3	18.3	0 52 0.7	15.8	2 30 16.5	48.8	5 31 7.2	68.8
17 30	5 13 14.2	59.4	2 33 40.6	44.7	0 56 42.0	18.1	0 52 16.5	16.0	2 31 5.3	48 9	5 32 16.0	68.9
17 40	5 12 14.8	59.4	2 32 55 9	44.6	0 56 23.9	17.9	0 52 32.5	16.2	2 31 54.2	49.1	5 33 24.9	68.9
17 50	5 11 15.4	59.3	2 32 11.3	44.4	0 56 6.0	17.7	0 52 48.7	16.4	2 32 43.3	49.2	5 34 33.8	68.9
18 0	5 10 16.1	59.3	2 31 26.9	44.4	0 55 48.3	17.5	0 53 5.1	16.5	2 33 32.5	49.4	5 35 42.7	69.0
18 10	5 9 16.8	59.3	2 30 42.5	44.2	0 55 30.8	17.4	0 53 21.6	16.8	2 34 21.9	49.6	5 36 51.7	69.1
18 20	5 8 17.5	59.2	2 29 58.3	44.1	0 55 13.4	17.3	0 53 38.4	17.0	2 35 11.5	49.7	5 38 0.8	69.1
18 30	5 7 18.3	59.1	2 29 14.2	44.0	0 54 56.1	17.0	0 53 55.4	17.2	2 36 1.2	49.8	5 39 9.9	69.1
18 40	5 6 19.2	59.1	2 28 30.2	43.8	0 54 39.1	16.9	0 54 12.6	17.3	2 36 51.0	50.0	5 40 19.0	69.2
18 50	5 5 20.1	59.0	2 27 46.4	43.8	0 54 22.2	16.6	0 54 29.9	17.6	2 37 41.0	50.2	5 41 28.2	69.3
19 0	5 4 21.1	59.0	2 27 2.6	43.6	0 54 5.6	16.5	0 54 47.5	17.7	2 38 31.2	50.3	5 42 37.5	69.3
19 10	5 3 22.1	59.0	2 26 19.0	43.5	0 53 49.1	16.4	0 55 5.2	18.0	2 39 21.5	50.5	5 43 46.8	69.3
19 20	5 2 23.1	58.9	2 25 35.5	43.4	0 53 32.7	16.1	0 55 23.2	18.1	2 40 12.0	50.7	5 44 56.1	69.4
19 30	5 1 24.2	58.8	2 24 52.1	43.3	0 53 16.6	16.0	0 55 41.3	18.4	2 41 2.7	50.8	5 46 5.5	69.5
19 40	5 0 25.4	58.8	2 24 8.8	43.1	0 53 0.6	15.8	0 55 59.7	18.5	2 41 53.5	50.9	5 47 15.0	69.4
19 50	4 59 26.6	58.8	2 23 25.7	43.0	0 52 44.8	15.6	0 56 18.2	18.8	2 42 44.4	51.1	5 48 24.4	69.6
20 0	4 58 27.8	58.7	2 22 42.7	42.9	0 52 29.2	15.5	0 56 37.0	18.9	2 43 35.5	51.2	5 49 34.0	69.5
20 10	4 57 29.1	58.6	2 21 59.8	42.8	0 52 13.7	15.2	0 56 55.9	19.1	2 44 26.7	51.4	5 50 43.5	69.6
20 20	4 56 30.5	58.6	2 21 17.0	42.6	0 51 58.5	15.1	0 57 15.0	19.3	2 45 18.1	51.6	5 51 53.1	69.7
20 30	4 55 31.9	58.6	2 20 34.4	42.5	0 51 43.4	14.9	0 57 34.3	19.6	2 46 9.7	51.7	5 53 2.8	69.7
20 40	4 54 33.3	58.5	2 19 51.9	42.4	0 51 28.5	14.8	0 57 53.9	19.7	2 47 1.4	51.8	5 54 12.5	69.7
20 50	4 53 34.8	58.4	2 19 9.5	42.3	0 51 13.7	14.5	0 58 13.6	19.9	2 47 53.2	52.0	5 55 22.2	69.7
21 0	4 52 36.4	58.4	2 18 27.2	42.2	0 50 59.2	14.4	0 58 33.5	20.1	2 48 45.2	52.2	5 56 31.9	69.9
21 10	4 51 38.0	58.3	2 17 45.0	42.0	0 50 44.8	14.2	0 58 53.6	20.3	2 49 37.4	52.3	5 57 41.8	69.9
21 20	4 50 39.7	58.3	2 17 3.0	41.9	0 50 30.6	14.0	0 59 13.9	20.5	2 50 29.7	52.4	5 58 51.7	69.9
21 30	4 49 41.4	58.2	2 16 21.1	41.7	0 50 16.6	13.8	0 59 34.4	20.7	2 51 22.1	52.6	6 0 1.6	70.0
21 40	4 48 43.2	58.2	2 15 39.4	41.7	0 50 2.8	13.6	0 59 55.1	20.9	2 52 14.7	52.7	6 1 11.6	69.9
21 50	4 47 45.0	58.1	2 14 57.7	41.5	0 49 49.2	13.5	1 0 16.0	21.1	2 53 7.4	52.9	6 2 21.5	70.0
22 0	4 46 46.9	58.1	2 14 16.2	41.4	0 49 35.7	13.3	1 0 37.1	21.3	2 54 0.3	53.1	6 3 31.5	70.1
22 10	4 45 48.8	58.0	2 13 34.8	41.3	0 49 22.4	13.1	1 0 58.4	21.4	2 54 53.4	53.1	6 4 41.6	70.1
22 20	4 44 50.8	57.9	2 12 53.5	41.1	0 49 9.3	13.0	1 1 19.8	21.7	2 55 46.5	53.4	6 5 51.7	70.1
22 30	4 43 52.9		2 12 12.4		0 48 56.3		1 1 41.5		2 56 39.9		6 7 1.8	

EQUATION OF THE CENTRE.

	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs	Diff.	Vs	Diff.
22° 30′	9° 34″ 29.″7	64.″2	12° 12′ 47.″1	38.″6	13° 17′ 44.″1	4.″1	12° 40′ 3.″1	28.″2	10° 38′ 17.″1	51.″1	7° 46′ 14.″2	61.″4
22 40	9 35 33.9	64.2	12 13 25.7	38.4	13 17 48.2	3.8	12 39 34.9	28.4	10 37 26.0	51.3	7 45 12.8	61.4
22 50	9 36 38.1	64.1	12 14 4.1	38.2	13 17 52.0	3.7	12 39 6.5	28.6	10 36 34.7	51.3	7 44 11.4	61.4
23 0	9 37 42.2	63.9	12 14 42.3	38.1	13 17 55.7	3.4	12 38 37.9	28.7	10 35 43.4	51.4	7 43 10.0	61.4
23 10	9 38 46.1	63.9	12 15 20.4	37.9	13 17 59.1	3.3	12 38 9.2	28.8	10 34 52.0	51.5	7 42 8.6	61.5
23 20	9 39 50.0	63.8	12 15 58.3	37.7	13 18 2.4	3.1	12 37 40.4	29.1	10 34 0.5	51.6	7 41 7.1	61.4
23 30	9 40 53.8	63.7	12 16 36.0	37.5	13 18 5.5	2.9	12 37 11.3	29.2	10 33 8.9	51.7	7 40 5.7	61.5
23 40	9 41 57.5	63.5	12 17 13.5	37.4	13 18 8.4	2.7	12 36 42.1	29.3	10 32 17.2	51.8	7 39 4.2	61.5
23 50	9 43 1.0	63.5	12 17 50.9	37.1	13 18 11.1	2.5	12 36 12.8	29.5	10 31 25.4	51.9	7 38 2.7	61.5
24 0	9 44 4.5	63.4	12 18 28.0	37.0	13 18 13.6	2.2	12 35 43.3	29.6	10 30 33.5	52.0	7 37 1.2	61.5
24 10	9 45 7.9	63.2	12 19 5.0	36.8	13 18 15.8	2.2	12 35 13.7	29.8	10 29 41.5	52.0	7 35 59.7	61.6
24 20	9 46 11.1	63.2	12 19 41.8	36.7	13 18 18.0	1.9	12 34 43.9	30.0	10 28 49.5	52.2	7 34 58.1	61.5
24 30	9 47 14.3	63.1	12 20 18.5	36.4	13 18 19.9	1.7	12 34 13.9	30.1	10 27 57.3	52.2	7 33 56.6	61 6
24 40	9 48 17.4	62.9	12 20 54.9	36.3	13 18 21.6	1.5	12 33 43.8	30.2	10 27 5.1	52.3	7 32 55.0	61.5
24 50	9 49 20.3	62.9	12 21 31.2	36.0	13 18 23.1	1.3	12 33 13.6	30.4	10 26 12.8	52.4	7 31 53.5	61.6
25 0	9 50 23.2	62.8	12 22 7.2	35.9	13 18 24.4	1.2	12 32 43.2	30.6	10 25 20.4	52.5	7 30 51.9	61.6
25 10	9 51 26.0	62.6	12 22 43.1	35.7	13 18 25.6	0.9	12 32 12.6	30.7	10 24 27.9	52.6	7 29 50.3	61.7
25 20	9 52 28.6	62.6	12 23 18.8	35.6	13 18 26.5	0.8	12 31 41.9	30.9	10 23 35.3	52.7	7 28 48.6	61.6
25 30	9 53 31.2	62.4	12 23 54.4	35.3	13 18 27.3	0.5	12 31 11.0	31.0	10 22 42.6	52.8	7 27 47.0	61.7
25 40	9 54 33.6	62.4	12 24 29.7	35.2	13 18 27.8	0.4	12 30 40.0	31.2	10 21 49.8	52.8	7 26 45.3	61.6
25 50	9 55 36.0	62.2	12 25 4.9	34.9	13 18 28.2	0.2	12 30 8.8	31.4	10 20 57.0	53.0	7 25 43.7	61.7
26 0	9 56 38.2	62.1	12 25 39.8	34.8	13 18 28.4	0.0	12 29 37.4	31.5	10 20 4.0	53.0	7 24 42.0	61.7
26 10	9 57 40.3	62.0	12 26 14.6	34.6	13 18 28.4	0.2	12 29 5.9	31.6	10 19 11.0	53.1	7 23 40.3	61.6
26 20	9 58 42.3	61.9	12 26 49.2	34.5	13 18 28.2	0.4	12 28 34.3	31.8	10 18 17.9	53.2	7 22 38.7	61.7
26 30	9 59 44.2	61.8	12 27 23.7	34.2	13 18 27.8	0.6	12 28 2.5	31.9	10 17 24.7	53.2	7 21 37.0	61.7
26 40	10 0 46.0	61.7	12 27 57.9	34.1	13 18 27.2	0.8	12 27 30.6	32.1	10 16 31.5	53.4	7 20 35.3	61.7
26 50	10 1 47.7	61.6	12 28 32.0	33.8	13 18 26.4	1.0	12 26 58.5	32.2	10 15 38.1	53.4	7 19 33.6	61.8
27 0	10 2 49.3	61.4	12 29 5.8	33.7	13 18 25.4	1.2	12 26 26.3	32.4	10 14 44.7	53.6	7 18 31.8	61.7
27 10	10 3 50.7	61.4	12 29 39.5	33.5	13 18 24.2	1.3	12 25 53.9	32.5	10 13 51.1	53.6	7 17 30.1	61.7
27 20	10 4 52.1	61.2	12 30 13.0	33.3	13 18 22.9	1.6	12 25 21.4	32.7	10 12 57.5	53.7	7 16 28.4	61.8
27 30	10 5 53.3	61.1	12 30 46.3	33.2	13 18 21.3	1.7	12 24 48.7	32.9	10 12 3.8	53.8	7 15 26.6	61.7
27 40	10 6 54.4	61.0	12 31 19.5	32.9	13 18 19.6	1.9	12 24 15.8	33.0	10 11 10.0	53.8	7 14 24.9	61.8
27 50	10 7 55.4	60.9	12 31 52.4	32.8	13 18 17.7	2.1	12 23 42.8	33.1	10 10 16.2	53.9	7 13 23.1	61.7
28 0	10 8 56.3	60.8	12 32 25.2	32.5	13 18 15.6	2.4	12 23 9.7	33.3	10 9 22.3	54.0	7 12 21.4	61.8
28 10	10 9 57.1	60.7	12 32 57.7	32.4	13 18 13.2	2.5	12 22 36.4	33.4	10 8 28.3	54.1	7 11 19.6	61.7
28 20	10 10 57.8	60.5	12 33 30.1	32.2	13 18 10.7	2.7	12 22 3.0	33.5	10 7 34.2	54.2	7 10 17.9	61.8
28 30	10 11 58.3	60.5	12 34 2.3	32.0	13 18 8.0	2.9	12 21 29.5	33.7	10 6 40.0	54.3	7 9 16.1	61.8
28 40	10 12 58.8	60.3	12 34 34.3	31.9	13 18 5.1	3.0	12 20 55.8	33.8	10 5 45.7	54.3	7 8 14.3	61.8
28 50	10 13 59.1	60.2	12 35 6.2	31.6	13 18 2.1	3.3	12 20 22.0	34.0	10 4 51.4	54.4	7 7 12.5	61.7
29 0	10 14 59.3	60.1	12 35 37.8	31.4	13 17 58.8	3.5	12 19 48.0	34.3	10 3 57.0	54.5	7 6 10.8	61.8
29 10	10 15 59.4	59.9	12 36 9.2	31.3	13 17 55.3	3.6	12 19 13.7	34.3	10 3 2.5	54.6	7 5 9.0	61.8
29 20	10 16 59.3	59.9	12 36 40.5	31.0	13 17 51.7	3.8	12 18 39.4	34.4	10 2 7.9	54.6	7 4 7.2	61.8
29 30	10 17 59.2	59.7	12 37 11.5	30.9	13 17 47.9	4.1	12 18 5.0	34.6	10 1 13.3	54.8	7 3 5.4	61.8
29 40	10 18 58.9	59.5	12 37 42.4	30.7	13 17 43.8	4.2	12 17 30.4	34.7	10 0 18.5	54.8	7 2 3.6	61.8
29 50	10 19 58.4	59.5	12 38 13.1	30.5	13 17 39.6	4.4	12 16 55.7	34.9	9 59 23.7	54.8	7 1 1.8	61.8
30 0	10 20 57.9		12 38 43.6		13 17 35.2		12 16 20.8		9 58 28.9		7 0 0.0	

End of TABLE XVIII.

	VI*s*	Diff.	VII*s*	Diff.	VIII*s*	Diff.	IX*s*	Diff.	X*s*	Diff.	XI*s*	Diff.
22 30	4 43 52.9	57.9	2 12 12.4	41.0	48 56.3	12.7	1 1 41.5	21.9	2 56 39.9	53.4	6 7 1.8	70.1
22 40	4 42 55.0	57.8	2 11 31.4	40.9	48 43.6	12.6	1 2 3.4	22.0	2 57 33.3	53.6	6 8 11.9	70.2
22 50	4 41 57.2	57.8	2 10 50.5	40.8	48 31.0	12.4	1 2 25.4	22.3	2 58 26.9	53.8	6 9 22.1	70.2
23 0	4 40 59.4	57.7	2 10 9.7	40.6	48 18.6	12.2	1 2 47.7	22.4	2 59 20.7	53.9	6 10 32.3	70.2
23 10	4 40 1.7	57.6	2 9 29.1	40.5	48 6.4	12.0	1 3 10.1	22.7	3 0 14.6	54.0	6 11 42.5	70.3
23 20	4 39 4.1	57.6	2 8 48.6	40.3	47 54.4	11.8	1 3 32.8	22.8	3 1 8.6	54.2	6 12 52.8	70.3
23 30	4 38 6.5	57.5	2 8 8.3	40.3	47 42.6	11.7	1 3 55.6	23.0	3 2 2.8	54.3	6 14 3.1	70.4
23 40	4 37 9.0	57.5	2 7 28.0	40.1	47 30.9	11.5	1 4 18.6	23.2	3 2 57.1	54.5	6 15 13.5	70.3
23 50	4 36 11.5	57.4	2 6 47.9	39.9	47 19.4	11.3	1 4 41.8	23.5	3 3 51.6	54.6	6 16 23.8	70.4
24 0	4 35 14.1	57.4	2 6 8.0	39.9	47 8.1	11.1	1 5 5.3	23.6	3 4 46.2	54.8	6 17 34.2	70.4
24 10	4 34 16.7	57.3	2 5 28.1	39.7	46 57.0	10.9	1 5 28.9	23.8	3 5 41.0	54.9	6 18 44.6	70.5
24 20	4 33 19.4	57.2	2 4 48.4	39.5	46 46.1	10.8	1 5 52.7	24.0	3 6 35.9	55.0	6 19 55.1	70.4
24 30	4 32 22.2	57.1	2 4 8.9	39.5	46 35.3	10.6	1 6 16.7	24.2	3 7 30.9	55.2	6 21 5.5	70.5
24 40	4 31 25.1	57.1	2 3 29.4	39.3	46 24.7	10.3	1 6 40.9	24.3	3 8 26.1	55.3	6 22 16.0	70.5
24 50	4 30 28.0	57.1	2 2 50.1	39.2	46 14.4	10.2	1 7 5.2	24.6	3 9 21.4	55.4	6 23 26.5	70.5
25 0	4 29 30.9	57.0	2 2 10.9	39.0	46 4.2	10.0	1 7 29.8	24.8	3 10 16.8	55.6	6 24 37.0	70.6
25 10	4 28 33.9	56.9	2 1 31.9	38.9	45 54.2	9.8	1 7 54.6	24.9	3 11 12.4	55.8	6 25 47.6	70.6
25 20	4 27 37.0	56.8	2 0 53.0	38.8	45 44.4	9.6	1 8 19.5	25.2	3 12 8.2	55.8	6 26 58.2	70.6
25 30	4 26 40.2	56.8	2 0 14.2	38.7	45 34.8	9.5	1 8 44.7	25.4	3 13 4.0	56.0	6 28 8.8	70.6
25 40	4 25 43.4	56.7	1 59 35.5	38.5	45 25.3	9.3	1 9 10.1	25.5	3 14 0.0	56.1	6 29 19.4	70.6
25 50	4 24 46.7	56.7	1 58 57.0	38.3	45 16.0	9.1	1 9 35 6	25.7	3 14 56.1	56.3	6 30 30.0	70.7
26 0	4 23 50.0	56.7	1 58 18.7	38.3	45 6.9	8.9	1 10 1 3	26.0	3 15 52.4	56.4	6 31 40.7	70.7
26 10	4 22 53.4	56.5	1 57 40.4	38.1	44 58.0	8.7	1 10 27 3	26.1	3 16 48.8	56.6	6 32 51.4	70.7
26 20	4 21 56.9	56.4	1 57 2.3	37.9	44 49.3	8.5	1 10 53 4	26.3	3 17 45.4	56.6	6 34 2.1	70.7
26 30	4 21 0.5	56.4	1 56 24.4	37.8	44 40.8	8.4	1 11 19 7	26.5	3 18 42.0	56.8	6 35 12.8	70.7
26 40	4 20 4.1	56.3	1 55 46.6	37.7	44 32.4	8.1	1 11 46.2	26.7	3 19 38.8	57.0	6 36 23.5	70.8
26 50	4 19 7.8	56.3	1 55 8.9	37.6	44 24.3	8.0	1 12 12.9	26.9	3 20 35.8	57.1	6 37 34.3	70.8
27 0	4 18 11.5	56.2	1 54 31.3	37.4	44 16.3	7.8	1 12 39.8	27.1	3 21 32.9	57.2	6 38 45.1	70.7
27 10	4 17 15.3	56.1	1 53 53.9	37.3	44 8.5	7.6	1 13 6.9	27.2	3 22 30.1	57.3	6 39 55.8	70.8
27 20	4 16 19.2	56 0	1 53 16.6	37.1	44 0.9	7.4	1 13 34.1	27.5	3 23 27.4	57.4	6 41 6.6	70.7
27 30	4 15 23.2	56.0	1 52 39.5	37.0	43 53.5	7.2	1 14 1.6	27.6	3 24 24.8	57.6	6 42 17.3	70.8
27 40	4 14 27.2	55.9	1 52 2.5	36.9	43 46.3	7.1	1 14 29.2	27.9	3 25 22.4	57.7	6 43 28.1	70.8
27 50	4 13 31.3	55.8	1 51 25.6	36.7	43 39.2	6.8	1 14 57.1	28.0	3 26 20.1	57.9	6 44 38.9	70.8
28 0	4 12 35.5	55.8	1 50 48.9	36.6	43 32.4	6.7	1 15 25.1	28.2	3 27 18.0	58.0	6 45 49.7	70.8
28 10	4 11 39.7	55.7	1 50 12.3	36.4	43 25.7	6.5	1 15 53.3	28.5	3 28 16.0	58.1	6 47 0.5	70.9
28 20	4 10 44.0	55.6	1 49 35 9	36.3	43 19.2	6.3	1 16 21.8	28.6	3 29 14.1	58.2	6 48 11.4	70.8
28 30	4 9 48.4	55.5	1 48 59 6	36.2	43 12.9	6.1	1 16 50.4	28.8	3 30 12.3	58.4	6 49 22.2	70.9
28 40	4 8 52.9	55.5	1 48 23 4	36.0	43 6.8	5.9	1 17 19.2	29.0	3 31 10.7	58.5	6 50 33.1	70.8
28 50	4 7 57.4	55.4	1 47 47 4	35.9	43 0.9	5.7	1 17 48.2	29.1	3 32 9.2	58.6	6 51 43.9	70.9
29 0	4 7 2.0	55.3	1 47 11 5	35.7	42 55.2	5.5	1 18 17.3	29.4	3 33 7.8	58.7	6 52 54.8	70.8
29 10	4 6 6.7	55.3	1 46 35.8	35.6	42 49.7	5.4	1 18 46.7	29.6	3 34 6.5	58.9	6 54 5.6	70.9
29 20	4 5 11.4	55.2	1 46 0.2	35.5	42 44.3	5.2	1 19 16.3	29.7	3 35 5.4	59.0	6 55 16.5	70.9
29 30	4 4 16.2	55.1	1 45 24.7	35.3	42 39.1	4.9	1 19 46.0	30.0	3 36 4.4	59.1	6 56 27.4	70.8
29 40	4 3 21.1	55.0	1 44 49.4	35.1	42 34.2	4.8	1 20 16.0	30.1	3 37 3.5	59.2	6 57 38.2	70.9
29 50	4 2 26.1	55.0	1 44 14.3	35.1	42 29.4	4.6	1 20 46.1	30.3	3 38 2.7	59.4	6 58 49.1	70.9
30 0	4 1 31.1		1 43 39.2		42 24.8		1 21 16.4		3 39 2.1		7 0 0.0	

TABLE XIX.
VARIATION.

Deg.	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff.	IVs.	Diff.	Vs	Diff.
0	0 38′ 0″.0	73″.3	1 8′ 1″.5	34″.0	1 6′ 57″.9	39″.9	35′ 54″.4	74″.0	5 29″.5	35″.3	6′ 1″.6	40″.0
1	0 39 13.3	73.2	1 8 35.5	31.7	1 6 18.0	42.1	34 40.4	73.8	4 54.2	32.9	6 41.6	42.3
2	0 40 26.5	73.0	1 9 7.2	29.3	1 5 35.9	44.2	33 26.6	73.6	4 21.3	30.7	7 23.9	44.5
3	0 41 39.5	72.7	1 9 36.5	26.9	1 4 51.7	46.2	32 13.0	73.4	3 50.6	28.3	8 8.4	46.6
4	0 42 52.2	72.3	1 10 3.4	24.5	1 4 5.5	48.2	30 59.6	72.9	3 22.3	25.8	8 55.0	48.7
5	0 44 4.5	71.9	1 10 27 9	22.0	1 3 17.3	50.1	29 46.7	72.4	2 56.5	23.4	9 43.7	50.8
6	0 45 16.4	71.3	1 10 49.9	19.5	1 2 27.2	51.9	28 34.3	71.9	2 33.1	21.0	10 34.5	52.8
7	0 46 27.7	70.7	1 11 9.4	17.0	1 1 35.3	53.7	27 22.4	71.2	2 12.1	18.4	11 27.3	54.7
8	0 47 38.4	69.9	1 11 26.4	14.5	1 0 41.6	55.5	26 11.2	70.5	1 53.7	15.9	12 22.0	56.6
9	0 48 48.3	69.1	1 11 40.9	12.0	0 59 46.1	57.1	25 0.7	69.6	1 37.8	13.3	13 18.6	58.3
10	0 49 57.4	68.2	1 11 52.9	9.3	0 58 49.0	58.8	23 51.1	68.8	1 24.5	10.8	14 16.9	60.1
11	0 51 5.6	67.2	1 12 2.2	6.8	0 57 50.2	60.2	22 42.3	67.8	1 13.7	8.2	15 17.0	61.7
12	0 52 12.8	66.1	1 12 9.0	4.2	0 56 50.0	61.7	21 34.5	66.6	1 5.5	5.5	16 18.7	63.3
13	0 53 18.9	64.9	1 12 13.2	1.6	0 55 48.3	63.1	20 27.9	65.6	1 0.0	3.0	17 22.0	64.9
14	0 54 23.8	63.7	1 12 14.8	0.9	0 54 45.2	64.3	19 22.3	64.3	0 57.0	0.3	18 26.9	66.2
15	0 55 27.5	62.3	1 12 13.9	3.6	0 53 40.9	65.6	18 18.0	63.0	0 56.7	2.3	19 33.1	67.6
16	0 56 29.8	60.9	1 12 10.3	6.1	0 52 35.3	66.8	17 15.0	61.6	0 59.0	4.9	20 40.7	68.9
17	0 57 30.7	59.4	1 12 4.2	8.7	0 51 28.5	67.8	16 13.4	60.2	1 3.9	7.6	21 49.6	70.0
18	0 58 30.1	57.9	1 11 55.5	11.3	0 50 20.7	68.8	15 13.2	58.6	1 11.5	10.1	22 59.6	71.2
19	0 59 28.0	56.2	1 11 44.2	13.7	0 49 11.9	69.7	14 14.6	57.1	1 21.6	12.8	24 10.8	72.1
20	1 0 24.2	54.5	1 11 30.5	16.4	0 48 2.2	70.5	13 17.5	55.3	1 34.4	15.4	25 22.9	73.0
21	1 1 18.7	52.7	1 11 14.1	18.8	0 46 51.7	71.2	12 22.2	53.7	1 49.8	18.0	26 35.9	73.9
22	1 2 11.4	50.9	1 10 55.3	21.3	0 45 40.5	71.9	11 28.5	51.8	2 7.8	20.5	27 49.8	74.7
23	1 3 2.3	48.9	1 10 34.0	23.8	0 44 28.6	72.5	10 36.7	49.9	2 28.3	23.1	29 4.5	75.2
24	1 3 51.2	47.0	1 10 10.2	26.2	0 43 16.1	72.9	9 46.8	48.0	2 51.4	25.5	30 19.7	75.9
25	1 4 38.2	44.9	1 9 44.0	28.6	0 42 3.2	73.3	8 58.8	46.1	3 16.9	28.1	31 35.6	76.3
26	1 5 23.1	42.9	1 9 15.4	30.9	0 40 49.9	73.7	8 12.7	44.0	3 45.0	30.6	32 51.9	76.7
27	1 6 6.0	40.7	1 8 44.5	33.3	0 39 36.2	73.8	7 28.7	41.9	4 15.6	32.9	34 8.6	77.0
28	1 6 46.7	38.5	1 8 11.2	35.5	0 38 22.4	74.0	6 46.8	39.7	4 48.5	35.4	35 25.6	77.1
29	1 7 25.2	36.3	1 7 35.7	37.8	0 37 8.4	74.0	6 7.1	37.6	5 23.9	37.7	36 42.7	77.3
30	1 8 1.5		1 6 57.9		0 35 54.4		5 29.5		6 1.6		38 0.0	

TABLE XX. REDUCTION.

Argu. = Supplem. of the Node + long. ☽ + the Sum of all the preceding Equations.

Deg.	Os VIs	Diff.	Is VIIs	Diff.	IIs VIIIs	Diff.	IIIs IXs	Diff.	IVs Xs	Diff.	Vs XIs	Diff.
0	7′ 0″.0	14″.4	1′ 3″.0	7″.0	1′ 3.0	7″.4	7′ 0″.0	14″.4	12 57.0	7″.0	12 57.0	7″.4
1	6 45.6	14.4	0 56.0	6.5	1 10.4	7.9	7 14.4	14.4	13 4.0	6.5	12 49.6	7.9
2	6 31.2	14.3	0 49.5	6.1	1 18.3	8.2	7 28.8	14.3	13 10.5	6.1	12 41.7	8.2
3	6 16.9	14.3	0 43.4	5.6	1 26.5	8.7	7 43.1	14.3	13 16.6	5.6	12 33.5	8.7
4	6 2.6	14.2	0 37.8	5.1	1 35.2	9.0	7 57.4	14.2	13 22.2	5.1	12 24.8	9.0
5	5 48.4	14.1	0 32.7	4.5	1 44.2	9.5	8 11.6	14.1	13 27.3	4.5	12 15.8	9.5
6	5 34.3	14.0	0 28.2	4.3	1 53.7	9.8	8 25.7	14.0	13 31.8	4.3	12 6.3	9.8
7	5 20.3	13.9	0 23.9	3.9	2 3.5	10.2	8 39.7	13.9	13 36.1	3.9	11 56.5	10.2
8	5 6.4	13.8	0 20.0	3.2	2 13.7	10.5	8 53.6	13.8	13 40.0	3.2	11 46.3	10.5
9	4 52.6	13.6	0 16.8	2.7	2 24.2	10.8	9 7.4	13.6	13 43.2	2.7	11 35.8	10.8
10	4 39.0	13.4	0 14.1	2.3	2 35.0	11.2	9 21.0	13.4	13 45.9	2.3	11 25.0	11.2
11	4 25.6	13.3	0 11.8	1.7	2 46.2	11.5	9 34.4	13.3	13 48.2	1.7	11 13.8	11.5
12	4 12.3	13.0	0 10.1	1.3	2 57.7	11.8	9 47.7	13.0	13 49.9	1.3	11 2.3	11.8
13	3 59.3	12.8	0 8.8	0.7	3 9.5	12.1	10 0.7	12.8	13 51.2	0.7	10 50.5	12.1
14	3 46.5	12.6	0 8.1	0.3	3 21.6	12.3	10 13.5	12.6	13 51.9	0.3	10 38.4	12.3
15	3 33.9		0 7.8		3 33.9		10 26.1		13 52.2		10 26.1	

TABLE XIX. continued.

Deg.	VI*s*	Diff.	VII*s*	Diff.	VIII*s*	Diff.	IX*s*	Diff.	X*s*	Diff.	XI*s*	Diff.
0	0°38′ 0″0	77″3	1° 9′ 58″4	37.7	1°10′ 30″5	37.6	40′ 5″6	74″0	9′ 2″1	37″8	7′ 58″5	36″3
1	0 39 17.3	77.1	1 10 36.1	35.4	1 9 52.9	39.7	38 51.6	74.0	8 24.3	35.5	8 34.8	38.5
2	0 40 34.4	77.0	1 11 11.5	32.9	1 9 13.2	41.9	37 37.6	73.8	7 48.8	33.3	9 13.3	40.7
3	0 41 51.4	76.7	1 11 44.4	30.6	1 8 31.3	44.0	36 23.8	73.7	7 15.5	30.9	9 54.0	42.9
4	0 43 8.1	76.3	1 12 15.0	28.1	1 7 47.3	46.1	35 10.1	73.3	6 44.6	28.6	10 36.9	44.9
5	0 44 24.4	75.9	1 12 43.1	25.5	1 7 1.2	48.0	33 56.8	72.9	6 16.0	26.2	11 21.8	47.0
6	0 45 40.3	75.2	1 13 8.6	23.1	1 6 13.2	49.9	32 43.9	72.5	5 49.8	23.8	12 8.8	48.9
7	0 46 55.5	74.7	1 13 31.7	20.5	1 5 23.3	51.8	31 31.4	71.9	5 26.0	21.3	12 57.7	50.9
8	0 48 10.2	73.9	1 13 52.2	18.0	1 4 31.5	53.7	30 19.5	71.2	5 4.7	18.8	13 48.6	52.7
9	0 49 24.1	73.0	1 14 10.2	15.4	1 3 37.8	55.3	29 8.3	70.5	4 45.9	16.4	14 41.3	54.5
10	0 50 37.1	72.1	1 14 25.6	12.8	1 2 42.5	57.1	27 57.8	69.7	4 29.5	13.7	15 35.8	56.2
11	0 51 49.2	71.2	1 14 38.4	10.1	1 1 45.4	58.6	26 48.1	68.8	4 15.8	11.3	16 32.0	57.9
12	0 53 0.4	70.0	1 14 48.5	7.6	1 0 46.8	60.2	25 39.3	67.8	4 4.5	8.7	17 29.9	59.4
13	0 54 10.4	68.9	1 14 56.1	4.9	0 59 46.6	61.6	24 31.5	66.8	3 55.8	6.1	18 29.3	60.9
14	0 55 19.3	67.6	1 15 1.0	2.3	0 58 45.0	63.0	23 24.7	65.6	3 49.7	3.6	19 30.2	62.3
15	0 56 26.9	66.2	1 15 3.3	0.3	0 57 42.0	64.3	22 19.1	64.3	3 46.1	0.9	20 32.5	63.7
16	0 57 33.1	64.9	1 15 3.0	3.0	0 56 37.7	65.6	21 14.8	63.1	3 45.2	1.6	21 36.2	64.9
17	0 58 38.0	63.3	1 15 0.0	5.5	0 55 32.1	66.6	20 11.7	61.7	3 46.8	4.2	22 41.1	66.1
18	0 59 41.3	61.7	1 14 54.5	8.2	0 54 25.5	67.8	19 10.0	60.2	3 51.0	6.8	23 47.2	67.2
19	1 0 43.0	60.1	1 14 46.3	10.8	0 53 17.7	68.8	18 9.8	58.8	3 57.8	9.3	24 54.4	68.2
20	1 1 43.1	58.3	1 14 35.5	13.3	0 52 8.9	69.6	17 11.0	57.1	4 7.1	12.0	26 2.6	69.1
21	1 2 41.4	56.6	1 14 22.2	15.9	0 50 59.3	70.5	16 13.9	55.5	4 19.1	14.5	27 11.7	69.9
22	1 3 38.0	54.7	1 14 6.3	18.4	0 49 48.8	71.2	15 18.4	53.7	4 33.6	17.0	28 21.6	70.7
23	1 4 32.7	52.8	1 13 47.9	21.0	0 48 37.6	71.9	14 24.7	51.9	4 50.6	19.5	29 32.3	71.3
24	1 5 25.5	50.8	1 13 26.9	23.4	0 47 25.7	72.4	13 32.8	50.1	5 10.1	22.0	30 43.6	71.9
25	1 6 16.3	48.7	1 13 3.5	25.8	0 46 13.3	72.9	12 42.7	48.2	5 32.1	24.5	31 55.5	72.3
26	1 7 5.0	46.6	1 12 37.7	28.3	0 45 0.4	73.4	11 54.5	46.2	5 56.6	26.9	33 7.8	72.7
27	1 7 51.6	44.5	1 12 9.4	30.7	0 43 47.0	73.6	11 8.3	44.2	6 23.5	29.3	34 20.5	73.0
28	1 8 36.1	42.3	1 11 38.7	32.9	0 42 33.4	73.8	10 24.1	42.1	6 52.8	31.7	35 33.5	73.2
29	1 9 18.4	40.0	1 11 5.8	35.3	0 41 19.6	74.0	9 42.0	39.9	7 24.5	34.0	36 46.7	73.3
30	1 9 58.4		1 10 30.5		0 40 5.6		9 2.1		7 58.5		38 0.0	

TABLE XX. continued.

Deg.	O*s* VI*s*	Diff.	I*s* VII*s*	Diff.	II*s* VIII*s*	Diff.	III*s* IX*s*	Diff.	IV*s* X*s*	Diff.	V*s* XI*s*	Diff.
15	3′ 33″9	12″3	0′ 7″8	0″3	3′ 33″9	12″6	10′ 26″1	12″3	13′ 52″2	0″3	10′ 26″1	12″6
16	3 21.6	12.1	0 8.1	0.7	3 46.5	12.8	10 38.4	12.1	13 51.9	0.7	10 13.5	12.8
17	3 9.5	11.8	0 8.8	1.3	3 59.3	13.0	10 50.5	11.8	13 51.2	1.3	10 0.7	13.0
18	2 57.7	11.5	0 10.1	1.7	4 12.3	13.3	11 2.3	11.5	13 49.9	1.7	9 47.7	13.3
19	2 46.2	11.2	0 11.8	2.3	4 25.6	13.4	11 13.8	11.2	13 48.2	2.3	9 34.4	13.4
20	2 35.0	10.8	0 14.1	2.7	4 39.0	13.6	11 25.0	10.8	13 45.9	2.7	9 21.0	13.6
21	2 24.2	10.5	0 16.8	3.2	4 52.6	13.8	11 35.8	10.5	13 43.2	3.2	9 7.4	13.8
22	2 13.7	10.2	0 20.0	3.9	5 6.4	13.9	11 46.3	10.2	13 40.0	3.9	8 53.6	13.9
23	2 3.5	9.8	0 23.9	4.3	5 20.3	14.0	11 56.5	9.8	13 36.1	4.3	8 39.7	14.0
24	1 53.7	9.5	0 28.2	4.5	5 34.3	14.1	12 6.3	9.5	13 31.8	4.5	8 25.7	14.1
25	1 44.2	9.0	0 32.7	5.1	5 48.4	14.2	12 15.8	9.0	13 27.3	5.1	8 11.6	14.2
26	1 35.2	8.7	0 37.8	5.6	6 2.6	14.3	12 24.8	8.7	13 22.2	5.6	7 57.4	14.3
27	1 26.5	8.2	0 43.4	6.1	6 16.9	14.3	12 33.5	8.2	13 16.6	6.1	7 43.1	14.3
28	1 18.3	7.9	0 49.5	6.5	6 31.2	14.4	12 41.7	7.9	13 10.5	6.5	7 28.8	14.4
29	1 10.4	7.4	0 56.0	7.0	6 45.6	14.4	12 49.6	7.4	13 4.0	7.0	7 14.4	14.4
30	1 3.0		1 3.0		7 0.0		12 57.0		12 57.0		7 0.0	

TABLE XXI.

Distance of the Moon from the North Pole of the Ecliptic, or for her Latitude.

Argum. = argum. preceding, which serves to find the Reduction.

	IIIs 84°	Diff.	IVs 85°	Diff.	Vs	Diff.	VIs	Diff.	VIIs	Diff.	VIIIs 94°	Diff.	
0 00	39 16.0	0.1	20 42.7	27.0	87 13 46.6	46.7	89 48 0.0	53.8	92 22 13.4	46.7	15 17.3	26.9	30 0
0 10	39 16.1	0.2	21 9.7	27.2	87 14 33.3	46.7	89 48 53.8	53.8	92 23 0.1	46.5	15 44.2	26.8	29 50
0 20	39 16.3	0.4	21 36.9	27.3	87 15 20.0	46.9	89 49 47.6	53.8	92 23 46.6	46.5	16 11.0	26.7	29 40
0 30	39 16.7	0.6	22 4.2	27.5	87 16 6.9	46.9	89 50 41.4	53.9	92 24 33.1	46.5	16 37.7	26.5	29 30
0 40	39 17.3	0.7	22 31.7	27.6	87 16 53.8	47.0	89 51 35.3	53.8	92 25 19.6	46.4	17 4.2	26.3	29 20
0 50	39 18.0	0.8	22 59.3	27.7	87 17 40.8	47.0	89 52 29.1	53.8	92 26 6.0	46.2	17 30.5	26.3	29 10
1 0	39 18.8	1.0	23 27.0	27.9	87 18 27.8	47.2	89 53 22.9	53.8	92 26 52.2	46.1	17 56.8	26.1	29 0
1 10	39 19.8	1.2	23 54.9	28.0	87 19 15.0	47.2	89 54 16.7	53.8	92 27 38.3	46.0	18 22.9	25.9	28 50
1 20	39 21.0	1.4	24 22.9	28.1	87 20 2.2	47.3	89 55 10.5	53.8	92 28 24.3	45.9	18 48.8	25.8	28 40
1 30	39 22.4	1.5	24 51.0	28.3	87 20 49.5	47.4	89 56 4.3	53.8	92 29 10.2	45.8	19 14.6	25.7	28 30
1 40	39 23.9	1.6	25 19.3	28.4	87 21 36.9	47.4	89 56 58.1	53.8	92 29 56.0	45.8	19 40.3	25.6	28 20
1 50	39 25.5	1.8	25 47.7	28.5	87 22 24.3	47.5	89 57 51.9	53.8	92 30 41.8	45.7	20 5.9	25.4	28 10
2 0	39 27.3	2.0	26 16.2	28.6	87 23 11.8	47.6	89 58 45.7	53.8	92 31 27.5	45.6	20 31.3	25.3	28 0
2 10	39 29.3	2.1	26 44.8	28.8	87 23 59.4	47.7	89 59 39.5	53.8	92 32 13.1	45.6	20 56.6	25.1	27 50
2 20	39 31.4	2.3	27 13.6	29.0	87 24 47.1	47.7	90 0 33.3	53.7	92 32 58.7	45.5	21 21.7	25.0	27 40
2 30	39 33.7	2.4	27 42.6	29.0	87 25 34.8	47.8	90 1 27.0	53.8	92 33 44.2	45.4	21 46.7	24.8	27 30
2 40	39 36.1	2.6	28 11.6	29.2	87 26 22.6	47.9	90 2 20.8	53.7	92 34 29.6	45.3	22 11.5	24.7	27 20
2 50	39 38.7	2.8	28 40.8	29.3	87 27 10.5	48.0	90 3 14.5	53.8	92 35 14.9	45.3	22 36.2	24.6	27 10
3 0	39 41.5	2.9	29 10.1	29.5	87 27 58.5	48.0	90 4 8.3	53.7	92 36 0.2	45.1	23 0.8	24.4	27 0
3 10	39 44.4	3.0	29 39.6	29.6	87 28 46.5	48.1	90 5 2.0	53.8	92 36 45.3	45.1	23 25.2	24.3	26 50
3 20	39 47.4	3.2	30 9.2	29.7	87 29 34.6	48.2	90 5 55.8	53.7	92 37 30.4	44.9	23 49.5	24.2	26 40
3 30	39 50.6	3.4	30 38.9	29.8	87 30 22.8	48.2	90 6 49.5	53.7	92 38 15.3	44.9	24 13.7	24.0	26 30
3 40	39 54.0	3.5	31 8.7	30.0	87 31 11.0	48.3	90 7 43.2	53.7	92 39 0.2	44.8	24 37.7	23.9	26 20
3 50	39 57.5	3.7	31 38.7	30.1	87 31 59.3	48.4	90 8 36.9	53.7	92 39 45.0	44.7	25 1.6	23.7	26 10
4 0	40 1.2	3.9	32 8.8	30.2	87 32 47.7	48.4	90 9 30.6	53.7	92 40 29.7	44.7	25 25.3	23.6	26 0
4 10	40 5.1	4.0	32 39.0	30.4	87 33 36.1	48.5	90 10 24.3	53.6	92 41 14.4	44.5	25 48.9	23.5	25 50
4 20	40 9.1	4.1	33 9.4	30.5	87 34 24.6	48.6	90 11 17.9	53.7	92 41 58.9	44.4	26 12.4	23.3	25 40
4 30	40 13.2	4.4	33 39.9	30.6	87 35 13.2	48.7	90 12 11.6	53.6	92 42 43.3	44.4	26 35.7	23.1	25 30
4 40	40 17.6	4.4	34 10.5	30.8	87 36 1.9	48.7	90 13 5.2	53.7	92 43 27.7	44.3	26 58.8	23.1	25 20
4 50	40 22.0	4.7	34 41.3	30.9	87 36 50.6	48.7	90 13 58.9	53.6	92 44 12.0	44.2	27 21.9	22.9	25 10
5 0	40 26.7	4.8	35 12.2	31.0	87 37 39.3	48.9	90 14 52.5	53.6	92 44 56.2	44.0	27 44.8	22.7	25 0
5 10	40 31.5	4.9	35 43.2	31.1	87 38 28.2	48.9	90 15 46.1	53.6	92 45 40.2	44.0	28 7.5	22.6	24 50
5 20	40 36.4	5.1	36 14.3	31.3	87 39 17.1	49.0	90 16 39.7	53.6	92 46 24.2	43.9	28 30.1	22.5	24 40
5 30	40 41.5	5.2	36 45.6	31.4	87 40 6.1	49.0	90 17 33.3	53.6	92 47 8.1	43.9	28 52.6	22.3	24 30
5 40	40 46.7	5.4	37 17.0	31.5	87 40 55.1	49.1	90 18 26.9	53.5	92 47 52.0	43.7	29 14.9	22.1	24 20
5 50	40 52.1	5.6	37 48.5	31.6	87 41 44.2	49.2	90 19 20.4	53.6	92 48 35.7	43.7	29 37.0	22.0	24 10
6 0	40 57.7	5.7	38 20.1	31.8	87 42 33.4	49.2	90 20 14.0	53.5	92 49 19.4	43.5	29 59.0	21.9	24 0
6 10	41 3.4	5.9	38 51.9	31.9	87 43 22.6	49.3	90 21 7.5	53.5	92 50 2.9	43.5	30 20.9	21.8	23 50
6 20	41 9.3	6.1	39 23.8	32.0	87 44 11.9	49.3	90 22 1.0	53.4	92 50 46.4	43.3	30 42.7	21.6	23 40
6 30	41 15.4	6.2	39 55.8	32.2	87 45 1.2	49.4	90 22 54.4	53.5	92 51 29.7	43.3	31 4.3	21.4	23 30
6 40	41 21.6	6.3	40 28.0	32.3	87 45 50.6	49.5	90 23 47.9	53.4	92 52 13.0	43.2	31 25.7	21.3	23 20
6 50	41 27.9	6.5	41 0.3	32.4	87 46 40.1	49.5	90 24 41.3	53.5	92 52 56.2	43.1	31 47.0	21.2	23 10
7 0	41 34.4	6.7	41 32.7	32.5	87 47 29.6	49.6	90 25 34.8	53.4	92 53 39.3	43.0	32 8.2	21.1	23 0
7 10	41 41.1	6.8	42 5.2	32.7	87 48 19.2	49.7	90 26 28.2	53.4	92 54 22.3	42.9	32 29.3	20.9	22 50
7 20	41 47.9	6.9	42 37.9	32.7	87 49 8.9	49.7	90 27 21.6	53.3	92 55 5.2	42.8	32 50.2	20.7	22 40
7 30	41 54.8		43 10.6		87 49 58.6		90 28 14.9		92 55 48.0		33 10.9		22 30
	IIs		Is		Os		XIs		Xs		IXs		

TABLE XXI. continued.

	IIIs 84°	Diff.	IVs	Diff.	Vs	Diff.	VIs	Diff.	VIIs	Diff.	VIIIs 94°	Diff.	
° ′	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	′ ″	″	° ′
7 30	41 54.8	7.2	85 43 10.6	32.9	87 49 58.6	49.8	90 28 14.9	53.4	92 55 48.0	42.7	33 10.9	20.5	22 30
7 40	42 2.0	7.3	85 43 43.5	33.0	87 50 48.4	49.8	90 29 8.3	53.3	92 56 30.7	42.6	33 31.4	20.5	22 20
7 50	42 9.3	7.4	85 44 16.5	33.2	87 51 38.2	49.9	90 30 1.6	53.3	92 57 13.3	42.5	33 51.9	20.3	22 10
8 0	42 16.7	7.6	85 44 49.7	33.3	87 52 28.1	50.0	90 30 54.9	53.3	92 57 55.8	42.4	34 12.2	20.1	22 0
8 10	42 24.3	7.7	85 45 23.0	33.4	87 53 18.1	50.0	90 31 48.2	53.3	92 58 38.2	42.3	34 32.3	20.0	21 50
8 20	42 32.0	7.9	85 45 56.4	33.5	87 54 8.1	50.1	90 32 41.5	53.2	92 59 20.5	42.3	34 52.3	19.9	21 40
8 30	42 39.9	8.1	85 46 29.9	33.6	87 54 58.2	50.1	90 33 34.7	53.2	93 0 2.8	42.1	35 12.2	19.7	21 30
8 40	42 48.0	8.2	85 47 3.5	33.8	87 55 48.3	50.2	90 34 27.9	53.2	93 0 44.9	42.0	35 31.9	19.6	21 20
8 50	42 56.2	8.4	85 47 37.3	33.9	87 56 38.5	50.2	90 35 21.1	53.2	93 1 26.9	42.0	35 51.5	19.4	21 10
9 0	43 4.6	8.5	85 48 11.2	34.0	87 57 28.7	50.3	90 36 14.3	53.2	93 2 8.9	41.8	36 10.9	19.3	21 0
9 10	43 13.1	8.7	85 48 45·2	34.1	87 58 19.0	50.4	90 37 7.5	53.1	93 2 50.7	41.7	36 30.2	19.1	20 50
9 20	43 21.8	8.8	85 49 19.3	34.2	87 59 9.4	50.4	90 38 0.6	53.1	93 3 32.4	41.7	36 49.3	19.0	20 40
9 30	43 30.6	9.0	85 49 53.5	34.4	87 59 59.8	50.4	90 38 53.7	53.1	93 4 14.1	41.5	37 8.3	18.8	20 30
9 40	43 39.6	9.2	85 50 27.9	34.5	88 0 50.2	50.5	90 39 46.8	53.0	93 4 55.6	41.5	37 27.1	18.7	20 20
9 50	43 48.8	9.3	85 51 2.4	34.6	88 1 40.7	50.6	90 40 39.8	53.0	93 5 37.1	41.3	37 45.8	18.6	20 10
10 0	43 58.1	9.4	85 51 37.0	34.7	88 2 31.3	50.6	90 41 32.8	53.0	93 6 18.4	41.2	38 4.4	18.4	20 0
10 10	44 7.5	9.6	85 52 11.7	34.9	88 3 21.9	50.7	90 42 25.8	53.0	93 6 59.6	41.2	38 22.8	18.2	19 50
10 20	44 17.1	9.8	85 52 46.6	35.0	88 4 12.6	50.7	90 43 18.8	53.0	93 7 40.8	41.0	38 41.0	18.1	19 40
10 30	44 26.9	9.9	85 53 21.6	35.0	88 5 3.3	50.8	90 44 11.7	52.9	93 8 21.8	40.9	38 59.1	18.0	19 30
10 40	44 36.8	10.1	85 53 56.6	35.2	88 5 54.1	50.8	90 45 4.6	52.9	93 9 2.7	40.9	39 17.1	17.8	19 20
10 50	44 46.9	10.2	85 54 31.8	35.3	88 6 44.9	50.9	90 45 57.5	52.9	93 9 43.6	40.7	39 34.9	17.6	19 10
11 0	44 57.1	10.4	85 55 7.1	35.5	88 7 35.8	50.9	90 46 50.4	52.8	93 10 24.3	40.6	39 52.5	17.5	19 0
11 10	45 7.5	10.6	85 55 42.6	35.5	88 8 26.7	51.0	90 47 43.2	52.8	93 11 4.9	40.6	40 10.0	17.4	18 50
11 20	45 18.1	10.7	85 56 18.1	35.7	88 9 17.7	51.1	90 48 36.0	52.7	93 11 45.5	40.4	40 27.4	17.2	18 40
11 30	45 28.8	10.8	85 56 53.8	35.8	88 10 8.8	51.0	90 49 28.7	52.8	93 12 25.9	40.3	40 44.6	17.0	18 30
11 40	45 39.6	11.0	85 57 29.6	35.9	88 10 59.8	51.2	90 50 21.5	52.7	93 13 6.2	40.2	41 1.6	16.9	18 20
11 50	45 50.6	11.2	85 58 5.5	36.1	88 11 51.0	51.1	90 51 14.2	52.6	93 13 46.4	40.2	41 18.5	16.8	18 10
12 0	46 1.8	11.3	85 58 41.6	36.1	88 12 42.1	51.2	90 52 6.8	52.6	93 14 26.6	40.0	41 35.3	16.6	18 0
12 10	46 13.1	11.4	85 59 17.7	36.3	88 13 33.3	51.3	90 52 59.4	52.6	93 15 6.6	39.9	41 51.9	16.5	17 50
12 20	46 24.5	11.6	85 59 54.0	36.3	88 14 24.6	51.4	90 53 52.0	52.6	93 15 46.5	39.8	42 8.4	16.3	17 40
12 30	46 36.1	11.8	86 0 30.3	36.5	88 15 16.0	51.3	90 54 44.6	52.5	93 16 26.3	39.6	42 24.7	16.2	17 30
12 40	46 47.9	11.9	86 1 6.8	36.6	88 16 7.3	51.5	90 55 37.1	52.5	93 17 5.9	39.6	42 40.9	16.0	17 20
12 50	46 59.8	12.1	86 1 43.4	36.7	88 16 58.8	51.4	90 56 29.6	52.5	93 17 45.5	39.5	42 56.9	15.8	17 10
13 0	47 11.9	12.2	86 2 20.1	36.9	88 17 50.2	51.5	90 57 22.1	52.5	93 18 25.0	39.4	43 12.7	15.7	17 0
13 10	47 24.1	12.4	86 2 57.0	36.9	88 18 41.7	51.6	90 58 14.6	52.5	93 19 4.4	39.3	43 28.4	15.6	16 50
13 20	47 36.5	12.5	86 3 33.9	37.1	88 19 33.3	51.6	90 59 7.0	52.4	93 19 43.7	39.1	43 44.0	15.4	16 40
13 30	47 49.0	12 7	86 4 11.0	37.2	88 20 24.9	51.6	90 59 59.3	52.3	93 20 22.8	39.1	43 59.4	15.3	16 30
13 40	48 1.7	12.8	86 4 48.2	37.3	88 21 16.5	51.7	91 0 51.6	52.3	93 21 1.9	38.9	44 14.7	15.1	16 20
13 50	48 14.5	13.0	86 5 25.5	37.4	88 22 8.2	51.7	91 1 43.9	52.3	93 21 40.8	38.9	44 29.8	14.9	16 10
14 0	48 27.5	13.2	86 6 2.9	37.5	88 22 59.9	51.8	91 2 36.2	52.3	93 22 19.7	38.7	44 44.7	14.9	16 0
14 10	48 40.7	13.3	86 6 40.4	37.6	88 23 51.7	51.8	91 3 28.4	52.2	93 22 58.4	38.6	44 59.6	14.6	15 50
14 20	48 54.0	13.4	86 7 18.0	37.7	88 24 43.5	51.8	91 4 20.6	52.2	93 23 37.0	38.5	45 14.2	14.5	15 40
14 30	49 7.4	13.6	86 7 55.7	37.9	88 25 35.3	51.9	91 5 12.7	52.1	93 24 15.5	38.4	45 28.7	14.4	15 30
14 40	49 21.0	13.8	86 8 33.6	38.0	88 26 27.2	52.0	91 6 4.8	52.1	93 24 53.9	38.3	45 43.1	14.2	15 20
14 50	49 34.8	13.9	86 9 11.6	38.0	88 27 19.2	51.9	91 6 56.9	52.1	93 25 32.2	38.2	45 57.3	14.0	15 10
15 0	49 48.7		86 9 49.6		88 28 11.1		91 7 48.9	52.0	93 26 10.4		46 11.3		15 0
	IIs.		Is		Os		XIs.		Xs.		IXs.		

Distance of the Moon from the North Pole of the Ecliptic, or for her Latitude.

	IIIs	Diff.	IVs 86°	Diff.	Vs	Diff.	VIs	Diff.	VIIs 93°	Diff.	VIIIs 94°	Diff.	
15 00	84 49 48.7	14.0	9 49.6	38.2	88 28 11.1	52.0	91 7 48.9	51.9	26 10.4	38.0	46 11.3	13.9	15 0
15 10	84 50 2.7	14.2	10 27.8	38.3	88 29 3.1	52.1	91 8 40.8	52.0	26 48.4	38.0	46 25.2	13.8	14 50
15 20	84 50 16.9	14.4	11 6.1	38.4	88 29 55.2	52.1	91 9 32.8	51.9	27 26.4	37.9	46 39.0	13.6	14 40
15 30	84 50 31.3	14.5	11 44.5	38.5	88 30 47.3	52.1	91 10 24.7	51.8	28 4.3	37.7	46 52.6	13.4	14 30
15 40	84 50 45.8	14.6	12 23.0	38.6	88 31 39.4	52.2	91 11 16.5	51.8	28 42.0	37.6	47 6.0	13.3	14 20
15 50	84 51 0.4	14.2	13 1.6	38.7	88 32 31.6	52.2	91 12 8.3	51.8	29 19.6	37.5	47 19.3	13.2	14 10
16 0	84 51 15.3	14.9	13 40.3	38.9	88 33 23.8	52.3	91 13 0.1	51.7	29 57.1	37.4	47 32.5	13.0	14 0
16 10	84 51 30.2	15.1	14 19.2	38.9	88 34 16.1	52.3	91 13 51.8	51.7	30 34.5	37.3	47 45.5	12.8	13 50
16 20	84 51 45.3	15.3	14 58.1	39.1	88 35 8.4	52.3	91 14 43.5	51.6	31 11.8	37.2	47 58.3	12.7	13 40
16 30	84 52 0.6	15.4	15 37.2	39.1	88 36 0.7	52.3	91 15 35.1	51.6	31 49.0	37.1	48 11.0	12.5	13 30
16 40	84 52 16.0	15.6	16 16.3	39.3	88 36 53.0	52.4	91 16 26.7	51.6	32 26.1	36.9	48 23.5	12.4	13 20
16 50	84 52 31.6	15.7	16 55.6	39.4	88 37 45.4	52.5	91 17 18.3	51.5	33 3.0	36.9	48 35.9	12.2	13 10
17 0	84 52 47.3	15.8	17 35.0	39.5	88 38 37.9	52.5	91 18 9.8	51.4	33 39.9	36.7	48 48.1	12.1	13 0
17 10	84 53 3.1	16.0	18 14.5	39.6	88 39 30.4	52.5	91 19 1.2	51.5	34 16.6	36.6	49 0.2	11.9	12 50
17 20	84 53 19.1	16.2	18 54.1	39.6	88 40 22.9	52.5	91 19 52.7	51.3	34 53.2	36.5	49 12.1	11.8	12 40
17 30	84 53 35.3	16.3	19 33.7	39.8	88 41 15.4	52.6	91 20 44.0	51.4	35 29.7	36.3	49 23.9	11.6	12 30
17 40	84 53 51.6	16.5	20 13.5	39.9	88 42 8.0	52.6	91 21 35.4	51.3	36 6.0	36.3	49 35.5	11.4	12 20
17 50	84 54 8.1	16.6	20 53.4	40.0	88 43 0.6	52.6	91 22 26.7	51.2	36 42.3	36.1	49 46.9	11.3	12 10
18 0	84 54 24.7	16.8	21 33.4	40.2	88 43 53.2	52.6	91 23 17.9	51.1	37 18.4	36.1	49 58.2	11.2	12 0
18 10	84 54 41.5	16.9	22 13.6	40.2	88 44 45.8	52.7	91 24 9.0	51.2	37 54.5	35.9	50 9.4	11.0	11 50
18 20	84 54 58.4	17.0	22 53.8	40.3	88 45 38.5	52.8	91 25 0.2	51.0	38 30.4	35.8	50 20.4	10.8	11 40
18 30	84 55 15.4	17.2	23 34.1	40.4	88 46 31.3	52.7	91 25 51.2	51.1	39 6.2	35.7	50 31.2	10.7	11 30
18 40	84 55 32.6	17.4	24 14.5	40.6	88 47 24.0	52.8	91 26 42.3	51.0	39 41.9	35.5	50 41.9	10.6	11 20
18 50	84 55 50.0	17.5	24 55.1	40.6	88 48 16.8	52.8	91 27 33.3	50.9	40 17.4	35.5	50 52.5	10.4	11 10
19 0	84 56 7.5	17.6	25 35.7	40.7	88 49 9.6	52.9	91 28 24.2	50.9	40 52.9	35.3	51 2.9	10.2	11 0
19 10	84 56 25.1	17.8	26 16.4	40.9	88 50 2.5	52.9	91 29 15.1	50.8	41 28.2	35.2	51 13.1	10.1	10 50
19 20	84 56 42.9	18.0	26 57.3	40.9	88 50 55.4	52.9	91 30 5.9	50.8	42 3.4	35.0	51 23.2	9.9	10 40
19 30	84 57 0.9	18.1	27 38.2	41.0	88 51 48.3	52.9	91 30 56.7	50.7	42 38.4	35.0	51 33.1	9.8	10 30
19 40	84 57 19.0	18.2	28 19.2	41.2	88 52 41.2	53.0	91 31 47.4	50.7	43 13.4	34.9	51 42.9	9.6	10 20
19 50	84 57 37.2	18.4	29 0.4	41.2	88 53 34.2	53.0	91 32 38.1	50.6	43 48.3	34.7	51 52.5	9.4	10 10
20 0	84 57 55.6	18.6	29 41.6	41.3	88 54 27.2	53.0	91 33 28.7	50.6	44 23.0	34.6	52 1.9	9.3	10 0
20 10	84 58 14.2	18.7	30 22.9	41.5	88 55 20.2	53.0	91 34 19.3	50.5	44 57.6	34.5	52 11.2	9.2	9 50
20 20	84 58 32.9	18.8	31 4.4	41.5	88 56 13·2	53.1	91 35 9.8	50.4	45 32.1	34.4	52 20.4	9.0	9 40
20 30	84 58 51.7	19.0	31 45.9	41.7	88 57 6.3	53.1	91 36 0.2	50.4	46 6.5	34.2	52 29.4	8.8	9 30
20 40	84 59 10.7	19.1	32 27.6	41.7	88 57 59.4	53.1	91 36 50.6	50.4	46 40.7	34.1	52 38.2	8.7	9 20
20 50	84 59 29.8	19.3	33 9.3	41.8	88 58 52.5	53.2	91 37 41.0	50.3	47 14.8	34.0	52 46.9	8.5	9 10
21 0	84 59 49.1	19.4	33 51.1	42.0	88 59 45.7	53.2	91 38 31.3	50.2	47 48.8	33.9	52 55.4	8.4	9 0
21 10	85 0 8.5	19.6	34 33.1	42.0	89 0 38.9	53.2	91 39 21.5	50.2	48 22.7	33.8	53 3.8	8.2	8 50
21 20	85 0 28.1	19.7	35 15.1	42.1	89 1 32.1	53.2	91 40 11.7	50.1	48 56.5	33.6	53 12.0	8.1	8 40
21 30	85 0 47.8	19.9	35 57.2	42.3	89 2 25.3	53.2	91 41 1.8	50.1	49 30.1	33.5	53 20.1	7.9	8 30
21 40	85 1 7.7	20.0	36 39.5	42.3	89 3 18.5	53.3	91 41 51.9	50.0	50 3.6	33.4	53 28.0	7.7	8 20
21 50	85 1 27.7	20.1	37 21.8	42.4	89 4 11.8	53.3	91 42 41.9	50.0	50 37.0	33.3	53 35.7	7.6	8 10
22 0	85 1 47.8	20.3	38 4.2	42.5	89 5 5.1	53.3	91 43 31.9	49.9	51 10.3	33.2	53 43·3	7.4	8 0
22 10	85 2 8.1	20.5	38 46.7	42.6	89 5 58.4	53.3	91 44 21.8	49.8	51 43.5	33.0	53 50.7	7.3	7 50
22 20	85 2 28.6	20.5	39 29.3	42.7	89 6 51.7	53.4	91 45 11.6	49.8	52 16.5	32.9	53 58.0	7.2	7 40
22 30	85 2 49.1		40 12 0		89 7 45.1		91 46 1.4		52 49.4		54 5.2		7 30
	IIs		Is		Os		XIs		Xs		IXs		

End of TABLE XXI.

	IIIs 85°	Diff.	IVs	Diff.	Vs 89°	Diff.	VIs	Diff.	VIIs	Diff.	VIIIs 94°	Diff.	
22° 30′	2′ 49.″1	20.″7	86° 40′ 12.″0	42.″8	7′ 45.″1	53.″3	91° 46′ 1.″4	49.″7	93° 52′ 49.″4	32.″7	54′ 5.″2	6.″9	7° 30′
22 40	3 9.8	20.9	86 40 54.8	42.9	8 38.4	53.4	91 46 51.1	49.7	93 53 22.1	32.7	54 12.1	6.8	7 20
22 50	3 30.7	21.1	86 41 37.7	43.0	9 31.8	53.4	91 47 40.8	49.6	93 53 54.8	32.5	54 18.9	6.7	7 10
23 0	3 51.8	21.2	86 42 20.7	43.1	10 25.2	53.5	91 48 30.4	49.5	93 54 27.3	32.4	54 25.6	6.5	7 0
23 10	4 13.0	21.3	86 43 3.8	43.2	11 18.7	53.4	91 49 19.9	49.5	93 54 59.7	32.3	54 32.1	6.3	6 50
23 20	4 34.3	21.4	86 43 47.0	43.3	12 12.1	53.5	91 50 9.4	49.4	93 55 32.0	32.2	54 38.4	6.2	6 40
23 30	4 55.7	21.6	86 44 30.3	43.3	13 5.6	53.4	91 50 58.8	49.3	93 56 4.2	32.0	54 44.6	6.1	6 30
23 40	5 17.3	21.8	86 45 13.6	43.5	13 59.0	53.5	91 51 48.1	49.3	93 56 36.2	31.9	54 50.7	5.9	6 20
23 50	5 39.1	21.9	86 45 57.1	43.5	14 52.5	53.5	91 52 37.4	49.2	93 57 8.1	31.8	54 56.6	5.7	6 10
24 0	6 1.0	22.0	86 46 40.6	43.7	15 46.0	53.6	91 53 26.6	49.2	93 57 39.9	31.6	55 2.3	5.6	6 0
24 10	6 23.0	22.1	86 47 24.3	43.7	16 39.6	53.5	91 54 15.8	49.1	93 58 11.5	31.5	55 7.9	5.4	5 50
24 20	6 45.1	22.3	86 48 8.0	43.9	17 33.1	53.6	91 55 4.9	49.0	93 58 43.0	31.4	55 13.3	5.2	5 40
24 30	7 7.4	22.5	86 48 51.9	43.9	18 26.7	53.6	91 55 53.9	49.0	93 59 14.4	31.3	55 18.5	5.1	5 30
24 40	7 29.9	22.6	86 49 35.8	44.0	19 20.3	53.6	91 56 42.9	48.9	93 59 45.7	31.1	55 23.6	4.9	5 20
24 50	7 52.5	22.7	86 50 19.8	44.0	20 13.9	53.6	91 57 31.8	48.9	94 0 16.8	31.0	55 28.5	4.8	5 10
25 0	8 15.2	22.9	86 51 3.8	44.2	21 7.5	53.6	91 58 20.7	48.7	94 0 47.8	30.9	55 33.3	4.7	5 0
25 10	8 38.1	23.1	86 51 48.0	44.3	22 1.1	53.7	91 59 9.4	48.7	94 1 18.7	30.8	55 38.0	4.4	4 50
25 20	9 1.2	23.1	86 52 32.3	44.4	22 54.8	53.6	91 59 58.1	48.7	94 1 49.5	30.6	55 42.4	4.4	4 40
25 30	9 24.3	23.3	86 53 16.7	44.4	23 48.4	53.7	92 0 46.8	48.6	94 2 20.1	30.5	55 46.8	4.1	4 30
25 40	9 47.6	23.5	86 54 1.1	44.5	24 42.1	53.6	92 1 35.4	48.5	94 2 50.6	30.4	55 50.9	4.0	4 20
25 50	10 11.1	23.6	86 54 45.6	44.7	25 35.7	53.7	92 2 23.9	48.4	94 3 21.0	30.2	55 54.9	3.9	4 10
26 0	10 34.7	23.7	86 55 30.3	44.7	26 29.4	53.7	92 3 12.3	48.4	94 3 51.2	30.1	55 58.8	3.7	4 0
26 10	10 58.4	23.9	86 56 15.0	44.8	27 23.1	53.7	92 4 0.7	48.3	94 4 21.3	30.0	56 2.5	3.5	3 50
26 20	11 22.3	24.0	86 56 59.8	44.9	28 16.8	53.7	92 4 49.0	48.2	94 4 51.3	29.8	56 6.0	3.4	3 40
26 30	11 46.3	24.2	86 57 44.7	44.9	29 10.5	53.7	92 5 37.2	48.2	94 5 21.1	29.7	56 9.4	3.2	3 30
26 40	12 10.5	24.3	86 58 29.6	45.1	30 4.2	53.8	92 6 25.4	48.1	94 5 50.8	29.6	56 12.6	3.0	3 20
26 50	12 34.8	24.4	86 59 14.7	45.1	30 58.0	53.7	92 7 13.5	48.0	94 6 20.4	29.5	56 15.6	2.9	3 10
27 0	12 59.2	24.6	86 59 59.8	45.3	31 51.7	53.8	92 8 1.5	48.0	94 6 49.9	29.3	56 18.5	2.8	3 0
27 10	13 23.8	24.7	87 0 45.1	45.3	32 45.5	53.7	92 8 49.5	47.9	94 7 19.2	29.2	56 21.3	2.6	2 50
27 20	13 48.5	24.8	87 1 30.4	45.4	33 39.2	53.8	92 9 37.4	47.8	94 7 48.4	29.0	56 23.9	2.4	2 40
27 30	14 13.3	25.0	87 2 15.8	45.5	34 33.0	53.7	92 10 25.2	47.7	94 8 17.4	29.0	56 26.3	2.3	2 30
27 40	14 38.3	25.1	87 3 1.3	45.6	35 26.7	53.8	92 11 12.9	47.7	94 8 46.4	28.8	56 28.6	2.1	2 20
27 50	15 3.4	25.3	87 3 46.9	45.6	36 20.5	53.8	92 12 0.6	47.6	94 9 15.2	28.6	56 30.7	2.0	2 10
28 0	15 28.7	25.4	87 4 32.5	45.7	37 14.3	53.8	92 12 48.2	47.5	94 9 43.8	28.5	56 32.7	1.8	2 0
28 10	15 54.1	25.6	87 5 18.2	45.8	38 8.1	53.8	92 13 35.7	47.4	94 10 12.3	28.4	56 34.5	1.6	1 50
28 20	16 19.7	25.7	87 6 4.0	45.8	39 1.9	53.8	92 14 23.1	47.4	94 10 40.7	28.3	56 36.1	1.5	1 40
28 30	16 45.4	25.8	87 6 49.8	45.9	39 55.7	53.8	92 15 10.5	47.3	94 11 9.0	28.1	56 37.6	1.4	1 30
28 40	17 11.2	25.9	87 7 35.7	46.0	40 49.5	53.8	92 15 57.8	47.2	94 11 37.1	28.0	56 39.0	1.2	1 20
28 50	17 37.1	26.1	87 8 21.7	46.1	41 43.3	53.8	92 16 45.0	47.2	94 12 5.1	27.9	56 40.2	1.0	1 10
29 0	18 3.2	26.3	87 9 7.8	46.2	42 37.1	53.8	92 17 32.2	47.0	94 12 33.0	27.7	56 41.2	0.8	1 0
29 10	18 29.5	26.3	87 9 54.0	46.4	43 30.9	53.8	92 18 19.2	47.0	94 13 0.7	27.6	56 42.0	0.7	0 50
29 20	18 55.8	26.5	87 10 40.4	46.5	44 24.7	53.9	92 19 6.2	46.9	94 13 28.3	27.5	56 42.7	0.6	0 40
29 30	19 22.3	26.7	87 11 26.9	46.5	45 18.6	53.8	92 19 53.1	46.9	94 13 55.8	27.3	56 43.3	0.4	0 30
29 40	19 49.0	26.8	87 12 13.4	46.5	46 12.4	53.8	92 20 40.0	46.7	94 14 23.1	27.2	56 43.7	0.2	0 20
29 50	20 15.8	26.9	87 12 59.9	46.7	47 6.2	53.8	92 21 26.7	46.7	94 14 50.3	27.0	56 43.9	0.1	0 10
30 0	20 42.7		87 13 46.6		48 0.0		92 22 13.4		94 15 17.3		56 44.0		0 0
	IIs		Is		Os		XIs		Xs		IXs		

TABLE XXII.

Equation II. of the Moon's Latitude. Arg. II. increased by the Sum of the Equations of Long.

Deg.	IIIs	Diff.	IVs	Diff.	Vs	Diff.	VIs	Diff.	VIIs	Diff.	VIIIs	Diff.	
0	0′ 13″8	0″1	1′ 24″4	4″6	4′ 36″9	8″0	9′ 0″0	9″2	13′ 23″1	7.9	16′ 35.6	4″6	30
1	0 13.9	0.2	1 29.0	4.8	4 44.9	8.1	9 9.2	9.2	13 31.0	7.8	16 40.2	4.4	29
2	0 14.1	0.4	1 33.8	4.9	4 53.0	8.1	9 18.4	9.1	13 38.8	7.8	16 44.6	4.3	28
3	0 14.5	0.6	1 38.7	5.1	5 1.1	8.2	9 27.5	9.2	13 46.6	7.6	16 48.9	4.1	27
4	0 15.1	0.7	1 43.8	5.2	5 9.3	8.3	9 36.7	9.2	13 54.2	7.6	16 53.0	3.9	26
5	0 15.8	0.9	1 49.0	5.3	5 17.6	8.4	9 45.9	9.1	14 1.8	7.5	16 56.9	3.8	25
6	0 16.7	1.0	1 54.3	5.5	5 26.0	8.4	9 55.0	9.1	14 9.3	7.4	17 0.7	3.7	24
7	0 17.7	1.2	1 59.8	5.6	5 34.4	8.5	10 4.1	9.1	14 16.7	7.3	17 4.4	3.5	23
8	0 18.9	1.4	2 5.4	5.7	5 42.9	8.5	10 13.2	9·1	14 24.0	7.2	17 7.9	3.4	22
9	0 20.3	1.5	2 11.1	5.8	5 51.4	8.6	10 22.3	9.1	14 31.2	7.0	17 11.3	3.2	21
10	0 21.8	1·7	2 16.9	6.0	6 0.0	8.7	10 31.4	9.0	14 38.2	7.0	17 14.5	3.0	20
11	0 23.5	1.8	2 22.9	6.1	6 8.7	8.7	10 40.4	9.0	14 45.2	6.9	17 17.5	2.9	19
12	0 25.3	2.0	2 29.0	6.2	6 17.4	8.8	10 49.4	9.0	14 52.1	6.8	17 20.4	2.8	18
13	0 27.3	2.1	2 35.2	6.3	6 26.2	8.8	10 58.4	8.9	14 58.9	6.6	17 23.2	2.6	17
14	0 29.4	2.3	2 41.5	6.4	6 35.0	8.8	11 7.3	8.9	15 5.5	6.6	17 25.8	2.5	16
15	0 31.7	2.5	2 47.9	6.6	6 43.8	8.9	11 16.2	8.8	15 12.1	6.4	17 28.3	2.3	15
16	0 34.2	2.6	2 54.5	6.6	6 52.7	8.9	11 25.0	8.8	15 18.5	6.3	17 30.6	2.1	14
17	0 36.8	2.8	3 1.1	6.8	7 1.6	9.0	11 33.8	8.8	15 24.8	6.2	17 32.7	2.0	13
18	0 39.6	2.9	3 7.9	6.9	7 10.6	9.0	11 42.6	8.7	15 31.0	6.1	17 34.7	1.8	12
19	0 42.5	3.0	3 14.8	7.0	7 19.6	9.0	11 51.3	8.7	15 37.1	6.0	17 36.5	1.7	11
20	0 45.5	3.2	3 21.8	7.0	7 28.6	9.1	12 0.0	8.6	15 43.1	5·8	17 38.2	1.5	10
21	0 48.7	3.4	3 28.8	7.2	7 37.7	9.1	12 8.6	8.5	15 48.9	5.7	17 39.7	1.4	9
22	0 52.1	3.5	3 36,0	7.3	7 46.8	9.1	12 17.1	8.5	15 54.6	5.6	17 41.1	1.2	8
23	0 55.6	3.7	3 43.3	7.4	7 55.9	9.1	12 25.6	8.4	16 0.2	5.5	17 42.3	1.0	7
24	0 59.3	3.8	3 50.7	7.5	8 5.0	9.1	12 34.0	8.4	16 5.7	5.3	17 43.3	0.9	6
25	1 3.1	3.9	3 58.2	7.6	8 14.1	9.2	12 42.4	8.3	16 11.0	5.2	17 44.2	0.7	5
26	1 7.0	4.1	4 5.8	7.6	8 23.3	9.2	12 50.7	8.2	16 16.2	5.1	17 44.9	0.6	4
27	1 11.1	4.3	4 13.4	7.8	8 32.5	9.1	12 58.9	8.1	16 21.3	4.9	17 45.5	0.4	3
28	1 15.4	4.4	4 21.2	7.8	8 41.6	9.2	13 7.0	8.1	16 26.2	4.8	17 45.9	0.2	2
29	1 19.8	4.6	4 29.0	7.9	8 50.8	9.2	13 15.1	8.0	16 31.0	4.6	17 46.1	0.1	1
30	1 24.4		4 36.9		9 0.0		13 23.1		16 35.6		17 46.2		0
	IIs		Is		Os		XIs		Xs		IXs		Deg.

TABLE XXIII.

Equation III of Latitude.

Arg. True Long. Moon.

	IIIs	IVs	Vs	VIs	VIIs	VIIIs	Deg.
0	16″0	14″9	12″0	8″0	4″0	1″1	30
3	16.0	14.7	11.6	7.6	3.6	0.9	27
6	16.0	14.5	11.3	7.2	3.3	0.7	24
9	15.9	14.2	10.9	6.7	3.0	0.5	21
12	15.8	13.9	10.5	6.3	2.6	0.4	18
15	15.7	13.7	10.1	5.9	2.3	0.3	15
18	15.6	13.4	9.7	5.5	2.1	0.2	12
21	15.5	13.0	9.3	5.1	1.8	0.1	9
24	15.3	12.7	8.8	4.7	1.5	0.0	6
27	15.1	12.4	8.4	4.4	1.3	0.0	3
30	14.9	12.0	8.0	4.0	1.1	0.0	0
	II	I	O	XI	X	IX	

TABLE XXIV.

Part. dec.		Part. dec.		Part. dec.		Part. dec.		Part. dec.	
003	1° 5	013	4°41′	023	8°17′	033	11°53′	043	15°29′
004	1 26	014	5 2	024	8 38	034	12 14	044	15 50
005	1 48	015	5 24	025	9 0	035	12 36	045	16 12
006	2 10	016	5 46	026	9 22	036	12 58	046	16 34
007	2 31	017	6 7	027	9 43	037	13 19	047	16 55
008	2 53	018	6 29	028	10 5	038	13 41	048	17 17
009	3 14	019	6 50	029	10 26	039	14 2	049	17 38
010	3 36	020	7 12	030	10 48	040	14 24	050	18 0
011	3 58	021	7 34	031	11 10	041	14 46	051	18 22
012	4 19	022	7 55	032	11 31	042	15 7	052	18 43
013	4 41	023	8 17	033	11 53	043	15 29	053	19 5

Table XXIV serves to turn the sum of the Equations of the Moon's Longitude into decimal parts of the circumference. This sum must be added to all the Arguments of Latitude, from V to XI.

TABLE XXV.
EQUATION OF LATITUDE.

Ar.	Arg. 20 Long.	Dif	V cor.	Dif	VI cor.	Dif	VII cor.	Dif	VIII cor.	Dif	IX cor.	Dif	X cor.	Dif	Arg
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
250	0.3	0.0	55.9	0.1	6.1	0.1	2.6	0.1	25.1	0.0	3.0	0.1	0.9	0.0	250
260	0.3	0.1	55.8	0.1	6.2	0.1	2.7	0.1	25.1	0.1	3.1	0.1	0.9	0.1	240
270	0.4	0.2	55.7	0.3	6.3	0.2	2.8	0.2	25.0	0.1	3.2	0.3	1.0	0.0	230
280	0.6	0.2	55.4	0.3	6.5	0.4	3.0	0.3	24.9	0.1	3.5	0.3	1.0	0.1	220
290	0.8	0.2	55.1	0.5	6.9	0.4	3.3	0.4	24.8	0.1	3.8	0.5	1.1	0.1	210
300	1.0	0.3	54.6	0.5	7.3	0.5	3.7	0.5	24.7	0.3	4.3	0.6	1.2	0.1	200
310	1.3	0.4	54.1	0.7	7.8	0.6	4.2	0.5	24.4	0.3	4.9	0.7	1.3	0.1	190
320	1.7	0.4	53.4	0.7	8.4	0.7	4.7	0.7	24.1	0.3	5.6	0.8	1.4	0.1	180
330	2.1	0.5	52.7	0.8	9.1	0.7	5.4	0.7	23.8	0.3	6.4	0.8	1.5	0.2	170
340	2.6	0.5	51.9	0.9	9.8	0.9	6.1	0.8	23.5	0.3	7.2	1.0	1.7	0.2	160
350	3.1	0.6	51.0	1.0	10.7	0.9	6.9	0.8	23.2	0.4	8.2	1.0	1.9	0.2	150
360	3.7	0.6	50.0	1.1	11.6	1.0	7.7	1.0	22.8	0.4	9.2	1.1	2.1	0.2	140
370	4.3	0.6	48.9	1.2	12.6	1.0	8.7	1.0	22.4	0.5	10.3	1.2	2.3	0.2	130
380	4.9	0.7	47.7	1.2	13.6	1.2	9.7	1.0	21.9	0.5	11.5	1.3	2.5	0.3	120
390	5.6	0.8	46.5	1.3	14.8	1.2	10.7	1.1	21.4	0.5	12.8	1.3	2.8	0.2	110
400	6.4	0.7	45.2	1.3	16.0	1.2	11.8	1.2	20.9	0.5	14.1	1.4	3.0	0.3	100
410	7.1	0.8	43.9	1.4	17.2	1.3	13.0	1.2	20.4	0.5	15.5	1.5	3.3	0.2	090
420	7.9	0.9	42.5	1.5	18.5	1.3	14.2	1.3	19.9	0.6	17.0	1.5	3.5	0.3	080
430	8.8	0.8	41.0	1.5	19.8	1.4	15.5	1.3	19.3	0.6	18.5	1.6	3.8	0.3	070
440	9.6	0.9	39.5	1.5	21.2	1.4	16.8	1.3	18.7	0.6	20.1	1.6	4.1	0.3	060
450	10.5	0.8	38.0	1.6	22.6	1.5	18.1	1.3	18.1	0.6	21.7	1.6	4.4	0.3	050
460	11.3	0.9	36.4	1.5	24.1	1.4	19.4	1.4	17.5	0.6	23.3	1.6	4.7	0.3	040
470	12.2	1.0	34.9	1.7	25.5	1.5	20.8	1.4	16.9	0.6	24.9	1.7	5.0	0.4	030
480	13.2	0.9	33.2	1.6	27.0	1.5	22.2	1.4	16.3	0.7	26.6	1.7	5.4	0.3	020
490	14.1	0.9	31.6	1.6	28.5	1.5	23.6	1.4	15.6	0.6	28.3	1.7	5.7	0.3	010
500	15.0	0.9	30.0	1.6	30.0	1.5	25.0	1.4	15.0	0.6	30.0	1.7	6.0	0.3	000
510	15.9	0.9	28.4	1.6	31.5	1.5	26.4	1.4	14.4	0.7	31.7	1.7	6.3	0.3	990
520	16.8	1.0	26.8	1.7	33.0	1.5	27.8	1.4	13.7	0.6	33.4	1.7	6.6	0.4	980
530	17.8	0.9	25.1	1.5	34.5	1.4	29.2	1.4	13.1	0.6	35.1	1.6	7.0	0.3	970
540	18.7	0.8	23.6	1.6	35.9	1.5	30.6	1.3	12.5	0.6	36.7	1.6	7.3	0.3	960
550	19.5	0.9	22.0	1.5	37.4	1.4	31.9	1.3	11.9	0.6	38.3	1.6	7.6	0.3	950
560	20.4	0.8	20.5	1.5	38.8	1.4	33.2	1.3	11.3	0.6	39.9	1.6	7.9	0.3	940
570	21.2	0.9	19.0	1.5	40.2	1.3	34.5	1.3	10.7	0.6	41.5	1.5	8.2	0.3	930
580	22.1	0.8	17.5	1.4	41.5	1.3	35.8	1.2	10.1	0.5	43.0	1.5	8.5	0.2	920
590	22.9	0.7	16.1	1.3	42.8	1.2	37.0	1.2	9.6	0.5	44.5	1.4	8.7	0.3	910
600	23.6	0.8	14.8	1.3	44.0	1.2	38.2	1.1	9.1	0.5	45.9	1.3	9.0	0.2	900
610	24.4	0.7	13.5	1.2	45.2	1.2	39.3	1.0	8.6	0.5	47.2	1.3	9.2	0.3	890
620	25.1	0.6	12.3	1.2	46.4	1.0	40.3	1.0	8.1	0.5	48.5	1.2	9.5	0.2	880
630	25.7	0.6	11.1	1.1	47.4	1.0	41.3	1.0	7.6	0.4	49.7	1.1	9.7	0.2	870
640	26.3	0.6	10.0	1.0	48.4	0.9	42.3	0.8	7.2	0.4	50.8	1.0	9.9	0.2	860
650	26.9	0.5	9.0	0.9	49.3	0.9	43.1	0.8	6.8	0.3	51.8	1.0	10.1	0.2	850
660	27.4	0.5	8.1	0.8	50.2	0.7	43.9	0.7	6.5	0.3	52.8	0.8	10.3	0.2	840
670	27.9	0.4	7.3	0.7	50.9	0.7	44.6	0.7	6.2	0.3	53.6	0.8	10.5	0.1	830
680	28.3	0.4	6.6	0.7	51.6	0.6	45.3	0.5	5.9	0.3	54.4	0.7	10.6	0.1	820
690	28.7	0.3	5.9	0.5	52.2	0.5	45.8	0.5	5.6	0.3	55.1	0.6	10.7	0.1	810
700	29.0	0.2	5.4	0.5	52.7	0.4	46.3	0.4	5.3	0.1	55.7	0.5	10.8	0.1	800
710	29.2	0.2	4.9	0.3	53.1	0.4	46.7	0.3	5.2	0.1	56.2	0.3	10.9	0.1	790
720	29.4	0.2	4.6	0.3	53.5	0.2	47.0	0.2	5.1	0.1	56.5	0.3	11.0	0.0	780
730	29.6	0.1	4.3	0.1	53.7	0.1	47.2	0.1	5.0	0.1	56.8	0.1	11.0	0.1	770
740	29.7	0.0	4.2	0.1	53.8	0.1	47.3	0.1	4.9	0.0	56.9	0.1	11.1	0.0	760
750	29.7		4.1		53.9		47.4		4.9		57.0		11.1		750

TABLE XXVI.

Arg	XI cor.	Arg
	″	
000	4.0	500
010	3.7	510
020	3.4	520
030	3.1	530
040	2.8	540
050	2.5	550
060	2.3	560
070	2.1	570
080	1.9	580
090	1.7	590
100	1.6	600
110	1.5	610
120	1.5	620
130	1.5	630
140	1.5	640
150	1.6	650
160	1.7	660
170	1.9	670
180	2.1	680
190	2.3	690
200	2.5	700
210	2.8	710
220	3.1	720
230	3.4	730
240	3.7	740
250	4.0	750
260	4.3	760
270	4.6	770
280	4.9	780
290	5.2	790
300	5.5	800
310	5.7	810
320	5.9	820
330	6.1	830
340	6.3	840
350	6.4	850
360	6.5	860
370	6.5	870
380	6.5	880
390	6.5	890
400	6.4	900
410	6.3	910
420	6.1	920
430	5.9	930
440	5.7	940
450	5.5	950
460	5.2	960
470	4.9	970
480	4.6	980
490	4.3	990
500	4.0	1000

TABLE XXVII.

Argu.	XII	Diff.	Argu.
		″	
250	0.7	0.0	250
260	0.7	0.1	240
270	0.8	0.2	230
280	1.0	0.2	220
290	1.2	0.3	210
300	1.5	0.3	200
310	1.8	0.4	190
320	2.2	0.5	180
330	2.7	0.5	170
340	3.2	0.6	160
350	3.8	0.6	150
360	4.4	0.7	140
370	5.1	0.7	130
380	5.8	0.8	120
390	6.6	0.8	110
400	7.4	0.9	100
410	8.3	0.8	090
420	9.1	1.0	080
430	10.1	0.9	070
440	11.0	1.0	060
450	12.0	0.9	050
460	12.9	1.0	040
470	13.9	1.1	030
480	15.0	1.0	020
490	16.0	1.0	010
500	17.0	1.0	000
510	18.0	1.0	990
520	19.0	1.1	980
530	20.1	1.0	970
540	21.1	0.9	960
550	22.0	1.0	950
560	23.0	0.9	940
570	23.9	1.0	930
580	24.9	0.8	920
590	25.7	0.9	910
600	26.6	0.8	900
610	27.4	0.8	890
620	28.2	0.7	880
630	28.9	0.7	870
640	29.6	0.6	860
650	30.2	0.6	850
660	30.8	0.5	840
670	31.3	0.5	830
680	31.8	0.4	820
690	32.2	0.3	810
700	32.5	0.3	800
710	32.8	0.2	790
720	33.0	0.2	780
730	33.2	0.1	770
740	33.3	0.0	760
750	33.3		750

TABLE XXVIII.

EQUATIONS OF THE MOON'S PARALLAX.

The Arguments are the same as those for the Moon's Longitude.

Ar	Equ. 1	Equ. 2	Equ. 4	Equ. 5	Equ. 6	Equ. 8	Equ. 9	Equ. 12	Equ. 13	Arg	Ar	Equ. 1	Equ. 2	Equ. 4	Equ. 5	Equ. 6	Equ 8	Equ 9	Equ 12	Equ 13	Arg
	″	″	″	″	″	″	″	″	″			″	″	″	″	″	″	″	″	″	
00	0.0	1.6	0.6	1.6	1.9	0.0	3.6	1.4	2.0	100	27	0.5	0.7	0.3	0.7	1.0	0.7	0.1	0.6	0.9	73
03	0.0	1.6	0.6	1.6	1.9	0.0	3.5	1.4	2.0	97	30	0.5	0.6	0.2	0.6	0.9	0.8	0.4	0.5	0.7	70
06	0.0	1.5	0.6	1.5	1.8	0.0	3.1	1.4	1.9	94	33	0.6	0.4	0.2	0.4	0.7	0.9	0.8	0.4	0.5	67
09	0.1	1.5	0.6	1.5	1.8	0.1	2.6	1.3	1.8	91	36	0.7	0.3	0.1	0.3	0.6	1.0	1.5	0.3	0.4	64
12	0.1	1.4	0.5	1.4	1.7	0.2	1.9	1.2	1.7	88	39	0.7	0.2	0.1	0.2	0.5	1.1	2.1	0.2	0.2	61
15	0.1	1.3	0.5	1.3	1.6	0.2	1.3	1.1	1.6	85	42	0.8	0.1	0.0	0.1	0.4	1.1	2.8	0.1	0.1	58
18	0.2	1.1	0.4	1.1	1.4	0.3	0.7	1.0	1.4	82	45	0.8	0.0	0.0	0.0	0.3	1.2	3.2	0.0	0.0	55
21	0.3	1.0	0.4	1.0	1.3	0.5	0.2	0.9	1.2	79	48	0.8	0.0	0.0	0.0	0.3	1.2	3.5	0.0	0.0	52
24	0.4	0.9	0.3	0.9	1.2	0.6	0.0	0.7	1.0	76	50	0.8	0.0	0.0	0.0	0.3	1.2	3.6	0.0	0.0	50
27	0.5	0.7	0.3	0.7	1.0	0.7	0.1	0.6	0.9	73											

TABLE XXIX.

ARGUMENT, THAT OF THE EVECTION.

Deg.	Os	Is	IIs	IIIs	IVs	Vs		Deg.	Os	Is	IIs	IIIs	IVs	Vs	
	′ ″	′ ″	′ ″	″	″	″			″	″	″	″	″	″	
0	1 20.8	1 15.6	1 1.5	42.6	24.1	10.8	30	0	55.6	42.3	16.0	3.7	17.6	44.0	30
1	1 20.8	1 15.2	1 0.9	41.9	23.6	10.5	29	1	55.6	41.5	15.3	3.8	18.5	44.8	29
2	1 20.8	1 14.9	1 0.3	41.3	23.0	10.2	28	2	55.5	40.7	14.5	3.8	19.3	45.6	28
3	1 20.7	1 14.5	59.7	40.6	22.5	9.9	27	3	55.5	39.8	13.8	3.9	20.1	46.3	27
4	1 20.7	1 14.2	59.2	40.0	21.9	9.6	26	4	55.3	39.0	13.1	4.1	21.0	47.0	26
5	1 20.6	1 13.8	58.6	39.4	21.4	9.4	25	5	55.2	38.1	12.4	4.3	21.9	47.7	25
6	1 20.6	1 13.4	57.9	38.7	20.9	9.1	24	6	55.0	37.2	11.7	4.5	22.7	48.4	24
7	1 20.5	1 13.0	57.3	38.1	20.4	8.8	23	7	54.8	36.3	11.1	4.7	23.6	49.1	23
8	1 20.4	1 12.6	56.7	37.4	19.9	8.6	22	8	54.6	35.5	10.4	5.0	24.5	49.7	22
9	1 20.3	1 12.2	56.1	36.8	19.4	8.4	21	9	54.3	34.6	9.8	5.3	25.4	50.3	21
10	1 20.2	1 11.7	55.5	36.1	18.9	8.2	20	10	54.0	33.7	9.2	5.6	26.3	50.9	20
11	1 20.1	1 11.3	54.9	35.5	18.4	8.0	19	11	53.7	32.7	8.7	6.0	27.2	51.5	19
12	1 19.9	1 10.8	54.2	34.9	17.9	7.8	18	12	53.3	31.8	8.2	6.3	28.2	52.1	18
13	1 19.8	1 10.4	53.6	34.2	17.5	7.6	17	13	52.9	30.9	7.7	6.8	29.1	52.6	17
14	1 19.6	1 9.9	53.0	33.6	17.0	7.4	16	14	52.5	30.0	7.2	7.2	30.0	53.1	16
15	1 19.5	1 9.4	52.3	33.0	16.6	7.2	15	15	52.0	29.1	6.7	7.7	30.9	53.5	15
16	1 19.3	1 9.0	51.7	32.4	16.1	7.1	14	16	51.5	28.2	6.3	8.2	31.8	54.0	14
17	1 19.1	1 8.5	51.1	31.7	15.7	6.9	13	17	51.0	27.2	5.9	8.7	32.8	54.4	13
18	1 18.9	1 8.0	50.4	31.1	15.2	6.8	12	18	50.5	26.3	5.6	9.3	33.7	54.8	12
19	1 18.7	1 7.5	49.8	30.5	14.8	6.7	11	19	49.9	25.4	5.3	9.8	34.6	55.1	11
20	1 18.4	1 7.0	49.1	29.9	14.4	6.5	10	20	49.4	24.5	5.0	10.5	35.5	55.4	10
21	1 18.2	1 6.5	48.5	29.3	14.0	6.4	9	21	48.8	23.6	4.7	11.1	36.4	55.7	9
22	1 18.0	1 5.9	47.8	28.7	13.6	6.3	8	22	48.1	22.7	4.5	11.7	37.3	56.0	8
23	1 17.7	1 5.4	47.2	28.1	13.2	6.3	7	23	47.4	21.9	4.3	12.4	38.2	56.2	7
24	1 17.4	1 4.8	46.5	27.5	12.9	6.2	6	24	46.8	21.0	4.1	13.1	39.0	56.4	6
25	1 17.1	1 4.3	45.9	26.9	12.5	6.1	5	25	46.1	20.1	3.9	13.8	39.9	56.6	5
26	1 16.9	1 3.8	45.2	26.3	12.1	6.1	4	26	45.4	19.3	3.8	14.5	40.8	56.8	4
27	1 16.6	1 3.2	44.6	25.8	11.8	6.1	3	27	44.6	18.5	3.7	15.3	41.6	56.9	3
28	1 16.2	1 2.6	43.9	25.2	11.5	6.0	2	28	43.9	17.6	3.7	16.1	42.4	56.9	2
29	1 15.9	1 2.1	43.3	24.7	11.1	6.0	1	29	43.1	16.8	3.7	16.8	43.2	57.0	1
30	1 15.6	1 1.5	42.6	24.1	10.8	6.0	0	30	42.3	16.0	3.7	17.6	44.0	57.0	0
	XIs	Xs	IXs	VIIIs	VIIs	VIs	Deg.		XIs	Xs	IXs	VIIIs	VIIs	VIs	Deg.

TABLE XXX.

ARGUMENT, THAT OF THE VARIATION.

TABLE XXXI.

EQUATORIAL PARALLAX OF THE MOON. *Argum.* Anomaly ☾.

Deg.	Os	Diff.	Is	Diff.	IIs	Diff.	IIIs	Diff	IVs	Diff.	Vs	Diff.	
0	58′ 57″7	0″0	58′ 27″0	2″0	57′ 7″9	3″1	55′ 29″8	3″2	54′ 1″9	2″5	53′ 3″2	1′4	30
1	58 57.7	0.1	58 25.0	2.0	57 4.8	3.2	55 26.6	3.2	53 59.4	2.5	53 1.8	1.3	29
2	58 57.6	0.2	58 23.0	2.1	57 1.6	3.2	55 23.4	3.2	53 56.9	2.4	53 0.5	1.2	28
3	58 57.4	0.3	58 20.9	2.2	56 58.4	3.2	55 20.2	3.2	53 54.5	2.4	52 59.3	1.2	27
4	58 57.1	0.3	58 18.7	2.2	56 55.2	3.2	55 17.0	3.2	53 52.1	2.4	52 58.1	1.1	26
5	58 56.8	0.4	58 16.5	2.2	56 52.0	3.2	55 13.8	3.2	53 49.7	2.3	52 57.0	1.2	25
6	58 56.4	0.4	58 14.3	2.3	56 48.8	3.3	55 10.6	3.1	53 47.4	2.3	52 55 8	1.0	24
7	58 56.0	0.6	58 12.0	2.4	56 45.5	3.2	55 7.5	3.1	53 45.1	2.2	52 54.8	1.0	23
8	58 55.4	0.6	58 9.6	2.4	56 42.3	3.3	55 4.4	3.1	53 42.9	2.3	52 53.8	1.0	22
9	58 54.8	0.6	58 7.2	2.4	56 39.0	3.3	55 1.3	3.1	53 40.6	2.1	52 52.8	0.9	21
10	58 54.2	0.8	58 4.8	2.5	56 35.7	3.3	54 58.2	3.1	53 38.5	2.2	52 51.9	0.9	20
11	58 53.4	0.8	58 2.3	2.5	56 32.4	3.3	54 55.1	3.0	53 36.3	2.1	52 51.0	0.9	19
12	58 52.6	0.8	57 59.8	2.6	56 29.1	3.3	54 52.1	3.0	53 34.2	2.1	52 50.1	0.8	18
13	58 51.8	1.0	57 57.2	2.6	56 25.8	3.3	54 49.1	3.0	53 32.1	2.0	52 49.3	0.7	17
14	58 50.8	1.0	57 54.6	2.7	56 22.5	3.3	54 46.1	3.0	53 30.1	2.0	52 48.6	0.7	16
15	58 49.8	1.1	57 51.9	2.7	56 19.2	3.3	54 43.1	2.9	53 28.1	1.9	52 47.9	0.7	15
16	58 48.7	1.1	57 49.2	2 8	56 15.9	3.3	54 40.2	2.9	53 26.2	1.9	52 47.2	0.6	14
17	58 47.6	1.2	57 46.4	2.7	56 12.6	3.3	54 37.3	2.9	53 24.3	1.9	52 46.6	0.6	13
18	58 46.4	1.3	57 43.7	2.9	56 9.3	3.3	54 34.4	2.9	53 22.4	1.8	52 46.0	0.5	12
19	58 45.1	1.3	57 40.8	2.8	56 6.0	3.3	54 31.5	2.8	53 20.6	1.8	52 45.5	0.5	11
20	58 43.8	1 4	57 38.0	2.9	56 2.7	3.4	54 28.7	2.8	53 18.8	1.8	52 45.0	0.4	10
21	58 42.4	1.5	57 35.1	2.9	55 59.3	3.3	54 25.9	2.8	53 17.0	1.7	52 44.6	0.4	9
22	58 40.9	1.5	57 32.2	2.9	55 56.0	3.3	54 23.1	2.8	53 15.3	1.6	52 44.2	0.4	8
23	58 39.4	1.6	57 29.3	3.0	55 52.7	3.3	54 20.3	2.7	53 13.7	1.7	52 43.8	0.3	7
24	58 37.8	1.6	57 26.3	3.0	55 49.4	3.3	54 17.6	2.7	53 12.0	1.6	52 43.5	0.2	6
25	58 36.2	1.8	57 23.3	3.0	55 46.1	3.3	54 14.9	2.7	53 10.4	1.5	52 43.3	0.2	5
26	58 34.4	1.7	57 20.2	3.0	55 42.8	3.2	54 12.2	2.6	53 8.9	1.5	52 43.1	0.2	4
27	58 32.7	1.8	57 17.2	3.1	55 39.6	3.2	54 9.6	2.6	53 7.4	1.5	52 42.9	0.1	3
28	58 30.9	1.9	57 14.1	3.1	55 36.4	3.3	54 7.0	2.6	53 5.9	1.4	52 42.8	0.1	2
29	58 29.0	2.0	57 11.0	3.1	55 33.1	3.3	54 4.4	2.6	53 4.5	1.3	52 42.7	0 0	1
30	58 27.0		57 7.9		55 29.8		54 1.9	2.5	53 3.2		52 42.7		0
	XIs		Xs		IXs		VIIIs		VIIs		VIs		Deg.

TABLE XXXII.

SEMI-DIAMETER OF THE MOON. *Argum.* The Equatorial Parallax.

Equatorial Parall.	Semi-diameter.	Equatorial Parall.	Semi-diameter.	Equatorial Parall.	Semi-diameter.	Equato. Parall.	Semi-diameter.	Secon.	Proport. parts.
53′ 0″	14′ 26″55	55′ 20″	15′ 4″70	57′ 40″	15′ 42″85	60′ 0″	16′ 21″00	1	0″27
53 10	14 29.27	55 30	15 7.42	57 50	15 45.57	60 10	16 23.72	2	0.54
53 20	14 32.00	55 40	15 10.15	58 0	15 48.30	60 20	16 26.45	3	0.82
53 30	14 34.72	55 50	15 12.87	58 10	15 51.02	60 30	16 29.17	4	1.09
53 40	14 37.45	56 0	15 15.60	58 20	15 53.75	60 40	16 31.90	5	1.36
53 50	14 40.17	56 10	15 18.32	58 30	15 56.47	60 50	16 34.62	6	1.63
54 0	14 42.90	56 20	15 21.05	58 40	15 59.20	61 0	16 37.35	7	1.91
54 10	14 45.62	56 30	15 23.77	58 50	16 1.92	61 10	16 40.07	8	2.18
54 20	14 48.35	56 40	15 26.50	59 0	16 4.65	61 20	16 42.80	9	2.45
54 30	14 51.07	56 50	15 29.22	59 10	16 7.37	61 30	16 45.52	10	2.72
54 40	14 53.80	57 0	15 31.95	59 20	16 10.10	61 40	16 48.25	log. $\left(\frac{\text{semi-diam.}}{\text{parall.}}\right)$	
54 50	14 56.52	57 10	15 34.67	59 30	16 12.82	61 50	16 50.97		
55 0	14 59.25	57 20	15 37.40	59 40	16 15.55	62 0	16 53.70	= 9.4353665.	
55 10	15 1.97	57 30	15 40.12	59 50	16 18.27	62 10	16 56.42		

TABLE XXXIII.

Logarithms of the Radii of the Earth for $\frac{1}{300}$ and $\frac{1}{330}$ of Displacement.

Lat.	Ratio of the Axes.				Lat.	Ratio of the Axes.			
	299 : 300	Diff.	329 : 330	Diff.		299 : 300	Diff.	329 : 330	Diff.
0	0.0000000		0.0000000		45	9.9992786		9.9993440	
		5		5			253		230
1	9.9999995		9.9999995		46	9.9992533		9.9993210	
		13		12			253		230
2	9.9999982		9.9999983		47	9.9992280		9.9992980	
		22		19			252		229
3	9.9999960		9.9999964		48	9.9992028		9.9992751	
		30		28			252		229
4	9.9999930		9.9999936		49	9.9991776		9.9992522	
		40		36			251		227
5	9.9999890		9.9999900		50	9.9991525		9.9992295	
		47		43			248		226
6	9.9999843		9.9999857		51	9.9991277		9.9992069	
		57		51			247		225
7	9.9999786		9.9999806		52	9.9991030		9.9991844	
		65		59			245		223
8	9.9999721		9.9999747		53	9.9990785		9.9991621	
		73		67			243		220
9	9.9999648		9.9999680		54	9.9990542		9.9991401	
		82		74			240		218
10	9.9999566		9.9999606		55	9.9990302		9.9991183	
		89		82			237		216
11	9.9999477		9.9999524		56	9.9990065		9.9990967	
		98		89			234		212
12	9.9999379		9.9999435		57	9.9989831		9.9990755	
		106		97			231		210
13	9.9999273		9.9999338		58	9.9989600		9.9990545	
		115		103			226		205
14	9.9999158		9.9999235		59	9.9989374		9.9990340	
		121		111			223		202
15	9.9999037		9.9999124		60	9.9989151		9.9990138	
		129		117			219		198
16	9.9998908		9.9999007		61	9.9988932		9.9989940	
		137		125			212		195
17	9.9998771		9.9998882		62	9.9988720		9.9989745	
		144		131			208		189
18	9.9998627		9.9998751		63	9.9988512		9.9989556	
		151		137			204		185
19	9.9998476		9.9998614		64	9.9988308		9.9989371	
		158		144			197		179
20	9.9998318		9.9998470		65	9.9988111		9.9989192	
		165		150			193		175
21	9.9998153		9.9998320		66	9.9987918		9.9989017	
		170		156			186		170
22	9.9997983		9.9998164		67	9.9987732		9.9988847	
		178		161			180		163
23	9.9997805		9.9998003		68	9.9987552		9.9988684	
		184		167			174		158
24	9.9997621		9.9997836		69	9.9987378		9.9988526	
		190		173			168		153
25	9.9997431		9.9997663		70	9.9987210		9.9988373	
		195		178			160		145
26	9.9997236		9.9997485		71	9.9987050		9.9988228	
		201		182			154		139
27	9.9997035		9.9997303		72	9.9986896		9.9988089	
		206		188			146		133
28	9.9996829		9.9997115		73	9.9986750		9.9987956	
		211		192			139		127
29	9.9996618		9.9996923		74	9.9986611		9.9987829	
		216		196			132		119
30	9.9996402		9.9996727		75	9.9986479		9.9987710	
		221		200			123		112
31	9.9996181		9.9996527		76	9.9986356		9.9987598	
		224		204			116		105
32	9.9995957		9.9996323		77	9.9986240		9.9987493	
		229		208			109		98
33	9.9995728		9.9996115		78	9.9986131		9.9987395	
		232		211			100		92
34	9.9995496		9.9995904		79	9.9986031		9.9987303	
		235		214			91		83
35	9.9995261		9.9995690		80	9.9985940		9.9987220	
		238		217			83		75
36	9.9995023		9.9995473		81	9.9985857		9.9987145	
		242		219			75		68
37	9.9994781		9.9995254		82	9.9985782		9.9987077	
		244		222			66		60
38	9.9994537		9.9995032		83	9.9985716		9.9987017	
		246		223			57		52
39	9.9994291		9.9994809		84	9.9985659		9.9986965	
		247		225			49		45
40	9.9994044		9.9994584		85	9.9985610		9.9986920	
		250		227			40		36
41	9.9993794		9.9994357		86	9.9985570		9.9986884	
		251		228			31		28
42	9.9993543		9.9994129		87	9.9985539		9.9986856	
		252		229			22		19
43	9.9993291		9.9993900		88	9.9985517		9.9986837	
		253		230			13		12
44	9.9993038		9.9993670		89	9.9985504		9.9986825	
		252		230			5		5
45	9.9992786		9.9993440		90	9.9985499		9.9986620	

TABLE XXXIV.

ANGLES OF THE VERTICAL WITH THE RADIUS.

Lat.	Ratio of the Axes. 299 : 300	Diff. +	329 : 330	Diff. +	Lat.	Ratio of the Axes. 299 : 300	Diff. −	329 : 330	Diff. −
0	0′ 0″0	24.0	0′ 0″0	21.8	45	11′ 28″7	0.3	10′ 26″0	0.3
1	0 24.0	23.9	0 21.8	21.7	46	11 28.4	1.1	10 25.7	1.1
2	0 47.9	23.9	0 43.5	21.7	47	11 27.3	2.1	10 24.6	1.9
3	1 11.8	23.7	1 5.2	21.7	48	11 25.2	2.9	10 22.7	2.6
4	1 35.5	23.7	1 26.9	21.5	49	11 22.3	3.7	10 20.1	3.3
5	1 59.2	23.5	1 48.4	21.4	50	11 18.6	4.5	10 16.8	4.1
6	2 22.7	23.4	2 9.8	21.2	51	11 14.1	5.3	10 12.7	4.9
7	2 46.1	23.1	2 31.0	21.0	52	11 8.8	6.2	10 7.8	5.6
8	3 9.2	22.9	2 52.0	20.9	53	11 2.6	6.9	10 2.2	6.3
9	3 32·1	22.7	3 12.9	20.6	54	10 55.7	7.8	9 55.9	7.1
10	3 54.8	22.4	3 33.5	20.3	55	10 47.9	8.5	9 48.8	7.8
11	4 17.2	22.1	3 53.8	20.1	56	10 39.4	9.4	9 41.0	8.4
12	4 39.3	21.7	4 13 9	19.8	57	10 30.0	10.1	9 32.6	9.2
13	5 1.0	21.4	4 33.7	19.4	58	10 19.9	10.9	9 23.4	10.0
14	5 22.4	21.0	4 53.1	19.1	59	10 9.0	11.6	9 13.4	10.6
15	5 43.4	20.5	5 12.2	18.7	60	9 57.4	12.3	9 2.8	11.2
16	6 3.9	20.2	5 30.9	18.3	61	9 45.1	13.1	8 51.6	11.8
17	6 24.1	19.6	5 49.2	17.8	62	9 32.0	13.7	8 39.8	12.5
18	6 43.7	19.2	6 7.0	17.5	63	9 18.3	14.5	8 27.3	13.1
19	7 2.9	18.7	6 24.5	16.9	64	9 3.8	15.1	8 14.2	13.8
20	7 21.6	18.1	6 41.4	16.5	65	8 48.7	15.8	8 0.4	14.3
21	7 39.7	17.6	6 57.9	16.0	66	8 32.9	16.3	7 46.1	14.9
22	7 57.3	16.9	7 13.9	15.4	67	8 16.6	17.0	7 31.2	15.4
23	8 14·2	16.5	7 29.3	14.9	68	7 59.6	17.6	7 15.8	16.0
24	8 30.7	15.8	7 44.2	14.4	69	7 42.0	18.2	6 59.8	16.5
25	8 46.5	15.1	7 58.6	13.8	70	7 23.8	18.7	6 43.3	17.0
26	9 1.6	14.5	8 12.4	13.1	71	7 5.1	19.2	6 26.3	17.4
27	9 16.1	13.8	8 25.5	12.6	72	6 45.9	19.7	6 8.9	18.0
28	9 29.9	13.1	8 38.1	11.9	73	6 26.2	20.2	5 50.9	18.3
29	9 43.0	12.4	8 50.0	11.3	74	6 6.0	20.6	5 32.6	18.8
30	9 55.4	11.8	9 1.3	10.6	75	5 45.4	21.1	5 13.8	19.2
31	10 7.2	10.9	9 11.9	10.0	76	5 24.3	21.5	4 54.6	19.5
32	10 18.1	10.2	9 21.9	9.2	77	5 2.8	21.8	4 35.1	19.8
33	10 28.3	9.5	9 31.1	8.6	78	4 41.0	22.2	4 15.3	20.1
34	10 37.8	8.6	9 39.7	7.9	79	4 18.8	22.5	3 55.2	20.5
35	10 46.4	7.9	9 47.6	7.2	80	3 56.3	22.8	3 34.7	20.7
36	10 54.3	7.1	9 54.8	6.4	81	3 33.5	23.1	3 14.0	21.0
37	11 1.4	6.3	10 1.2	5.7	82	3 10.4	23.2	2 53·0	21.1
38	11 7.7	5.5	10 6.9	5.0	83	2 47.2	23.5	2 31.9	21.4
39	11 13.2	4.7	10 11.9	4.2	84	2 23.7	23.7	2 10.5	21.5
40	11 17.9	3.8	10 16.1	3.5	85	2 0.0	23.8	1 49.0	21.6
41	11 21.7	3.0	10 19.6	2.8	86	1 36.2	23.9	1 27.4	21.8
42	11 24.7	2.2	10 22.4	1.9	87	1 12.3	24.1	1 5.6	21.8
43	11 26.9	1.3	10 24.3	1.2	88	0 48.2	24.1	0 43.8	21.9
44	11 28.2	0.5	10 25.5	0.5	89	0 24.1	24.1	0 21.9	21.9
45	11 28.7		10 26.0		90	0 0.0		0 0.0	

TABLE XXXV.

Augmentation ☾'s Semi-Diameter.

App. alt.	*Argum.* ☾'s Horizontal Semi-Diameter.					
	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″
0	0″0	0″0	0″0	0″0	0″0	0.0
3	0.7	0.8	0.8	0.9	0.9	1.0
6	1.4	1.5	1.6	1.7	1.8	1.9
9	2.1	2.3	2.4	2.6	2.7	2.9
12	2.8	3.0	3.2	3.4	3.6	3.9
15	3.5	3.7	4.0	4.3	4.5	4.8
18	4.2	4.5	4.8	5.1	5.4	5.7
21	4.8	5.2	5.5	5.9	6.3	6.6
24	5.5	5.9	6.3	6.7	7.1	7.5
27	6.1	6.6	7.0	7·5	7.9	8.4
30	6.8	7.2	7.7	8.2	8.7	9.3
33	7.4	7.9	8.4	9.0	9.5	10.1
36	7.9	8.5	9.1	9.7	10.3	10.9
39	8.5	9.1	9.7	10.4	11.0	11.7
42	9.0	9.7	10.3	11.0	11.7	12.4
45	9.6	10.2	10.9	11.7	12.4	13.2
48	10.1	10.8	11.5	12.3	13.0	13.8
51	10.5	11.3	12.0	12.8	13.6	14.5
54	11.0	11.7	12.5	13.3	14.2	15.1
57	11.4	12.2	13.0	13.8	14.7	15.6
60	11.7	12.6	13.4	14.3	15.2	16.1
63	12.1	12.9	13.8	14.7	15.6	16.6
66	12.4	13.2	14.1	15.1	16.0	17.0
69	12.6	13.5	14.5	15.4	16.4	17.4
72	12.9	13.8	14.7	15.7	16.7	17.7
75	13.1	14.0	15.0	16.0	17.0	18.0
78	13.2	14.2	15.2	16.2	17.2	18.2
81	13.4	14.3	15.3	16.3	17.4	18.4
84	13.5	14.4	15.4	16.4	17.5	18.6
87	13.5	14.5	15.5	16.5	17.5	18.6
90	13.5	14.5	15.5	16.5	17.6	18.7

The figure of the Disc of the Sun or Moon when near the horizon appears elliptical, by means of the refraction; so that the vertical diameter is less than the horizontal one by the difference of the refractions of the upper and lower limbs of either object. In altitudes above 45° the effect is less than $\frac{1}{4}$ of a second in the semidiameter of the Sun or Moon, and hence it is too small to be attended to in this case; but in small altitudes the effect is sensible; and it is greatest upon the vertical semidiameter, but nothing upon the horizontal one. And if the difference between the refraction for the centre and that for the limb, be multiplied by the natural cosine of the inclination of the diameter and vertical line, the product is the diminution of the semidiameter answering to the given altitude and inclination.

TABLE XXXVI.

To find the enlightened part of the Diameter of the Moon or Venus; supposing the Diameter divided into 200 equal Parts.

For the Moon. *Argum.* 1.—Diff. Long. ☉ and ☾. *Argum.* 2.—Dist. between ☉ and ☾. *Argum.* 3.—The angle contained between two lines drawn from the ☾ to the Sun and Earth.

D	0*s*	1*s*	2*s*	3*s*	4*s*	5*s*	D
0	0.0	13.4	50.0	100.0	150.0	186.6	30
1	0.0	14.3	51.5	101.8	151.5	187.5	29
2	0.1	15.2	53.1	103.5	153.0	188.3	28
3	0.1	16.1	54.6	105.2	154.5	188.1	27
4	0.2	17.1	56.2	106.9	155.9	189.9	26
5	0.4	18.1	57.7	108.7	157.4	190.6	25
6	0.5	19.1	59.3	110.5	158.8	191.4	24
7	0.7	20.1	60.9	112.2	160.2	192.1	23
8	1.0	21.2	62.5	113.9	161.6	192.7	22
9	1.2	22.3	64.2	115.6	162.9	193.4	21
10	1.5	23.4	65.8	117.4	164.3	194.0	20
11	1.8	24.5	67.4	119.1	165.6	194.6	19
12	2.2	25.7	69.1	120.8	166.9	195.1	18
13	2.6	26.9	70.8	122.5	168.2	195.6	17
14	3.0	28.1	72.4	124.2	169.5	196.1	16
15	3.4	29.3	74.1	125.9	170.7	196.6	15
16	3.9	30.5	75.8	127.6	171.9	197.0	14
17	4.4	31.8	77.5	129.2	173.1	197.4	13
18	4.9	33.1	79.2	130.9	174.3	197.8	12
19	5.4	34.4	80.9	132.6	175.5	198.2	11
20	6.0	35.7	82.6	134.2	176.6	198.5	10
21	6.6	37.1	84.4	135.8	177.7	198.8	9
22	7.3	38.4	86.1	137.5	178.8	199.0	8
23	7.9	39.8	87.8	139.1	179.9	199.3	7
24	8.6	41.2	89.5	140.7	180.9	199.5	6
25	9.4	42.6	91.3	142.3	181.9	199.6	5
26	10.1	44.1	93.0	143.8	182.9	199.8	4
27	11.0	45.5	94.8	145.4	183.9	199.9	3
28	11.7	47.0	96.5	146.9	184.8	199.9	2
29	12.5	48.5	98.2	148.5	185.7	200.0	1
30	13.4	50.0	100.0	150.0	186.6	200.0	0
	5*s*	4*s*	3*s*	2*s*	1*s*	0*s*	

For Venus. *Argument.*—The angle formed at Venus by two lines drawn from Venus to the Sun and Earth.

The first arg. for the Moon is not strictly true, because it supposes the diff of Long. to be the same as the dist.; and again, because it considers the lines drawn from the Sun to the Earth and Moon to be parallel to each o her. The second arg. is very near the truth; but supposes the rays of the Sun falling on the radius vector of the Moon to move in lines parallel to each other. The third arg. and also that for Venus, are strictly true; and the signs are to be looked for at the bottom of the table, and the degrees in the right hand column; the other two arguments must be taken at the top, and down the left hand column, and the number found in the angle of meeting is the breadth of the part illuminated.

The table is constructed by taking the natural versed sines of the arcs at the top of the table to the radius 100.

TABLE XXXVII.

Equation of Second Difference. *Argument.*—Second Difference.

Whole Interval, 12 Hours. Time after Noon or Midn.		Whole Interval, 60 Minutes, or 1 Degree. *Argument.*		0 Minute.						1 Minute.						2 Minutes.					
h. m.	*h. m.*	′	″	′	″	0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″
0 0	12 0	0	0	60	0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0
0 10	11 50	0	50	59	10	0.0	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2
0 20	11 40	1	40	58	20	0.0	0.1	0.3	0.4	0.5	0.7	0.8	0.9	1.1	1.2	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3
0 30	11 30	2	30	57	30	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
0 40	11 20	3	20	56	40	0.0	0.3	0.5	0.8	1.0	1.3	1.6	1.8	2.1	2.4	2.6	2.9	3.1	3.4	3.7	3.9	4.2	4.5
0 50	11 10	4	10	55	50	0.0	0.3	0.6	1.0	1.3	1.6	1.9	2.3	2.6	2.9	3.2	3.6	3.9	4.2	4.5	4.8	5.2	5.5
1 0	11 0	5	0	55	0	0.0	0.4	0.8	1.1	1.5	1.9	2.3	2.7	3.1	3.4	3.8	4.2	4.6	5.0	5.3	5.7	6.1	6.5
1 10	10 50	5	50	54	10	0.0	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	3.9	4.4	4.8	5.3	5.7	6.1	6.6	7.0	7.5
1 20	10 40	6	40	53	20	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.9	8.4
1 30	10 30	7	30	52	30	0.0	0.5	1.1	1.6	2.2	2.7	3.3	3.8	4.4	4.9	5.5	6.0	6.6	7.1	7.7	8.2	8.8	9.3
1 40	10 20	8	20	51	40	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0	6.6	7.2	7.8	8.4	9.0	9.6	10.2
1 50	10 10	9	10	50	50	0.0	0.6	1.3	1.9	2.6	3.2	3.9	4.5	5.2	5.8	6.5	7.1	7.8	8.4	9.1	9.7	10.4	11.0
2 0	10 0	10	0	50	0	0.0	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	6.9	7.6	8.3	9.0	9.7	10.4	11.1	11.8
2 10	9 50	10	50	49	10	0.0	0.7	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.7	7.4	8.1	8.9	9.6	10.4	11.1	11.8	12.6
2 20	9 40	11	40	48	20	0.0	0.8	1.6	2.3	3.1	3.9	4.7	5.5	6.3	7.0	7.8	8.6	9.4	10.2	11.0	11.7	12.5	13.3
2 30	9 30	12	30	47	30	0.0	0.8	1.6	2.5	3.3	4.1	4.9	5.8	6.6	7.4	8.2	9.1	9.9	10.7	11.5	12.4	13.2	14.0
2 40	9 20	13	20	46	40	0.0	0.9	1.7	2.6	3.5	4.3	5.2	6.0	6.9	7.8	8.6	9.5	10.4	11.2	12.1	13.0	13.8	14.7
2 50	9 10	14	10	45	50	0.0	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8	11.7	12.6	13.5	14.4	15.3
3 0	9 0	15	0	45	0	0.0	0.9	1.9	2.8	3.8	4.7	5.6	6.6	7.5	8.4	9.4	10.3	11.3	12.2	13.1	14.1	15.0	15.9
3 10	8 50	15	50	44	10	0.0	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.7	9.7	10.7	11.7	12.6	13.6	14.6	15.5	16.5
3 20	8 40	16	40	43	20	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.1
3 30	8 30	17	30	42	30	0.0	1.0	2.1	3.1	4.1	5.2	6.2	7.2	8.3	9.3	10.3	11.4	12.4	13.4	14.5	15.5	16.5	17.6
3 40	8 20	18	20	41	40	0.0	1.1	2.1	3.2	4.2	5.3	6.4	7.4	8.5	9.5	10.6	11.7	12.7	13.8	14.9	15.9	17.0	18.0
3 50	8 10	19	10	40	50	0.0	1.1	2.2	3.3	4.3	5.4	6.5	7.6	8.7	9.8	10.9	12.0	13.0	14.1	15.2	16.3	17.4	18.5
4 0	8 0	20	0	40	0	0.0	1.1	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.0	11.1	12.2	13.3	14.4	15.6	16.7	17.8	18.9
4 10	7 50	20	50	39	10	0.0	1.1	2.3	3.4	4.5	5.7	6.8	7.9	9.1	10.2	11.3	12.5	13.6	14.7	15.9	17.0	18.1	19.3
4 20	7 40	21	40	38	20	0.0	1.2	2.3	3.5	4.6	5.8	6.9	8.1	9.2	10.4	11.5	12.7	13.8	15.0	16.1	17.3	18.5	19.6
4 30	7 30	22	30	37	30	0.0	1.2	2.3	3.5	4.7	5.9	7.0	8.2	9.4	10.5	11.7	12.9	14.1	15.2	16.4	17.6	18.8	19.9
4 40	7 20	23	20	36	40	0.0	1.2	2.4	3.6	4.8	5.9	7.1	8.3	9.5	10.7	11.9	13.1	14.3	15.4	16.6	17.8	19.0	20.2
4 50	7 10	24	10	35	50	0.0	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8	12.0	13.2	14.4	15.6	16.8	18.0	19.2	20.4
5 0	7 0	25	0	35	0	0.0	1.2	2.4	3.6	4.9	6.1	7.3	8.5	9.7	10.9	12.2	13.4	14.6	15.8	17.0	18.2	19.4	20.7
5 10	6 50	25	50	34	10	0.0	1.2	2.5	3.7	4.9	6.1	7.4	8.6	9.8	11.0	12.3	13.5	14.7	15.9	17.2	18.4	19.6	20.8
5 20	6 40	26	40	33	20	0.0	1.2	2.5	3.7	4.9	6.2	7.4	8.6	9.9	11.1	12.3	13.6	14.8	16.0	17.3	18.5	19.8	21.0
5 30	6 30	27	30	32	30	0.0	1.2	2.5	3.7	5.0	6.2	7.4	8.7	9.9	11.2	12.4	13.7	14.9	16.1	17.4	18.6	19.9	21.1
5 40	6 20	28	20	31	40	0.0	1.2	2.5	3.7	5.0	6.2	7.5	8.7	10.0	11.2	12.5	13.7	15.0	16.2	17.4	18.7	19.9	21.2
5 50	6 10	29	10	30	50	0.0	1.2	2.5	3.7	5.0	6.2	7.5	8.7	10.0	11.2	12.5	13.7	15.0	16.2	17.5	18.7	20.0	21.2
6 0	6 0	30	0	30	0	0.0	1.3	2.5	3.8	5.0	6.3	7.5	8.8	10.0	11.3	12.5	13.8	15.0	16.3	17.5	18.8	20.0	21.3

TABLE XXXVII.

Equation of Second Difference. *Argument.*---Second Difference.

Whole Interval, 12 Hours. Time after Noon or Midn.		Whole Interval, 60 Minutes, or 1 Degree. *Argument.*		3 Minutes.						4 Minutes.						5 Minutes.						6*m*.
				0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″	0″
h. *m.*	*h.* *m.*	*t* ″	*t* ″																			
0 0	12 0	0 0	60 0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0″0	0.0	0″0	0″0	0.0	0.0	0″0
0 10	11 50	0 50	59 10	1.2	1.3	1.4	1.4	1.5	1.6	1.6	1.7	1.8	1.8	1.9	2.0	2.1	2.1	2.2	2.3	2.3	2.4	2.5
0 20	11 40	1 40	58 20	2.4	2.6	2.7	2.8	3.0	3.1	3.2	3.4	3.5	3.6	3.8	3.9	4.1	4.2	4.3	4.5	4.6	4.7	4.9
0 30	11 30	2 30	57 30	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2
0 40	11 20	3 20	56 40	4.7	5.0	5.2	5.5	5.8	6.0	6.3	6.6	6.8	7.1	7.3	7.6	7.9	8.1	8.4	8.7	8.9	9.2	9.4
0 50	11 10	4 10	55 50	5.8	6.1	6.5	6.8	7.1	7.4	7.8	8.1	8.4	8.7	9.0	9.4	9.7	10.0	10.3	10.7	11.0	11.3	11.6
1 0	11 0	5 0	55 0	6.9	7.3	7.6	8.0	8.4	8.8	9.2	9.5	9.9	10.3	10.7	11.1	11.5	11.8	12.2	12.6	13.0	13.4	13.8
1 10	10 50	5 50	54 10	7.9	8.3	8.8	9.2	9.7	10.1	10.5	11.9	11.4	11.8	12.3	12.7	13.2	13.6	14.0	14.5	14.9	15.4	15.8
1 20	10 40	6 40	53 20	8.9	9.4	9.9	10.4	10.9	11.4	11.9	12.3	12.8	13.3	13.8	14.3	14.8	15.3	15.8	16.3	16.8	17.3	17.8
1 30	10 30	7 30	52 30	9.8	10.4	10.9	11.5	12.0	12.6	13.1	13.7	14.2	14.8	15.3	15.9	16.4	17.0	17.5	18.0	18.6	19.1	19.7
1 40	10 20	8 20	51 40	10.8	11.4	12.0	12.6	13.2	13.8	14.4	14.9	15.5	16.1	16.7	17.3	17.9	18.5	19.1	19.7	20.3	20.9	21.5
1 50	10 10	9 10	50 50	11.6	12.3	12.9	13.6	14.2	14.9	15.5	16.2	16.8	17.5	18.1	18.8	19.4	20.1	20.7	21.4	22.0	22.7	23.3
2 0	10 0	10 0	50 0	12.5	13.2	13.9	14.6	15.3	16.0	16.7	17.4	18.1	18.8	19.4	20.1	20.8	21.5	22.2	22.9	23.6	24.3	25.0
2 10	9 50	10 50	49 10	13.3	14.1	14.8	15.5	16.3	17.0	17.8	18.5	19.2	20.0	20.7	21.5	22.2	22.9	23.6	24.4	25.2	25.9	26.6
2 20	9 40	11 40	48 20	14.1	14.9	15.7	16.4	17.2	18.0	18.8	19.6	20.4	21.1	21.9	22.7	23.5	24.3	25.1	25.8	26.6	27.4	28.2
2 30	9 30	12 30	47 30	14.8	15.7	16.5	17.3	18.1	19.0	19.8	20.6	21.4	22.3	23.1	23.9	24.7	25.6	26.4	27.2	28.0	28.9	29.7
2 40	9 20	13 20	46 40	15.6	16.4	17.3	18.1	19.0	19.9	20.7	21.6	22.5	23.3	24.2	25.1	25.9	26.8	27.7	28.5	29.4	30.2	31.1
2 50	9 10	14 10	45 50	16.2	17.1	18.0	18.9	19.8	20.7	21.6	22.5	23.4	24.3	25.3	26.2	27.1	28.0	28.9	29.8	30.7	31.6	32.5
3 0	9 0	15 0	45 0	16.9	17.8	18.8	19.7	20.6	21.6	22.5	23.4	24.4	25.3	26.3	27.2	28.1	29.1	30.0	30.9	31.9	32.8	33.8
3 10	8 50	15 50	44 10	17.5	18.5	19.4	20.4	21.4	22.3	23.3	24.3	25.3	26.2	27.2	28.2	29.1	30.1	31.1	32.1	33.0	34.0	35.0
3 20	8 40	16 40	43 20	18.1	19.1	20.1	21.1	22.1	23.1	24.1	25.1	26.1	27.1	28.1	29.1	30.1	31.1	32.1	33.1	34.1	35.1	36.1
3 30	8 30	17 30	42 30	18.6	19.6	20.7	21.7	22.7	23.8	24.8	25.8	26.9	27.9	28.9	30.0	31.0	32.0	33.1	34.1	35.1	36.2	37.2
3 40	8 20	18 20	41 40	19.1	20.2	21.2	22.3	23.3	24.4	25.5	26.5	27.6	28.6	29.7	30.8	31.8	32.9	34.0	35.0	36.1	37.1	38.2
3 50	8 10	19 10	40 50	19.6	20.7	21.7	22.8	23.9	25.0	26.1	27.2	28.3	29.3	30.4	31.5	32.6	33.7	34.8	35.9	37.0	38.0	39.1
4 0	8 0	20 0	40 0	20.0	21.1	22.2	23.3	24.4	25.6	26.7	27.8	28.9	30.0	31.1	32.2	33.3	34.4	35.6	36.7	37.8	38.9	40.0
4 10	7 50	20 50	39 10	20.4	21.5	22.7	23.8	24.9	26.1	27.2	28.3	29.5	30.6	31.7	32.9	34.0	35.1	36.3	37.4	38.5	39.7	40.8
4 20	7 40	21 40	38 20	20.8	21.9	23.1	24.2	25.4	26.5	27.7	28.8	30.0	31.1	32.3	33.5	34.6	35.8	36.9	38.1	39.2	40.4	41.5
4 30	7 30	22 30	37 30	21.1	22.3	23.4	24.6	25.8	27.0	28.1	29.3	30.5	31.6	32.8	34.0	35.2	36.3	37.5	38.7	39.8	41.0	42.2
4 40	7 20	23 20	36 40	21.4	22.6	23.8	25.0	26.1	27.3	28.5	29.7	30.9	32.1	33.3	34.5	35.6	36.8	38.0	39.2	40.4	41.6	42.8
4 50	7 10	24 10	35 50	21.6	22.9	24.1	25.3	26.5	27.7	28.9	30.1	31.3	32.5	33.7	34.9	36.1	37.3	38.5	39.7	40.9	42.1	43.3
5 0	7 0	25 0	35 0	21.9	23.1	24.3	25.5	26.7	28.0	29.2	30.4	31.6	32.8	34.0	35.2	36.5	37.7	38.9	40.1	41.3	42.5	43.8
5 10	6 50	25 50	34 10	22.1	23.3	24.5	25.7	27.0	28.2	29.4	30.6	31.9	23.1	34.3	35.6	36.8	38.0	39.2	40.5	41.7	42.9	44.1
5 20	6 40	26 40	33 20	22.2	23.5	24.7	25.9	27.2	28.4	29.6	30.9	32.1	33.3	34.6	35.8	37.0	38.3	39.5	40.7	42.0	43.2	44.4
5 30	6 30	27 30	32 30	22.3	23.6	24.8	26.1	27.3	28.6	29.8	31.0	32.3	33.5	34.8	36.0	37.2	38.5	39.7	41.0	42.2	43.4	44.7
5 40	6 20	28 20	31 40	22.4	23.7	24.9	26.2	27.4	28.7	29.9	31.2	32.4	33.6	34.9	36.1	37.4	38.6	39.9	41.1	42.4	43.6	44.9
5 50	6 10	29 10	30 50	22.5	23.7	25.0	26.2	27.5	28.7	30.0	31.2	32.5	33.7	35.0	36.2	37.5	38.7	40.0	41.2	42.5	43.7	45.0
6 0	6 0	30 0	30 0	22.5	23.8	25.0	26.3	27.5	28.8	30.0	31.3	32.5	33.8	35.0	36.3	37.5	38.8	40.0	41.3	42.5	43.8	45.0

TABLES OF THE PLANETS.

TABLE I.

Epochs of the Mean Longitude of Mercury.

	Years.	Mean. Long. S.	D.	M.	S.	Aphelion. S.	D.	M.	S.	Node. S.	D.	M.	S.
Nat. J. C.	0	8	24	0	44	7	16	13	21	0	24	17	48
	100	11	8	5	4	7	17	47	6	0	25	29	58
	1400	7	11	1	24	8	8	5	51	1	11	8	8
	1500	9	25	5	44	8	9	39	36	1	12	20	18
B. N. S.	1600	10	28	14	38	8	11	13	21	1	13	32	28
C.	1700	1	8	13	26	8	12	47	6	1	14	44	38
B.	1740	2	7	51	10	8	13	24	36	1	15	13	30
B.	1760	2	22	40	2	8	13	43	21	1	15	27	56
B.	1780	3	7	28	52	8	14	2	6	1	15	42	22
	1786	2	3	52	47	8	14	7	44	1	15	46	42
	1787	3	27	35	50	8	14	8	40	1	15	47	25
B.	1788	5	25	24	26	8	14	9	36	1	15	48	8
	1789	7	19	7	30	8	14	10	32	1	15	48	51
	1790	9	12	50	33	8	14	11	28	1	15	49	35
	1791	11	6	33	37	8	14	12	25	1	15	50	18
B.	1792	1	4	22	13	8	14	13	21	1	15	51	1
	1793	2	28	5	16	8	14	14	17	1	15	51	45
	1794	4	21	48	20	8	14	15	13	1	15	52	28
	1795	6	15	31	23	8	14	16	10	1	15	53	11
B.	1796	8	13	19	59	8	14	17	6	1	15	53	55
	1797	10	7	3	3	8	14	18	2	1	15	54	38
	1798	0	0	46	6	8	14	18	58	1	15	55	21
	1799	1	24	29	10	8	14	19	55	1	15	56	4
C.	1800	3	18	12	13	8	14	20	51	1	15	56	48
	1801	5	11	55	16	8	14	21	47	1	15	57	31
	1802	7	5	38	20	8	14	22	43	1	15	58	14
	1803	8	29	21	23	8	14	23	40	1	15	58	57
B.	1804	10	27	9	59	8	14	24	36	1	15	59	41
	1805	0	20	53	3	8	14	25	32	1	16	0	24
	1806	2	14	36	6	8	14	26	29	1	16	1	8
	1807	4	8	19	10	8	14	27	25	1	16	1	51
B.	1808	6	6	7	46	8	14	28	21	1	16	2	34
	1809	7	29	50	49	8	14	29	17	1	16	3	17
	1810	9	23	33	53	8	14	30	13	1	16	4	1
	1811	11	17	16	56	8	14	31	10	1	16	4	44
B.	1812	1	15	5	32	8	14	32	6	1	16	5	27
	1813	3	8	48	36	8	14	33	2	1	16	6	10
	1814	5	2	31	39	8	14	33	58	1	16	6	54
	1815	6	26	14	43	8	14	34	55	1	16	7	37
B.	1816	8	24	3	19	8	14	35	51	1	16	8	20
	1817	10	17	46	22	8	14	36	47	1	16	9	4
	1818	0	11	29	26	8	14	37	43	1	16	9	47
	1819	2	5	12	29	8	14	38	40	1	16	10	30
B.	1820	4	3	1	5	8	14	39	36	1	16	11	14

TABLE II.

Mean Motion of Mercury for *Years*.

	Years.	Mot. in Long. S.	D.	M.	S.	Mot. Aphelion S.	D.	M.	S.	Mot. Node. D.	M.	S.
	1	1	23	43	3	0	0	0	56	0	0	43
	2	3	17	26	7	0	0	1	52	0	1	27
	3	5	11	9	10	0	0	2	49	0	2	10
B.	4	7	8	57	46	0	0	3	45	0	2	53
	5	9	2	40	50	0	0	4	41	0	3	36
	6	10	26	23	53	0	0	5	37	0	4	20
	7	0	20	6	57	0	0	6	34	0	5	3
B.	8	2	17	55	33	0	0	7	30	0	5	46
	9	4	11	38	36	0	0	8	26	0	6	30
	10	6	5	21	40	0	0	9	22	0	7	13
	11	7	29	4	43	0	0	10	19	0	7	56
B.	12	9	26	53	19	0	0	11	15	0	8	40
	13	11	20	36	23	0	0	12	11	0	9	23
	14	1	14	19	26	0	0	13	8	0	10	6
	15	3	8	2	29	0	0	14	4	0	10	49
B.	16	5	5	51	6	0	0	15	0	0	11	33
	17	6	29	34	9	0	0	15	56	0	12	16
	18	8	23	17	12	0	0	16	52	0	12	59
	19	10	17	0	16	0	0	17	49	0	13	43
B.	20	0	14	48	52	0	0	18	45	0	14	26
	40	0	29	37	44	0	0	37	30	0	28	52
	60	1	14	26	36	0	0	56	15	0	43	18
	80	1	29	15	28	0	1	15	0	0	57	44
	100	2	14	4	20	0	1	33	45	1	12	10
	200	4	28	8	40	0	3	7	30	2	24	20
	300	7	12	13	0	0	4	41	15	3	36	30
	400	9	26	17	20	0	6	15	0	4	48	40
	500	0	10	21	40	0	7	48	45	6	0	50
	600	2	24	26	0	0	9	22	30	7	13	0
	700	5	8	30	20	0	10	56	15	8	25	10
	800	7	22	34	40	0	12	30	0	9	37	20
	900	10	6	39	0	0	14	3	45	10	49	30
	1000	0	20	43	20	0	15	37	30	12	1	40
	1100	3	4	47	40	0	17	11	15	13	13	50
	1200	5	18	52	0	0	18	45	0	14	26	0
	1300	8	2	56	20	0	20	18	45	15	38	10
	1400	10	17	0	40	0	21	52	30	16	50	20
	1500	1	1	5	0	0	23	26	15	18	2	30
	2000	1	11	26	40	1	1	15	0	24	3	20

Bissextiles.

TABLE III.

Mean Motion of Mercury for Days.

Days of Month.	JANUARY. Mot. Long. S.	D.	M.	S.	Mot Aph. SEC.	Mot. Node. SEC.	Days of Month.	FEBRUARY. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.	Days of Month.	MARCH. Mot Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	0	4	5	33	0	0	1	4	10	57	22	5	4	1	8	5	52	33	9	7
2	0	8	11	5	0	0	2	4	15	2	54	5	4	2	8	9	38	6	9	7
3	0	12	16	38	1	0	3	4	19	8	27	5	4	3	8	13	43	39	10	7
4	0	16	22	10	1	1	4	4	23	13	59	6	4	4	8	17	49	11	10	8
5	0	20	27	43	1	1	5	4	27	19	32	6	4	5	8	21	54	44	10	8
6	0	24	33	15	1	1	6	5	1	25	5	6	5	6	8	26	0	16	10	8
7	0	28	38	48	1	1	7	5	5	30	38	6	5	7	9	0	5	49	10	8
8	1	2	44	20	1	1	8	5	9	36	10	6	5	8	9	4	11	21	10	8
9	1	6	49	53	2	1	9	5	13	41	42	6	5	9	9	8	16	54	10	8
10	1	10	55	26	2	1	10	5	17	47	15	6	5	10	9	12	22	26	11	8
11	1	15	0	58	2	1	11	5	21	52	47	6	5	11	9	16	27	59	11	8
12	1	19	6	31	2	2	12	5	25	58	20	7	5	12	9	20	33	32	11	9
13	1	23	12	3	2	2	13	6	0	3	53	7	5	13	9	24	39	4	11	9
14	1	27	17	36	2	2	14	6	4	9	25	7	5	14	9	28	44	37	11	9
15	2	1	23	8	2	2	15	6	8	14	58	7	6	15	10	2	50	9	11	9
16	2	5	28	41	3	2	16	6	12	20	30	7	6	16	10	6	55	42	12	9
17	2	9	34	13	3	2	17	6	16	26	3	7	6	17	10	11	1	14	12	9
18	2	13	39	46	3	2	18	6	20	31	35	8	6	18	10	15	6	47	12	9
19	2	17	45	19	3	2	19	6	24	37	8	8	6	19	10	19	12	19	12	9
20	2	21	50	51	3	2	20	6	28	42	40	8	6	20	10	23	17	52	12	9
21	2	25	56	24	3	3	21	7	2	48	13	8	6	21	10	27	23	25	12	10
22	3	0	1	56	4	3	22	7	6	53	46	8	6	22	11	1	28	57	12	10
23	3	4	7	29	4	3	23	7	10	59	18	8	6	23	11	5	34	30	13	10
24	3	8	13	2	4	3	24	7	15	4	51	8	7	24	11	9	40	2	13	10
25	3	12	18	34	4	3	25	7	19	10	23	9	7	25	11	13	45	35	13	10
26	3	16	24	7	4	3	26	7	23	15	56	9	7	26	11	17	51	7	13	10
27	3	20	29	39	4	3	27	7	27	21	28	9	7	27	11	21	56	40	13	10
28	3	24	35	12	4	3	28	8	1	27	1	9	7	28	11	26	2	12	13	10
29	3	28	40	44	4	3								29	0	0	7	45	14	10
30	4	2	46	17	5	4								30	0	4	13	18	14	11
31	4	6	51	50	5	4								31	0	8	18	50	14	11

In the Months JANUARY and FEBRUARY of a Bissextile Year, subtract 1 from the given Day of the Month.

TABLE III.

Mean Motion of Mercury for Days.

Days of Month.	APRIL. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.
1	0	12	24	23	14	11
2	0	16	29	55	14	11
3	0	20	35	28	14	11
4	0	24	41	0	14	11
5	0	28	46	33	15	11
6	1	2	52	6	15	11
7	1	6	57	38	15	12
8	1	11	3	11	15	12
9	1	15	8	43	15	12
10	1	19	14	16	15	12
11	1	23	19	48	16	12
12	1	27	25	21	16	12
13	2	1	30	53	16	12
14	2	5	36	26	16	12
15	2	9	41	59	16	13
16	2	13	47	31	16	13
17	2	17	53	4	16	13
18	2	21	58	36	17	13
19	2	26	4	9	17	13
20	3	0	9	41	17	13
21	3	4	15	14	17	13
22	3	8	20	46	17	13
23	3	12	26	19	17	13
24	3	16	31	52	18	14
25	3	20	37	24	18	14
26	3	24	42	57	18	14
27	3	28	48	29	18	14
28	4	2	54	2	18	14
29	4	6	59	34	18	14
30	4	11	5	7	18	14

Days of Month.	MAY. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.
1	4	15	10	39	19	14
2	4	19	16	12	19	15
3	4	23	21	45	19	15
4	4	27	27	17	19	15
5	5	1	32	50	19	15
6	5	5	38	22	19	15
7	5	9	43	55	20	15
8	5	13	49	27	20	15
9	5	17	55	0	20	15
10	5	22	0	32	20	16
11	5	26	6	5	20	16
12	6	0	11	38	20	16
13	6	4	17	10	20	16
14	6	8	22	43	21	16
15	6	12	28	15	21	16
16	6	16	33	48	21	16
17	6	20	39	20	21	16
18	6	24	44	53	21	16
19	6	28	50	25	21	17
20	7	2	55	58	22	17
21	7	7	1	31	22	17
22	7	11	7	3	22	17
23	7	15	12	36	22	17
24	7	19	18	8	22	17
25	7	23	23	41	22	17
26	7	27	29	13	23	17
27	8	1	34	46	23	17
28	8	5	40	18	23	18
29	8	9	45	51	23	18
30	8	13	51	24	23	18
31	8	17	56	56	23	18

Days of Month.	JUNE. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.
1	8	22	2	29	23	18
2	8	26	8	1	24	18
3	9	0	13	34	24	18
4	9	4	19	6	24	18
5	9	8	24	39	24	19
6	9	12	30	12	24	19
7	9	16	35	44	24	19
8	9	20	41	17	25	19
9	9	24	46	49	25	19
10	9	28	52	22	25	19
11	10	2	57	54	25	19
12	10	7	3	27	25	19
13	10	11	8	59	25	19
14	10	15	14	32	25	20
15	10	19	20	5	26	20
16	10	23	25	37	26	20
17	10	27	31	10	26	20
18	11	1	36	42	26	20
19	11	5	42	15	26	20
20	11	9	47	47	26	20
21	11	13	53	20	27	20
22	11	17	58	52	27	21
23	11	22	4	25	27	21
24	11	26	9	57	27	21
25	0	0	15	30	27	21
26	0	4	21	3	27	21
27	0	8	26	35	27	21
28	0	12	32	8	28	21
29	0	16	37	40	28	21
30	0	20	43	13	28	22

TABLE III.

Mean Motion of Mercury for Days.

Days of Month.	JULY. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.	Days of Month.	AUGUST. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.	Days of Month.	SEPTEMBER. Mot. Long. S.	D.	M.	S.	Mot. Aphe. SEC.	Mot. Node. SEC.
1	0	24	48	45	28	22	1	5	1	40	35	33	25	1	9	8	32	24	38	29
2	0	28	54	18	28	22	2	5	5	46	7	33	25	2	9	12	37	57	38	29
3	1	2	59	51	28	22	3	5	9	51	40	33	26	3	9	16	43	29	38	29
4	1	7	5	23	29	22	4	5	13	57	12	33	26	4	9	20	49	2	38	29
5	1	11	10	56	29	22	5	5	18	2	45	33	26	5	9	24	54	34	38	29
6	1	15	16	28	29	22	6	5	22	8	17	34	26	6	9	29	0	7	38	30
7	1	19	22	1	29	22	7	5	26	13	50	34	26	7	10	3	5	39	39	30
8	1	23	27	33	29	22	8	6	0	19	23	34	26	8	10	7	11	12	39	30
9	1	27	33	6	29	23	9	6	4	24	55	34	26	9	10	11	16	44	29	30
10	2	1	38	38	29	23	10	6	8	30	28	34	26	10	10	15	22	17	39	30
11	2	5	44	11	30	23	11	6	12	36	0	34	26	11	10	19	27	50	39	30
12	2	9	49	44	30	23	12	6	16	41	33	35	27	12	10	23	33	22	39	30
13	2	13	55	16	30	23	13	6	20	47	5	35	27	13	10	27	38	55	39	30
14	2	18	0	49	30	23	14	6	24	52	38	35	27	14	11	1	44	27	40	31
15	2	22	6	21	30	23	15	6	28	58	11	35	27	15	11	5	50	0	40	31
16	2	26	11	54	30	23	16	7	3	3	43	35	27	16	11	9	55	32	40	31
17	3	0	17	26	31	24	17	7	7	9	16	35	27	17	11	14	1	5	40	31
18	3	4	22	59	31	24	18	7	11	14	48	35	27	18	11	18	6	37	40	31
19	3	8	28	31	31	24	19	7	15	20	21	36	27	19	11	22	12	10	40	31
20	3	12	34	4	31	24	20	7	19	25	53	36	28	20	11	26	17	43	41	31
21	3	16	39	37	31	24	21	7	23	31	26	36	28	21	0	0	23	16	41	31
22	3	20	45	9	31	24	22	7	27	36	58	36	28	22	0	4	28	48	41	32
23	3	24	50	42	31	24	23	8	1	42	31	36	28	23	0	8	34	20	41	32
24	3	28	56	14	32	24	24	8	5	48	4	36	28	24	0	12	39	53	41	32
25	4	3	1	47	32	25	25	8	9	53	36	37	28	25	0	16	45	25	41	32
26	4	7	7	19	32	25	26	8	13	59	9	37	28	26	0	20	50	58	41	32
27	4	11	12	52	32	25	27	8	18	4	41	37	28	27	0	24	56	30	42	32
28	4	15	18	24	32	25	28	8	22	10	14	37	29	28	0	29	2	3	42	32
29	4	19	23	57	32	25	29	8	26	15	46	37	29	29	1	3	7	36	42	32
30	4	23	29	30	33	25	30	9	0	21	19	37	29	30	1	7	13	8	42	32
31	4	27	35	2	33	25	31	9	4	26	51	37	29							

TABLE III.

Mean Motion of Mercury for Days.

Days of Month.	OCTOBER. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.	Days of Month.	NOVEMBER. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.	Days of Month.	DECEMBER. Mot Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	1	11	18	41	42	33	1	5	18	10	30	47	36	1	9	20	56	47	52	40
2	1	15	24	13	42	33	2	5	22	16	3	47	36	2	9	25	2	19	52	40
3	1	19	29	46	43	33	3	5	26	21	35	47	36	3	9	29	7	52	52	40
4	1	23	35	19	43	33	4	6	0	27	8	47	37	4	10	3	13	24	52	40
5	1	27	40	51	43	33	5	6	4	32	40	48	37	5	10	7	18	57	52	40
6	2	1	46	23	43	33	6	6	8	38	13	48	37	6	10	11	24	30	52	40
7	2	5	51	56	43	33	7	6	12	43	45	48	37	7	10	15	30	2	53	41
8	2	9	57	29	43	33	8	6	16	49	18	48	37	8	10	19	35	35	53	41
9	2	14	3	1	44	34	9	6	20	54	50	48	37	9	10	23	41	7	53	41
10	2	18	8	34	44	34	10	6	25	0	23	48	37	10	10	27	46	40	53	41
11	2	22	14	6	44	34	11	6	29	5	56	49	37	11	11	1	52	12	53	41
12	2	26	19	39	44	34	12	7	3	11	28	49	37	12	11	5	57	45	53	41
13	3	0	25	11	44	34	13	7	7	17	1	49	38	13	11	10	3	17	53	41
14	3	4	30	44	44	34	14	7	11	22	33	49	38	14	11	14	8	50	54	41
15	3	8	36	17	44	34	15	7	15	28	6	49	38	15	11	18	14	22	54	41
16	3	12	41	49	45	34	16	7	19	33	38	49	38	16	11	22	19	55	54	42
17	3	16	47	22	45	35	17	7	23	39	11	49	38	17	11	26	25	28	54	42
18	3	20	52	54	45	35	18	7	27	44	43	50	38	18	0	0	31	0	54	42
19	3	24	58	27	45	35	19	8	1	50	16	50	38	19	0	4	36	33	54	42
20	3	29	3	59	45	35	20	8	5	55	49	50	38	20	0	8	42	5	55	42
21	4	3	9	32	45	35	21	8	10	1	21	50	38	21	0	12	47	38	55	42
22	4	7	15	4	45	35	22	8	14	6	54	50	39	22	0	16	53	10	55	42
23	4	11	20	37	46	35	23	8	18	12	26	50	39	23	0	20	58	43	55	42
24	4	15	26	10	46	35	24	8	22	17	59	51	39	24	0	25	4	16	55	42
25	4	19	31	42	46	35	25	8	26	23	31	51	39	25	0	29	9	48	55	43
26	4	23	37	15	46	35	26	9	0	29	4	51	39	26	1	3	15	21	55	43
27	4	27	42	47	46	36	27	9	4	34	36	51	39	27	1	7	20	53	56	43
28	5	1	48	20	46	36	28	9	8	40	9	51	40	28	1	11	26	26	56	43
29	5	5	53	52	47	36	29	9	12	45	42	51	40	29	1	15	31	58	56	43
30	5	9	59	25	47	36	30	9	16	51	14	51	40	30	1	19	37	31	56	43
31	5	14	4	57	47	36								31	1	23	43	3	56	43

TABLE IV.

Mean Motion of Mercury for *Hours*, *Minutes*, and *Seconds*.

	Mot. Long.				Mot. Long.			Mot. Long.	
Hours.				Min.	′	″	Min.	′	″
	D.	M.	S.	Sec.	″	‴	Sec.	″	‴
1	0	10	14	1	0	10	31	5	17
2	0	20	28	2	0	20	32	5	27
3	0	30	41	3	0	31	33	5	38
4	0	40	55	4	0	41	34	5	48
5	0	51	9	5	0	51	35	5	58
6	1	1	23	6	1	1	36	6	8
7	1	11	37	7	1	12	37	6	18
8	1	21	51	8	1	22	38	6	29
9	1	32	5	9	1	32	39	6	39
10	1	42	18	10	1	42	40	6	49
11	1	52	32	11	1	52	41	6	59
12	2	2	46	12	2	3	42	7	10
13	2	13	0	13	2	13	43	7	20
14	2	23	14	14	2	23	44	7	30
15	2	33	28	15	2	33	45	7	40
16	2	43	42	16	2	44	46	7	51
17	2	53	55	17	2	54	47	8	1
18	3	4	9	18	3	4	48	8	11
19	3	14	23	19	3	14	49	8	21
20	3	24	38	20	3	24	50	8	31
21	3	34	52	21	3	35	51	8	42
22	3	45	5	22	3	45	52	8	52
23	3	55	19	23	3	55	53	9	2
24	4	5	32	24	4	5	54	9	12
				25	4	16	55	9	23
				26	4	26	56	9	33
				27	4	36	57	9	43
				28	4	46	58	9	53
				29	4	57	59	10	4
				30	5	7	60	10	14

TABLE V.

The Equation of the Orbit of Mercury for every Degree of Anomaly.

ARGUMENT. *Mean Anomaly of Mercury.*

	Sig. O. —			Differ.		Sig. I.—			Differ.		Sig. II. —			Differ		
D.	D.	M.	S.	M.	S.	D.	M.	S.	M.	S.	D.	M.	S.	M.	S.	D.
0	0	0	0			9	34	42			17	46	53			30
				19	36				18	14				13	53	
1	0	19	36			9	52	56			18	0	46			29
				19	35				18	8				13	41	
2	0	39	11			10	11	4			18	14	27			28
				19	35				18	2				13	29	
3	0	58	46			10	29	6			18	27	56			27
				19	35				17	56				13	16	
4	1	18	21			10	47	2			18	41	12			26
				19	34				17	50				13	2	
5	1	37	55			11	4	52			18	54	14			25
				19	33				17	43				12	49	
6	1	57	28			11	22	35			19	7	3			24
				19	32				17	37				12	36	
7	2	17	0			11	40	12			19	19	39			23
				19	31				17	30				12	22	
8	2	36	31			11	57	42			19	32	1			22
				19	29				17	23				12	7	
9	2	56	0			12	15	5			19	44	8			21
				19	28				17	16				11	53	
10	3	15	28			12	32	21			19	56	1			20
				19	26				17	9				11	39	
11	3	34	54			12	49	30			20	7	40			19
				19	24				17	1				11	23	
12	3	54	18			13	6	31			20	19	3			18
				19	22				16	53				11	8	
13	4	13	40			13	23	24			20	30	11			17
				19	20				16	45				10	52	
14	4	33	0			13	40	9			20	41	3			16
				19	18				16	37				10	37	
15	4	52	18			13	56	46			20	51	40			15
				19	14				16	28				10	20	
16	5	11	32			14	13	14			21	2	0			14
				19	12				16	20				10	4	
17	5	30	44			14	29	34			21	12	4			13
				19	9				16	11				9	47	
18	5	49	53			14	45	45			21	21	51			12
				19	6				16	1				9	30	
19	6	8	59			15	1	46			21	31	21			11
				19	3				15	52				9	12	
20	6	28	2			15	17	38			21	40	33			10
				18	59				15	43				8	54	
21	6	47	1			15	33	21			21	49	27			9
				18	55				15	33				8	37	
22	7	5	56			15	48	54			21	58	4			8
				18	51				15	23				8	18	
23	7	24	47			16	4	17			22	6	22			7
				18	47				15	12				7	59	
24	7	43	34			16	19	29			22	14	21			6
				18	43				15	2				7	41	
25	8	2	17			16	34	31			22	22	2			5
				18	39				14	51				7	21	
26	8	20	56			16	49	22			22	29	23			4
				18	34				14	40				7	1	
27	8	39	30			17	4	2			22	36	24			3
				18	29				14	29				6	41	
28	8	57	59			17	18	31			22	43	5			2
				18	24				14	17				6	21	
29	9	16	23			17	32	48			22	49	26			1
				18	19				14	5				6	0	
30	9	34	42			17	46	53			22	55	26			0
	Sig XI. +					Sig X. +					Sig. IX. +					

TABLE V.

The Equation of the Orbit of Mercury.

ARGUMENT. *Mean Anomaly of Mercury.*

	Sig. III. —			Differ.		Sig. IV.—			Differ.		Sig. V. —			Differ		
D.	D.	M.	S.	M.	S.	D.	M.	S.	M.	S.	D.	M.	S.	M.	S.	D.
0	22	55	26	5	39	22	45	28	7	38	14	55	11	24	16	30
1	23	1	5	5	17	22	37	50	8	10	14	30	55	24	46	29
2	23	6	22	4	56	22	29	40	8	42	14	6	9	25	16	28
3	23	11	18	4	34	22	20	58	9	15	13	40	53	25	46	27
4	23	15	52	4	11	22	11	43	9	46	13	15	7	26	15	26
5	23	20	3	3	48	22	1	57	10	20	12	48	52	26	43	25
6	23	23	51	3	25	21	51	37	10	53	12	22	9	27	11	24
7	23	27	16	3	2	21	40	44	11	26	11	54	58	27	37	23
8	23	30	18	2	37	21	29	18	11	59	11	27	21	28	4	22
9	23	32	55	2	14	21	17	19	12	33	10	59	17	28	29	21
10	23	35	9	1	48	21	4	46	13	6	10	30	48	28	53	20
11	23	36	57	1	24	20	51	40	13	41	10	1	55	29	16	19
12	23	38	21	0	59	20	37	59	14	14	9	32	39	29	39	18
13	23	39	20	0	33	20	23	45	14	49	9	3	0	30	1	17
14	23	39	53	+ 0	6	20	8	56	15	23	8	32	59	30	21	16
15	23	39	59	— 0	20	19	53	33	15	56	8	2	38	30	40	15
16	23	39	39	0	46	19	37	37	16	31	7	31	58	30	58	14
17	23	38	53	1	14	19	21	6	17	6	7	1	0	31	16	13
18	23	37	39	1	42	19	4	0	17	39	6	29	44	31	32	12
19	23	35	57	2	9	18	46	21	18	14	5	58	12	31	46	11
20	23	33	48	2	37	18	28	7	18	47	5	26	26	32	0	10
21	23	31	11	3	6	18	9	20	19	22	4	54	26	32	11	9
22	23	28	5	3	35	17	49	58	19	55	4	22	15	32	23	8
23	23	24	30	4	4	17	30	3	20	29	3	49	52	32	32	7
24	23	20	26	4	34	17	9	34	21	3	3	17	20	32	41	6
25	23	15	52	5	4	16	48	31	21	35	2	44	39	32	48	5
26	23	10	48	5	34	16	26	56	22	8	2	11	51	32	53	4
27	23	5	14	6	4	16	4	48	22	40	1	38	58	32	57	3
28	22	59	10	6	36	15	42	8	23	13	1	6	1	33	0	2
29	22	52	34	7	6	15	18	55	23	44	0	33	1	33	1	1
30	22	45	28			14	55	11			0	0	0			0
	Sig VIII. +					Sig VII. +					Sig. VI. +					

TABLE VI.

Logarithms of the Distance of Mercury from the Sun.

ARGUMENT. *Mean Anomaly of Mercury.*

	Sig. O.		Sig. I.		Sig. II.		
D.	Logarithms.	Differ.	Logarithms.	Differ.	Logarithms.	Differ.	D.
0	9.668993		9.661990		9.640787		30
		8		476		955	
1	9.668985		9.661514		9.639832		29
		23		492		970	
2	9.668962		9.661022		9.638862		28
		38		507		987	
3	9.668924		9.660515		9.637875		27
		54		524		1003	
4	9.668870		9.659991		9.636872		26
		70		539		1019	
5	9.668800		9.659452		9.635853		25
		85		555		1034	
6	9.668715		9.658897		9.634819		24
		101		571		1051	
7	9.668614		9.658326		9.633768		23
		117		587		1066	
8	9.668497		9.657739		9.632702		22
		132		603		1083	
9	9.668365		9.657136		9.631619		21
		147		619		1098	
10	9.668218		9.656517		9.630521		20
		163		635		1114	
11	9.668055		9.655882		9.629407		19
		178		650		1130	
12	9.667877		9.655232		9.628277		18
		194		667		1146	
13	9.667683		9.654565		9.627131		17
		210		682		1161	
14	9.667473		9.653883		9.625970		16
		226		698		1177	
15	9.667247		9.653185		9.624793		15
		241		714		1193	
16	9.667006		9.652471		9.623600		14
		256		731		1208	
17	9.666760		9.651740		9.622392		13
		272		746		1223	
18	9.666478		9.650994		9.621169		12
		287		763		1239	
19	9.666191		9.650231		9.619930		11
		304		778		1254	
20	9.665887		9.649453		9.618676		10
		319		795		1269	
21	9.665568		9.648658		9.617407		9
		335		810		1284	
22	9.665233		9.647848		9.616123		8
		350		827		1300	
23	9.664883		9.647021		9.614823		7
		366		842		1314	
24	9.664517		9.646179		9.613509		6
		383		858		1330	
25	9.664134		9.645321		9.612179		5
		398		875		1344	
26	9.663736		9.644446		9.610835		4
		412		891		1358	
27	9.663324		9.643555		9.609477		3
		429		908		1373	
28	9.662895		9.642647		9.608104		2
		445		923		1387	
29	9.662450		9.641724		9.606717		1
		460		937		1401	
30	9.661990		9.640787		9.605316		0
	Sig. XI		Sig. X		Sig. IX		

TABLE VI.

Logarithms of the Distance of Mercury from the Sun.

ARGUMENT. *Mean Anomaly of Mercury.*

	Sig. III.		Sig. IV.		Sig. V.		
D.	Logarithms.	Differ.	Logarithms.	Differ.	Logarithms.	Differ.	D.
0	9.605316		9.557972		9.510077		30
1	9.603901	1415	9.556280	1692	9.508747	1330	29
2	9.602472	1429	9.554586	1694	9.507447	1300	28
3	9.601029	1443	9.552890	1696	9.506179	1268	27
4	9.599573	1456	9.551195	1695	9.504943	1236	26
5	9.598104	1469	9.549500	1695	9.503742	1201	25
6	9.596622	1482	9.547807	1693	9.502575	1167	24
7	9.595127	1495	9.546116	1691	9.501445	1130	23
8	9.593620	1507	9.544428	1688	9.500353	1092	22
9	9.592101	1519	9.542743	1685	9.499299	1054	21
10	9.590569	1532	9.541064	1679	9.498285	1014	20
11	9.589026	1543	9.539390	1674	9.497313	972	19
12	9.587472	1554	9.537723	1667	9.496383	930	18
13	9.585906	1566	9.536063	1660	9.495496	887	17
14	9.584329	1577	9.534412	1651	9.494654	842	16
15	9.582743	1586	9.532770	1642	9.493857	797	15
16	9.581146	1597	9.531139	1631	9.493107	750	14
17	9.579539	1607	9.529521	1618	9.492404	703	13
18	9.577923	1616	9.527915	1606	9.491749	655	12
19	9.576298	1625	9.526323	1592	9.491144	605	11
20	9.574664	1634	9.524746	1577	9.490589	555	10
21	9.573022	1642	9.523186	1560	9.490085	504	9
22	9.571373	1649	9.521644	1542	9.489632	453	8
23	9.569717	1656	9.520120	1524	9.489230	402	7
24	9.568054	1663	9.518615	1505	9.488880	350	6
25	9.566386	1668	9.517131	1484	9.488584	296	5
26	9.564712	1674	9.515670	1461	9.488341	243	4
27	9.563033	1679	9.514233	1437	9.488152	189	3
28	9.561349	1684	9.512821	1412	9.488017	135	2
29	9.559662	1687	9.511435	1386	9.487935	82	1
30	9.557972	1690	9.510077	1358	9.487907	28	0
	Sig. VIII.		Sig. VII.		Sig. VI.		

TABLE VII.

Reduction to the Ecliptic both in Longitude and Distance.

ARGUMENT. *The Longitude of Mercury* minus *the Longitude of the Node.*

Degrees.	Sig. O. — Sig. VI.— M. S.	Sub. from Log.	Sig. I. — Sig. VII.— M. S.	Sub. from Log.	Sig. II. — Sig. VIII.— M. S.	Sub. from Log.	Degrees.
0	0 0	0	11 7	808	11 9	2432	30
1	0 26	1	11 20	858	10 56	2480	29
2	0 53	4	11 32	908	10 41	2528	28
3	1 20	9	11 43	959	10 26	2574	27
4	1 47	16	11 54	1011	10 9	2621	26
5	2 13	24	12 4	1063	9 53	2665	25
6	2 39	35	12 13	1118	9 34	2708	24
7	3 5	48	12 21	1172	9 16	2749	23
8	3 31	62	12 28	1226	8 57	2789	22
9	3 57	79	12 34	1281	8 37	2829	21
10	4 23	98	12 40	1337	8 17	2867	20
11	4 48	118	12 45	1392	7 56	2903	19
12	5 13	140	12 48	1448	7 34	2937	18
13	5 37	164	12 50	1506	7 12	2969	17
14	6 1	189	12 51	1562	6 50	3000	16
15	6 25	216	12 52	1619	6 27	3030	15
16	6 48	246	12 51	1675	6 3	3058	14
17	7 11	277	12 50	1731	5 39	3084	13
18	7 33	309	12 48	1788	5 15	3108	12
19	7 54	343	12 45	1845	4 50	3130	11
20	8 14	378	12 40	1901	4 25	3151	10
21	8 34	415	12 35	1957	3 59	3169	9
22	8 53	454	12 29	2012	3 34	3186	8
23	9 12	493	12 22	2066	3 8	3201	7
24	9 31	534	12 15	2120	2 41	3213	6
25	9 50	577	12 6	2173	2 15	3224	5
26	10 7	622	11 56	2227	1 47	3233	4
27	10 23	667	11 45	2280	1 21	3240	3
28	10 39	713	11 35	2332	0 54	3245	2
29	10 54	760	11 22	2382	0 27	3248	1
30	11 7	808	11 9	2432	0 0	3249	0
	Sig. XI. + Sig. V. +	Sub. from Log.	Sig. X. + Sig. IV. +	Sub. from Log.	Sig. IX. + Sig. III. +	Sub. from Log.	

TABLE VIII.

Heliocentric Latitude of Mercury.

ARGUMENT. *The Longitude of Mercury* minus *the Longitude of the Node.*

Degrees.	Sig. O. *N.* / Sig. VI. *S.* D.	M.	S.	Differ. M.	S.	Sig. I. *N.* / Sig. VII. *S.* D.	M.	S.	Differ. M.	S.	Sig. II. *N.* / Sig. VIII. *S.* D.	M.	S.	Differ M.	S.	Degrees.
0	0	0	0	7	18	3	29	37	6	20	6	3	30	3	37	30
1	0	7	18	7	19	3	35	57	6	15	6	7	7	3	30	29
2	0	14	37	7	18	3	42	12	6	11	6	10	37	3	23	28
3	0	21	55	7	18	3	48	23	6	6	6	14	0	3	18	27
4	0	29	13	7	18	3	54	29	6	1	6	17	18	3	11	26
5	0	36	31	7	17	4	0	30	5	58	6	20	29	3	3	25
6	0	43	48	7	16	4	6	28	5	54	6	23	32	2	56	24
7	0	51	4	7	15	4	12	22	5	49	6	26	28	2	49	23
8	0	58	19	7	14	4	18	11	5	44	6	29	17	2	42	22
9	1	5	33	7	12	4	23	55	5	40	6	31	59	2	35	21
10	1	12	45	7	11	4	29	35	5	35	6	34	34	2	27	20
11	1	19	56	7	10	4	35	10	5	30	6	37	1	2	20	19
12	1	27	6	7	9	4	40	40	5	24	6	39	21	2	13	18
13	1	34	15	7	7	4	46	4	5	19	6	41	34	2	5	17
14	1	41	22	7	5	4	51	23	5	14	6	43	39	1	58	16
15	1	48	27	7	4	4	56	37	5	8	6	45	37	1	50	15
16	1	55	31	7	2	5	1	45	5	3	6	47	27	1	44	14
17	2	2	33	6	59	5	6	48	4	58	6	49	11	1	36	13
18	2	9	32	6	55	5	11	46	4	52	6	50	47	1	28	12
19	2	16	27	6	53	5	16	38	4	46	6	52	15	1	20	11
20	2	23	20	6	50	5	21	24	4	40	6	53	35	1	12	10
21	2	30	10	6	48	5	26	4	4	34	6	54	47	1	5	9
22	2	36	58	6	46	5	30	38	4	28	6	55	52	0	57	8
23	2	43	44	6	42	5	35	6	4	22	6	56	49	0	50	7
24	2	50	26	6	40	5	39	28	4	17	6	57	39	0	44	6
25	2	57	6	6	37	5	43	45	4	11	6	58	23	0	36	5
26	3	3	43	6	33	5	47	56	4	4	6	58	59	0	27	4
27	3	10	16	6	30	5	52	0	3	57	6	59	26	0	19	3
28	3	16	46	6	27	5	55	57	3	50	6	59	45	0	11	2
29	3	23	13	6	24	5	59	47	3	43	6	59	56	0	4	1
30	3	29	37			6	3	30			7	0	0			0
	Sig. XI. *S.* / Sig. V. *N.*					Sig. X. *S.* / Sig. IV. *N.*					Sig. IX. *S.* / Sig. III. *N.*					

TABLE I.

Epochs of the Mean Longitude of Venus.

	YEARS.	Mean Long. S.	D.	M.	S.	Aphelion. S.	D.	M.	S.	Node. S.	D.	M.	S.
Nat. J. C.	0	5	28	29	10	9	14	8	12	1	29	22	9
	100	0	17	42	10	9	15	39	12	2	0	13	49
	1400	2	27	31	10	10	3	12	12	2	11	25	29
	1500	9	16	44	10	10	4	33	12	2	12	17	9
B. N. S.	1600	3	19	55	52	10	5	54	12	2	13	8	48
C.	1700	10	7	32	45	10	7	15	12	2	14	0	28
B.	1740	10	15	13	58	10	7	46	36	2	14	21	8
B.	1760	4	19	4	34	10	8	3	48	2	14	31	28
B.	1780	10	22	55	10	10	8	20	0	2	14	41	48
	1790	1	24	2	24	10	8	28	6	2	14	46	58
C.	1800	4	25	9	38	10	8	36	12	2	14	52	8
	1810	7	26	16	52	10	8	44	18	2	14	57	18
B.	1820	10	29	0	14	10	8	52	24	2	15	2	28
	1821	6	13	47	44	10	8	53	13	2	15	2	59
	1822	1	28	35	14	10	8	54	1	2	15	3	30
	1823	9	13	22	44	10	8	54	50	2	15	4	1
B.	1824	4	29	46	21	10	8	55	38	2	15	4	32
	1825	0	14	33	51	10	8	56	27	2	15	5	3
	1826	7	29	21	21	10	8	57	16	2	15	5	34
	1827	3	14	8	51	10	8	58	4	2	15	6	5
B.	1828	11	0	32	28	10	8	58	53	2	15	6	36
	1829	6	15	19	58	10	8	59	41	2	15	7	7
	1830	2	0	7	28	10	9	0	30	2	15	7	38
	1831	9	14	54	58	10	9	1	19	2	15	8	9
B.	1832	5	1	18	36	10	9	2	7	2	15	8	40
	1833	0	16	6	5	10	9	2	56	2	15	9	11
	1834	8	0	53	35	10	9	3	44	2	15	9	42
	1835	3	15	41	5	10	9	4	33	2	15	10	13
B.	1836	11	2	4	43	10	9	5	21	2	15	10	44
	1837	6	16	52	13	10	9	6	10	2	15	11	15
	1838	2	1	39	42	10	9	6	59	2	15	11	46
	1839	9	16	27	12	10	9	7	47	2	15	12	17
B.	1840	5	2	50	50	10	9	8	36	2	15	12	48
	1841	0	17	38	20	10	9	9	24	2	15	13	19
	1842	8	2	25	50	10	9	10	13	2	15	13	50
	1843	3	17	13	20	10	9	11	2	2	15	14	21
B.	1844	11	3	36	57	10	9	11	50	2	15	14	52
	1845	6	18	24	27	10	9	12	39	2	15	15	23
	1846	2	3	11	57	10	9	13	27	2	15	15	54
	1847	9	17	59	27	10	9	14	16	2	15	16	25
B.	1848	5	4	23	4	10	9	15	5	2	15	16	56
	1849	0	19	10	34	10	9	15	53	2	15	17	27
	1850	8	3	58	4	10	9	16	42	2	15	17	58

TABLE II.

Mean Motion of Venus for *Years*.

	YEARS.	Mot. in Long. S.	D.	M.	S.	Mot. Aph. D.	M.	S.	Mot. Node. D.	M.	S.
	1	7	14	47	30	0	0	49	0	0	31
	2	2	29	35	0	0	1	37	0	1	2
	3	10	14	22	30	0	2	26	0	1	33
B.	4	6	0	46	7	0	3	14	0	2	4
	5	1	15	33	37	0	4	3	0	2	35
	6	9	0	21	7	0	4	52	0	3	6
	7	4	15	8	37	0	5	40	0	3	37
B.	8	0	1	32	14	0	6	29	0	4	8
	9	7	16	19	44	0	7	17	0	4	39
	10	3	1	7	14	0	8	6	0	5	10
	11	10	15	54	44	0	8	55	0	5	41
B.	12	6	2	18	22	0	9	43	0	6	12
	13	1	17	5	51	0	10	32	0	6	43
	14	9	1	53	21	0	11	20	0	7	14
	15	4	16	40	51	0	12	9	0	7	45
B.	16	0	3	4	29	0	12	58	0	8	16
	17	7	17	51	59	0	13	46	0	8	47
	18	3	2	39	28	0	14	35	0	9	18
	19	10	17	26	58	0	15	23	0	9	49
B.	20	6	3	50	36	0	16	12	0	10	20
	40	0	7	41	12	0	32	24	0	20	40
	60	6	11	31	48	0	48	36	0	31	0
	80	0	15	22	24	1	4	48	0	41	20
	100	6	19	13	0	1	21	0	0	51	40
	200	1	8	26	0	2	42	0	1	43	20
	300	7	27	39	0	4	3	0	2	35	0
	400	2	16	52	0	5	24	0	3	26	40
	500	9	6	5	0	6	45	0	4	18	20
	600	3	25	18	0	8	6	0	5	10	0
	700	10	14	31	0	9	27	0	6	1	40
	800	5	3	44	0	10	48	0	6	53	20
	900	11	22	57	0	12	9	0	7	45	0
	1000	6	12	10	0	13	30	0	8	36	40
	1100	1	1	23	0	14	51	0	9	28	20
	1200	7	20	36	0	16	12	0	10	20	0
	1300	2	9	49	0	17	33	0	11	11	40
	1400	8	29	2	0	18	54	0	12	3	20
	1500	3	18	15	0	20	15	0	12	55	0
	2000	0	24	20	0	27	0	0	17	13	20

Bissextiles.

TABLE III.

Mean Motion of Venus for *Days*.

Days of Month.	JANUARY. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	0	1	36	8	0	0
2	0	3	12	16	0	0
3	0	4	48	23	1	0
4	0	6	24	31	1	0
5	0	8	0	39	1	0
6	0	9	36	47	1	0
7	0	11	12	55	1	1
8	0	12	49	2	1	1
9	0	14	25	10	1	1
10	0	16	1	18	1	1
11	0	17	37	26	2	1
12	0	19	13	34	2	1
13	0	20	49	41	2	1
14	0	22	25	49	2	1
15	0	24	1	57	2	1
16	0	25	38	5	2	1
17	0	27	14	13	2	1
18	0	28	50	21	3	2
19	1	0	26	28	3	2
20	1	2	2	36	3	2
21	1	3	38	44	3	2
22	1	5	14	52	3	2
23	1	6	51	0	3	2
24	1	8	27	7	3	2
25	1	10	3	15	3	2
26	1	11	39	23	3	2
27	1	13	15	31	4	2
28	1	14	51	39	4	2
29	1	16	27	46	4	2
30	1	18	3	54	4	3
31	1	19	40	2	4	3

Days of Month.	FEBRUARY. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	1	21	16	10	4	3
2	1	22	52	18	4	3
3	1	24	28	25	5	3
4	1	26	4	33	5	3
5	1	27	40	41	5	3
6	1	29	16	49	5	3
7	2	0	52	57	5	3
8	2	2	29	4	5	3
9	2	4	5	12	5	3
10	2	5	41	20	5	3
11	2	7	17	28	6	4
12	2	8	53	36	6	4
13	2	10	29	43	6	4
14	2	12	5	51	6	4
15	2	13	41	59	6	4
16	2	15	18	7	6	4
17	2	16	54	15	6	4
18	2	18	30	23	7	4
19	2	20	6	30	7	4
20	2	21	42	38	7	4
21	2	23	18	46	7	4
22	2	24	54	54	7	4
23	2	26	31	2	7	5
24	2	28	7	9	7	5
25	2	29	43	17	7	5
26	3	1	19	25	8	5
27	3	2	55	33	8	5
28	3	4	31	41	8	5

Days of Month.	MARCH. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	3	6	7	48	8	5
2	3	7	43	56	8	5
3	3	9	20	4	8	5
4	3	10	56	12	8	5
5	3	12	32	20	9	5
6	3	14	8	27	9	6
7	3	15	44	35	9	6
8	3	17	20	43	9	6
9	3	18	56	51	9	6
10	3	20	32	59	9	6
11	3	22	9	7	9	6
12	3	23	45	14	9	6
13	3	25	21	22	10	6
14	3	26	57	30	10	6
15	3	28	33	38	10	6
16	4	0	9	46	10	6
17	4	1	45	53	10	6
18	4	3	22	1	10	7
19	4	4	58	9	10	7
20	4	6	34	17	11	7
21	4	8	10	25	11	7
22	4	9	46	32	11	7
23	4	11	22	40	11	7
24	4	12	58	48	11	7
25	4	14	34	56	11	7
26	4	16	11	4	11	7
27	4	17	47	11	11	7
28	4	19	23	19	12	7
29	4	20	59	27	12	7
30	4	22	35	35	12	8
31	4	24	11	43	12	8

In the Months JANUARY and FEBRUARY of a Bissextile Year, subtract 1 from the given Day of the M onth.

TABLE III.

Mean Motion of Venus for *Days*.

Days of Month.	APRIL. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	4	25	47	50	12	8
2	4	27	23	58	12	8
3	4	29	0	6	12	8
4	5	0	36	14	13	8
5	5	2	12	22	13	8
6	5	3	48	30	13	8
7	5	5	24	37	13	8
8	5	7	0	45	13	8
9	5	8	36	53	13	8
10	5	10	13	1	13	8
11	5	11	49	9	13	9
12	5	13	25	16	14	9
13	5	15	1	24	14	9
14	5	16	37	32	14	9
15	5	18	13	40	14	9
16	5	19	49	48	14	9
17	5	21	25	55	14	9
18	5	23	2	3	14	9
19	5	24	38	11	14	9
20	5	26	14	19	15	9
21	5	27	50	27	15	9
22	5	29	26	34	15	10
23	6	1	2	42	15	10
24	6	2	38	50	15	10
25	6	4	14	58	15	10
26	6	5	51	6	15	10
27	6	7	27	13	16	10
28	6	9	3	21	16	10
29	6	10	39	29	16	10
30	6	12	15	37	16	10

Days of Month.	MAY. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	6	13	51	45	16	10
2	6	15	27	52	16	10
3	6	17	4	0	16	10
4	6	18	40	8	16	11
5	6	20	16	16	17	11
6	6	21	52	24	17	11
7	6	23	28	31	17	11
8	6	25	4	39	17	11
9	6	26	40	47	17	11
10	6	28	16	55	17	11
11	6	29	53	3	17	11
12	7	1	29	10	18	11
13	7	3	5	18	18	11
14	7	4	41	26	18	11
15	7	6	17	34	18	11
16	7	7	53	42	18	12
17	7	9	29	50	18	12
18	7	11	5	57	18	12
19	7	12	42	5	18	12
20	7	14	18	13	19	12
21	7	15	54	21	19	12
22	7	17	30	29	19	12
23	7	19	6	36	19	12
24	7	20	42	44	19	12
25	7	22	18	52	19	12
26	7	23	55	0	19	12
27	7	25	31	8	20	12
28	7	27	7	15	20	13
29	7	28	43	23	20	13
30	8	0	19	31	20	13
31	8	1	55	39	20	13

Days of Month.	JUNE. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	8	3	31	47	20	13
2	8	5	7	54	20	13
3	8	6	44	2	20	13
4	8	8	20	10	21	13
5	8	9	56	18	21	13
6	8	11	32	26	21	13
7	8	13	8	33	21	13
8	8	14	44	41	21	13
9	8	16	20	49	21	14
10	8	17	56	57	21	14
11	8	19	33	5	22	14
12	8	21	9	12	22	14
13	8	22	45	20	22	14
14	8	24	21	28	22	14
15	8	25	57	36	22	14
16	8	27	33	44	22	14
17	8	29	9	51	22	14
18	9	0	45	59	22	14
19	9	2	22	7	23	14
20	9	3	58	15	23	15
21	9	5	34	23	23	15
22	9	7	10	30	23	15
23	9	8	46	38	23	15
24	9	10	22	46	23	15
25	9	11	58	54	23	15
26	9	13	35	2	24	15
27	9	15	11	10	24	15
28	9	16	47	17	24	15
29	9	18	23	25	24	15
30	9	19	59	33	24	15

TABLE III.

Mean Motion of Venus for *Days*.

Days of Month.	JULY. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot Node. SEC.	Days of Month.	AUGUST. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.	Days of Month.	SEPTEMBER. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	9	21	35	41	24	15	1	11	11	15	43	28	18	1	1	0	55	45	32	21
2	9	23	11	49	24	16	2	11	12	51	51	28	18	2	1	2	31	53	33	21
3	9	24	47	56	24	16	3	11	14	27	59	29	18	3	1	4	8	0	33	21
4	9	26	24	4	25	16	4	11	16	4	6	29	18	4	1	5	44	8	33	21
5	9	28	0	12	25	16	5	11	17	40	14	29	18	5	1	7	20	16	33	21
6	9	29	36	20	25	16	6	11	19	16	22	29	19	6	1	8	56	24	33	21
7	10	1	12	28	25	16	7	11	20	52	30	29	19	7	1	10	32	32	33	21
8	10	2	48	35	25	16	8	11	22	28	38	29	19	8	1	12	8	40	33	21
9	10	4	24	43	25	16	9	11	24	4	45	29	19	9	1	13	44	47	34	21
10	10	6	0	51	25	16	10	11	25	40	53	30	19	10	1	15	20	55	34	21
11	10	7	36	59	26	16	11	11	27	17	1	30	19	11	1	16	57	3	34	22
12	10	9	13	7	26	16	12	11	28	53	9	30	19	12	1	18	33	11	34	22
13	10	10	49	14	26	16	13	0	0	29	17	30	19	13	1	20	9	19	34	22
14	10	12	25	22	26	17	14	0	2	5	24	30	19	14	1	21	45	26	34	22
15	10	14	1	30	26	17	15	0	3	41	32	30	19	15	1	23	21	34	34	22
16	10	15	37	38	26	17	16	0	5	17	40	30	19	16	1	24	57	42	34	22
17	10	17	13	46	26	17	17	0	6	53	48	30	19	17	1	26	33	50	35	22
18	10	18	49	53	26	17	18	0	8	29	56	31	20	18	1	28	9	58	35	22
19	10	20	26	1	27	17	19	0	10	6	3	31	20	19	1	29	46	5	35	22
20	10	22	2	9	27	17	20	0	11	42	11	31	20	20	2	1	22	13	35	22
21	10	23	38	17	27	17	21	0	13	18	19	31	20	21	2	2	58	21	35	22
22	10	25	14	25	27	17	22	0	14	54	27	31	20	22	2	4	34	29	35	22
23	10	26	50	33	27	17	23	0	16	30	35	31	20	23	2	6	10	37	35	23
24	10	28	26	40	27	17	24	0	18	6	42	31	20	24	2	7	46	44	36	23
25	11	0	2	48	27	17	25	0	19	42	50	32	20	25	2	9	22	52	36	23
26	11	1	38	56	28	18	26	0	21	18	58	32	20	26	2	10	59	0	36	23
27	11	3	15	4	28	18	27	0	22	55	6	32	20	27	2	12	35	8	36	23
28	11	4	51	12	28	18	28	0	24	31	14	32	20	28	2	14	11	16	36	23
29	11	6	27	19	28	18	29	0	26	7	21	32	20	29	2	15	47	23	36	23
30	11	8	3	27	28	18	30	0	27	43	29	32	21	30	2	17	23	31	36	23
31	11	9	39	35	28	18	31	0	29	19	37	32	21							

TABLE III.

Mean Motion of Venus for *Days*.

Days of Month.	OCTOBER. Mot. Long. S.	D.	M.	S.	Mot Aph. SEC.	Mot Node. SEC.
1	2	18	58	39	36	23
2	2	20	35	47	37	23
3	2	22	11	55	37	23
4	2	23	48	2	37	24
5	2	25	24	10	37	24
6	2	27	0	18	37	24
7	2	28	36	26	37	24
8	3	0	12	34	37	24
9	3	1	48	41	38	24
10	3	3	24	49	38	24
11	3	5	0	57	38	24
12	3	6	37	5	38	24
13	3	8	13	13	38	24
14	3	9	49	21	38	24
15	3	11	25	28	38	24
16	3	13	1	36	38	25
17	3	14	37	44	39	25
18	3	16	13	52	39	25
19	3	17	50	0	39	25
20	3	19	26	7	39	25
21	3	21	2	15	39	25
22	3	22	38	23	39	25
23	3	24	14	31	39	25
24	3	25	50	39	40	25
25	3	27	26	47	40	25
26	3	29	2	54	40	25
27	4	0	39	2	40	25
28	4	2	15	10	40	26
29	4	3	51	18	40	26
30	4	5	27	26	40	26
31	4	7	3	33	40	26

Days of Month.	NOVEMBER. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	4	8	39	41	41	26
2	4	10	15	49	41	26
3	4	11	51	57	41	26
4	4	13	28	5	41	26
5	4	15	4	12	41	26
6	4	16	40	20	41	26
7	4	18	16	28	41	26
8	4	19	52	36	41	26
9	4	21	28	44	42	27
10	4	23	4	51	42	27
11	4	24	40	59	42	27
12	4	26	17	7	42	27
13	4	27	53	15	42	27
14	4	29	29	22	42	27
15	5	1	5	30	42	27
16	5	2	41	38	43	27
17	5	4	17	46	43	27
18	5	5	53	54	43	27
19	5	7	30	2	43	27
20	5	9	6	10	43	28
21	5	10	42	17	43	28
22	5	12	18	25	43	28
23	5	13	54	33	43	28
24	5	15	30	41	44	28
25	5	17	6	49	44	28
26	5	18	42	56	44	28
27	5	20	19	4	44	28
28	5	21	55	12	44	28
29	5	23	31	20	44	28
30	5	25	7	28	44	28

Days of Month.	DECEMBER. Mot. Long. S.	D.	M.	S.	Mot. Aph. SEC.	Mot. Node. SEC.
1	5	26	43	35	45	28
2	5	28	19	43	45	29
3	5	29	55	51	45	29
4	6	1	31	59	45	29
5	6	3	8	7	45	29
6	6	4	44	14	45	29
7	6	6	20	22	45	29
8	6	7	56	30	45	29
9	6	9	32	38	46	29
10	6	11	8	46	46	29
11	6	12	44	53	46	29
12	6	14	21	1	46	29
13	6	15	57	9	46	29
14	6	17	33	17	46	30
15	6	19	9	25	46	30
16	6	20	45	32	47	30
17	6	22	21	40	47	30
18	6	23	57	48	47	30
19	6	25	33	56	47	30
20	6	27	10	4	47	30
21	6	28	46	12	47	30
22	7	0	22	19	47	30
23	7	1	58	27	47	30
24	7	3	34	35	47	30
25	7	5	10	43	48	30
26	7	6	46	51	48	31
27	7	8	22	58	48	31
28	7	9	59	6	48	31
29	7	11	35	14	48	31
30	7	13	11	22	48	31
31	7	14	47	30	49	31

TABLE IV.

Mean Motion of Venus for *Hours*, *Minutes*, and *Seconds*.

Hours.	Mot. Long. D.	M.	S.	Min. Sec.	Mot. Long. ′ ″ ‴	Min. Sec.	Mot. Long. ′ ″ ‴
1	0	4	0	1	0 4	31	2 4
2	0	8	1	2	0 8	32	2 8
3	0	12	1	3	0 12	33	2 12
4	0	16	1	4	0 16	34	2 16
5	0	20	2	5	0 20	35	2 20
6	0	24	2	6	0 24	36	2 24
7	0	28	2	7	0 28	37	2 28
8	0	32	3	8	0 32	38	2 32
9	0	36	3	9	0 36	39	2 36
10	0	40	3	10	0 40	40	2 40
11	0	44	4	11	0 44	41	2 44
12	0	48	4	12	0 48	42	2 48
13	0	52	4	13	0 52	43	2 52
14	0	56	5	14	0 56	44	2 56
15	1	0	5	15	1 0	45	3 0
16	1	4	5	16	1 4	46	3 4
17	1	8	6	17	1 8	47	3 8
18	1	12	6	18	1 12	48	3 12
19	1	16	6	19	1 16	49	3 16
20	1	20	6	20	1 20	50	3 20
21	1	24	7	21	1 24	51	3 24
22	1	28	7	22	1 28	52	3 28
23	1	32	7	23	1 32	53	3 32
24	1	36	8	24	1 36	54	3 36
				25	1 40	55	3 40
				26	1 44	56	3 44
				27	1 48	57	3 48
				28	1 52	58	3 52
				29	1 56	59	3 56
				30	2 0	60	4 0

TABLE V.

Equation of the Orbit of Venus for 1780.

ARGUMENT. *Mean Anomaly of Venus.*

	Sig. O. —		Sig. I. —		Sig. II. —		Sig. III. —		Sig. IV. —		Sig. V. —		
D.	M. S.	Diff	M. S.	Diff.	M. S.	Diff.	M. S.	Diff.	M. S.	Diff.	M. S.	Diff.	D.
0	0 0	″	23 29	″	40 49	″	47 19	″	41 10	″	23 51	″	30
1	0 49	49	24 12	43	41 13	24	47 20	1	40 45	25	23 7	44	29
2	1 38	49	24 54	42	41 37	24	47 19	1	40 19	26	22 23	44	28
3	2 27	49	25 35	41	42 0	23	47 17	2	39 53	26	21 39	44	27
4	3 16	49	26 16	41	42 23	23	47 14	3	39 26	27	20 55	44	26
5	4 5	49	26 57	41	42 44	21	47 11	2	38 58	28	20 10	45	25
6	4 54	49	27 37	40	43 5	21	47 7	4	38 29	29	19 24	46	24
7	5 43	49	28 17	40	43 25	20	47 2	5	38 0	29	18 38	46	23
8	6 32	49	28 56	39	43 45	20	46 56	6	37 30	30	17 52	46	22
9	7 20	48	29 35	39	44 3	18	46 49	7	36 59	31	17 6	46	21
10	8 9	49	30 13	38	44 21	18	46 41	8	36 27	32	16 19	47	20
11	8 58	49	30 51	38	44 38	17	46 32	9	35 55	32	15 32	47	19
12	9 46	48	31 28	37	44 54	16	46 22	10	35 23	32	14 45	47	18
13	10 33	47	32 5	37	45 9	15	46 12	10	34 49	34	13 57	48	17
14	11 21	48	32 41	36	45 23	14	46 1	11	34 15	34	13 9	48	16
15	12 9	48	33 16	35	45 37	14	45 49	12	33 40	35	12 21	48	15
16	12 56	47	33 51	35	45 50	13	45 36	13	33 5	35	11 33	49	14
17	13 44	48	34 25	34	46 2	12	45 23	13	32 29	36	10 44	49	13
18	14 31	47	34 58	33	46 13	11	45 8	15	31 52	37	9 55	49	12
19	15 17	46	35 32	34	46 23	10	44 53	15	31 15	37	9 6	49	11
20	16 3	46	36 4	32	46 32	9	44 36	17	30 37	38	8 17	49	10
21	16 49	46	36 35	31	46 41	9	44 19	17	29 59	38	7 28	49	9
22	17 35	46	37 6	31	46 49	8	44 2	17	29 20	39	6 39	50	8
23	18 21	46	37 36	30	46 56	7	43 43	19	28 41	39	5 49	50	7
24	19 6	45	38 6	30	47 2	6	43 23	20	28 1	40	4 59	50	6
25	19 51	45	38 35	29	47 7	5	43 3	20	27 21	40	4 10	49	5
26	20 35	44	39 3	28	47 11	4	42 42	21	26 40	41	3 20	50	4
27	21 19	44	39 31	28	47 14	3	42 20	22	25 59	41	2 30	50	3
28	22 3	44	39 57	26	47 17	3	41 58	22	25 17	42	1 40	50	2
29	22 46	43	40 23	26	47 19	2	41 34	24	24 34	43	0 50	50	1
30	23 29	43	40 49	26	47 19	0	41 10	24	23 51	43	0 0	50	0
	Sig. XI. +		Sig. X. +		Sig. IX. +		Sig. VIII. +		Sig. VII. +		Sig. VI. +		

The greatest equation diminishes 25″ in 100 years. The others are proportional at the rate of 0″.5 for a minute.

TABLE VI.

Logarithms of the Distance of Venus from the Sun for 1780.

ARGUMENT. *Mean Anomaly of Venus.*

	Sig. O.		Sig. I.		Sig. II.		
D.	Logarithms.	Differ.	Logarithms.	Differ.	Logarithms.	Differ.	D.
0	9.862318		9.861924		9.860845		30
		1		26		44	
1	9.862317		9.861898		9.860801		29
		2		26		46	
2	9.862315		9.861872		9.860755		28
		2		28		46	
3	9.862313		9.861844		9.860709		27
		3		28		46	
4	9.862310		9.861816		9.860663		26
		4		30		47	
5	9.862306		9.861786		9.860616		25
		5		29		47	
6	9.862301		9.861757		9.860569		24
		6		31		47	
7	9.862295		9.861726		9.860522		23
		6		31		48	
8	9.862289		9.861695		9.860474		22
		8		32		48	
9	9.862281		9.861663		9.860426		21
		8		33		49	
10	9.862273		9.861630		9.860377		20
		9		33		49	
11	9.862264		9.861597		9.860328		19
		11		34		49	
12	9.862253		9.861563		9.860279		18
		11		35		49	
13	9.862242		9.861528		9.860230		17
		12		35		50	
14	9.862230		9.861493		9.860180		16
		13		36		50	
15	9.862217		9.861457		9.860130		15
		13		37		50	
16	9.862204		9.861420		9.860080		14
		15		37		51	
17	9.862189		9.861383		9.860029		13
		15		38		50	
18	9.862174		9.861345		9.859979		12
		17		39		51	
19	9.862157		9.861306		9.859928		11
		17		39		51	
20	9.862140		9.861267		9.859877		10
		18		39		52	
21	9.862122		9.861228		9.859825		9
		18		41		51	
22	9.862104		9.861187		9.859774		8
		20		41		52	
23	9.862084		9.861146		9.859722		7
		20		41		51	
24	9.862064		9.861105		9.859671		6
		21		42		52	
25	9.862043		9.861063		9.859619		5
		22		43		52	
26	9.862021		9.861020		9.859567		4
		23		42		52	
27	9.861998		9.860978		9.859515		3
		24		44		52	
28	9.861974		9.860934		9.859463		2
		25		44		52	
29	9.861949		9.860890		9.859411		1
		25		45		52	
30	9.861924		9.860845		9.859359		0
	Sig. XI.		Sig. X.		Sig. IX.		

Corrections of the Logarithms after 100 Years.

SIG.	D.	COR.	SIG.	D.
O	0	— 26	XII.	0
	10	26		20
	20	24		10
I.	0	22	XI.	0
	10	20		20
	20	17		10
II.	0	13	X.	0
	10	9		20
	20	— 5		10
III.	0	0	IX.	0
	10	+ 5		20
	20	9		10
IV.	0	13	VIII.	0
	10	17		20
	20	20		10
V.	0	22	VII.	0
	10	24		20
	20	26		10
VI.	0	+ 26	VI.	0

This Equation of the Distance arises from the Diminution of the Equation of Venus. It changes its Sign for any time before 1780.

TABLE VI.

Logarithms of the Distance of Venus from the Sun for 1780.

ARGUMENT. *Mean Anomaly of Venus.*

	Sig. III.		Sig. IV.		Sig. V.		
D.	Logarithms.	Differ.	Logarithms.	Differ.	Logarithms.	Differ.	D.
0	9.859359	53	9.857856	45	9.856746	26	30
1	9.859306	52	9.857811	45	9.856720	25	29
2	9.859254	52	9.857766	45	9.856695	25	28
3	9.859202	52	9.857721	44	9.856670	23	27
4	9.859150	52	9.857677	43	9.856647	23	26
5	9.859098	52	9.857634	43	9.856624	22	25
6	9.859046	52	9.857591	43	9.856602	22	24
7	9.858994	52	9.857548	42	9.856580	20	23
8	9.858942	52	9.857506	41	9.856560	19	22
9	9.858890	51	9.857465	41	9.856541	19	21
10	9.858839	52	9.857424	40	9.856522	18	20
11	9.858787	51	9.857384	40	9.856504	17	19
12	9.858736	52	9.857344	39	9.856487	16	18
13	9.858684	50	9.857305	38	9.856471	15	17
14	9.858634	51	9.857267	38	9.856456	14	16
15	9.858583	51	9.857229	37	9.856442	13	15
16	9.858532	50	9.857192	36	9.856429	13	14
17	9.858482	50	9.857156	36	9.856416	11	13
18	9.858432	50	9.857120	35	9.856405	11	12
19	9.858382	49	9.857085	35	9.856394	10	11
20	9.858333	50	9.857050	33	9.856384	8	10
21	9.858283	49	9.857017	33	9.856376	8	9
22	9.858234	48	9.856984	33	9.856368	7	8
23	9.858186	48	9.856951	31	9.856361	6	7
24	9.858138	48	9.856920	31	9.856355	5	6
25	9.858090	48	9.856889	30	9.856350	5	5
26	9.858042	47	9.856859	29	9.856345	3	4
27	9.857995	47	9.856830	29	9.856342	2	3
28	9.857948	46	9.856801	28	9.856340	2	2
29	9.857902	46	9.856773	27	9.856338	1	1
30	9.857856		9.856746		9.856337		0
	Sig. VIII.		Sig. VII.		Sig. VI.		

TABLE VII.

Heliocentric Latitude of Venus, with the Reduction to the Ecliptic.

ARGUMENT. *The Longitude of Venus—that of the Node.*

Sig.	O. Lat. *N.* VI. lat. *S.*	O. VI.		I. lat. *N.* VII. lat. *S.*	I. VII.		II. lat. *N.* VIII. lat. *S*	II. VIII.		
Degrees.	Latitude.	Sub. Long.	Sub. from Log.	Latitude.	Sub. Long.	Sub. from Log.	Latitude.	Sub. Long.	Sub. from Log.	Degrees.
	D. M. S.	M. S.		D. M. S.	M. S.		D. M. S.	M. S.		
0	0 0 0	0 0	0	1 41 45	2 37	190	2 56 17	2 37	571	30
1	0 3 33	0 6	0	1 44 48	2 40	202	2 58 2	2 33	583	29
2	0 7 6	0 13	1	1 47 50	2 43	214	2 59 44	2 30	594	28
3	0 10 39	0 19	2	1 50 50	2 45	226	3 1 22	2 26	605	27
4	0 14 12	0 25	4	1 53 48	2 47	238	3 2 57	2 23	615	26
5	0 17 44	0 31	6	1 56 44	2 50	250	3 4 29	2 19	626	25
6	0 21 16	0 38	8	1 59 37	2 52	263	3 5 58	2 15	636	24
7	0 24 48	0 44	11	2 2 28	2 54	276	3 7 23	2 10	645	23
8	0 28 19	0 50	15	2 5 17	2 55	288	3 8 45	2 6	655	22
9	0 31 50	0 56	19	2 8 4	2 57	301	3 10 3	2 1	664	21
10	0 35 20	1 2	23	2 10 49	2 58	314	3 11 18	1 56	673	20
11	0 38 49	1 8	28	2 13 31	2 59	328	3 12 29	1 51	681	19
12	0 42 18	1 13	33	2 16 11	3 0	341	3 13 37	1 46	689	18
13	0 45 46	1 19	38	2 18 48	3 1	354	3 14 41	1 41	697	17
14	0 49 13	1 25	44	2 21 23	3 1	367	3 15 41	1 36	704	16
15	0 52 40	1 30	51	2 23 55	3 1	380	3 16 38	1 30	711	15
16	0 56 5	1 36	58	2 26 24	3 1	394	3 17 32	1 25	717	14
17	0 59 29	1 41	65	2 28 51	3 1	407	3 18 22	1 19	723	13
18	1 2 53	1 46	73	2 31 15	3 0	420	3 19 7	1 13	729	12
19	1 6 15	1 51	80	2 33 36	2 59	434	3 19 50	1 8	734	11
20	1 9 35	1 56	89	2 35 55	2 58	447	3 20 29	1 2	739	10
21	1 12 54	2 1	98	2 38 11	2 57	460	3 21 4	0 56	743	9
22	1 16 13	2 6	107	2 40 23	2 55	473	3 21 36	0 50	747	8
23	1 19 31	2 10	116	2 42 33	2 54	486	3 22 4	0 44	751	7
24	1 22 46	2 15	126	2 44 40	2 52	498	3 22 28	0 38	754	6
25	1 26 0	2 19	136	2 46 44	2 50	511	3 22 48	0 31	756	5
26	1 29 12	2 23	146	2 48 45	2 47	523	3 23 5	0 25	758	4
27	1 32 23	2 26	157	2 50 42	2 45	536	3 23 18	0 19	760	3
28	1 35 32	2 30	168	2 52 37	2 43	548	3 23 27	0 13	761	2
29	1 38 39	2 33	179	2 54 29	2 40	560	3 23 33	0 6	762	1
30	1 41 45	2 37	190	2 56 17	2 37	571	3 23 35	0 0	762	0
	XI. lat. *S.* V. lat. *N.*	Add to Long.	Sub. Log.	X. lat. *S.* IV. lat. *N.*	Add to Long.	Sub. Log.	IX. lat. *S.* III. lat. *N.*	Add to Long.	Sub. Log.	
		XI. V.			X. IV.			IX. III.		Sig.

DEGREES OF LATITUDE.

Dec.	50	51	52	53	54	55	56	57	58
Deg.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*	*h. m. s.*
1	6 4 46	6 4 56	6 5 7	6 5 19	6 5 30	6 5 43	6 5 56	6 6 10	6 6 24
2	6 9 32	6 9 53	6 10 15	6 10 37	6 11 1	6 11 24	6 11 52	6 12 20	6 12 49
3	6 14 19	6 14 51	6 15 23	6 15 57	6 16 33	6 17 9	6 17 49	6 18 31	6 19 15
4	6 19 7	6 19 49	6 20 32	6 21 18	6 22 6	6 22 56	6 23 48	6 24 44	6 25 42
5	6 23 56	6 24 49	6 25 43	6 26 40	6 27 40	6 28 43	6 29 49	6 30 58	6 32 12
6	6 28 46	6 29 50	6 30 56	6 32 4	6 33 16	6 34 32	6 35 52	6 37 15	6 38 44
7	6 33 39	6 34 53	6 36 10	6 37 31	6 38 55	6 40 24	6 41 57	6 43 36	6 45 20
8	6 38 34	6 39 59	6 41 27	6 43 0	6 44 37	6 46 19	6 48 6	6 50 0	6 52 0
9	6 43 31	6 45 7	6 46 47	6 48 32	6 50 22	6 52 18	6 54 19	6 56 28	6 58 44
10	6 48 31	6 50 18	6 52 10	6 54 8	6 56 11	6 58 21	7 0 37	7 3 1	7 5 34
11	6 53 34	6 55 32	6 57 36	6 59 48	7 2 4	7 4 28	7 7 0	7 9 40	7 12 30
12	6 58 40	7 0 52	7 3 8	7 5 32	7 8 3	7 10 40	7 13 28	7 16 24	7 19 26
13	7 3 52	7 6 16	7 8 44	7 11 20	7 14 6	7 17 0	7 20 4	7 23 16	7 26 44
14	7 9 8	7 11 44	7 14 28	7 17 16	7 20 16	7 23 28	7 26 48	7 30 20	7 34 4
15	7 14 28	7 17 16	7 20 12	7 23 18	7 26 32	7 30 0	7 33 36	7 37 28	7 41 32
16	7 19 56	7 22 56	7 26 8	7 29 28	7 33 0	7 36 40	7 40 36	7 44 48	7 49 16
17	7 25 28	7 28 44	7 32 8	7 35 44	7 39 32	7 43 32	7 47 48	7 52 20	7 57 12
18	7 31 8	7 34 36	7 38 16	7 42 9	7 46 16	7 50 36	7 55 12	8 0 4	8 5 20
19	7 36 54	7 40 40	7 44 36	7 48 44	7 53 8	7 57 48	8 2 48	8 8 4	8 13 44
20	7 42 48	7 46 52	7 51 4	7 55 32	8 0 16	8 5 16	8 10 36	8 16 20	8 22 28
21	7 48 52	7 53 11	7 57 43	8 2 30	8 7 36	8 12 59	8 18 44	8 24 56	8 31 36
22	7 55 8	7 59 44	8 4 32	8 9 40	8 15 8	8 20 56	8 27 12	8 33 52	8 41 8
23	8 1 32	8 6 28	8 11 36	8 17 4	8 23 0	8 29 16	8 35 56	8 42 44	8 51 8
24	8 8 12	8 13 24	8 18 56	8 24 52	8 31 12	8 37 56	8 45 12	8 53 8	9 1 48
25	8 15 4	8 20 38	8 26 36	8 32 56	8 39 44	8 47 0	8 54 56	9 3 32	9 13 4
26	8 22 12	8 28 8	8 34 32	8 41 20	8 48 40	8 56 36	9 5 16	9 14 40	9 25 12
27	8 29 32	8 35 56	8 42 48	8 50 12	8 58 8	9 6 44	9 16 16	9 26 44	9 38 28
28	8 37 16	8 44 8	8 51 32	8 59 32	9 8 8	9 17 38	9 28 8	9 39 52	9 53 16
29	8 45 24	8 52 48	9 0 48	9 9 24	9 18 52	9 29 20	9 41 4	9 54 24	10 10 0
30	8 53 56	9 1 56	9 10 36	9 20 4	9 30 28	9 42 10	9 55 28	10 11 0	10 30 4
31	9 2 52	9 11 32	9 21 4	9 31 32	9 43 12	9 56 24	10 11 56	10 30 48	10 56 16
32	9 12 32	9 22 0	9 32 28	9 44 4	9 57 16	10 12 42	10 31 32	10 56 48	12 0 0
33	9 22 52	9 33 16	9 44 52	9 58 4	10 13 24	10 32 8	10 57 16	12 0 0	
34	9 34 0	9 45 36	9 58 48	10 14 4	10 32 44	10 57 44	12 0 0		
35	9 46 16	9 59 24	10 14 40	10 33 16	10 58 8	12 0 0			
36	9 59 56	10 15 12	10 33 40	10 58 28	12 0 0				

When the latitude and declination have the same name, *i. e.* both north or both south, the arc must be taken from the table: but, when they have different names the number found above must be taken from 12 hours, and the remainder will be the semi-arc of apparition.

By this table, 1st, the times of the Sun's rising and setting may be found in any part of *Great Britain* or *Ireland*, after the following manner: Where the latitude of the place is known, look for the declination in the left hand column, and for the latitude of the place at the top, and in the angle of meeting you will find the time of his half duration above the horizon, when the lat. and dec. have the same name. But when the lat. and dec. have contrary names, the number in the table gives half the time of his continuance below the horizon.

In the summer half year the numbers in the table shew the times of the Sun's setting, and in the winter those of his rising; in both cases a small allowance must be made for refraction. When the lat. or dec. consists of deg. and min. proportional parts must be applied, if great accuracy be required. 2dly, Within the limits of the table, as to dec. and lat. it will serve to determine the time of semi-duration of a star, above the horizon; and if the right ascension be known, the times of rising and setting may be known.

TABLE I.

Epochs of the Mean Longitude of Mars, with the Arguments of the Equations.

	YEARS.	Mean Long. S.	D.	M.	S.	Aphelion. S.	D.	M.	S.	Node. S.	D.	M.	S.
B.	O. S. 1500	1	23	46	9.4	4	26	48	19	1	15	44	22
B.	N. S. 1600	3	20	13	33.6	4	28	39	57	1	16	29	54
B.	1620	11	8	23	55.8	4	29	2	17	1	16	39	1
B.	1640	6	26	54	17.9	4	29	24	37	1	16	48	7
B.	1660	2	15	14	40.1	4	29	46	57	1	16	57	14
B.	1680	10	3	35	2.2	5	0	9	17	1	17	6	21
C.	1700	5	21	23	57.7	5	0	31	37	1	17	15	27
B.	1720	1	9	44	19.8	5	0	53	57	1	17	24	34
B.	1740	8	28	4	42.0	5	1	16	17	1	17	33	40
B.	1760	4	16	25	4.2	5	1	38	37	1	17	42	47
B.	1780	0	4	45	26.3	5	2	0	57	1	17	51	54
C.	1800	7	22	34	21.8	5	2	23	17	1	18	1	1
	1810	11	16	28	49.6	5	2	34	27	1	18	5	34
B.	1820	3	10	54	44	5	2	45	37	1	18	10	7
	1821	9	22	11	53.4	5	2	46	44	1	18	10	34
	1822	4	3	29	2.9	5	2	47	51	1	18	11	2
	1823	10	14	46	12.3	5	2	48	58	1	18	11	29
B.	1824	4	26	34	48.4	5	2	50	5	1	18	11	56
	1825	11	7	51	57.8	5	2	51	12	1	18	12	24
	1826	5	19	9	7.3	5	2	52	19	1	18	12	51
	1827	0	0	26	16.7	5	2	53	26	1	18	13	18
B.	1828	6	12	14	52.9	5	2	54	33	1	18	13	46
	1829	0	23	32	2.3	5	2	55	40	1	18	14	13
	1830	7	4	49	11.8	5	2	56	47	1	18	14	40
	1831	1	16	6	21.2	5	2	57	54	1	18	15	8
B.	1832	7	27	54	57.3	5	2	59	1	1	18	15	35
	1833	2	9	12	6.7	5	3	0	8	1	18	21	3
	1834	8	20	29	16.2	5	3	1	15	1	18	21	31
	1835	3	1	46	25.6	5	3	2	22	1	18	21	58
B.	1836	9	10	45	1.7	5	3	3	29	1	18	22	25
	1837	3	24	52	11.1	5	3	4	36	1	18	22	53
	1838	10	6	0	20.6	5	3	5	43	1	18	23	20
	1839	4	17	17	30	5	3	6	50	1	18	23	47
B.	1840	10	29	6	6.2	5	3	7	57	1	18	24	15
	1841	5	10	23	15.6	5	3	9	4	1	18	24	42
	1842	11	21	40	25.1	5	3	10	11	1	18	25	10
	1843	6	2	57	34.5	5	3	11	18	1	18	25	37
B.	1844	0	14	46	10.6	5	3	12	25	1	18	26	4
	1845	6	26	3	20	5	3	13	32	1	18	26	32
	1846	1	7	20	29.5	5	3	14	39	1	18	26	59
	1847	7	18	37	38.9	5	3	15	46	1	18	27	26
B.	1848	2	0	26	15.1	5	3	16	53	1	18	27	54
	1849	8	11	43	24.5	5	3	18	0	1	18	28	21
	1850	2	23	0	34	5	3	19	7	1	18	28	48

TABLE I.

Epochs of the Mean Longitude of Mars, with the Arguments of the Equations.

YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.	ARG. IX.
B. O. S. 1500	221	271	493	952	155	005	149	145
B. N. S. 1600	946	573	520	374	972	339	632	192
B. 1620	893	836	730	059	337	606	731	801
B. 1640	841	098	940	744	703	874	829	411
B. 1660	788	361	150	429	070	141	928	020
B. 1680	736	623	360	113	436	408	027	630
C. 1700	683	885	569	798	802	676	126	239
B. 1720	630	147	777	483	167	943	224	848
B. 1740	578	410	987	168	533	211	323	457
B. 1760	525	672	197	853	900	478	422	067
B. 1780	472	935	407	538	266	745	521	676
C. 1800	420	197	617	223	632	013	619	286
1810	891	827	719	065	312	646	667	591
B. 1820	366	459	825	908	997	280	717	896
1821	843	822	635	992	465	343	122	926
1822	260	185	445	76	932	407	526	957
1823	707	547	255	161	400	470	931	987
B. 1824	155	911	67	245	870	533	337	18
1825	602	274	877	329	338	596	741	48
1826	49	537	687	413	805	660	146	79
1827	496	999	497	498	273	723	551	109
B. 1828	945	364	309	582	743	787	957	140
1829	392	726	119	666	211	850	362	170
1830	803	89	929	750	678	914	766	201
1831	286	452	739	835	146	977	171	231
B. 1832	734	816	551	920	617	40	576	262
1833	171	179	361	4	85	103	981	292
1834	628	542	171	88	552	167	385	323
1835	75	904	981	172	20	230	790	353
B. 1836	524	269	793	256	490	294	196	384
1837	971	632	603	340	958	357	601	414
1838	418	995	413	424	425	421	6	445
1839	863	357	223	509	893	484	411	475
B. 1840	313	722	35	593	363	547	816	505
1841	760	85	845	677	831	610	221	535
1842	207	458	655	761	298	674	625	566
1843	654	810	465	846	766	737	30	596
B. 1844	102	174	277	930	236	800	436	627
1845	549	537	87	14	704	863	840	657
1846	996	800	895	98	171	927	245	688
1847	443	262	707	183	639	990	650	718
B. 1848	892	627	519	267	109	54	56	749
1849	339	989	329	351	577	117	461	779
1850	786	352	139	435	44	181	865	810

TABLE II.

Mean Motion of Mars for *Years*, with the Arguments of the Equations.

		Mot. Longitude.				Mot. Aphe.			Mot. Node.	
YEARS.		S.	D.	M.	S.	D.	M	S.	M.	S.
	1	6	11	17	9.4	0	1	7	0	27
	2	0	22	34	18.9	0	2	14	0	55
	3	7	3	51	28.3	0	3	21	1	22
B.	4	1	15	40	4.4	0	4	28	1	49
	5	7	26	57	13.8	0	5	35	2	17
	6	2	8	14	23.3	0	6	42	2	44
	7	8	19	31	32.7	0	7	49	3	11
B.	8	3	1	20	8.9	0	8	56	3	39
	9	9	12	37	18.3	0	10	3	4	6
	10	3	23	54	27.8	0	11	10	4	33
	11	10	5	11	37.2	0	12	17	5	1
B.	12	4	17	0	13.3	0	13	24	5	28
	13	10	28	17	22.7	0	14	31	5	55
	14	5	9	34	32.2	0	15	38	6	23
	15	11	20	51	41.6	0	16	45	6	50
B.	16	6	2	40	17.7	0	17	52	7	17
	17	0	13	57	27.1	0	18	59	7	45
	18	6	25	14	36.6	0	20	6	8	12
	19	1	6	31	46.0	0	21	13	8	39
B.	20	7	18	20	22.2	0	22	20	9	7
B.	40	3	6	40	44.3	0	44	40	18	13
B.	60	10	25	1	6.5	1	7	0	27	20
B.	80	6	13	21	28.6	1	29	20	36	27
B.	100	2	1	41	50.7	1	51	40	45	33
C.	100	2	1	10	24.1	1	51	40	45	33

TABLE II.

Mean Motion of Mars for *Years*, with the Arguments of the Equations.

	YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.	ARG. IX.
	1	447	363	810	084	468	063	405	030
	2	894	726	620	168	935	127	809	061
	3	341	088	430	253	403	190	214	091
B.	4	789	452	242	337	873	253	620	122
	5	236	815	052	421	341	316	024	152
	6	683	078	862	505	808	380	429	183
	7	130	540	672	590	276	443	834	213
B.	8	579	905	484	674	746	507	240	244
	9	026	267	294	758	214	570	645	274
	10	473	630	104	842	681	634	049	305
	11	920	993	914	927	149	697	454	335
B.	12	368	357	726	012	620	760	859	366
	13	815	720	536	096	088	823	264	396
	14	262	083	346	180	555	887	668	427
	15	709	445	156	264	023	950	073	457
B.	16	158	810	968	348	493	014	479	488
	17	605	173	778	432	961	077	884	518
	18	052	536	588	516	428	141	289	549
	19	497	898	398	601	896	204	694	579
B.	20	947	263	210	685	366	267	099	609
B.	40	895	525	420	370	732	535	198	219
B.	60	842	788	629	055	099	802	296	828
B.	80	790	050	839	740	465	069	395	438
B.	100	737	312	049	425	831	337	494	047
C.	100	736	311	047	424	830	337	493	047

TABLE III.

Mean Motion of Mars for *Months*, with the Arguments of the Equations.

Months.	Mot. Long.	Mot. Aphelion.	Mot. Node.
	S. D. M. S.	SEC.	SEC.
January	0 0 0 0.0	0.0	0.0
February	0 16 14 46.3	5.7	2.3
March	1 0 55 12.7	10.8	4.4
April	1 17 9 59.0	16.5	6.8
May	2 2 53 18.7	22.0	9.0
June	2 19 8 5.1	27.7	11.3
July	3 4 51 24.7	33.2	13.6
August	3 21 6 11.1	38.9	15.9
September . . .	4 7 20 57.4	44.6	18.2
October	4 23 4 17.1	50.1	20.5
November . . .	5 9 19 3.4	55.8	22.8
December . . .	5 25 2 23.1	61.4	25.1

TABLE III.

Mean Motion of Mars for *Months*, with the Arguments of the Equations.

Months.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
January	000	000	000	000	000	000	000	000
February	038	031	069	007	040	005	034	003
March	072	059	131	014	076	010	066	005
April	110	090	200	021	115	015	100	008
May	147	120	267	028	154	021	133	010
June	185	150	336	035	194	026	168	013
July	221	180	401	042	232	031	201	015
August	259	211	470	049	272	036	235	018
September . . .	298	242	539	056	311	042	269	020
October	334	272	606	063	350	047	303	023
November . . .	372	302	674	070	389	053	337	026
December. . . .	409	332	741	077	428	058	370	028

TABLE IV.

Mean Motion of Mars for *Days*, with the Arguments of the Equations.

Days of Month.	Mot. Longitude.			Mot. Aphelion.	Mot. Node.
	D.	M.	S.	SEC.	SEC.
1	0	31	26.7	0.2	0.1
2	1	2	53.3	0.4	0.2
3	1	34	20.0	0.5	0.2
4	2	5	46.6	0.7	0.3
5	2	37	13.3	0.9	0.4
6	3	8	39.9	1.1	0.5
7	3	40	6.6	1.3	0.5
8	4	11	33.2	1.5	0.6
9	4	42	59.9	1.6	0.7
10	5	14	26.6	1.8	0.8
11	5	45	53.2	2.0	0.8
12	6	17	19.9	2.2	0.9
13	6	48	46.5	2.4	1.0
14	7	20	13.2	2.6	1.1
15	7	51	39.8	2.7	1.1
16	8	23	6.5	2.9	1.2
17	8	54	33.2	3.1	1.3
18	9	25	59.8	3.3	1.4
19	9	57	26.5	3.5	1.4
20	10	28	53.1	3.7	1.5
21	11	0	19.8	3.8	1.6
22	11	31	46.4	4.0	1.7
23	12	3	13.1	4.2	1.7
24	12	34	39.7	4.4	1.8
25	13	6	6.4	4.6	1.9
26	13	37	33.1	4.8	2.0
27	14	8	59.7	4.9	2.0
28	14	40	26.4	5.1	2.1
29	15	11	53.0	5.3	2.2
30	15	43	19.7	5.5	2.3
31	16	14	46.3	5.7	2.3

In the Months JANUARY and FEBRUARY of a Bissextile Year, subtract 1 from the given Day of the Month.

TABLE IV.

Mean Motion of Mars for *Days*, with the Arguments of the Equations.

Days of Month.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
1	001	001	002	000	001	000	001	000
2	002	002	004	000	003	000	002	000
3	004	003	007	001	004	000	003	000
4	005	004	009	001	005	001	004	000
5	006	005	011	001	006	001	006	000
6	007	006	013	001	008	001	007	001
7	009	007	016	002	009	001	008	001
8	010	008	018	002	010	001	009	001
9	011	009	020	002	012	002	010	001
10	012	010	022	002	013	002	011	001
11	014	011	024	003	014	002	012	001
12	015	012	027	003	015	002	013	001
13	016	013	029	003	017	002	014	001
14	017	014	031	003	018	002	016	001
15	018	015	033	003	019	003	017	001
16	020	016	036	064	020	003	018	001
17	021	017	038	004	022	003	019	001
18	022	018	040	004	023	003	020	002
19	023	019	042	004	024	003	021	002
20	025	020	044	005	026	003	022	002
21	026	021	047	005	027	004	023	002
22	027	022	009	005	028	004	024	002
23	028	023	051	005	030	004	026	002
24	029	024	053	006	031	004	027	002
25	031	025	056	006	032	004	028	002
26	032	026	058	006	033	004	029	002
27	033	027	060	006	035	005	030	002
28	034	028	062	006	036	005	031	002
29	036	029	064	007	037	005	032	002
30	037	030	067	007	038	005	033	003
31	038	031	069	007	040	005	034	003

TABLE V.

Mean Motion of Mars for *Hours*, with the Arguments of the Equations.

For Hours.

Hours.	Mot. Long. M.	s.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
1	1	18.6	0	0	0	0	0	0	0	0
2	2	37.2	0	0	0	0	0	0	0	0
3	3	55.8	0	0	0	0	0	0	0	0
4	5	14.4	0	0	0	0	0	0	0	0
5	6	33.1	0	0	0	0	0	0	0	0
6	7	51.7	0	0	1	0	0	0	0	0
7	9	10.3	0	0	1	0	0	0	0	0
8	10	28.9	0	0	1	0	0	0	0	0
9	11	47.5	0	0	1	0	0	0	0	0
10	13	6.1	0	0	1	0	0	0	0	0
11	14	24.7	1	0	1	0	1	0	0	0
12	15	43.3	1	0	1	0	1	0	1	0
13	17	1.9	1	1	1	0	1	0	1	0
14	18	20.5	1	1	1	0	1	0	1	0
15	19	39.1	1	1	1	0	1	0	1	0
16	20	57.8	1	1	1	0	1	0	1	0
17	22	16.4	1	1	2	0	1	0	1	0
18	23	35.0	1	1	2	0	1	0	1	0
19	24	53.6	1	1	2	0	1	0	1	0
20	26	12.2	1	1	2	0	1	0	1	0
21	27	30.8	1	1	2	0	1	0	1	0
22	28	49.4	1	1	2	0	1	0	1	0
23	30	8.0	1	1	2	0	1	0	1	0
24	31	26.7	1	1	2	0	1	0	1	0

TABLE VI.

Mean Motion of Mars for *Minutes* and *Seconds*.

For Minutes.						*For Seconds.*	
Min.	Mot. Long. M.	S.	Min.	Mot. Long. M.	S.	Sec.	Mot. Long. SEC.
1	0	1.3	31	0	40.6	1	0.02
2	0	2.6	32	0	41.9	2	0.04
3	0	3.9	33	0	43.2	3	0.07
4	0	5.2	34	0	44.5	4	0.09
5	0	6.6	35	0	45.9	5	0.11
6	0	7.9	36	0	47.2	6	0.13
7	0	9.2	37	0	48.5	7	0.15
8	0	10.5	38	0	49.8	8	0.17
9	0	11.8	39	0	51.1	9	0.20
10	0	13.1	40	0	52.4	10	0.22
11	0	14.4	41	0	53.7	20	0.44
12	0	15.7	42	0	55.0	30	0.65
13	0	17.0	43	0	56.3	40	0.87
14	0	18.3	44	0	57.6	50	1.09
15	0	19.7	45	0	59.0		
16	0	21.0	46	1	0.3		
17	0	22.3	47	1	1.6		
18	0	23.6	48	1	2.9		
19	0	24.9	49	1	4.2		
20	0	26.2	50	1	5.5		
21	0	27.5	51	1	6.8		
22	0	28.8	52	1	8.1		
23	0	30.1	53	1	9.4		
24	0	31.4	54	1	10.7		
25	0	32.8	55	1	12.1		
26	0	34.1	56	1	13.4		
27	0	35.4	57	1	14.7		
28	0	36.7	58	1	16.0		
29	0	38.0	59	1	17.3		
30	0	39.3	60	1	18.6		

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. O. —	Diff.	Var. —	Sig. I. —	Diff.	Var. —	Sig. II. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
0. 0	0 0 0.0	100.1	0.0	4 50 39.5	90.3	15.2	8 42 26.8	60.9	28.2	30. 0
10	0 1 40.2	100.1	0.1	4 52 9.8	90.2	15.2	8 43 27.7	60.6	28.2	29.50
20	0 3 20.3	100.1	0.2	4 53 40.0	90.1	15.3	8 44 28.3	60.4	28.3	40
30	0 5 0.4	100.2	0.2	4 55 10.1	90.0	15.4	8 45 28.7	60.1	28.4	30
40	0 6 40.6	100.1	0.3	4 56 40.1	89.8	15.5	8 46 28.8	59.9	28.4	20
50	0 8 20.7	100.1	0.4	4 58 9.9	89.8	15.6	8 47 28.7	59.6	28.5	10
1. 0	0 10 0.8	100.1	0.5	4 59 39.7	89.7	15.7	8 48 28.3	59.4	28.6	29. 0
10	0 11 40.9	100.1	0.6	5 1 9.4	89.5	15.7	8 49 27.7	59.2	28.6	28.50
20	0 13 21.0	100.1	0.6	5 2 38.9	89.4	15.8	8 50 26.9	59.0	28.7	40
30	0 15 1.1	100.1	0.7	5 4 8.3	89.3	15.9	8 51 25.9	58.8	28.8	30
40	0 16 41.2	100.1	0.8	5 5 37.6	89.2	16.0	8 52 24.7	58.6	28.8	20
50	0 18 21.4	100.2	0.9	5 7 6.8	89.1	16.1	8 52 23.3	58.4	28.9	10
2. 0	0 20 1.5	100.1	1.0	5 8 35.9	89.0	16.2	8 54 21.7	58.2	28.9	28. 0
10	0 21 41.5	100.0	1.0	5 10 4.9	88.9	16.2	8 55 19.9	58.0	29.0	27.50
20	0 23 21.6	100.1	1.1	5 11 33.8	88.7	16.3	8 56 17.9	57.7	29.1	40
30	0 25 1.7	100.1	1.2	5 13 2.5	88.6	16.4	8 57 15.6	57.5	29.1	30
40	0 26 41.7	100.0	1.3	5 14 31.1	88.4	16.4	8 58 13.1	57.2	29.2	20
50	0 28 21.8	100.1	1.4	5 15 59.5	88.2	16.5	8 59 10.3	56.9	29.2	10
3. 0	0 30 1.9	100.1	1.5	5 17 27.7	88.1	16.6	9 0 7.2	56.7	29.3	27. 0
10	0 31 41.9	100.0	1.5	5 18 55.8	88.1	16.6	9 1 3.9	56.5	29.4	26.50
20	0 33 21.9	100.0	1.6	5 20 23.9	88.0	16.7	9 2 0.4	56.3	29.4	40
30	0 35 2.0	100.1	1.7	5 21 51.9	87.9	16.8	9 2 56.7	56.1	29.5	30
40	0 36 42.0	100.0	1.8	5 23 19.8	87.8	16.9	9 3 52.8	55.9	29.5	20
50	0 38 22.0	100.0	1.9	5 24 47.6	87.6	17.0	9 4 48.7	55.6	29.6	10
4. 0	0 40 2.1	100.1	2.0	5 26 15.2	87.5	17.1	9 5 44.3	55.4	29.6	26. 0
10	0 41 42.1	100.0	2.1	5 27 42.7	87.4	17.1	9 6 39.7	55.2	29.7	25.50
20	0 43 22.0	99.9	2.2	5 29 10.1	87.3	17.2	9 7 34.9	54.9	29.8	40
30	0 45 1.9	99.9	2.3	5 30 37.4	87.1	17.3	9 8 29.8	54.7	29.8	30
40	0 46 41.8	99.9	2.4	5 32 4.5	87.0	17.4	9 9 24.5	54.5	29.9	20
50	0 48 21.8	100.0	2.5	5 33 31.5	86.9	17.5	9 10 19.0	54.3	30.0	10
5. 0	0 50 1.7	99.9	2.6	5 34 58.4	86.8	17.6	9 11 13.3	54.1	30.0	25. 0
10	0 51 41.5	99.8	2.6	5 36 25.2	86.7	17.6	9 12 7.4	53.8	30.1	24.50
20	0 53 21.4	99.9	2.7	5 37 51.9	86.5	17.7	9 13 1.2	53.5	30.1	40
30	0 55 1.3	99.9	2.8	5 39 18.4	86.3	17.8	9 13 54.7	53.2	30.2	30
40	0 56 41.1	99.8	2.9	5 40 44.7	86.2	17.9	9 14 47.9	53.0	30.2	20
50	0 58 20.9	99.8	3.0	5 42 10.9	86.1	18.0	9 15 40.9	52.7	30.3	10
6. 0	1 0 0.7	99.8	3.1	5 43 37.0		18.1	9 16 33.6		30.3	24. 0
	Sig. XI. +		+	Sig. X. +		+	Sig. IX. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMEMT. *The Mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. III. —	Diff.	Var. —	Sig. IV. —	Diff.	Var. —	Sig. V. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
0. 0	10 37 16.4	12.6	36.4	9 46 26.6	48.0	35.6	5 55 36.7	103.3	22.6	30. 0
10	10 37 29.0	12.3	36.4	9 45 38.6	48.3	35.6	5 53 53.4	103.5	22.6	29. 50
20	10 37 41.3	12.0	36.4	9 44 50.3	48.6	35.5	5 52 9.9	103.7	22.5	40
30	10 37 53.3	11.7	36.4	9 44 1.7	48.9	35.5	5 50 26.2	103.9	22.4	30
40	10 38 5.0	11.3	36.4	9 43 12.8	49.2	35.4	5 48 42.3	104.2	22.3	20
50	10 38 16.3	11.1	36.4	9 42 23.6	49.5	35.4	5 46 58.1	104.4	22.2	10
1. 0	10 38 27.4	10.8	36.5	9 41 34.1	49.9	35.3	5 45 13.7	104.7	22.1	29. 0
10	10 38 38.2	10.4	36.5	9 40 44.2	50.3	35.3	5 43 29.0	105.0	21.9	28. 50
20	10 38 48.6	10.1	36.5	9 39 53.9	50.7	35.2	5 41 44.0	105.2	21.8	40
30	10 38 58.7	9.8	36.5	9 39 3.2	51.1	35.2	5 39 58.8	105.4	21.7	30
40	10 39 8.5	9.5	36.5	9 38 12.1	51.4	35.1	5 38 13.4	105.7	21.6	20
50	10 39 18.0	9.2	36.5	9 37 20.7	51.7	35.1	5 36 27.7	106.0	21.5	10
2. 0	10 39 27.2	8.9	36.6	9 36 29.0	52.0	35.0	5 34 41.7	106.2	21.4	28. 0
10	10 39 36.1	8.6	36.6	9 35 37.0	52.4	35.0	5 32 55.5	106.4	21.3	27. 50
20	10 39 44.7	8.3	36.6	9 34 44.6	52.7	34.9	5 31 9.1	106.6	21.2	40
30	10 39 53.0	7.9	36.6	9 33 51.9	53.0	34.9	5 29 22.5	106.8	21.1	30
40	10 40 0.9	7.5	36.6	9 32 58.9	53.4	34.9	5 27 35.7	106.9	21.0	20
50	10 40 8.4	7.2	36.6	9 32 5.5	53.7	34.8	5 25 48.8	107.1	20.9	10
3. 0	10 40 15.6	6.9	36.7	9 31 11.8	54.0	34.8	5 24 1.7	107.4	20.8	27. 0
10	10 40 22.5	6.6	36.7	9 30 17.8	54.4	34.8	5 22 14.3	107.7	20.7	26. 50
20	10 40 29.1	6.3	36.7	9 29 23.4	54.7	34.7	5 20 26.6	107.9	20.5	40
30	10 40 35.4	6.0	36.7	9 28 28.7	55.1	34.7	5 18 38.7	108.2	20.4	30
40	10 40 41.4	5.7	36.7	9 27 33.6	55.5	34.7	5 16 50.5	108.4	20.3	20
50	10 40 47.1	5.5	36.7	9 26 38.1	55.8	34.6	5 15 2.1	108.6	20.2	10
4. 0	10 40 52.6	5.1	36.8	9 25 42.3	56.1	34.6	5 13 13.5	108.8	20.1	26. 0
10	10 40 57.7	4.8	36.8	9 24 46.2	56.5	34.6	5 11 24.7	109.0	20.0	25. 50
20	10 41 2.5	4.4	36.8	9 23 49.7	56.8	34.5	5 9 35.7	109.2	19.8	40
30	10 41 6.9	4.1	36.8	9 22 52.9	57.1	34.5	5 7 46.5	109.5	19.7	30
40	10 41 11.0	3.8	36.8	9 21 55.8	57.4	34.4	5 5 57.0	109.7	19.6	20
50	10 41 14.8	3.4	36.8	9 20 58.4	57.8	34.4	5 4 7.3	110.0	19.5	10
5. 0	10 41 18.2	3.1	36.9	9 20 0.6	58.1	34.3	5 2 17.3	110.1	19.4	25. 0
10	10 41 21.3	2.8	36.9	9 19 2.5	58.5	34.3	5 0 27.2	110.3	19.3	24. 50
20	10 41 24.1	2.5	36.9	9 18 4.0	58.8	34.2	4 58 36.9	110.5	19.1	40
30	10 41 26.6	2.1	36.9	9 17 5.2	59.2	34.2	4 56 46.4	110.7	19.0	30
40	10 41 28.7	1.8	36.9	9 16 6.0	59.6	34.1	4 54 55.7	110.9	18.9	20
50	10 41 30.5	1.5	36.9	9 15 6.4	59.9	34.1	4 53 4.8	111.1	18.8	10
6. 0	10 41 32.0		37.0	9 14 6.5		34.0	4 51 13.7		18.7	24. 0
	Sig. VIII +		+	Sig. VII. +		+	Sig. VI. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. O. —	Diff.	Var. —	Sig. I. —	Diff.	Var. —	Sig. II. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
6. 0	1 0 0.7	99.7	3.1	5 43 37.0	86.0	18.1	9 16 33.6	52.5	30.3	24. 0
10	1 1 40.4	99.7	3.1	5 45 3.0	85.9	18.1	9 17 26.1	52.3	30.3	23. 50
20	1 3 20.1	99.6	3.2	5 46 28.9	85.7	18 2	9 18 18.4	52.1	30.4	40
30	1 4 59.7	99.7	3.3	5 47 54.6	85.6	18.3	9 19 10.5	51.9	30.4	30
40	1 6 39.4	99.7	3.4	5 49 20.2	85.4	18.3	9 20 2.4	51.7	30.5	20
50	1 8 19.1	99.6	3.5	5 50 45.6	85.2	18.4	9 20 54.1	51.4	30.6	10
7. 0	1 9 58.7	99.6	3.6	5 52 10.8	85.1	18.5	9 21 45.5	51.2	30.7	23. 0
10	1 11 38.3	99.6	3.6	5 53 35.9	85.0	18.5	9 22 36.7	50.9	30.7	22. 50
20	1 13 17.9	99.6	3.7	5 55 0.9	84.9	18.6	9 23 27.6	50.7	30.8	40
30	1 14 57.5	99.6	3.8	5 56 25.8	84.8	18.7	9 24 18.3	50.4	30.8	30
40	1 16 37.0	99.5	3.9	5 57 50.6	84.7	18.8	9 25 8.7	50.1	30.9	20
50	1 18 16.5	99.5	4.0	5 59 15.3	84.6	18.9	9 25 58.8	49.8	30.9	10
8. 0	1 19 56.0	99.4	4.1	6 0 39.9	84.4	19.0	9 26 48.6	49.6	31.0	22. 0
10	1 21 35.4	99.4	4.1	6 2 4.3	84.2	19.0	9 27 38.2	49.4	31.0	21. 50
20	1 23 14.8	99.4	4.2	6 3 28.5	84.0	19.1	9 28 27.6	49.2	31.1	40
30	1 24 54.2	99.3	4.3	6 4 52.5	83.9	19.2	9 29 16.8	48.9	31.1	30
40	1 26 33.5	99.3	4.4	6 6 16.4	83.8	19.3	9 30 5.7	48.7	31.2	20
50	1 28 12.8	99.4	4.5	6 7 40.2	83.6	19.3	9 30 54.4	48.4	31.2	10
9. 0	1 29 52.2	99.2	4.6	6 9 3.8	83.5	19.4	9 31 42.8	48.2	31.3	21. 0
10	1 31 31.4	99.2	4.7	6 10 27.3	83.4	19.5	9 32 31.0	48.0	31.3	20. 50
20	1 33 10.6	99.2	4.8	6 11 50.7	83.2	19.6	9 33 19.0	47.7	31.4	40
30	1 34 49.8	99.2	4.9	6 13 13.9	83.1	19.6	9 34 6.7	47.4	31.4	30
40	1 36 29.0	99.1	5.0	6 14 37.0	83.0	19.7	9 34 54.1	47.1	31.5	20
50	1 38 8.1	99.1	5.1	6 16 0.0	82.8	19.8	9 35 41.2	46.9	31.5	10
10. 0	1 39 47.2	99.1	5.2	6 17 22.8	82.6	19.9	9 36 28.1	46.6	31.6	20. 0
10	1 41 26.3	99.0	5.2	6 18 45.4	82.5	19.9	9 37 14.7	46.4	31.6	19. 50
20	1 43 5.3	98.9	5.3	6 20 7.9	82.4	20.0	9 38 1.1	46.2	31.7	40
30	1 44 44.2	98.9	5.3	6 21 30.3	82.2	20.1	9 38 47.3	45.9	31.7	30
40	1 46 23.1	98.9	5.4	6 22 52.5	82.0	20.2	9 39 33.2	45.7	31.8	20
50	1 48 2.0	98.8	5.5	6 24 14.5	81.8	20.2	9 40 18.9	45.5	31.8	10
11. 0	1 49 40.8	98.8	5.6	6 25 36.3	81.7	20.3	9 41 4.4	45.2	31.9	19. 0
10	1 51 19.6	98.8	5.6	6 26 58.0	81.6	20.4	9 41 49.6	44.9	31.9	18. 50
20	1 52 58.4	98.8	5.7	6 28 19.6	81.5	20.5	9 42 34.5	44.6	32.0	40
30	1 54 37.2	98.7	5.8	6 29 41.1	81.4	20.5	9 43 19.1	44.4	32.0	30
40	1 56 15.9	98.7	5.9	6 31 2.5	81.2	20.6	9 44 3.5	43.2	32.1	20
50	1 57 54.6	98.6	6.0	6 32 23.7	81.0	20.7	9 44 47.7	43.9	32.1	10
12. 0	1 59 33.2		6.1	6 33 44.8		20.8	9 45 31.6		32.2	18. 0
	Sig. XI. +		+	Sig. X. +		+	Sig. IX. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. III. —	Diff.	Var. —	Sig. IV. —	Diff.	Var. —	Sig. V. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
6. 0	10 41 32.0	1.2	37.0	9 14 6.5	60.2	34.0	4 51 13.7	111.3	18.7	24. 0
10	10 41 33.2	0.9	37.0	9 13 6.3	60.6	34.0	4 49 22.4	111.5	18.6	23. 50
20	10 41 34.1	0.5	37.0	9 12 5.7	60.9	33.9	4 47 30.9	111.7	18.5	40
30	10 41 34.6	0.2	37.0	9 11 4.8	61.2	33.9	4 45 39.2	111.9	18.4	30
40	10 41 34.8	0.2	37.0	9 10 3.6	61.5	33.8	4 43 47.3	112.2	18.3	20
50	10 41 34.6	0.5	37.0	9 9 2.1	61.8	33.8	4 41 55.1	112.4	18.2	10
7. 0	10 41 34.1	0.8	37.1	9 8 0.3	62.2	33.7	4 40 2.7	112.5	18.0	23. 0
10	10 41 33.3	1.1	37.1	9 6 58.1	62.5	33.7	4 38 10.2	112.7	17.9	22. 50
20	10 41 32.2	1.4	37.1	9 5 55.6	62.9	33.6	4 36 17.5	112.9	17.8	40
30	10 41 30.8	1.7	37.1	9 4 52.7	63.3	33.6	4 34 24.6	113.1	17.7	30
40	10 41 29.1	2.1	37.1	9 3 49.4	63.6	33.5	4 32 31.5	113.3	17.6	20
50	10 41 27.0	2.4	37.1	9 2 45.8	63.8	33.4	4 30 38.2	113.6	17.5	10
8. 0	10 41 24.6	2.7	37.2	9 1 42.0	64.2	33.3	4 28 44.6	113.7	17.3	22. 0
10	10 41 21.9	3.0	37.2	9 0 37.8	64.5	33.3	4 26 50.9	113.9	17.2	21. 50
20	10 41 18.9	3.4	37.2	8 59 33.3	64.8	33.2	4 24 57.0	114.0	17.1	40
30	10 41 15.5	3.8	37.2	8 58 28.5	65.2	33.2	4 23 3.0	114.2	17.0	30
40	10 41 11.7	4.1	37.2	8 57 23.3	65.6	33.1	4 21 8.8	114.4	16.9	20
50	10 41 7.6	4.5	37.2	8 56 17.7	65.9	33.1	4 19 14.2	114.6	16.8	10
9. 0	10 41 3.1	4.8	37.2	8 55 11.8	66.2	33.0	4 17 19.8	114.8	16.6	21. 0
10	10 40 58.3	5.1	37.2	8 54 5.6	66.5	32.9	4 15 25.0	115.0	16.5	20. 50
20	10 40 53.2	5.4	37.2	8 52 59.1	66.8	32.9	4 13 30.0	115.1	16.4	40
30	10 40 47.8	5.7	37.2	8 51 52.3	67.2	32.8	4 11 34.9	115.3	16.2	30
40	10 40 42.1	6.1	37.2	8 50 45.1	67.6	32.8	4 9 36.6	115.5	16.1	20
50	10 40 36.0	6.4	37.2	8 49 37.5	67.9	32.7	4 7 44.1	115.8	16.0	10
10. 0	10 40 29.6	6.7	37.3	8 48 29.6	68.2	32.7	4 5 48.3	115.9	15.8	20. 0
10	10 40 22.9	7.1	37.3	8 47 21.4	68.5	32.6	4 3 52.4	116.0	15.7	19. 50
20	10 40 15.8	7.4	37.3	8 46 12.9	68.9	32.6	4 1 56.4	116.2	15.6	40
30	10 40 8.4	7.7	37.3	8 45 4.0	69.2	32.5	4 0 0.2	116.4	15.5	30
40	10 40 0.7	8.1	37.3	8 43 54.8	69.5	32.4	3 58 3.8	116.5	15.4	20
50	10 39 52.6	8.4	37.3	8 42 45.3	69.8	32.3	3 56 7.3	116.7	15.3	10
11. 0	10 39 44.2	8.7	37.3	8 41 35.5	70.1	32.3	3 54 10.6	116.9	15.1	19. 0
10	10 39 35.5	9.1	37.3	8 40 25.4	70.5	32.2	3 52 13.7	117.1	15.0	18. 50
20	10 39 26.4	9.4	37.3	8 39 14.9	70.8	32.1	3 50 16.6	117.2	14.9	40
30	10 39 17.0	9.8	37.3	8 38 4.1	71.2	32.1	3 48 19.4	117.3	14.8	30
40	10 39 7.2	10.1	37.3	8 36 52.9	71.5	32.0	3 46 22.1	117.5	14.7	20
50	10 38 57.1	10.5	37.3	8 35 41.4	71.8	31.9	3 44 24.6	117.7	14.6	10
12. 0	10 38 46.6		37.4	8 34 29.6		31.8	3 42 26.9		14.4	18. 0
	Sig. VIII +		+	Sig. VII +		+	Sig. VI. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. O. —	Diff.	Var. —	Sig. I. —	Diff.	Var. —	Sig. II. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
12. 0	1 59 33.2	98.5	6.1	6 33 44.8	80.9	20.8	9 45 31.6	43.6	32.2	18. 0
10	2 1 11.7	98.5	6.2	6 35 5.7	80.7	20.9	9 46 15.2	43.3	32.2	17.50
20	2 2 50.2	98.5	6.3	6 36 26.4	80.6	21.0	9 46 58.5	43.1	32.3	40
30	2 4 28.7	98.4	6.4	6 37 47.0	80.4	21.0	9 47 41.6	42.8	32.4	30
40	2 6 7.1	98.4	6.5	6 39 7.4	80.2	21.1	9 48 24.4	42.5	32.4	20
50	2 7 45.5	98.4	6.6	6 40 27.6	80.0	21.2	9 49 6.9	42.2	32.5	10
13. 0	2 9 23.9	98.3	6.7	6 41 47.6	79.9	21.3	9 49 49.1	42.0	32.5	17. 0
10	2 11 2.2	98.3	6.8	6 43 7.5	79.7	21.3	9 50 31.1	41.7	32.6	16.50
20	2 12 40.5	98.2	6.9	6 44 27.2	79.6	21.4	9 51 12.8	41.5	32.6	40
30	2 14 18.7	98.1	7.0	6 45 46.8	79.4	21.5	9 51 54.3	41.3	32.7	30
40	2 15 56.8	98.1	7.1	6 47 6.2	79.3	21.6	9 52 35.6	41.1	32.7	20
50	2 17 34.9	98.1	7.2	6 48 25.5	79.1	21.6	9 53 16.7	40.8	32.8	10
14. 0	2 19 13.0	98.0	7.3	6 49 44.6	78.9	21.7	9 53 57.5	40.5	32.8	16. 0
10	2 20 51.0	97.9	7.3	6 51 3.5	78.8	21.7	9 54 38.0	40.2	32.9	15.50
20	2 22 28.9	97.9	7.4	6 52 22.3	78.7	21.8	9 55 18.2	39.9	32.9	40
30	2 24 6.8	97.9	7.5	6 53 41.0	78.5	21.9	9 55 58.1	39.6	32.9	30
40	2 25 44.7	97.8	7.6	6 54 59.5	78.4	21.9	9 56 37.7	39.4	33.0	20
50	2 27 22.5	97.7	7.7	6 56 17.9	78.3	22.1	9 57 17.1	39.2	33.0	10
15. 0	2 29 0.2	97.6	7.8	6 57 36.2	78.1	22.2	9 57 56.3	38.9	33.1	15. 0
10	2 30 37.8	97.5	7.8	6 58 54.3	77.9	22.2	9 58 35.2	38.6	33.1	14.50
20	2 32 15.3	97.5	7.9	7 0 12.2	77.7	22.3	9 59 13.8	38.3	33.2	40
30	2 33 52.8	97.5	7.9	7 1 29.9	77.5	22.4	9 59 52.1	38.0	33.2	30
40	2 35 30.3	97.5	8.0	7 2 47.4	77.2	22.4	10 0 30.1	37.8	33.2	20
50	2 37 7.8	97.4	8.1	7 4 4.6	77.0	22.5	10 1 7.9	37.5	33.3	10
16. 0	2 38 45.2	97.3	8.2	7 5 21.6	76.9	22.5	10 1 45.4	37.2	33.3	14. 0
10	2 40 22.5	97.3	8.2	7 6 38.5	76.8	22.6	10 2 22.6	36.9	33.3	13.50
20	2 41 59.8	97.3	8.3	7 7 55.3	76.7	22.7	10 2 59.5	36.7	33.4	40
30	2 43 37.1	97.2	8.4	7 9 12.0	76.6	22.8	10 3 36.2	36.4	33.4	30
40	2 45 14.3	97.1	8.5	7 10 28.6	76.4	22.9	10 4 12.6	36.1	33.5	20
50	2 46 51.4	97 0	8.6	7 11 45.0	76.1	23.0	10 4 48.7	35.9	33.5	10
17. 0	2 48 28.4	96 9	8.7	7 13 1.1	75.9	23.1	10 5 24.6	35.6	33.6	13. 0
10	2 50 5.3	96.9	8.7	7 14 17.0	75.8	23.1	10 6 0.2	35.3	33.6	12.50
20	2 51 42.2	96.8	8.8	7 15 32.8	75.6	23.2	10 6 35.5	35.0	33.7	40
30	2 53 19.0	96.8	8.9	7 16 48.4	75.5	23.3	10 7 10.5	34.8	33.7	30
40	2 54 55.8	96.8	9.0	7 18 3.9	75.3	23.3	10 7 45.3	34.5	33.7	20
50	2 56 32.6	96.7	9.1	7 19 19.2	75.2	23.4	10 8 19.8	34.2	33.8	10
18. 0	2 58 9.3		9.2	7 20 34.4			10 8 54.0		33.8	12. 0
	Sig. XI. +		+	Sig. X. +		+	Sig. IX. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMEMT. *The Mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. III. —	Diff.	Var. —	Sig. IV. —	Diff.	Var. —	Sig. V. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
12. 0	10 38 46.6	10.8	37.4	8 34 29.6	72.1	31.8	3 42 26.9	117.8	14.4	18. 0
10	10 38 35.8	11.1	37.4	8 33 17.5	72.4	31.8	3 40 29.1	118.0	14.2	17. 50
20	10 38 24.7	11.4	37.4	8 32 5.1	72.8	31.7	3 38 31.1	118.2	14.1	40
30	10 38 13.3	11.7	37.4	8 30 52.3	73.1	31.6	3 36 32.9	118.3	14.0	30
40	10 38 1.6	12.1	37.4	8 29 39.2	73.4	31.5	3 34 34.6	118.5	13.9	20
50	10 37 49.5	12.4	37.4	8 28 25.8	73.7	31.4	3 32 36.1	118.7	13.8	10
13. 0	10 37 37.1	12.8	37.4	8 27 12.1	74.0	31.4	3 30 37.4	118.8	13.6	17. 0
10	10 37 24.3	13.1	37.4	8 25 58.1	74.3	31.3	3 28 38.6	119.0	13.5	16. 50
20	10 37 11.2	13.5	37.4	8 24 43.8	74.6	31.2	3 26 39.6	119.1	13.4	40
30	10 36 57.7	13.8	37.4	8 23 29.2	75.0	31.2	3 24 40.5	119.2	13.2	30
40	10 36 43.9	14.1	37.4	8 22 14.2	75.3	31.1	3 22 41.3	119.3	13.1	20
50	10 36 29.8	14.4	37.4	8 20 58.9	75.7	31.0	3 20 42.0	119.4	13.0	10
14. 0	10 36 15.4	14.7	37.4	8 19 43.2	76.0	31.0	3 18 42.6	119.6	12.9	16. 0
10	10 36 0.7	15.1	37.4	8 18 27.2	76.3	30.9	3 16 43.0	119.7	12.7	15. 50
20	10 35 45.6	15.5	37.4	8 17 10.9	76.6	30.8	3 14 43.3	119.9	12.6	40
30	10 35 30.1	15.8	37.4	8 15 54.3	76.9	30.8	3 12 43.4	120.1	12.4	30
40	10 35 14.3	16.2	37.4	8 14 37.4	77.2	30.7	3 10 43.3	120.3	12.2	20
50	10 34 58.1	16.6	37.4	8 13 20.2	77.5	30.6	3 8 43.0	120.4	12.1	10
15. 0	10 34 41.5	16.9	37.4	8 12 2.7	77.8	30.6	3 6 42.6	120.5	12.0	15. 0
10	10 34 24.6	17.2	37.4	8 10 44.9	78.1	30.5	3 4 42.1	120.6	11.9	14. 50
20	10 34 7.4	17.6	37.4	8 9 26.8	78.5	30.5	3 2 41.5	120.7	11.8	40
30	10 33 49.8	17.9	37.4	8 8 8.3	78.8	30.4	3 0 40.8	120.8	11.7	30
40	10 33 31.9	18.2	37.4	8 6 49.5	79.1	30.3	2 58 40.0	121.0	11.6	20
50	10 33 13.7	18.5	37.4	8 5 30.4	79.4	30.2	2 56 39.0	121.1	11.5	10
16. 0	10 32 55.2	18.8	37.3	8 4 11.0	79.7	30.2	2 54 37.9	121.2	11.3	14. 0
10	10 32 36.4	19.2	37.3	8 2 51.3	80.0	30.1	2 52 36.7	121.4	11.1	13. 50
20	10 32 17.2	19.6	37.3	8 1 31.3	80.4	30.0	2 50 35.3	121.5	11.0	40
30	10 31 57.6	19.9	37.3	8 0 10.9	80.7	29.9	2 48 33.8	121.6	10.9	30
40	10 31 37.7	20.3	37.3	7 58 50.2	81.0	29.9	2 46 32.2	121.8	10.7	20
50	10 31 17.4	20.7	37.3	7 57 29.2	81.3	29.8	2 44 30.4	121.9	10.6	10
17. 0	10 30 56.7	21.0	37.3	7 56 7.9	81.6	29.7	2 42 28.5	122.0	10.5	13. 0
10	10 30 35.7	21.3	37.3	7 54 46.3	81.9	29.6	2 40 26.5	122.1	10.4	12. 50
20	10 30 14.4	21.6	37.3	7 53 24.4	82.2	29.6	2 38 24.4	122.2	10.2	40
30	10 29 52.8	22.0	37.3	7 52 2.2	82.5	29.5	2 36 22.2	122.3	10.1	30
40	10 29 30.8	22.3	37.3	7 50 39.7	82.8	29.4	2 34 19.9	122.4	10.0	20
50	10 29 8.5	22.6	37.3	7 49 16.9	83.1	29.4	2 32 17.5	122.5	9.9	10
18. 0	10 28 45.9		37.3	7 47 53.8		29.3	2 30 15.0		9.7	12. 0
	Sig. VIII +		+	Sig. VII. +		+	Sig. VI. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. O. —	Diff.	Var. —	Sig. I. —	Diff.	Var. —	Sig. II. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
18. 0	2 58 9.3	96.7	9.2	7 20 34.4	75.0	23.4	10 8 54.0	33.9	33.8	12. 0
10	2 59 46.0	96.6	9.3	7 21 49.4	74.8	23.5	10 9 27.9	33.7	33.8	11.50
20	3 1 22.6	96.4	9.4	7 23 42.	74.6	23.5	10 10 1.6	33.4	33.9	40
30	3 2 59.0	96.3	9.5	7 24 18.8	74.4	23.6	10 10 35.0	33.1	33.9	30
40	3 4 35.3	96.2	9.6	7 25 33.2	74.2	23.6	10 11 8.1	32.8	34.0	20
50	3 6 11.5	96.2	9.6	7 26 47.4	74.0	23.7	10 11 40.9	32.5	34.0	10
19. 0	3 7 47.7	96.2	9.7	7 28 1.4	73.9	23.8	10 12 13.4	32.3	34.1	11. 0
10	3 9 23.9	96.1	9.8	7 29 15.3	73.8	23.9	10 12 45.7	32.0	34.1	10.50
20	3 11 0.1	96.1	9.9	7 30 29.1	73.6	24.0	10 13 17.7	31.7	34.2	40
30	3 12 36.2	96.0	10.0	7 31 42.7	73.4	24.1	10 13 49.4	31.4	34.2	30
40	3 14 12.2	95.9	10.1	7 32 56.1	73.2	24.1	10 14 20.8	31.1	34.3	20
50	3 15 48.1	95.9	10.2	7 34 9.3	72.9	24.2	10 14 51.9	30.9	34.3	10
20. 0	3 17 24.0	95 8	10.3	7 35 22.2	72.8	24.3	10 15 22.8	30.6	34.4	10. 0
10	3 18 59.8	95.6	10.3	7 36 35.0	72.6	24.3	10 15 53.4	30.3	34.4	9.50
20	3 20 35.4	95.5	10.4	7 37 47.6	72.5	24.4	10 16 23.7	30.0	34.4	40
30	3 22 10.9	95.5	10.5	7 39 0.1	72.3	24.5	10 16 53.7	29.7	34.5	30
40	3 23 46.4	95.5	10.6	7 40 12.4	72.1	24.5	10 17 23.4	29.4	34.5	20
50	3 25 21.9	95.4	10.6	7 41 24.5	71.9	24.6	10 17 52.8	29.2	34.6	10
21. 0	3 26 57.3	95.4	10.7	7 42 36.4	71.7	24.7	10 18 22.0	28.8	3.46	9. 0
10	3 28 32.7	95.4	10.8	7 43 48.1	71.5	24.8	10 18 50.8	28.5	34.7	8. 50
20	3 30 8.1	95.3	10.9	7 44 59.6	71.3	24 8	10 19 19.3	28.2	34.7	40
30	3 31 43.4	95.2	11.0	7 46 10.9	71.1	24.9	10 19 47.5	28.0	34.7	30
40	3 33 18.6	95.2	11.1	7 47 22.0	70.9	25.0	10 20 15.5	27.7	34.8	20
50	3 34 53.8	95.1	11.1	7 48 32.9	70.6	25.0	10 20 43.2	27.4	34.8	10
22. 0	3 36 28.9	94.8	11.2	7 49 43.5	70.5	25.1	10 21 10.6	27.1	34.8	8. 0
10	3 38 3.7	94.7	11.3	7 50 54.0	70.3	25.2	10 21 37.7	26.8	34.8	7.50
20	3 39 38.4	94.5	11.4	7 52 4.3	70.2	25.2	10 22 4.5	26.5	34.9	40
30	3 41 12.9	94.4	11.5	7 53 14.5	70.0	25.3	10 22 31.0	26.2	34.9	30
40	3 42 47.3	94.3	11.6	7 54 24.5	69.9	25.3	10 22 57.2	26.0	34.9	20
50	3 44 21.6	94 3	11.6	7 55 34.4	69.8	25.4	10 23 23.2	25.7	35.0	10
23. 0	3 45 55.9	94 3	11.7	7 56 44.2	69.5	25.5	10 23 48.9	25.4	35.0	7. 0
10	3 47 30.2	94.2	11.8	7 57 53.7	69.3	25.5	10 24 14.3	25.1	35.0	6.50
20	3 49 4.4	94.2	11.9	7 59 3.0	69.1	25.6	10 24 39.4	24.8	35.1	40
30	3 50 38.6	94.1	12.0	8 0 12.1	68.8	25.7	10 25 4.2	24.5	35.1	30
40	3 52 12.7	94.0	12.1	8 1 20.9	68.5	25.7	10 25 28.7	24.2	35.2	20
50	3 53 46.7	93.9	12.1	8 2 29.4	68.3	25.8	10 25 52.9	23.9	35.2	10
24. 0	3 55 20.6		12.2	8 3 37.7		25.9	10 26 16.8		35.3	6. 0
	Sig. XI. +		+	Sig. X. +		+	Sig. IX. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. III. —	Diff.	Var. —	Sig. IV. —	Diff.	Var. —	Sig. V. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
18. 0	10 28 45.9	22.9	37.3	7 47 53.8	83.4	29.3	2 30 15.0	122.6	9.7	12. 0
10	10 28 23.0	23.3	37.3	7 46 30.4	83.7	29.3	2 28 12.4	122.7	9.6	11.50
20	10 27 59.7	23.7	37.3	7 45 6.7	84.1	29 2	2 26 9.7	122.8	9.5	40
30	10 27 36.0	24.1	37.3	7 43 42.6	84.3	29.1	2 24 6.9	122.9	9.3	30
40	10 27 11.9	24.4	37.3	7 42 18.3	84.6	29.0	2 22 4.0	123.1	9.2	20
50	10 26 47.5	24.8	37.3	7 40 53.7	84.9	28.9	2 20 0.9	123.3	9.1	10
19. 0	10 26 22.7	25.1	37.2	7 39 28.8	85.2	28.8	2 17 57.6	123.3	8.9	11. 0
10	10 25 57.6	25.4	37.2	7 38 3.6	85.5	28.8	2 15 54.3	123.4	8.8	10.50
20	10 25 32.2	25.7	37.2	7 36 38.1	85.8	28.7	2 13 50.9	123.5	8.6	40
30	10 25 6.5	26.1	37.2	7 35 12.3	86.1	28.6	2 11 47.4	123.6	8.5	30
40	10 24 40.4	26.4	37.2	7 33 46.2	86.5	28.5	2 9 43.8	123.6	8.4	20
50	10 24 14.0	26.8	37.2	7 32 19.7	86.8	28.4	2 7 40.2	123.7	8.2	10
20. 0	10 23 47.2	27.1	37.1	7 30 52.9	87.0	28.4	2 5 36.5	123.8	8.1	10. 0
10	10 23 20.1	27.4	37.1	7 29 25.9	87.4	28.3	2 3 32.7	123.9	8.0	9.50
20	10 22 52.7	27.8	37.1	7 27 58.5	87.6	28.2	2 1 28.8	124.0	7.8	40
30	10 22 24.9	28.2	37.1	7 26 30.9	87.9	28.1	1 59 24.8	124.1	7.7	30
40	10 21 56.7	28.6	37.1	7 25 3.0	88.2	28.0	1 57 20.7	124.2	7.6	20
50	10 21 28.1	28.9	37.1	7 23 34.8	88.5	27.9	1 55 16.5	124.2	7.4	10
21. 0	10 20 59.2	29.2	37.0	7 22 6.3	88.8	27.8	1 53 12.3	124.3	7.3	9. 0
10	10 20 30.0	29.6	37.0	7 20 37.5	89.1	27.7	1 51 8.0	124.4	7.2	8.50
20	10 20 0.4	29.9	37.0	7 19 8.4	89.3	27.6	1 49 3.6	124.5	7.1	40
30	10 19 30.5	30.3	37.0	7 17 39.1	89.6	27.5	1 46 59.1	124.6	6.9	30
40	10 19 0.2	30.6	37.0	7 16 9.5	89.9	27.4	1 44 54.5	124.7	6.8	20
50	10 18 29.6	30.9	37.0	7 14 39.6	90.2	27.3	1 42 49.8	124.7	6.6	10
22. 0	10 17 58.7	31.3	36.9	7 13 9.4	90.5	27.2	1 40 45.1	124.7	6.5	8. 0
10	10 17 27.4	31.7	36.9	7 11 38.9	90.8	27.2	1 38 40.4	124.8	6.4	7.50
20	10 16 55.7	32.0	36.9	7 10 8.1	91.1	27.1	1 36 35.6	124.9	6.2	40
30	10 16 23.7	32.3	36.9	7 8 37.0	91.4	27.0	1 34 30.7	125.0	6.1	30
40	10 15 51.4	32.6	36.9	7 7 5.6	91.6	26.9	1 32 25.7	125.2	6.0	20
50	10 15 18.8	33.0	36.9	7 5 34.0	91.9	26.8	1 30 20.5	125.3	5.9	10
23. 0	10 14 45.8	33.3	36.8	7 4 2.1	92.2	26.7	1 28 15.2	125.3	5.7	7. 0
10	10 14 12.5	33.6	36.8	7 2 29.9	92.5	26.6	1 26 9.9	125.3	5.6	6.50
20	10 13 38.9	34.0	36.8	7 0 57.4	92.8	26.5	1 24 4.6	125.3	5.4	40
30	10 13 4.9	34.4	36.8	6 59 24.6	93.1	26.4	1 21 59.3	125.3	5.3	30
40	10 12 30.5	34.8	36.8	6 57 51.5	93.3	26.3	1 19 53.9	125.4	5.2	20
50	10 11 55.7	35.1	36.8	6 56 18.2	93.6	26.2	1 17 48.5	125.4	5.1	10
24. 0	10 11 20.6		36.7	6 54 44.6		26.2	1 15 43.1	125.4	4.9	6. 0
	Sig. VIII.+		+	Sig. VII.+		+	Sig. VI.+		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1, 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMENT. *The mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. O. —	Diff.	Var. —	Sig. I. —	Diff.	Var. —	Sig. II. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
24. 0	3 55 20.6	93.8	12.2	8 3 37.7	68.2	25.9	10 26 16.8	23.6	35.3	6. 0
10	3 56 54.4	93.8	12.3	8 4 45.9	68.0	25.9	10 26 40.4	23.3	35.3	5. 50
20	3 58 28.2	93.7	12.4	8 5 53.9	67.9	26.1	10 27 3.7	23.0	35.3	40
30	4 0 1.9	93.6	12.5	8 7 1.8	67.7	26.1	10 27 26.7	22.7	35.4	30
40	4 1 35.5	93.5	12.6	8 8 9.5	67.5	26.1	10 27 49.4	22.5	35.4	20
50	4 3 9.0	93.4	12.7	8 9 17.0	67.4	26.1	10 28 11.9	22.1	35.4	10
25. 0	4 4 42.4	93.3	12.8	8 10 24.4	67.2	26.3	10 28 34.0	21.8	35.5	5. 0
10	4 6 15.7	93.2	12.8	8 11 31.6	67.0	26.3	10 28 55.8	21.5	35.5	4. 50
20	4 7 48.9	93.1	12.9	8 12 38.6	66.7	26.3	10 29 17.3	21.2	35.5	40
30	4 9 22.0	93.0	12.9	8 13 45.3	66.4	26.5	10 29 38.5	20.9	35.6	30
40	4 10 55.0	92.9	13.0	8 14 51.7	66.2	26.5	10 29 59.4	20.6	35.6	20
50	4 12 27.9	92.9	13.1	8 15 57.9	65.9	26.5	10 30 20.0	20.3	35.6	10
26. 0	4 14 0.8	92.8	13.2	8 17 3.8	65.8	26.7	10 30 40.3	20.0	35.7	4. 0
10	4 15 33.6	92.7	13.2	8 18 9.6	65.6	26.7	10 31 0.3	19.7	35.7	3. 50
20	4 17 6.3	92.6	13.3	8 19 15.2	65.4	26.9	10 31 20.0	19.4	35.7	40
30	4 18 38.9	92.5	13.4	8 20 20.6	65.3	26.9	10 31 39.4	19.1	35.8	30
40	4 20 11.4	92.3	13.5	8 21 25.9	65.1	26.9	10 31 58.5	18.8	35.8	20
50	4 21 43.7	92.2	13.6	8 22 31.0	64.8	26.9	10 32 17.3	18.5	35.8	10
27. 0	4 23 15.9	92.1	13.7	8 23 35.8	64.6	27.1	10 32 35.8	18.1	35.9	3. 0
10	4 24 48.0	92.0	13.7	8 24 40.4	64.4	27.1	10 32 53.9	17.8	35.9	2. 50
20	4 26 20.0	92.0	13.8	8 25 44.8	64.2	27.3	10 33 11.7	17.5	35.9	40
30	4 27 52.0	91.9	13.9	8 26 49.0	64.0	27.3	10 33 29.2	17.3	36.0	30
40	4 29 23.9	91.8	14.0	8 27 53.0	63.8	27.3	10 33 46.5	17.0	36.0	20
50	4 30 55.7	91.7	14.1	8 28 56.8	63.6	27.3	10 34 3.5	16.8	36.0	10
28. 0	4 32 27.4	91.6	14.2	8 30 0.4	63.4	27.5	10 34 20.3	16.4	36.1	2. 0
10	4 33 59.0	91.5	14.2	8 31 3.8	63.2	27.5	10 34 36.7	16.1	36.1	1. 50
20	4 35 30.5	91.4	14.3	8 32 7.0	63.0	27.5	10 34 52.8	15.7	36.1	40
30	4 37 1.9	91.3	14.4	8 33 10.0	62.7	27.7	10 35 8.5	15.4	36.2	30
40	4 38 33.2	91.1	14.5	8 34 12.7	62.5	27.7	10 35 23.9	15.2	36.2	20
50	4 40 4.3	91.0	14.6	8 35 15.2	62.2	27.7	10 35 39.1	14.9	36.2	10
29. 0	4 41 35.3	90.9	14.7	8 36 17.4	62.0	27.9	10 35 54.0	14.6	36.3	1. 0
10	4 43 6.2	90.8	14.7	8 37 19.4	61.8	27.9	10 36 8.6	14.3	36.3	0. 50
20	4 44 37.0	90.7	14.8	8 38 21.2	61.6	28.0	10 36 22.9	13.9	36.3	40
30	4 46 7.7	90.7	14.9	8 39 22.8	61.5	28.0	10 36 36.8	13.5	36.4	30
40	4 47 38.4	90.6	15.0	8 40 24.3	61.3	28.1	10 36 50.3	13.2	36.4	20
50	4 49 9.0	90.5	15.1	8 41 25.6	61.2	28.1	10 37 3.5	12.9	36.4	10
30. 0	4 50 39.5		15.2	8 42 26.8		28.2	10 37 16.4		36.4	0. 0
	Sig. XI. +		+	Sig. X. +		+	Sig. IX. +		+	

TABLE VII.

Equation of the Center of Mars for Jan. 1800, with the Secular Variation, to be applied to the Longitude.

ARGUMEMT. *The Mean Anomaly of Mars, or mean Longitude of Mars — Longitude of the Aphelion.*

Deg.	Sig. III. —	Diff.	Var. —	Sig, IV. —	Diff.	Var. —	Sig. V. —	Diff.	Var. —	Deg.
	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	D. M. S.	SEC.	SEC.	
24. 0	10 11 20.6	35.5	36.7	6 54 44.6	93.9	26.2	1 15 43.1	125.5	4.9	6. 0
10	10 10 45.1	35.8	36.7	6 53 10.7	94.2	26.1	1 13 37.6	125.6	4.8	5. 50
20	10 10 9.3	36.1	36.7	6 51 36.5	94.5	26.0	1 11 32.0	125.7	4.7	40
30	10 9 33.2	36.4	36.7	6 50 2.0	94.7	25.9	1 9 26.3	125.7	4.5	30
40	10 8 56.8	36.8	36.7	6 48 27.3	94.9	25.8	1 7 20.6	125.8	4.4	20
50	10 8 20.0	37.2	36.6	6 46 52.4	95.2	25.8	1 5 14.8	125.8	4.3	10
25. 0	10 7 42.8	37.5	36.6	6 45 17.2	95.5	25.7	1 3 9.0	125.8	4.1	5. 0
10	10 7 5.3	37.9	36.6	6 43 41.7	95.8	25.6	1 1 3.2	125.9	4.0	4. 50
20	10 6 27.4	38.3	36.5	6 42 5.9	96.1	25.5	0 58 57.3	125.9	3.9	40
30	10 5 49.1	38.6	36.5	6 40 29.8	96.4	25.4	0 56 51.4	126.0	3.7	30
40	10 5 10.5	38.9	36.5	6 38 53.4	96.6	25.3	0 54 45.4	126.1	3.6	20
50	10 4 31.6	39.2	36.5	6 37 16.8	96.9	25.2	0 52 39.3	126.1	3.5	10
26. 0	10 3 52.4	39.6	36.4	6 35 39.9	97.2	25.1	0 50 33.2	126.1	3.3	4. 0
10	10 3 12.8	39.9	36.4	6 34 2.7	97.4	25.0	0 48 27.1	126.1	3.2	3. 50
20	10 2 32.9	40.3	36.3	6 32 25.3	97.7	24.9	0 46 21.0	126.2	3.1	40
30	10 1 52.6	40.7	36.3	6 30 47.6	97.9	24.8	0 44 14.8	126.2	2.9	30
40	10 1 11.9	41.0	36.3	6 29 9.7	98.2	24.7	0 42 8.6	126.3	2.8	20
50	10 0 30.9	41.3	36.3	6 27 31.5	98.4	24.6	0 40 2.3	126.3	2.7	10
27. 0	9 59 49.6	41.6	36.2	6 25 53.1	98.7	24.5	0 37 56.0	126.3	2.5	3. 0
10	9 59 8.0	42.0	36.2	6 24 14.4	99.0	24.4	0 35 49.7	126.3	2.4	2. 50
20	9 58 26.0	42.4	36.2	6 22 35.4	99.3	24.3	0 33 43.4	126.3	2.3	40
30	9 57 43.6	42.7	36.1	6 20 56.1	99.5	24.2	0 31 37.1	126.3	2.1	30
40	9 57 0.9	43.1	36.1	6 19 16.6	99.8	24.1	0 29 30.7	126.4	2.0	20
50	9 56 17.8	43.4	36.1	6 17 36.8	100.0	24.0	0 27 24.3	126.4	1.9	10
28. 0	9 55 34.4	43.7	36.0	6 15 56.8	100.3	23.9	0 25 17.9	126.4	1.7	2. 0
10	9 54 50.7	44.1	36.0	6 14 16.5	100.6	23.8	0 23 11.5	126.4	1.6	1. 50
20	9 54 6.6	44.5	36.0	6 12 35.9	100.8	23.7	0 21 5.1	126.4	1.5	40
30	9 53 22.1	44.8	35.9	6 10 55.1	101.0	23.6	0 18 58.7	126.5	1.3	30
40	9 52 37.3	45.1	35.9	6 9 14.1	101.3	23.5	0 16 52.2	126.5	1.2	20
50	9 51 52.2	45.4	35.9	6 7 32.8	101.6	23.4	0 14 45.7	126.5	1.1	10
29. 0	9 51 6.8	45.8	35.8	6 5 51.2	101.9	23.3	0 12 39.2	126.5	0.9	1. 0
10	9 50 21.0	46.1	35.8	6 4 9.3	102.1	23.2	0 10 32.7	126.5	0.8	0. 50
20	9 49 34.9	46.5	35.8	6 2 27.2	102.3	23.1	0 8 26.2	126.5	0.7	40
30	9 48 48.4	46.9	35.7	6 0 44.9	102.5	23.0	0 6 19.7	126.5	0.5	30
40	9 48 1.5	47.3	35.7	5 59 2.4	102.7	22.9	0 4 13.2	126.6	0.3	20
50	9 47 14.2	47.6	35.7	5 57 19.7	103.0	22.8	0 2 6.6	126.6	0.1	10
30. 0	9 46 26.6		35.6	5 55 36.7		22.7	0 0 0.0		0.0	0. 0
	Sig. VIII +		+	Sig. VII. +		+	Sig. VI. +		+	

TABLE VIII.

ARGUMENT II. *Long.* ♂ — *Long.* ♃.

The Equations are all positive, but the sum must be diminished 2 minutes.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	35.0	34.8	19.0	3.2	8.7	35.0	61.3	66.8	51.0	35.2
10	35.4	33.9	17.0	2.7	10.6	38.1	63.0	65.9	49.0	34.5
20	35.8	32.8	15.0	2.3	12.8	41.1	64.4	64.9	47.0	34.0
30	36.2	31.5	13.0	2.3	15.0	44.1	65.7	63.6	45.2	33.6
40	36.4	30.0	11.1	2.3	17.6	47.0	66.6	62.2	43.3	33.4
50	36.6	28.5	9.4	2.7	20.2	49.8	67.3	60.6	41.5	33.4
60	36.6	26.7	7.8	3.4	23.0	52.4	67.7	58.9	39.9	33.6
70	36.4	24.8	6.4	4.3	25.9	55.0	67.7	57.0	38.5	33.8
80	36.0	23.0	5.1	5.6	28.9	57.2	67.7	55.0	37.2	33.2
90	35.5	21.0	4.1	7.0	31.9	59.4	67.3	53.0	36.1	34.6
100	34.8	19.0	3.2	8.7	35.0	61.3	66.8	51.0	35.2	35.0

TABLE IX.

ARGUMENT III. *Long.* ♂ — 2 *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	18.4	31.3	43.6	50.9	50.1	41.6	28.7	16.4	9.1	9.9
10	19.6	32.6	44.6	51.1	49.5	40.4	27.4	15.4	8.9	10.5
20	20.7	33.9	45.6	51.3	48.8	39.3	26.1	14.4	8.7	11.3
30	22.0	35.2	46.5	51.4	48.1	38.0	24.8	13.5	8.6	11.9
40	23.2	36.6	47.3	51.5	47.4	36.8	23.4	12.7	8.5	12.6
50	24.6	37.8	48.0	51.4	46.6	35.4	22.2	12.0	8.6	13.4
60	25.9	39.1	48.6	51.3	45.8	34.1	20.9	11.4	8.7	14.2
70	27.2	40.2	49.4	51.1	44.8	32.8	19.8	10.6	8.9	15.2
80	28.5	41.4	50.9	50.0	43.8	31.5	18.6	10.0	9.1	16.2
90	29.9	42.5	50.5	50.6	42.7	30.1	17.5	9.5	9.4	17.3
100	31.3	43.6	50.9	50.1	41.6	28.7	16.4	9.1	9.9	18.4

TABLE X.

ARGUMENT IV. 2 *Long.* ♂ — 3 *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	6.8	7.3	7.0	5.9	4.5	3.2	2.7	3.0	4.1	5.5
10	6.9	7.3	6.9	5.8	4.3	3.1	2.7	3.1	4.2	5.7
20	7.0	7.3	6.8	5.6	4.2	3.0	2.7	3.2	4.4	5.8
30	7.0	7.3	6.7	5.5	4.1	3.0	2.7	3.3	4.5	5.9
40	7.1	7.3	6.6	5.3	3.9	2.9	2.7	3.4	4.7	6.1
50	7.2	7.3	6.5	5.2	3.8	2.8	2.7	3.5	4.8	6.2
60	7.2	7.2	6.4	5.0	3.7	2.8	2.8	3.6	5.0	6.3
70	7.2	7.2	6.3	4.9	3.5	2.8	2.8	3.7	5.1	6.5
80	7.3	7.1	6.2	4.8	3.4	2.7	2.9	3.8	5.2	6.6
90	7.3	7.1	6.1	4.6	3.3	2.7	2.9	3.9	5.4	6.7
100	7.3	7.0	5.9	4.5	3.2	2.7	3.0	4.1	5.5	6.8

TABLE XI.

ARGUMENT V. *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	8.6	7.7	5.7	3.5	1.9	1.4	2.4	4.3	6.5	8.1
10	8.6	7.5	5.4	3.3	1.8	1.5	2.5	4.6	6.7	8.2
20	8.5	7.3	5.2	3.1	1.7	1.5	2.7	4.8	6.9	8.3
30	8.5	7.1	5.0	2.9	1.6	1.6	2.9	5.0	7.1	8.4
40	8.4	6.9	4.8	2.7	1.5	1.7	3.1	5.2	7.3	8.5
50	8.3	6.7	4.6	2.5	1.5	1.8	3.3	5.4	7.5	8.5
60	8.2	6.5	4.3	2.4	1.4	1.9	3.5	5.7	7.6	8.6
70	8.1	6.3	4.1	2.3	1.4	2.0	3.7	5.9	7.7	8.6
80	8.0	6.1	3.9	2.1	1.4	2.1	3.9	6.1	7.9	8.6
90	7.9	5.9	3.7	2.0	1.4	2.3	4.1	6.3	8.0	8.6
100	7.7	5.7	3.5	1.9	1.4	2.4	4.3	6.5	8.1	8.6

TABLE XII.

ARGUMENT VI. *Long.* ☉ — *Long.* ♂.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	10.0	13.2	16.5	17.3	15.2	10.0	4.8	2.7	3.5	6.8
10	10.4	13.5	16.7	17.2	14.7	9.4	4.4	2.7	3.8	7.1
20	10.7	13.8	16.8	17.2	14.3	8.9	4.1	2.7	4.2	7.5
30	11.0	14.1	17.0	17.1	13.8	8.3	3.8	2.7	4.5	7.8
40	11.3	14.4	17.1	17.0	13.3	7.7	3.5	2.7	4.8	8.1
50	11.6	14.8	17.2	16.8	12.8	7.2	3.2	2.8	5.2	8.4
60	11.9	15.2	17.3	16.5	12.3	6.7	3.0	2.9	5.6	8.7
70	12.2	15.5	17.3	16.2	11.7	6.2	2.9	3.0	5.9	9.0
80	12.5	15.8	17.3	15.9	11.1	5.7	2.8	3.2	6.2	9.3
90	12.9	16.2	17.3	15.6	10.6	5.3	2.8	3.3	6.5	9.6
100	13.2	16.5	17.3	15.2	10.0	4.8	2.7	3.5	6.8	10.0

TABLE XIII.

ARGUMENT VII. 2 *Long.* ♂ — *Long.* ☉.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	5.4	1.7	3.0	8.8	17.2	24.6	28.3	27.0	21.2	12.8
10	4.8	1.5	3.4	9.6	18.1	25.2	28.5	26.6	20.4	11.9
20	4.3	1.4	3.8	10.3	18.9	25.7	28.6	26.2	19.7	11.1
30	3.8	1.4	4.3	11.1	19.7	26.2	28.6	25.7	18.9	10.3
40	3.4	1.5	4.8	11.9	20.4	26.6	28.5	25.2	18.1	9.6
50	3.0	1.7	5.4	12.8	21.2	27.0	28.3	24.6	17.2	8.8
60	2.6	1.9	6.0	13.7	21.9	27.4	28.1	24.0	16.3	8.1
70	2.3	2.1	6.6	14.5	22.7	27.7	27.9	23.4	15.5	7.3
80	2.1	2.3	7.3	15.5	23.4	27.9	27.7	22.7	14.5	6.6
90	1.9	2.6	8.1	16.3	24.0	28.1	27.4	21.9	13.7	6.0
100	1.7	3.0	8.8	17.2	24.6	28.3	27.0	21.2	12.8	5.4

TABLE XIV.

ARGUMENT VIII. 2 *Long.* ☉ — *Long.* ♂.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	6.5	10.1	13.7	15.9	15.9	13.5	9.9	6.3	4.1	4.1
10	6.8	10.5	14.0	16.0	15.7	13.2	9.5	6.0	4.0	4.3
20	7.1	10.9	14.3	16.0	15.5	12.9	9.1	5.7	4.0	4.5
30	7.5	11.2	14.5	16.1	15.3	12.5	8.8	5.5	3.9	4.7
40	7.9	11.6	14.8	16.2	15.1	12.2	8.4	5.2	3.8	4.9
50	8.3	11.9	15.1	16.2	14.9	11.8	8.1	4.9	3.8	5.1
60	8.6	12.4	15.3	16.2	14.6	11.4	7.7	4.7	3.8	5.4
70	9.0	12.7	15.5	16.1	14.3	11.0	7.3	4.5	3.9	5.7
80	9.3	13.1	15.7	16.0	14.1	10.7	6.9	4.3	4.0	5.9
90	9.7	13.4	15.8	16.0	13.8	10.3	6.6	4.2	4.0	6.2
100	10.1	13.7	15.9	15.9	13.5	9.9	6.3	4.1	4.1	6.5

TABLE XV.

ARGUMENT IX. *Long.* ♀ — 3 *Long.* ♂.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	10.0	13.4	15.4	15.4	13.4	10.0	6.6	4.6	4.6	6.6
10	10.4	13.6	15.5	15.3	13.1	9.6	6.4	4.5	4.7	6.9
20	10.8	13.9	15.5	15.2	12.8	9.2	6.1	4.5	4.8	7.2
30	11.1	14.1	15.6	15.0	12.4	8.9	5.9	4.4	5.0	7.6
40	11.5	14.4	15.7	14.8	12.1	8.5	5.6	4.3	5.2	7.9
50	11.8	14.6	15.7	14.6	11.8	8.2	5.4	4.3	5.4	8.2
60	12.1	14.8	15.7	14.4	11.5	7.9	5.2	4.3	5.6	8.5
70	12.4	15.0	15.6	14.1	11.1	7.6	5.0	4.4	5.9	8.9
80	12.8	15.2	15.5	13.9	10.8	7.2	4.8	4.5	6.1	9.2
90	13.1	15.3	15.5	13.6	10.4	6.9	4.7	4.5	6.4	9.6
100	13.4	15.4	15.4	13.4	10.0	6.6	4.6	4.6	6.6	10.0

TABLE XVI.

Logarithms of the Radius Vector of the Orbit of Mars for Jan. 1, 1800, with the Secular Variation.

ARGUMENT. *The mean Anomaly of Mars.*

D.	Sig. O.	Diff.	Sec. Var.	Sig. I.	Diff.	Sec. Var.	Sig. II.	Diff.	Sec. Var.	D.
0	0.221606		+35.8	0.217392		+31.7	0.205193		+21.8	30
1	0.221601	005	35.8	0.217109	283	31.5	0.204663	530	21.4	29
2	0.221587	014	35.8	0.216817	292	31.3	0.204126	537	21.0	28
3	0.221564	023	35.7	0.216516	301	31.0	0.203583	543	20.6	27
4	0.221531	033	35.7	0.216206	310	30.8	0.203032	551	20.1	26
5	0.221488	043	35.6	0.215888	318	30.5	0.202474	558	19.6	25
6	0.221437	051	35.6	0.215560	328	30.2	0.201909	565	19.1	24
7	0.221375	062	35.5	0.215224	336	30.0	0.201338	571	18.6	23
8	0.221305	070	35.4	0.214879	345	29.7	0.200760	578	18.1	22
9	0.221224	081	35.3	0.214525	354	29.5	0.200176	584	17.6	21
10	0.221135	089	35.2	0.214163	362	29.2	0.199585	591	17.0	20
11	0.221036	099	35.1	0.213792	371	28.9	0.198988	597	16.5	19
12	0.220927	109	35.0	0.213413	379	28.6	0.198385	603	16.0	18
13	0.220810	117	34.9	0.213025	388	28.3	0.197776	609	15.5	17
14	0.220683	127	34.7	0.212629	396	28.0	0.197162	614	15.0	16
15	0.220546	137	34.6	0.212224	405	27.7	0.196541	621	14.5	15
16	0.220401	145	34.4	0.211812	412	27.3	0.195915	626	13.9	14
17	0.220246	155	34.3	0.211390	422	27.0	0.195283	632	13.4	13
18	0.220081	165	34.2	0.210961	429	26.6	0.194646	637	12·8	12
19	0.219908	173	34.1	0.210523	438	26.2	0.194003	643	12.3	11
20	0.219725	183	33.9	0.210078	445	25.8	0.193356	647	11.8	10
21	0.219533	192	33.7	0.209624	454	25.4	0.192703	653	11.2	9
22	0.219332	201	33.5	0.209163	461	25.0	0.192046	657	10.6	8
23	0.219121	211	33.3	0.208693	470	24.6	0.191384	662	10.0	7
24	0.218901	220	33.1	0.208216	477	24.2	0.190718	666	9.4	6
25	0.218673	228	32.9	0.207731	485	23.8	0.190048	670	8.8	5
26	0.218435	238	32.7	0.207238	493	23.4	0.189373	675	8.2	4
27	0.218188	247	32.5	0.206738	500	23.0	0.188694	679	7.6	3
28	0.217932	256	32.2	0.206231	507	22.6	0.188011	683	7.0	2
29	0.217667	265	32.0	0.205715	516	22.2	0.187324	687	6.4	1
30	0.217392	275	31.7	0.205193	522	21.8	0.186635	689	5.8	0
	Sig. XI.			Sig. X.			Sig. IX.			

TABLE XVI.

Logarithms of the Radius Vector of the Orbit of Mars for Jan. 1, 1800, with the Secular Variation.

ARGUMEMT. *The Mean Anomaly of Mars,*

D.	Sig. III.	Diff.	Sec. Var.	Sig. IV.	Diff.	Sec. Var.	Sig. V.	Diff.	Sec. Var.	D.
0	0.186635	694	+5.8	0.165331	692	—15.4	0.147503	446	—34.8	30
1	0.185941	696	5.2	0.164639	687	16.1	0.147057	433	35.3	29
2	0.185245	699	4.5	0.163952	683	16.8	0.146624	420	35.8	28
3	0.184546	702	3.8	0.163269	678	17.6	0.146204	408	36.3	27
4	0.183844	704	3.3	0.162591	674	18.4	0.145796	394	36.8	26
5	0.183140	707	2.6	0.161917	669	19.1	0.145402	381	37.3	25
6	0.182433	709	1.9	0.161248	663	19.8	0.145021	367	37.8	24
7	0.181724	712	1.2	0.160585	657	20.5	0.144654	353	38.2	23
8	0.181012	713	+ 0.5	0.159928	652	21.2	0.144301	338	38.6	22
9	0.180299	714	— 0.2	0.159276	645	21.9	0.143963	324	39.0	21
10	0.179585	716	0.9	0.158631	639	22.7	0.143639	310	39.3	20
11	0.178869	717	1.6	0.157992	632	23.5	0.143329	295	39.6	19
12	0.178152	717	2.3	0.157360	625	24.2	0.143034	280	39.9	18
13	0.177435	718	3.0	0.156735	618	25.0	0.142754	265	40.2	17
14	0.176717	719	3.7	0.156117	607	25.7	0.142489	250	40.5	16
15	0.175998	720	4.4	0.155510	600	26.3	0.142239	234	40.8	15
16	0.175279	720	5.1	0.154910	593	27.0	0.142005	220	41.1	14
17	0.174559	719	5.8	0.154317	585	27.6	0.141785	204	41.3	13
18	0.173840	718	6.6	0.153732	576	28.3	0.141581	188	41.5	12
19	0.173122	717	7.3	0.153156	565	29.0	0.141393	171	41.7	11
20	0.172405	716	8.0	0.152591	557	29.7	0.141222	155	41.9	10
21	0.171689	715	8.8	0.152034	547	30.3	0.141067	139	42.1	9
22	0.170974	713	9.5	0.151487	537	30.9	0.140928	123	42.3	8
23	0.170261	712	10.3	0.150950	526	31.4	0.140805	108	42.4	7
24	0.169549	709	11.0	0.150424	515	31.9	0.140697	091	42.5	6
25	0.168840	707	11.7	0.149909	504	32.4	0.140606	074	42.6	5
26	0.168133	705	12.4	0.149405	493	32.9	0.140532	058	42.7	4
27	0.167428	702	13.2	0.148912	481	33.4	0.140474	041	42.8	3
28	0.166726	699	14.0	0.148431	470	33.8	0.140433	025	42.8	2
29	0.166027	696	14.7	0.147961	458	34.3	0.140408	009	42.9	1
30	0.165331		15.4	0.147503		34.8	0.140399		42.9	0
	Sig. VIII.			Sig. VII.			Sig. VI.			

TABLE XVII.

Equation of the Radius Vector.

ARGUMENT II. *Long.* ♂ — *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	"	"	"
0	142	185	224	165	55	0	55	165	224	185
10	143	192	223	155	46	0	65	174	224	178
20	144	198	221	144	36	2	76	184	223	172
30	147	204	217	133	28	5	87	193	221	166
40	150	210	213	121	21	9	98	200	218	160
50	155	214	207	109	15	15	109	207	214	155
60	160	218	200	98	9	21	121	213	210	150
70	166	221	193	87	5	28	133	217	204	147
80	172	223	184	76	2	36	144	221	198	144
90	178	224	174	65	0	46	155	223	192	143
100	185	224	165	55	0	55	165	224	185	142

TABLE XVIII.

ARGUMENT III. *Long.* ♂ — 2 *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
	"	"	"	"	"	"	"	′	"	"
0	9	0	12	41	75	101	110	98	69	35
10	7	0	15	44	78	103	110	95	66	32
20	6	1	17	48	81	104	109	93	62	29
30	4	2	20	51	84	106	108	90	59	26
40	3	3	22	54	87	107	107	88	56	23
50	2	4	25	58	90	108	106	85	52	20
60	1	5	28	61	92	109	105	82	49	18
70	1	7	31	65	95	109	103	79	45	15
80	0	9	34	68	97	110	101	76	42	13
90	0	10	38	71	99	110	100	72	39	11
100	0	12	41	75	101	110	98	69	35	9

TABLE XIX.

ARGUMENT IV. 2 *Long.* ♂ — 3 *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
0	4	11	17	21	22	18	11	5	1	0
10	5	11	18	21	21	17	11	4	1	1
20	5	12	18	22	21	17	10	4	0	1
30	6	13	19	22	21	16	9	3	0	1
40	7	13	19	22	21	15	9	3	0	1
50	7	14	20	22	20	15	8	2	0	2
60	8	15	20	22	20	14	7	2	0	2
70	9	15	20	22	19	13	7	2	0	3
80	9	16	21	22	19	13	6	1	0	3
90	10	17	21	22	18	12	5	1	0	4
100	11	17	21	22	18	11	5	1	0	4

TABLE XX.

ARGUMENT V. *Long.* ♃.

ARG.	0	100	200	300	400	500	600	700	800	900
0	2	7	13	17	18	16	11	5	1	0
10	3	8	13	17	18	15	10	5	1	0
20	3	8	14	17	18	15	10	4	1	0
30	4	9	14	17	18	14	9	4	1	0
40	4	9	15	18	17	14	9	3	0	1
50	5	10	15	18	17	13	8	3	0	1
60	5	11	15	18	17	13	7	3	0	1
70	6	11	16	18	17	12	7	2	0	1
80	6	12	16	18	16	12	6	2	0	2
90	7	12	16	18	16	11	6	2	0	2
100	7	13	17	18	16	11	5	1	0	2

TABLE XXI.

ARGUMENT VI. *Long.* ☉ — *Long.* ♂.

ARG.	0	100	200	300	400	500	600	700	800	900
0	1	0	3	17	31	37	31	17	3	0
10	1	0	4	18	32	37	30	15	2	0
20	1	0	5	20	33	37	29	13	1	0
30	1	0	6	21	34	37	27	12	1	0
40	1	0	8	23	35	36	26	10	1	1
50	1	1	9	25	36	36	25	9	1	1
60	1	1	10	26	36	35	23	8	0	1
70	0	1	12	27	37	34	21	6	0	1
80	0	2	13	29	37	33	20	5	0	1
90	0	2	15	30	37	32	18	4	0	1
100	0	3	17	31	37	31	17	3	0	1

TABLE XXII.

ARGUMENT VII. *Long.* ♂ — *Long.* ☉.

ARG.	0	100	200	300	400	500	600	700	800	900
0	19	13	6	2	0	3	9	16	20	22
10	19	13	6	1	0	3	9	16	21	22
20	18	12	5	1	0	4	10	17	21	22
30	18	11	5	1	1	4	11	17	21	21
40	17	10	4	0	1	5	12	18	22	21
50	17	9	4	0	1	5	13	18	22	21
60	16	9	3	0	1	6	13	19	22	21
70	16	8	3	0	2	6	14	19	22	20
80	15	7	2	0	2	7	15	20	22	20
90	14	7	2	0	2	8	15	20	22	20
100	13	6	2	0	3	9	16	20	22	19

TABLE XXIII.

ARGUMENT VIII. 2 *Long.* ☉ — 3 *Long.* ♂.

ARG.	0	100	200	300	400	500	600	700	800	900
0	4	0	4	13	25	34	38	34	25	13
10	4	0	5	14	26	34	38	33	24	12
20	3	0	5	16	27	35	38	33	23	11
30	3	1	6	17	28	36	37	32	22	10
40	2	1	7	18	29	36	37	31	20	9
50	2	2	8	19	30	36	36	30	19	8
60	2	2	9	20	31	37	36	29	18	7
70	1	2	10	21	32	37	36	28	17	6
80	1	3	11	22	33	38	35	27	16	5
90	0	3	12	24	33	38	35	26	14	5
100	0	4	13	25	34	38	34	25	13	4

TABLE XXIV.

The Equation of the Radius Vector in parts of its Logarithm.

ARGUMENT. *At the Side, the Log. of the Radius Vector; and at the Top, the Sum of the preceding Equations diminished by 263.*

ARG. *Log. of Rad. Vect.*	100	200	300	400	500	600	700	800	900
0.220000	26.2	52.3	78	105	131	157	183	209	235
0.215	26.5	53.0	79	106	133	159	185	211	238
0.210	26.8	53.6	80	107	134	161	187	214	241
0.205	27.0	54.1	81	108	135	162	189	216	243
0.200	27.4	54.8	82	109	137	164	192	219	246
0.195	27.7	55.4	83	111	139	166	194	221	249
0.190	28.0	56.1	84	112	140	168	196	224	252
0.185	28.3	56.6	85	113	142	170	198	226	255
0.180	28.7	57.4	86	115	144	172	201	229	258
0.175	29.0	58.0	87	116	145	174	203	232	261
0.170	29.3	58.7	88	117	147	176	205	234	264
0.165	29.7	59.4	89	119	148	178	208	237	267
0.160	30.0	60.0	90	120	150	180	210	240	270
0.155	30.4	60.7	91	121	152	182	213	243	273
0.150	30.7	61.4	92	123	153	184	215	246	276
0.145	31.1	62.2	93	124	155	186	218	249	280
0.140	31.4	62.8	94	126	157	188	220	252	283

TABLE XXV.

Heliocentric Latitude of Mars.

ARGUMENT. *The Longitude upon the Orbit—the Longitude of the Node.*

Degrees.	Sig. O. *N.* / Sig. VI. *S.* D. M. S.	Differ. SEC.	Sig. I. *N.* / Sig. VII. *S.* D. M. S.	Differ. SEC.	Sig. II. *N.* / Sig. VIII. *S.* D. M. S.	Differ. SEC.	Degrees.
0	0 0 0.0	116.3	0 55 31.3	100.2	1 36 10.5	57.3	30
1	0 1 56.3	116.3	0 57 11.5	99.2	1 37 7.8	55.5	29
2	0 3 52.5	116.2	0 58 50.7	98.1	1 38 3.3	53.7	28
3	0 5 48.7	116.0	1 0 28.8	97.0	1 38 57.0	51.9	27
4	0 7 44.7	115.9	1 2 5.8	95.8	1 39 48.9	50.1	26
5	0 9 40.6	115.8	1 3 41.6	94.7	1 40 39.0	48.2	25
6	0 11 36.4	115.6	1 5 16.3	93.5	1 41 27.2	46.4	24
7	0 13 32.0	115.3	1 6 49.8	92.2	1 42 13.6	44.6	23
8	0 15 27.3	114.9	1 8 22.0	91.0	1 42 58.2	42.7	22
9	0 17 22.2	114.7	1 9 53.0	89.7	1 43 40.9	40.7	21
10	0 19 16.9	114.3	1 11 22.7	88.5	1 44 21.6	38.8	20
11	0 21 11.2	114.0	1 12 51.2	87.1	1 45 0.4	36.9	19
12	0 23 5.2	113.5	1 14 18.3	85.8	1 45 37.3	35.0	18
13	0 24 58.7	113.1	1 15 44.1	84.4	1 46 12.3	33.0	17
14	0 26 51.8	112.6	1 17 8.5	82.9	1 46 45.3	31.1	16
15	0 28 44.4	112.0	1 18 31.4	81.5	1 47 16.4	29.1	15
16	0 30 36.4	111.5	1 19 52.9	80.1	1 47 45.5	27.1	14
17	0 32 27.9	110.9	1 21 13.0	78.6	1 48 12.6	25.2	13
18	0 34 18.8	110.3	1 22 31.6	77.0	1 48 37.8	23.2	12
19	0 36 9.1	109.6	1 23 48.6	75.5	1 49 1.0	21.2	11
20	0 37 58.7	108.9	1 25 4.1	74.0	1 49 22.2	19.2	10
21	0 39 47.6	108.2	1 26 18.1	72.5	1 49 41.4	17.2	9
22	0 41 35.8	107.4	1 27 30.6	70.9	1 49 58.6	15.2	8
23	0 43 23.2	106.7	1 28 41.5	69.1	1 50 13.8	13.2	7
24	0 45 9.9	105.8	1 29 50.6	67.6	1 50 27.0	11.2	6
25	0 46 55.7	104.9	1 30 58.2	65.9	1 50 38.2	9.1	5
26	0 48 40.6	104.1	1 32 4.1	64.1	1 50 47.3	7.1	4
27	0 50 24.7	103.2	1 33 8.2	62.5	1 50 54.4	5.1	3
28	0 52 7.9	102.2	1 34 10.7	60.8	1 50 59.5	3.0	2
29	0 53 50.1	101.2	1 35 11.5	59.0	1 51 2.5	1.0	1
30	0 55 31.3		1 36 10.5		1 51 3.5		0
	Sig. XI. *S.* / Sig. V. *N.*		Sig. X. *S.* / Sig. IV. *N.*		Sig. IX. *S.* / Sig. III. *N.*		

TABLE XXVI.

Reduction to the Ecliptic both in Longitude and for the Radius Vector.

The Longitude upon the Orbit — the Longitude of the Node.

Degrees.	Sig. O. — / Sig. VI. —	Diff.	Log. —	Sig. I. — / Sig. VII. —	Diff.	Log. —	Sig. II. — / Sig. VIII —	Diff.	Log. —	Degrees.
	SEC.	SEC.		SEC.	SEC.		SEC.	SEC.		
0	0.0	1.9	0.0	46.6	0.9	56.7	46.6	1.0	169.9	30
1	1.9	1.9	0.1	47.5	0.8	60.0	45.6	1.0	173.4	29
2	3.8	1.9	0.3	48.3	0.8	63.6	44.6	1.1	176.3	28
3	5.7	2.0	0.6	49.1	0.7	67.3	43.5	1.1	179·9	27
4	7.7	1.8	1.1	49.8	0.7	70.8	42.4	1.2	183.1	26
5	9.5	1.7	1.7	50.5	0.7	74.6	41.2	1.2	186.1	25
6	11.2	1.8	2.5	51.1	0.6	78.2	40.0	1.3	189.1	24
7	13.0	1.8	3.3	51.7	0.5	82.1	38.7	1.3	191.6	23
8	14.8	1.8	4.4	52.2	0.4	85.9	37.4	1.4	194.8	22
9	16.6	1.8	5.5	52.6	0.3	89.7	36.0	1.4	197.5	21
10	18.4	1.8	6.8	52.9	0.3	93.6	34.6	1.5	199.9	20
11	20.2	1.7	8.2	53.2	0.3	97.5	33.1	1.5	202.6	19
12	21.9	1.7	9.8	53.5	0.2	101.4	31.6	1.5	205.0	18
13	23.6	1.7	11.5	53.7	0.1	105.4	30.1	1.6	207.2	17
14	25.3	1.6	13.2	53.8	0.0	109.3	28.5	1.6	209.4	16
15	26.9	1.6	15.2	53.8	0.0	113.3	26.9	1.6	211.5	15
16	28.5	1.6	17.2	53.8	0.1	117.2	25.3	1·7	213.4	14
17	30.1	1.5	19.3	53.7	0.2	121.2	23.6	1.7	215.1	13
18	31.6	1.5	21.7	53.5	0.3	125.2	21.9	1.7	216.8	12
19	33.1	1.5	24.0	53.2	0.3	129.1	20.2	1.8	218.2	11
20	34.6	1.4	26.5	52.9	0.3	133.0	18.4	1.8	219.6	10
21	36.0	1.4	29.1	52.6	0.4	136.9	16.6	1.8	221.1	9
22	37.4	1.3	31.8	52.2	0.5	140.7	14.8	1.8	222.3	8
23	38.7	1.3	34.6	51.7	0.6	144.6	13.0	1.8	223.2	7
24	40.0	1.2	37.5	51.1	0.6	148.3	11.2	1.7	224.2	6
25	41.2	1.2	40.5	50.5	0.7	152.1	9.5	1.8	225.0	5
26	42.4	1.1	43.5	49.8	0.7	155.7	7.7	2.0	225.5	4
27	43.5	1.1	46.7	49.1	0.8	159.4	5.7	1.9	226.0	3
28	44.6	1.0	49.9	48.3	0.8	163.0	3.8	1.9	226.4	2
29	45.6	1.0	53.3	47.5	0.9	166.5	1.9	1.9	226.6	1
30	46.6		56.7	46.6		169.9	0.0		226.6	0
	Sig. XI. + / Sig. V. +		—	Sig. X. + / Sig. IV. +		—	Sig. IX. + / Sig. III. +		—	

TABLE I.

Epochs of the Mean Longitude of Jupiter, with the Arguments of the Equations.

	YEARS.	Mean Long. S.	D.	M.	S.	Aphelion. S.	D.	M.	S.	Node. S.	D.	M.	S.
Nat. J. C.	0	4	29	32	41.8	5	12	46	27	2	20	33	8
	100	10	5	50	15.0	5	14	21	0	2	21	32	38
	1400	5	27	38	26.4	6	4	50	10	3	4	26	8
	1500	11	3	55	59.6	6	6	24	43	3	5	25	38
B. N. S.	1600	4	9	23	40.2	6	7	59	14	3	6	25	7
C.	1700	9	15	36	14.2	6	9	33	47	3	7	24	37
B.	1740	2	0	7	15.3	6	10	11	37	3	7	48	25
B.	1760	10	7	22	46.0	6	10	30	31	3	8	0	19
B.	1780	6	14	38	16.6	6	10	49	26	3	8	12	13
C.	1800	2	21	48	48.1	6	11	8	21	3	8	24	7
B.	1820	10	29	4	18.8	6	11	27	16	3	8	36	1
	1821	11	29	24	50.5	6	11	28	13	3	8	36	37
	1822	0	29	45	22.3	6	11	29	9	3	8	37	12
	1823	2	0	5	54.0	6	11	30	6	3	8	37	48
B.	1824	3	0	31	24.9	6	11	31	3	3	8	38	24
	1825	4	0	51	56.7	6	11	32	0	3	8	38	59
	1826	5	1	12	28.4	6	11	32	56	3	8	39	35
	1827	6	1	33	0.2	6	11	33	53	3	8	40	11
B.	1828	7	1	58	31.1	6	11	34	50	3	8	40	47
	1829	8	2	19	2.8	6	11	35	47	3	8	41	22
	1830	9	2	39	34.6	6	11	36	43	3	8	41	58
	1831	10	3	0	6.3	6	11	37	40	3	8	42	34
B.	1832	11	3	25	37.2	6	11	38	37	3	8	43	9
	1833	0	3	46	8.9	6	11	39	34	3	8	43	45
	1834	1	4	6	40.7	6	11	40	30	3	8	44	21
	1835	2	4	27	12.4	6	11	41	27	3	8	44	57
B.	1836	3	4	52	43.3	6	11	42	24	3	8	45	32
	1837	4	5	13	15.1	6	11	43	20	3	8	46	8
	1838	5	5	33	46.8	6	11	44	17	3	8	46	44
	1839	6	5	54	18.6	6	11	45	14	3	8	47	19
B.	1840	7	6	19	49.4	6	11	46	11	3	8	47	55
	1841	8	6	40	21.1	6	11	47	8	3	8	48	31
	1842	9	7	0	52.9	6	11	48	4	3	8	49	6
	1843	10	7	21	24.6	6	11	49	1	3	8	49	42
B.	1844	11	7	46	55.5	6	11	49	58	3	8	50	18
	1845	0	8	7	27.3	6	11	50	55	3	8	50	53
	1846	1	8	27	59.0	6	11	51	51	3	8	51	29
	1847	2	8	48	30.8	6	11	52	48	3	8	52	5
B.	1848	3	9	14	1.7	6	11	53	45	3	8	52	41
	1849	4	9	34	33.4	6	11	54	42	3	8	53	16
	1850	5	9	55	5.2	6	11	55	38	3	8	53	52
	1851	6	10	15	36.9	6	11	56	35	3	8	54	28
B.	1852	7	10	41	7.8	6	11	57	32	3	8	55	3

TABLE I.

Epochs of the Mean Longitude of Jupiter with the Arguments of the Equations.

	YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.	ARG. IX.
Nat. J. C.	0	2487	9957	2467	2190	511	981	220	234
	100	2841	6374	9237	5391	223	482	974	629
	1400	7451	9796	7270	7002	473	997	779	762
	1500	7806	6213	4040	0203	185	498	533	157
B. N. S.	1600	8145	2626	0772	3381	893	995	286	550
C.	1700	8500	9043	7543	6579	605	495	040	945
B.	1740	8641	5610	4251	9860	289	896	742	303
B.	1760	8712	8893	7605	6500	632	596	093	982
B.	1780	8783	2177	0950	3140	974	296	444	661
C.	1800	8854	5459	4313	9778	316	996	795	340
B.	1820	8925	8742	7667	6418	659	696	145	19
	1821	9429	8906	8335	7249	776	781	163	53
	1822	9932	9070	9002	8081	893	66	180	87
	1823	436	9235	9670	8912	10	251	198	121
B.	1824	939	9399	338	9746	127	430	215	155
	1825	1443	9563	1006	577	245	621	233	189
	1826	1946	9727	1673	1409	362	806	250	223
	1827	2450	9891	2341	2240	479	991	268	257
B.	1828	2953	55	3009	3074	596	176	285	291
	1829	3457	220	3676	3906	713	361	303	325
	1830	3960	384	4344	4737	830	546	320	358
	1831	4464	548	5012	5568	947	731	338	402
B.	1832	4968	712	5679	6402	64	916	355	426
	1833	5471	876	6347	7234	182	101	373	460
	1834	5975	1040	7015	8065	299	286	390	494
	1835	6478	1205	7683	8896	416	471	408	528
B.	1836	6982	1369	8350	9730	533	656	426	562
	1837	7485	1533	9018	562	650	841	443	596
	1838	7989	1697	9686	1393	767	26	461	630
	1839	8492	1861	353	2224	884	211	478	664
B.	1840	8996	2025	1021	3058	1	396	496	698
	1841	9500	2189	1689	3889	118	581	514	732
	1842	3	2353	2356	4721	235	766	531	766
	1843	507	2518	3024	5552	252	951	549	800
B.	1844	1010	2682	3692	6386	469	136	566	834
	1845	1514	2846	4360	7217	587	321	584	868
	1846	2017	3010	5027	8049	704	506	601	902
	1847	2521	3174	5695	8880	821	691	619	936
B.	1848	3024	3338	6363	9714	938	876	636	970
	1849	3528	3503	7030	546	55	61	654	4
	1850	4031	3667	7698	1377	172	246	671	37
	1851	4535	3831	8366	2208	289	431	689	71
B.	1852	5039	3995	9033	3042	406	616	706	105

TABLE II.

Mean Motion of Jupiter for *Julian Years*, with the Arguments of the Equations.

	YEARS.	Mot. in Long. S.	D.	M.	S.	Mot. Aphelion. S.	D.	M.	S.	Mot. Node. S.	D.	M.	S.
	1	1	0	20	31.7	0	0	0	57	0	0	0	36
	2	2	0	41	3.5	0	0	1	53	0	0	1	11
	3	3	1	1	35.2	0	0	2	50	0	0	1	47
B.	4	4	1	27	6.1	0	0	3	47	0	0	2	23
	5	5	1	47	37.9	0	0	4	44	0	0	2	58
	6	6	2	8	9.6	0	0	5	40	0	0	3	34
	7	7	2	28	41.4	0	0	6	37	0	0	4	10
B.	8	8	2	54	12.3	0	0	7	34	0	0	4	46
	9	9	3	14	44.0	0	0	8	31	0	0	5	21
	10	10	3	35	15.8	0	0	9	27	0	0	5	57
	11	11	3	55	47.5	0	0	10	24	0	0	6	33
B.	12	0	4	21	18.4	0	0	11	21	0	0	7	8
	13	1	4	41	50.1	0	0	12	18	0	0	7	44
	14	2	5	2	21.9	0	0	13	14	0	0	8	20
	15	3	5	22	53.6	0	0	14	11	0	0	8	56
B.	16	4	5	48	24.5	0	0	15	8	0	0	9	31
	17	5	6	8	56.3	0	0	16	4	0	0	10	7
	18	6	6	29	28.0	0	0	17	1	0	0	10	43
	19	7	6	49	59.8	0	0	17	58	0	0	11	18
B.	20	8	7	15	30.6	0	0	18	55	0	0	11	54
	40	4	14	31	1.3	0	0	37	49	0	0	23	48
	60	0	21	46	31.9	0	0	56	44	0	0	35	42
	80	8	29	2	2.5	0	1	15	38	0	0	47	36
	100	5	6	17	33.2	0	1	34	33	0	0	59	30
	200	10	12	35	6.4	0	3	9	6	0	1	59	0
	300	3	18	52	39.6	0	4	43	39	0	2	58	30
	400	8	25	10	12.7	0	6	18	12	0	3	58	0
	500	2	1	27	45.9	0	7	52	45	0	4	57	30
	600	7	7	45	19.1	0	9	27	19	0	5	57	0
	700	0	14	2	52.3	0	11	1	52	0	6	56	30
	800	5	20	20	25.5	0	12	36	25	0	7	56	0
	900	10	26	37	58.7	0	14	10	58	0	8	55	30
	1000	4	2	55	31.8	0	15	45	31	0	9	55	0
	2000	8	5	51	3.7	1	1	31	2	0	19	50	0
	3000	0	8	46	35.5	1	17	16	33	0	29	45	0
	4000	4	11	42	7.4	2	3	2	4	1	9	49	0
	5000	8	14	37	39.2	2	18	47	35	1	19	35	0
	6000	0	17	33	11.0	3	4	33	6	1	29	30	0

Bissextiles.

TABLE II.

Mean Motion of Jupiter for *Julian Years*, with the Arguments of the Equations.

	YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.	ARG. IX.
	1	0504	0164	0668	0831	117	185	018	034
	2	1007	0328	1335	1663	234	370	035	068
	3	1511	0493	2003	2494	351	555	053	102
B.	4	2014	0657	2671	3328	468	740	070	136
	5	2518	0821	3339	4159	586	925	088	170
	6	3021	0985	4006	4991	703	110	105	204
	7	3525	1149	4674	5822	820	295	123	238
B.	8	4028	1313	5342	6656	937	480	140	272
	9	4532	1478	6009	7488	054	665	158	306
	10	5035	1642	6677	8319	171	850	175	339
	11	5539	1806	7345	9150	288	035	193	373
B.	12	6043	1970	8012	9984	405	220	210	407
	13	6546	2134	8680	0816	523	405	228	441
	14	7050	2298	9348	1647	640	590	245	475
	15	7553	2463	0016	2478	757	775	263	509
B.	16	8057	2627	0683	3312	874	960	281	543
	17	8560	2791	1351	4144	991	145	298	577
	18	9064	2955	2019	4975	108	330	316	611
	19	9567	3119	2686	5806	225	515	333	645
B.	20	0071	3283	3354	6640	342	700	351	679
	40	0142	6567	6708	3280	685	400	702	358
	60	0213	9850	0062	9920	027	101	052	037
	80	0284	3134	3416	6561	369	801	403	716
	100	0355	6417	6770	3201	712	501	754	395
	200	0709	2834	3541	6402	423	002	508	790
	300	1064	9251	0311	9602	135	504	262	184
	400	1418	5668	7081	2803	846	005	017	579
	500	1773	2085	3852	6004	558	506	771	974
	600	2128	8502	0622	9205	269	007	525	369
	700	2482	4920	7392	2406	981	508	279	764
	800	2837	1337	4162	5607	693	010	033	158
	900	3191	7754	0933	8807	404	511	787	553
	1000	3546	4171	7703	2008	116	012	542	948
	2000	7092	8342	5406	4017	231	024	083	896
	3000	0638	2513	3109	6025	347	036	625	844
	4000	4184	6684	0812	8033	463	048	166	792
	5000	7730	0856	8515	0042	579	059	708	740
	6000	1276	5027	6219	2050	694	071	249	688

Bissextiles.

TABLE III.

Mean Motion of Jupiter for *Months*, with the Arguments of the Equations.

Months.	Mot. Long. D. M. S.	Aph. SEC.	Node. SEC.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.	ARG. IX.
January	0 0 0.0	0	0	0	0	0	0	0	0	0	0
February	2 34 37.2	5	3	43	14	57	71	10	16	1	3
March	4 54 16.6	9	6	81	27	108	134	19	30	3	6
April	7 28 53.9	14	9	124	41	165	205	29	45	4	8
May	9 58 31.8	19	12	165	54	219	273	38	61	6	11
June	12 33 9.0	23	15	208	68	276	344	48	76	7	14
July	15 2 47.0	28	18	250	81	331	412	58	91	9	17
August	17 37 24.2	33	21	292	95	388	483	68	107	10	20
Septembr.	20 12 1.4	38	24	335	109	444	553	78	123	12	23
October	22 41 39.4	42	27	376	123	499	622	88	138	13	25
Novembr.	25 16 16.6	47	30	419	137	556	692	97	154	15	28
December	27 45 54.5	52	33	460	150	611	761	107	169	16	31

TABLE IV.

Mean Motion of Jupiter for *Hours* and *Minutes*.

Hours.	Mot. Long. M. S.	Hours.	Mot. Long. M. S.	Min.	Mt. Lon. SEC.	Min.	Mt. Lon. SEC.
1	0 12.5	13	2 42.1	1	0.2	27	5.6
2	0 24.9	14	2 54.6	2	0.4	30	6.2
3	0 37.4	15	3 7.0	3	0.6	33	6.9
4	0 49.9	16	3 19.5	4	0.8	36	7.5
5	1 2.3	17	3 32.0	5	1.0	39	8.1
6	1 14.8	18	3 44.4	6	1.2	42	8.7
7	1 27.3	19	3 56.9	9	1.9	45	9.4
8	1 39.8	20	4 9.4	12	2.5	48	10.0
9	1 52.2	21	4 21.9	15	3.1	51	10.6
10	2 4.7	22	4 34.3	18	3.7	54	11.2
11	2 17.2	23	4 46.8	21	4.4	57	11.8
12	2 29.6	24	4 59.3	24	5.0	60	12.5

TABLE V.

Mean Motion of Jupiter for *Days* with the Arguments of the Equations.

Days of Month.	Motion in Longitude. D. M. S.	Mot. Aphel. SEC.	Mot. Node. SEC.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
1	0 4 59.3	0	0	1	0	2	2	0	1	0	0
2	0 9 58.5	0	0	3	1	4	5	1	1	0	0
3	0 14 57.8	0	0	4	1	5	7	1	2	0	0
4	0 19 57.1	1	0	6	2	7	9	1	2	0	0
5	0 24 56.3	1	0	7	2	9	11	2	3	0	0
6	0 29 55.6	1	1	8	3	11	14	2	3	0	1
7	0 34 54.9	1	1	10	3	13	16	2	4	0	1
8	0 39 54.1	1	1	11	4	15	18	3	4	0	1
9	0 44 53.4	1	1	12	4	16	20	3	5	0	1
10	0 49 52.7	2	1	14	5	18	23	3	5	0	1
11	0 54 51.9	2	1	15	5	20	25	4	6	1	1
12	0 59 51.2	2	1	16	5	22	27	4	6	1	1
13	1 4 50.4	2	1	17	6	24	30	4	7	1	1
14	1 9 49.7	2	1	19	6	26	32	4	7	1	1
15	1 14 49.0	2	1	21	7	27	34	5	8	1	1
16	1 19 48.2	3	2	22	7	29	36	5	8	1	2
17	1 24 47.5	3	2	23	8	31	39	5	9	1	2
18	1 29 46.8	3	2	25	8	33	41	6	9	1	2
19	1 34 46.0	3	2	26	9	35	43	6	10	1	2
20	1 39 45.3	3	2	28	9	37	46	6	10	1	2
21	1 44 44.6	3	2	29	9	38	48	7	11	1	2
22	1 49 43.8	3	2	30	10	40	50	7	11	1	2
23	1 54 43.1	4	2	32	10	42	52	7	12	1	2
24	1 59 42.4	4	2	33	11	44	55	8	12	1	2
25	2 4 41·6	4	2	35	11	46	57	8	13	1	2
26	2 9 40.9	4	3	36	12	48	59	8	13	1	2
27	2 14 40.2	4	3	37	12	49	61	9	14	1	3
28	2 19 39.4	4	3	39	13	51	64	9	14	1	3
29	2 24 38.7	5	3	40	13	53	66	9	15	1	3
30	2 29 38.0	5	3	41	13	55	68	10	15	1	3
31	2 34 37.2	5	3	43	14	57	71	10	16	1	3

In the Months JANUARY and FEBRUARY of a Bissextile Year, subtract one from the given Day of the Month.

TABLE VI.

The great Inequality of the Motion of Jupiter, with the Corrections of the Arguments which regulate the other Inequalities.

Years.	Equations.	Diff.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
	M. S.	SEC.								
1550	— 1 18.5	86.2	— 2	— 3	— 5	— 9	— 1	— 0	+ 0	+ 0
1560	+ 0 7.7	86.0	+ 0	+ 0	+ 0	+ 1	+ 0	+ 0	— 0	— 0
1570	1 33.7		2	4	6	11	1	1	— 1	0
		85.4								
1580	2 59.1	84.6	5	8	12	18	2	1	— 1	0
1590	4 23.7	83.3	7	12	18	30	3	2	— 2	0
1600	5 47.0		9	15	24	39	3	2	— 2	— 1
		81.6								
1610	7 8.6	79.6	11	19	30	49	4	3	— 3	— 1
1620	8 28.2	77.1	13	22	35	58	5	3	— 3	— 1
1630	9 45.3		15	27	41	66	6	3	— 4	— 1
		74.3								
1640	10 59.6	71.2	17	29	46	75	6	4	— 4	— 1
1650	12 10.8	67.7	19	32	51	83	7	4	— 4	— 1
1660	13 18.5		21	35	56	91	8	5	— 5	— 1
		63.9								
1670	14 22.4	59.9	22	38	60	98	8	5	— 5	— 2
1680	15 22.3	55.4	24	40	64	105	9	5	— 6	— 2
1690	16 17.7		25	43	68	111	9	6	— 6	— 2
		50.9								
1700	17 8.6	46.0	27	45	72	117	10	6	— 6	— 2
1710	17 54.6	41.0	28	47	75	122	10	6	— 7	— 2
1720	18 35.6		29	49	78	127	11	7	— 7	— 2
		35.6								
1730	19 11.2	30.3	30	50	80	131	11	7	— 7	— 2
1740	19 41.5	24.7	30	52	82	134	11	7	— 7	— 2
1750	20 6.2		31	53	84	137	12	7	— 7	— 2
		19.1								
1760	20 25.3	13.3	32	54	85	139	12	7	— 8	— 2
1770	20 38.6	7.4	32	54	86	141	12	7	— 8	— 2
1780	20 46.0		32	55	87	141	12	7	— 8	— 2
		1.7								
1790	+20 47.7		32	55	87	142	12	7	— 8	— 2

TABLE VI.

The great Inequality of the Motion of Jupiter, with the Corrections of the Arguments which regulate the other Inequalities.

Years.	Equations.	Diff.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.	Arg. IX.
	M. S.	SEC.								
1790	+20 47.7		+32	+55	+87	+142	+12	+7	— 8	— 2
		—4.2								
1800	20 43.5		32	54	87	141	12	7	— 8	— 2
		10.0								
1810	20 33.5		32	54	86	140	12	7	— 8	— 2
		15.8								
1820	20 17.7		31	53	85	138	12	7	— 7	— 2
		21.5								
1830	19 56.2		31	52	83	136	11	7	— 7	— 2
		27.1								
1840	19 29.1		30	51	81	133	11	7	— 7	— 2
		32.5								
1850	18 56.6		29	50	79	129	11	7	— 7	— 2
		37.8								
1860	18 18.8		28	48	77	125	10	6	— 7	— 2
		42.9								
1870	17 35.9		27	46	74	120	10	6	— 6	— 2
		47 9								
1880	16 48.0		26	44	70	114	10	6	— 6	— 2
		52.7								
1890	15 55.3		25	42	67	109	9	6	— 6	— 2
		56.9								
1900	14 58.4		23	39	62	102	9	5	— 5	— 2
		61.1								
1910	13 57.3		22	37	58	95	8	5	— 5	— 2
		65.1								
1920	12 52.2		20	34	54	88	7	5	— 5	— 2
		68.6								
1930	11 43.6		18	31	49	80	7	4	— 4	— 1
		71.8								
1940	10 31.8		16	28	44	72	6	4	— 4	— 1
		74.8								
1950	9 17.0		14	25	39	64	5	3	— 3	— 1
		77.3								
1960	7 59.7		12	21	33	54	5	3	— 3	— 1
		79.5								
1970	6 40.2		10	18	28	45	4	2	— 2	— 1
		81.4								
1980	5 18.8		8	14	22	36	3	2	— 2	— 1
		82.8								
1990	3 56.0		6	10	16	27	2	1	— 1	— 0
		83.8								
2000	2 32.2		4	7	10	17	1	1	— 1	— 0
		84.5								
2010	+ 1 7.7		+ 2	+ 3	+ 5	+ 8	+ 1	+0	— 0	— 0
		84.7								
2020	— 0 17.0		— 0	— 1	— 1	— 2	— 0	—0	+ 0	+ 0

TABLE VII.

Equation of Jupiter's Orbit for 1750, with the Secular Variation.

ARGUMENT. *Long.* ♃ *cor. by Tab. VI.* — *Aphelion, or mean Anomaly.*

Degrees.	Sig. O. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Sig. I. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Degrees.
0	0 0 0.0	5 26.5	0.00	2 37 1.8	4 48.1	25.00	30
1	0 5 26.5	5 26.4	0.86	2 41 49.9	4 45.5	25.79	29
2	0 10 52.9	5 26.2	1.72	2 46 35.4	4 43.0	26.55	28
3	0 16 19.1	5 26.0	2.58	2 51 18.4	4 40.3	27.31	27
4	0 21 45.1	5 25.6	3.44	2 55 58.7	4 37.5	28.08	26
5	0 27 10.7	5 25.2	4.31	3 0 36.2	4 34.6	28.83	25
6	0 32 35.9	5 24.7	5.16	3 5 10.8	4 31.7	29.57	24
7	0 38 0.6	5 24.2	6.01	3 9 42.5	4 28.7	30.32	23
8	0 43 24.8	5 23.5	6.88	3 14 11.2	4 25.6	31.04	22
9	0 48 48.3	5 22.7	7.73	3 18 36.8	4 22.6	31.77	21
10	0 54 11.0	5 21.9	8.57	3 22 59.4	4 19.3	32.48	20
11	0 59 32.9	5 21.0	9.42	3 27 18.7	4 16.0	33.20	19
12	1 4 53.9	5 20.1	10.27	3 31 34.7	4 12.6	33.90	18
13	1 10 14.0	5 18.9	11.12	3 35 47.3	4 9.2	34.60	17
14	1 15 32.9	5 17.6	11.97	3 39 56.5	4 5.6	35.29	16
15	1 20 50.5	5 16.5	12.81	3 44 2.1	4 2.0	35.97	15
16	1 26 7.0	5 15.3	13.65	3 48 4.1	3 58.4	36.63	14
17	1 31 22.3	5 13.8	14.49	3 52 2.5	3 54.8	37.30	13
18	1 36 36.1	5 12.2	15.32	3 55 57.3	3 50.8	37.95	12
19	1 41 48.3	5 10.7	16.15	3 59 48.1	3 46.9	38.59	11
20	1 46 59.0	5 9.0	16.98	4 3 35.0	3 43.0	39.23	10
21	1 52 8.0	5 7.4	17.80	4 7 18.0	3 39.0	39.86	9
22	1 57 15.4	5 5.5	18.62	4 10 57.0	3 34.9	40.49	8
23	2 2 20.9	5 3.6	19.43	4 14 31.9	3 30.7	41.08	7
24	2 7 24.5	5 1.6	20.24	4 18 2.6	3 26.5	41.68	6
25	2 12 26.1	4 59.6	21.05	4 21 29.1	3 22.3	42.27	5
26	2 17 25.7	4 57.4	21.85	4 24 51.4	3 17.9	42.85	4
27	2 22 23.1	4 55.2	22.65	4 28 9.3	3 13.5	43.41	3
28	2 27 18.3	4 52.9	23.44	4 31 22.8	3 9.0	43.97	2
29	2 32 11.2	4 50.6	24.22	4 34 31.8	3 4.5	44.52	1
30	2 37 1.8		25.00	4 37 36.3		45.06	0
	+ Sig. XI.		+	+ Sig. X.		+	

TABLE VII.

Equation of Jupiter's Orbit for 1750, with the Secular Variation.

ARGUMENT. *Long. ♃ cor. by Tab. VI. — Aphelion, or mean Anomaly.*

Degrees.	Sig. II. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Sig. III. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Degrees.
0	4 37 36.3	2 59.8	45.06	5 30 2.6	0 17.7	55.15	30
1	4 40 36.1	2 55.2	45.58	5 30 20.3	0 11.7	55.26	29
2	4 43 31.3	2 50.5	46.10	5 30 32.0	+0 5.7	55.35	28
3	4 46 21.8	2 45.7	46.61	5 30 37.7	—0 0.3	55.43	27
4	4 49 7.5	2 40.8	47.09	5 30 37.4	0 6.4	55.48	26
5	4 51 48.3	2 35.9	47.57	5 30 31.0	0 12.5	55.52	25
6	4 54 24.2	2 30.9	48.03	5 30 18.5	0 18.6	55.55	24
7	4 56 55.1	2 25.9	48.49	5 29 59.9	0 24.6	55.56	23
8	4 59 21.0	2 20.9	48.94	5 29 35.3	0 30.8	55.55	22
9	5 1 41.9	2 15.7	49.37	5 29 4.5	0 37.0	55.52	21
10	5 3 57.6	2 10.6	49.78	5 28 27.5	0 43.0	55.48	20
11	5 6 8.2	2 5.3	50.18	5 27 44.5	0 49.1	55.41	19
12	5 8 13.5	2 0.0	50.57	5 26 55.4	0 55.3	55.33	18
13	5 10 13.5	1 54.7	50.95	5 26 0.1	1 1.6	55.24	17
14	5 12 8.2	1 49.3	51.31	5 44 58.5	1 7.6	55.11	16
15	5 13 57.5	1 43.9	51.65	5 23 50.9	1 13.8	54.99	15
16	5 15 41.4	1 38.4	51.99	5 22 37.1	1 19.8	54.85	14
17	5 17 19.8	1 32.9	52.32	5 21 17.3	1 26.0	54.67	13
18	5 18 52.7	1 27.3	52.62	5 19 51.3	1 32.2	54.49	12
19	5 20 20.0	1 21.8	52.91	5 18 19.1	1 38.3	54.28	11
20	5 21 41.8	1 16.1	53.20	5 16 40.8	1 44.4	54.06	10
21	5 22 57.9	1 10.4	53.46	5 14 56.4	1 50.5	53.81	9
22	5 24 8.3	1 4.7	53.71	5 13 5.9	1 56.5	53.56	8
23	5 25 13.0	0 58.9	53.94	5 11 9.4	2 2.6	53.29	7
24	5 26 11.9	0 53.1	54.17	5 9 6 8	2 8.6	52.99	6
25	5 27 5.0	0 47.3	54.38	5 6 58.2	2 14.7	52.67	5
26	5 27 52.3	0 41.4	54.56	5 4 43.5	2 20.5	52.35	4
27	5 28 33.7	0 35.5	54.73	5 2 23.0	2 26.6	51.99	3
28	5 29 9.2	0 29.7	54.89	4 59 56.4	2 32.5	51.62	2
29	5 29 38.9	0 23.7	55.02	4 57 23.9	2 38.3	51.22	1
30	5 30 2.6		55.15	4 54 45.6		50.82	0
	+ Sig. IX.		+	+ Sig. VIII.		+	

TABLE VII.

Equation of Jupiter's Orbit for 1750, with the Secular Variation.

ARGUMENT. *Long. ♃ cor. by Tab. VI. — Aphelion, or mean Anomaly.*

Degrees.	Sig. IV. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Sig. V. Equation. — D. M. S.	Diff. M. S.	Sec. Var. — SEC.	Degrees.
0	4 54 45.6	2 44.2	50.75	2 54 15.1	5 11.8	30.78	30
1	4 52 1.4	2 50.1	50.40	2 49 3.3	5 15.5	29.88	29
2	4 49 11.3	2 55.8	49.97	2 43 47.8	5 19.0	28.96	28
3	4 46 15.5	3 1.5	49.51	2 38 28.8	5 22.4	28.03	27
4	4 43 14.0	3 7.3	49.03	2 33 6.4	5 25.7	27.09	26
5	4 40 6.7	3 12.8	48.53	2 27 40.7	5 28.9	26.15	25
6	4 36 53.9	3 18.5	48.02	2 22 11.8	5 32.0	25.19	24
7	4 33 35.4	3 24.0	47.48	2 16 39.8	5 34.9	24.23	23
8	4 30 11.4	3 29.5	46.95	2 11 4.9	5 37.7	23.25	22
9	4 26 41.9	3 34.9	46.38	2 5 27.2	5 40.4	22.26	21
10	4 23 7.0	3 40.3	45.79	1 59 46.8	5 43.0	21.26	20
11	4 19 26.7	3 45.7	45.19	1 54 3.8	5 45.5	20.25	19
12	4 15 41.0	3 50.9	44.57	1 48 18.3	5 47.9	19.24	18
13	4 11 50.1	3 55.9	43.92	1 42 30.4	5 50.2	18.21	17
14	4 7 54.2	4 1.1	43.29	1 36 40.2	5 52.2	17.19	16
15	4 3 53.1	4 6.3	42.63	1 30 48.0	5 54.2	16.15	15
16	3 59 46.8	4 11.2	41.95	1 24 53.8	5 56.0	15.10	14
17	3 55 35.6	4 16.1	41.24	1 18 57.8	5 57.8	14.04	13
18	3 51 19.5	4 20.9	40.52	1 13 0.0	5 59.4	12.98	12
19	3 46 58.6	4 25.6	39.80	1 7 0.6	6 0.8	11.92	11
20	3 42 33.0	4 30.3	39.04	1 0 59.8	6 2.1	10.85	10
21	3 38 2.7	4 34.8	38.29	0 54 57.7	6 3.4	9.79	9
22	3 33 27.9	4 39.4	37.50	0 48 54.3	6 4.5	8.72	8
23	3 28 48.5	4 43.7	36.72	0 42 49.8	6 5.3	7.63	7
24	3 24 4.8	4 48.0	35.91	0 36 44.5	6 6.1	6.54	6
25	3 19 16.8	4 52.3	35.08	0 30 38.4	6 6.8	5.45	5
26	3 14 24.5	4 56.4	34.26	0 24 31.6	6 7.4	4.36	4
27	3 9 28.1	5 0.5	33.40	0 18 24.2	6 7.9	3.28	3
28	3 4 27.6	5 4.4	32.55	0 12 16.3	6 8.1	2.18	2
29	2 59 23.2	5 8.1	31.67	0 6 8.2	6 8.2	1.10	1
30	2 54 15.1		30.78	0 0 0.0		0.00	0
	+ Sig. VII.		+	+ Sig. VI.		+	

TABLE VIII. Equation II.

Argument II.

Arg. II.	Equation II. M. S.	Diff. SEC.	Arg. II.	Equation II. M. S.	Diff. SEC.	Arg. II.	Equation II. M. S.	Diff. SEC.
0	4 14.7	24.6	3400	0 5.2	4.2	6700	8 17.2	9.9
100	4 39.3	24.0	3500	0 1.0	1.0	6800	8 7.3	12.6
200	5 3.3	22.9	3600	0 0.0	1.9	6900	7 54.7	15.0
300	5 26.2	21.2	3700	0 1.9	4.9	7000	7 39.7	17.4
400	5 47.4	19.3	3800	0 6.8	8.0	7100	7 22.3	19.5
500	6 6.7	16.7	3900	0 14.8	10.9	7200	7 2.8	21.4
600	6 23.4	13.7	4000	0 25.7	13.7	7300	6 41.4	22.8
700	6 37.1	10.9	4100	0 39.4	16.4	7400	6 18.6	24.2
800	6 48.0	7.1	4200	0 55.8	18.9	7500	5 54.4	25.0
900	6 55.1	3.7	4300	1 14.7	21.2	7600	5 29.4	25.7
1000	6 58.8	0.2	4400	1 35.9	23.2	7700	5 3.7	26.0
1100	6 59.0	3.5	4500	1 59.1	25.0	7800	4 37.7	25.6
1200	6 55.5	6.9	4600	2 24.1	26.3	7900	4 12.1	25.1
1300	6 48.6	10.2	4700	2 50.4	27.5	8000	3 47.0	23.9
1400	6 38.4	13.4	4800	3 17.9	28.3	8100	3 23.1	22.9
1500	6 25.0	16.2	4900	3 46.2	28.5	8200	3 0.2	20.9
1600	6 8.8	18.7	5000	4 14.7	28.5	8300	2 39.3	18.7
1700	5 50.1	20.9	5100	4 43.2	28.3	8400	2 20.6	16.2
1800	5 29.2	22.9	5200	5 11.5	27.5	8500	2 4.4	13.4
1900	5 6.3	23.9	5300	5 39.0	26.3	8600	1 51.0	10.2
2000	4 42.4	25.1	5400	6 5.3	25.0	8700	1 40.8	6.9
2100	4 17.3	25.6	5500	6 30.3	23.2	8800	1 33.9	3.5
2200	3 51.7	26.0	5600	6 53.5	21.2	8900	1 30.4	0.2
2300	3 25.7	25.7	5700	7 14.7	18.9	9000	1 30.6	3.7
2400	3 0.0	25.0	5800	7 33.6	16.4	9100	1 34.3	7.1
2500	2 35.0	24.2	5900	7 50.0	13.7	9200	1 41.4	10.9
2600	2 10.8	22.8	6000	8 3.7	10.9	9300	1 52.3	13.7
2700	1 48.0	21.4	6100	8 14.6	8.0	9400	2 6.0	16.7
2800	1 26.6	19.5	6200	8 22.6	4.9	9500	2 22.7	19.3
2900	1 7.1	17.4	6300	8 27.5	1.9	9600	2 42.0	21.2
3000	0 49.7	15.0	6400	8 29.4	1.0	9700	3 3.2	22.9
3100	0 34.7	12.6	6500	8 28.4	4.2	9800	3 26.1	24.0
3200	0 22.1	9.9	6600	8 24.2	7.0	9900	3 50.1	24.6
3300	0 12.2	7.0	6700	8 17.2		10000	4 14.7	
3400	0 5.2							

TABLE IX. Equation III.

Argument III.

Arg. III.	Equation III. M. S.	Diff. SEC.	Arg. III.	Equation III. M. S.	Diff. SEC.	Arg. III.	Equation III. M. S.	Diff. SEC.
0	1 46.0	8.5	3400	4 29.4	3.1	6700	0 36.1	5.7
100	1 54.5	8.6	3500	4 26.3	3.6	6800	0 30.4	5.2
200	2 3.1	8.6	3600	4 22.7	4.0	6900	0 25.2	4.8
300	2 11.7	8.7	3700	4 18.7	4.5	7000	0 20.4	4.3
400	2 20.4	8.7	3800	4 14.2	5.0	7100	0 16.1	3.8
500	2 29.1	8.7	3900	4 9.2	5.5	7200	0 12.3	3.4
600	2 37.8	8.6	4000	4 3.7	5.8	7300	0 8.9	2.8
700	2 46.4	8.4	4100	3 57.9	6.2	7400	0 6.1	2.3
800	2 54.8	8.3	4200	3 51.7	6.6	7500	0 3.8	1.7
900	3 3.1	8.2	4300	3 45.1	7.0	7600	0 2.1	1.2
1000	3 11.3	7.9	4400	3 38.1	7.3	7700	0 0.9	0.7
1100	3 19.2	7.7	4500	3 30.8	7.6	7800	0 0.2	0.2
1200	3 26.9	7.4	4600	3 23.2	7.8	7900	0 0.0	0.4
1300	3 34.3	7.1	4700	3 15.4	8.0	8000	0 0.4	1.0
1400	3 41.4	6.8	4800	3 7.4	8.2	8100	0 1.4	1.5
1500	3 48.2	6.4	4900	2 59.2	8.4	8200	0 2.9	2.0
1600	3 54.6	6.1	5000	2 50.8	8.5	8300	0 4.9	2.5
1700	4 0.7	5.7	5100	2 42.3	8.6	8400	0 7.4	3.1
1800	4 6.4	5.2	5200	2 33.7	8.6	8500	0 10.5	3.6
1900	4 11.6	4.8	5300	2 25.1	8.7	8600	0 14.1	4.0
2000	4 16.4	4.3	5400	2 16.4	8.7	8700	0 18.1	4.5
2100	4 20.7	3.8	5500	2 7.7	8.7	8800	0 22.6	5.0
2200	4 24.5	3.4	5600	1 59.0	8.6	8900	0 27.6	5.5
2300	4 27.9	2.8	5700	1 50.4	8.4	9000	0 33.1	5.8
2400	4 30.7	2.3	5800	1 42.0	8.3	9100	0 38.9	6.2
2500	4 33.0	1.7	5900	1 33.7	8.2	9200	0 45.1	6.6
2600	4 34.7	1.2	6000	1 25.5	7.9	9300	0 51.7	7.0
2700	4 35.9	0.7	6100	1 17.6	7.7	9400	0 58.7	7.3
2800	4 36.6	0.2	6200	1 9.9	7.4	9500	1 6.0	7.6
2900	4 36.8	0.4	6300	1 2.5	7.1	9600	1 13.6	7.8
3000	4 36.4	1.0	6400	0 55.4	6.8	9700	1 21.4	8.0
3100	4 35.4	1.5	6500	0 48.6	6.4	9800	1 29.4	8.2
3200	4 33.9	2.0	6600	0 42.2	6.1	9900	1 37.6	8.4
3300	4 31.9	2.5	6700	0 36.1		10000	1 46.0	
3400	4 29.4							

TABLE X. Equation IV.

Argument IV.

Arg. IV.	Equation IV. M. S.	Diff. SEC.	Arg. IV.	Equation IV. M. S.	Diff. SEC.	Arg. IV.	Equation IV. M. S.	Diff. SEC.
0	0 10.2	2.8	3400	2 43.4	2.5	6700	1 28.6	5.5
100	0 13.0	3.0	3500	2 45.9	2.3	6800	1 23.1	5.4
200	0 16.0	3.3	3600	2 48.2	2.0	6900	1 17.7	5.5
300	0 19.3	3.6	3700	2 50.2	1.5	7000	1 12.2	5.3
400	0 22.9	3.8	3800	2 51.7	1.3	7100	1 6.9	5.3
500	0 26.7	4.1	3900	2 53.0	0.9	7200	1 1.6	5.2
600	0 30.8	4.2	4000	2 53.9	0.6	7300	0 56.4	5.1
700	0 35.0	4.5	4100	2 54.5	0.3	7400	0 51.3	4.9
800	0 39.5	4.7	4200	2 54.8	0.0	7500	0 46.4	4.8
900	0 44.2	4.9	4300	2 54.8	0.4	7600	0 41.6	4.6
1000	0 49.1	5.0	4400	2 54.4	0.8	7700	0 37.0	4.3
1100	0 54.1	5.2	4500	2 53.6	1.2	7800	0 32.7	4.2
1200	0 59.3	5.2	4600	2 52.4	1.5	7900	0 28.5	4.0
1300	1 4.5	5.3	4700	2 50.9	1.8	8000	0 24.5	3.7
1400	1 9.8	5.4	4800	2 49.1	2.1	8100	0 20.8	3.4
1500	1 15.2	5.5	4900	2 47.0	2.4	8200	0 17.4	3.1
1600	1 20.7	5.5	5000	2 44.6	2.8	8300	0 14.3	2.9
1700	1 26.2	5.5	5100	2 41.8	3.0	8400	0 11.4	2.5
1800	1 31.7	5.4	5200	2 38.8	3.3	8500	0 8.9	2.3
1900	1 37.1	5.5	5300	2 35.5	3.6	8600	0 6.6	2.0
2000	1 42.6	5.3	5400	2 31.9	3.8	8700	0 4.6	1.5
2100	1 47.9	5.3	5500	2 28.1	4.1	8800	0 3.1	1.3
2200	1 53.2	5.2	5600	2 24.0	4.2	8900	0 1.8	0.9
2300	1 58.4	5.1	5700	2 19.8	4.5	9000	0 0.9	0.6
2400	2 3.5	4.9	5800	2 15.3	4.7	9100	0 0.3	0.3
2500	2 8.4	4.8	5900	2 10.6	4.9	9200	0 0.0	0.0
2600	2 13.2	4.6	6000	2 5.7	5.0	9300	0 0.0	0.4
2700	2 17.8	4.3	6100	2 0.7	5.2	9400	0 0.4	0.8
2800	2 22.1	4.2	6200	1 55.5	5.2	9500	0 1.2	1.2
2900	2 26.3	4.0	6300	1 50.3	5.3	9600	0 2.4	1.5
3000	2 30.3	3.7	6400	1 45.0	5.4	9700	0 3.9	1.8
3100	2 34.0	3.4	6500	1 39.6	5.5	9800	0 5.7	2.1
3200	2 37.4	3.1	6600	1 34.1	5.5	9900	0 7.8	2.4
3300	2 40.5	2.9	6700	1 28.6		10000	0 10.2	
3400	2 43.4							

TABLE XI. Equation V.

Argument V.

Arg. V.	Equation V. M. S.	Diff. SEC.	Arg. V.	Equation V. M. S.	Diff. SEC.	Arg. V.	Equation V. M. S.	Diff. SEC.
0	5 4.3	5.7	3400	2 53.7	10.5	6700	0 17.6	4.9
100	5 10.0	5.2	3500	2 43.2	10.5	6800	0 22.5	5.6
200	5 15.2	4.5	3600	2 32.7	10.4	6900	0 28.1	6.1
300	5 19.7	3.9	3700	2 22.3	10.3	7000	0 34.2	6.6
400	5 23.6	3.4	3800	2 12.0	10.2	7100	0 40.8	7.2
500	5 27.0	2.7	3900	2 1.8	10.0	7200	0 48.0	7.6
600	5 29.7	2.0	4000	1 51.8	9.8	7300	0 55.6	8.0
700	5 31.7	1.3	4100	1 42.0	9.5	7400	1 3.6	8.4
800	5 33.0	0.8	4200	1 32.5	9.2	7500	1 12.0	8.8
900	5 33.8	0.2	4300	1 23.3	9.0	7600	1 20.8	9.1
1000	5 34.0	0.6	4400	1 14.3	8.5	7700	1 29.9	9.5
1100	5 33.4	1.3	4500	1 5.8	8.1	7800	1 39.4	9.7
1200	5 32.1	1.8	4600	0 57.7	7.7	7900	1 49.1	9.9
1300	5 30.3	2.5	4700	0 50.0	7.3	8000	1 59.0	10.2
1400	5 27.8	3.2	4800	0 42.7	6.8	8100	2 9.2	10.4
1500	5 24.6	3.8	4900	0 35.9	6.2	8200	2 19.6	10.3
1600	5 20.8	4.4	5000	0 29.7	5.7	8300	2 29.9	10.4
1700	5 16.4	4.9	5100	0 24.0	5.2	8400	2 40.3	10.5
1800	5 11.5	5.6	5200	0 18.8	4.5	8500	2 50.8	10.5
1900	5 5.9	6.1	5300	0 14.3	3.9	8600	3 1.3	10.4
2000	4 59.8	6.6	5400	0 10.4	3.4	8700	3 11.7	10.3
2100	4 53.2	7.2	5500	0 7.0	2.7	8800	3 22.0	10.2
2200	4 46.0	7.6	5600	0 4.3	2.0	8900	3 32.2	10.0
2300	4 38.4	8.0	5700	0 2.3	1.3	9000	3 42.2	9.8
2400	4 30.4	8.4	5800	0 1.0	0.8	9100	3 52.0	9.5
2500	4 22.0	8.8	5900	0 0.2	0.2	9200	4 1.5	9.2
2600	4 13.2	9.1	6000	0 0.0	0.6	9300	4 10.7	9.0
2700	4 4.1	9.5	6100	0 0.6	1.3	9400	4 19.7	8.5
2800	3 54.6	9.7	6200	0 1.9	1.8	9500	4 28.2	8.1
2900	3 44.9	9.9	6300	0 3.7	2.5	9600	4 36.3	7.7
3000	3 35.0	10.2	6400	0 6.2	3.2	9700	4 44.0	7.3
3100	3 24.8	10.4	6500	0 9.4	3.8	9800	4 51.3	6.8
3200	3 14.4	10.3	6600	0 13.2	4.4	9900	4 58.1	6.2
3300	3 4.1	10.4	6700	0 17.6		10000	5 4.3	
3400	2 53.7							

TABLE XII. Equation VI.

Argument VI.

Arg. VI.	Equat. VI. sec.	Arg. VI.	Equat. VI. sec.	Arg VI.	Equat. VI. sec.
0	30.2	340	2.2	670	15.5
10	29.7	350	1.7	680	16.5
20	29.2	360	1.3	690	17.5
30	28.6	370	1.0	700	18.5
40	28.0	380	0.7	710	19.5
50	27.3	390	0.4	720	20.5
60	26.5	400	0.2	730	21.5
70	25.8	410	0.1	740	22.4
80	25.0	420	0.0	750	23.3
90	24.1	430	0.0	760	24.2
100	23.2	440	0.1	770	25.1
110	22.3	450	0.2	780	25.9
120	21.4	460	0.4	790	26.6
130	20.4	470	0.7	800	27.4
140	19.5	480	1.0	810	28.1
150	18.5	490	1.3	820	28.7
160	17.5	500	1.8	830	29.3
170	16.5	510	2.3	840	29.8
180	15.5	520	2.8	850	30.3
190	14.5	530	3.4	860	30.7
200	13.5	540	4.0	870	31.0
210	12.5	550	4.7	880	31.3
220	11.5	560	5.5	890	31.6
230	10.5	570	6.2	900	31.8
240	9.6	580	7.0	910	31.9
250	8.7	590	7.9	920	32.0
260	7.8	600	8.8	930	32.0
270	6.9	610	9.7	940	31.9
280	6.1	620	10.6	950	31.8
290	5.4	630	11.6	960	31.6
300	4.6	640	12.5	970	31.3
310	3.9	650	13.5	980	31.0
320	3.3	660	14.5	990	30.7
330	2.7	670	15.5	1000	30.2
340	2.2				

TABLE XIII. Equation VII.

Argument VII.

Arg VII.	Equat. VII. sec.	Arg. VII.	Equat. VII. sec.	Arg. VII.	Equat. VII. sec.
0	10.9	340	24.5	670	2.6
10	11.7	350	24.1	680	2.2
20	12.5	360	23.7	690	1.8
30	13.3	370	23.4	700	1.4
40	14.1	380	22.9	710	1.0
50	14.9	390	22.3	720	0.7
60	15.7	400	21.8	730	0.5
70	16.5	410	21.2	740	0.2
80	17.3	420	20.6	750	0.1
90	18.0	430	19.9	760	0.0
100	18.7	440	19.2	770	0.0
110	19.4	450	18.5	780	0.0
120	20.0	460	18.0	790	0.1
130	20.7	470	17.1	800	0.2
140	21.3	480	16.2	810	0.4
150	21.9	490	15.4	820	0.6
160	22.5	500	14.7	830	0.8
170	23.0	510	13.9	840	1.1
180	23.4	520	13.1	850	1.5
190	23.8	530	12.3	860	1.9
200	24.2	540	11.5	870	2.2
210	24.6	550	10.7	880	2.7
220	24.9	560	9.9	890	3.3
230	25.1	570	9.1	900	3.8
240	25.4	580	8.3	910	4.4
250	25.5	590	7.6	920	5.0
260	25.6	600	6.9	930	5.7
270	25.6	610	6.2	940	6.4
280	25.6	620	5.6	950	7.1
290	25.5	630	4.9	960	7.8
300	25.4	640	4.3	970	8.5
310	25.2	650	3.7	980	9.4
320	25.0	660	3.1	990	10.2
330	24.8	670	2.6	1000	10.9
340	24.5				

TABLE XIV. Equation VIII.

Argument VIII.

Arg. VIII.	Equat. VIII. sec.	Arg. VIII.	Equat. VIII. sec.	Arg. VIII.	Equat. VIII. sec.
0	24.1	340	1.4	670	13.6
10	23.6	350	1.0	680	14.5
20	23.1	360	0.7	690	15.3
30	22.5	370	0.5	700	16.1
40	21.9	380	0.3	710	16.8
50	21.4	390	0.1	720	17.6
60	20.8	400	0.1	730	18.4
70	20.1	410	0.0	740	19.1
80	19.4	420	0.0	750	19.8
90	18.7	430	0.1	760	20.4
100	18.0	440	0.2	770	21.0
110	17.2	450	0.4	780	21.7
120	16.4	460	0.6	790	22.3
130	15.6	470	0.8	800	22.9
140	14.8	480	1.2	810	23.4
150	14.0	490	1.5	820	23.9
160	13.2	500	1.9	830	24.3
170	12.4	510	2.4	840	24.6
180	11.5	520	2.9	850	25.0
190	10.7	530	3.5	860	25.3
200	9.9	540	4.0	870	25.5
210	9.2	550	4.6	880	25.7
220	8.4	560	5.2	890	25.9
230	7.6	570	5.9	900	25.9
240	6.9	580	6.6	910	26.0
250	6.2	590	7.3	920	26.0
260	5.6	600	8.0	930	25.9
270	5.0	610	8.8	940	25.8
280	4.3	620	9.6	950	25.6
290	3.7	630	10.4	960	25.5
300	3.1	640	11.2	970	25.2
310	2.6	650	12.0	980	24.8
320	2.1	660	12.8	990	24.5
330	1.7	670	13.6	1000	24.1
340	1.4				

TABLE XV. Equation IX.

Argument IX.

Arg. IX.	Equat. IX. sec.	Arg. IX.	Equat. IX. sec.	Arg. IX.	Equat. IX. sec.
0	19.8	340	14.1	670	0.5
10	20.3	350	13.4	680	0.8
20	20.8	360	12.7	690	1.0
30	21.2	370	12.0	700	1.3
40	21.5	380	11.2	710	1.7
50	21.9	390	10.5	720	2.0
60	22.2	400	9.8	730	2.4
70	22.5	410	9.1	740	2.9
80	22.8	420	8.4	750	3.4
90	23.0	430	7.7	760	3.9
100	23.1	440	7.0	770	4.5
110	23.1	450	6.3	780	5.0
120	23.2	460	5.6	790	5.6
130	23.2	470	5.0	800	6.3
140	23.1	480	4.5	810	7.0
150	23.1	490	3.9	820	7.7
160	23.0	500	3.4	830	8.4
170	22.7	510	2.9	840	9.1
180	22.4	520	2.4	850	9.8
190	22.2	530	2.0	860	10.5
200	21.9	540	1.7	870	11.2
210	21.5	550	1.3	880	12.0
220	21.2	560	1.0	890	12.7
230	20.8	570	0.8	900	13.4
240	20.3	580	0.4	910	14.1
250	19.8	590	0.2	920	14.8
260	19.3	600	0.1	930	15.5
270	18.7	610	0.1	940	16.2
280	18.2	620	0.0	950	16.9
290	17.6	630	0.0	960	17.6
300	16.9	640	0.1	970	18.2
310	16.2	650	0.1	980	18.7
320	15.5	660	0.2	990	19.3
330	14.8	670	0.5	1000	19.8
340	14.1				

TABLE XVI. Equation X.

Argument X. = VII. — VIII.

Arg. X.	Equat. X. sec.	Arg. X.	Equat. X. sec.	Arg. X.	Equat. X. sec.
0	17.1	340	12.2	670	0.4
10	17.5	350	11.6	680	0.7
20	17.9	360	10.9	690	0.9
30	18.3	370	10.3	700	1.1
40	18.6	380	9.7	710	1.4
50	18.9	390	9.1	720	1.8
60	19.1	400	8.4	730	2.1
70	19.4	410	7.8	740	2.5
80	19.6	420	7.2	750	2.9
90	19.8	430	6.6	760	3.3
100	19.9	440	6.1	770	3.8
110	19.9	450	5.5	780	4.4
120	20.0	460	4.9	790	4.9
130	20.0	470	4.4	800	5.5
140	20.0	480	3.8	810	6.0
150	19.9	490	3.3	820	6.6
160	19.8	500	2.9	830	7.2
170	19.6	510	2.5	840	7.8
180	19.3	520	2.1	850	8.4
190	19.1	530	1.7	860	9.1
200	18.9	540	1.4	870	9.7
210	18.6	550	1.1	880	10.3
220	18.2	560	0.9	890	10.9
230	17.9	570	0.6	900	11.6
240	17.5	580	0.4	910	12.2
250	17.1	590	0.2	920	12.8
260	16.7	600	0.1	930	13.4
270	16.2	610	0.1	940	13.9
280	15.6	620	0.0	950	14.5
290	15.1	630	0.0	960	15.1
300	14.5	640	0.0	970	15.6
310	14.0	650	0.1	980	16.2
320	13.4	660	0.2	990	16.7
330	12.8	670	0.4	1000	17.1
340	12.2				

TABLE XVII. Equation XI.

Argument XI. = VI. + VIII.

Arg. XI.	Equat. XI. sec.	Arg. XI.	Equat. XI. sec.	Arg. XI.	Equat. XI. sec.
0	3.9	340	1.8	670	10.7
10	3.5	350	2.1	680	10.8
20	3.2	360	2.4	690	10.8
30	3.0	370	2.7	700	10.8
40	2.7	380	3.0	710	10.8
50	2.4	390	3.3	720	10.8
60	2.1	400	3.6	730	10.7
70	1.8	410	3.9	740	10.7
80	1.6	420	4.2	750	10.6
90	1.3	430	4.6	760	10.6
100	1.1	440	4.9	770	10.4
110	0.9	450	5.2	780	10.2
120	0.8	460	5.6	790	10.0
130	0.6	470	5.9	800	9.9
140	0.4	480	6.2	810	9.7
150	0.3	490	6.6	820	9.5
160	0.2	500	6.9	830	9.2
170	0.1	510	7.3	840	9.0
180	0.0	520	7.6	850	8.7
190	0.0	530	7.8	860	8.4
200	0.0	540	8.1	870	8.1
210	0.0	550	8.4	880	7.8
220	0.0	560	8.7	890	7.6
230	0.1	570	9.0	900	7.2
240	0.1	580	9.2	910	6.9
250	0.2	590	9.5	920	6.6
260	0.3	600	9.7	930	6.2
270	0.4	610	9.9	940	5.9
280	0.6	620	10.0	950	5.6
290	0.8	630	10.2	960	5.2
300	0.9	640	10.4	970	4.9
310	1.1	650	10.5	980	4.6
320	1.3	660	10.6	990	4.2
330	1.6	670	10.7	1000	3.9
340	1.8				

TABLE XVIII.

Radius Vector of Jupiter's Orbit for 1750, with the Secular Variation.

ARGUMENT I. *The Mean Anomaly of Jupiter.*

Degrees.	Sig. O. Rad. Vec.	Diff.	Secular Var. + 0.000	Sig. I. Rad. Vec.	Diff.	Secular Var. + 0.000	Sig. II. Rad. Vec.	Diff.	Secular Var. + 0.000	Degrees.
0	5.44648	3	69.64	5.41579	204	61.99	5.33011	362	39.86	30
1	5.44645	11	69.63	5.41375	210	61.49	5.32649	366	38.90	29
2	5.44634	17	69.61	5.41165	216	60.95	5.32283	369	37.93	28
3	5.44617	24	69.57	5.40949	223	60.41	5.31914	374	36.95	27
4	5.44593	32	69.51	5.40726	228	59.83	5.31540	377	35.96	26
5	5.44561	38	69.43	5.40498	235	59.25	5.31163	381	34.95	25
6	5.44523	45	69.34	5.40263	240	58.65	5.30782	384	33.93	24
7	5.44478	52	69.22	5.40023	247	58.05	5.30398	388	32.90	23
8	5.44426	58	69.09	5.39776	252	57.43	5.30010	391	31.85	22
9	5.44368	66	68.95	5.39524	257	56.78	5.29619	394	30.81	21
10	5.44302	73	68.79	5.39267	264	56.12	5.29225	398	29.75	20
11	5.44229	79	68.61	5.39003	269	55.44	5.28827	400	28.68	19
12	5.44150	86	68.40	5.38734	275	54.77	5.28427	404	27.60	18
13	5.44064	93	68.19	5.38459	280	54.06	5.28023	406	26.50	17
14	5.43971	100	67.98	5.38179	286	53.33	5.27617	408	25.40	16
15	5.43871	107	67.74	5.37893	291	52.60	5.27209	412	24.29	15
16	5.43764	113	67.45	5.37602	296	51.85	5.26797	414	23.17	14
17	5.43651	120	67.18	5.37306	301	51.09	5.26383	416	22.05	13
18	5.43531	126	66.87	5.37005	307	50.31	5.25967	418	20.92	12
19	5.43405	134	66.56	5.36698	311	49.52	5.25549	421	19.76	11
20	5.43271	140	66.23	5.36387	317	48.70	5.25128	422	18.60	10
21	5.43131	146	65.88	5.36070	321	47.88	5.24706	424	17.44	9
22	5.42985	153	65.52	5.35749	327	47.05	5.24282	426	16.27	8
23	5.42832	160	65.13	5.35422	331	46.20	5.23856	428	15.11	7
24	5.42672	166	64.74	5.35091	335	45.33	5.23428	429	13.92	6
25	5.42506	173	64.32	5.34756	340	44.45	5.22999	430	12.73	5
26	5.42333	179	63.88	5.34416	345	43.56	5.22569	432	11.54	4
27	5.42154	185	63.44	5.34071	349	42.65	5.22137	433	10.34	3
28	5.41969	192	62.98	5.33722	354	41.73	5.21704	434	9.14	2
29	5.41777	198	62.49	5.33368	357	40.79	5.21270	434	7.93	1
30	5.41579		61.99	5.33011		39.86	5.20836		6.72	0
	Sig. XI.		+	Sig. X.		+	Sig. IX.		+	

TABLE XVIII.

Radius Vector of Jupiter's Orbit for 1750, with the Secular Variation.

ARGUMENT I. *The Mean Anomaly of Jupiter.*

Degrees.	Sig. III. Rad. Vec.	Diff.	Secular Var. + 0.000	Sig. IV. Rad. Vec.	Diff.	Secular Var. − 0.000	Sig. V. Rad. Vec.	Diff.	Secular Var. − 0.000	Degrees.
0	5.20836	436	06.72	5.08063	394	29.78	4.98293	234	58.63	30
1	5.20400	436	05.49	5.07669	391	30.93	4.98059	227	59.33	29
2	5.19964	436	04.28	5.07278	387	32.07	4.97832	220	60.02	28
3	5.19528	437	03.06	5.06891	383	33.20	4.97612	212	60.67	27
4	5.19091	437	01.82	5.06508	380	34.32	4.97400	205	61.30	26
5	5.18654	437	00.61	5.06128	375	35.43	4.97195	198	61.92	25
6	5.18217	437	00.64	5.05753	371	36.53	4.96997	191	62.50	24
7	5.17780	437	01.87	5.05382	366	37.62	4.96806	182	63.07	23
8	5.17343	436	03.09	5.05016	363	38.70	4.96624	176	63.62	22
9	5.16907	436	04.34	5.04653	357	39.76	4.96448	167	64.14	21
10	5.16471	436	05.58	5.04296	353	40.82	4.96281	160	64.65	20
11	5.16035	434	06.82	5.03943	348	41.86	4.96121	152	65.14	19
12	5.15601	434	08.06	5.03595	343	42.89	4.95969	144	65.59	18
13	5.15167	433	09.29	5.03252	338	43.91	4.95825	136	66.02	17
14	5.14734	431	10.53	5.02914	333	44.89	4.95689	129	66.43	16
15	5.14303	431	11.75	5.02581	328	45.88	4.95560	120	66.82	15
16	5.13872	428	12.98	5.02253	322	46.85	4.95440	112	67.18	14
17	5.13444	428	14.22	5.01931	316	47.80	4.95328	104	67.51	13
18	5.13016	425	15.44	5.01615	311	48.75	4.95224	96	67.82	12
19	5.12591	424	16.66	5.01304	305	49.66	4.95128	88	68.11	11
20	5.12167	422	17.89	5.00999	299	50.56	4.95040	79	68.37	10
21	5.11745	419	19.11	5.00700	293	51.46	4.94961	71	68.61	9
22	5.11326	418	20.31	5.00407	287	52.33	4.94890	63	68.83	8
23	5.10908	415	21.52	5.00120	281	53.19	4.94827	54	69.02	7
24	5.10493	412	22.73	4.99839	274	54.02	4.94773	46	69.18	6
25	5.10081	409	23.91	4.99565	267	54.83	4.94727	38	69.32	5
26	5.09672	407	25.10	4.99298	262	55.63	4.94689	30	69.43	4
27	5.09265	404	26.28	4.99036	254	56.41	4.94659	21	69.51	3
28	5.08861	401	27.45	4.98782	248	57.17	4.94638	12	69.58	2
29	5.08460	397	28.62	4.98534	241	57.91	4.94626	4	69.62	1
30	5.08063		29.78	4.98293		58.63	4.94622		69.64	0
	Sig. VIII.		—	Sig. VII.		—	Sig. VI.		—	

Equations of the Radius Vector, always additive.

TABLE XIX.

Arg. II.	Equation. II.		Arg. II.	Arg. II.	Equation. II.		Arg. II.
	0.00	Diff.			0.00	Diff.	
0	075	3	10000	2500	617	12	7500
100	078	9	9900	2600	605	16	7400
200	087	14	9800	2700	589	20	7300
300	101	19	9700	2800	569	22	7200
400	120	25	9600	2900	547	26	7100
500	145	28	9500	3000	521	28	7000
600	173	32	9400	3100	493	30	6900
700	205	35	9300	3200	463	32	6800
800	240	37	9200	3300	431	34	6700
900	277	39	9100	3400	397	35	6600
1000	316	39	9000	3500	362	35	6500
1100	355	38	8900	3600	327	35	6400
1200	393	37	8800	3700	292	36	6300
1300	430	36	8700	3800	256	35	6200
1400	466	33	8600	3900	221	33	6100
1500	499	30	8500	4000	188	32	6000
1600	529	27	8400	4100	156	30	5900
1700	556	23	8300	4200	126	28	5800
1800	579	19	8200	4300	098	25	5700
1900	598	14	8100	4400	073	21	5600
2000	612	10	8000	4500	052	18	5500
2100	622	5	7900	4600	034	14	5400
2200	627	1	7800	4700	020	11	5300
2300	628	4	7700	4800	009	7	5200
2400	624	7	7600	4900	002	2	5100
2500	617		7500	5000	000		5000

Equations of the Radius Vector, always additive.

TABLE XX.

Arg. IV.	Equation IV.		Arg. IV.	Equation IV.		Arg. IV.	Equation IV.		Arg. IV.	Equation IV.	
	0.00	Diff.		0.00	Diff.		0.00	Diff.		0.00	Diff.
0	049	5	2500	011	3	5000	136	5	7500	175	3
100	044	5	2600	014	3	5100	141	5	7600	172	4
200	039	4	2700	017	3	5200	146	5	7700	168	3
300	035	5	2800	020	4	5300	151	4	7800	165	4
400	030	4	2900	024	4	5400	155	4	7900	161	4
500	026	4	3000	028	5	5500	159	4	8000	157	4
600	022	4	3100	033	4	5600	163	4	8100	153	5
700	018	3	3200	037	5	5700	167	3	8200	148	5
800	015	3	3300	042	5	5800	170	3	8300	143	5
900	012	3	3400	047	5	5900	173	2	8400	138	5
1000	009	2	3500	052	5	6000	176	2	8500	133	5
1100	007	2	3600	057	6	6100	178	2	8600	128	5
1200	005	2	3700	063	5	6200	180	2	8700	123	6
1300	003	1	3800	068	6	6300	182	1	8800	117	6
1400	002	1	3900	074	6	6400	183	1	8900	111	5
1500	001	0	4000	080	6	6500	184	1	9000	106	6
1600	000	0	4100	086	5	6600	185	0	9100	100	6
1700	000	0	4200	091	6	6700	185	0	9200	094	6
1800	000	1	4300	097	6	6800	185	0	9300	088	6
1900	001	1	4400	103	6	6900	185	1	9400	082	5
2000	002	1	4500	109	6	7000	184	1	9500	077	6
2100	003	1	4600	115	5	7100	183	2	9600	071	6
2200	004	2	4700	120	6	7200	181	2	9700	065	5
2300	006	2	4800	126	5	7300	179	2	9800	060	6
2400	008	3	4900	131	5	7400	177	2	9900	054	5
2500	011		5000	136		7500	175		10000	049	

Equation of the Radius Vector, always additive.

TABLE XXI.

Arg. V.	Equation. V.		Arg. V.	Equation. V.		Arg. V.	Equation. V.		Arg. V.	Equation V.	
	0.00	Diff.		0.00	Diff.		0.00	Diff.		0.00	Diff.
0	087		2500	381		5000	334		7500	040	
		11			7			11			7
100	098		2600	388		5100	323		7600	033	
		11			7			11			7
200	109		2700	395		5200	312		7700	026	
		12			6			12			6
300	121		2800	401		5300	300		7800	020	
		13			5			12			5
400	134		2900	406		5400	288		7900	015	
		12			5			13			5
500	146		3000	411		5500	275		8000	010	
		13			4			13			4
600	159		3100	415		5600	262		8100	006	
		13			2			13			2
700	172		3200	417		5700	249		8200	004	
		13			2			13			2
800	185		3300	419		5800	236		8300	002	
		13			2			13			2
900	198		3400	421		5900	223		8400	000	
		13			0			13			0
1000	211		3500	421		6000	210		8500	000	
		13			1			13			1
1100	224		3600	420		6100	197		8600	001	
		13			1			13			1
1200	237		3700	419		6200	184		8700	002	
		13			2			13			2
1300	250		3800	417		6300	171		8800	004	
		13			3			13			3
1400	263		3900	414		6400	158		8900	007	
		13			3			13			3
1500	276		4000	411		6500	145		9000	010	
		12			5			13			5
1600	288		4100	406		6600	132		9100	015	
		13			5			12			5
1700	301		4200	401		6700	120		9200	020	
		11			6			11			6
1800	312		4300	395		6800	109		9300	026	
		12			7			12			7
1900	324		4400	388		6900	097		9400	033	
		11			8			11			8
2000	335		4500	380		7000	086		9500	041	
		10			8			10			8
2100	345		4600	372		7100	076		9600	049	
		10			8			10			8
2200	355		4700	364		7200	066		9700	057	
		9			10			9			10
2300	364		4800	354		7300	057		9800	067	
		9			10			9			10
2400	373		4900	344		7400	048		9900	077	
		8			10			8			10
2500	381		5000	334		7500	040		10000	087	

TABLE XXII.

Heliocentric Latitude for 1750, with the Secular Variation.

ARGUMENT XII. *Longitude of Jupiter — that of the Node.*

Degrees.	Latitude. Sig. O. *N.* / Sig. VI. *S.* D. M. S.	Diff. M. S.	Sec. Var. — / — SEC.	Latitude. Sig. I. *N.* / Sig. VII. *S.* D. M. S.	Diff. M. S.	Sec. Var. — / — SEC.	Latitude. Sig. II. *N.* / Sig. VIII. *S.* D. M. S.	Diff. SEC.	Sec. Var. — / — SEC.	Degrees.
0	0 0 0	1 23	0.0	0 39 31	1 11	11.0	1 8 27	40	19.0	30
1	0 1 23	1 22	0·4	0 40 42	1 11	11.4	1 9 7	40	19.2	29
2	0 2 45	1 23	0.8	0 41 53	1 10	11.7	1 9 47	38	19.4	28
3	0 4 8	1 23	1.1	0 43 3	1 9	12.0	1 10 25	37	19.6	27
4	0 5 31	1 22	1.5	0 44 12	1 8	12.3	1 11 2	36	19.8	26
5	0 6 53	1 23	1.9	0 45 20	1 7	12.6	1 11 38	34	20.0	25
6	0 8 16	1 22	2.3	0 46 27	1 7	12.9	1 12 12	33	20.1	24
7	0 9 38	1 22	2.7	0 47 34	1 5	13.2	1 12 45	32	20.3	23
8	0 11 0	1 22	3.1	0 48 39	1 5	13·5	1 13 17	30	20.4	22
9	0 12 22	1 21	3.4	0 49 44	1 4	13.8	1 13 47	29	20.5	21
10	0 13 43	1 22	3.8	0 50 48	1 3	14.1	1 14 16	28	20.7	20
11	0 15 5	1 21	4.2	0 51 51	1 2	14.4	1 14 44	26	20.8	19
12	0 16 26	1 21	4.6	0 52 53	1 1	14.7	1 15 10	25	20.9	18
13	0 17 47	1 20	5.0	0 53 54	1 0	15.0	1 15 35	23	21.1	17
14	0 19 7	1 20	5.4	0 54 54	0 59	15.3	1 15 58	22	21.2	16
15	0 20 27	1 20	5.7	0 55 53	0 58	15.5	1 16 20	21	21.3	15
16	0 21 47	1 19	6.1	0 56 51	0 57	15.8	1 16 41	19	21.4	14
17	0 23 6	1 19	6.5	0 57 48	0 56	16.1	1 17 0	18	21.5	13
18	0 24 25	1 19	6.8	0 58 44	0 55	16.3	1 17 18	17	21.5	12
19	0 25 44	1 18	7.2	0 59 39	0 54	16.6	1 17 35	15	21.6	11
20	0 27 2	1 17	7.6	1 0 33	0 52	16.9	1 17 50	14	21.6	10
21	0 28 19	1 17	7.9	1 1 25	0 52	17.1	1 18 4	12	21.7	9
22	0 29 36	1 17	8.3	1 2 17	0 50	17.4	1 18 16	11	21.8	8
23	0 30 53	1 17	8.7	1 3 7	0 49	17.6	1 18 27	9	21.8	7
24	0 32 9	1 16	9.0	1 3 56	0 48	17.8	1 18 36	8	21.9	6
25	0 33 24	1 15	9.4	1 4 44	0 47	18.1	1 18 44	7	21.9	5
26	0 34 39	1 15	9.7	1 5 31	0 45	18.3	1 18 51	5	22.0	4
27	0 35 53	1 14	10.0	1 6 16	0 45	18.5	1 18 56	3	22.0	3
28	0 37 6	1 13	10.4	1 7 1	0 44	18.7	1 18 59	2	22.0	2
29	0 38 19	1 13	10.7	1 7 45	0 42	18.9	1 19 1	1	22.0	1
30	0 39 31	1 12	11.0	1 8 27		19.0	1 19 2		22.0	0
	Sig. XI. *S.* / Sig. V. *N.*		— / —	Sig. X. *S.* / Sig. IV. *N.*		— / —	Sig. IX. *S.* / Sig. III. *N.*		— / —	

For any time before 1750, the Secular Variation changes the Sign.

TABLE XXIII.

Reduction to the Ecliptic, and the Logarithm of the Cosine of the Heliocentric Latitude.

ARGUMENT XII.

Degrees.	Reduct. to Eclipt. Sig. O. — Sig. VI. — SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. I. — Sig. VII. — SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. II. — Sig. VIII — SEC.	Log. cosin. Lat. helioc.	Degrees.
0	0.0	0.000000	23.6	9.999971	23.6	9.999914	30
1	0.9	0.000000	24.0	9.999970	23.1	9.999912	29
2	1.9	0.000000	24.5	9.999968	22.6	9.999910	28
3	2.8	0.000000	24.9	9.999966	22.1	9.999909	27
4	3.8	9.999999	25.3	9.999964	21.5	9.999907	26
5	4.7	9.999999	25.6	9.999962	20.9	9.999906	25
6	5.7	9.999999	25.9	9.999960	20.3	9.999904	24
7	6.6	9.999998	26.2	9.999958	19.6	9.999903	23
8	7.5	9.999998	26.4	9.999957	19.0	9.999901	22
9	8.4	9.999997	26.7	9.999955	18.2	9.999900	21
10	9.3	9.999997	26.8	9.999953	17.5	9.999899	20
11	10.2	9.999996	27.0	9.999951	16.8	9.999897	19
12	11.1	9.999995	27.1	9.999949	16.0	9.999896	18
13	12.0	9.999994	27.2	9.999947	15.2	9.999895	17
14	12.8	9.999993	27.2	9.999945	14.4	9.999895	16
15	13.6	9.999992	27.2	9.999943	13.6	9.999893	15
16	14.4	9.999991	27.2	9.999941	12.8	9.999892	14
17	15.2	9.999990	27.2	9.999939	12.0	9.999891	13
18	16.0	9.999989	27.1	9.999937	11.1	9.999890	12
19	16.8	9.999988	27.0	9.999935	10.2	9.999889	11
20	17.5	9.999987	26.8	9.999933	9.3	9.999889	10
21	18.2	9.999985	26.7	9.999931	8.4	9.999888	9
22	19.0	9.999984	26.4	9.999929	7.5	9.999887	8
23	19.6	9.999982	26.2	9.999927	6.6	9.999887	7
24	20.3	9.999981	25.9	9.999925	5.7	9.999887	6
25	20.9	9.999980	25.6	9.999923	4.7	9.999886	5
26	21.5	9.999978	25.3	9.999921	3.8	9.999886	4
27	22.1	9.999976	24.9	9.999919	2.8	9.999886	3
28	22.6	9.999974	24.5	9.999917	1.9	9.999885	2
29	23.1	9.999973	24.0	9.999916	0.9	9.999885	1
30	23.6	9.999971	23.6	9.999914	0.0	9.999885	0
	Sig. XI. + Sig. V. +		Sig. X. + Sig. IV. +		Sig. IX. + Sig. III. +		

TABLE I.

Epochs of the Mean Longitude of Saturn, with the Arguments of the Equations.

	YEARS.	Mean Long. S.	D.	M.	S.	Aphelion. S.	D.	M.	S.	Node. S.	D.	M.	S.
Nat. J. C.	0	2	0	0	22.4	7	26	2	7	3	6	10	10
	100	6	23	31	58.8	7	27	52	14	3	7	2	45
	1400	8	29	22	52.1	8	21	43	44	3	18	26	20
	1500	1	22	54	28.5	8	23	33	51	3	19	18	55
B. N. S	1600	6	16	5	59.0	8	25	23	56	3	20	11	29
C.	1700	11	9	35	34.8	8	27	14	4	3	21	4	5
B.	1740	3	19	0	13.4	8	27	58	6	3	21	25	7
B.	1760	11	23	42	32.6	8	28	20	8	3	21	35	38
B.	1780	7	28	24	51.9	8	28	42	9	3	21	46	9
C.	1800	4	3	5	10.6	8	29	4	10	3	21	56	40
B.	1820	0	7	47	29.9	8	29	26	12	3	22	7	10
	1821	0	20	1	6.7	8	29	27	18	3	22	7	42
	1822	1	2	14	43.5	8	29	28	24	3	22	8	13
	1823	1	14	28	20.3	8	29	29	30	3	22	8	45
B.	1824	1	26	43	57.8	8	29	30	36	3	22	9	16
	1825	2	8	57	34.6	8	29	31	42	3	22	9	48
	1826	2	21	11	11.4	8	29	32	48	3	22	10	19
	1827	3	3	24	48.2	8	29	33	52	3	22	10	51
B.	1828	3	15	40	25.6	8	29	35	1	3	22	11	22
	1829	3	27	54	2.4	8	29	36	7	3	22	11	54
	1830	4	10	7	39.2	8	29	37	13	3	22	12	26
	1831	4	22	21	16.1	8	29	38	19	3	22	12	57
B.	1832	5	4	36	53.5	8	29	39	25	3	22	13	29
	1833	5	16	50	30.3	8	29	40	31	3	22	14	0
	1834	5	29	4	7.1	8	29	41	37	3	22	14	32
	1835	6	11	17	43.9	8	29	42	43	3	22	15	3
B.	1836	6	23	33	21.3	8	29	43	49	3	22	15	35
	1837	7	5	46	58.1	8	29	44	55	3	22	16	6
	1838	7	18	0	35.0	8	29	46	1	3	22	16	38
	1839	8	0	14	11.8	8	29	47	7	3	22	17	9
B.	1840	8	12	29	49.2	8	29	48	13	3	22	17	41
	1841	8	24	43	26.0	8	29	49	19	3	22	18	11
	1842	9	6	57	2.8	8	29	50	25	3	22	18	44
	1843	9	19	10	39.6	8	29	51	31	3	22	19	16
B.	1844	10	1	26	17.1	8	29	52	37	3	22	19	47
	1845	10	13	39	53.9	8	29	53	43	3	22	20	19
	1846	10	25	52	30.7	8	29	54	49	3	22	20	50
	1847	11	8	7	7.5	8	29	55	55	3	22	21	22
B.	1848	11	20	22	44.9	8	29	57	2	3	22	21	53
	1849	0	2	36	21.7	8	29	58	8	3	22	22	25
	1850	0	14	49	58.5	8	29	59	14	3	22	22	57
	1851	0	27	3	35.4	9	0	0	20	3	22	23	28
B.	1852	1	9	19	12.8	9	0	1	26	3	22	24	0

TABLE I.

Epochs of the Mean Longitude of Saturn, with the Arguments of the Equations.

YEARS.		Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.
Nat. J. C.	0	2487	9949	9703	220	266
	100	2841	6367	2549	974	942
	1400	7451	9794	9549	779	732
	1500	7806	6212	2395	533	408
B. N. S	1600	8145	2625	5232	286	079
C.	1700	8500	9043	8079	040	755
B.	1740	8641	5610	1217	742	425
B.	1760	8712	8893	7787	093	760
B.	1780	8783	2177	4356	444	096
C.	1800	8854	5459	0925	795	431
B.	1820	8925	8742	7494	145	766
	1821	9429	8906	7822	163	833
	1822	9932	9070	8151	180	900
	1823	0436	9235	8479	198	966
B.	1824	0939	9399	8808	215	033
	1825	1443	9563	9136	233	100
	1826	1946	9727	9465	250	167
	1827	2450	9891	9793	268	233
B.	1828	2953	0055	0122	285	300
	1829	3457	0220	0450	303	367
	1830	3960	0384	0779	320	434
	1831	4464	0548	1107	338	500
B.	1832	4968	0712	1436	355	567
	1833	5471	0876	1764	373	634
	1834	5975	1040	2092	390	701
	1835	6478	1205	2421	408	767
B.	1836	6982	1369	2749	426	834
	1837	7485	1533	3078	443	901
	1838	7989	1697	3406	461	968
	1839	8492	1861	3735	478	034
B.	1840	8996	2025	4063	496	101
	1841	9500	2189	4391	514	168
	1842	0003	2353	4720	531	235
	1843	0507	2518	5048	549	301
B.	1844	1010	2682	5377	566	368
	1845	1514	2846	5705	584	435
	1846	2017	3010	6034	601	502
	1847	2521	3174	6362	619	568
B.	1848	3024	3338	6691	636	635
	1849	3528	3503	7019	654	702
	1850	4031	3667	7348	671	769
	1851	4535	3831	7676	689	835
B.	1852	5039	3995	8005	706	902

TABLE II.

Mean Motion of Saturn for *Julian Years*, with the Arguments of the Equations.

	YEARS.	Mot. in Long. S.	D.	M.	S.	Mot. Aphelion S.	D.	M.	S.	Mot. Node. S.	D.	M.	S.
	1	0	12	13	36.8	0	0	1	6	0	0	0	32
	2	0	24	27	13.6	0	0	2	12	0	0	1	3
	3	1	6	40	50.4	0	0	3	18	0	0	1	35
B.	4	1	18	56	27.9	0	0	4	24	0	0	2	6
	5	2	1	10	4.7	0	0	5	30	0	0	2	38
	6	2	13	23	41.5	0	0	6	36	0	0	3	9
	7	2	25	37	18.3	0	0	7	42	0	0	3	41
B.	8	3	7	52	55.7	0	0	8	49	0	0	4	12
	9	3	20	6	32.5	0	0	9	55	0	0	4	44
	10	4	2	20	9.3	0	0	11	1	0	0	5	16
	11	4	14	33	46.2	0	0	12	7	0	0	5	47
B.	12	4	26	49	23.6	0	0	13	13	0	0	6	19
	13	5	9	3	0.4	0	0	14	19	0	0	6	50
	14	5	21	16	37.2	0	0	15	25	0	0	7	22
	15	6	3	30	14.0	0	0	16	31	0	0	7	53
B.	16	6	15	45	51.4	0	0	17	37	0	0	8	25
	17	6	27	59	28.2	0	0	18	43	0	0	8	56
	18	7	10	13	5.1	0	0	19	49	0	0	9	28
	19	7	22	26	41.9	0	0	20	55	0	0	9	59
B.	20	8	4	42	19.3	0	0	22	1	0	0	10	31
	40	4	9	24	38.6	0	0	44	3	0	0	21	2
	60	0	14	6	57.8	0	1	6	4	0	0	31	33
	80	8	18	49	17.1	0	1	28	6	0	0	42	4
	100	4	23	31	36.4	0	1	50	7	0	0	52	35
	200	9	17	3	12.8	0	3	40	14	0	1	45	10
	300	2	10	34	49.2	0	5	30	21	0	2	37	45
	400	7	4	6	25.6	0	7	20	28	0	3	30	20
	500	11	27	38	2.0	0	9	10	35	0	4	22	55
	600	4	21	9	38.5	0	11	0	42	0	5	15	30
	700	9	14	41	14.9	0	12	50	49	0	6	8	5
	800	2	8	12	51.3	0	14	40	56	0	7	0	40
	900	7	1	44	27.7	0	16	31	3	0	7	53	15
	1000	11	25	16	4.1	0	18	21	10	0	8	45	50
	2000	11	20	32	8.2	1	6	42	20	0	17	31	40
	3000	11	15	48	12.3	1	25	3	29	0	26	17	30
	4000	11	11	4	16.4	2	13	24	39	1	5	3	20
	5000	11	6	20	20.5	3	1	45	49	1	13	49	10
	6000	11	1	36	24.6	3	20	6	58	1	22	35	0

Bissextiles.

TABLE II.

Mean Motion of Saturn for *Julian Years*, with the Arguments of the Equations.

	YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.
	1	0504	0164	0328	018	067
	2	1007	0328	0657	035	134
	3	1511	0493	0985	053	200
B.	4	2014	0657	1314	070	267
	5	2518	0821	1642	088	334
	6	3021	0985	1971	105	401
	7	3525	1149	2299	123	467
B.	8	4028	1313	2628	140	534
	9	4532	1478	2956	158	601
	10	5035	1642	3285	175	668
	11	5539	1806	3613	193	734
B.	12	6043	1970	3942	210	801
	13	6546	2134	4270	228	868
	14	7050	2298	4598	245	935
	15	7553	2463	4927	263	001
B.	16	8057	2627	5255	281	068
	17	8560	2791	5584	298	135
	18	9064	2955	5912	316	202
	19	9567	3119	6241	333	268
B.	20	0071	3283	6569	351	335
Bissextiles.	40	0142	6567	3138	702	670
	60	0213	9850	9707	052	006
	80	0284	3134	6277	403	341
	100	0355	6417	2846	754	676
	200	0709	2835	5692	508	352
	300	1064	9252	8538	262	028
	400	1418	5670	1385	017	705
	500	1773	2087	4230	771	381
	600	2128	8505	7077	525	057
	700	2482	4922	9922	279	733
	800	2837	1340	2769	033	409
	900	3191	7757	5615	787	085
	1000	3546	4175	8461	542	761
	2000	7092	8350	6923	083	523
	3000	0638	2525	5384	625	284
	4000	4184	6700	3845	166	045
	5000	7730	0875	2307	708	806
	6000	1276	5049	0768	249	568

TABLE III.

Mean Motion of Saturn for *Months*, with the Arguments of the Equations.

Months.	Mot. Long. D.	M.	S.	Aphel. SEC.	Node. SEC.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.
January	0	0	0.0	0	0	0	0	00	0	0
February	1	2	18.4	6	3	43	14	28	1	6
March	1	58	35.0	10	5	81	27	53	3	11
April	3	0	53.5	17	8	124	41	81	4	17
May	4	1	11.3	21	10	165	54	108	6	22
June	5	3	29.7	27	13	208	68	136	7	28
July	6	3	47.5	33	16	250	81	163	9	33
August	7	6	5.9	39	18	292	95	191	10	39
September	8	8	24.3	44	21	335	109	219	12	45
October	9	8	42.2	49	24	376	123	246	13	50
November	10	11	0.6	55	26	419	137	273	15	56
December	11	11	18.4	60	29	460	150	301	16	61

TABLE IV.

Mean Motion of Saturn for *Hours* and *Minutes*.

Hours.	Mot. Long. M.	S.	Hours.	Mot. Long. M.	S.	Min.	Mot. Lon SEC.	Min.	Mot. Lon SEC.
1	0	5.0	13	1	5.3	1	0.1	27	2.2
2	0	10.0	14	1	10.3	2	0.2	30	2.5
3	0	15.1	15	1	15.4	3	0.3	33	2.7
4	0	20.1	16	1	20.4	4	0.3	36	3.0
5	0	25.1	17	1	25.4	5	0.4	39	3.2
6	0	30.1	18	1	30.4	6	0.5	42	3.5
7	0	35.2	19	1	35.5	9	0.7	45	3.7
8	0	40.2	20	1	40.5	12	1.0	48	4.0
9	0	45.2	21	1	45.5	15	1.3	51	4.2
10	0	50.2	22	1	50.5	18	1.5	54	4.5
11	0	55.3	23	1	55.6	21	1.8	57	4.7
12	1	0.3	24	2	0.6	24	2.0	60	5.0

TABLE V.

Mean Motion of Saturn for *Days*, with the Arguments of the Equations.

Days of Month.	Motion in Longitude. D.	M.	S.	Mot. Aphel. SEC.	Mot. Node. SEC.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.
1	0	2	0.6	0	0	1	0	1	0	0
2	0	4	1.2	0	0	3	1	2	0	0
3	0	6	1.8	1	0	4	1	3	0	1
4	0	8	2.4	1	0	6	2	4	0	1
5	0	10	3.0	1	0	7	2	5	0	1
6	0	12	3.6	1	1	8	3	5	0	1
7	0	14	4.2	1	1	10	3	6	0	1
8	0	16	4.8	2	1	11	4	7	0	2
9	0	18	5.3	2	1	12	4	8	0	2
10	0	20	5.9	2	1	14	5	9	0	2
11	0	22	6.5	2	1	15	5	10	1	2
12	0	24	7.1	2	1	17	5	11	1	2
13	0	26	7.7	2	1	18	6	12	1	2
14	0	28	8.3	3	1	19	6	13	1	2
15	0	30	8.9	3	1	21	7	13	1	3
16	0	32	9.5	3	1	22	7	14	1	3
17	0	34	10.1	3	1	23	8	15	1	3
18	0	36	10.7	3	2	25	8	16	1	3
19	0	38	11.3	4	2	26	9	17	1	3
20	0	40	11.9	4	2	28	9	18	1	4
21	0	42	12.5	4	2	29	9	19	1	4
22	0	44	13.1	4	2	30	10	20	1	4
23	0	46	13.7	4	2	32	10	21	1	4
24	0	48	14.3	4	2	33	11	22	1	4
25	0	50	14.8	5	2	35	11	23	1	5
26	0	52	15.4	5	2	36	12	23	1	5
27	0	54	16.0	5	2	37	12	24	1	5
28	0	56	16.6	5	2	39	13	25	1	5
29	0	58	17.2	5	3	40	13	26	1	5
30	1	0	17.8	6	3	41	13	27	1	5
31	1	2	18.4	6	3	43	14	28	1	6

In the Months JANUARY and FEBRUARY in a Bissextile Year, subtract 1 from the given day of the Month.

TABLE VI.

The great Inequality of the Motion of Saturn, with the Corrections of the Arguments which regulate the other Inequalities.

Years.	Equations.	Diff.	Change for half period.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.
	M. S.	SEC.	SEC.					
1550	+ 3 3.8							
		201.8	2.86	— 2	— 3	— 7	+ 0	— 1
1560	— 0 18.0	201.3	0.28	+ 0	+ 0	+ 1	— 0	+ 0
1570	3 39.3	200.0	3.42	2	4	8	+ 1	+ 1
1580	— 6 59.2	198.0	6.54	5	8	16	— 1	1
1590	10 17.2		9.63	7	12	23	— 2	2
		194.9						
1600	13 32.1		12.68	9	15	30	— 2	2
		190.9						
1610	— 16 43.0	186.2	15.67	11	19	38	— 3	3
1620	19 49.2	180.5	18.58	13	22	45	— 3	4
1630	22 49.7	173.9	21.41	15	26	51	— 4	4
1640	— 25 43.6		24.13	17	29	57	— 4	5
		166.5						
1650	28 30.1		26.75	19	32	64	— 4	5
		158.5						
1660	31 8.6	149.6	29.29	21	35	70	— 5	6
1670	— 33 38.2	140.0	31.59	22	38	76	— 5	6
1680	35 58.2	129.9	33.79	24	40	81	— 6	6
1690	38 8.1		35.83	25	43	86	— 6	7
		119.0						
1700	— 40 7.1		37.71	27	45	90	— 6	7
		107.6						
1710	41 54.7	95.8	39.41	28	47	94	— 7	7
1720	43 30.5	83.5	40.93	29	49	98	— 7	8
1730	— 44 54.0	70.9	42.25	30	50	101	— 7	8
1740	46 4.9		43.38	30	52	104	— 7	8
		57.9						
1750	47 2.8		44.30	31	53	106	— 7	8
		44.6						
1760	— 47 47.4	31.1	45.02	32	54	107	— 8	9
1770	48 18.5	17.5	45.52	32	54	109	— 8	9
1780	48 36.0	3.9	45.81	32	55	109	— 8	9
1790	48 39.9		45.89	32	55	109	— 8	9

TABLE VI.

The great Inequality of the Motion of Saturn, with the Corrections of the Arguments which regulate the other Inequalities.

Years.	Equation.	Diff.	Change for half period.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.
	M. S.	SEC.	SEC.					
1790	— 48 39.9		45.89	+32	+55	+109	— 8	+ 9
		9.9						
1800	48 30.0		45.75	32	54	109	— 8	9
		23.4						
1810	48 6.6		45.39	32	54	108	— 8	9
		37.0						
1820	47 29.6		44.83	31	53	107	— 7	8
		50.2						
1830	— 46 39.4		44.05	31	52	105	— 7	8
		63.4						
1840	45 36.0		43.07	30	51	103	— 7	8
		76.1						
1850	— 44 19.9		41.88	29	50	100	— 7	8
		88.6						
1860	42 51.3		40.51	28	48	96	— 6	8
		100.4						
1870	41 10.9		38.94	27	46	93	— 6	7
		112.1						
1880	— 39 18.8		37.18	26	44	88	— 6	7
		122.9						
1890	37 15.9		35.26	25	42	84	— 5	7
		133.5						
1900	35 2.4		33.17	23	40	79	— 5	6
		143.0						
1910	— 32 39.4		30.92	22	37	73	— 5	5
		152.2						
1920	30 7.2		28.53	20	34	68	— 4	5
		160.6						
1930	27 26.6		26.00	18	31	62	— 4	4
		168.2						
1940	— 24 38.4		23.35	16	28	55	— 3	4
		174.9						
1950	21 43.5		20.60	14	25	49	— 3	3
		181.0						
1960	18 42.5		17.74	12	21	42	— 3	3
		186.1						
1970	— 15 36.4		14.81	10	18	35	— 2	2
		190.4						
1980	12 26.0		11.80	8	14	28	— 2	2
		193.7						
1990	9 12 3		8.74	6	10	21	— 1	2
		196.1						
2000	5 56.2		5.64	4	7	13	— 1	+ 1
		197.8						
2010	— 2 38.4		2.51	+ 2	+ 3	+ 6	— 0	+ 0
		198.3						
2020	+ 0 39.9		0.63	— 0	— 1	— 2	+ 0	— 1

TABLE VII.

Equation of Saturn's Orbit for 1750, with the Secular Variations.

ARGUMENT I. *Long.* ♄ *cor. by Tab. VI.—Aphelion, or mean Anomaly.*

Degrees.	Sig. O. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Sig. I. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Degrees.
0	0 0 0.0	6 18.1	0.00	3 2 5.0	5 34.9	48.84	30
1	0 6 18.1	6 18.1	1.68	3 7 39.9	5 32.0	50.36	29
2	0 12 36.2	6 17.9	3.35	3 13 11.9	5 29.1	51.88	28
3	0 18 54.1	6 17.5	5.03	3 18 41.0	5 26.0	53.38	27
4	0 25 11.6	6 17.2	6.71	3 24 7.0	5 22.9	54.87	26
5	0 31 28.8	6 16.8	8.38	3 29 29.9	5 19.7	56.35	25
6	0 37 45.6	6 16.2	10.05	3 34 49.6	5 16.4	57.82	24
7	0 44 1.8	6 15.5	11.72	3 40 6.0	5 13.0	59.28	23
8	0 50 17.3	6 14.8	13.39	3 45 19.0	5 9.5	60.72	22
9	0 56 32.1	6 13.9	15.05	3 50 28.5	5 5.9	62.16	21
10	1 2 46.0	6 13.0	16.71	3 55 34.4	5 2.3	63.57	20
11	1 8 59.0	6 12.0	18.37	4 0 36.7	4 58.6	64.98	19
12	1 15 11.0	6 10.8	20.03	4 5 35.3	4 54.7	66.37	18
13	1 21 21.8	6 9.6	21.68	4 10 30.0	4 50.8	67.75	17
14	1 27 31.4	6 8.4	23.33	4 15 20.8	4 46.9	69.11	16
15	1 33 39.8	6 6.9	24.98	4 20 7.7	4 42.7	70.46	15
16	1 39 46.7	6 5.4	26.61	4 24 50.4	4 38.6	71.79	14
17	1 45 52.1	6 3.9	28.24	4 29 29.0	4 34.3	73.10	13
18	1 51 56.0	6 2.1	29.87	4 34 3.3	4 30.1	74.40	12
19	1 57 58.1	6 0.4	31.49	4 38 33.4	4 25.6	75.69	11
20	2 3 58.5	5 58.6	33.09	4 42 59.0	4 21.1	76.96	10
21	2 9 57.1	5 56.5	34.71	4 47 20.1	4 16.5	78.21	9
22	2 15 53.6	5 54.6	36.32	4 51 36.6	4 11.9	79.44	8
23	2 21 48.2	5 52.4	37.91	4 55 48.5	4 7.2	80.65	7
24	2 27 40.6	5 50.1	39.50	4 59 55.7	4 2.3	81.85	6
25	2 33 30.7	5 47.9	41.07	5 3 58.0	3 57.5	83.02	5
26	2 39 18.6	5 45.4	42.65	5 7 55.5	3 52.5	84.18	4
27	2 45 4.0	5 42.9	44.21	5 11 48.0	3 47.5	85.32	3
28	2 50 46.9	5 40.3	45.76	5 15 35.5	3 42.3	86.44	2
29	2 56 27.2	5 37.8	47.31	5 19 17.8	3 37.2	87.54	1
30	3 2 5.0		48.84	5 22 55.0		88.61	0
	+ Sig. XI.		—	+ Sig. X.		—	

TABLE VII.

Equation of Saturn's Orbit for 1750, with the Secular Variation.

ARGUMENT I. *Long.* ♄ *cor. by Tab. VI.* — *Aphelion or mean Anomaly.*

Degrees.	Sig. II. Equation. —	Diff.	Sec. Var. +	Sig. III. Equation. —	Diff.	Sec. Var. +	Degrees.
	D. M. S.	M. S.	SEC.	D. M. S.	M. S.	SEC.	
0	5 22 55.0		88.61	6 25 44.7		109.44	30
		3 31.8			0 24.7		
1	5 26 26.8		89.66	6 26 9.4		109.69	29
		3 26.6			0 17.9		
2	5 29 53.4		90.70	6 26 27.3		109.91	28
		3 21.2			0 10.7		
3	5 33 14.6		91.72	6 26 38.0		110.10	27
		3 15.6			0 3.7		
4	5 36 30.2		92.71	6 26 41.7		110.24	26
		3 10.1			0 3.3		
5	5 39 40.3		93.67	6 26 38.4		110.37	25
		3 4.6			0 10.4		
6	5 42 44.9		94.62	6 26 28.0		110.45	24
		2 58.7			0 17.6		
7	5 45 43.6		95.54	6 26 10.4		110.51	23
		2 53.0			0 24.7		
8	5 48 36.6		96.44	6 25 45.7		110.53	22
		2 47.2			0 31.9		
9	5 51 23.8		97.33	6 25 13.8		110.51	21
		2 41.3			0 39.1		
10	5 54 5.1		98.16	6 24 34.7		110.46	20
		2 35.4			0 46.2		
11	5 56 40.5		98.99	6 23 48.5		110.37	19
		2 29.3			0 53.5		
12	5 59 9.8		99.79	6 22 55.0		110.25	18
		2 23.3			1 0.6		
13	6 1 33.1		100.56	6 21 54.4		110.10	17
		2 17.1			1 7.9		
14	6 3 50.2		101.31	6 20 46.5		109.91	16
		2 10.9			1 15.2		
15	6 6 1.1		102.03	6 19 31.3		109.68	15
		2 4.7			1 22.4		
16	6 8 5.8		102.73	6 18 8.9		109.42	14
		1 58.3			1 29.5		
17	6 10 4.1		103.39	6 16 39.4		109.12	13
		1 52.0			1 36.8		
18	6 11 56.1		104.03	6 15 2.6		108.79	12
		1 45.6			1 44.1		
19	6 13 41.7		104.64	6 13 18.5		108.41	11
		1 39.0			1 51.2		
20	6 15 20.7		105.23	6 11 27.3		108.01	10
		1 32.6			1 58.5		
21	6 16 53.3		105.78	6 9 28.8		107.59	9
		1 25.9			2 5.7		
22	6 18 19.2		106.31	6 7 23.1		107.09	8
		1 19.4			2 12.8		
23	6 19 38.6		106.81	6 5 10.3		106.57	7
		1 12.7			2 20.0		
24	6 20 51.3		107.27	6 2 50.3		106.01	6
		1 5.9			2 27.1		
25	6 21 57.2		107.71	6 0 23.2		105.42	5
		0 59.2			2 34.3		
26	6 22 56.4		108.12	5 57 48.9		104.79	4
		0 52.4			2 41.4		
27	6 23 48.8		108.50	5 55 7.5		104.13	3
		0 45.5			2 48.4		
28	6 24 34.3		108.84	5 52 19.1		103.43	2
		0 38.7			2 55.4		
29	6 25 13.0		109.16	5 49 23.7		102.69	1
		0 31.7			3 2.6		
30	6 25 44.7		109.44	5 46 21.1		101.91	0
	+ Sig. IX.		—	+ Sig. VIII.		—	

TABLE VII.

Equation of Saturn's Orbit for 1750, with the Secular Variation.

ARGUMENT I. *Long.* ♄ *cor. by Tab. VI.* — *Aphelion, or mean Anomaly.*

Degrees.	Sig. IV. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Sig. V. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Degrees.
0	5 46 21.1	3 9.4	101.91	3 25 38.8	6 6.7	62.28	30
1	5 43 11.7	3 16.4	101.10	3 19 32.1	6 11.3	60.47	29
2	5 39 55.3	3 23.2	100.25	3 13 20.8	6 15.4	58.64	28
3	5 36 32.1	3 30.1	99.36	3 7 5.4	6 19.6	56.78	27
4	5 33 2.0	3 36.9	98.44	3 0 45.8	6 23.5	54.89	26
5	5 29 25.1	3 43.6	97.48	2 54 22.3	6 27.4	52.98	25
6	5 25 41.5	3 50.3	96.48	2 47 54.9	6 31.1	51.05	24
7	5 21 51.2	3 56.9	95.45	2 41 23.8	6 34.7	49.10	23
8	5 17 54.3	4 3.5	94.38	2 34 49.1	6 38.2	47.12	22
9	5 13 50.8	4 10.2	93.27	2 28 10.9	6 41.5	45.12	21
10	5 9 40.6	4 16.2	92.13	2 21 29.4	6 44.6	43.09	20
11	5 5 24.4	4 22.8	90.96	2 14 44.8	6 47.6	41.07	19
12	5 1 1.6	4 29.1	89.74	2 7 57.2	6 50.5	39.02	18
13	4 56 32.5	4 35.3	88.49	2 1 6.7	6 53.2	36.95	17
14	4 51 57.2	4 41.4	87.21	1 54 13.5	6 55.8	34.86	16
15	4 47 15.8	4 47.5	85.90	1 47 17.7	6 58.2	32.76	15
16	4 42 28.3	4 53.5	84.55	1 40 19.5	7 0.5	30.64	14
17	4 37 34.8	4 59.4	83.16	1 33 19.0	7 2.5	28.51	13
18	4 32 35.4	5 5.2	81.74	1 26 16.5	7 4.5	26.37	12
19	4 27 30.2	5 10.9	80.30	1 19 12.0	7 6.3	24.22	11
20	4 22 19.3	5 16.5	78.81	1 12 5.7	7 7.9	22.05	10
21	4 17 2.8	5 22.0	77.29	1 4 57.8	7 9.4	19.87	9
22	4 11 40.8	5 27.5	75.74	0 57 48.4	7 10.7	17.69	8
23	4 6 13.3	5 32.7	74.16	0 50 37.7	7 11.8	15.49	7
24	4 0 40.6	5 37.9	72.56	0 43 25.9	7 12.8	13.29	6
25	3 55 2.7	5 43.1	70.92	0 36 13.1	7 13.7	11.09	5
26	3 49 19.6	5 48.0	69.25	0 28 59.4	7 14.2	8.87	4
27	3 43 31.6	5 52.9	67.55	0 21 45.2	7 14.8	6.66	3
28	3 37 38.7	5 57.6	65.82	0 14 30.4	7 15.1	4.43	2
29	3 31 41.1	6 2.3	64.06	0 7 15.3	7 15.3	2.22	1
30	3 25 38.8		62.28	0 0 00.0		0.00	0
	+ Sig. VII.		—	+ Sig. VI.		—	

Equation of Saturn, always additive.

TABLE VIII. Equation II.

Argument II.

Arg. II.	Equation II. M. S.	Diff. SEC.	Arg. II.	Equation II. M. S.	Diff. SEC.	Arg. II.	Equation II. M. S.	Diff. SEC.
0	1 3.8	4.7	3400	1 8.6	1.0	6700	0 3.0	1.9
100	0 59.1	4.8	3500	1 7.6	1.3	6800	0 4.9	2.5
200	0 54.3	4.6	3600	1 6.3	1.7	6900	0 7.4	2.9
300	0 49.7	4.4	3700	1 4.6	2.1	7000	0 10.3	3.4
400	0 45.3	4.1	3800	1 2.5	2.3	7100	0 13.7	3.9
500	0 41.2	3.6	3900	1 0.2	2.5	7200	0 17.6	4.3
600	0 37.6	3.2	4000	0 57.7	2.8	7300	0 21.9	4.6
700	0 34.4	2.7	4100	0 54.9	2.9	7400	0 26.5	5.0
800	0 31.7	2.1	4200	0 52.0	3.0	7500	0 31.5	5.3
900	0 29.6	1.5	4300	0 49.0	3.2	7600	0 36.8	5.4
1000	0 28.1	0.8	4400	0 45.8	3.2	7700	0 42.2	5.6
1100	0 27.3	0.3	4500	0 42.6	3.3	7800	0 47.8	5.6
1200	0 27.0	0.4	4600	0 39.3	3.3	7900	0 53.4	5.5
1300	0 27.4	0.9	4700	0 36.0	3.2	8000	0 58.9	5.4
1400	0 28.3	1.5	4800	0 32.8	3.3	8100	1 4.3	5.1
1500	0 29.8	1.9	4900	0 29.5	3.1	8290	1 9.4	4.8
1600	0 31.7	2.3	5000	0 26.4	3.2	8300	1 14.2	4.4
1700	0 34.0	2.7	5100	0 23.2	3.0	8400	1 18.6	3.9
1800	0 36.7	3.0	5200	0 20.2	2.9	8500	1 22.5	3.3
1900	0 39.7	3.2	5300	0 17.3	2.8	8600	1 25.8	2.7
2000	0 42.9	3.2	5400	0 14.5	2.6	8700	1 28.5	2.0
2100	0 46.1	3.3	5500	0 11.9	2.4	8800	1 30.5	1.3
2200	0 49.4	3.2	5600	0 9.5	2.2	8900	1 31.8	0.6
2300	0 52.6	3.1	5700	0 7.3	2.0	9000	1 32.4	0.2
2400	0 55.7	2.9	5800	0 5.3	1.7	9100	1 32.2	0.9
2500	0 58.6	2.7	5900	0 3.6	1.4	9200	1 31.3	1.5
2600	1 1.3	2.3	6000	0 2.2	1.1	9300	1 29.8	2.3
2700	1 3.6	2.0	6100	0 1.1	0.7	9400	1 27.5	2.9
2800	1 5.6	1.6	6200	0 0.4	0.4	9500	1 24.6	3.4
2900	1 7.2	1.1	6300	0 0.0	0.0	9600	1 21.2	3.9
3000	1 8.3	0.7	6400	0 0.0	0.5	9700	1 17.3	4.2
3100	1 9.0	0.3	6500	0 0.5	1.0	9800	1 13.1	4.5
3200	1 9.3	0.2	6600	0 1.5	1.5	9900	1 8.6	4.8
3300	1 9.1	0.5	6700	0 3.0		10000	1 3.8	
3400	1 8.6							

Equation of Saturn, always additive.

TABLE IX. Equation III.

Argument III.

Arg. III.	Equation III. M. S.	Diff. SEC.	Arg. III.	Equation III. M. S.	Diff. SEC.	Arg. III.	Equation III. M. S.	Diff. SEC.
0	8 47.8	25.6	3400	0 19.1	8.7	6700	12 1.8	17.6
100	8 22.2	26.0	3500	0 27.8	10.2	6800	12 19.4	16.4
200	7 56.2	26.2	3600	0 38.0	11.7	6900	12 35.8	15.0
300	7 30.0	26.3	3700	0 49.7	13.2	7000	12 50.8	13.7
400	7 3.7	26.3	3800	1 2.9	14.5	7100	13 4.5	12.2
500	6 37.4	26.2	3900	1 17.4	15.9	7200	13 16.7	10.7
600	6 11.2	26.1	4000	1 33.3	17.2	7300	13 27.4	9.2
700	5 45.1	25.8	4100	1 50.5	18.4	7400	13 36.6	7.6
800	5 19.3	25.4	4200	2 8.9	19.6	7500	13 44.2	6.0
900	4 53.9	24.9	4300	2 28.5	20.6	7600	13 50.2	4.4
1000	4 29.0	24.2	4400	2 49.1	21.6	7700	13 54.6	2.8
1100	4 4.8	23.7	4500	3 10.7	22.5	7800	13 57.4	1.0
1200	3 41.1	22.8	4600	3 33.2	23.4	7900	13 58.4	0.5
1300	3 18.3	21.9	4700	3 56.6	24.1	8000	13 57.9	2.2
1400	2 56.4	21.0	4800	4 20.7	24.7	8100	13 55.7	3.8
1500	2 35.4	19.9	4900	4 45.4	25.2	8200	13 51.9	5.6
1600	2 15.5	18.9	5000	5 10.6	25.6	8300	13 46.3	7.0
1700	1 56.6	17.6	5100	5 36.2	26.0	8400	13 39.3	8.7
1800	1 39.0	16.4	5200	6 2.2	26.2	8500	13 30.6	10.2
1900	1 22.6	15.0	5300	6 28.4	26.3	8600	13 20.4	11.7
2000	1 7.6	13.7	5400	6 54.7	26.3	8700	13 8.7	13.2
2100	0 53.9	12.2	5500	7 21.0	26.3	8800	12 55.5	14.5
2200	0 41.7	10.7	5600	7 47.3	26.0	8900	12 41.0	15.9
2300	0 31.0	9.2	5700	8 13.3	25.8	9000	12 25.1	17.2
2400	0 21.8	7.6	5800	8 39.1	25.4	9100	12 7.9	18.4
2500	0 14.2	6.0	5900	9 4.5	24.9	9200	11 49.5	19.6
2600	0 8.2	4.4	6000	9 29.4	24.2	9300	11 29.9	20.6
2700	0 3.8	2.8	6100	9 53.6	23.7	9400	11 9.3	21.6
2800	0 1.0	1.0	6200	10 17.3	22.8	9500	10 47.7	22.5
2900	0 0.0	0.6	6300	10 40.1	21.9	9600	10 25,2	23.4
3000	0 0.6	2.2	6400	11 2.0	21.0	9700	10 1.8	24.1
3100	0 2.8	3.8	6500	11 23.0	19.9	9800	9 37.7	24.6
3200	0 6.6	5.5	6600	11 42.9	18.9	9900	9 13.1	25.3
3300	0 12.1	7.0	6700	12 1.8		10000	8 47.8	
3400	0 19.1							

Equation of Saturn, always additive.

TABLE X. Equation IV.

Argument IV.

Arg. IV.	Equation IV. M. S.	Diff. SEC.	Arg. IV.	Equation IV. M. S.	Diff. SEC.	Arg. IV.	Equation IV. M. S.	Diff. SEC.
0	1 51.7	21.9	3400	10 30.9	40.9	6700	20 30.2	19.8
100	1 29.8	19.6	3500	11 11.8	40.8	6800	20 10.4	22.2
200	1 10.2	17.4	3600	11 52.6	40.6	6900	19 48.2	23.8
300	0 52.8	14.6	3700	12 33.2	40.2	7000	19 24.4	26.1
400	0 38.2	13.0	3800	13 13.4	39.6	7100	18 58.3	27.9
500	0 25.2	10.1	3900	13 53.0	38.9	7200	18 30.4	29.9
600	0 15.1	7.6	4000	14 31.9	37.9	7300	18 0.5	31.5
700	0 7.5	5.0	4100	15 9.8	36.9	7400	17 29.0	33.2
800	0 2.5	2.5	4200	15 46.7	35.9	7500	16 55.8	34.5
900	0 0.0	0.0	4300	16 22.6	34.6	7600	16 21.3	35.8
1000	0 0.0	2.7	4400	16 57.2	33.0	7700	15 45.5	37.0
1100	0 2.7	5.1	4500	17 30.2	31.5	7800	15 8.5	38.2
1200	0 7.8	7.7	4600	18 1.7	29.8	7900	14 30.3	38.9
1300	0 15.5	10.2	4700	18 31.5	27.8	8000	13 51.4	39.7
1400	0 25.7	12.8	4800	18 59.3	26.2	8100	13 11.7	40.1
1500	0 38.5	15.1	4900	19 25.5	24.0	8200	12 31.6	40.6
1600	0 53.6	17.4	5000	19 49.5	21.9	8300	11 51.0	40.7
1700	1 11.0	19.8	5100	20 11.4	19.6	8400	11 10.3	40.9
1800	1 30.8	22.2	5200	20 31.0	17.4	8500	10 29.4	40.8
1900	1 53.0	23.8	5300	20 48.4	14.6	8600	9 48.6	40.6
2000	2 16.8	26.1	5400	21 3.0	13.0	8700	9 8.0	40.2
2100	2 42.9	27.9	5500	21 16.0	10.1	8800	8 27.8	39.6
2200	3 10.8	29.9	5600	21 26.1	7.6	8900	7 48.2	38.9
2300	3 40.7	31.5	5700	21 33.7	5.0	9000	7 9.3	37.9
2400	4 12.2	33.2	5800	21 38.7	2.5	9100	6 31.4	36.9
2500	4 45.4	34.5	5900	21 41.2	0.0	9200	5 54.5	35.9
2600	5 19.9	35.8	6000	21 41.2	2.7	9300	5 18.6	34.6
2700	5 55.7	37.0	6100	21 38.5	5.1	9400	4 44.0	33.0
2800	6 32.7	38.2	6200	21 33.4	7.7	9500	4 11.0	31.5
2900	7 10.9	38.9	6300	21 25.7	10.2	9600	3 39.5	29.8
3000	7 49.8	39.7	6400	21 15.5	12.8	9700	3 9.7	27.8
3100	8 29.5	40.1	6500	21 2.7	15.1	9800	2 41.9	26.2
3200	9 9.6	40.6	6600	20 47.6	17.4	9900	2 15.7	24.0
3300	9 50.2	40.7	6700	20 30.2		10000	1 51.7	
3400	10 30.9							

Equation of Saturn, always additive.

TABLE XI. Equation V.

Argument V.

Arg. V.	Equation V. M.	S.	Diff. SEC.	Arg. V.	Equation V. M.	S.	Diff. SEC.	Arg. V.	Equation V. M.	S.	Diff. SEC.
0	0	0.0	0.0	340	1	14.9	2.7	670	1	14.8	2.7
10	0	0.0	0.2	350	1	17.6	2.5	680	1	12.1	2.9
20	0	0.2	0.5	360	1	20.1	2.4	690	1	9.2	2.9
30	0	0.7	0.6	370	1	22.5	2.3	700	1	6.3	3.0
40	0	1.3	0.7	380	1	24.8	2.1	710	1	3.3	3.0
50	0	2.0	1.0	390	1	26.9	2.0	720	1	0.3	3.1
60	0	3.0	1.2	400	1	28.9	1.8	730	0	57.2	3.1
70	0	4.2	1.3	410	1	30.7	1.7	740	0	54.1	3.1
80	0	5.5	1.5	420	1	32.4	1.5	750	0	51.0	3.1
90	0	7.0	1.7	430	1	33.9	1.3	760	0	47.9	3.1
100	0	8.7	1.8	440	1	35.2	1.1	770	0	44.8	3.1
110	0	10.5	1.9	450	1	36.3	1.0	780	0	41.7	3.1
120	0	12.4	2.2	460	1	37.3	0.8	790	0	38.6	3.0
130	0	14.6	2.2	470	1	38.1	0.5	800	0	35.6	2.9
140	0	16.8	2.4	480	1	38.6	0.3	810	0	32.7	2.9
150	0	19.2	2.5	490	1	38.9	0.3	820	0	29.8	2.8
160	0	21.7	2.7	500	1	39.2	0.0	830	0	27.0	2.7
170	0	24.4	2.7	510	1	39.2	0.2	840	0	24.3	2.7
180	0	27.1	2.9	520	1	39.0	0.5	850	0	21.6	2.5
190	0	30.0	2.9	530	1	38.5	0.6	860	0	19.1	2.4
200	0	32.9	3.0	540	1	37.9	0.7	870	0	16.7	2.3
210	0	35.9	3.0	550	1	37.2	1.0	880	0	14.4	2.1
220	0	38.9	3.1	560	1	36.2	1.2	890	0	12.3	2.0
230	0	42.0	3.1	570	1	35.0	1.3	900	0	10.3	1.8
240	0	45.1	3.1	580	1	33.7	1.5	910	0	8.5	1.7
250	0	48.2	3.1	590	1	32.2	1.7	920	0	6.8	1.5
260	0	51.3	3.1	600	1	30.5	1.8	930	0	5.3	1.3
270	0	54.4	3.1	610	1	28.7	1.9	940	0	4.0	1.1
280	0	57.5	3.1	620	1	26.8	2.2	950	0	2.9	1.0
290	1	0.6	3.0	630	1	24.6	2.2	960	0	1.9	0.8
300	1	3.6	2.9	640	1	22.4	2.4	970	0	1.1	0.5
310	1	6.5	2.9	650	1	20.0	2.5	980	0	0.6	0.3
320	1	9.4	2.8	660	1	17.5	2.7	990	0	0.3	0.3
330	1	12.2	2.7	670	1	14.8		1000	0	0.0	
340	1	14.9									

Equation of Saturn, always additive.

TABLE XII. Equation VI.

Argument VI.

Arg. VI.	Equat. VI. sec.	Arg. VI.	Equat. VI. sec.	Arg. VI.	Equat. VI. sec.
0	13.7	340	9.0	670	43.5
10	13.5	350	10.2	680	43.6
20	11.2	360	11.4	690	43.6
30	10.1	370	12.6	700	43.6
40	9.0	380	13.8	710	43.4
50	7.9	390	15.1	720	43.2
60	6.8	400	16.4	730	42.9
70	5.8	410	17.7	740	42.5
80	4.9	420	19.1	750	42.0
90	4.1	430	20.5	760	41.5
100	3.3	440	21.8	770	40.9
110	2.6	450	23.2	780	40.1
120	2.0	460	24.6	790	39.3
130	1.4	470	25.9	800	38.5
140	1.0	480	27.2	810	37.6
150	0.6	490	28.6	820	36.7
160	0.3	500	29.9	830	35.7
170	0.1	510	31.1	840	34.6
180	0.0	520	32.4	850	33.4
190	0.0	530	33.5	860	32.2
200	0.0	540	34.6	870	31.0
210	0.2	550	35.7	880	29.8
220	0.4	560	36.8	890	28.5
230	0.7	570	37.8	900	27.2
240	1.1	580	38.7	910	25.9
250	1.6	590	39.5	920	24.5
260	2.1	600	40.3	930	23.1
270	2.7	610	41.0	940	21.8
280	3.5	620	41.6	950	20.4
290	4.3	630	42.2	960	19.0
300	5.1	640	42.6	970	17.7
310	6.0	650	43.0	980	16.4
320	6.9	660	43.3	990	15.0
330	7.9	670	43.5	1000	13.7
340	9.0				

TABLE XIII. Equation VII.

Argument. VII.

Arg. VII.	Equat. VII. sec.	Arg. VII.	Equat. VII. sec.	Arg. VII.	Equat. VII. sec.
0	22.0	340	5.1	670	5.7
10	22.0	350	4.6	680	6.4
20	21.9	360	4.0	690	7.0
30	21.8	370	3.5	700	7.6
40	21.7	380	3.0	710	8.3
50	21.5	390	2.5	720	9.0
60	21.2	400	2.1	730	9.7
70	21.0	410	1.7	740	10.3
80	20.7	420	1.3	750	11.0
90	20.3	430	1.0	760	11.7
100	19.9	440	0.8	770	12.3
110	19.5	450	0.5	780	13.0
120	19.0	460	0.3	790	13.7
130	18.5	470	0.2	800	14.4
140	18.0	480	0.1	810	15.0
150	17.4	490	0.0	820	15.6
160	16.9	500	0.0	830	16.3
170	16.3	510	0.0	840	16.9
180	15.6	520	0.1	850	17.4
190	15.0	530	0.2	860	18.0
200	14.4	540	0.3	870	18.5
210	13.7	550	0.5	880	19.0
220	13.0	560	0.8	890	19.5
230	12.3	570	1.0	900	19.9
240	11.7	580	1.3	910	20.3
250	11.0	590	1.7	920	20.7
260	10.3	600	2.1	930	21.0
270	9.7	610	2.5	940	21.2
280	9.0	620	3.0	950	21.5
290	8.3	630	3.5	960	21.7
300	7.6	640	4.0	970	21.8
310	7.0	650	4.6	980	21.9
320	6.4	660	5.1	990	22.0
330	5.7	670	5.7	1000	22.0
340	5.1				

TABLE XIV.

Radius Vector of Saturn's Orbit for 1750, with the Secular Variation.

ARGUMENT. I. *The mean Anomaly.*

Degrees.	Sig. O. Rad. Vect.	Diff.	Secular Variation. — 0.00	Sig. I. Rad. Vect.	Diff.	Secular Variation. — 0.00	Sig. II. Rad. Vect.	Diff.	Secular Variation — 0.00	Degrees.
0	10.05389	7	253.5	9.98904	431	226.8	9.80735	769	148.2	30
1	10.05382	22	253.5	9.98473	445	224.9	9.79966	777	144.8	29
2	10.05360	37	253.4	9.98028	458	223.0	9.79189	786	141.3	28
3	10.05323	51	253.3	9.97570	471	221.1	9.78403	795	137.8	27
4	10.05272	66	253.0	9.97099	484	219.0	9.77608	803	134.2	26
5	10.05206	80	252.7	9.96615	496	217.1	9.76805	810	130.6	25
6	10.05126	95	252.4	9.96119	509	215.0	9.75995	818	127.0	24
7	10.05031	110	252.0	9.95610	521	212.9	9.75177	826	123.3	23
8	10.04921	124	251.5	9.95089	534	210.6	9.74351	833	119.5	22
9	10.04797	139	251.0	9.94555	547	208.3	9.73518	840	115.8	21
10	10.04658	153	250.5	9.94008	558	206.0	9.72678	847	112.0	20
11	10.04505	168	249.8	9.93450	570	203.6	9.71831	853	108.1	19
12	10.04337	182	249.1	9.92880	582	201.2	9.70978	860	104.2	18
13	10.04155	196	248.4	9.92298	594	198.7	9.70118	866	100.3	17
14	10.03959	211	247.5	9.91704	605	196.1	9.69252	872	096.3	16
15	10.03748	225	246.8	9.91099	617	193.5	9.68380	878	092.2	15
16	10.03523	240	245.9	9.90482	628	190.9	9.67502	883	088.2	14
17	10.03283	253	244.8	9.89854	639	188.2	9.66619	888	084.2	13
18	10.03030	268	243.8	9.89215	650	185.4	9.65731	892	080.1	12
19	10.02762	281	242.7	9.88565	661	182.6	9.64839	899	076.0	11
20	10.02481	296	241.5	9.87904	672	179.8	9.63940	904	071.8	10
21	10.02185	310	240.4	9.87232	683	176.8	9.63036	906	067.6	9
22	10.01875	323	239.0	9.86549	692	173.9	9.62130	910	063.4	8
23	10.01552	337	237.6	9.85857	702	170.8	9.61220	914	059.1	7
24	10.01215	352	236.3	9.85155	713	167.7	9.60306	918	054.8	6
25	10.00863	365	234.9	9.84442	722	164.6	9.59388	920	050.5	5
26	10.00498	378	233.4	9.83720	732	161.5	9.58468	924	046.2	4
27	10.00120	393	231.8	9.82988	742	158.2	9.57544	926	041.8	3
28	9.99727	405	230.1	9.82246	751	154.9	9.56618	929	037.4	2
29	9.99322	418	228.4	9.81495	760	151.6	9.55689	931	033.0	1
30	9.98904		226.8	9.80735		148.2	9.54758		028.6	0
	Sig. XI.		—	Sig. X.		—	Sig. IX.		—	

TABLE XIV.

Radius Vector of Saturn's Orbit for 1750, with the Secular Variation.

ARGUMENT. I. *The mean Anomaly.*

Degrees.	Sig. III. Rad. Vect.	Diff.	Secular Variation. — 0.00	Sig. IV. Rad. Vect.	Diff.	Secular Variation. + 0.00	Sig. V. Rad. Vect.	Diff.	Secular Variation + 0.00	Degrees.
0	9.54758	936	028.6	9.27222	851	105.2	9.06107	505	212.4	30
1	9.53822	938	024.2	9.26371	844	109.6	9.05602	490	215.0	29
2	9.52884	938	019.8	9.25527	836	113.7	9.05112	476	217.5	28
3	9.51946	940	015.2	9.24691	828	118.0	9.04636	460	220.0	27
4	9.51006	940	010.8	9.23863	820	122.1	9.04176	443	222.2	26
5	9.50066	941	006.3	9.23043	811	126.2	9.03733	428	224.7	25
6	9.49125	941	001.8	9.22232	801	130.3	9.03305	411	226.9	24
7	9.48184	940	002.7	9.21431	793	134.4	9.02894	395	229.0	23
8	9.47244	940	007.2	9.20638	783	138.3	9.02499	378	231.0	22
9	9.46304	939	011.8	9.19855	773	142.2	9.02121	363	233.0	21
10	9.45365	939	016.3	9.19082	763	146.2	9.01758	346	234.9	20
11	9.44426	936	020.8	9.18319	752	150.0	9.01412	328	236.7	19
12	9.43490	935	025.3	9.17567	741	153.8	9.01084	311	238.4	18
13	9.42555	933	029.8	9.16826	731	157.6	9.00773	295	240.0	17
14	9.41622	931	034.3	9.16095	720	161.3	9.00478	277	241.5	16
15	9.40691	928	038.9	9.15375	708	164.9	9.00201	260	243.0	15
16	9.39763	925	043.5	9.14667	696	168.5	8.99941	243	244.3	14
17	9.38838	921	047.9	9.13971	684	172.1	8.99698	224	245.6	13
18	9.37917	918	052.4	9.13287	672	175.6	8.99474	207	246.7	12
19	9.36999	915	056.9	9.12615	659	179.0	8.99267	190	247.8	11
20	9.36084	910	061.4	9.11956	646	182.4	8.99077	171	248.7	10
21	9.35174	905	066.0	9.11310	633	185.7	8.98906	154	249.7	9
22	9.34269	901	070.3	9.10677	620	189.0	8.98752	136	250.5	8
23	9.33368	896	074.7	9.10057	607	192.1	8.98616	117	251.2	7
24	9.32472	890	079.2	9.09450	593	195.2	8.98499	100	251.8	6
25	9.31582	885	083.6	9.08857	579	198.3	8.98399	82	252.3	5
26	9.30697	879	088.0	9.08278	565	201.2	8.98317	64	252.8	4
27	9.29818	872	092.4	9.07713	550	204.1	8.98253	45	253.1	3
28	9.28946	865	096.7	9.07163	535	207.0	8 98208	27	253.4	2
29	9.28081	859	101.0	9.06628	521	209.7	8.98181	9	253.5	1
30	9.27222		105.2	9.06107		212.4	8.98172		253.5	0
	Sig. VIII.		0.00 +	Sig. VII.		0.00 +	Sig. VI.		0.00 +	

Equation of the Radius Vector, always additive.

TABLE XV.

Arg. II.	Equation. II.		Arg. II.	Arg. II.	Equation II.		Arg. II.
	0.0	Diff.			0.0	Diff.	
0	1629	3	10000	2500	0538	50	7500
100	1626	8	9900	2600	0488	48	7400
200	1618	13	9800	2700	0440	45	7300
300	1605	19	9700	2800	0395	43	7200
400	1586	24	9600	2900	0352	40	7100
500	1562	28	9500	3000	0312	37	7000
600	1534	33	9400	3100	0275	34	6900
700	1501	37	9300	3200	0241	31	6800
800	1464	41	9200	3300	0210	29	6700
900	1423	45	9100	3400	0181	26	6600
1000	1378	49	9000	3500	0155	24	6500
1100	1329	51	8900	3600	0131	21	6400
1200	1278	53	8800	3700	0110	19	6300
1300	1225	56	8700	3800	0091	17	6200
1400	1169	57	8600	3900	0074	14	6100
1500	1112	59	8500	4000	0060	13	6000
1600	1053	59	8400	4100	0047	11	5900
1700	0994	59	8300	4200	0036	9	5800
1800	0935	60	8200	4300	0027	7	5700
1900	0875	59	8100	4400	0020	7	5600
2000	0816	59	8000	4500	0013	5	5500
2100	0757	57	7900	4600	0008	3	5400
2200	0700	56	7800	4700	0005	3	5300
2300	0644	54	7700	4800	0002	2	5200
2400	0590	52	7600	4900	0000	0	5100
2500	0538		7500	5000	0000		5000

Equation of the Radius Vector, always additive.

TABLE XV.

Arg. III.	Equation III.		Arg. III.	Equation. III.		Arg. III.	Equation. III.		Arg. III.	Equation III.	
	0.0	Diff.		0.0	Diff.		0.0	Diff.		0.0	Diff.
0	1053	8	2500	0675	33	5000	0018	8	7500	0397	33
100	1061	6	2600	0642	33	5100	0010	5	7600	0430	33
200	1067	3	2700	0609	34	5200	0005	4	7700	0463	33
300	1070	2	2800	0575	33	5300	0001	1	7800	0496	34
400	1072	1	2900	0542	34	5400	0000	1	7900	0530	34
500	1071	3	3000	0508	34	5500	0001	3	8000	0564	33
600	1068	5	3100	0474	33	5600	0004	4	8100	0597	34
700	1063	7	3200	0441	33	5700	0008	7	8200	0631	33
800	1056	9	3300	0408	32	5800	0015	9	8300	0664	32
900	1047	11	3400	0376	32	5900	0024	11	8400	0696	32
1000	1036	13	3500	0344	31	6000	0035	14	8500	0728	31
1100	1023	15	3600	0313	30	6100	0049	15	8600	0759	30
1200	1008	17	3700	0283	29	6200	0064	16	8700	0789	29
1300	0991	18	3800	0254	28	6300	0080	19	8800	0818	28
1400	0973	21	3900	0226	27	6400	0099	20	8900	0846	27
1500	0952	21	4000	0199	26	6500	0119	22	9000	0873	26
1600	0931	24	4100	0173	24	6600	0141	24	9100	0899	24
1700	0907	25	4200	0149	22	6700	0165	25	9200	0923	22
1800	0882	26	4300	0127	21	6800	0190	26	9300	0945	21
1900	0856	28	4400	0106	19	6900	0216	28	9400	0966	19
2000	0828	29	4500	0087	18	7000	0244	29	9500	0985	18
2100	0799	30	4600	0069	16	7100	0273	30	9600	1003	15
2200	0769	31	4700	0053	13	7200	0303	31	9700	1018	14
2300	0738	31	4800	0040	12	7300	0334	31	9800	1032	12
2400	0707	32	4900	0028	10	7400	0365	32	9900	1044	10
2500	0675		5000	0018		7500	0397		10000	1054	

Equation of the Radius Vector, always additive.

TABLE XV.

Arg. IV.	Equation IV.		Arg. IV.	Equation IV.		Arg. IV.	Equation IV.		Arg. IV.	Equation IV.	
	0.0	Diff.		0.0	Diff.		0.0	Diff.		0.0	Diff.
0	2346	80	2500	0258	50	5000	0659	80	7500	2746	51
100	2266	83	2600	0208	46	5100	0739	83	7600	2797	45
200	2183	85	2700	0162	40	5200	0822	85	7700	2842	40
300	2098	88	2800	0122	34	5300	0907	88	7800	2882	35
400	2010	90	2900	0088	29	5400	0995	90	7900	2917	29
500	1920	92	3000	0059	24	5500	1085	91	8000	2946	23
600	1828	92	3100	0035	17	5600	1176	93	8100	2969	18
700	1736	94	3200	0018	12	5700	1269	94	8200	2987	12
800	1642	94	3300	0006	6	5800	1363	94	8300	2999	6
900	1548	95	3400	0000	0	5900	1457	94	8400	3005	1
1000	1453	94	3500	0000	6	6000	1551	95	8500	3004	6
1100	1359	94	3600	0006	12	6100	1646	93	8600	2998	11
1200	1265	92	3700	0018	18	6200	1739	93	8700	2987	18
1300	1173	92	3800	0036	24	6300	1832	91	8800	2969	24
1400	1081	90	3900	0080	29	6400	1923	90	8900	2945	29
1500	0991	87	4000	0089	35	6500	2013	88	9000	2916	35
1600	0904	86	4100	0124	40	6600	2101	85	9100	2881	40
1700	0818	82	4200	0164	46	6700	2186	83	9200	2841	46
1800	0736	80	4300	0210	50	6800	2269	80	9300	2795	51
1900	0656	76	4400	0260	56	6900	2349	76	9400	2744	55
2000	0580	73	4500	0316	60	7000	2425	73	9500	2689	60
2100	0507	69	4600	0376	65	7100	2498	68	9600	2629	65
2200	0438	64	4700	0441	69	7200	2566	65	9700	2564	69
2300	0374	60	4800	0510	73	7300	2631	60	9800	2495	73
2400	0314	56	4900	0583	76	7400	2691	55	9900	2422	76
2500	0258		5000	0659		7500	2746		10000	2346	

TABLE XVI.

Heliocentric Latitude for 1750, with the Secular Variation.

ARGUMENT VIII. *Longitude of Saturn — that of the Node.*

Degrees.	Latitude. Sig. O. *N.* / Sig. VI. *S.* D. M. S.	Differ. M. S.	Sec. Var. — / — SEC.	Latitude. Sig. I. *N.* / Sig. VII. *S.* D. M. S.	Differ. M. S.	Sec. Var. — / — SEC.	Latitude. Sig. II. *N.* / Sig. VIII. *S.* D. M. S.	Differ. M. S.	Sec. Var. — / — SEC.	Degrees.
0	0 0 0	2 37	0.0	1 14 56	2 16	8.0	2 9 49	1 17	13.9	30
1	0 2 37	2 37	0.3	1 17 12	2 14	8.3	2 11 6	1 15	14.1	29
2	0 5 14	2 37	0.6	1 19 26	2 12	8.5	2 12 21	1 13	14.2	28
3	0 7 51	2 36	0.9	1 21 38	2 11	8.7	2 13 34	1 10	14.3	27
4	0 10 27	2 37	1.2	1 23 49	2 9	8.9	2 14 44	1 8	14.4	26
5	0 13 4	2 36	1.5	1 25 58	2 8	9.2	2 15 52	1 5	14.5	25
6	0 15 40	2 36	1.7	1 28 6	2 6	9.4	2 16 57	1 3	14.6	24
7	0 18 16	2 36	2.0	1 30 12	2 5	9.7	2 18 0	1 0	14.7	23
8	0 20 52	2 35	2.3	1 32 17	2 3	9.9	2 19 0	0 57	14.8	22
9	0 23 27	2 35	2.5	1 34 20	2 1	10.1	2 19 57	0 55	14.9	21
10	0 26 2	2 34	2.8	1 36 21	1 59	10.3	2 20 52	0 53	15.0	20
11	0 28 36	2 34	3.1	1 38 20	1 58	10.5	2 21 45	0 50	15.1	19
12	0 31 10	2 33	3.3	1 40 18	1 56	10.7	2 22 35	0 47	15.2	18
13	0 33 43	2 32	3.6	1 42 14	1 54	10.9	2 23 22	0 44	15.2	17
14	0 36 15	2 32	3.9	1 44 8	1 51	11.9	2 24 6	0 42	15.3	16
15	0 38 47	2 32	4.1	1 45 59	1 50	11.3	2 24 48	0 40	15.4	15
16	0 41 19	2 30	4.4	1 47 49	1 49	11.5	2 25 28	0 36	15.5	14
17	0 43 49	2 30	4.7	1 49 38	1 46	11.7	2 26 4	0 34	15.5	13
18	0 46 19	2 29	4.9	1 51 24	1 44	11.9	2 26 38	0 32	15.6	12
19	0 48 48	2 28	5.2	1 53 8	1 42	12.1	2 27 10	0 28	15.6	11
20	0 51 16	2 27	5.5	1 54 50	1 40	12.3	2 27 38	0 26	15.7	10
21	0 53 43	2 26	5.7	1 56 30	1 37	12.4	2 28 4	0 23	15.7	9
22	0 56 9	2 25	6.0	1 58 7	1 36	12.6	2 28 27	0 21	15.8	8
23	0 58 34	2 24	6.3	1 59 43	1 33	12.8	2 28 48	0 18	15.8	7
24	1 0 58	2 23	6.5	2 1 16	1 31	12.9	2 29 6	0 15	15.9	6
25	1 3 21	2 21	6.8	2 2 47	1 29	13.1	2 29 21	0 12	15.9	5
26	1 5 42	2 21	7.1	2 4 16	1 27	13.3	2 29 33	0 10	15.9	4
27	1 8 3	2 19	7.3	2 5 43	1 24	13.4	2 29 43	0 7	16.0	3
28	1 10 22	2 18	7.6	2 7 7	1 22	13.6	2 29 50	0 4	16.0	2
29	1 12 40	2 16	7.8	2 8 29	1 20	13.8	2 29 54	0 1	16.0	1
30	1 14 56		8.0	2 9 49		13.9	2 29 55		16.0	0
	Sig. XI. *S.* / Sig. V. *N.*		— / —	Sig. X. *S.* / Sig. IV. *N.*		— / —	Sig. IX. *S.* / Sig. III. *N.*		— / —	

For any time before 1750, the Secular Variation changes the Sign.

TABLE XVII.

Reduction to the Ecliptic, and the Logarithm of the Cosine of the Heliocentric Latitude.

ARGUMENT VIII. *Longitude of Saturn — that of the Node.*

Degrees.	Reduct. to Eclipt. Sig. O. — Sig. VI. SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. I. — Sig. VII. SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. II. — Sig. VIII. SEC.	Log. cosin. Lat. helioc.	Degrees.
0	0.0	0.000000	84.9	9.999897	84.9	9.999690	30
1	3.4	0.000000	86.6	9.999891	83.2	9.999683	29
2	6.8	9.999999	88.1	9.999884	81.3	9.999678	28
3	10.2	9.999999	89.6	9.999877	79.3	9.999672	27
4	13.6	9.999998	90.9	9.999871	77.3	9.999666	26
5	17.0	9.999997	92.1	9.999864	75.1	9.999661	25
6	20.3	9.999996	93.2	9.999857	72.9	9.999655	24
7	23.7	9.999994	94.3	9.999851	70.5	9.999650	23
8	27.0	9.999992	95.2	9.999843	68.1	9.999645	22
9	30.3	9.999990	95.9	9.999836	65.6	9.999640	21
10	33.5	9.999988	96.6	9.999829	63.0	9.999635	20
11	36.7	9.999985	97.1	9.999822	60.4	9.999631	19
12	39.9	9.999982	97.5	9.999815	57.6	9.999626	18
13	43.0	9.999979	97.8	9.999808	54.8	9.999622	17
14	46.0	9.999976	98.0	9.999801	52.0	9.999618	16
15	49.0	9.999972	98.1	9.999794	49.0	9.999615	15
16	52.0	9.999969	98.0	9.999786	46.0	9.999611	14
17	54.8	9.999965	97.8	9.999779	43.0	9.999608	13
18	57.6	9.999961	97.5	9.999772	39.9	9.999605	12
19	60.4	9.999956	97.1	9.999765	36.7	9.999602	11
20	63.0	9.999952	96.6	9.999758	33.5	9.999599	10
21	65.6	9.999947	95.9	9.999751	30.3	9.999597	9
22	68.1	9.999942	95.2	9.999743	27.0	9.999595	8
23	70.5	9.999937	94.3	9.999737	23.7	9.999593	7
24	72.9	9.999932	93.2	9.999730	20.3	9.999591	6
25	75.1	9.999926	92.1	9.999723	17.0	9.999590	5
26	77.3	9.999921	90.9	9.999716	13.6	9.999589	4
27	79.3	9.999914	89.6	9.999710	10.2	9.999588	3
28	81.3	9.999909	88.1	9.999703	6.8	9.999587	2
29	83.2	9.999903	86.6	9.999697	3.4	9.999587	1
30	84.9	9.999897	84.9	9.999690	0.0	9.999587	0
	Sig. XI. + Sig. V. +		Sig. X. + Sig. IV. +		Sig. IX. + Sig. III. +		

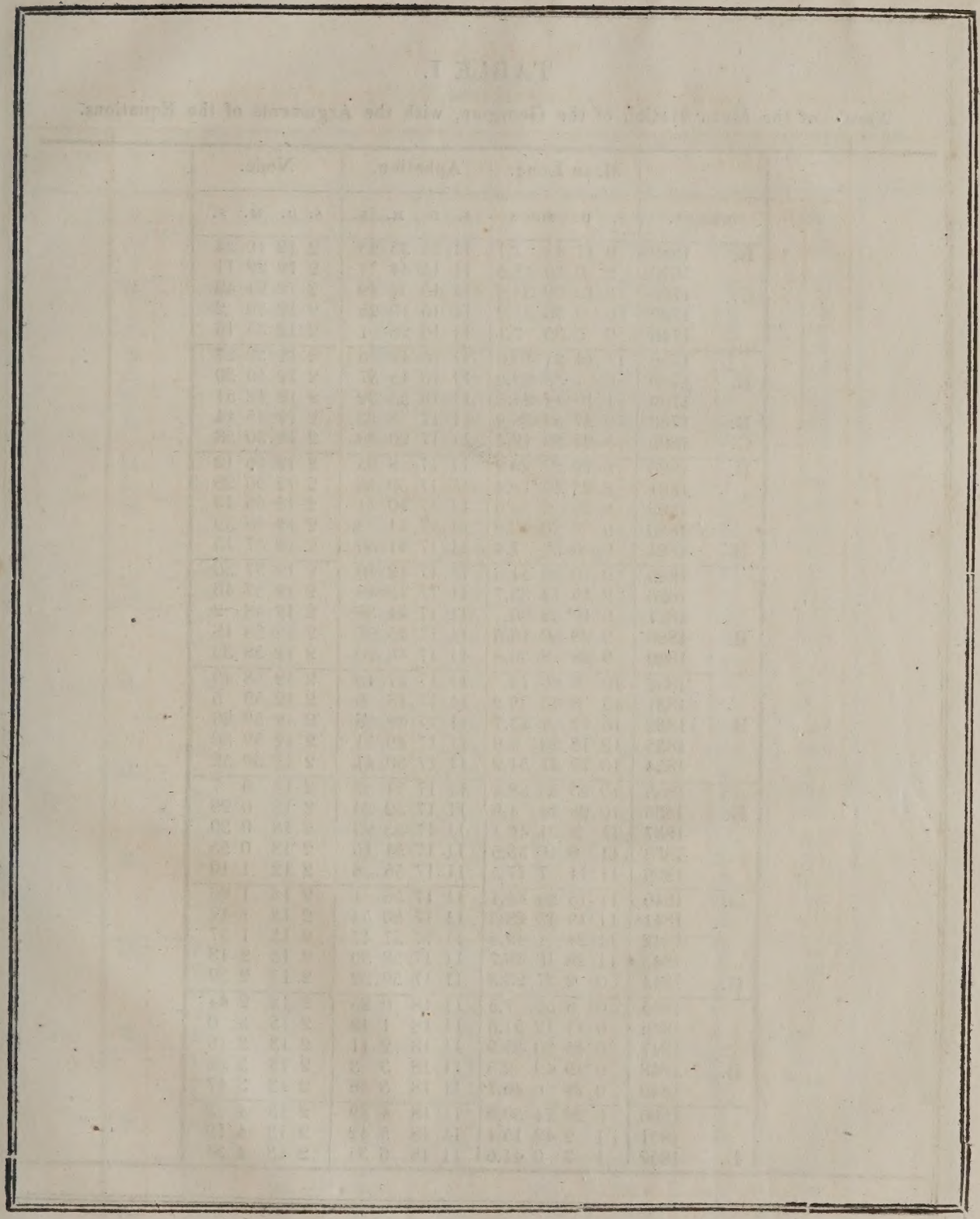

TABLE I.

Epochs of the Mean Motion of the Georgian, with the Arguments of the Equations.

		Mean Long.				Aphelion.				Node.			
YEARS.		S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.
B.	1680	0	17	41	1.7	11	15	35	13	2	12	19	34
	1690	2	0	39	48.5	11	15	44	1	2	12	22	11
C.	1700	3	13	38	35.2	11	15	52	49	2	12	24	48
	1720	6	9	36	51.2	11	16	10	25	2	12	30	2
	1740	9	5	35	7.1	11	16	28	1	2	12	35	16
	1756	11	14	21	43.8	11	16	42	6	2	12	39	27
B.	1760	0	1	33	23.0	11	16	45	37	2	12	40	30
	1769	1	10	14	24.5	11	16	53	32	2	12	42	51
B.	1780	2	27	31	38.9	11	17	3	13	2	12	45	44
C.	1800	5	23	29	12.4	11	17	20	49	2	12	50	58
B.	1820	8	19	27	28.2	11	17	38	25	2	12	56	12
	1821	8	23	45	12.4	11	17	39	18	2	12	56	28
	1822	8	28	2	56.6	11	17	40	11	2	12	56	43
	1823	9	2	20	40.8	11	17	41	3	2	12	56	59
B.	1824	9	6	39	7.4	11	17	41	56	2	12	57	15
	1825	9	10	56	51.6	11	17	42	49	2	12	57	30
	1826	9	15	14	35.7	11	17	43	42	2	12	57	46
	1827	9	19	32	20.	11	17	44	35	2	12	58	2
B.	1828	9	23	50	46.6	11	17	45	27	2	12	58	18
	1829	9	28	8	30.8	11	17	46	20	2	12	58	33
	1830	10	2	26	15.	11	17	47	13	2	12	58	49
	1831	10	6	43	59.2	11	17	48	6	2	12	59	5
B.	1832	10	11	2	25.7	11	17	48	58	2	12	59	20
	1833	10	15	20	9.9	11	17	49	51	2	12	59	36
	1834	10	19	37	54 2	11	17	50	44	2	12	59	52
	1835	10	23	55	38.4	11	17	51	37	2	13	0	7
B.	1836	10	28	14	4.9	11	17	52	30	2	13	0	23
	1837	11	2	31	49.1	11	17	53	23	2	13	0	39
	1838	11	6	49	33.5	11	17	54	15	2	13	0	55
	1839	11	11	7	17.5	11	17	55	8	2	13	1	10
B.	1840	11	15	25	44.1	11	17	56	1	2	13	1	26
	1841	11	19	43	28.3	11	17	56	54	2	13	1	42
	1842	11	24	1	12.5	11	17	57	47	2	13	1	57
	1843	11	28	18	56.7	11	17	58	39	2	13	2	13
B.	1844	0	2	37	23.3	11	17	59	32	2	13	2	39
	1845	0	6	55	7.5	11	18	0	25	2	13	2	44
	1846	0	11	12	51.6	11	18	1	18	2	13	3	0
	1847	0	15	30	35.9	11	18	2	11	2	13	3	16
B.	1848	0	19	49	2.5	11	18	3	3	2	13	3	32
	1849	0	24	6	46.7	11	18	3	56	2	13	3	47
	1850	0	28	24	30.9	11	18	4	49	2	13	4	3
	1851	1	2	42	15.1	11	18	5	42	2	13	4	19
B.	1852	1	7	0	41.6	11	18	6	35	2	13	4	34

TABLE I.

Epochs of the Mean Longitude of the Georgian, with the Arguments of the Equations.

YEARS.		ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.
B.	1680	215	126	9068	376	8907	057	969
	1690	435	228	0085	698	9084	781	574
C.	1700	656	329	1100	020	9260	505	179
B.	1720	096	532	3132	664	9612	953	389
B.	1740	537	735	5162	308	9963	401	597
B.	1756	890	898	6788	823	0245	560	567
B.	1760	978	938	7194	951	0315	849	809
	1769	177	029	8108	241	0473	501	353
B.	1780	419	141	9225	595	0666	297	019
C.	1800	860	344	1256	239	1018	745	229
B.	1820	301	547	3287	883	1370	193	439
	1821	323	557	3389	915	1388	265	499
	1822	345	567	3490	947	1405	338	560
	1823	367	577	3592	980	1423	410	620
B.	1824	389	588	3693	012	1440	483	681
	1825	412	598	3795	044	1458	555	741
	1826	433	608	3896	076	1475	627	802
	1827	455	618	3998	108	1493	700	862
B.	1828	477	628	4099	141	1511	772	923
	1829	499	638	4201	173	1528	845	983
	1830	521	648	4303	205	1546	917	044
	1831	544	659	4404	237	1563	989	104
B.	1832	566	669	4506	269	1581	062	165
	1833	588	679	4607	301	1599	134	225
	1834	610	689	4709	334	1616	207	286
	1835	632	699	4810	366	1634	279	346
B.	1836	654	709	4912	398	1651	351	407
	1837	676	719	5014	430	1669	424	467
	1838	698	730	5115	462	1686	496	528
	1839	720	740	5217	495	1704	569	589
B.	1840	742	750	5318	527	1722	641	649
	1841	764	760	5420	559	1740	713	709
	1842	786	770	5521	591	1757	786	770
	1843	808	780	5623	624	1775	858	830
B.	1844	830	791	5724	656	1792	930	891
	1845	852	801	5826	688	1810	003	951
	1846	874	811	5927	720	1827	075	012
	1847	896	821	6029	752	1845	148	072
B.	1848	918	831	6130	785	1863	220	133
	1849	940	841	6232	817	1880	293	193
	1850	962	851	6334	849	1898	365	254
	1851	985	862	6435	881	1915	437	314
B.	1852	007	872	6537	913	1933	510	375

TABLE II.

Mean Motion of the Georgian for *Julian Years*, with the Arguments of the Equations.

	YEARS.	Mot. Long. S.	D.	M.	S.	Mot. Aphelion. S.	D.	M.	S.	Mot. Node. S.	D.	M.	S.
	1	0	4	17	44.2	0	0	0	53	0	0	0	16
	2	0	8	35	28.4	0	0	1	46	0	0	0	31
	3	0	12	53	12.6	0	0	2	38	0	0	0	47
B.	4	0	17	11	39.2	0	0	3	31	0	0	1	3
	5	0	21	29	23.4	0	0	4	24	0	0	1	18
	6	0	25	47	7.5	0	0	5	17	0	0	1	34
	7	1	0	4	51.8	0	0	6	10	0	0	1	50
B.	8	1	4	23	18.4	0	0	7	2	0	0	2	6
	9	1	8	41	2.6	0	0	7	55	0	0	2	21
	10	1	12	58	46.8	0	0	8	48	0	0	2	37
	11	1	17	16	31.0	0	0	9	41	0	0	2	53
B.	12	1	21	34	57.5	0	0	10	34	0	0	3	8
	13	1	25	52	41.7	0	0	11	26	0	0	3	24
	14	2	0	10	26.0	0	0	12	19	0	0	3	40
	15	2	4	28	10.2	0	0	13	12	0	0	3	55
B.	16	2	8	46	36.7	0	0	14	5	0	0	4	11
	17	2	13	4	20.9	0	0	14	58	0	0	4	27
	18	2	17	22	5.1	0	0	15	50	0	0	4	43
	19	2	21	39	49.3	0	0	16	43	0	0	4	58
	20	2	25	58	15.9	0	0	17	36	0	0	5	14
Bissextiles.	40	5	21	56	31.8	0	0	35	12	0	0	10	28
	60	8	17	54	47.7	0	0	52	48	0	0	15	42
	80	11	13	53	3.6	0	1	10	24	0	0	20	56
	100	2	9	51	19.5	0	1	28	0	0	0	26	10
	200	4	19	42	39.1	0	2	56	6	0	0	52	20
	300	6	29	33	58.6	0	4	24	0	0	1	18	30
	400	9	9	25	18.1	0	5	52	0	0	1	44	40
	500	11	19	16	37.7	0	7	20	0	0	2	10	50
	600	1	29	7	57.2	0	8	48	0	0	2	37	0
	700	4	8	59	16.7	0	10	16	0	0	3	3	10
	800	6	18	50	36.3	0	11	44	0	0	3	29	20
	900	8	28	41	55.8	0	13	12	0	0	3	55	30
	1000	11	8	33	15.4	0	14	40	0	0	4	21	34

TABLE II.

Mean Motion of the Georgian for *Julian Years*, with the Arguments of the Equations.

	YEARS.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.
	1	022	010	0102	032	0018	072	060
	2	044	020	0203	064	0035	145	121
	3	066	030	0305	097	0053	217	181
B.	4	088	041	0406	129	0070	290	242
	5	110	051	0508	161	0088	362	302
	6	132	061	0609	193	0105	434	363
	7	154	071	0711	225	0123	507	423
B.	8	176	081	0812	258	0141	579	484
	9	198	091	0914	290	0158	652	544
	10	220	101	1016	322	0176	724	605
	11	243	112	1117	354	0193	796	665
B.	12	265	122	1219	386	0211	869	726
	13	287	132	1320	418	0229	941	786
	14	309	142	1422	451	0246	014	847
	15	331	152	1523	483	0264	086	907
B.	16	353	162	1625	515	0281	158	968
	17	375	172	1727	547	0299	231	028
	18	397	183	1828	579	0316	303	089
	19	419	193	1930	612	0334	376	150
	20	441	203	2031	644	0352	448	210
	40	882	406	4062	288	0703	896	420
	60	323	609	6094	931	1055	344	630
	80	764	812	8125	575	1406	792	840
	100	205	015	0156	219	1758	240	050
	200	410	030	0312	438	3516	480	100
	300	615	045	0468	657	5274	720	150
	400	820	060	0624	876	7032	960	200
	500	025	075	0780	095	8790	200	250
	600	230	090	0936	314	0548	440	300
	700	435	105	1092	533	2306	680	350
	800	640	120	1248	752	4064	920	400
	900	845	135	1404	971	5822	160	450
	1000	050	150	1560	190	7580	400	500

Bissextiles.

TABLE III.

Mean Motion of the Georgian for *Months*, with the Arguments of the Equations.

Months.	Mot. Long. D. M. S.	Aphel. SEC.	Node. SEC.	ARG. II.	ARG. III.	ARG. IV.	ARG. V.	ARG. VI.	ARG. VII.	ARG. VIII.
January	0 0 0.0	0	0	0	0	0	0	0	0	0
February	0 21 53.4	4	1	2	1	9	3	1	6	5
March	0 41 39.7	9	2	4	2	17	5	3	12	10
April	1 3 33.1	13	4	6	2	25	8	4	18	15
May	1 24 44.2	17	5	7	3	33	11	6	24	20
June	1 46 37.5	22	6	9	4	42	13	7	30	25
July	2 7 48.6	26	8	11	5	50	16	9	36	30
August	2 29 42.0	31	9	13	6	59	19	10	42	35
September	2 51 35.4	35	10	15	7	68	22	12	48	40
October	3 12 46.4	39	12	17	8	76	24	13	54	45
November	3 34 39.8	44	13	18	8	85	27	15	60	50
December	3 55 50.8	48	15	20	9	93	30	16	66	55

TABLE IV.

Mean Motion of the Georgian for *Hours* and *Minutes*.

Hours.	Mot. Lon. SEC.	Hours.	Mot. Lon. SEC.	Min.	Mot. Lon. SEC.	Min.	Mot. Lon. SEC.
1	1.8	13	22.9	1	0.0	27	0.8
2	3.5	14	24.7	2	0.0	30	0.9
3	5.3	15	26.5	3	0.1	33	1.0
4	7.1	16	28.2	4	0.1	36	1.1
5	8.8	17	30.0	5	0.1	39	1.1
6	10.6	18	31.8	6	0.2	42	1.2
7	12.4	19	33.5	9	0.3	45	1.3
8	14.1	20	35.3	12	0.4	48	1.4
9	15.9	21	37.1	15	0.4	51	1.5
10	17.7	22	38.8	18	0.5	54	1.6
11	19.4	23	40.6	21	0.6	57	1.7
12	21.2	24	42.4	24	0.7	60	1.8

TABLE V.

Mean Motion of the Georgian for *Days*, with the Arguments of the Equations.

Days of the Month.	Motion in Longitude.	Mot. Aphel.	Mot. Node.	Arg. II.	Arg. III.	Arg. IV.	Arg. V.	Arg. VI.	Arg. VII.	Arg. VIII.
	D. M. S.	SEC.	SEC.							
1	0 0 42.4	0	0	0	0	0	0	0	0	0
2	0 1 24.7	0	0	0	0	0	0	0	0	0
3	0 2 7.1	0	0	0	0	1	0	0	1	0
4	0 2 49.5	1	0	0	0	1	0	0	1	1
5	0 3 31.8	1	0	0	0	1	0	0	1	1
6	0 4 14.2	1	0	0	0	2	1	0	1	1
7	0 4 56.6	1	0	0	0	2	1	0	1	1
8	0 5 38.9	1	0	0	0	2	1	0	1	1
9	0 6 21.3	1	0	0	0	2	1	0	2	1
10	0 7 3.7	1	0	0	0	3	1	0	2	2
11	0 7 46.0	2	0	1	0	3	1	1	2	2
12	0 8 28.4	2	1	1	0	3	1	1	2	2
13	0 9 10.8	2	1	1	0	4	1	1	2	2
14	0 9 53.1	2	1	1	0	4	1	1	3	2
15	0 10 35.5	2	1	1	0	4	1	1	3	2
16	0 11 17.9	2	1	1	0	4	1	1	3	3
17	0 12 0.2	2	1	1	0	5	1	1	3	3
18	0 12 42.6	3	1	1	0	5	2	1	3	3
19	0 13 25.0	3	1	1	1	5	2	1	4	3
20	0 14 7.4	3	1	1	1	5	2	1	4	3
21	0 14 49.7	3	1	1	1	6	2	1	4	3
22	0 15 32.1	3	1	1	1	6	2	1	4	4
23	0 16 14.5	3	1	1	1	6	2	1	4	4
24	0 16 56.8	3	1	1	1	6	2	1	5	4
25	0 17 39.2	4	1	2	1	7	2	1	5	4
26	0 18 21.6	4	1	2	1	7	2	1	5	4
27	0 19 3.9	4	1	2	1	7	2	1	5	4
28	0 19 46.3	4	1	2	1	8	2	1	5	5
29	0 20 28.7	4	1	2	1	8	3	1	6	5
30	0 21 11.0	4	1	2	1	8	3	1	6	5
31	0 21 53.4	4	1	2	1	9	3	1	6	5

TABLE VI.

Equation of the Orbit of the Georgian for 1780, with the Secular Variation.

ARGUMENT I. *(Mean Long.—Aphelion), or mean Anomaly.*

Degrees.	Sig. O. Equation. −	Diff.	Sec. Var. +	Sig. I. Equation. −	Diff.	Sec. Var. +	Degrees.
	D. M. S.	M. S.	SEC.	D. M. S.	M. S.	SEC.	
0	0 0 0.0		0.00	2 32 42.3		4.97	30
1	0 5 17.5	5 17.5	0.17	2 37 22.3	4 40.0	5.13	29
2	0 10 34.9	5 17.4	0.34	2 41 59.8	4 37.5	5.28	28
3	0 15 52.3	5 17.4	0.51	2 46 34.8	4 35.0	5.43	27
4	0 21 9.4	5 17.1	0.68	2 51 7.2	4 32.4	5.58	26
5	0 26 26.1	5 16.7	0.85	2 55 36.9	4 29.7	5.73	25
6	0 31 42.4	5 16.3	1.02	3 0 3.9	4 27.0	5.88	24
7	0 36 58.2	5 15.8	1.19	3 4 27.9	4 24.0	6.02	23
8	0 42 13.5	5 15.3	1.36	3 8 49.0	4 21.1	6.17	22
9	0 47 28.1	5 14.6	1.53	3 13 7.1	4 18.1	6.31	21
10	0 52 42.0	5 13.9	1.70	3 17 22.2	4 15.1	6.46	20
11	0 57 55.0	5 13.0	1.87	3 21 34.0	4 11.8	6.60	19
12	1 3 7.2	5 12.2	2.04	3 25 42.7	4 8.7	6.74	18
13	1 8 18.4	5 11.2	2.21	3 29 48.1	4 5.4	6.87	17
14	1 13 28.6	5 10.2	2.38	3 33 50.1	4 2.0	7.01	16
15	1 18 37.6	5 9.0	2.55	3 37 48.8	3 58.7	7.15	15
16	1 23 45.4	5 7.8	2.71	3 41 43.8	3 55.0	7.28	14
17	1 28 51.9	5 6.5	2.88	3 45 35.3	3 51.5	7.41	13
18	1 33 57.0	5 5.1	3.05	3 49 23.2	3 47.9	7.54	12
19	1 39 0.7	5 3.7	3.21	3 53 7.3	3 44.1	7.67	11
20	1 44 2.8	5 2.1	3.38	3 56 47.6	3 40.3	7.79	10
21	1 49 3.3	5 0.5	3.54	4 0 24.1	3 36.5	7.92	9
22	1 54 2.1	4 58.8	3.70	4 3 56.7	3 32.6	8.04	8
23	1 58 59.2	4 57.1	3.86	4 7 25.3	3 28.6	8.16	7
24	2 3 54.4	4 55.2	4.02	4 10 49.9	3 24.6	8.28	6
25	2 8 47.7	4 53.3	4.18	4 14 10.3	3 20.4	8.40	5
26	2 13 38.9	4 51.2	4.34	4 17 26.6	3 16.3	8.51	4
27	2 18 28.0	4 49.1	4.50	4 20 38.6	3 12.0	8.63	3
28	2 23 15.0	4 47.0	4.66	4 23 46.3	3 7.7	8.74	2
29	2 27 59.8	4 44.8	4.81	4 26 49.6	3 3.3	8.84	1
30	2 32 42.3	4 42.5	4.97	4 29 48.5	2 58.9	8.95	0
	+ Sig. XI.		−	+ Sig. X.		−	

TABLE VI.

Equation of the Orbit of the Georgian for 1780, with the Secular Variation.

ARGUMENT I. *(Mean Long. — Aphelion), or mean Anomaly.*

Degrees.	Sig. II. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Sig. III. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Degrees.
0	4 29 48.5	2 54.4	8.95	5 20 30.6	0 16.3	10.95	30
1	4 32 42.9	2 49.9	9.06	5 20 46.9	0 11.0	10.97	29
2	4 35 32.8	2 45.2	9.16	5 20 57.9	0 4.8	10.99	28
3	4 38 18.0	2 40.6	9.26	5 21 2.7	0 1.0	11.00	27
4	4 40 58.6	2 35.9	9.35	5 21 1.7	0 6.7	11.02	26
5	4 43 34.5	2 31.1	9.45	5 20 55.0	0 12.7	11.02	25
6	4 46 5.6	2 26.2	9.54	5 20 42.3	0 18.7	11.03	24
7	4 48 31.8	2 21.3	9.63	5 20 23.6	0 24.5	11.03	23
8	4 50 53.1	2 16.4	9.72	5 19 59.1	0 30.5	11.03	22
9	4 53 9.5	2 11.5	9.81	5 19 28.6	0 36.3	11.02	21
10	4 55 21.0	2 6.4	9.89	5 18 52.3	0 42.3	11.01	20
11	4 57 27.4	2 1.2	9.97	5 18 10.0	0 48.5	11.00	19
12	4 59 28.6	1 56.2	10.04	5 17 21.5	0 54.3	10.98	18
13	5 1 24.8	1 50.8	10.11	5 16 27.2	1 0.2	10.96	17
14	5 3 15.6	1 45.8	10.19	5 15 27.0	1 6.2	10.94	16
15	5 5 1.4	1 40.4	10.26	5 14 20.8	1 12.2	10.91	15
16	5 6 41.8	1 35.1	10.33	5 13 8.6	1 18.2	10.88	14
17	5 8 16.9	1 29.8	10.39	5 11 50.4	1 24.0	10.85	13
18	5 9 46.7	1 24.2	10.45	5 10 26.4	1 30.0	10.81	12
19	5 11 10.9	1 18.9	10.51	5 8 56.4	1 36.0	10.77	11
20	5 12 29.8	1 13.4	10.56	5 7 20.4	1 41.8	10.73	10
21	5 13 43.2	1 7.8	10.61	5 5 38.6	1 47.8	10.68	9
22	5 14 51.0	1 2.3	10.66	5 3 50.8	1 53.6	10.63	8
23	5 15 53.3	0 56.6	10.71	5 1 57.2	1 59.5	10.57	7
24	5 16 49.9	0 51.1	10.76	4 59 57.7	2 5.3	10.51	6
25	5 17 41.0	0 45.3	10.80	4 57 52.4	2 11.2	10.45	5
26	5 18 26.3	0 39.7	10.83	4 55 41.2	2 16.9	10.38	4
27	5 19 6.0	0 34.0	10.87	4 53 24.3	2 22.7	10.31	3
28	5 19 40.0	0 28.2	10.90	4 51 1.6	2 28.5	10.24	2
29	5 20 8.2	0 22.4	10.93	4 48 33.1	2 34.1	10.16	1
30	5 20 30.6		10.95	4 45 59.0		10.08	0
	+ Sig. IX.		—	+ Sig. VIII.		—	

For any time before 1780, the Secular Variation changes the Sign.

TABLE VI.

Equation of the Orbit of the Georgian for 1780, with the Secular Variation.

ARGUMENT I. *(Mean Long. — Aphelion), or mean Anomaly.*

Degrees.	Sig. IV. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Sig. V. Equation. — D. M. S.	Diff. M. S.	Sec. Var. + SEC.	Degrees.
0	4 45 59.0	2 39.9	10.08	2 48 56.7	5 2.5	6.10	30
1	4 43 19.1	2 45.5	10.00	2 43 54.2	5 6.1	5.92	29
2	4 40 33.6	2 51.0	9.91	2 38 48.1	5 9.5	5.74	28
3	4 37 42.6	2 56.5	9.82	2 33 38.6	5 12.7	5.56	27
4	4 34 46.1	3 2.1	9.72	2 28 25.9	5 15.9	5.37	26
5	4 31 44.0	3 7.6	9.63	2 23 10.0	5 18.9	5.19	25
6	4 28 36.5	3 13.0	9.53	2 17 51.1	5 21.9	4.99	24
7	4 25 23.5	3 18.3	9.42	2 12 29.2	5 24.9	4.80	23
8	4 22 5.2	3 23.6	9.31	2 7 4.3	5 27.5	4.61	22
9	4 18 41.6	3 28.9	9.20	2 1 36.8	5 30.0	4.41	21
10	4 15 12.7	3 34.1	9.08	1 56 6.8	5 32.6	4.21	20
11	4 11 38.6	3 39.1	8.96	1 50 34.2	5 34.9	4.01	19
12	4 7 59.5	3 44.3	8.84	1 44 59.3	5 37.4	3.81	18
13	4 4 15.2	3 49.3	8.72	1 39 21.9	5 39.4	3.61	17
14	4 0 25.9	3 54.3	8.58	1 33 42.5	5 41.5	3.40	16
15	3 56 31.6	3 59.0	8.45	1 28 1.0	5 43.4	3.20	15
16	3 52 32.6	4 4.0	8.32	1 22 17.6	5 45.1	2.99	14
17	3 48 28.6	4 8.6	8.18	1 16 32.5	5 46.8	2.78	13
18	3 44 20.0	4 13.3	8.03	1 10 45.7	5 48.4	2.57	12
19	3 40 6.7	4 17.9	7.89	1 4 57.3	5 49.8	2.36	11
20	3 35 48.8	4 22.4	7.74	0 59 7.5	5 51.1	2.15	10
21	3 31 26.4	4 26.8	7.59	0 53 16.4	5 52.2	1.94	9
22	3 26 59.6	4 31.1	7.44	0 47 24.2	5 53.2	1.73	8
23	3 22 28.5	4 35.4	7.28	0 41 31.0	5 54.3	1.51	7
24	3 17 53.1	4 39.5	7.12	0 35 36.7	5 54.9	1.30	6
25	3 13 13.6	4 43.6	6.96	0 29 41.8	5 55.6	1.08	5
26	3 8 30.0	4 47.6	6.79	0 23 46.2	5 56.1	0.86	4
27	3 3 42.4	4 51.5	6.62	0 17 50.1	5 56.5	0.65	3
28	2 58 50.9	4 55.3	6.45	0 11 53.6	5 56.7	0.43	2
29	2 53 55.6	4 58.9	6.27	0 5 56.9	5 56.9	0.21	1
30	2 48 56.7		6.10	0 0 0.0		0.00	0
	+ Sig. VII.		—	+ Sig. VI.		—	

TABLE VII. Equation II.

Argument II.

Arg. II.	Equat. II. sec.	Arg. II.	Equat. II. sec.	Arg. II.	Equat. II. sec.
0	38.4	340	47.7	670	3.3
10	39.0	350	46.5	680	3.4
20	39.6	360	45.2	690	3.5
30	40.1	370	43.9	700	3.8
40	40.7	380	42.5	710	4.2
50	41.3	390	41.0	720	4.8
60	41.9	400	39.4	730	5.5
70	42.5	410	37.8	740	6.3
80	43.2	420	36.1	750	7.2
90	43.9	430	34.4	760	8.3
100	44.7	440	32.7	770	9.4
110	45.4	450	30.8	780	10.7
120	46.2	460	29.0	790	12.1
130	47.0	470	27.1	800	13.5
140	47.8	480	25.2	810	15.0
150	48.5	490	23.4	820	16.6
160	49.2	500	21.6	830	18.1
170	49.9	510	19.8	840	19.7
180	50.5	520	18.1	850	21.3
190	51.1	530	16.3	860	22.9
200	51.7	540	14.8	870	24.5
210	52.1	550	13.3	880	26.0
220	52 4	560	11.7	890	27.5
230	52.7	570	10.4	900	28.9
240	52.8	580	9.2	910	30.1
250	52.8	590	8.0	920	31.4
260	52.7	600	7.0	930	32.6
270	52.4	610	6.1	940	33.6
280	52.1	620	5.3	950	34.6
290	51.6	630	4.7	960	35.4
300	51.0	640	4.0	970	36.3
310	50.3	650	3.5	980	37.0
320	49.5	660	3.4	990	37.7
330	48.6	670	3.3	1000	38.4
340	47.7				

TABLE VIII. Equation III.

Argument III.

Arg. III.	Equat. III. sec.	Arg. III.	Equat. III. sec.	Arg. III.	Equat. III. sec.
0	50.0	340	12.7	670	88.7
10	47.3	350	14.3	680	90.0
20	44.5	360	16.0	690	91.1
30	41.7	370	17.8	700	92.0
40	39.0	380	19.8	710	92.8
50	36.4	390	21.8	720	93.4
60	33.7	400	24.0	730	93.8
70	31.2	410	26.3	740	94.1
80	28.7	420	28.7	750	94.2
90	26.3	430	31.2	760	94.1
100	24.0	440	33.7	770	93.8
110	21.7	450	36.4	780	93.4
120	19.8	460	39.0	790	92.8
130	17.8	470	41.7	800	92.0
140	16.0	480	44.5	810	91.1
150	14.3	490	47.3	820	90.0
160	12.7	500	50.0	830	88.7
170	11.3	510	52.7	840	87.3
180	10.0	520	55.5	850	85.7
190	8.9	530	58.3	860	84.0
200	8.0	540	61.0	870	82.2
210	7.2	550	63.6	880	80.2
220	6.6	560	66.3	890	78.2
230	6.2	570	68.8	900	76.0
240	5.9	580	71.3	910	73.7
250	5.8	590	73.7	920	71.3
260	5.9	600	76.0	930	68.8
270	6.2	610	78.2	940	66.3
280	6.6	620	80.2	950	63.6
290	7.2	630	82.2	960	61.0
300	8.0	640	84.0	970	58.3
310	8.9	650	85.7	980	55.5
320	10.0	660	87.3	990	52.7
330	11.3	670	88.7	1000	50.0
340	12.7				

TABLE IX. Equation IV.

Argument IV.

Arg. IV.	Arg. IV.	Equation IV. M. s.	Diff.	Arg. IV.	Arg. IV.	Equation IV. M. s.	Diff.
2500	2500	5 0.0	0.3	5900	9100	1 9.6	7.8
2600	2400	4 59.7	0.9	6000	9000	1 1.8	7.4
2700	2300	4 58.8	1.5	6100	8900	0 54.4	7.1
2800	2200	4 57.3	2.0	6200	8800	0 47.3	6.6
2900	2100	4 55.3	2.7	6300	8700	0 40.7	6.2
3000	2000	4 52.6	3.2	6400	8600	0 34.5	5.9
3100	1900	4 49.4	3.7	6500	8500	0 28.6	5.2
3200	1800	4 45.7	4.3	6600	8400	0 23.4	4.8
3300	1700	4 41.4	4.8	6700	8300	0 18.6	4.3
3400	1600	4 36.6	5.2	6800	8200	0 14.3	3.7
3500	1500	4 31.4	5.9	6900	8100	0 10.6	3.2
3600	1400	4 25.5	6.2	7000	8000	0 7.4	2.7
3700	1300	4 19.3	6.6	7100	7900	0 4.7	2.0
3800	1200	4 12.7	7.1	7200	7800	0 2.7	1.5
3900	1100	4 5.6	7.4	7300	7700	0 1.2	0.9
4000	1000	3 58.2	7.8	7400	7600	0 0.3	0.3
4100	900	3 50.4	8.1	7500	7500	0 0.0	
4200	800	3 42.3	8.4				
4300	700	3 33.9	8.7				
4400	600	3 25.2	8.8				
4500	500	3 16.4	9.1				
4600	400	3 7.3	9.2				
4700	300	2 58.1	9.3				
4800	200	2 48.8	9.4				
4900	100	2 39.4	9.4				
5000	0	2 30.0	9.4				
5100	9900	2 20.6	9.4				
5200	9800	2 11.2	9.3				
5300	9700	2 1.9	9.2				
5400	9600	1 52.7	9.1				
5500	9500	1 43.6	8.8				
5600	9400	1 34.8	8.7				
5700	9300	1 26.1	8.4				
5800	9200	1 17.7	8.1				
5900	9100	1 9.6					

TABLE X. Equation V.

Argument V.					
Arg. V.	Equat. V. sec.	Arg. V.	Equat. V. sec.	Arg. V.	Equat. V. sec.
0	3.5	340	8.1	670	3.4
10	3.6	350	8.1	680	3.2
20	3.8	360	8.0	690	2.9
30	4.0	370	8.0	700	2.5
40	4.2	380	7.9	710	2.4
50	4.4	390	7.9	720	2.4
60	4.5	400	7.8	730	2.3
70	4.7	410	7.7	740	2.3
80	4.9	420	7.7	750	2.3
90	5.1	430	7.5	760	2.3
100	5.3	440	7.4	770	2.2
110	5.5	450	7.3	780	2.1
120	5.7	460	7.2	790	2.1
130	5.9	470	7.0	800	2.0
140	6.1	480	6.9	810	2.0
150	6.3	490	6.7	820	2.0
160	6.4	500	6.5	830	1.9
170	6.6	510	6.4	840	1.9
180	6.8	520	6.2	850	1.9
190	7.1	530	6.0	860	2.0
200	7.5	540	5.8	870	2.1
210	7.6	550	5.6	880	2.1
220	7.6	560	5.5	890	2.1
230	7.6	570	5.3	900	2.2
240	7.7	580	5.1	910	2.3
250	7.7	590	4.9	920	2.4
260	7.7	600	4.7	930	2.5
270	7.8	610	4.5	940	2.6
280	7.9	620	4.3	950	2.7
290	7.9	630	4.1	960	2.8
300	8.0	640	3.9	970	3.0
310	8.0	650	3.7	980	3.1
320	8.1	660	3.6	990	3.2
330	8.1	670	3.4	1000	3.5
340	8.1				

TABLE XI. Equation VI.

Argument VI.

Arg. VI.	Equation VI. M. S.	Diff. SEC.	Arg. VI.	Equation VI. M. S.	Diff. SEC.	Arg. VI.	Equation VI. M. S.	Diff. SEC.
0	4 33.1	0.6	3400	1 4.1	6.7	6700	1 20.6	7.6
100	4 32.5	1.1	3500	0 57.4	6.4	6800	1 28.2	7.8
200	4 31.4	1.7	3600	0 51.0	6.1	6900	1 36.0	8.0
300	4 29.7	2.1	3700	0 44.9	5.7	7000	1 44.0	8.1
400	4 27.6	2.7	3800	0 39.2	5.2	7100	1 52.1	8.3
500	4 24.9	3.1	3900	0 34.0	4.9	7200	2 0.4	8.3
600	4 21.8	3.6	4000	0 29.1	4.4	7300	2 8.7	8.4
700	4 18.2	4.2	4100	0 24.7	4.0	7400	2 17.1	8.3
800	4 14.0	4.5	4200	0 20.7	3.5	7500	2 25.4	8.4
900	4 09.5	5.0	4300	0 17.2	3.0	7600	2 33.8	8.3
1000	4 04.5	5.4	4400	0 14.2	2.5	7700	2 42.1	8.2
1100	3 59.1	5.8	4500	0 11.7	2.0	7800	2 50.3	8.1
1200	3 53.3	6.1	4600	0 9.7	1.5	7900	2 58.4	7.9
1300	3 47.2	6.5	4700	0 8.2	1.0	8000	3 6.3	7.7
1400	3 40.7	6.8	4800	0 7.2	0.4	8100	3 14.0	7.6
1500	3 33.9	7.1	4900	0 6.8	0.1	8200	3 21.6	7.3
1600	3 26.8	7.4	5000	0 6.9	0.6	8300	3 28.9	7.0
1700	3 19.4	7.6	5100	0 7.5	1.1	8400	3 35.9	6.7
1800	3 11.8	7.8	5200	0 8.6	1.7	8500	3 42.6	6.4
1900	3 4.0	8.0	5300	0 10.3	2.1	8600	3 49.0	6.1
2000	2 56.0	8.1	5400	0 12.4	2.7	8700	3 55.1	5.7
2100	2 47.9	8.3	5500	0 15.1	3.1	8800	4 0.8	5.2
2200	2 39.6	8.3	5600	0 18.2	3.6	8900	4 6.0	4.9
2300	2 31.3	8.4	5700	0 21.8	4.2	9000	4 10.9	4.4
2400	2 22.9	8.3	5800	0 26.0	4.5	9100	4 15.3	4.0
2500	2 14.6	8.4	5900	0 30.5	5.0	9200	4 19.3	3.5
2600	2 6.2	8.3	6000	0 35.5	5.4	9300	4 22.8	3.0
2700	1 57.9	8.2	6100	0 40.9	5.8	9400	4 25.8	2.5
2800	1 49.7	8.1	6200	0 46.7	6.1	9500	4 28.3	2.0
2900	1 41.6	7.9	6300	0 52.8	6.5	9600	4 30.3	1.5
3000	1 33.7	7.7	6400	0 59.3	6.8	9700	4 31.8	1.0
3100	1 26.0	7.6	6500	1 6.1	7.1	9800	4 32.8	0.4
3200	1 18.4	7.3	6600	1 13.2	7.4	9900	4 33.2	0.1
3300	1 11.1	7.0	6700	1 20.6		10000	4 33.1	
3400	1 4.1							

TABLE XII. Equation VII.

Argument VII.

Arg. VII.	Equation VII. M. S.	Arg. VII.	Equation VII. M. S.	Arg. VII.	Equation VII. M. S.
0	1 0.0	340	1 44.5	670	0 13.8
10	1 3.3	350	1 42.7	680	0 12.4
20	1 6.6	360	1 40.7	690	0 11.1
30	1 9.7	370	1 38.5	700	0 10.0
40	1 12.9	380	1 36.1	710	0 9.0
50	1 16.1	390	1 33.6	720	0 8.3
60	1 19.2	400	1 31.0	730	0 7.9
70	1 22.3	410	1 28.3	740	0 7.6
80	1 25.1	420	1 25.5	750	0 7.5
90	1 27.9	430	1 22.5	760	0 7.6
100	1 30.6	440	1 19.4	770	0 7.9
110	1 33.2	450	1 16.3	780	0 8.5
120	1 35.7	460	1 13.2	790	0 9.2
130	1 38.1	470	1 9.9	800	0 10.2
140	1 40.2	480	1 6.6	810	0 11.3
150	1 42.3	490	1 3.3	820	0 12.6
160	1 44.1	500	1 0.0	830	0 14.2
170	1 45.8	510	0 56.7	840	0 15.9
180	1 47.4	520	0 53.4	850	0 17.7
190	1 48.7	530	0 50.1	860	0 19.7
200	1 49.8	540	0 46.8	870	0 21.9
210	1 50.8	550	0 43.7	880	0 24.3
220	1 51.5	560	0 40.6	890	0 26.8
230	1 52.1	570	0 37.5	900	0 29.4
240	1 52.4	580	0 34.5	910	0 32.1
250	1 52.5	590	0 31.7	920	0 34.9
260	1 52.4	600	0 29.0	930	0 37.7
270	1 52.1	610	0 26.4	940	0 40.8
280	1 51.7	620	0 23.9	950	0 43.9
290	1 51.0	630	0 21.5	960	0 47.0
300	1 50.0	640	0 19.4	970	0 50.3
310	1 48.9	650	0 17.3	980	0 53.4
320	1 47.6	660	0 15.5	990	0 56.7
330	1 46.2	670	0 13.8	1000	1 0.0
340	1 44.5				

TAB XIII. Equat. VIII

Argument VIII.

Arg. VIII.	Arg. VIII.	Equat. VIII. sec.	Arg. VIII.	Equat. VIII. sec.
250	250	1.3	590	7.0
260	240	1.3	600	7.2
270	230	1.3	610	7.4
280	220	1.4	620	7.5
290	210	1.4	630	7.7
300	200	1.5	640	7.8
310	190	1.6	650	8.0
320	180	1.7	660	8.1
330	170	1.8	670	8.2
340	160	1.9	680	8.3
350	150	2.0	690	8.4
360	140	2.2	700	8.5
370	130	2.3	710	8.6
380	120	2.5	720	8.6
390	110	2.6	730	8.7
400	100	2.8	740	8.7
410	90	3.0	750	8.7
420	80	3.2	760	8.7
430	70	3.4	770	8.7
440	60	3.6	780	8.6
450	50	3.9	790	8.6
460	40	4.1	800	8.5
470	30	4.3	810	8.4
480	20	4.5	820	8.3
490	10	4.8	830	8.2
500	0	5.0	840	8.1
510	990	5.2	850	8.0
520	980	5.5	860	7.8
530	970	5.7	870	7.7
540	960	5.9	880	7.5
550	950	6.1	890	7.4
560	940	6.4	900	7.2
570	930	6.6	910	7.0
580	920	6.8		
590	910	7.0		

TABLE XIV.

Radius Vector for 1780, with the Secular Variation.

ARGUMENT I. *The mean Anomaly.*

Degrees.	Sig. O. Rad. Vec.	Diff.	Secular Variation — 0.0	Sig. I. Rad. Vec.	Diff.	Secular Variation — 0.0	Sig. II. Rad. Vec.	Diff.	Secular Variation — 0.0	Degrees.
0	20.0722	1	005.2	19.9620	73	004.5	19.6547	130	002.9	30
1	20.0721	4	005.2	19.9547	75	004.5	19.6417	131	002.9	29
2	20.0717	6	005.2	19.9472	78	004.5	19.6286	133	002.8	28
3	20.0711	9	005.2	19.9394	80	004.4	19.6153	134	002.7	27
4	20.0702	11	005.2	19.9314	82	004.4	19.6019	135	002.6	26
5	20.0691	14	005.1	19.9232	84	004.3	19.5884	136	002.5	25
6	20.0677	16	005.1	19.9148	86	004.3	19.5748	138	002.5	24
7	20.0661	18	005.1	19.9062	89	004.3	19.5610	140	002.4	23
8	20.0643	21	005.1	19.8973	91	004.2	19.5470	140	002.4	22
9	20.0622	24	005.1	19.8882	92	004.2	19.5330	141	002.3	21
10	20.0598	26	005.0	19.8790	94	004.1	19.5189	142	002.2	20
11	20.0572	29	005.0	19.8696	97	004.1	19.5047	143	002.2	19
12	20.0543	31	005.0	19.8599	98	004.0	19.4904	145	002.1	18
13	20.0512	33	005.0	19.8501	101	004.0	19.4759	146	002.0	17
14	20.0479	36	005.0	19.8400	102	003.9	19.4613	146	001.9	16
15	20.0443	39	005.0	19.8298	105	003.8	19.4467	148	001.8	15
16	20.0404	40	005.0	19.8193	106	003.8	19.4319	148	001.7	14
17	20.0364	43	005.0	19.8087	108	003.8	19.4171	149	001.6	13
18	20.0321	45	005.0	19.7979	110	003.7	19.4022	150	001.5	12
19	20.0276	48	005.0	19.7869	112	003.6	19.3872	150	001.4	11
20	20.0228	50	004.9	19.7757	114	003.5	19.3722	152	001.3	10
21	20.0178	53	004.9	19.7643	115	003.5	19.3570	152	001.3	9
22	20.0125	56	004.8	19.7528	117	003.4	19.3418	152	001.2	8
23	20.0069	57	004.8	19.7411	118	003.3	19.3266	153	001.1	7
24	20.0012	59	004.8	19.7293	121	003.2	19.3113	154	001.0	6
25	19.9953	62	004.7	19.7172	122	003.1	19.2959	155	000.9	5
26	19.9891	64	004.7	19.7050	124	003.1	19.2804	154	000.9	4
27	19.9827	67	004.6	19.6926	125	003.1	19.2650	155	000.8	3
28	19.9760	69	004.6	19.6801	126	003.0	19.2495	155	000.7	2
29	19.9691	71	004.6	19.6675	128	003.0	19.2340	156	000.6	1
30	19.9620		004.5	19.6547		002.9	19.2184		000.5	0
	Sig. XI.		0.0 —	Sig. X.		0.0 —	Sig. IX.		0.0 —	

TABLE XIV.

Radius Vector for 1780, with the Secular Variation.

ARGUMENT I. *The mean Anomaly.*

Degrees.	Sig. III. Rad. Vec.	Diff.	Secular Variation — 0.0	Sig. IV. Rad. Vec.	Diff.	Secular Variation. + 0.0	Sig. V. Rad. Vec.	Diff.	Secular Variation. + 0.0	Degrees.
0	19.2184	156	000.5	18.7613	141	002.2	18.4122	83	004.3	30
1	19.2028	156	000.3	18.7472	140	002.2	18.4038	81	004.3	29
2	19.1872	156	000.1	18.7332	138	002.3	18.3957	78	004.4	28
3	19.1716	157	000.0	18.7194	137	002.4	18.3879	76	004.4	27
4	19.1559	156	000.0	18.7057	136	002.5	18.3803	74	004.4	26
5	19.1403	157	000.1	18.6921	134	002.6	18.3729	70	004.5	25
6	19.1246	156	000.2	18.6787	132	002.6	18.3659	68	004.5	24
7	19.1090	156	000.2	18.6655	131	002.7	18.3591	65	004.5	23
8	19.0934	157	000.3	18.6524	130	002.8	18.3526	63	004.6	22
9	19.0777	156	000.3	18.6394	128	002.9	18.3463	60	004.6	21
10	19.0621	155	000.4	18.6266	126	003.0	18.3403	57	004.7	20
11	19.0466	156	000.5	18.6140	124	003.0	18.3346	54	004.7	19
12	19.0310	155	000.6	18.6016	123	003.1	18.3292	52	004.7	18
13	19.0155	155	000.7	18.5893	121	003.2	18.3240	48	004.8	17
14	19.0000	154	000.8	18.5772	119	003.3	18.3192	45	004.8	16
15	18.9846	155	000.9	18.5653	117	003.4	18.3147	44	004.9	15
16	18.9691	153	000.9	18.5536	115	003.5	18.3103	40	004.9	14
17	18.9538	153	001.0	18.5421	113	003.5	18.3063	37	004.9	13
18	18.9385	152	001.1	18.5308	111	003.6	18.3026	34	004.9	12
19	18.9233	151	001.2	18.5197	109	003.6	18.2992	31	004.9	11
20	18.9082	152	001.3	18.5088	107	003.7	18.2961	29	005.0	10
21	18.8930	150	001.4	18.4981	104	003.7	18.2932	25	005.0	9
22	18.8780	149	001.5	18.4877	103	003.8	18.2907	23	005.0	8
23	18.8631	148	001.6	18.4774	100	003.9	18.2884	19	005.0	7
24	18.8483	148	001.7	18.4674	98	003.9	18.2865	16	005.0	6
25	18.8335	147	001.7	18.4576	96	004.0	18.2849	14	005.1	5
26	18.8188	146	001.8	18.4480	93	004.0	18.2835	11	005.1	4
27	18.8042	144	001.9	18.4387	91	004.1	18.2824	7	005.1	3
28	18.7898	143	002.0	18.4296	88	004.1	18.2817	4	005.1	2
29	18.7755	142	002.1	18.4208	86	004.2	18.2813	2	005.1	1
30	18.7613		002.2	18.4122		004.3	18.2811		005.1	0
	Sig. VIII.		0.0 +	Sig. VII.		0.0 +	Sig. VI.		0.0 +	

TABLE XV.

Equations of the Radius Vector.

ARGUMENT.

ARG.	Equation II.	Equation III.	Equation IV.	Equation VII.	ARG.
0	0.0070	0.0032	0.0000	0.0098	1000
10	0.0070	0.0032	0.0000	0.0098	990
20	0.0070	0.0032	0.0000	0.0098	980
30	0.0069	0.0032	0.0001	0.0097	970
40	0.0069	0.0032	0.0002	0.0096	960
50	0.0068	0.0032	0.0003	0.0096	950
60	0.0067	0.0031	0.0004	0.0095	940
70	0.0066	0.0031	0.0006	0.0093	930
80	0.0065	0.0031	0.0007	0.0092	920
90	0.0063	0.0030	0.0009	0.0090	910
100	0.0061	0.0030	0.0011	0.0089	900
110	0.0060	0.0029	0.0014	0.0087	890
120	0.0058	0.0029	0.0017	0.0085	880
130	0.0056	0.0028	0.0019	0.0082	870
140	0.0053	0.0027	0.0022	0.0080	860
150	0.0051	0.0027	0.0025	0.0078	850
160	0.0049	0.0026	0.0028	0.0075	840
170	0.0046	0.0025	0.0032	0.0073	830
180	0.0044	0.0024	0.0035	0.0070	820
190	0.0042	0.0023	0.0039	0.0067	810
200	0.0039	0.0022	0.0043	0.0064	800
210	0.0037	0.0021	0.0046	0.0061	790
220	0.0034	0.0020	0.0050	0.0058	780
230	0.0030	0.0019	0.0054	0.0054	770
240	0.0028	0.0018	0.0058	0.0052	760
250	0.0026	0.0016	0.0062	0.0049	750
260	0.0024	0.0014	0.0058	0.0046	740
270	0.0022	0.0013	0.0054	0.0043	730
280	0.0020	0.0012	0.0050	0.0040	720
290	0.0018	0.0011	0.0046	0.0037	710
300	0.0016	0.0010	0.0043	0.0034	700
310	0.0015	0.0009	0.0039	0.0031	690
320	0.0013	0.0008	0.0035	0.0028	680
330	0.0012	0.0007	0.0032	0.0025	670
340	0.0010	0.0006	0.0028	0.0023	660

ARG.	Equation II.	Equation III.	ARG.
350	0.0009	0.0005	650
360	0.0007	0.0005	640
370	0.0006	0.0004	630
380	0.0005	0.0003	620
390	0.0004	0.0003	610
400	0.0003	0.0002	600
410	0.0002	0.0002	590
420	0.0001	0.0001	580
430	0.0001	0.0001	570
440	0.0001	0.0001	560
450	0.0000	0.0000	550
460	0.0000	0.0000	540
470	0.0000	0.0000	530
480	0.0000	0.0000	520
490	0.0000	0.0000	510
500	0.0000	0.0000	500

ARG.	Equation IV.	Equation VII.	ARG.
350	0.0025	0.0020	650
360	0.0022	0.0018	640
370	0.0019	0.0016	630
380	0.0017	0.0013	620
390	0.0014	0.0011	610
400	0.0011	0.0009	600
410	0.0009	0.0008	590
420	0.0007	0.0006	580
430	0.0006	0.0005	570
440	0.0004	0.0004	560
450	0.0003	0.0003	550
460	0.0002	0.0002	540
470	0.0001	0.0001	530
480	0.0000	0.0000	520
490	0.0000	0.0000	510
500	0.0000	0.0000	500

For Equation IV, take away the last Cypher of the Argument.

TABLE XVI.

Heliocentric Latitude for 1780, with the Secular Variation.

ARGUMENT IX. *Longitude of the Georgian — that of the Node.*

Degrees.	Latitude. Sig. O. *N.* / Sig. VI. *S.* M. S.	Differ.	Sec. Var.	Latitude. Sig. I. *N.* / Sig. VII. *S.* M. S.	Differ.	Sec. Var.	Latitude. Sig. II. *N.* / Sig. VIII *S.* M. S.	Differ.	Sec. Var.	Degrees.
		SEC.	SEC.		SEC.	SEC.		SEC.	SEC.	
0	0 0.0	48.4	0.0	23 8.0	41.7	5.0	40 4.1	23.8	8.7	30
1	0 48.4	48.5	0.2	23 49.7	41.4	5.2	40 27.9	23.2	8.7	29
2	1 36.9	48.4	0.3	24 31.1	40.8	5.3	40 51.1	22.3	8.8	28
3	2 25.3	48.4	0.5	25 11.9	40.4	5.4	41 13.4	21.6	8.9	27
4	3 13.7	48.2	0.7	25 52.3	40.0	5.6	41 35.0	20.9	9.0	26
5	4 1.9	48.2	0.9	26 32.3	39.4	5.7	41 55.9	20.1	9.1	25
6	4 50.1	48.2	1.0	27 11.7	38.9	5.9	42 16.0	19.3	9.1	24
7	5 38.3	48.0	1.2	27 50.6	38.5	6.0	42 35.3	18.6	9.2	23
8	6 26.3	47.9	1.4	28 29.1	37.9	6.2	42 53.9	17.7	9.3	22
9	7 14.2	47.8	1.6	29 7.0	37.4	6.3	43 11.6	17.0	9.3	21
10	8 2.0	47.7	1.7	29 44.4	36.8	6.4	43 28.6	16.2	9.4	20
11	8 49.7	47.5	1.9	30 21.2	36.3	6.6	43 44.8	15.3	9.5	19
12	9 37.2	47.3	2.1	30 57.5	35.7	6.7	44 0.1	14.6	9.5	18
13	10 24.5	47.1	2.2	31 33.2	35.2	6.8	44 14.7	13.8	9.6	17
14	11 11.6	46.8	2.4	32 8.4	34.5	6.9	44 28.5	12.9	9.6	16
15	11 58.4	46.7	2.6	32 42.9	34.0	7.1	44 41.4	12.1	9.7	15
16	12 45.1	46.5	2.8	33 16.9	33.3	7.2	44 53.5	11.3	9.7	14
17	13 31.6	46.2	2.9	33 50.2	32.8	7.3	45 4.8	10.5	9.7	13
18	14 17.8	46.0	3.1	34 23.0	32.1	7.4	45 15.3	9.7	9.8	12
19	15 3.8	45.6	3.3	34 55.1	31.4	7.5	45 25.0	8.8	9.8	11
20	15 49.4	45.4	3.4	35 26.5	30.8	7.7	45 33.8	8.0	9.8	10
21	16 34.8	45.1	3.6	35 57.3	30.2	7.8	45 41.8	7.2	9.9	9
22	17 19.9	44.8	3.7	36 27.5	29.5	7.9	45 49.0	6.3	9.9	8
23	18 4.7	44.4	3.9	36 57.0	28.8	8.0	45 55.3	5.5	9.9	7
24	18 49.1	44.1	4.1	37 25.8	28.2	8.1	46 0.8	4.6	9.9	6
25	19 33.2	43.7	4.2	37 54.0	27.4	8.2	46 5.4	3.8	10.0	5
26	20 16.9	43.4	4.4	38 21.4	26.7	8.3	46 9.2	3.0	10.0	4
27	21 0.3	43.0	4.5	38 48.1	26.1	8.4	46 12.2	2.1	10.0	3
28	21 43.3	42.6	4.7	39 14.2	25.3	8.5	46 14.3	1.3	10.0	2
29	22 25.9	42.1	4.8	39 39.5	24.6	8.6	46 15.6	0.4	10.0	1
30	23 8.0		5.0	40 4.1		8.7	46 16.0		10.0	0
	Sig. XI. *S.* / Sig. V. *N*			Sig. X. *S.* / Sig. IV. *N.*			Sig. IX. *S.* / Sig. III. *N.*			

TABLE XVII.

Reduction to the Ecliptic, and the Logarithm of the Cosine of the Heliocentric Latitude.

ARGUMENT IX. *Longitude of the Georgian — that of the Node.*

Degrees.	Reduct. to Eclipt. Sig. O. — Sig. VI. SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. I. — Sig. VII. SEC.	Log. cosin. Lat. helioc.	Reduct. to Eclipt. Sig. II. — Sig. VIII. SEC.	Log. cosin. Lat. helioc.	Degrees.
0	0.0	10.000000	8.1	9.999990	8.1	9.999971	30
1	0.4	10.000000	8.3	9.999990	8.0	9.999970	29
2	0.7	10.000000	8.4	9.999989	7.8	9.999969	28
3	1.0	10.000000	8.6	9.999988	7.6	9.999969	27
4	1.3	10.000000	8.7	9.999988	7.4	9.999969	26
5	1.6	10.000000	8.8	9.999987	7.2	9.999968	25
6	1.9	9.999999	8.9	9.999986	6.9	9.999967	24
7	2.3	9.999999	9.0	9.999985	6.7	9.999967	23
8	2.6	9.999999	9.1	9.999984	6.5	9.999966	22
9	2.9	9.999999	9.2	9.999984	6.3	9.999966	21
10	3.2	9.999999	9.2	9.999983	6.0	9.999965	20
11	3.5	9.999999	9.3	9.999982	5.8	9.999965	19
12	3.8	9.999998	9.3	9.999982	5.5	9.999965	18
13	4.1	9.999998	9.3	9.999981	5.2	9.999964	17
14	4.4	9.999998	9.3	9.999980	4.9	9.999964	16
15	4.7	9.999998	9.3	9.999980	4.7	9.999963	15
16	4.9	9.999997	9.3	9.999979	4.4	9.999963	14
17	5.2	9.999997	9.3	9.999978	4.1	9.999963	13
18	5.5	9.999997	9.3	9.999978	3.8	9.999962	12
19	5.8	9.999996	9.3	9.999977	3.5	9.999962	11
20	6.0	9.999996	9.2	9.999976	3.2	9.999962	10
21	6.3	9.999995	9.2	9.999976	2.9	9.999962	9
22	6.5	9.999995	9.1	9.999975	2.6	9.999961	8
23	6.7	9.999994	9.0	9.999974	2.3	9.999961	7
24	6.9	9.999993	8.9	9.999974	1.9	9.999961	6
25	7.2	9.999993	8.8	9.999973	1.6	9.999961	5
26	7.4	9.999992	8.7	9.999972	1.3	9.999961	4
27	7.6	9.999992	8.6	9.999972	1.0	9.999961	3
28	7.8	9.999991	8.4	9.999972	0.7	9.999961	2
29	8.0	9.999991	8.3	9.999971	0.4	9.999961	1
30	8.1	9.999990	8.1	9.999971	0.0	9.999961	0
	Sig. XI. + Sig. V. +		Sig. X. + Sig. IV. +		Sig. IX. + Sig. III. +		

END OF THE TABLES OF THE SUN, MOON, AND PLANETS.

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7. PORTABLE MATHEMATICAL TABLES, containing Logarithms of Numbers, Proportional parts; artificial sines and tangents; natural sines and tangents, to every degree and minute of the quadrant; and a table of square and cube roots of all numbers from 1 to 180, by *Thomas Whiting*.—Price 4*s*. in boards. The Author begs leave to remark, that these tables are so correct, that he does not know of one material error in them.

Mr. Whiting has a few copies of Mr. John Davison's Algebra. Price 6*s*. neatly half-bound in calf.

Plate 1.

Uranus or Herschal ♅
Part of a comets orbit
Saturn ♄
SOLAR SYSTEM
Jupiter ♃
Pallas ⚴
Ceres ⚳
Juno ⚵
Vesta ⚶
Mars ♂
Earth ♁
Venus ♀
Mercury ☿
S.

Plate 2.

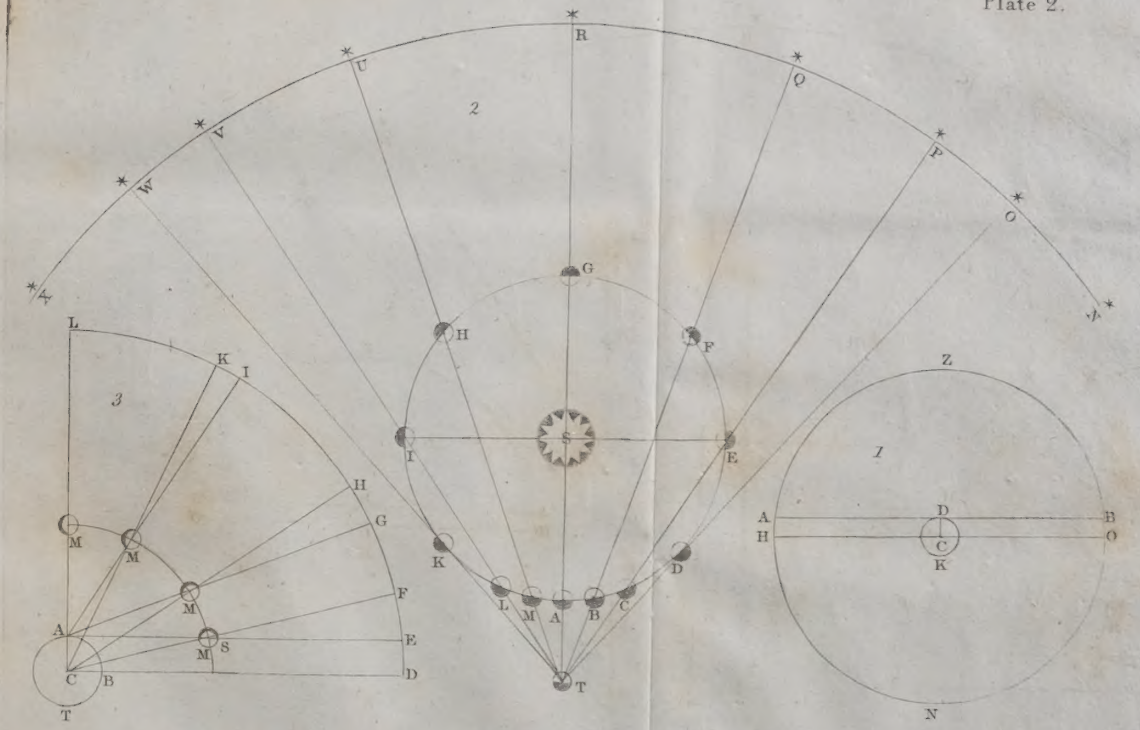
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Plate 5.

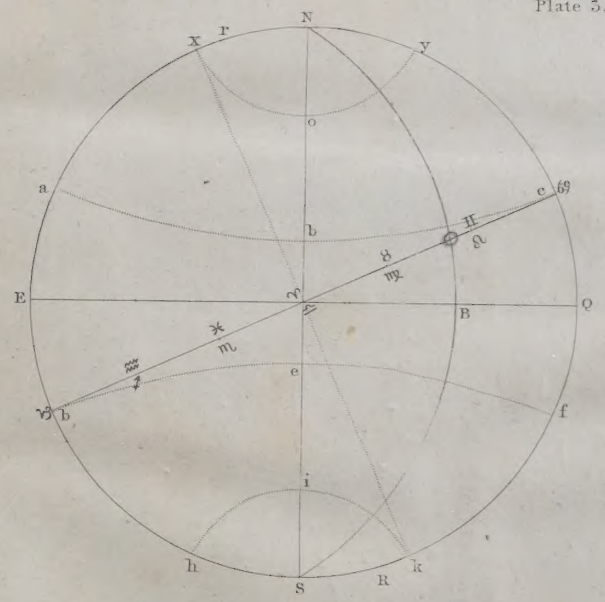

Plate 4.

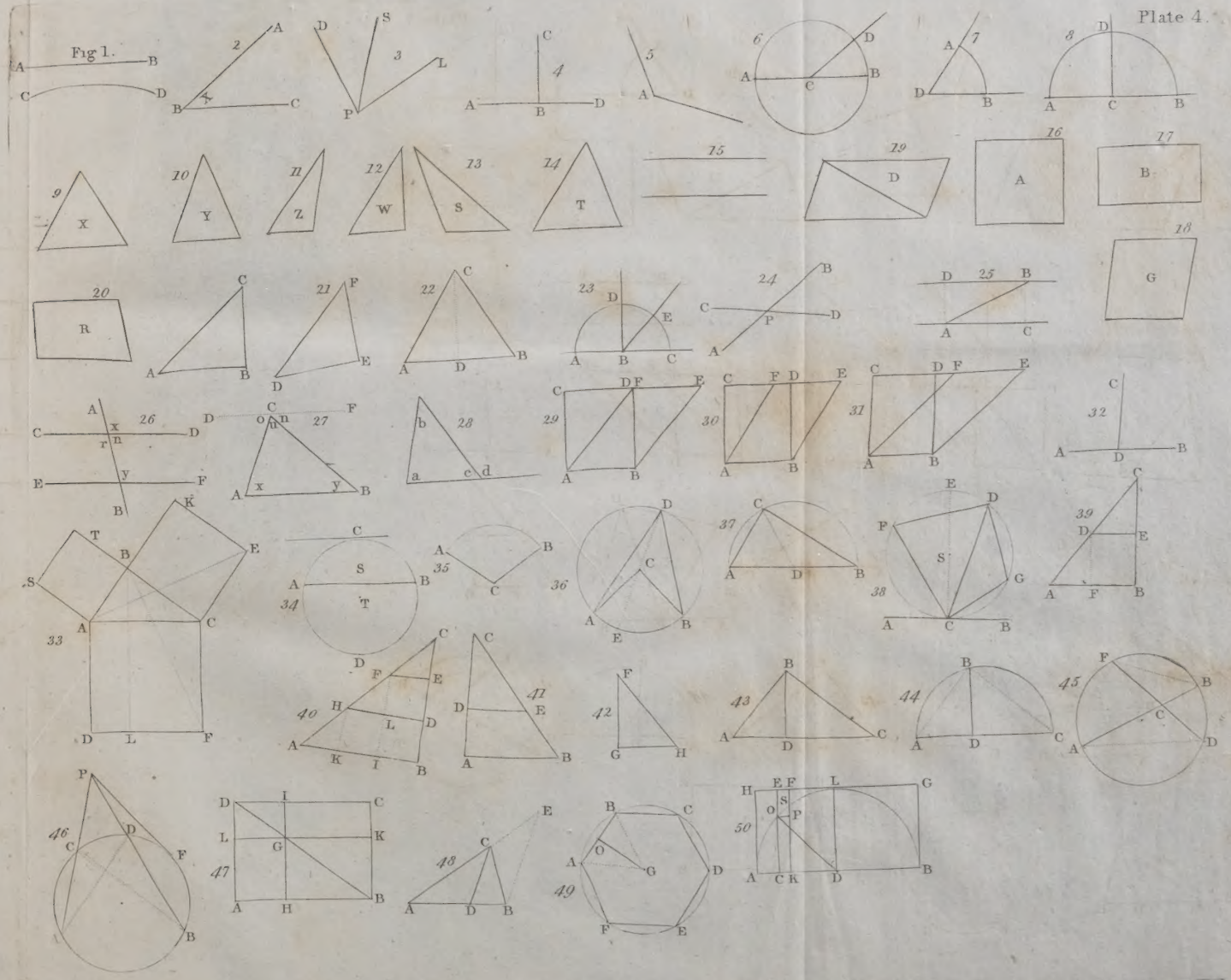
Fig 1.
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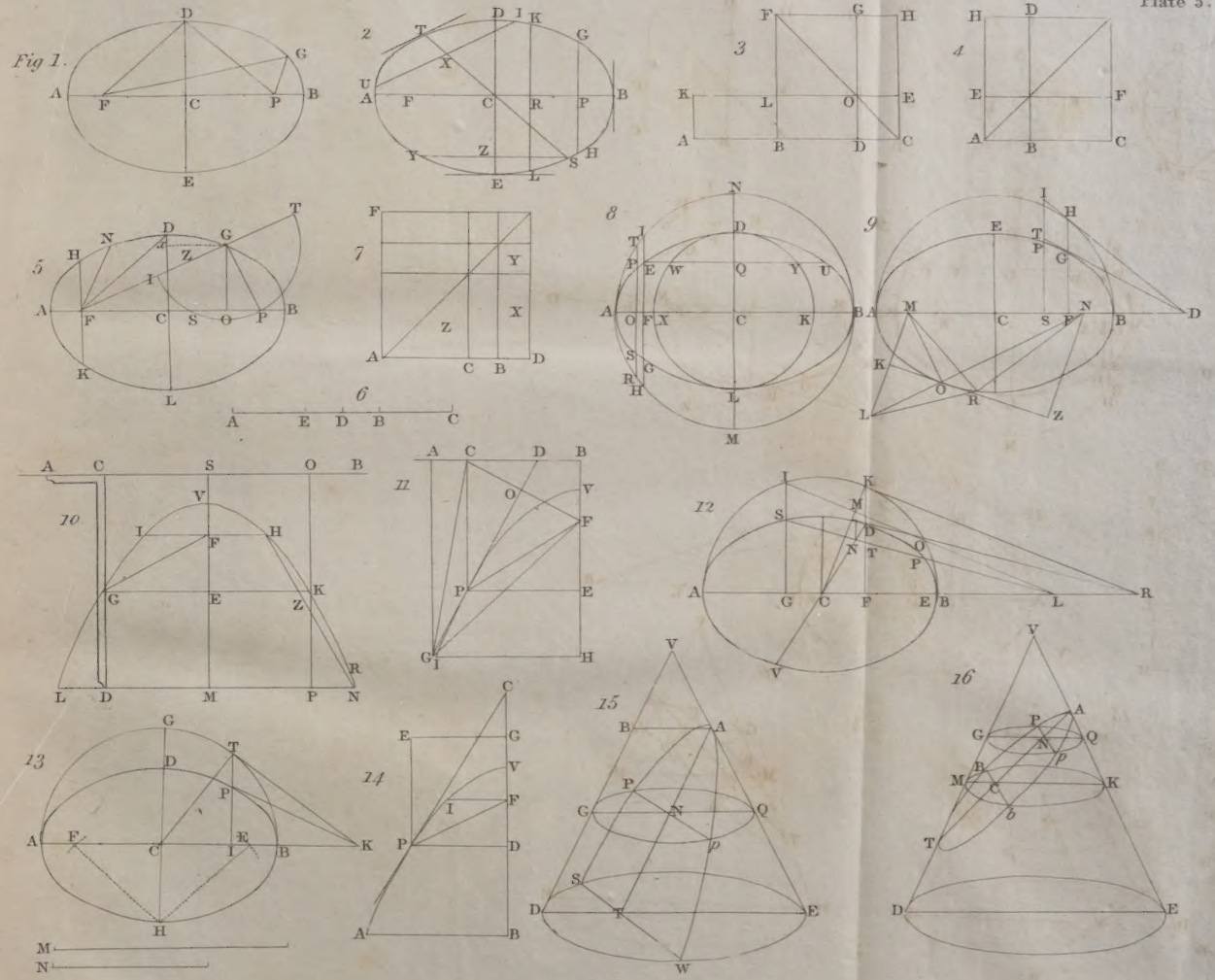
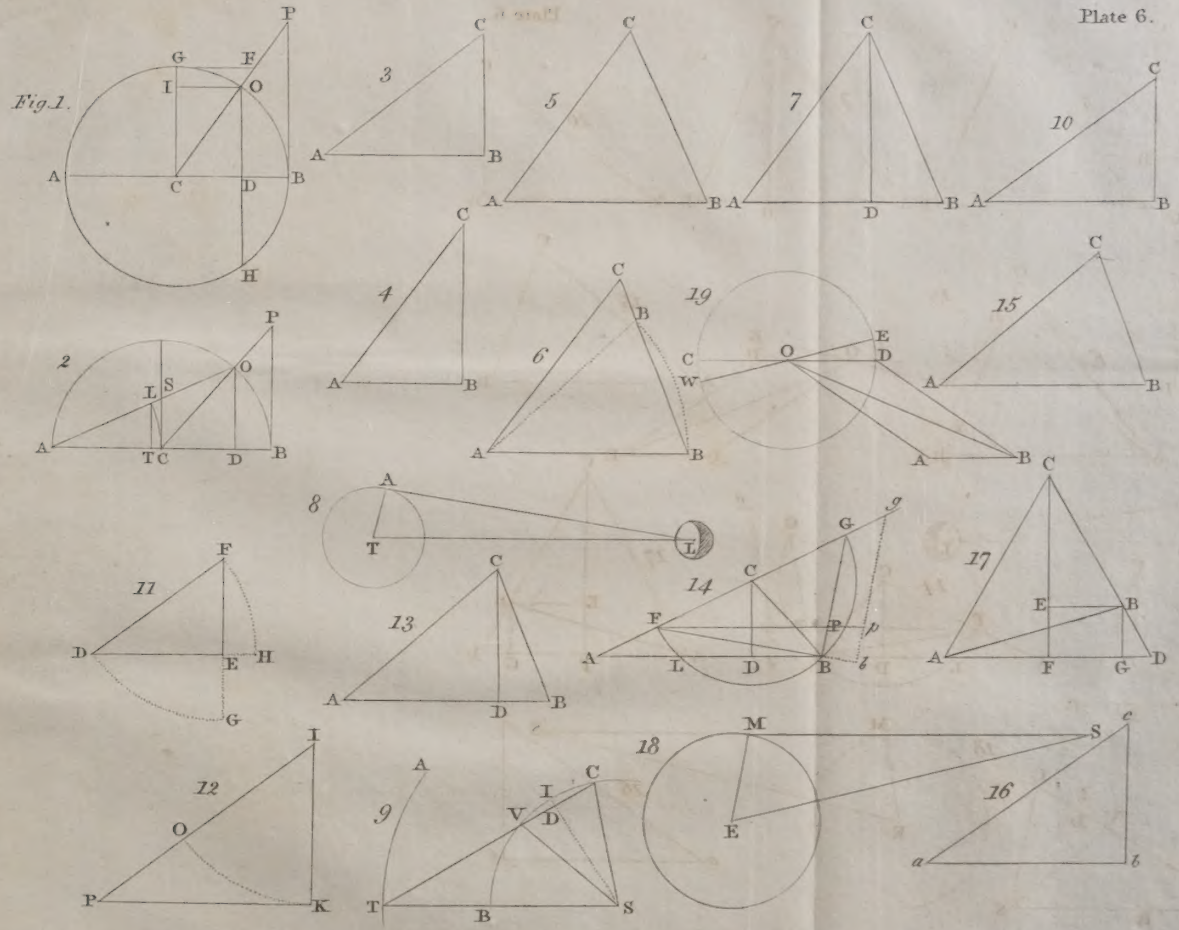
Plate 5.

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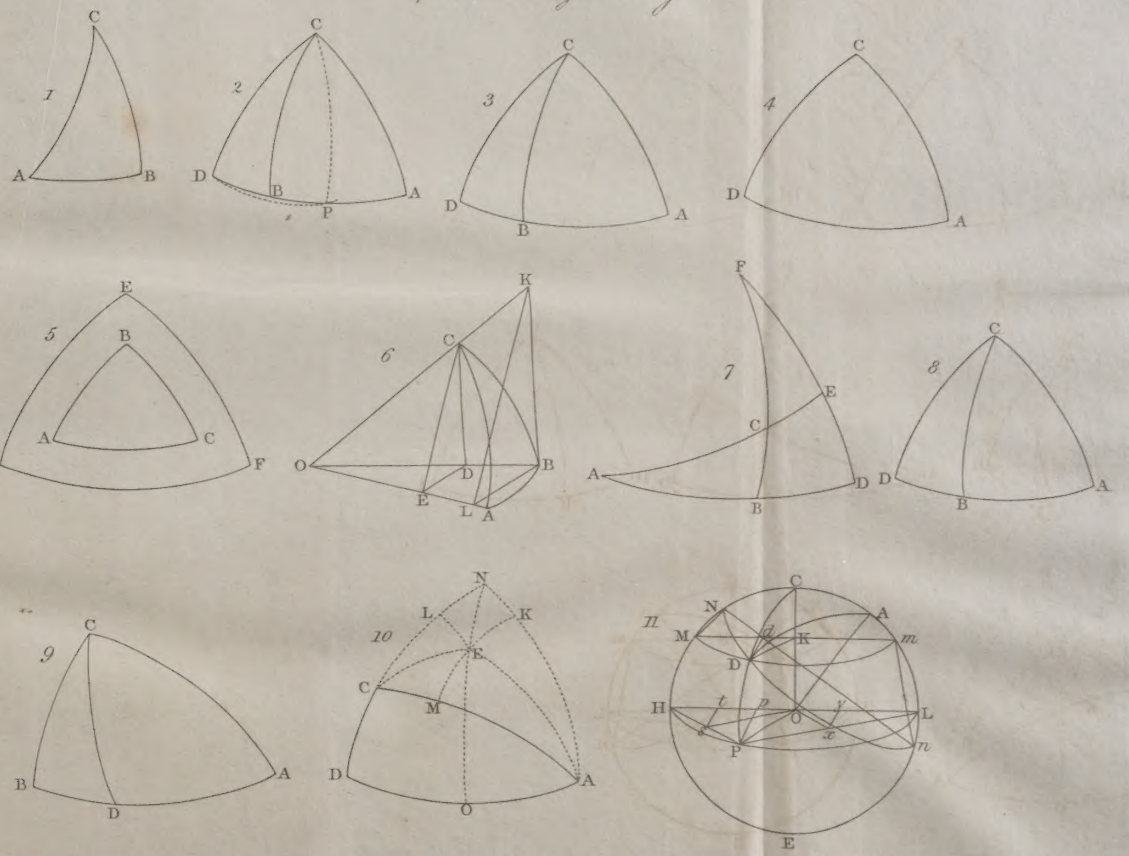
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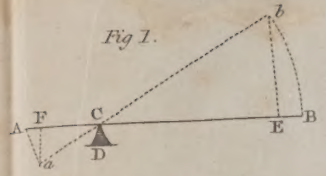
Plate 8.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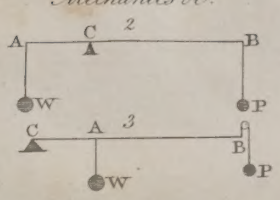
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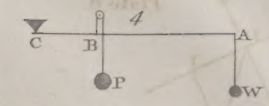
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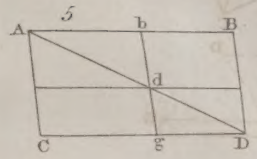
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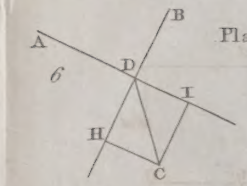
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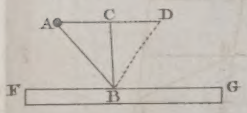
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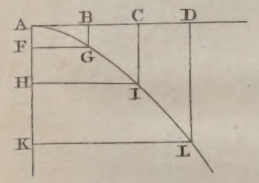
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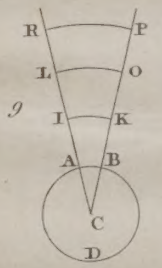
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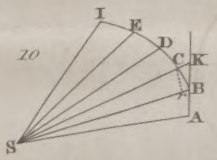
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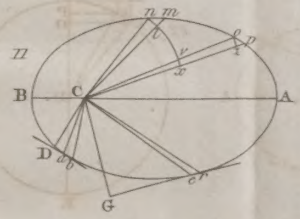
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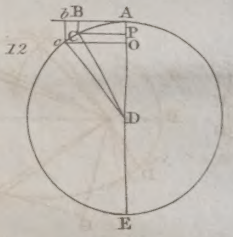
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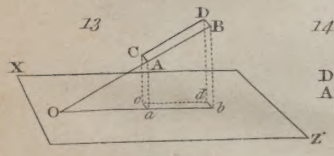
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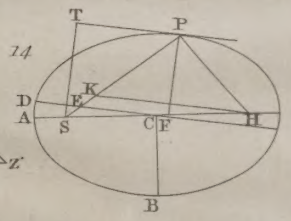
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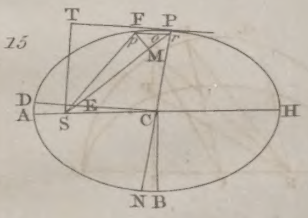
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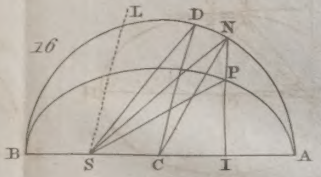
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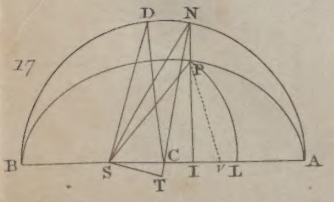
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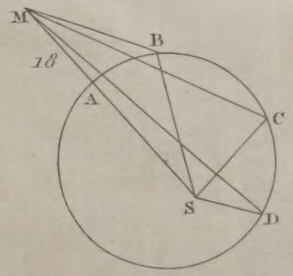
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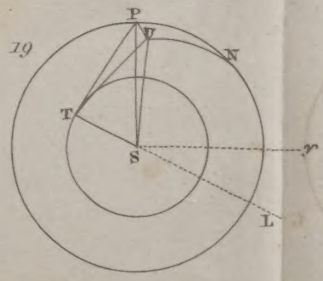
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